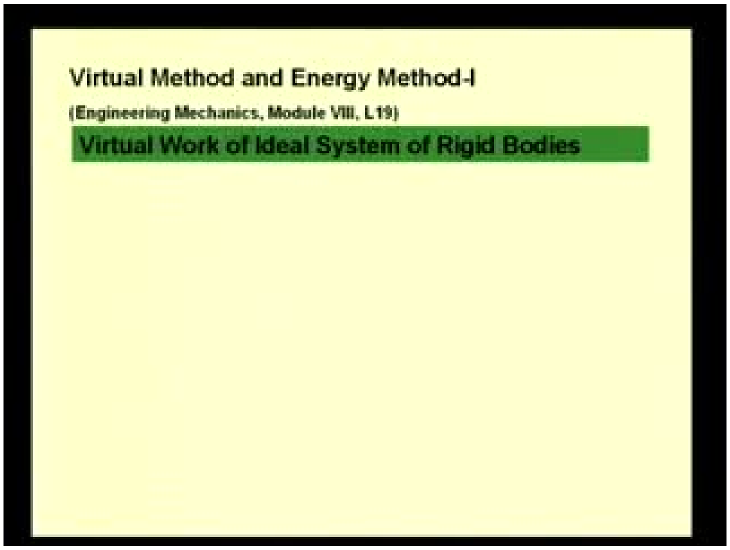
Engineering Mechanics

Prof. Dr. Saravana
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

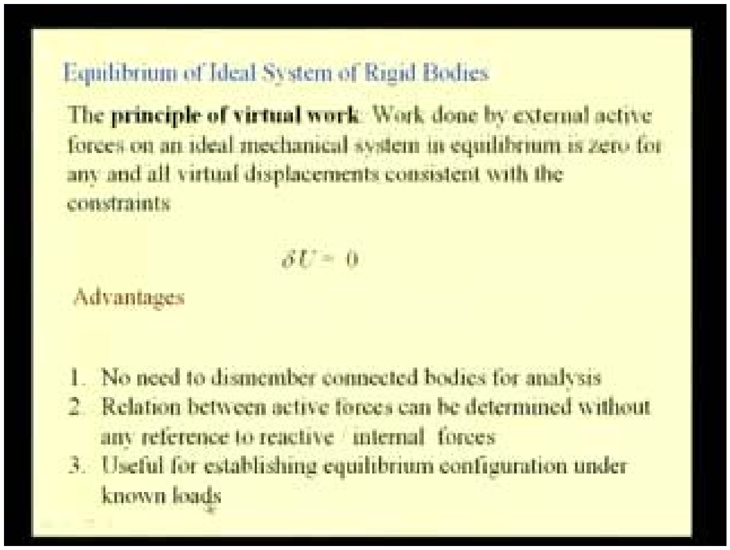
Module No. # 08
Virtual Method and Energy Method 1
Lecture No. # 02
Virtual Work of Ideal System of Rigid Bodies

An IITG person promises only what he can deliver, and an IITG person, delivers what he promises.

(Refer Slide Time: 01:21)



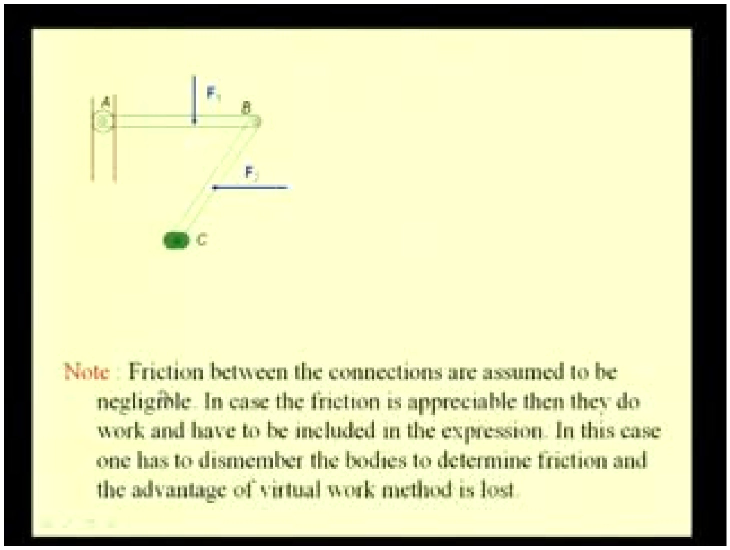
(Refer Slide Time: 02:01)



