Engineering Mechanics

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Virtual method and energy method-1

Module 8 Lecture - 20
Systems with Friction

Today, we will continue our lecture on the method of virtual work as applied to system of connected rigid bodies.

In the earlier lecture, we saw a system which has been idealized; that means, we have removed the elements that absorb energy or the work by means of elongation or compression. Also, we neglected any friction if it is there, so that the negative work done by this friction was not considered in our virtual work equations.

Today, we will see systems where friction is considerable; we have to consider it for this method of virtual work. As we discussed in the last lecture, the method of virtual work loses much of its benefits when we dismember the entity or the structure in order to analyze the equilibrium. But for systems with friction, this method provides a convenient way of determining the efficiency of the system in the presence of the friction. We can define the efficiency of a system by this method of virtual work.

We define efficiency of a mechanical system as the amount of available energy when some input energy is given. Certain energy is consumed by the system by dissipation of heat because of the friction existing in the various connections. We will see how to apply this method of virtual work to analyze such problems and to determine the efficiency of a real mechanical system.

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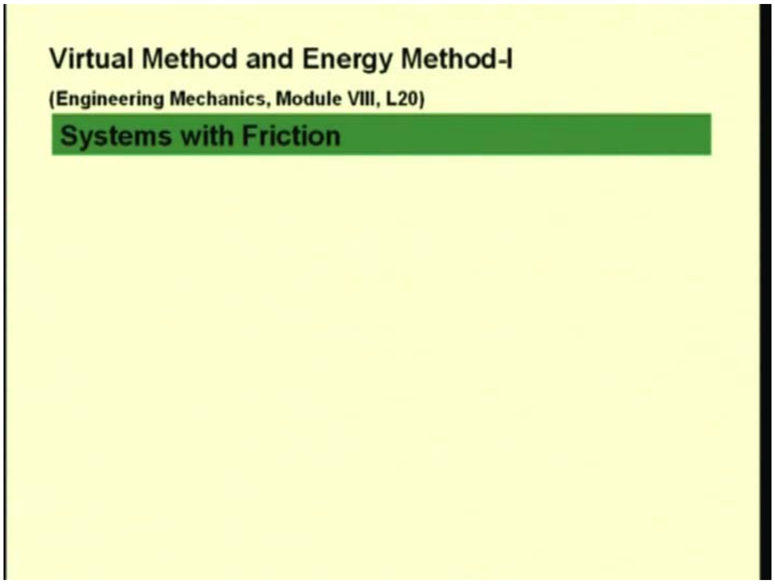

For your reference this is module 8, lecture 20 of the engineering mechanics course.

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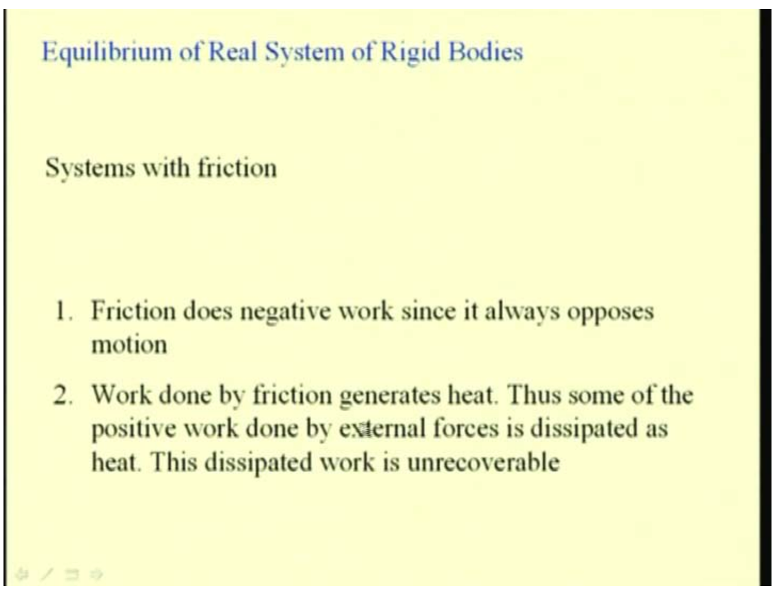

The real systems of rigid bodies are systems with friction. Here, we consider the effect of the friction when analyzing for equilibrium by this method of virtual work. Friction does negative work since it always opposes motion irrespective of the direction of the motion. We have seen that irrespective of the nature of the motion the friction always tends to oppose the motion. Thus, any virtual displacement if we consider the friction will be against that virtual displacement and thus will cause a negative work.

The work done by the friction generates heat. Thus, some of the positive work done by the external forces is dissipated as heat. This dissipated work is unrecoverable, and thus causes the inefficiency of a system because of friction and the work done by the external applied forces. Some of this work is dissipated as heat and this is unrecoverable.

