Engineering Mechanics

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Module 10

Kinematics

Lecture - 23

Kinematics of a particle

Today, I am going to discuss about the kinematics of a particle.

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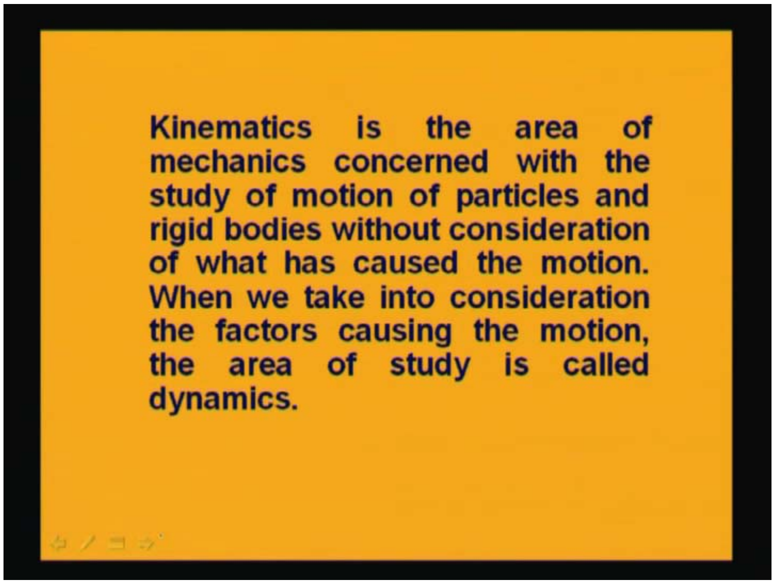

What is kinematics?

Kinematics is the area of mechanics concerned with the study of motion of particles and rigid bodies without consideration of what has caused the motion. So, when we take into consideration the factors causing the motion, the area of study is called dynamics.

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You just see this slider crank mechanism. Here, when I am moving the crank then this slider is moving. If I know what is the speed of the slider I need to have, then for that how much rpm crank should rotate? If this type of question is posed to us then we must know kinematics. See here, if I am moving the crank the slider is moving. If I am rotating it by 3 rpm then, what is the slider speed? Or conversely, If I am moving the slider at certain speed what will be the crank speed? Such types of questions are answered by kinematics. In kinematics, we are not concerned with the forces; we are concerned with the displacement, velocity and acceleration - all of you are familiar with these terms.

Now, quite often, we only deal with the kinematics. For example, if I want to design this mechanism, let us say this is just a toy mechanism, then probably we need not carry out extensive force analysis; nevertheless, we have to do kinematic analysis in order to find out the relation between the motion of various things. Similarly, suppose you are going from Pan Bazaar to Paltan Bazaar and you are going in a cycle rickshaw, you are concerned at what speed the rickshaw moves or if you are going by train from Delhi to Bombay, you must know the speed of the train. These types of questions you take interest. Generally, when we design certain machines then only we pay attention to these forces. Therefore, an engineer is usually concerned with the forces whereas kinematics has got much wider application. Another point is that when we take into account the forces then we must know the motion also. Therefore, dynamics encompasses kinematics also; dynamics encompasses kinematics because Newton's law says force is equal to mass into acceleration. If you want to calculate the force, you must know the acceleration also. Therefore, the study of kinematics is very important.

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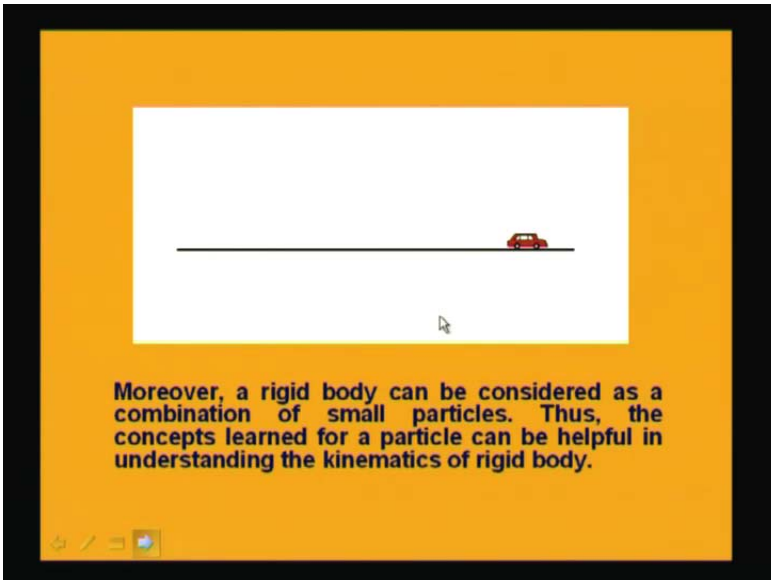

Why are we talking about kinematics of a particle? In kinematics of a particle, what is a particle? Particle is the concept which is having a mass but not having any size. A rigid body can be considered as a combination of small particles. Therefore, the concepts learned for a particle can be helpful in understanding the kinematics of a rigid body also. Moreover, sometimes, what happens is that bodies can be treated as particles. For example, I play this animation for you. Let me just play this car animation.

See, this is another thing I observed. One minute. You know you cannot continue kinematics. In the area one slide, you missed that particle I said. Know that you have to take the velocity and acceleration. There was one slide in which the definition of a particle was given but that slide is missed. But anyway, I have spoken about it. But one slide we missed. I said that you know that in one, kinematics is the area of this one. After that second point was there. That slide he missed. When this type of thing happens, I become nervous and this is the thing . This is why, I am not able to do. See, he has missed one and since, here I was not able to move to this one. See this one, now I am doing. See, it is very difficult. Now I am able to do. I am continuing from here.

Let me play the animation here. This is the animation and this is a car. Why this pen, sometimes it comes sometimes it does not come. **(Conversation in Hindi)**.

Now, I am playing the animation. See, this car is moving on a long highway. Therefore, compared to the distance it travels, the size of the car can be treated as a small particle. Therefore, we can apply kinematics of particles to this problem.

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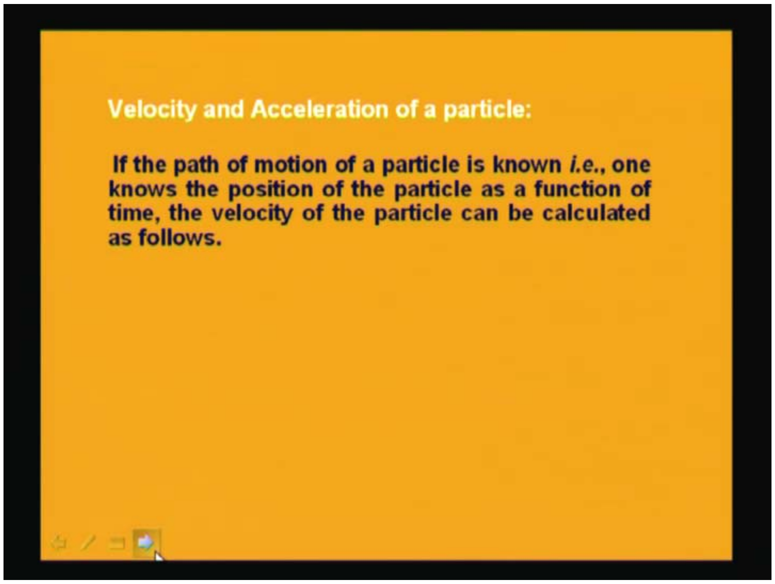

Next, I will discuss about how we can find out velocity and acceleration of a particle. If path of motion of a particle is known and one knows the position of the particle as a function of time, then the velocity of the particle can be easily calculated.

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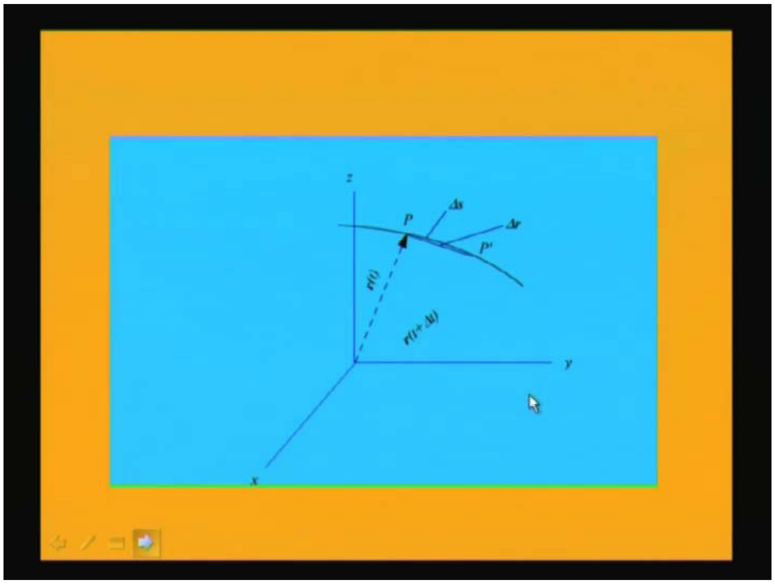

How do we calculate this? This was done during the Newton's time. Let us say that we have got a article P. This particle goes into P dash in time delta t. I will play this animation. **(Conversation in Hindi**) This, in delta t time, it has gone to P dash. It has gone very fast, animation should have been somewhat slow. Here, this distance P P dash, this line will be basically delta r. It is a vector and it is the difference of two vectors. One vector is OP dash. This is O; this point is origin O and this is P dash and this is OP. The difference between the two vectors r t plus delta t minus r t will be P P dash. P P dash is a vector. It is denoted by delta r. I will always be denoting the vectors by boldfaced letters. In your notebook, probably, you can put an arrow on top of the alphabet to indicate that it is a vector.

