Engineering Mechanics

Prof. U. S. Dixit

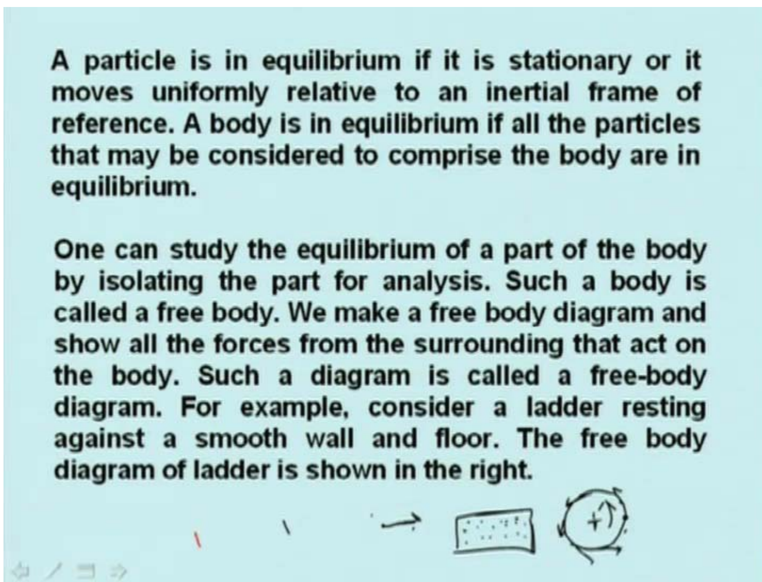
Department of Mechanical Engineering

Indian Institute of Technology, Guwahati

Module 1 Lecture 2
Equations of Equilibrium

Now we will discuss the equations of equilibrium; we have discussed that on a body forces and moments act.

(Refer Slide Time: 01:17)



Moment can be called as a generalized force. It has a unit of Newton meter so moment times the angular displacement gives the unit of work. Similarly, the force times the displacement gives the unit of work. Moment can also be called a generalized force. We know that moment is produced when there is a force present, that there is a force, you take its moment about a point O. So, the fundamental concept is that force, there are forces acting and therefore they produce the moment. It is possible to replace a system of forces by some resultant force acting about at a point and moment.

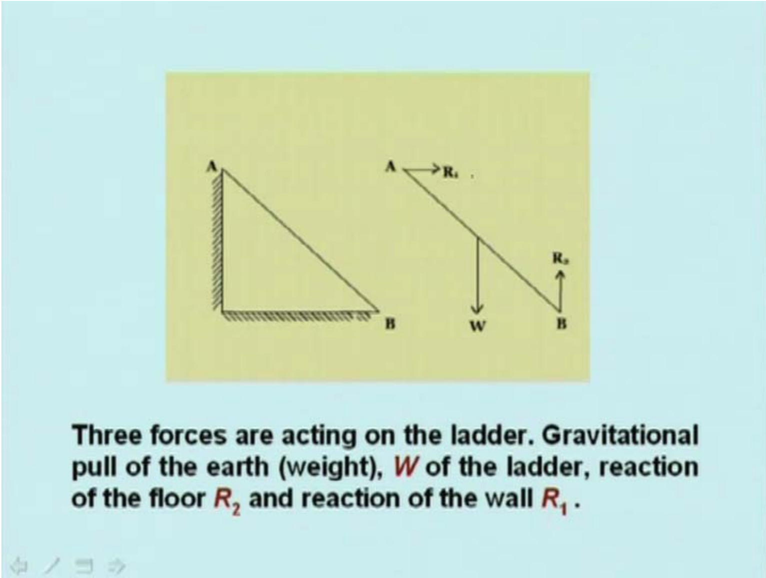
Now, we will discuss about what are the basic equations of equilibrium, which can help us to find out the relation between the forces and moments, when the body is in equilibrium. What is a particle? We have already discussed in the previous lecture that a particle does not have a size but it may have mass. So a particle is just like a point, which is not having a size. Therefore, a particle is in equilibrium, if it is stationary or it moves uniformly relative to an inertial reference frame, and a body is in equilibrium if all the particles that make the body maybe considered in equilibrium.

If this particle is moving with a constant velocity then it is in equilibrium. Similarly, there is a lock, there are a number of particles, in fact, an infinite number of particles, all of them are moving with a constant velocity. Therefore, this body is in equilibrium. However, it is not true for a rotating body. If this disc is there and it is revolving about a perpendicular axis, then if we consider a typical particle here, this particle is not moving with a constant velocity because the direction of its velocity keeps changing, although the magnitude may remain same if the angular velocity is the same. Therefore, naturally, this particle is not in equilibrium, although the particles which are at the center are in equilibrium, it is not moving; therefore, this body cannot be considered in equilibrium because all the particles are not in equilibrium.

So a body is in equilibrium, if all the particles that maybe considered to comprise the body are in equilibrium. Now, one can study the equilibrium of a part of the body whether this part is in equilibrium or not, by isolating that part for the analysis. Such a body will be called a free body.

We make a free body diagram and then show all the forces from the surrounding that act on the body, then that diagram is called a free body diagram.

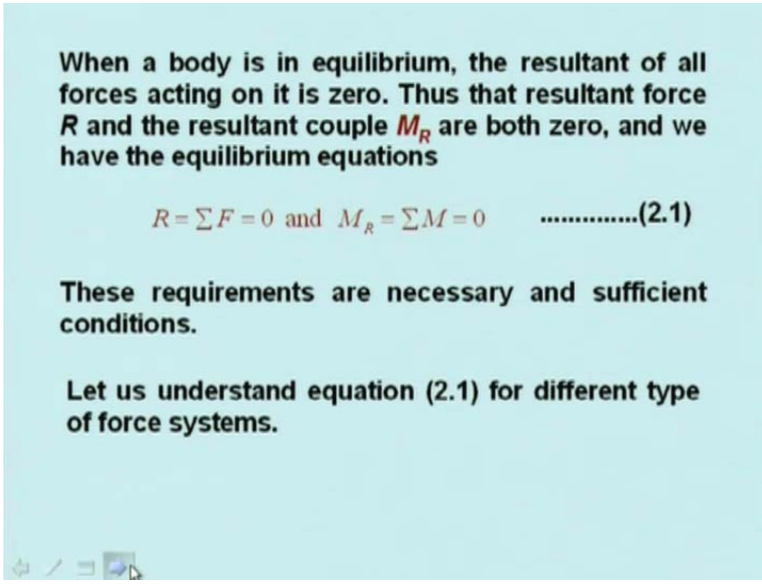
(Refer Slide Time: 05:14)



For example, if you consider a ladder AB which is resting against the wall and against the floor. Here, in this case, it is assumed that they are smooth; there may be friction also does not matter. This ladder AB is shown here separated from the wall and floor. Wall and floor have not been shown; we only concentrate our attention on the body AB. We only view these things and then we see what are the forces acting on this ladder? One is its own weight due to gravity; therefore, W is the force passing through the center of gravity. Then you have the reaction of the floor, that is, R_2; that is reaction of the wall R_1. These three forces are acting on the ladder. This is a free body diagram of the complete ladder.

I could have made the free body diagram of half the ladder that could also have been a free body diagram. Like that, we make the free body diagram, it becomes very easy to analyze that what are the forces acting. You see only the weight here. I am not aware that this floor is putting a reaction here because if the floor puts a reaction here, this also puts equal and opposite reaction on that things, so free body but when we separate that, we only see that this is R_2.

(Refer Slide Time: 06:52)

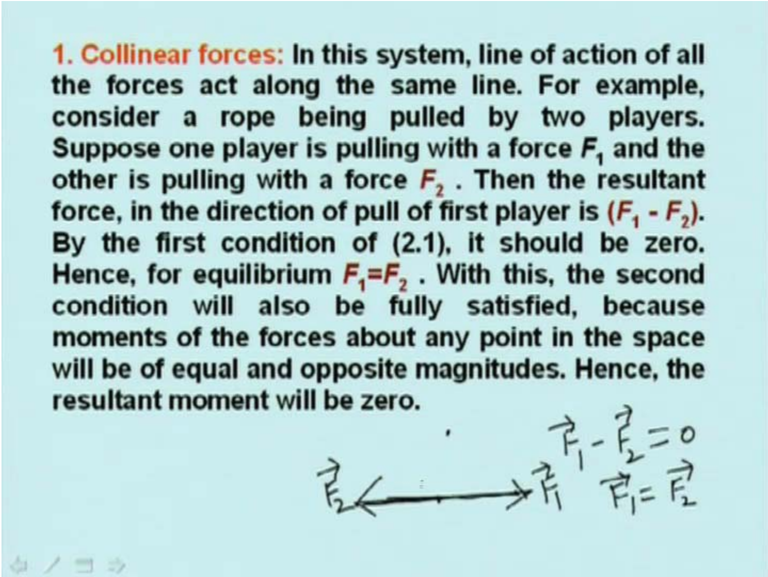


Now when a body is in equilibrium, the resultant of all forces acting on it is 0. Thus, the resultant force R and also the resultant couple M_R both will be 0. Therefore, we have the equilibrium equations as R is equal to sigma F equal to 0 and M_R is equal to sigma M this is also equal to 0.

These requirements are necessary and sufficient conditions. They are necessary, that means, if a body has to be in equilibrium then R must be 0 and resultant moment also must be 0; this is necessary. If R is 0, and then moment is 0, then the body has to be in equilibrium; that means these conditions are sufficient. We do not require any other condition.

Let us understand equation 2.1 for different types of force system. What type of forces can act on the body?

(Refer Slide Time: 07:57)

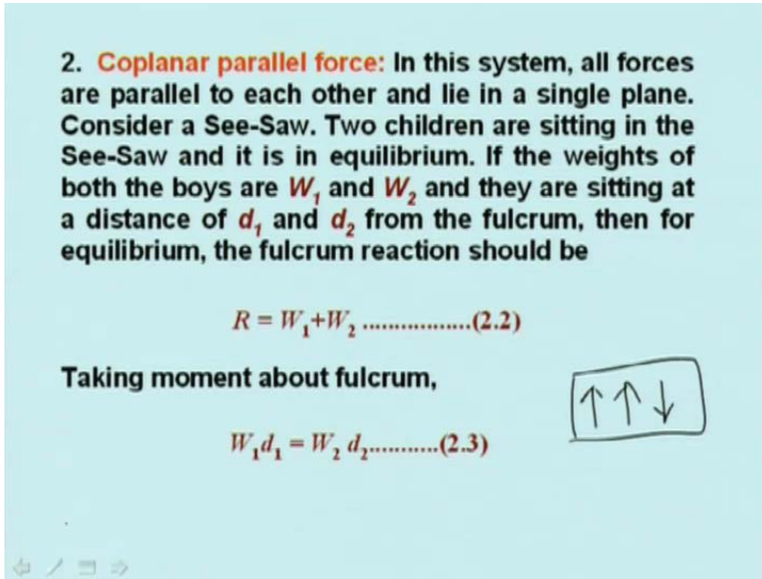


One is the system of collinear forces. In this system, line of action of all the forces act along the same line. For example, consider a rope being pulled by two players. Suppose this is a rope and it is being pulled; one player is pulling like this, other is pulling like that. In this case, that line of action of the two forces is same. The forces act along the same line; so, it is collinear.