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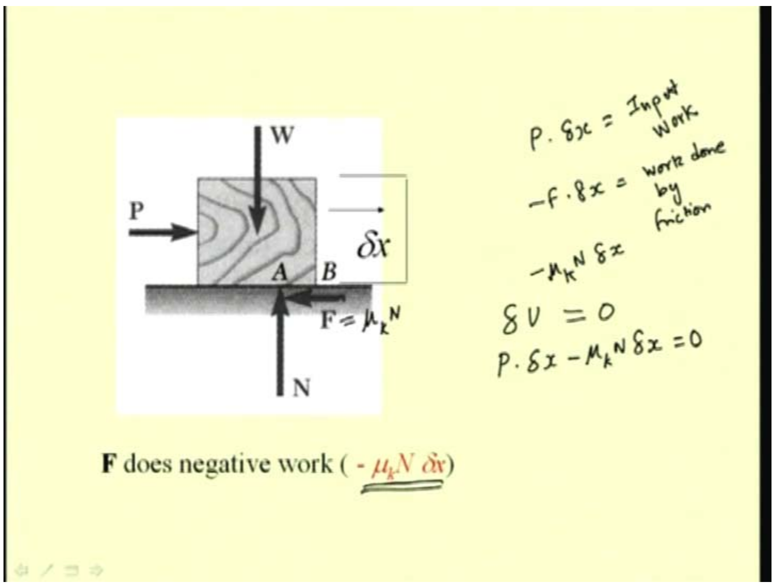

Let us consider the block with friction sliding on a surface which we have used to analyze the problems of friction. Here, we see a block with weight W. A force P is being applied to move this block against the surface. The contact has friction and that is mentioned by this force F. This is the normal reaction N to support this weight.

Let us assume that this body is under equilibrium. That means this force P is less than the maximum allowable force that exists as friction at this contact surface. In that case, we can analyze this problem for finding out this unknown force F by considering a virtual displacement. But, if the problem is to analyze the efficiency of a system with friction or if we are interested to find the efficiency of this way of pushing this block against this surface in order to move this object from one place to another, then the object is actually in motion. The force P is more than the limiting value of the friction that is available in this contact. So, this frictional force F, in such case, will be the dynamic friction or the kinetic friction which will be equal to mu_k times of N. So, this force F is equal to mu_k times of N.

If you would like to know the work done by this frictional force, let us consider a virtual displacement delta x in the forward direction or in the direction of the applied force. The work done by this applied force is P times delta x. This is the input work which is virtual because we are considering a virtual displacement delta x. This force which is in the opposite direction of this applied force for the same virtual displacement does this negative work which is equal to minus F times delta x, which is the work done by friction; in this case it is minus mu_k times N delta x.

Our equation of virtual work where delta u equal to 0 states that P dot delta x minus this work done by the friction mu_k times N delta x is equal to 0. Here, we see that this negative work term comes into the picture. The input work is consumed by the friction that exists between the surface and the block.

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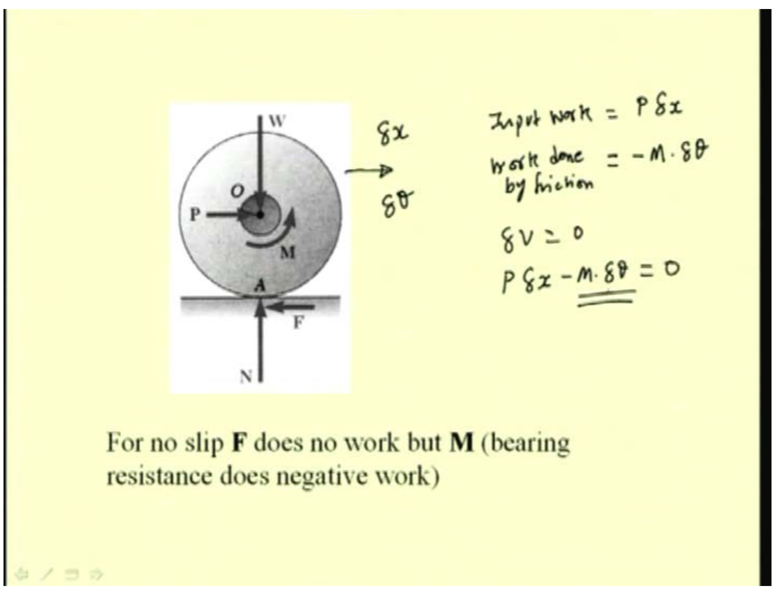

Let us consider another case where friction occurs in the bearing. So, we have already seen that the bearing resistance has to be overcome when a wheel is rolled on a surface. We have already seen that for this M to exist there will be a frictional force F at the point of contact between the wheel and the surface. But if we assume that this point A, if it does not slide, then for any movement of this wheel this force F does not do any work because there is no sliding that is taking place at A. But we see that this frictional moment M has to be overcome at this bearing surface. Thus, the negative work is done not by this frictional force F but by the frictional moment that exist in the bearing.

If we consider that this wheel moves forward by a distance delta x for this virtual displacement, this wheel let it rotate by an angle delta theta. So, for a virtual displacement delta x it is possible to find this virtual angular displacement delta theta if we know the radius of this wheel. Then we get the relation between this delta x and delta theta. Again, this is single degree of freedom system.

The input work is equal to P times delta x and the work done by friction; in this case the bearing friction is equal to this frictional moment M times delta theta. The negative sign comes from the fact that the wheel is rotating in the clockwise direction and M is in the anticlockwise direction; so the work done is negative.

We have from our virtual work principle this total work done is 0. So, we have minus M dot delta theta equal to 0 and this is the negative work done by the friction. In real systems, we see that a negative component of virtual work exists for each of the frictional F_x either sliding or in case of pure rolling motion.