Now, arc P P dash is the total distance travelled by the particle and that is denoted by delta s.

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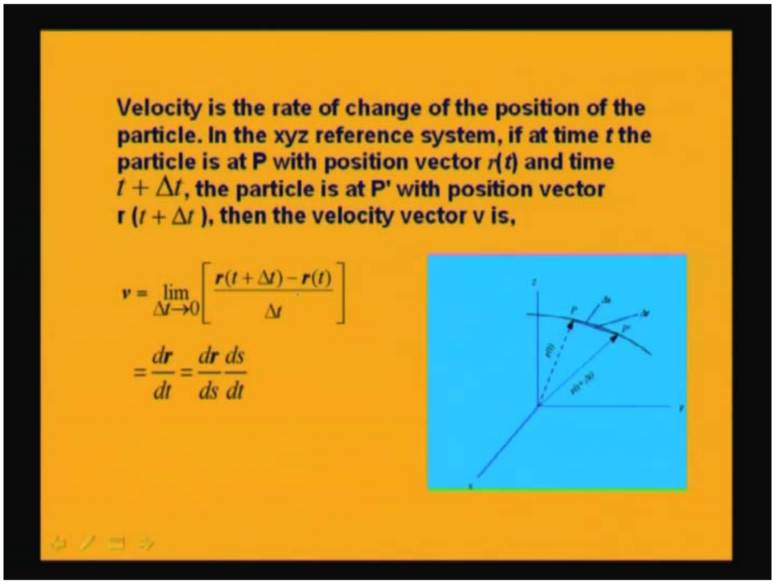

One has to understand the difference between a vector and this distance delta s. Delta s is not a vector whereas delta r is a vector. So, velocity is the rate of change of the position of the particle. Calculus was invented in order to define velocity properly by Newton. In the xyz reference system, if at time t the particle is at P with position vector r(t) and at time t plus delta t, the particle is at P dash with position vector r(t plus delta t) then the velocity vector V will be defined as V limit delta t tends to 0, that means the time interval is very small, r (t plus delta t) minus r(t) divided by delta t. Now r (t plus delta t) minus r(t) is nothing but the vector P P dash delta r. Therefore, this is a vector. If I divide it by a scalar quantity delta t entire thing is a vector. Therefore, the velocity is a vector only.

In the language of calculus, this is called dr by dt, where dr does not mean the product of d and r. It simply indicates a differential operator. So, you cannot cancel d and d here and say that it is equal to r by t. Just like sin theta does not mean sin into theta, it is basically symbol for the operator. Similarly, here your d is basically one operator. This can also be written as dr by ds into ds by dt, where I have multiplied by a small change in the arc length divided by the same quantity. This is basically a type of …… of derivatives.

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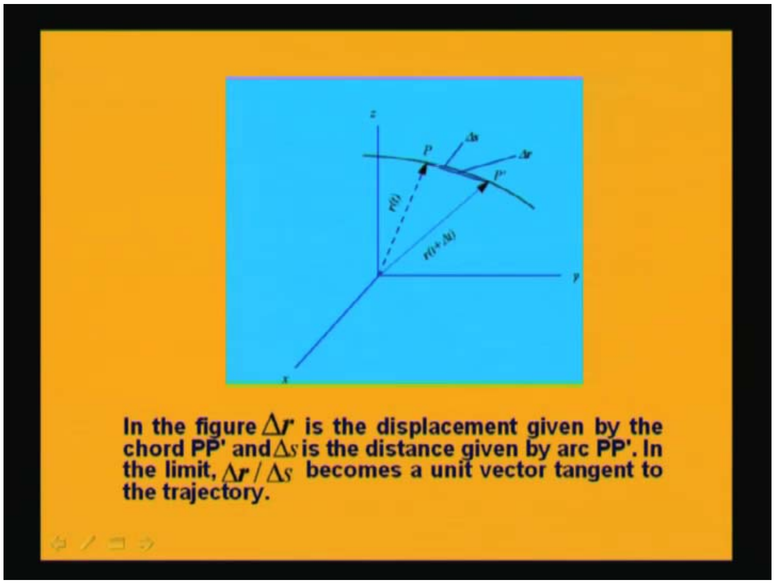

Next in this figure, delta r is the displacement given by the chord P P dash and delta s is the distance given by arc P P dash. In the limit, delta r divided by delta s becomes a unit vector tangent to the trajectory because in the unit, if you know that time is very small and distance between P and P dash is very small, in that case this delta length chord will be equal to arc. Therefore, the magnitude of delta r is same as delta s. Nevertheless, delta r is a vector and its direction is tangent to the direction at the point P. If you find out the tangent at P then this delta r is in the direction of the tangent. That is how we define the tangent also.

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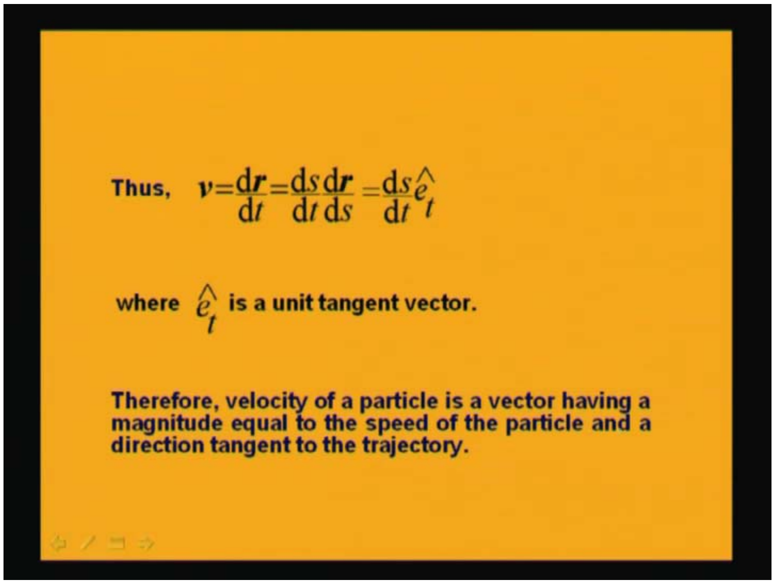

Therefore, we can write velocity equal to dr by dt. Same thing can be written as ds by dt into dr by ds. ds by dt can be called as speed. It is a scalar quantity; rate of change of distance with respect to time multiplied by unit tangent vector. Here cap e_t is a unit tangent vector. Therefore, velocity of a particle is a vector having a magnitude equal to the speed of a particle and direction tangent to the trajectory. That means, path of the motion of the particle.

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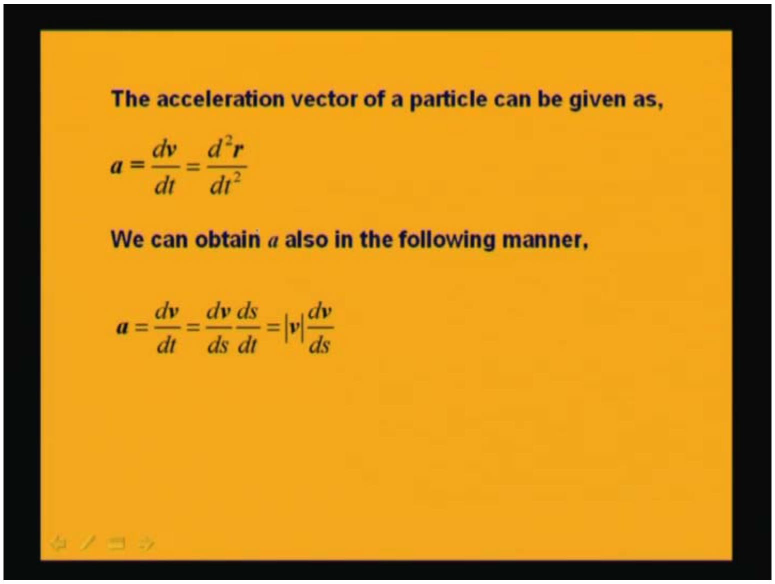

Let me discuss the acceleration. The acceleration vector of a particle is basically the rate of change of the velocity vector. a is equal to dv by dt which can also be written as d square r by dt square where d square is not indicating any square; but rather, it is indicating the double derivative. These symbols have been used - one must be very clear about that. d square r by dt square means the double derivative of position vector with respect to time. We can obtain a also in the following manner. We can write a is equal to dv by dt. Again, dv by ds into ds by dt; ds by dt is nothing but the magnitude of the velocity which is speed. So, this is the magnitude of the velocity into dv by ds.