Now this force may be F_1, this may be F_2. Then the resultant force in the direction of pull of first player is F_1 minus F_2. F_1 minus F_2 is the resultant force; it should be 0. Hence, for the equilibrium F_1 is equal to F_2.

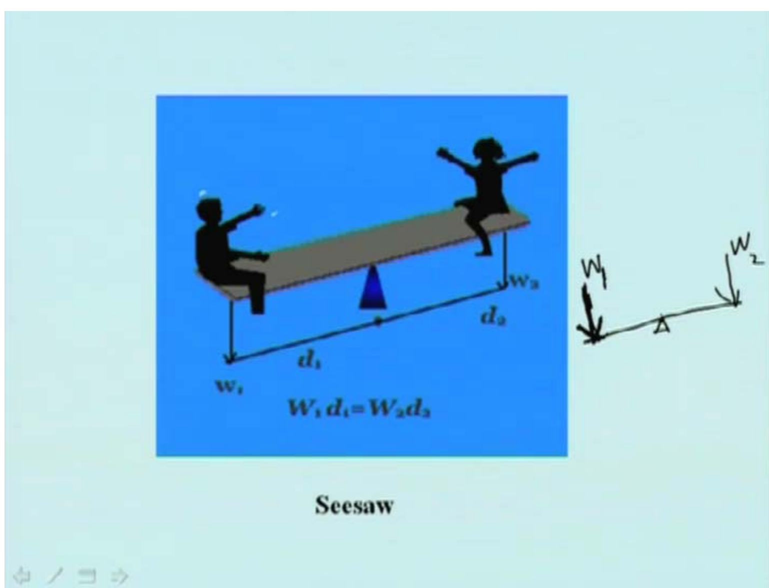
With this the second condition that is moment should also be 0 is fully satisfied because moment of the forces about any point in the space will be of equal and opposite magnitude .Suppose rotate the moment about this point, then also it is equal and opposite. Hence, the resultant moment will be 0. This is the thing.

(Refer Slide Time: 09:20)



Then we talk about coplanar parallel forces. Coplanar means forces acting on a particular plane. We can make a plane on that only and they are parallel. Like you have a plane on which the forces are acting like this, all forces are parallel to each other and lie in a single plane.

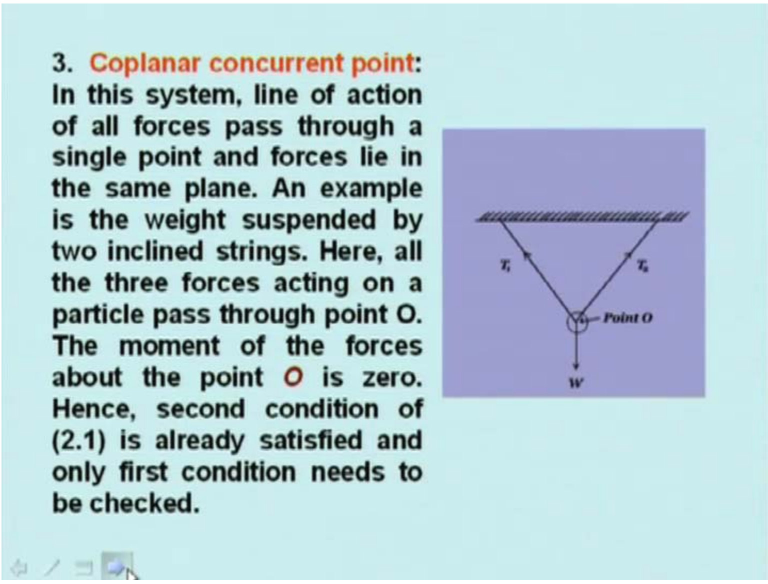
(Refer Slide Time: 09:46)



For example, you see this seesaw here; one boy and one girl, both are sitting on the opposite sides of seesaw. If we just consider the equilibrium of that seesaw; therefore, on the equilibrium, if we assume that seesaw is not having its own weight and then there is a fulcrum here. In that case, boy's weight is W_1; so, it is putting a force on that. That is, seesaw is putting a reaction on the boy. So, if we make a free body of diagram of seesaw, therefore this is acting, in turn the boy is providing a force W_1 on the seesaw. That is the thing so this is W_1. Similarly, the girl is putting a force W_2 on seesaw. Then we will have at the fulcrum the reaction force of the fulcrum. But these two forces are parallel; that means, W_1 and W_2 is vertical, therefore, the reaction should also be vertical. This system is coplanar parallel forces. If the children are sitting at a distance of W_1 and W_2 and they are sitting at a distance of d_1 and d_2 from the fulcrum, then for equilibrium the fulcrum reaction should be R is equal to W_1 plus W_2. That reaction of the fulcrum will be of that its magnitude will be R is equal to W_1 plus W_2.

If you take the moment about fulcrum that is W_1 into d_1 is equal to W_2 into d_2, this must also be satisfied. Otherwise, seesaw will not be in equilibrium. Therefore, this is the condition W_1 d_1, W_2 d_2 is equal to this one. If a child's weight is less, he can move a distance further and then he can balance.

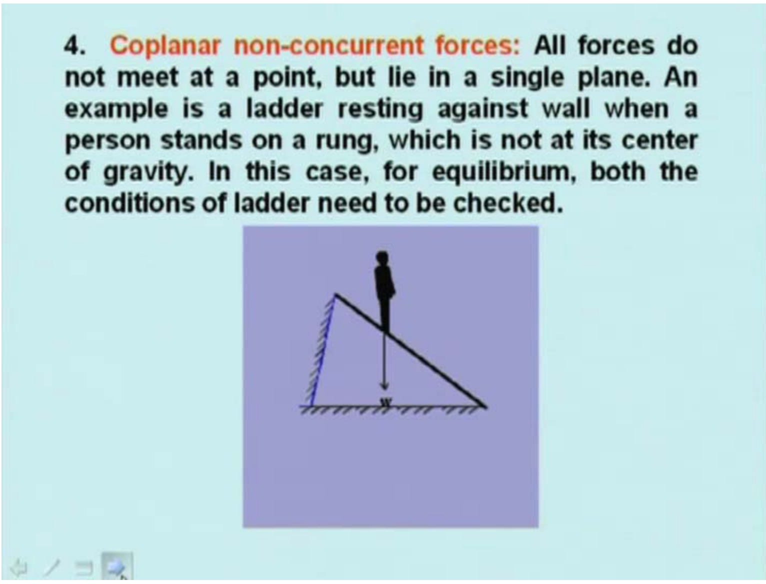
(Refer Slide Time: 12:25)



Then discuss about the coplanar and concurrent point. In this system, line of action of all forces passes through a single point and forces lie in the same plane. An example is the weight suspended by two inclined strings. In this case, all the forces are in plane. If we can construct a vertical plane which passes through the strings T_1 and T_2 and in that case, we have got that weight W is acting here and this is point O.

In that case, if this is the weight suspended by two inclined systems, all the forces are passing through point O. Therefore, they are concurrent. The moment of the forces about the point O is 0 there. Hence, second condition is already satisfied and only now, you can use the first condition for this one. You have constraint on the forces given by the first this one that vertical component of the forces should be 0 and horizontal component of the forces should be 0. So these can be found out.

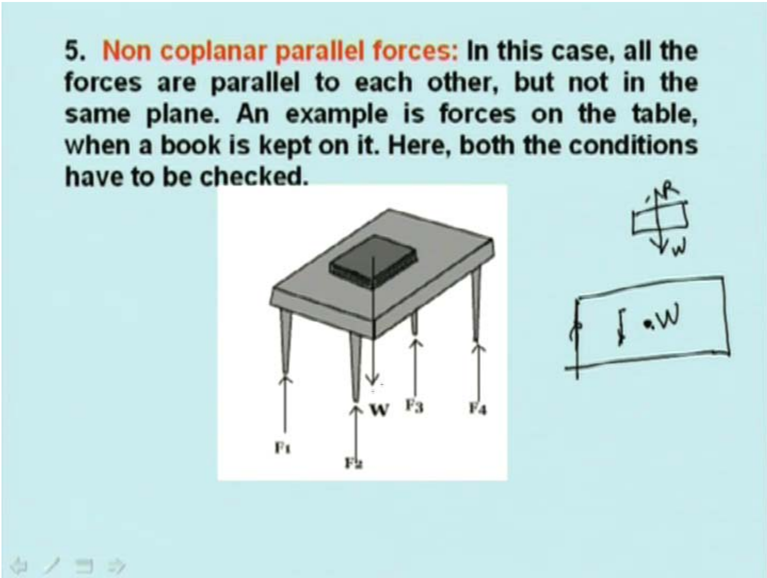
(Refer Slide Time: 13:36)



The fourth condition is that coplanar non-concurrent forces can be at one plane but they need not pass through a point. All forces do not meet a point but lie in a single plane. An example is a ladder resting against the wall and a person stands on a rung which is not at the center of gravity. In this case, both the conditions for the ladder need to be checked. So that in case you have center of gravity of the ladder maybe different. Therefore, ladder will also have some force acting.

If we make the free body diagram of the ladder, we have to show the reactions of the wall, then reaction of this thing, then at the same time this weight, and then we have to check both the conditions - that is resultant force should be 0 and the moment should be 0.

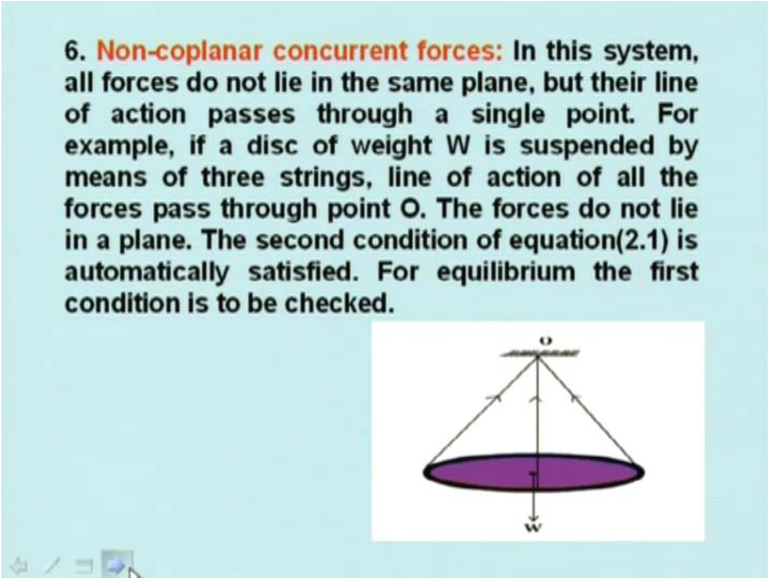
(Refer Slide Time: 14:45)



Then we have non-coplanar parallel forces. In this case all the forces are parallel to each other but not in the same plane. An example is force on the table when a book is kept on it. Here if you see on the table, you know that if you make a free body diagram of the table, I am just drawing this one. Let us say, the top view of table, here one has kept a book but you replace the book then you will be getting that the force applied by the book that is basically W on the table; because if we first consider the equilibrium of the book, then on the book there is a weight, and then there is a reaction of the table.