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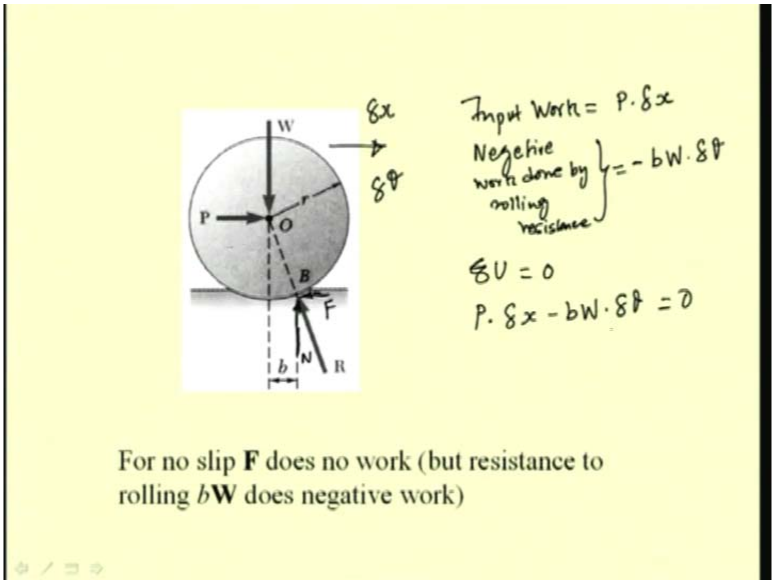

Let us see another case where we have this rolling resistance. We have already seen that real objects are deformable; so when a wheel or a free wheel tries to roll on a surface because of the deformation there is a rolling resistance and that rolling resistance is given by bW. Again, if we assume that no slip occurs at this point b, then the frictional force F that exists here, because we know that at this point we have the frictional force and the normal reaction N and this is a frictional force F. This N compensates for this weight and this frictional force F is equal and opposite to the applied force P.

If we assume that there is no slipping at this point b, then this force F does not do any work, but this moment, that is Wb, which is the rolling resistance, it does a negative work. If we again assume that this wheel moves by a virtual distance delta x and for that virtual displacement this wheel rotates by delta theta. Then in that case, the input work is equal to P dot delta x; the negative work done by rolling resistance is given by the rolling resistance bW times delta theta which is a negative work. This comes from the fact that the rotation is in the positive direction; that is, delta theta is in the clockwise direction and we have this moment in the anticlockwise direction. The moment of this force is in the anticlockwise direction about this point b.

We have the total work that is delta U equal to 0, P dot delta x minus bW dot delta theta equal to 0. In this way, we can see that the work associated with friction or rolling resistance is always a negative work; it is to be accounted for in the virtual work equation if we are interested to determine the equilibrium condition or we are interested to find some unknowns. This negative work is not recoverable and it is lost as heat. In case of this rolling resistance, again this heat this released by the expansion of this compressed material or deformable material, again this work is not available.

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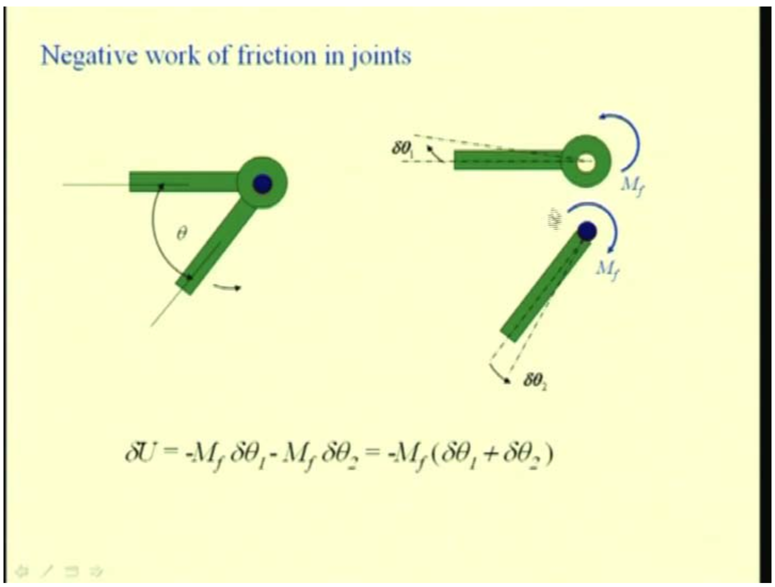

With this understanding, let us try to see how friction acts in the joints of connected rigid bodies. Here, you have these links, two links, connected at this joint.

Let the two links for this particular position be at an angle of theta. Let us assume that there is a virtual displacement of this link in this direction. So this is the positive direction of the virtual displacement. If the friction exists here, if we are considering the effect of diffraction at this joint, then the moment due to this friction does the negative work when we consider this virtual displacement. For a better understanding, here in this picture, we have dismembered in order to account for this frictional moment because if we take the free body diagram of this complete assembly, then the frictional moment cannot be accounted for.

We have to dismember in order to account for these forces which become now active forces thus doing the negative work in each of these bodies. We have this frictional moment M_f. So, if it is counterclockwise on this link, then the same is clockwise on the other link and it always opposes the motion. As we see that here the link tends to move from this equilibrium position in this direction, it tends to move by this angle delta $theta_1$, then this frictional moment M is in the opposite sense of the virtual displacement.

Same way, the link that is shown here moves in this direction that is delta $theta_2$ for the positive virtual displacement. This frictional moment M_f is again against this delta theta where M_f is in the clockwise direction and delta $theta_2$ is in the counterclockwise direction.

So, we see that always this frictional moment is against the virtual angular displacement that we are considering. If you would like to find the total work done by this frictional moment that is delta U is equal to… (Refer Slide Time: 20:02), since both these are against this virtual displacement they have this negative sign. We have minus M_f delta theta one minus M_f or the frictional moment times delta $theta_2$ which is equal to minus M_f times delta $theta_1$ plus delta $theta_2$ which is the total virtual displacement that we are taking for the particular joint.

This has to be accounted for in the systems when we analyze real systems and when we consider friction to exist in the connections between the various links.