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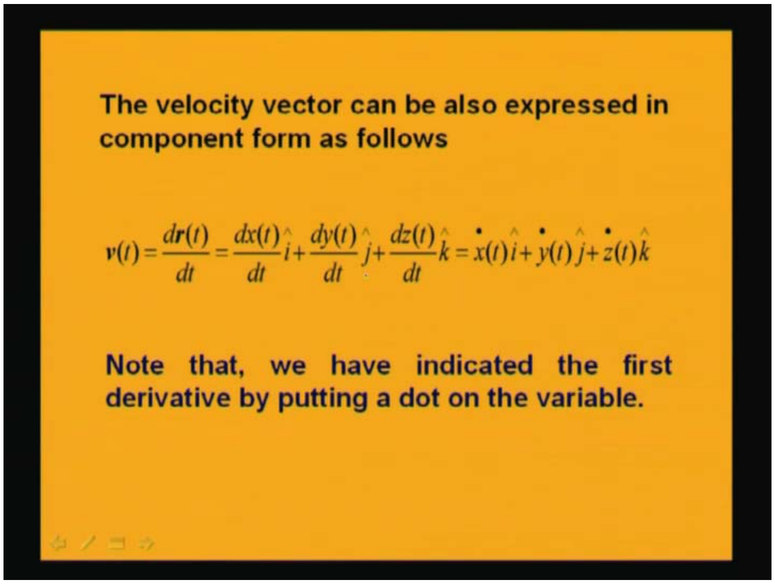

Since the velocity is a vector, it can be expressed in component forms also very easily. V(t) is equal to dr(t) by dt, rate of change of the position vector, r is basically xi plus yj plus zk, where unit vector i is along x-axis, unit vector j is along y-axis and unit vector k is along z-axis. Therefore, dr(t) by dt is dx by dt i plus dy by dt j plus dz by dt k which can be written as x dot (t)i y dot (t)j and z dot (t)k. x dot t indicates that this component is a function of time. Similarly, the other component is also a function of time. Dot indicates the derivative with respect to time; single dot will indicate first derivative and two dots on the alphabet will indicate double derivative.

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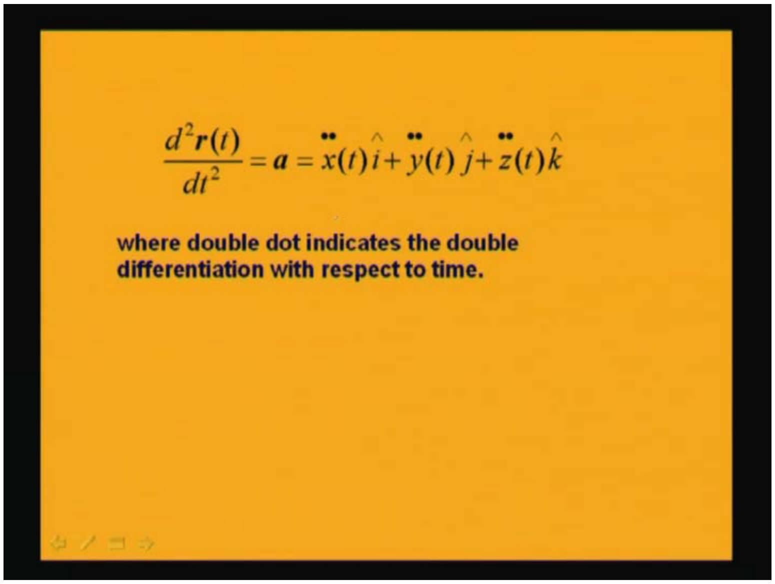

Acceleration can be written as d square rt by dt square is equal to a. This is equal to x double dot (t)i plus y double dot (t)j plus z double dot (t)k where double dot indicates the double differentiation with respect to time.

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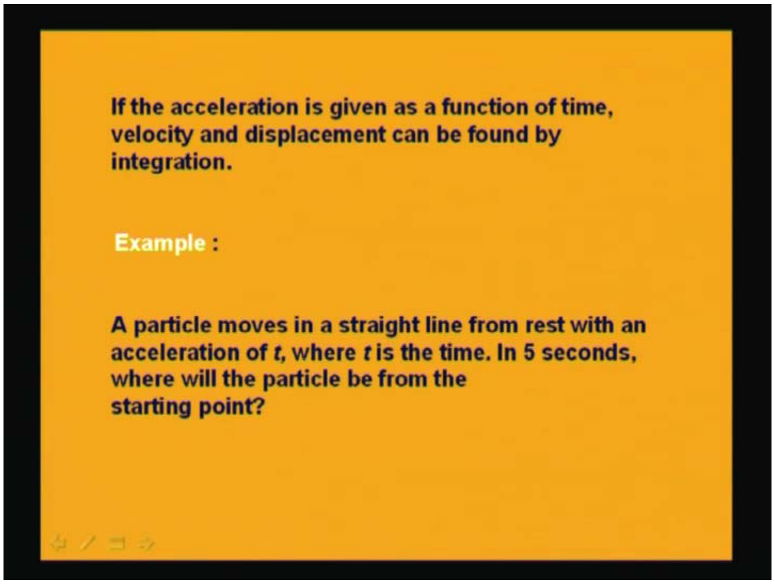

If the acceleration is given as a function of time, then velocity and displacements can be found by integration. Integration is the reverse process of differentiation. For example, suppose a particle moves in a straight line from rest with an acceleration of t, where t is the time; in 5 seconds, where will the particle be from the starting point?

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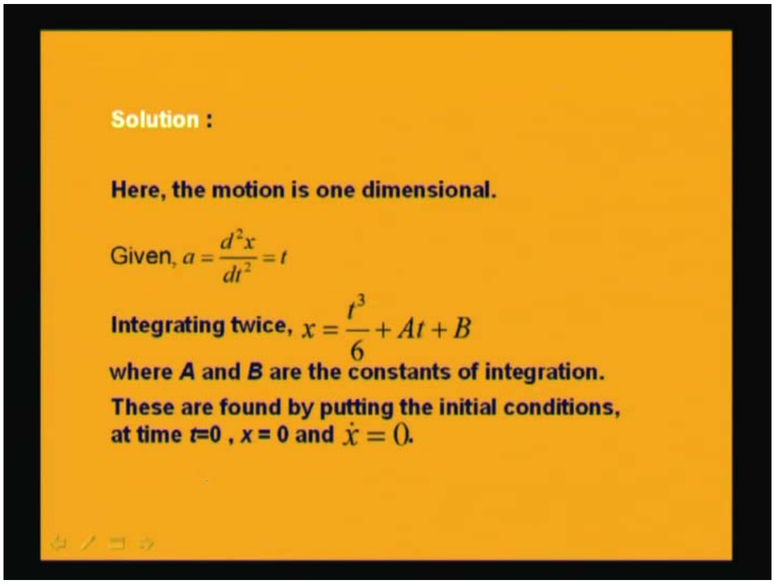

If this is the problem then we can solve it like this. Here, the motion is one-dimensional. So we can do away with the vectors. In one dimension only algebraic quantities are good enough. x can be positive or negative. We need not give any directions. So, given a is equal to d square x by dt square, which is basically the x component of the vector or if the motion is one dimensional, then the acceleration itself that has been given as a time t. If we integrate it twice, we get x is equal to t cube by 6 plus At plus B, where A and B are the constants of integration. The constants of integration can be found by putting the initial conditions. Since the particle starts from rest at the origin, at time t is equal to 0, x is equal to 0 and x dot is equal to 0.