For equilibrium of the book, reaction on the book must be equal to W. Therefore, the book also applies load W here. At the same time, similarly, you have wall reactions F_1, F_2, F_3, F_4 acting on this one and then there is the weight of the table itself. So all these forces are there but they are not in a single plane. You cannot make a single plane in which all the forces are parallel because all the forces are vertical. So that type of situation is also there. Here you have to check for both the conditions that means sum of all the forces should be 0 at the same time the moment should also be 0.

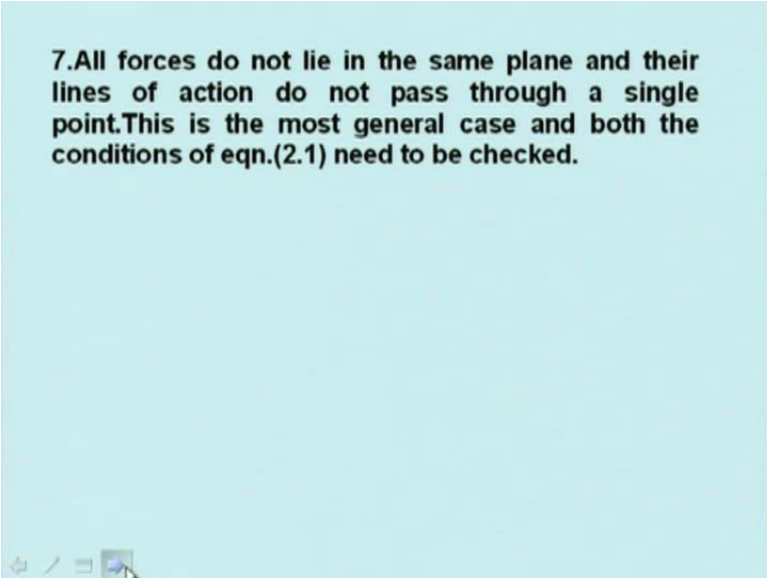
(Refer Slide Time: 16:17)



Then we may have a situation that there is a non-coplanar concurrent forces. In this system, all forces do not lie in the same plane but their line of action passes through a single point. For example, consider a weight W suspended by means of three strings, line of action of all the forces pass through point O. The forces do not lie in a plane. Therefore, the second condition of equation is automatically satisfied because all the forces are passing through point O.

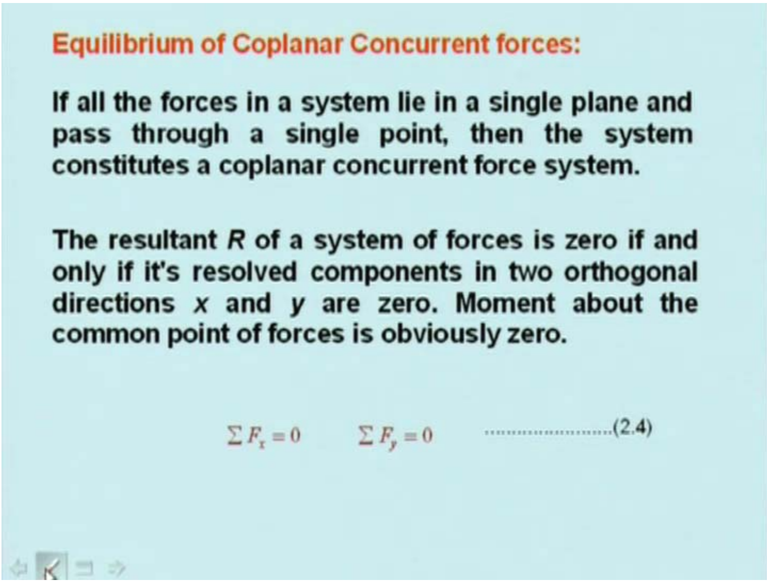
For equilibrium, the first condition has to be checked; that means, the resultant of the forces must be 0. Therefore, if you resolve the forces into three components, xyz, then the resolved sum of the resolved components in x direction should be 0, sum of the resolved components in y direction should be 0 and sum of the resolved components in the z direction should be 0.

(Refer Slide Time: 17:36)



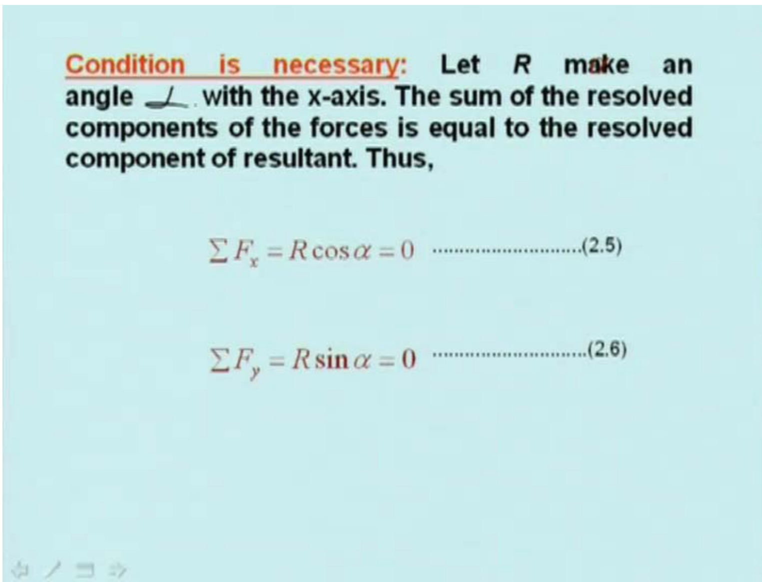
The last condition is that all forces do not lie in the same plane and their lines of action do not pass through a single point; they may not be parallel; this is the most general case and both the conditions of equation 2.1 that needs to be checked. That means resultant force must be 0 and its moment should also be 0.

(Refer Slide Time: 18:02)



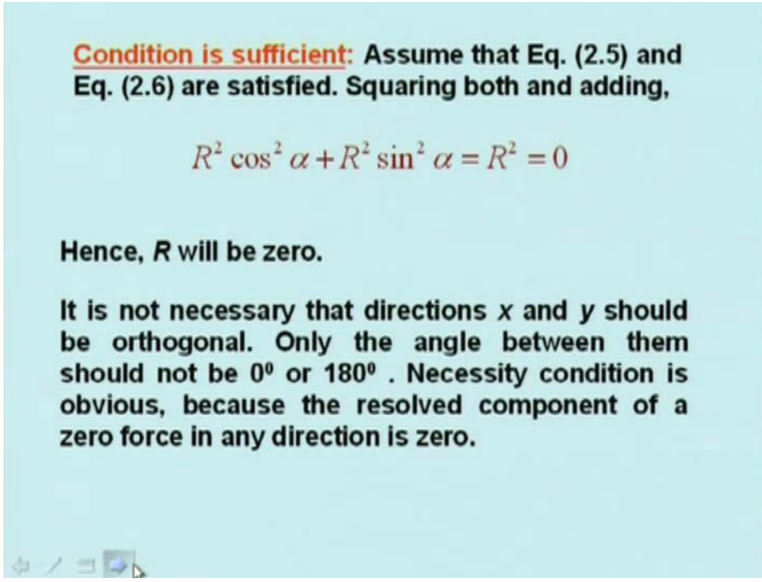
Let us discuss the equilibrium of coplanar concurrent forces. If all the forces in a system lie in a single plane and pass through the single point then the system constitutes a coplanar concurrent force system. The resultant R of a system of forces is 0 if and only if its resolved components in two orthogonal directions x and y are 0. Moment about the common point of forces is obviously 0 because the forces are passing through the point. So if you take the moment about the point O then they will be 0. Therefore, the first condition has to be seen that you have to see sigma F_x is equal to 0 sigma F_y is equal to 0. These two conditions have to be satisfied.

(Refer Slide Time: 19:04)



Now these conditions are necessary; that means, if the body is in equilibrium, then sigma F_x and sigma F_y has to be 0. Assume that there is a force R acting which makes an angle alpha with the x-axis. The sum of the resolved components of the forces is equal to the resolved component of resultant. Thus, sigma F_x is equal to R times cos alpha which is equal to 0 and sigma F_y is equal to R times sin alpha that also should be 0 because R is 0. Therefore, this condition is necessary because if R is 0 then naturally the resolved component should also be 0, sum of the resolved components that means sigma F_x is 0 and sigma F_y is 0.

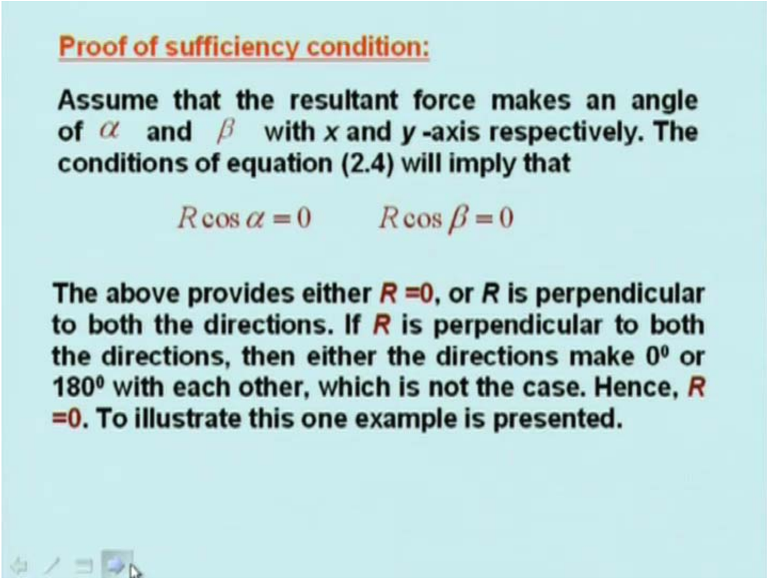
(Refer Slide Time: 20:02)



This condition is sufficient also because assume that equation… these are satisfied. That means for certain case R cos alpha is 0, R sin alpha is 0, which implies that if you square both and add, then R square cos square alpha plus R square sin square alpha is equal to R square that must be 0. Therefore, hence R will be 0.

It is not necessary that directions x and y should be orthogonal; only the angle between them should not be 0 degree or 180 degree. Although, it is always that when we resolved components we generally take orthogonal direction. Even if that angle is different, necessity condition is obvious because the resolved component of a 0 force in any direction is 0.

(Refer Slide Time: 21:08)



Assume that the resultant force makes an angle of alpha and beta with x and y-axis respectively. The condition of equation 2.4 will imply that R cos alpha equal to 0 and R cos beta equal to 0. This provides either R is equal to 0 or R is perpendicular to both the direction; that means either R is equal to 0 or cos alpha is equal to 0; if cos alpha is equal to 0 then alpha is 90 degree. Similarly, here R cos beta equal to 0, that means beta is equal to 90 degree. Therefore, if R is perpendicular to both the directions, then either the directions make 0 degree or 180 degree with each other which is not the case; hence R is equal to 0.