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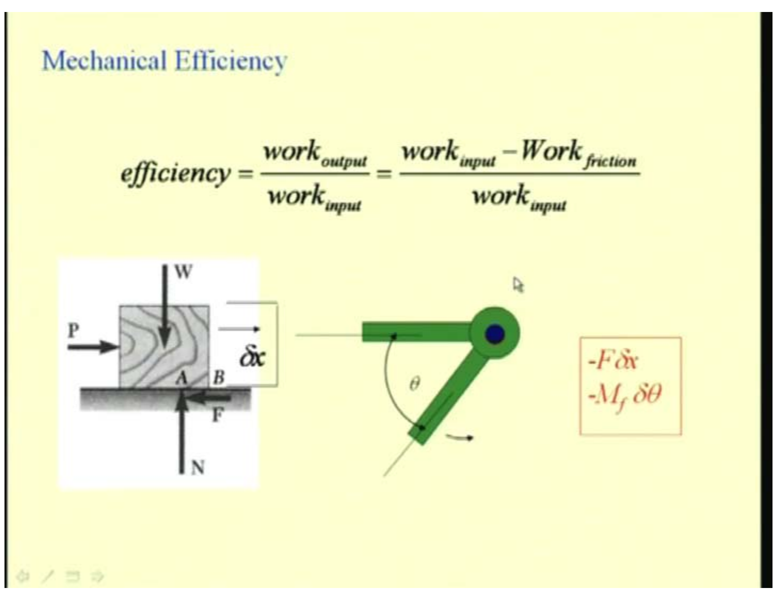

We have seen that the friction dissipates energy in the form of heat. We can define, now the efficiency of a mechanical system as the ratio of the available energy to the input energy where the available energy is nothing but the input energy minus the energy lost because of the heat dissipation due to the presence of friction.

In all systems where we have either the sliding action or where we are considering the bearing friction as in this case of the joints, we can account for the work done by these frictional forces and compute efficiency as the work output divided by work input, where the work output or the work that is available is equal to work input minus work done by the friction. This work done by this friction is minus F delta x as in this case where the block is sliding. Here as we see in this revolute joint, where we are considering the bearing friction in the joints, we have the work done by the frictional moment M_f and we have the work done by the friction as minus M_f delta theta.

These terms have to be accounted for and we can find the efficiency of the mechanical system.

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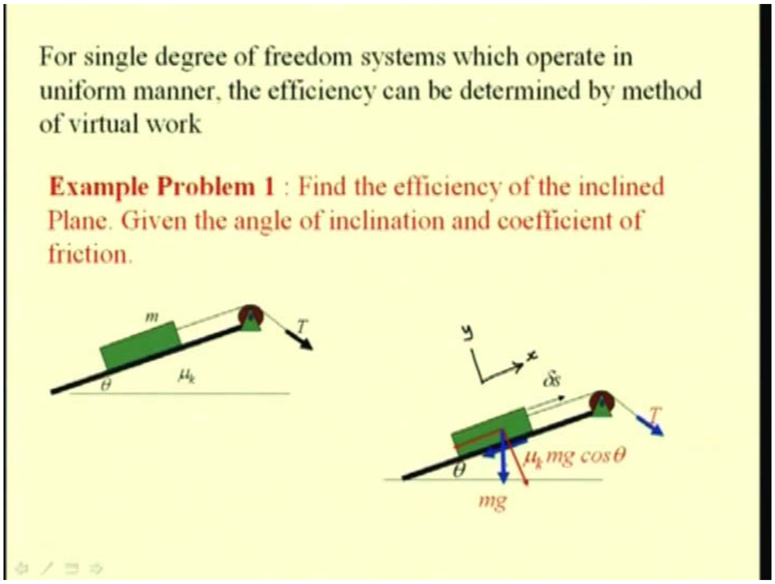

For single degree of freedom system, in these discussions we will limit our analysis of the single degree of freedom systems that operate in a uniform manner. The efficiency can be determined by this method of virtual work.

We will see few examples. Let us consider the first example where we have an inclined plane. We are interested to find the efficiency of this inclined plane. The plane is inclined at an angle of theta and let mu_k be the kinetic coefficient of friction between this block and this inclined plane. We have a smooth pulley and a cable passes over the pulley and the cable is pulled. Let T be the tension in the cable; we are interested to know the efficiency of this process of raising the load in this inclined plane.

Let us consider all these forces that are acting and also assume a virtual displacement consistent with the constraint. If we assume that the cord or the cable is being pulled by a distance of delta s which is along this cable on the block this delta s, the virtual displacement is along the cable or which is parallel to this inclined plane. We have this frictional force which is in the negative direction of this moment virtual moment that we are considering that is delta s and the weight of this body mg which has the two components; one along the inclined plane and one perpendicular to the inclined plane. If we consider our analysis with these as our coordinates say if this is my x coordinate and this is my y coordinate then this force or the component of the weight perpendicular to the inclined plane does not do any work but the component of the weight along the plane does the work. This force T does a work by displacing this cable by this virtual displacement delta x.

We have accounted for all the forces that is the tension or the force that is being applied and the weight of the block being raised as well as the friction between the block and the inclined plane surface.