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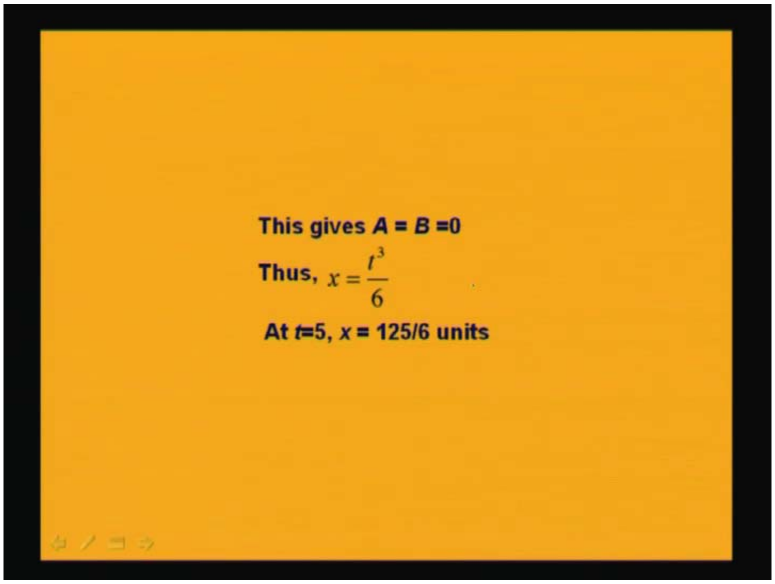

This gives A is equal to B is equal to 0. Thus, x is equal to t cube by 6. We have found the position of the particle with respect to time by this formula. With respect to the origin at what distance the particle is, we can substitute the value of t. For example, at t is equal to 5, x is equal to 125 by 6 units. In a psi system, it will be meter.

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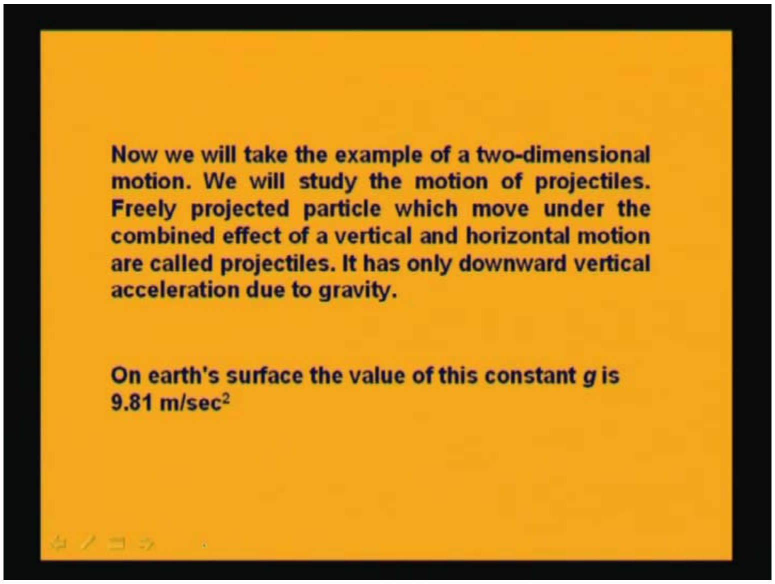

Let us discuss the example of a two dimensional motion. We will study the motion of projectiles. Freely projected particles which move under the combined effect of a vertical and horizontal motion are called projectiles. It has only downward vertical acceleration due to gravity. On earth's surface, the value of this gravitational acceleration is 9.81m per second square.

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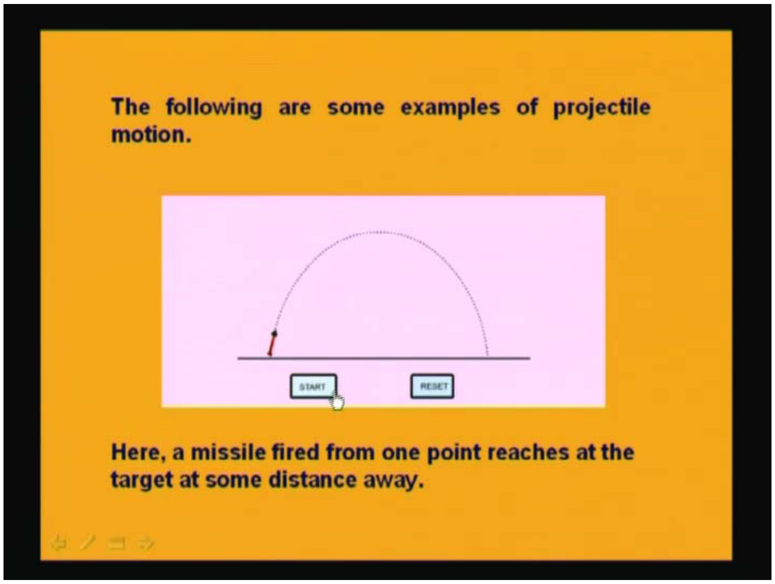

The motion of the projectile on the earth's surface is of great interest. This finds application in the defense and in the space applications. Following are some examples of a projectile motion. Suppose, we have got a missile starting from one point. I am starting the animation here. I am starting, missile is going and now it is reaching the top, then it is falling down and it has gone there. I will play it again. This is starting; it is reaching the top maximum height, coming down again. If we know our target, then we must properly know at what velocity and what angle should I project the missile so that it properly falls on the target. Here, a missile fired from one point reaches at the target at some distance away.

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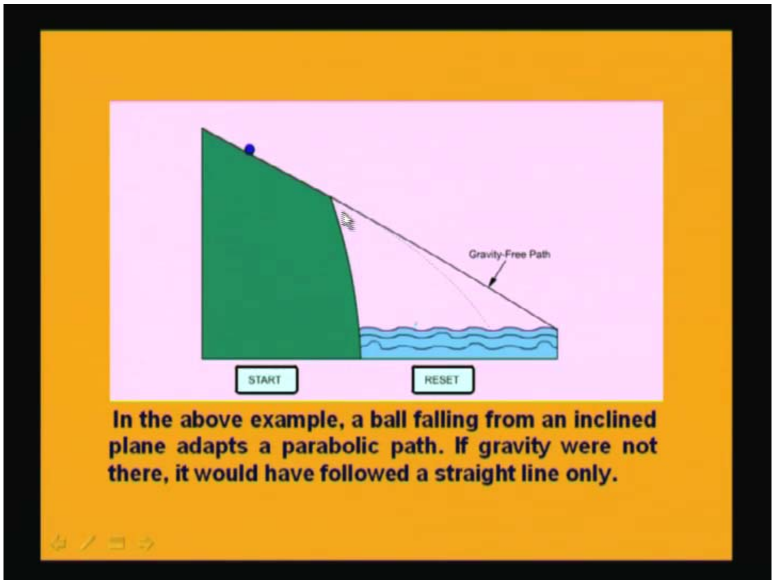

Another example: if we see this sloping surface, on this I am putting a ball and if the ball starts rolling down, if there was no gravity then the motion of the ball would have been in straight path. This is gravity free path in which ball moves from this point and it goes up to this point. However, because there is gravity, it will deviate from that straight path and ball will start moving downward. Therefore, its path will not be straight, rather it will be forwarding in x direction but at the same time it will come down in the y direction. Let us see this in animation.. So, this is the path of this one and it has fallen here. We can play the animation again. See it is falling down.

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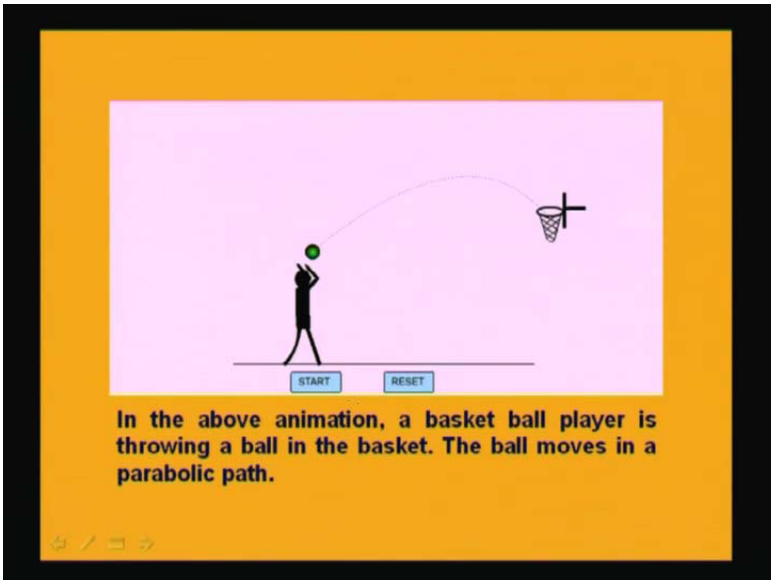

Similarly, this one; a basketball player is throwing a ball in the basket. He must know, with what velocity and what angle he should throw the ball so that it falls into the basket. He does not do any calculation. He does not study the kinematics but he does that mental calculation and by that he is able to throw the ball. Let me see this animation, it is going like this. We can play this animation again, yes this has fallen there. These are the examples of the projectile motion.