(Refer Slide Time: 22:10)

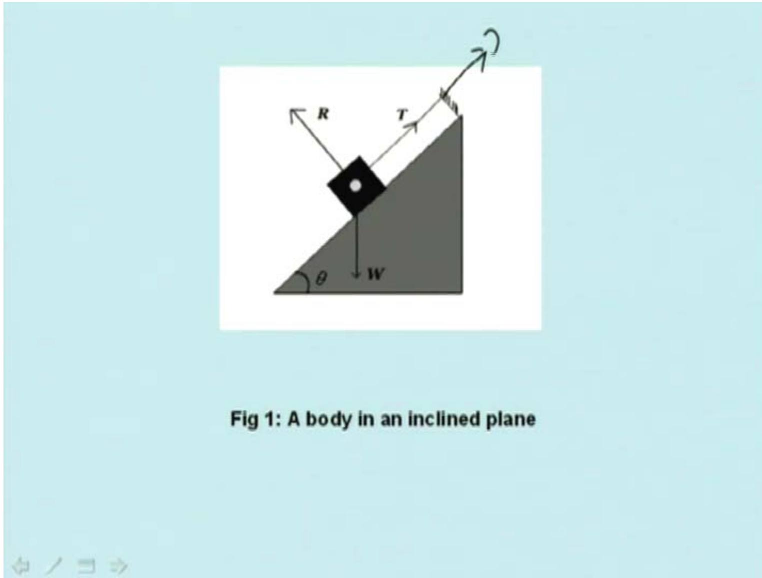
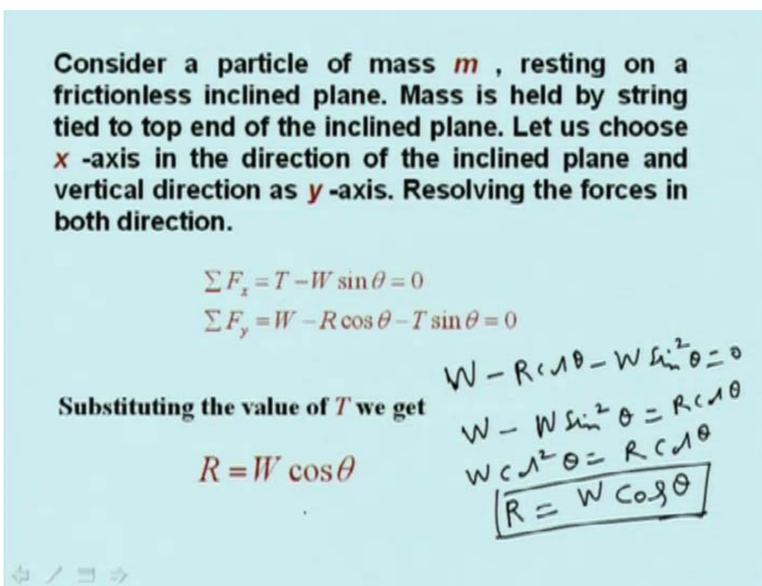


Fig 1: A body in an inclined plane

To illustrate this point, I present one example. Let us consider a body in an inclined plane. This body is kept here and it is acting on an inclined plane. R is this string and string tension is T and this is R here.

(Refer Slide Time: 22:34)



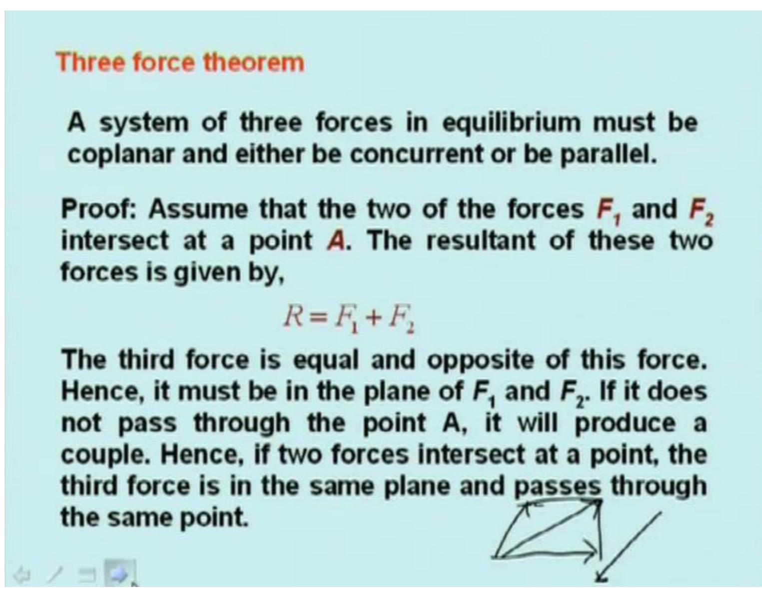
In this case, consider a particle of mass m, weight W, resting on a frictionless inclined plane. Now, mass is held by string tied to top end of the inclined plane. Let us choose x-axis in the direction of the inclined plane that means x-axis is in this direction but the y-axis is vertical. So y-axis maybe like this; then what happens?

If we resolve the forces in both the directions then sigma F_x resolving the forces in x direction we get T minus W sin theta is equal to 0. Resolving the forces in y direction, we get W minus R cos theta minus T sin theta is equal to 0.

If we substitute the value of T from the first equation into the second equation we get W minus R cos theta minus W sin squared theta equal to 0. This gives us W minus W sin square theta is equal to R cos theta. This gives you W cos square theta is equal to R cos theta. Therefore, R is equal to W cos theta. Therefore we have got R is equal to W cos theta, which we know that is correct; you have been doing this problem so many times. So resultant R will in this case be naturally that W cos theta.

So, here we see that we have resolved in two different directions, although, most of the time we will be resolving the forces in orthogonal directions.

(Refer Slide Time: 24:39)



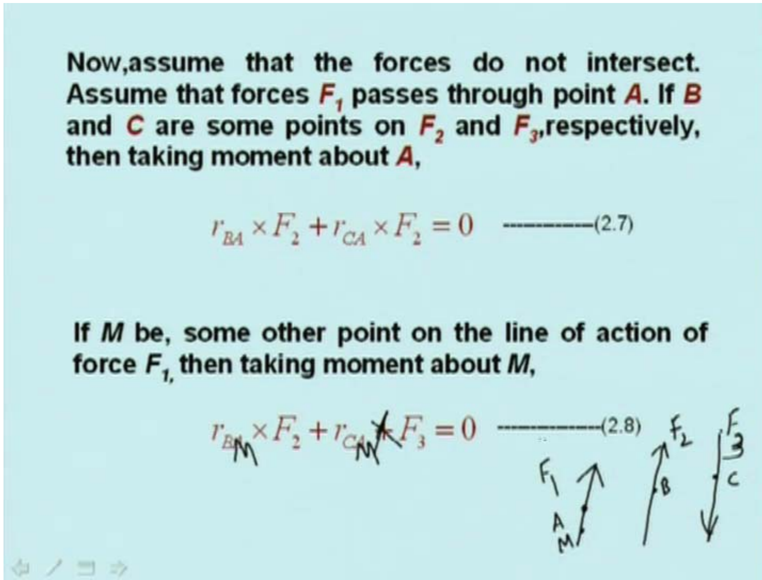
If the three forces are acting on a body and then the system of three forces in equilibrium, must be coplanar, and either be concurrent or be parallel - that is a three force theorem. If out of the two forces, if just one force is acting on a body, then the body cannot be kept in equilibrium because just one forces, so resultant cannot be 0.

If the two forces are acting on the body, they must be equal and opposite and they also must be collinear. If they are not collinear then they will produce movement. Therefore two force theorem is that if there are two forces acting on the body they must be collinear.

Now we present the three force theorem: a system of three forces in equilibrium must be coplanar and either be concurrent or be parallel. Now proof of this is as follows: assume that the two of the forces F_1 and F_2 intersect at a point A. Supposing they are not parallel and two forces can intersect at a point A. Then the resultant of these two forces is given by R is equal to F_1 plus $F_{2.}$ So resultant of these are the two vectors, therefore, it must be in the same plane because one can construct a parallelogram which lies in that plane. Third force is equal and opposite of this force. It must be in the planar containing F_1 and F_2.

If it does not pass through the point A then it will produce a couple. Hence, if two forces intersect at a point, the third force is in the same point and passes through the same point; that means, the third point should also intersect. Otherwise, if you have two forces they are there and you take their resultant. Now the third force maybe here then it will produce the couple. Therefore, if the forces are non-parallel but they are in the same plane then they must be concurrent.

(Refer Slide Time: 27:12)



Now, assume that the forces do not intersect. If they do not intersect, they must be parallel. Assume that force F_1 passes through point A, if B and C are some points on F_2 and F_3 respectively, then taking moment about A.

If we have a force $F_{1,}$ this may be F_2 and let us say this is F_3. Now this point is A, here you have got point B, then here you have got point C. Let us take the moment about A, moment of the force F_1 about A is 0, moment of force F_2 about this is r_{BA} plus F_2 plus r_{CA} plus F_2 is equal to 0. If M be some other point on the line of action of force F_1, then taking moment about M… suppose, there is M is this one, then we can have r_{BM} into F_2 plus r_{CM} into F_3 that is equal to 0.

(Refer Slide Time: 29:00)

Subtracting (2.8) from (2.7) ,

$$r_{AM} \times (F_2 + F_3) = 0$$

Since r_{am} is in the direction of F_1 , we can write

$$F_1 \times (F_2 + F_3) = 0$$

This implies that F_1 lies in the plane containing F_2 and F_3.Since they do not interact with each other they , they must be parallel.

Subtracting this from this one you get r_{AM} cross F_2 plus F_3 is equal to 0 because about this one and this one. Therefore, one can always say r_{BM}, so this is r_{AM} cross F_2 plus F_3 is equal to F_2 plus F_3 is equal to 0. Since r_{AM} is in the direction of F_1, we can write F_1 cross F_2 is equal to 0. This implies that F_1 lies in the plane containing F_2 and F_3. Since they do not interact with each other, they must be parallel

(Refer Slide Time: 30:11)

Lami's theorem: If three forces coplanar and concurrent forces acting on a particle keep it in equilibrium, then each force is proportional to the sine of the angle between the other two and the constant of proportionality is the same.