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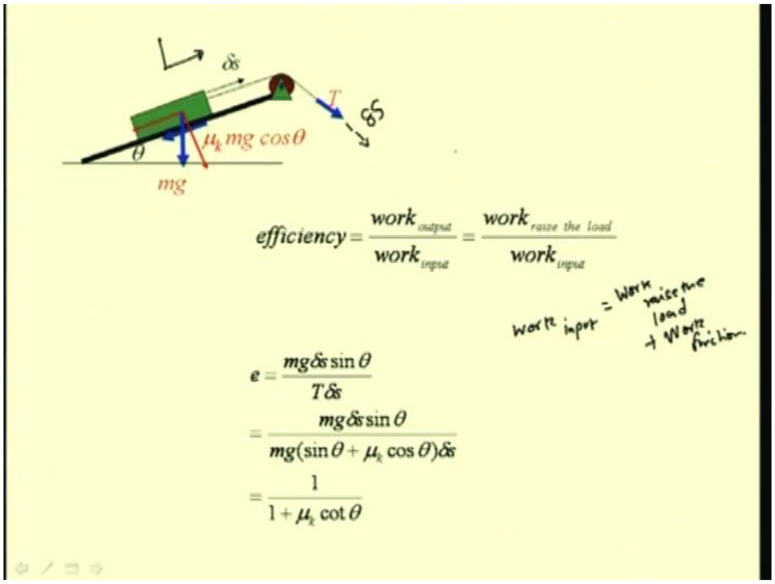

Let us write the virtual works that are involved for this virtual displacement and find this efficiency by writing that work output which is nothing but the work that is used to raise the load and work input is nothing but the work done by this force T for raising this load.

We can write the work done to raise this load as mg sin theta delta s, which is the component of this weight mg, in the direction of delta s, divided by the input work which is T times delta s; because, if delta is the is the virtual displacement of this block then this force T is also virtually displaced by an amount of delta s.

We have T times delta s as the work input; work output is nothing but the work done to raise this weight which is mg delta s sin theta divided by this T times delta s which is the input work. It can be written as the work used to raise the load plus the frictional work that is lost. That is the input work.

What are we doing? We are writing work input as the work done to raise the load plus the work of friction. The work done to raise the load is mg delta s sin theta and the work done by this frictional force is mu_k mg cos theta which is the frictional force which is in the negative direction of this positive virtual displacement delta s. So we have plus mu_k cos theta mg delta s. This mg and delta s being common terms have been taken out of this expression.

We can remove these common terms and we find this efficiency as 1 by 1 plus mu_k cot theta. So, we see that this method of virtual work can be used to find the efficiency of the system.

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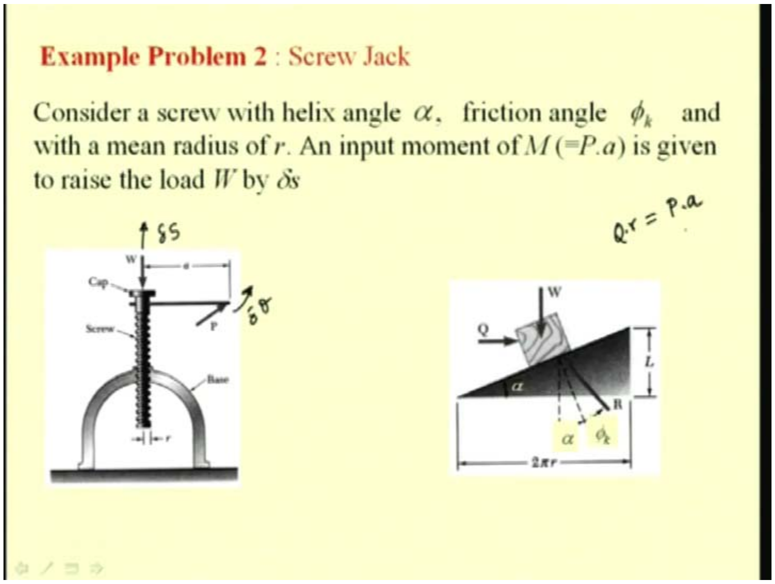

Let us consider another example where the working principle of this screw jack depends upon the friction that is available between the square threads and the base. We have already solved problems of on this screw jack but here we are interested to find the efficiency of this screw jack in raising this load.

Let us consider here a screw jack with a helix angle of alpha, the friction angle; let it be phi_k. The coefficient of kinetic friction is mu_k and the corresponding friction angle is phi_k. The radius of the screw is r. An input moment M which is equal to P times this momentum for the force that is being applied on the lever is given to raise this load W by a virtual displacement delta s.

Let us consider that this weight is being raised by a distance of delta s when this moment M acts for an angular displacement of delta theta. We are interested to find the efficiency of this screw jack in raising this load when this force P is applied on this lever arm. We have already seen that we can analogize an equivalent inclined plane for the screw jack and can perform the analysis.

Here you see an equivalent inclined plane where the inclination of the inclined plane is equal to the helix angle. This is the lead of this screw which is equal to the pitch for a single thread. For multiple threads it is equal to n times the pitch. The load is raised by this lead for one rotation that is 2 pi r. For the raising load condition, we have this Q which is the equivalent force for this moment P that is Q times r is equal to P times A.

This we have seen earlier when we solved this problem on screw jack by using an analogical inclined plane.

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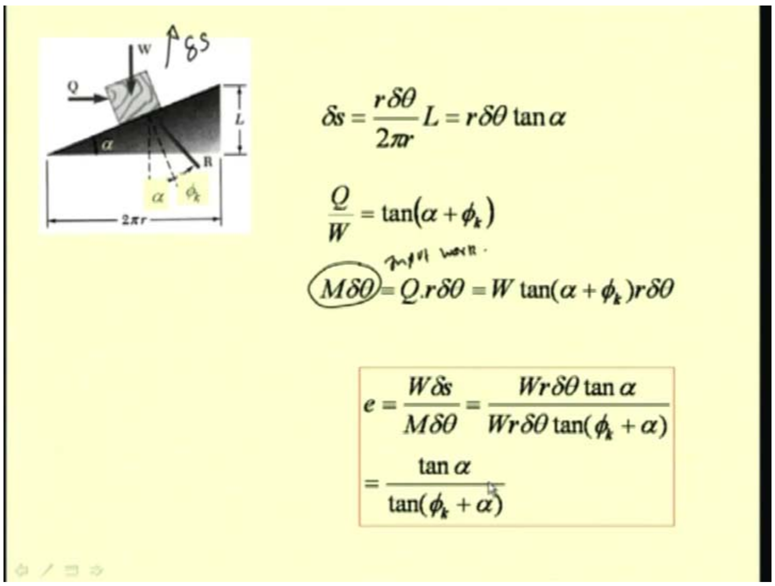

Let us write the virtual displacement.