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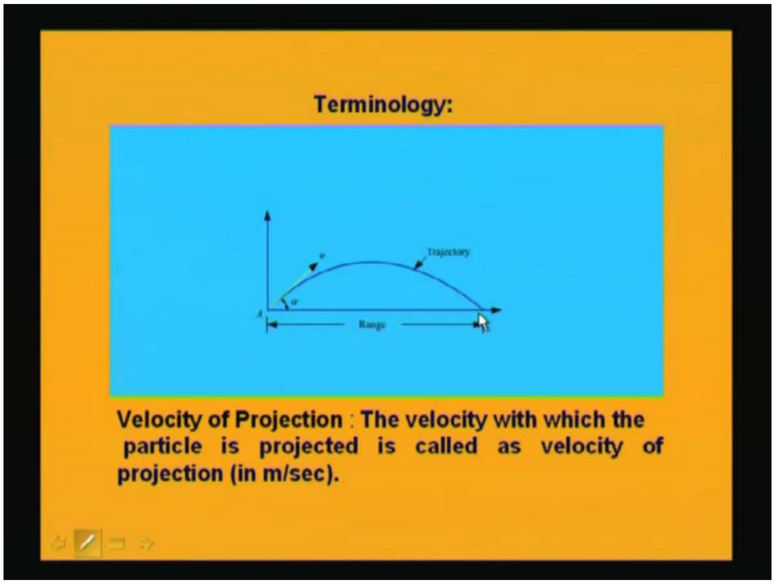

Now, we will discuss some terminology of the projectile motion. Here, this shows the trajectory of a projectile. One particle has started from point A and it has reached another point. Now, velocity of projection is the velocity with which the particle is projected, this is in meter per second.

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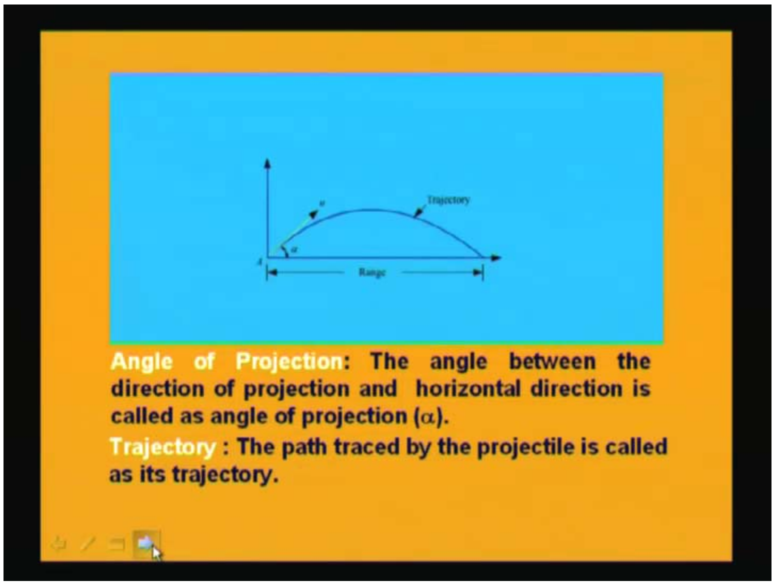

Suppose in this case, u is the velocity of the projection. Then here, the range of the particle is also shown. Range is the horizontal distance it travels, so horizontal range. Angle of projection is the angle between the direction of projection and horizontal direction is called as angle of projection alpha. The trajectory is the path traced by the projectile. Here, it is the trajectory and generally, it has parabolic shape.

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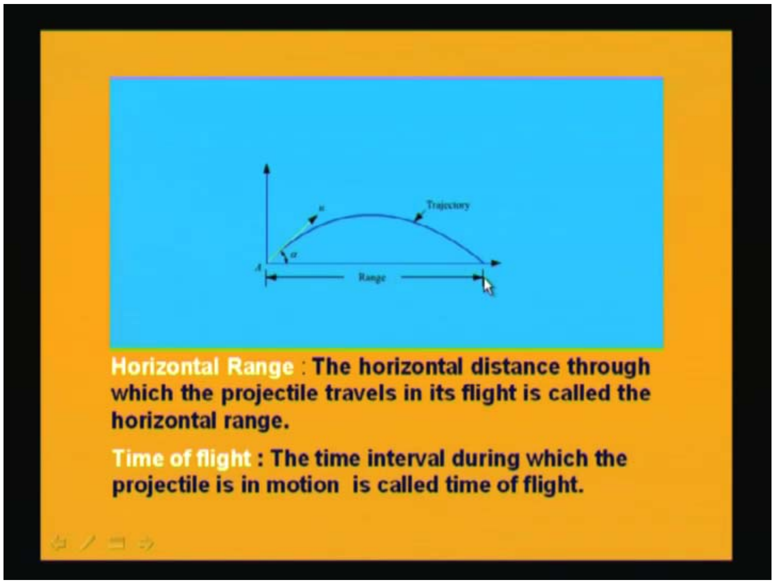

Then, horizontal range - the horizontal distance through which the projectile travels in its flight is called the horizontal range. In this case, the range is this. Then, time of flight; the time interval during which the projectile is in motion is called time of flight.

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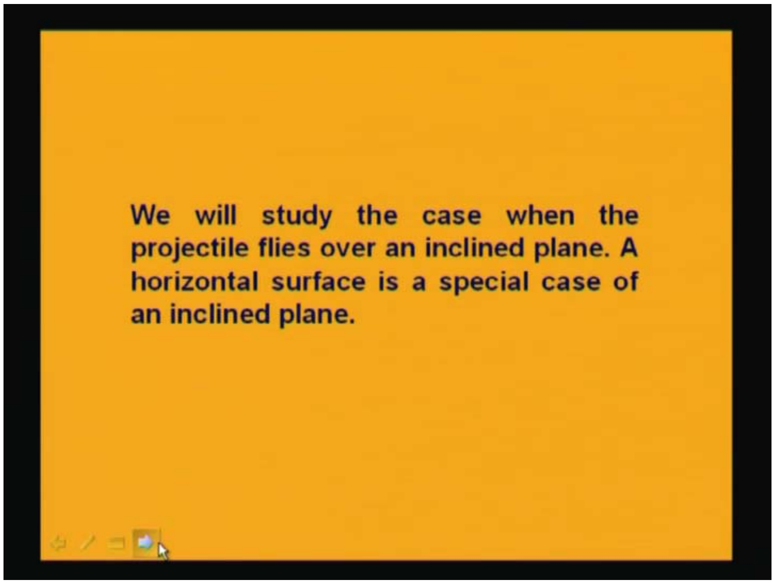

Next, we will now study the case when the projectile flies over an inclined plane, because moving on the horizontal plane is a special case of moving on inclined plane.

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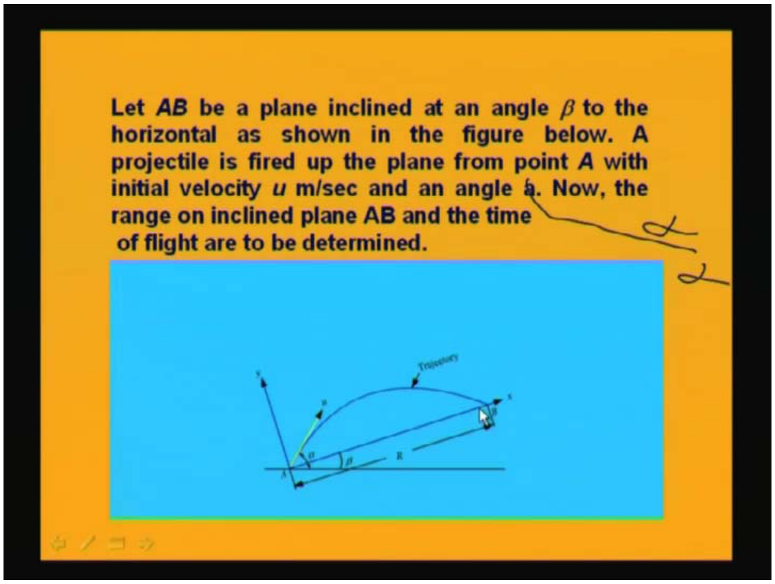

Horizontal plane is also an inclined plane with angle of inclination from horizontal equal to 0. So, a horizontal surface is a special case of an inclined plane. Let AB be a plane inclined at an angle beta to the horizontal as shown. Now, a projectile is fired up the plane from point A with initial velocity u meter per second and angle alpha. This is actually angle alpha. This will be angle alpha; actually this is not A, this is alpha. The range on an inclined plane AB and the time of flight are to be determined if you want to find out the range. Suppose, I know the initial velocity u that is velocity of projection, then I know the projection angle alpha, the angle which it makes from the horizontal surface.