So, we will continue with our discussions that we had in the last lecture on the principle of virtual work as applied to connected system of rigid bodies. So, for your reference, this is module 8, lecture number 19 of the Engineering Mechanics course. Before taking up some problems, we will see how to do define what we call as the degree of freedom of a system, which defines the number of, you know, individual coordinates that is required to define the position of a or a configuration of a rigid body or a system of connected rigid bodies. Before going into the details, let us recapitulate what we studied in the last lecture. We defined this equilibrium of ideal system of rigid bodies; we defined ideal systems as systems, which do not absorb energy and also do not dissipate energy because of friction.

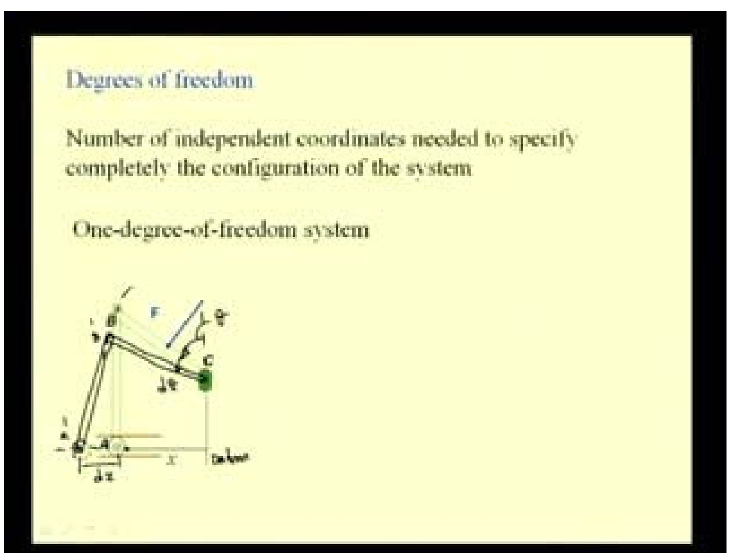
So, the principle of virtual work as applied to the ideal system of rigid bodies is that, the work done by the external active forces on an ideal mechanical system in equilibrium is 0, for all virtual displacement consistent with constraints that is delta U is equal to 0. And we also saw, that it is not needed to dismember the connected bodies for analysis. The relation between active forces can determined without any reference to the reactive or internal forces; that means, we do not need to find the reaction forces also and also it is useful in establishing equilibrium configuration under known loads; so, we will see some problems in order to realize these advantages.

(Refer Slide Time: 03:13)



You should note that we are neglecting the friction, because we are idealizing the system and we are assuming that friction does not exist, and in case, the friction is appreciable, then they do work and have to be included in the expression. In this case, one has to dismember the bodies to determine friction and the advantage of this virtual work method is lost, because in order to determine the forces, the internal forces, we have to dismember and once we do that, the advantage of the virtual work is method is lost.