Let this weight W be raised by a virtual displacement delta s. This delta s is equal to r times delta theta divided by 2 pi r times L because we know that if the screw rotates for one complete rotation that is for 2 pi r, the load is raised by a distance of L.

If the screw takes a rotation of r delta theta then this is the virtual displacement or it is equal to r delta theta tan alpha. We also know, for the raising load load on this inclined plane we have this Q by W is equal to tan alpha plus phi_k the coefficient of kinetic friction.

We write that M delta theta which is the input work which is also equal to Q times r delta theta is equal to W times tan alpha plus phi_k times r delta theta because this Q can be written in terms of

W tan alpha plus ϕ_k from this equation. We know that this M which is equal to P times A is also equal to Q times r. So, this is the input work.

Efficiency, we can define as the work done to raise this load that is W times delta s divided by the input work which is M delta theta. We have W delta s as W, delta s is r delta theta tan alpha; we have substituted that value divided by M delta theta, we have just found it to be W tan alpha plus ϕ_k times r delta theta. From this, if we remove these common terms we have this efficiency as tan alpha by tan ϕ_k plus alpha. Similarly, it is possible to find the efficiency of the screw jack when the load is being lowered in the same way as we have proceeded for this problem.

We see that this concept of virtual work can be effectively used to compute the efficiency of the mechanical system.

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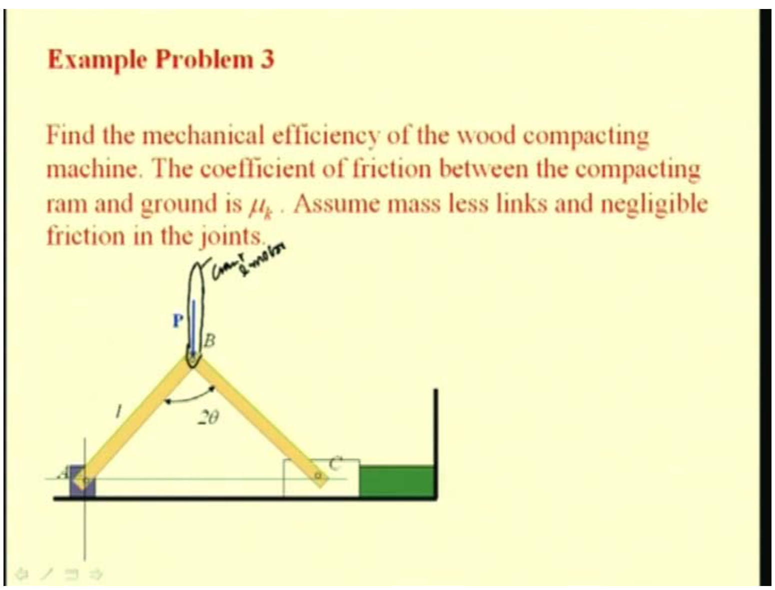

Let us take another example.

Here this picture shows a wood compacting machine. The problem could be also of that of a machine which is used to crush the rocks where there is a ram which is connected by this rod BC which is known as the connecting rod to this crank which is this link AB. Normally, these kinds of machines are powered by motors which rotate this crank AB. In some other cases, a force can be applied to this joint B by another connecting rod. So, here one can have another connecting rod which is in turn connected to a crank and a motor.

The force is applied through this connecting rod at this joint B in order to crush or in this case compact the wood. What you see as this green block is the wood that has to be compacted. Let there be friction and let mu_k be the friction between this ram and the surface or the ground which causes the input work or certain part of the input work to be dissipated in the form of heat.

We are interested to find this efficiency of the compacting machine in the presence of this friction between the ram and the ground. For the simplicity of the analysis, we are assuming mass less links. Also, we are assuming that the friction in the joints is negligible. That means the bearing frictions that have to be overcome at this joint B and joint A and also at joint C are negligible.

How do we proceed with this problem?

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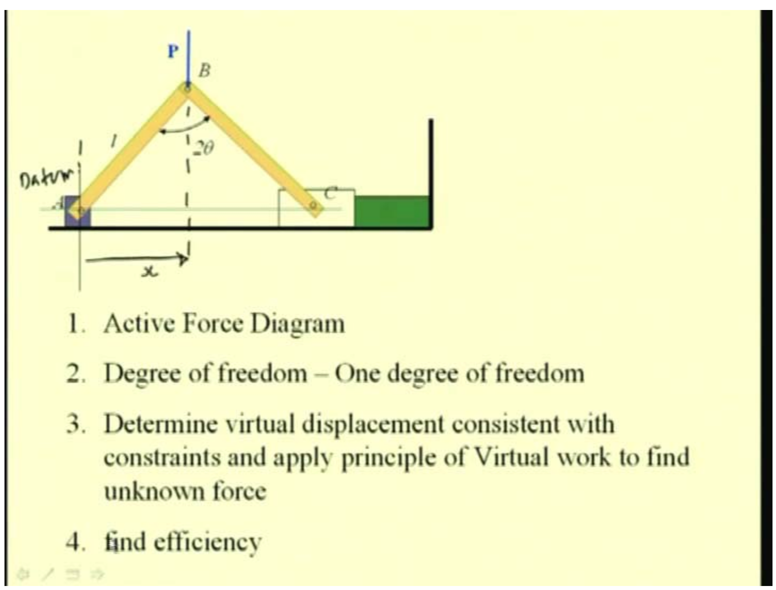