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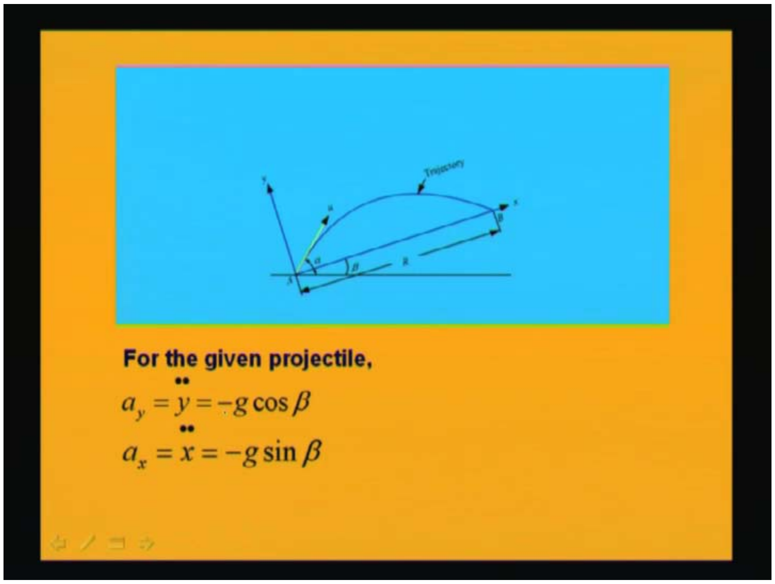

I also know the inclination angle beta. My task is to find out the time of flight and also the range AB. For that purpose, for the given projectile, if we take the x-axis along the inclined plane and y-axis perpendicular to that then we can study the motion in x y plane. The acceleration is downward and that is of about minus g but it has the components along the plane and normal to the plane. Normal to the plane, the component of the acceleration a_y that is indicated by y double dot is equal to minus g cos beta; y double dot means d square y by dt square is the y component of the acceleration.

Similarly, in the horizontal direction a_x is equal to x double dot is equal to minus g sin beta. If you square them, a square plus a_x square will be equal to g square. Therefore, the magnitude of the acceleration is g only.

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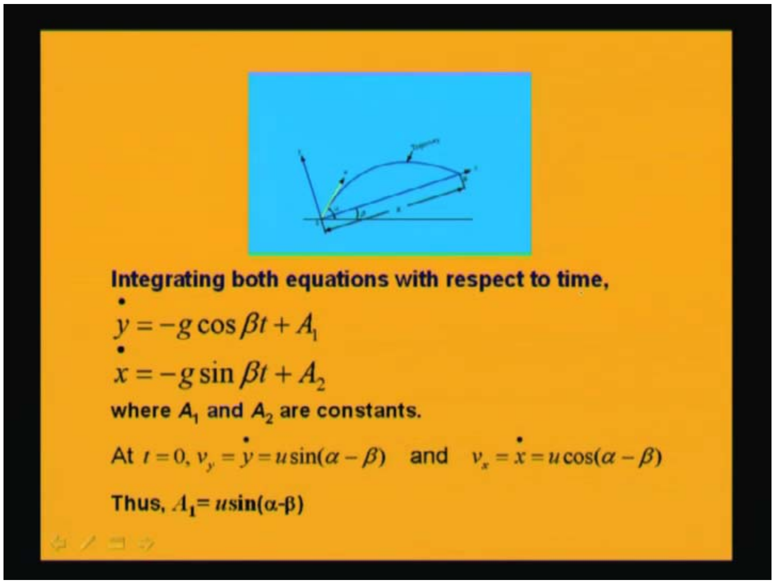

We can integrate both these equations with respect to time. We get the equation y dot which is basically the velocity component in the vertical direction that is equal to minus g cos beta into t plus A_1 where capital A_1 is a constant. Similarly, x dot is equal to minus g sin beta t plus A_2 where A_2 is another constant. At time t is equal to 0, v_y that is the vertical velocity y dot is equal to u sin alpha minus beta because this is the component which is normal to the plane and the horizontal component along the plane will be v_x, which is equal to x dot which is equal to u cos alpha minus beta.

Therefore, if we put these values here then we can get the values of the constants. We get A_1 is equal to u sin alpha minus beta and A_2 is equal to u cos alpha minus beta.

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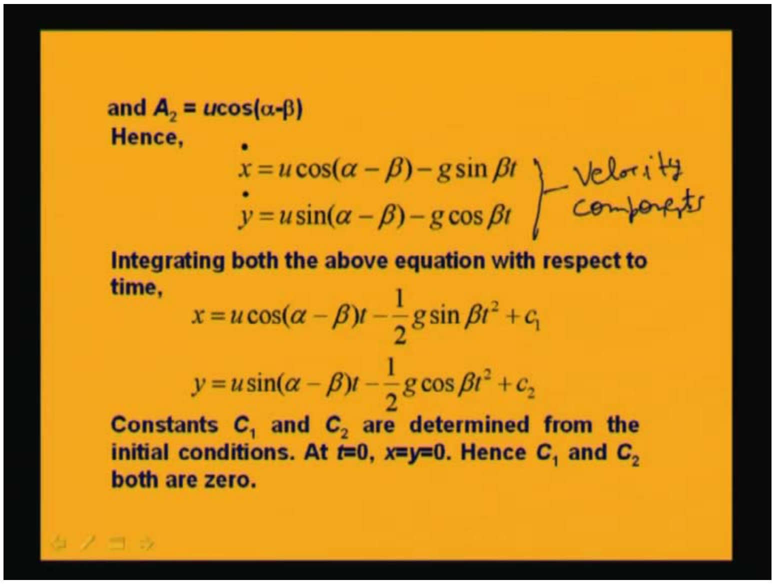

Hence, x dot is equal to u cos alpha minus beta minus g sin beta into t and y dot is equal to u sin alpha minus beta minus g cos beta into t. These are the two equations in the form of the velocity components. Therefore, the velocity as a function of time has been attained up to this point. If we integrate both the above equations with respect to time, then we get x is equal to u cos alpha minus beta into t minus half g sin beta into t square plus C_1. Similarly, y is equal to u sin alpha minus beta into t minus half g cos beta t square plus C_2 where constant C_1 and C_2 are determined from the initial conditions. At time t is equal to 0, x is equal to y is equal to 0 because we are taking that as the origin. Hence, C_1 and C_2 both are zeroes.

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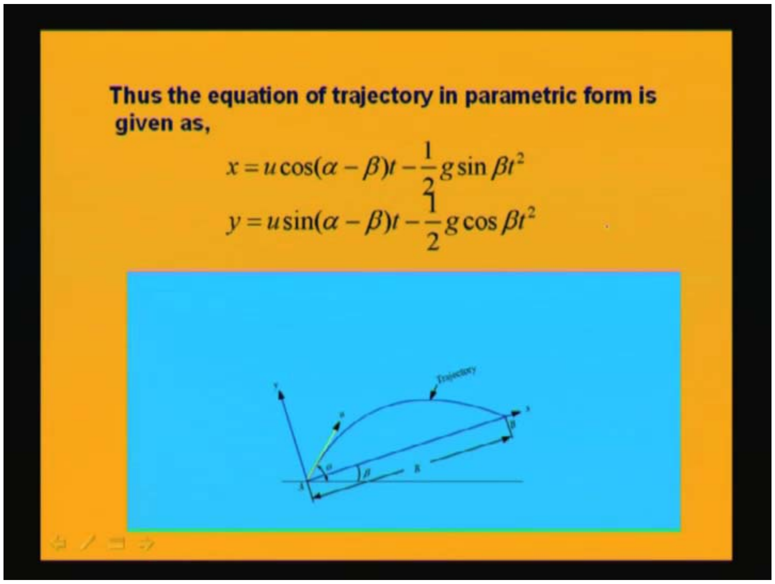

The equation of the trajectory in parametric form is given as x is equal to u cos alpha minus beta t minus half g sin beta t square and y is equal to u sin alpha minus beta t minus half g cos beta t square. Here, x and y both are functions of t. So, t is the parameter knowing which I can find out x and y. Therefore, this form is called parametric form.

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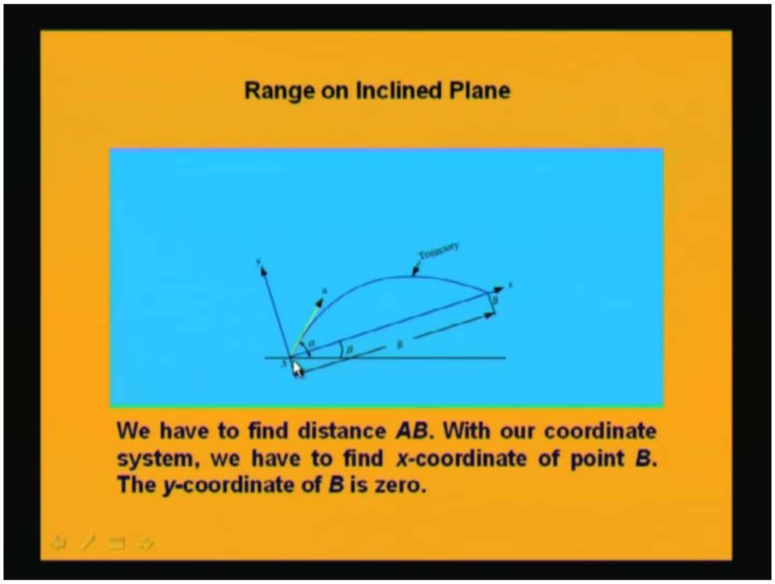

After we have found out the equations of this motion, let me discuss the range on inclined plane. In this figure, AB is the range of the particle on inclined plane. It starts from A, reaches some distance here, again it falls back to B. We have to find distance AB with our coordinate system. We have to find x-coordinate of point B when the y-coordinate is 0 because at A and B, at both the places y-coordinate is 0.