For the system shown in figure 2,

$$\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$$

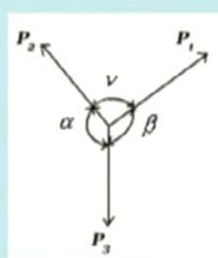
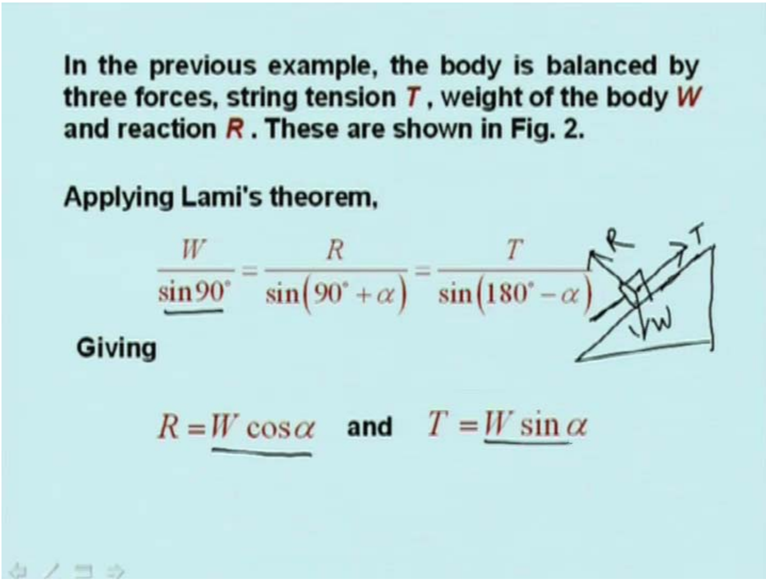


Fig 2: Coplanar and concurrent forces

We discuss about Lami's theorem: if the three forces are coplanar and concurrent, if three forces coplanar and concurrent forces acting on a particle keep it in equilibrium, then each force is proportional to the sine of the angle between the other two and the constant of proportionality is the same. For the system, shown in this one figure, P_1 divided by sin alpha P_2 divided by sin beta P_3 divided by sin gamma must hold good.

This can be easily proved that if you have that forces P_1, P_2, P_3. So here P_1 divided by sin alpha P_2 divided by sin beta P_3 divided by sin gamma must be holding good.

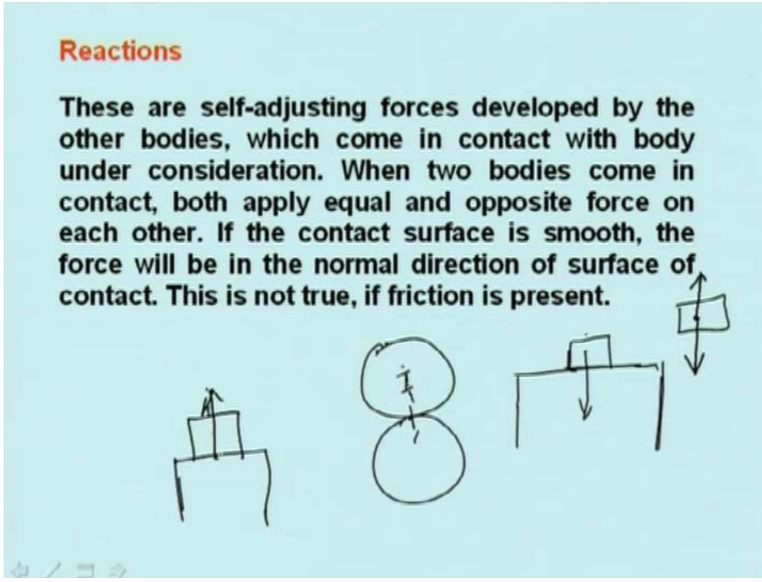
(Refer Slide Time: 31:15)



If we consider the same problem of an inclined block and this is the block kept like this. This is attached by string T, this is W and this is R. If you apply Lami's theorem, then W divided by the angle between R and T that is W divided by sin 90, then R divided by the angle between W and T that is sin 90 plus alpha.

Then T divided by sin 180 minus alpha that means T divided by this is a this is the thing between R and this one. Naturally, this angle must be T divided by angle between this and this. Therefore this is sin 180 minus alpha. Therefore it gives us R is equal to W cos alpha and T is equal to W sin alpha.

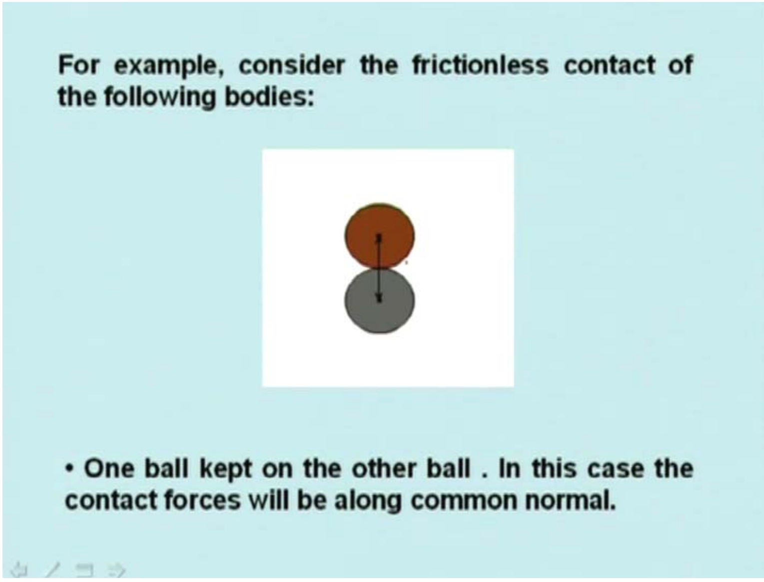
(Refer Slide Time: 32:37)



When we apply the forces, when the body is there it also experiences reactions. These are the self-adjusting forces developed by the other bodies which come in contact with body under consideration. Suppose, you put a body on the other body, then the first body applies some force on the second body; therefore, second body in the reaction supplies the force on the other body. Suppose, you put a weight on a table, this is being pulled by the gravitational force, it has tendency to go down, but as it tries to go, it will compress this table. Therefore, table in turn will apply the force that will be called reaction force. Therefore, body will not move, otherwise the body would have fallen, but here the weight W is of the body is acting like this and the reaction of that table is also like this. Therefore, this balances each other.

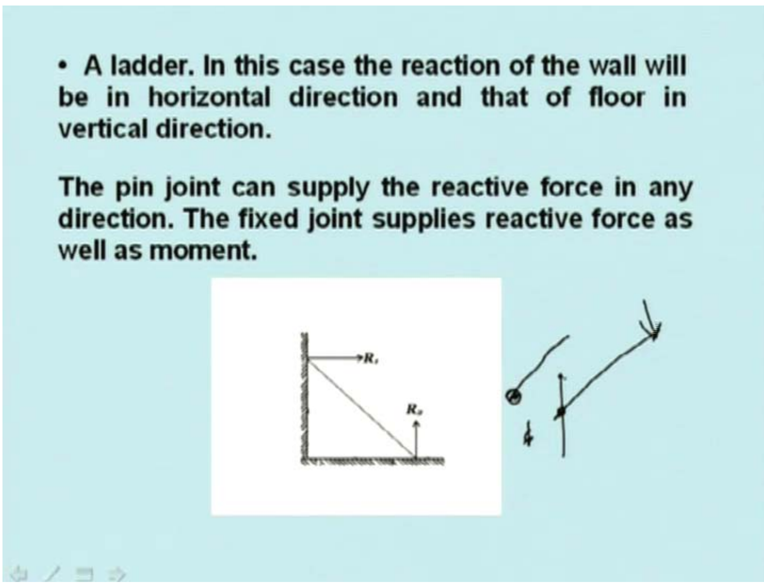
When two bodies come in contact, both apply equal and opposite forces on each other - that comes from Newton's third law. If the contact surface is smooth, the force will be in the normal direction of surface of contact because if the contact is smooth there is no frictional force acting here. Suppose, this is a block so no force can come from the side only the force can be normal. This is not true if friction is present because there will be a frictional force that will also be acting here, but otherwise, if there is no friction, then when the bodies come into contact, then the force has to be normal; otherwise, the body will start moving in the another direction because there is no friction to constrain the motion.

(Refer Slide Time: 35:00)



Consider the frictionless contact of the following two bodies: one ball is kept on the other ball. In this case, the contact force will be along common normal. You have common normal of the balls and the contact force will be along that.

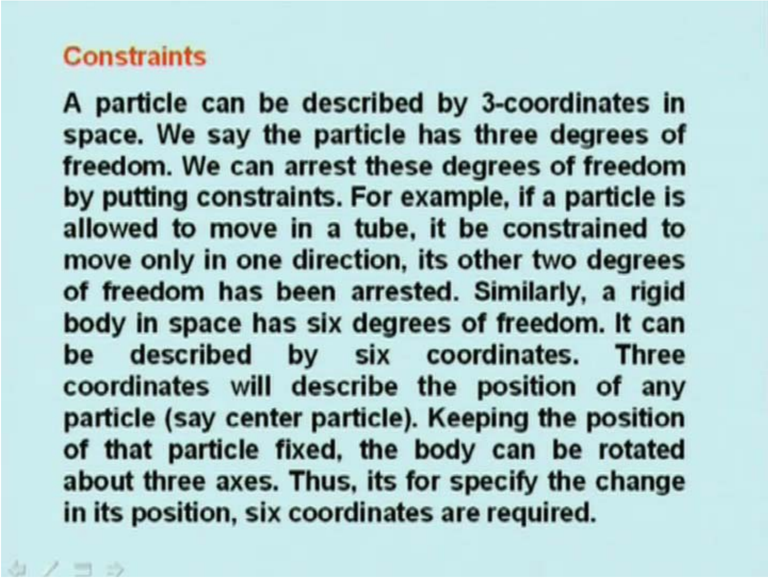
(Refer Slide Time: 35:20)



A ladder example is there. In this case, if I put a ladder, which maybe having some weight also; in this case the reaction of the wall will be in the horizontal direction and that of the floor in vertical direction. So, here, the reaction of the wall is shown like this, R_1 reaction of the floor is in the vertical direction.

Suppose, we just take a pin joint, we take a rod and then we pin it somewhere so that it can rotate. If you have a floor here and on this, we fix up this rod and make a pin. So this is a pin joint. It can supply the reactive forces in any direction. It can constraint the motion in xy, but it cannot prevent the moment. But if you have fixed joint, it gives you reactive forces as well as moment; because it does not allow it to move. In the pin joint, if I put a moment that means if I act a force like this, then you know that the pin is… this link starts rotating.

(Refer Slide Time: 36:58)



We discuss about constraints. Now a particle can be described by three coordinates in a space. If you have say, in any, like Cartesian coordinates system a particle can be described by xyz. So therefore, we say the particle has 3 degrees of freedom. Particle can have some x coordinate, some y coordinate and some z coordinate. We can arrest these degrees of freedom by putting constraints.