First, let us construct what is called as the active force diagram where we will consider all the forces that will do the work when a virtual displacement is given. So you have to remember that is case of real systems, you have to also consider the frictional forces that exist in the active force diagram because these frictional forces cause a negative work. Then obviously, we have to determine the degree of the freedom. In this case, we see that this system is a single degree of freedom system that means the position of these various links AB, BC the position of this ram or the position of this joint B can be uniquely defined if we specify this angle.

Let it be 2 theta or we can take another coordinate; the coordinate of this B from this datum. We define this as our datum. Then if we define this x then also the configuration of the various links, the position of this ram can be determined.

We see that this system is clearly one degree freedom system. We determine the virtual displacement consistent with the constraints and apply the principle of virtual work to find the unknown forces. In this case, we are interested to find the efficiency. Once we have found the unknown force or in this case the frictional force, we can find the efficiency of the system.

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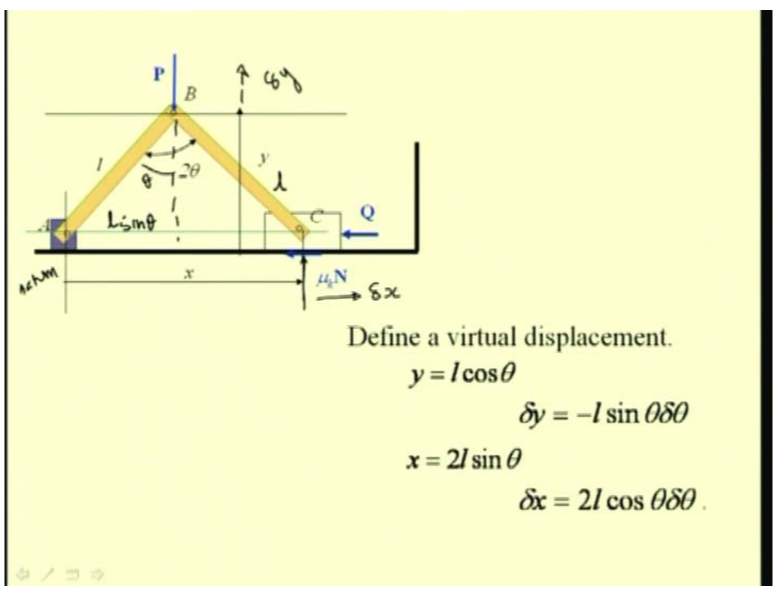

This diagram shows all the forces that are doing work.

This is the active force diagram for this problem. We have this force P that is applied at this joint B which is the input force. We have the reaction Q or the force of resistance from the wooden block and the frictional force which is equal to mu$_k$ N.

If we consider that N is the normal reaction at this point, then mu_k N is the frictional force which does negative work. This has to be also taken in the virtual work displacement. Here we are defining all the displacement in terms of this datum and for the y direction this is the datum. So, the position of this joint B is given by y and this gives also the positive virtual displacement for the joint B. This dimension x gives the location of this point C where these forces are acting. The direction gives the positive virtual displacement delta x.

Let us try to write these dimensions; that are y and x in terms of this theta so as to relate the various virtual displacements. We write first y; this is equal to l times cos theta because this is the half angle. By symmetry we are assuming that these links are of equal length. So, by symmetry this is theta and so we have the height y as l cos theta. Differentiating this equation, we get the virtual displacement delta y as minus l sin theta delta theta. This negative sign can be physically correlated that when this link expands or this theta increases, this point P comes down. The virtual displacement delta y is in the negative direction that is what this negative sign tells. So, both we can verify from the mathematical differentiation as well as the physical correlation the relevance of this negative sign.

Let us now write the virtual displacement for the joint C. We have this value x is equal to two l sin theta because this displacement is l sin theta and these two are having same link lengths l and l. So, we have the total distance x as two l sin theta. If we differentiate this, we have delta x equal to two l cos theta delta theta. Again we see that this delta x and delta theta are positively correlated. In the sense, if this theta increases or the links expand this C moves in the forward x direction, or the delta x the virtual displacement is in the positive direction. This kind of a physical correlation has to be done in order to check whether the virtual displacements have been correctly written.

We have these virtual displacements delta y and delta x.

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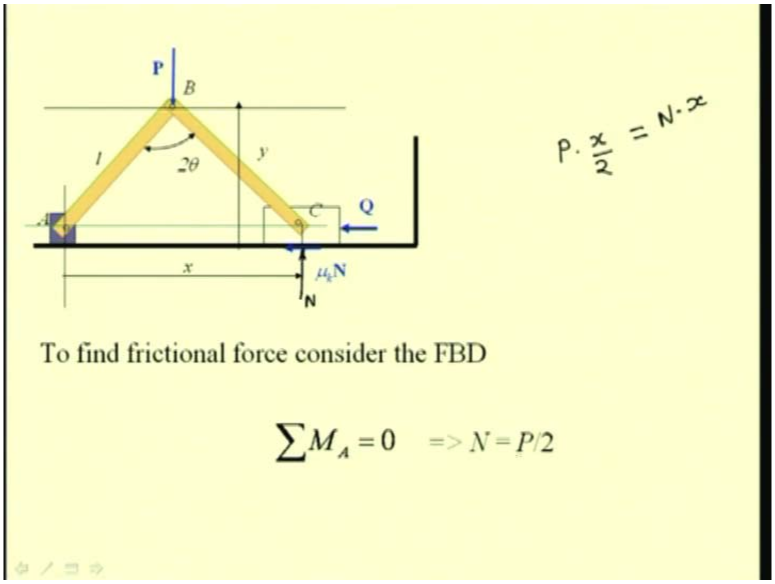

In order to find this frictional force that is mu$_k$ N which is the unknown force, we have to consider the free body diagram and solve by considering the equilibrium equations. We have already seen that when we have friction in the system, we have to go by our traditional methods of equilibrium to solve for the unknown because the virtual method work does not give any additional advantage.