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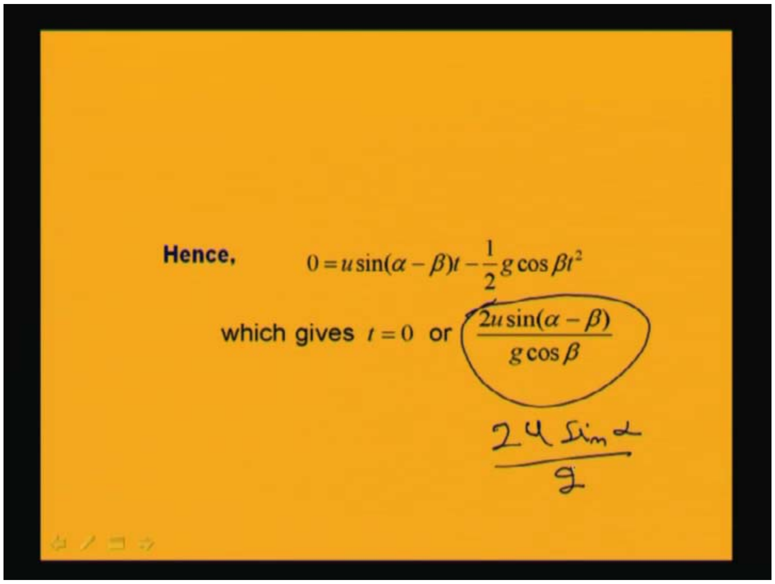

If we put y is equal to 0 is equal to u sin alpha minus beta t minus half g cos beta t square which gives two solutions t is equal to 0 or t is equal to 2u sin alpha minus beta divided by g cos beta. That means y component is 0 at the beginning, at time t is equal to 0 and then later on at time 2u sin alpha minus beta divided by g cos beta. Therefore, this must be the time of flight. Time of flight is given by 2u sin alpha minus beta by g cos beta. In this, beta is the inclination of the plane. If it is horizontal surface then beta will be 0 and time of flight will be 2u sin alpha divided by g. So, for horizontal plane, the time of flight is 2u sin alpha divided by g which you have studied in your class twelve.

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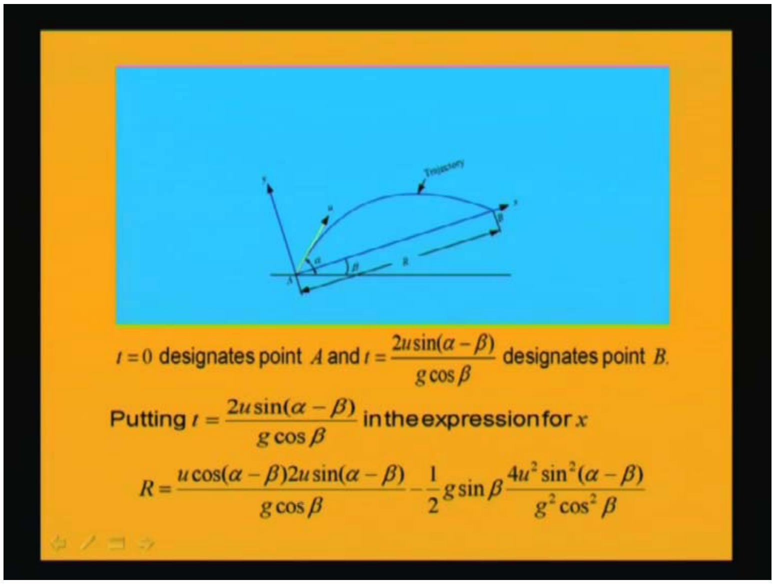

Once we have found time t, we can put this expression in the expression for x. Then we get R is equal to u cos alpha minus beta into 2u sin alpha minus beta divided by g cos beta minus half g sin beta 4 u square sin square alpha minus beta divided by g square cos square beta. We get this lengthy expression.

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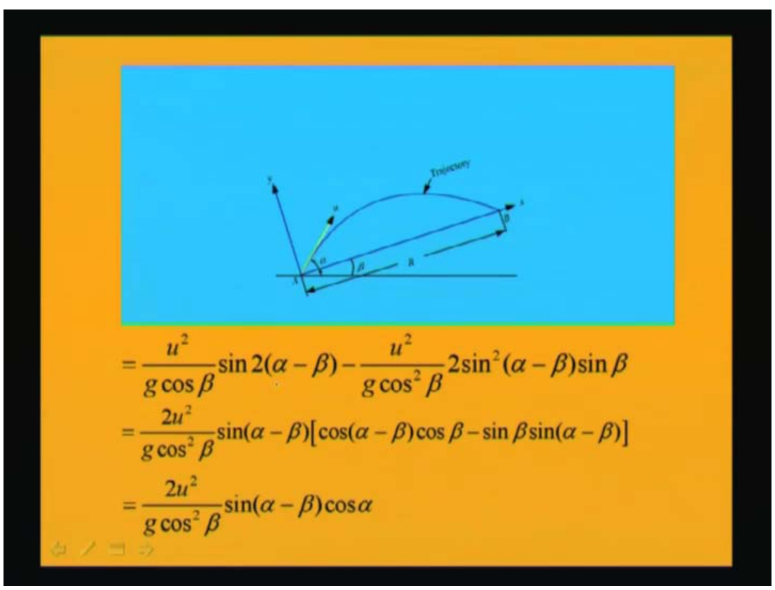

Let us see if we can simplify this. This can be written as u square by g cos beta into sin 2 alpha minus beta. We know the formula where sin 2a is equal to 2 sin a into cos a. Then similarly, the other part can be written as minus u square g cos square beta 2 sin square alpha minus beta into sin beta. Here, I can take sin alpha minus beta as common, u square by g cos square beta as common. Then, this will become cos alpha minus beta into cos beta minus sin beta into sin alpha minus beta. Using some trigonometric formula, we try to simplify this and this is equal to 2 u square divided by g cos square beta into sin alpha minus beta into cos alpha. This is the expression for the range.

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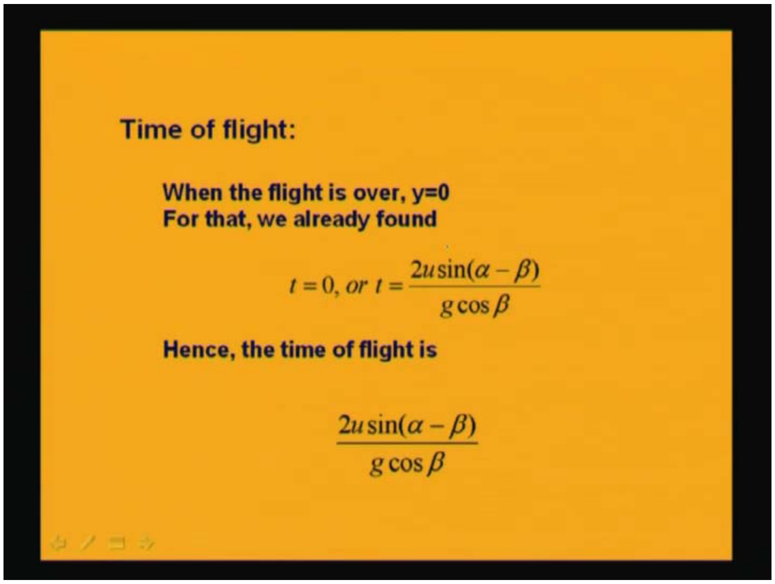

If you repeat it again, time of flight is basically when y is equal to 0. In the beginning y is equal to 0; when the particle has fallen down, then again the time of flight is 0. We have already found that t is equal to 0 or t is equal to this thing. Hence, the time of flight is 2u sin alpha minus beta divided by g cos beta.