We can constrain, for example, suppose we say that particle is allowed to move in a tube, a very thin tube, it is a R tube or on a surface of rod, and it is constrained to move only in one direction;

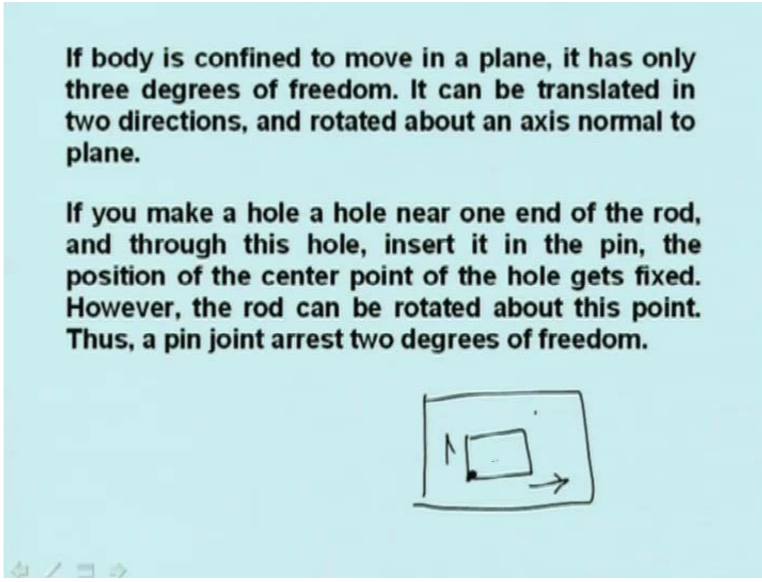
its other two degrees of freedom have been arrested. So we have arrested two degrees of freedom.

Similarly, in a rigid body in space has 6 degrees of freedom because it can be described by 6 coordinates. A rigid body can actually translate, that you know, like this or it can translate like this. (Refer Slide Time: 00:38:22 min) Suppose this is a rigid body, this block can move like this, it can also move in this direction, second y direction, it can also go up and down in the z direction, this is the third direction. But at the same time, the rigid body can rotate about the vertical axis; it can rotate about this axis and this x-axis, it can also rotate about the y-axis. So therefore it has 6 degrees of freedom.

If you want to describe the degrees of freedom of the body, we need to tell we can fix up any point here. Let us say this corner point. We have to say that what is its xyz coordinate, at the same time keeping it fixed; I can still change the angles' orientation. So this way I can keep the body, I can rotate the body like this. So we can still change the orientation, that means, I can still give the rotation about either x y and z axis. So that means a rigid body has got 6 degrees of freedom.

It can be specified by xyz and another three angles. Three coordinates will describe the position of any particles. Say, center particle, keeping the position of that particle is fixed, the body can be rotated about three axis, thus six coordinates are required.

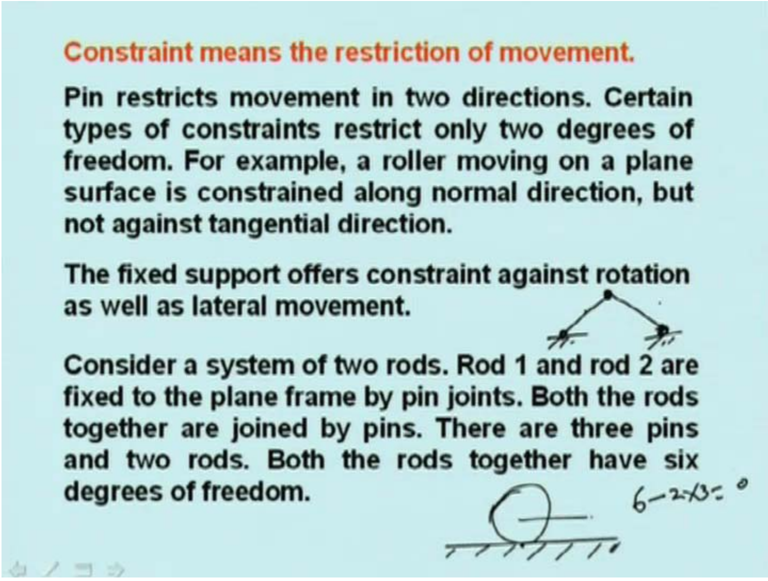
(Refer Slide Time: 40:14)



If a body is confined to move in a plane, it has only 3 degrees of freedom. If we say that body has to move in a plane only, like if we have the blackboard. If I say I am moving my duster but it should be only on the blackboard, then naturally, that it is a motion in a plane; the duster can be moved along either x direction, or y direction or it can be rotated in the plane itself. Therefore, it has got 3 degrees of freedom. Other 3 degrees of freedom you have arrested because you confined it to move in a plane.

If you make a hole near one end of the one rod and through this hole insert a pin. Suppose, you insert a pin on this duster, the position of this point gets fixed, how we hold that duster or rod can be rotated about this point. Thus, a pin joint arrests 2 degrees of freedom in a plane. If you fix up a pin joint in a plane, it is arresting two degrees of freedom.

(Refer Slide Time: 41:28)

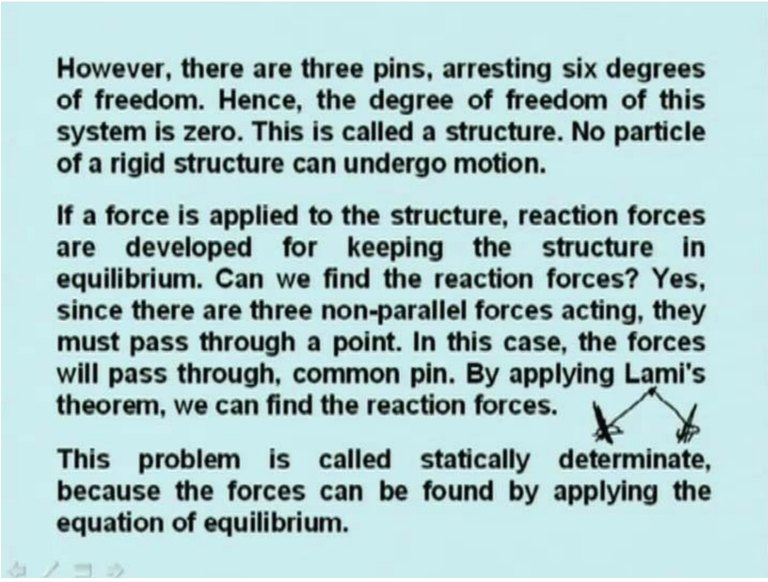


Constraint means the restriction of movement. Pin restricts movement in two directions. Certain types of constraints restrict only 2 degrees of freedom. For example, a roller moving on a plane surface is constrained along normal direction but not against tangential direction. So you can have only 2 degrees or it can be having that only one direction. Suppose, you have like this, now a roller is constrained to move against tangential direction; it is not constrained. It is constrained to move only along tangential direction.

So it cannot move in the normal direction, but it can move in the tangential direction. In the normal direction it cannot move but in tangential direction, it can move. You have arrested 1 degree of freedom. The fixed support offers constraint against rotation as well as lateral movement, so they are arrested. If you have a fixed support, it arrests all 6 degrees of freedom in three dimensions or in two dimensions, it arrests 3 degrees of freedom.

If you consider two rods which are fixed by this one. Let us say that this is the rod, and this is another rod and these are the pin joints (Refer Slide Time: 00:43:19 min). Now in each rod… this degree of freedom of this rod is 3, the degree of freedom of this rod is also 3, but you have 3 pins 1 2 3. Each one has arrested 2 degrees of freedom. So, therefore, you had total 6 degrees of freedom but they have been arrested. So, 2 into 3 that means degree of freedom of this is 0. Therefore, both the rods are having a system of zero. Hence, the degree of freedom of this system will be zero. Therefore, this becomes a structure and no particles of this structure can undergo a motion.

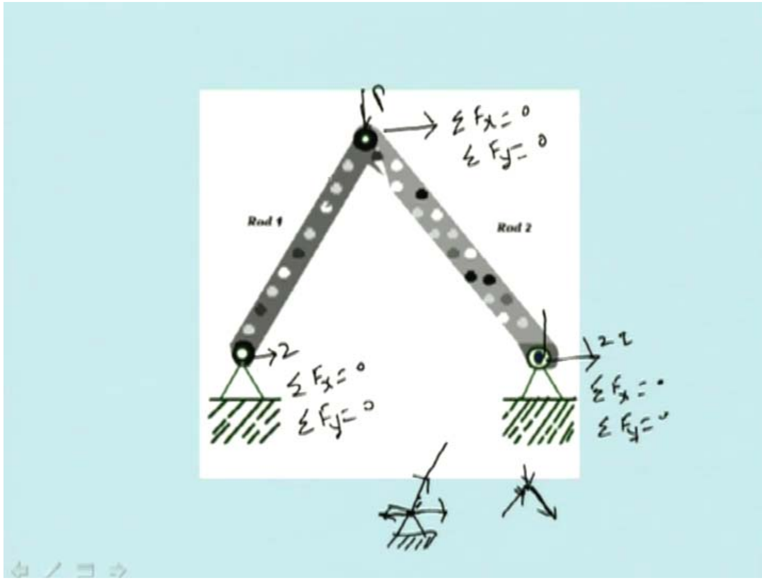
(Refer Slide Time: 44:02)



If a force is applied to the structure, reaction forces are developed for keeping the structure in equilibrium. If any force is applied to this structure, it will move. So therefore, for this structure now can we find out the reaction forces? Yes. Therefore, we can find out this one. If we have on this structure, simple structure like this, in this case this is the pin joint; this is another pin. Can we find out this? Because these are three non-parallel forces on a body and we can find out these reactions.

Here, there are reactions on this pin, that if we consider that there may be some reaction here, then there maybe reaction here, so these forces… in this case, we can find out the reaction forces. The problems which can be found out by statics are called statically determinate because the forces can be found by applying the equation of equilibrium. These things will be taught in detail in the next lecture on truss.

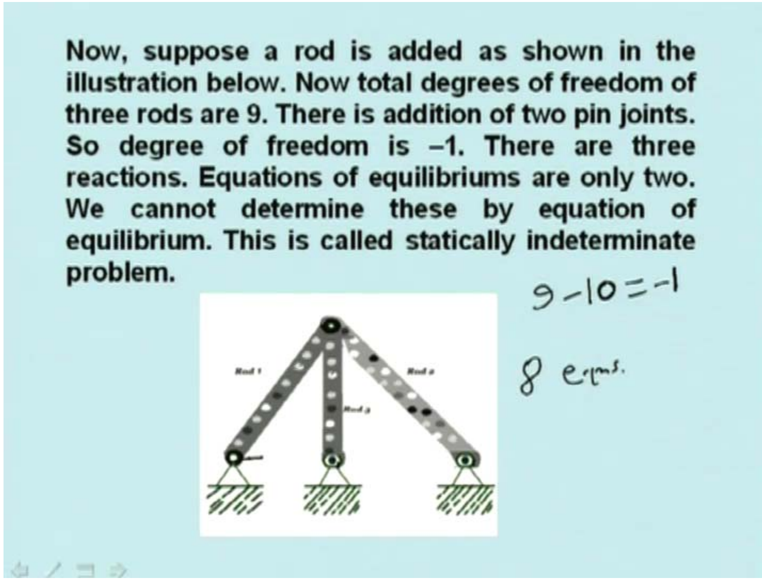
(Refer Slide Time: 46:09)



Here this is the example: this is rod one, this is rod two. If you want to find out the reaction then the rod one here, that you have to see like this, and on this, you have to find out reactions. This is the force acting; on this rod, only two forces are acting. Therefore, they must be collinear that means they must act along the rod. Here, you that know it maybe some force, maybe here and here; the reaction components may be F and maybe other component maybe there. There will be one equation of equilibrium here. Similarly, at this point there are rods; one force is like this and then you may have another one. If there is a force applied here that force will also be shown and these forces will balance. Therefore, you get another equation but then the third one, here also you have forces, and the other component is like this and then you have this one. Actually, you have got see three pins you can write.