We determine this frictional force mu$_k$ by considering the free body diagram and since we are interested to determine only this mu$_k$, let us consider the moment about this point A and equate it to 0. We have N is equal to P by 2 because these two forces are passing through this joint A. So, they do not have any moment component.

We have, when we sum this and equate it to 0, we have P times this distance that is x by 2 is equal to the normal reaction N times x. From this, we have this normal reaction N as P by 2. Once we know this normal reaction, we can determine the frictional force as mu$_k$ times N.

Now we know this unknown force F.

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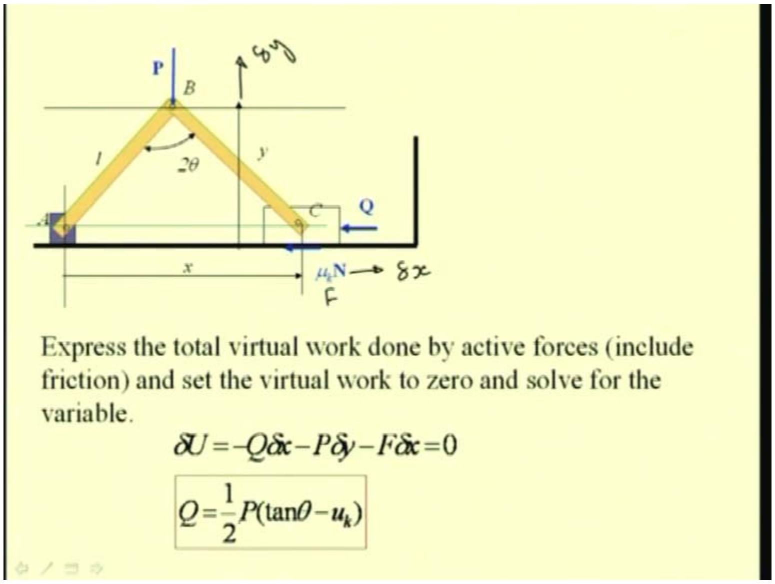

So now we can write the virtual work done by all these forces that is this input force P, this resistance force Q and the frictional force mu_k N and the total virtual work, we equate it to zero to solve for the variable. We have delta U the total virtual work as minus Q delta x because the positive virtual displacement for x is in this direction. This force Q is opposite to this positive virtual displacement delta x; so, we have minus Q delta x and this P is also in the opposite direction of the positive virtual displacement delta y; so we have minus P times delta y. This frictional force F is also in the negative sense with respect to the direction of this positive virtual displacement delta x; so we have minus F times delta x, equal to zero.

Let us now write the expressions for delta x delta y and then solve to get this force Q which is the unknown force. We have it as 1 by 2 times P times tan theta minus mu_k or the coefficient of kinetic friction. Now we know this force Q which was the unknown force.

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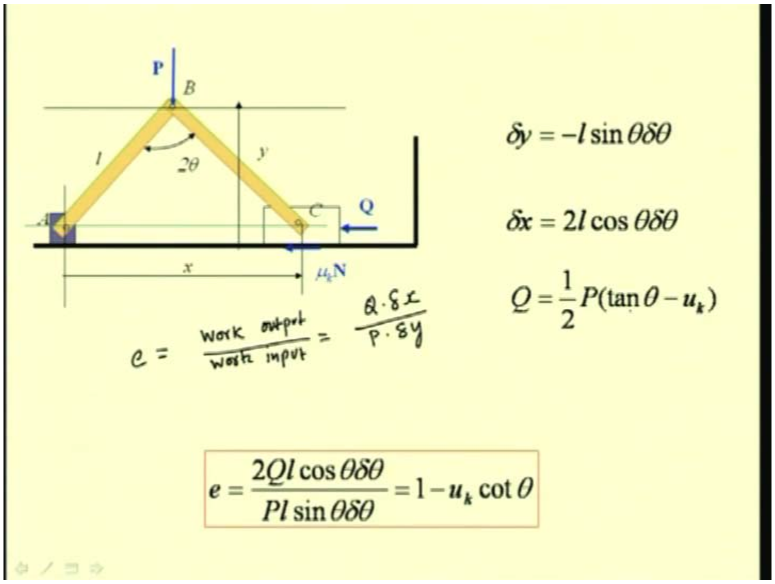

Let us try to write the efficiency.

These are the values of delta y delta x and Q that we have just found out. We can write efficiency e as the work output divided by the work input which is equal to the work done in the compaction; that is Q times delta x divided by the input work which is P times delta y. If we substitute these values, that is delta x delta y and for Q we have already found it to be 1 by 2 P times tan theta minus mu_k.

We have this as one minus mu_k times cot theta. This problem illustrated how this method of virtual work can be used to both solve for the equilibrium or we can determine some unknown forces for the equilibrium to exist. Also, it is very helpful in finding the efficiency of a real system where friction is involved.