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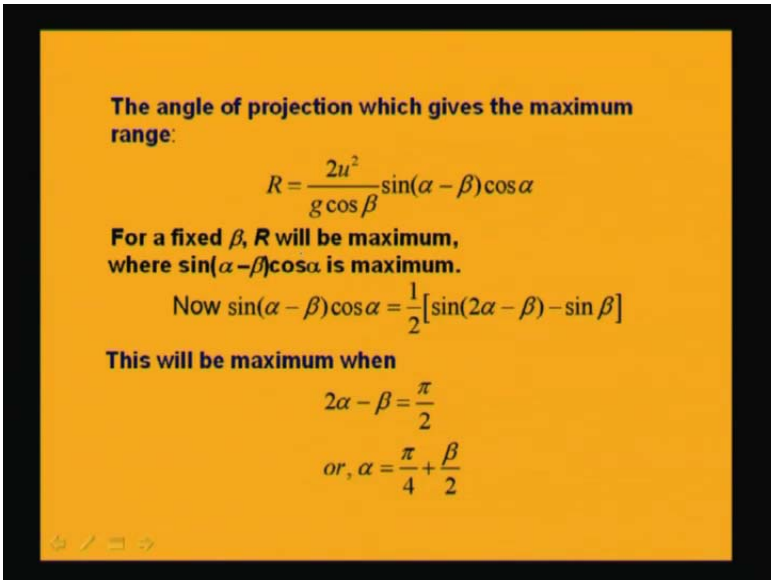

We are going to discuss the angle of projection which gives the maximum range. We have the expression for the range R which is given by R is equal to 2 u square g cos beta into sin alpha minus beta into cos alpha. For a fixed beta, R will be maximum. If beta is fixed then that means the plane is fixed. Therefore, sin alpha minus beta into cos alpha should be maximum. When sin alpha minus beta into cos alpha is maximum then you get the maximum range. Sin alpha minus beta into cos alpha is basically half sin 2 alpha minus beta minus sin beta. Now, sin beta is fixed. This portion will be maximum when 2 alpha minus beta is equal to 90 degree or pi by 2 radian because sin is maximum at an angle of pi by 2. So, 2 alpha minus beta is equal to pi by 2 or alpha is equal to pi by 4 plus beta by 2. Therefore, this is the angle of projection for getting the maximum range. If it is a horizontal surface then beta will be 0. Therefore, the alpha will come out to be pi by 4. Therefore, on a horizontal surface the angle of projection is kept 45 degrees for getting the maximum range.

If you fire something at an angle of 45 degrees, then the range will be maximum. This is about kinematics of a particle. Basically, in essence, in kinematics of a particle you need to carryout vector calculus. If you are given the displacement as vector then you can differentiate it and apply vector calculus. Thus, you can find out the velocity and acceleration of the particle. If somebody has given you acceleration as a function of time, then you can integrate and you can find out the velocity and you can find out the displacement also.

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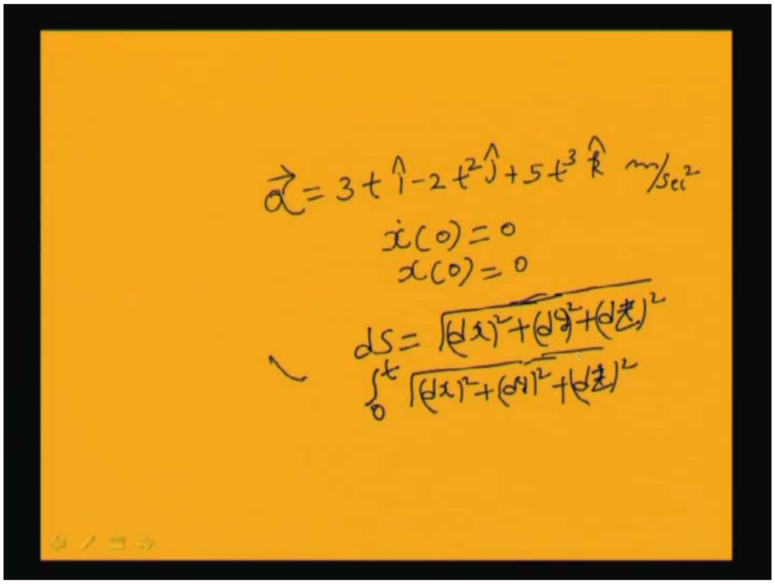

You can solve number of problems. For example, suppose somebody gives you acceleration of a particle which is a vector a is equal to 3 t i; i is a unit vector minus 2 t square j plus 5 t cube k meter per second square. If we say particle starts with 0 velocity at the origin, we write the condition that x dot at 0 is equal to 0, x is 0 at 0. Therefore, if somebody wants to find out particles position, displacement, distance travelled after 5 seconds, then we can integrate this expression and find out the particle's position. We can find out the particle's position displacement and distance. Suppose, somebody wants to find out the distance travelled, then we know that small distance ds will be equal to dx square plus dy square plus dt square.

Therefore, between two time limits, now dx can be expressed in terms of dt. You will be getting some expression for x which will come in the form of dt. Then, you have to integrate between 0 to t; this quantity dx square plus dy square plus dt square which will basically come dz square. This is dz square which will come in the form of dt. Then, you will be able to find out the distance travelled also. You can find out velocity, you can find out speed and acceleration.

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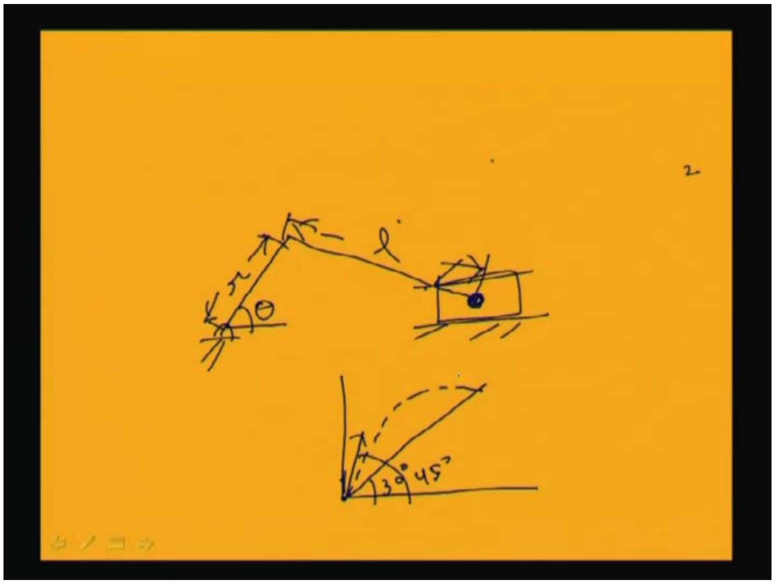

Another problem can be of this type. Here, if you have a slider crank mechanism, this is a crank then this is a connecting rod and this is a slider which is moving. Now, you want to find out the velocity and acceleration of a slider crank mechanism as a function of theta. That can be easily done, provided we will be moving the crank radius r and we will be moving this length as connecting rod length l. Then, we can find out the distance of this particle, which is in the slider, from the origin as a function of r and as a function of theta. Finally, we can differentiate this expression to find out the velocity also as a function of theta.

Velocity will contain the term of d theta by dt which is nothing but the angular velocity of the link. Similarly, there you can solve the problems of the projectiles. For example, this is the projectile. If this is at 30 degrees, it is fired at say 45 degrees from the horizontal. When you know how it falls and what is the range, these types of problems you can solve. Similarly, there may be problems of this type.

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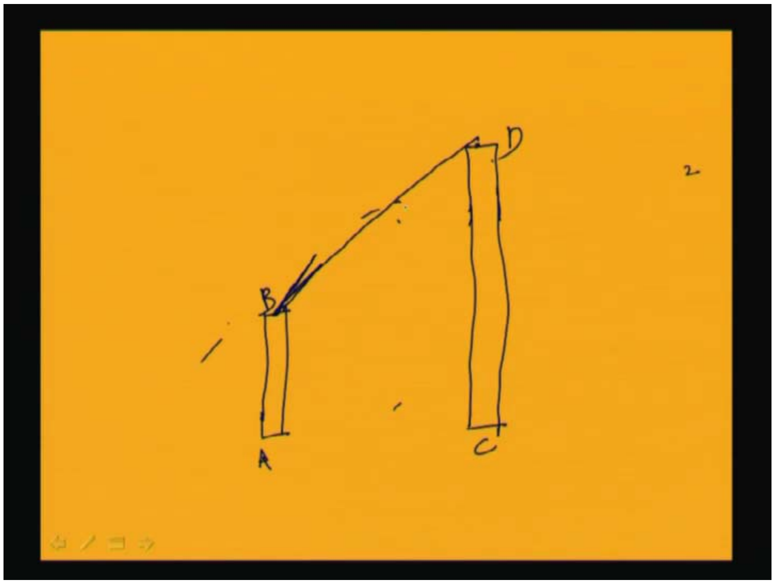

If you look, there are two columns; this is this one and this is column B, this is B and this is D; A B C D. Now, suppose you fire some shot from here, what angle should it be projected so that it hits here? What should be the angle of projection? Naturally, if you hit along this line, it will not touch D. So, therefore we have to decide the angle of projection. So, the similar types of problems can be solved.

Now, in my next lecture, I will be talking about kinematics of a particle moving on a curve. If the particle is constrained to move on a particular curve then how we study the kinematics?

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For example, in this case the slider is constrained to move in this guide way. This slider is not free to move anywhere; it is just constrained to move. This is a straight surface but sometimes the slider maybe constrained to move on a curved surface also. These type of things also we will study in the next lecture.