You can resolve the components of the forces in two directions and you can find out… say you can get two equations sigma F_x equal to 0 for this two equation. Then two equations sigma F_x equal 0 and sigma F_y equal to 0. Similarly, here sigma F_x equal to 0, sigma F_y equal to 0 and this is at this point. You go here also you can find out sigma F_x equal to 0, sigma F_y equal to 0. So there are basically 6 equations. You have how many unknowns? 2 reactions, x component, y component, to this one another x component, y component, and at the same time here that this force itself and the force on this. So six unknowns and six equations, that means, you can solve this problem.

(Refer Slide Time: 48:59)



Suppose a rod is added as shown in this slide. The total degrees of freedom of all the rods combined are 9, because each rod has 3 degree of freedom if it is unconstrained. There is an addition of two pin joints because this is constrained here; then again, it is constrained at this end also; so it can be considered as separate pin.

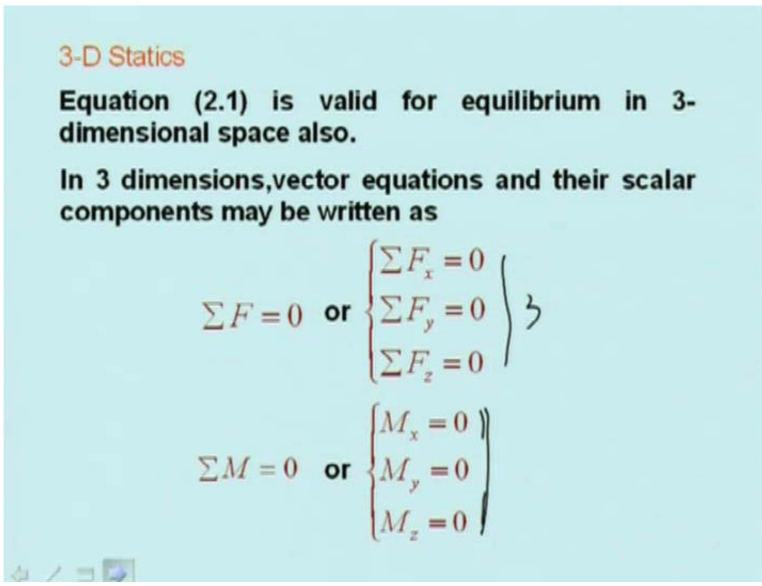
So degree of freedom is basically -1. Why? Because see we have total 9 degrees of freedom but now we have 1, 2, 3, 4, 5 pins. Therefore, each one will arrest 2 degrees of freedom. Therefore, we have 9 minus 10 is equal to -1: this is -1. There are three reactions here that you know.

For the whole structure, also, there are only three reactions; equations of equilibrium are only two; that means that, therefore, we cannot determine these by equations of equilibrium. This problem is called statically indeterminate problem. If we consider the component wise also, that you know, that this one; so you have at we can write sigma F_{x0}, sigma y_0 here. So we can have that this thing at this point also.

Therefore, what happened? We have two equations for these, two equations; so six and we have two equations at this pin joint (Refer Slide Time: 00:50:45 min). Therefore; totally, we have eight equations, but let us see how many unknowns you have to find out.

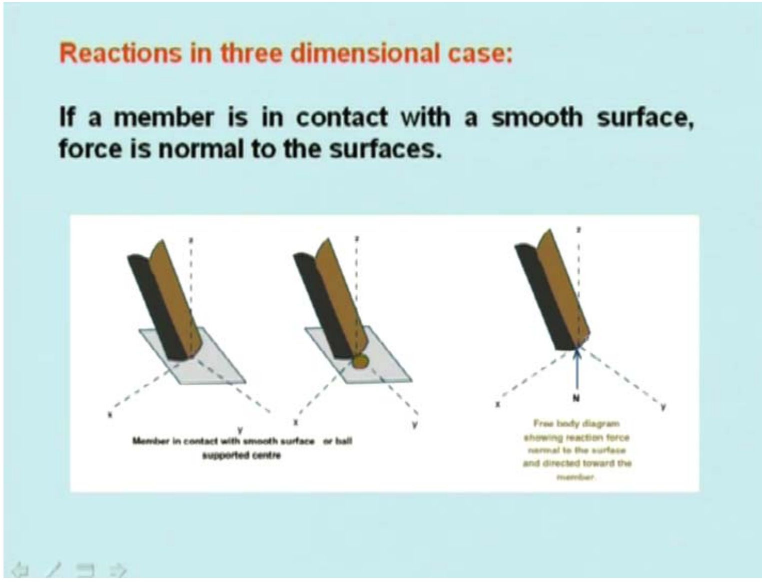
You have to find out, that x component of that reaction, y component of the reaction, so two here two here, two, two, four, two, six and plus these three; so you have to find out nine components, but you have got only 8 equations (Refer Slide Time: 00:51:55 min). Therefore, naturally this cannot be solved by statics. Therefore, this problem is called statically indeterminate problem.

(Refer Slide Time: 51:39)



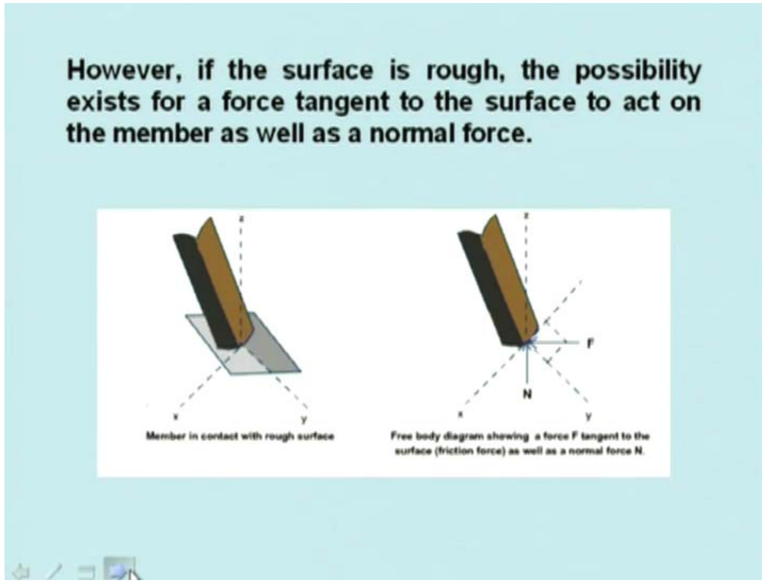
We have given some simple example based on the two-dimensional statics. Now, let us discuss something about three-D statics. Equation 2.1 is valid for equilibrium in three-dimensional space also. This means there also basically sigma F must be equal to 0, sigma M equal to 0. In three dimensions, vector equations and their scalar components maybe written as sigma F is equal 0, or sigma F_x is equal to 0 ,or sigma F_y equal to 0, sigma F_z is equal to 0. This is in the scalar form. That means, this is sigma F is equal to 0 but we write that sigma F_x 0, sigma F_y; so we get three equations here. Then we get sigma M is equal to 0, or M_x is equal to 0, M_y is equal to 0, M_z is equal to 0. We are obtaining these six equations and we can have this thing.

(Refer Slide Time: 52:43)



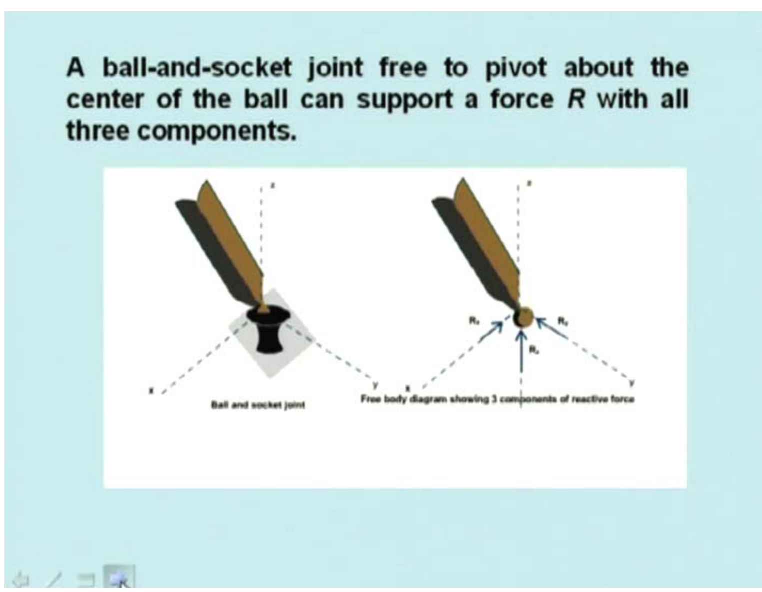
Now reactions in three-dimensional cases will be like this. If a member is in contact with a smooth surface, then the force will be normal to that surface. In this case, if a member is in contact with a smooth surface or if it is supported by a ball, in that case we can make the free body diagram of that structure. Free body diagram shows reaction force normal to the surface and directed towards this one member. So, on a smooth surface the reaction has to be this one. Suppose, we put that some object on a smooth floor, then the reaction will be normal to the floor only.

(Refer Slide Time: 53:44)



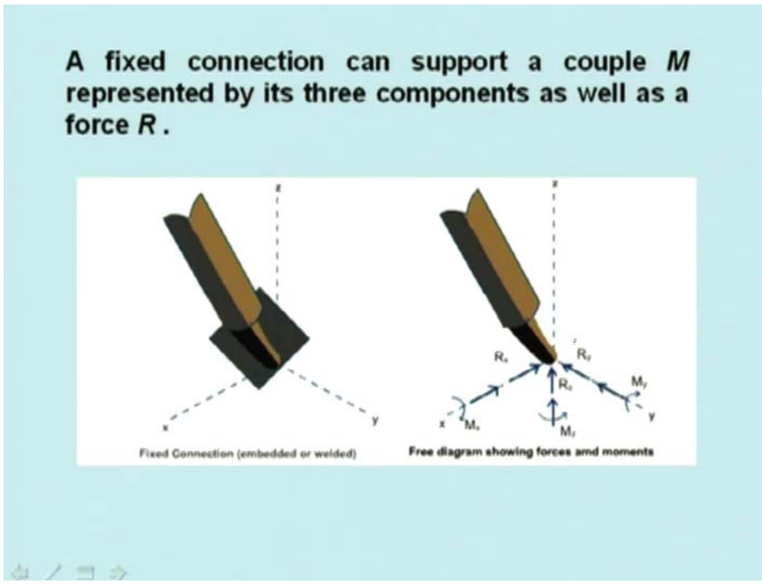
However, if the surface is rough, the possibility exists for a force tangent to the surface to act on the member as well as a normal force. If member is in contact with rough surface… this is a to three body diagram showing a force F tangent to the surface as well as normal force N. This is force F is acting here and there is a normal force N acting here. So, this is F and this is N. So, that reaction can be instead of normal, it can be at other angle also.

(Refer Slide Time: 54:25)



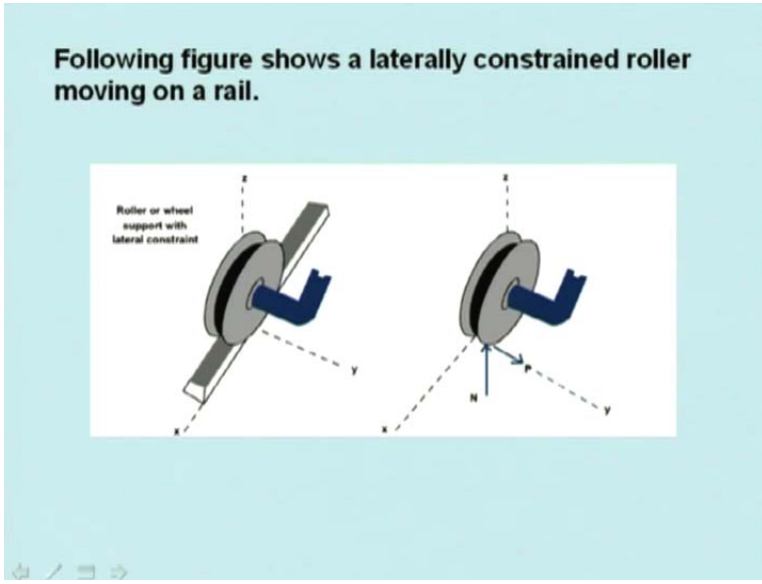
Now a ball and socket joint like this is free to pivot about the center of the ball can support a force R with three components. Now ball and socket joint, is this one, it can support three components which provides three components of reaction R_z,, R_x and R_y. So, free body diagram is shown here.

(Refer Slide Time: 54:59)



Fixed connection can support a couple M represented by its three components as well as a force R. So, force R is also provided and couple is also provided. Couple can have three components. So this is R_x, R_y, R_z, M_x, M_z and M_y. You have to know that what are the reactions coming on this one the object. If you make the free body diagram, you must know not only the applied forces, but also the reaction forces which the other body supplies because of the interaction of that object with the other body.

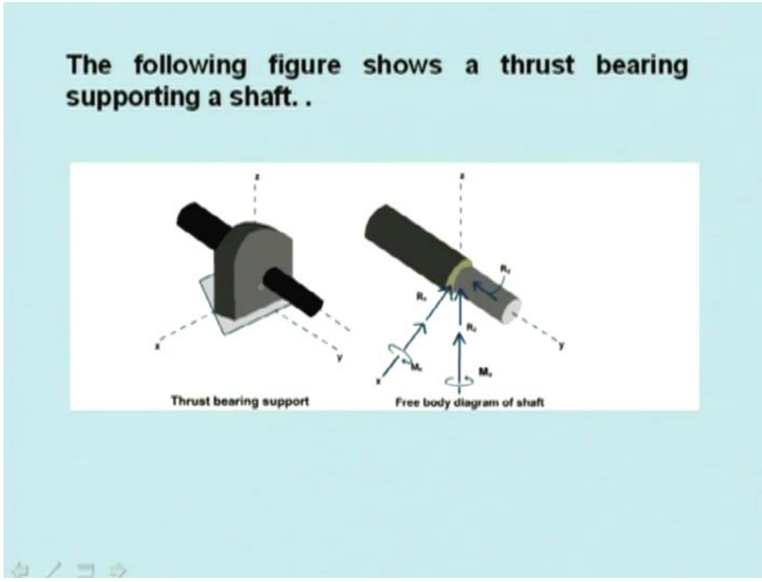
(Refer Slide Time: 55:39)



This figure shows a laterally constrained roller moving on a rail roller or wheel supported with lateral constraint. It is constrained to move that in this direction. It can move in this direction but it cannot move down because of the constraint of the rail and this cannot move this one. Therefore, roller or support with lateral constraint and it is also supported with a…. It cannot move sidewise as well. Because, they are you all know that here supports have been provided. Therefore, what happens that this is the reaction force is in the direction of upward direction N.

As it constrained, it is not allowed to move in either direction. This rail is not allowing it to move that means rail is applying some reaction that is why it is not able to move down. Similarly, it is not allowing it to move sideways, even if we apply some force. Therefore, that means rail is applying some force P in this direction; that this is the P. So, these are acting on this one. So in this thing but in this direction you do not see any reaction force because if I apply a force it will simply move, there is no constraint; so nobody is applying any reaction; that is why it is moving in this x direction.

(Refer Slide Time: 57:25)

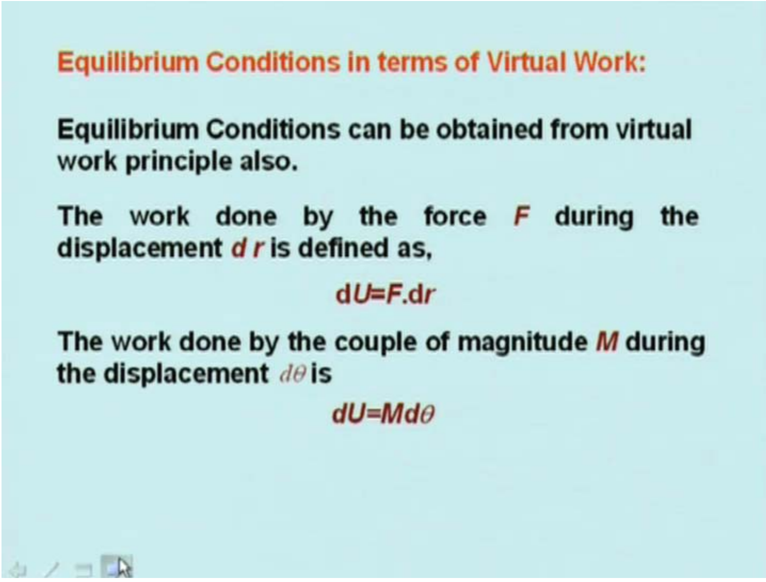


This figure shows a thrust bearing supported on a shaft. This is a thrust bearing support. This is x, y and z. In this case, it is constrained not to move along the x-axis. This rod cannot move in the x-axis; even if I apply some force: it will not move. Therefore, what happens, that there is a reaction R_x which the walls of that hole or this one is bearing or supplying. Therefore, bearing gives a reaction in direction x; so therefore this is R_x.

Similarly, it cannot move in the z direction either. Therefore, there must be another reaction $R_{z;}$ that is shown here, although I have shown it upward, in some cases it can be downward also. So, this is what that this thing is there.

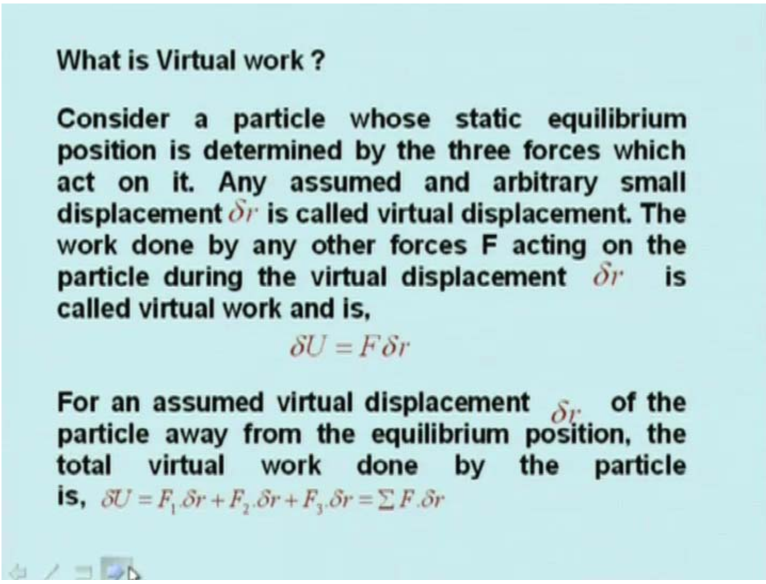
Similarly, but in this direction if I do not allow it to move in that direction, if I put some support then there will be a reaction R_y also. At the same time, this is not allowed to rotate about x-axis; therefore, reaction moment M_x is there, it is not allowed to rotate about z-axis. Therefore, reaction moment M will be z but it is allowed to move along y. Therefore, there is no reaction movement in that direction, it can only rotate. So this is a thrust bearing support and this is the free body diagram of the shaft shown.

(Refer Slide Time: 59:16)



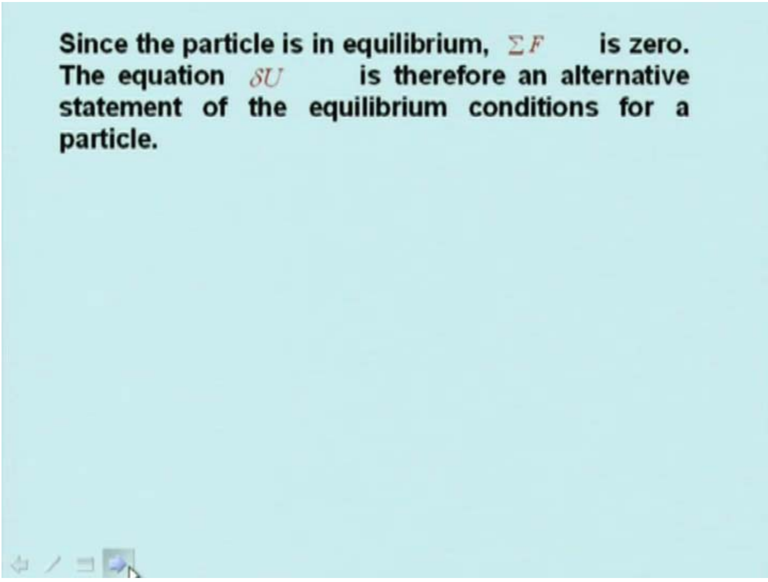
Apart from that, we have discussed about equilibrium conditions. Very briefly, we discussed about another type of method that is concept that virtual work; equilibrium conditions can be obtained from virtual work principle also. The work done by the force F during the displacement dr is defined as dU is equal to F times dr. The work done by the couple of magnitude M during the displacement d theta is dU is equal to Md theta.

(Refer Slide Time: 59:54)



What is a virtual work? Consider a particle whose static equilibrium position is determined by the three forces which act on it. Any assumed and arbitrary small displacement delta is called virtual displacement. The work done by any other force, say F, acting on the particle during the virtual displacement delta r is called virtual work and that is called delta U is equal to F times delta r. For an assumed virtual displacement delta r of the particle away from the equilibrium position, the total virtual work done by the particle is this one sigma Fdr. That is the condition.

(Refer Slide Time: 01:00:35)



Since the particle is in equilibrium, sigma F is 0. The equation delta U is therefore an alternative statement of the equilibrium condition for a particle. That means virtual work done on the particle must be 0. This is for a particle I worked out about the virtual work principle also.

So today, we discussed about the equations of equilibrium.