(Refer Slide Time: 03:52)



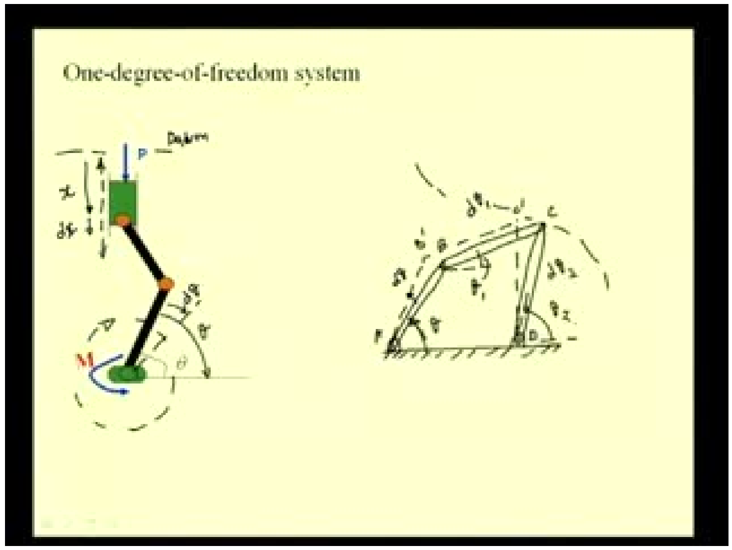
Now, let us define what we mean by the degrees of freedom of a system; the degree of freedom is nothing but the number of independent coordinates needed to specify the configuration of a system completely.

So, let us take one example of a 1 degree of freedom system, where we define the configuration of the system completely by specifying only one coordinate. So, here you see a link B C linked at C and the link A B connected by a roller at A and say some external forces are acting on the system and system undergoes displacement.

So, if assume that the system undergoes displacement consistent with the constraints, then this point A can move along this roller, say in this direction and can take up some new position like this, and since this is constrained at this point C, it, this point B can trace a circular path and so any new position, say B prime will be such that, this virtual displacement, because of the application of these external forces is consistent with the constrains, that means, this point A moves along this direction and this link undergoes a pure rotation.

Now, this new configuration can be defined, if we know this point say A prime and say this angle of rotation of this link B C, that means, if we know the coordinates of A prime, B prime, then we know the new configuration completely. Since, these bodies are rigid, these distances does not change, that is A prime, B prime or B prime C, the lengths do not change. So, if we define this as the coordinates say x, the distance, let us this is our datum, from this I define this distance x, then it is possible for me to obtain say the position theta of this link B C in terms of this x and so for any displacement say dx, I can know what is this d theta; so, that means, it is possible to get the required coordinates for defining the configuration of this system, if we specify say a single coordinate x.

(Refer Slide Time: 07:24)



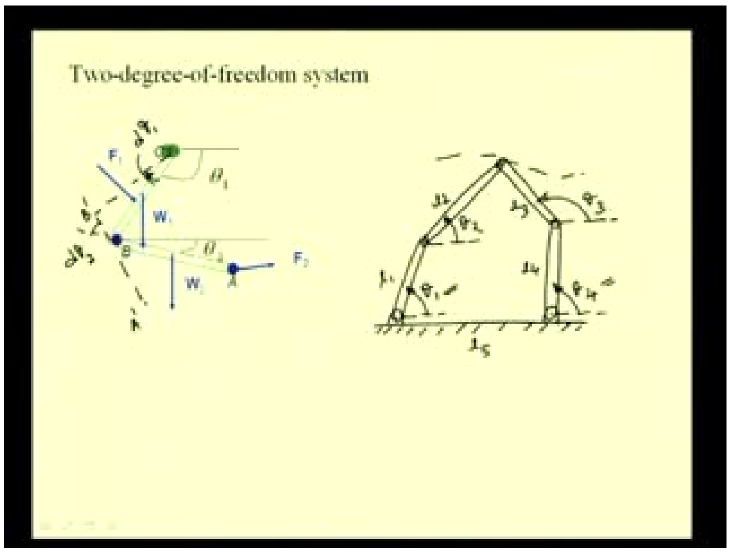
So, we call this as a single degree of freedom system, where we just need one coordinate to specify the configuration completely. So, we can see an another example, like the piston and the connecting rod and the crank, that you normally see in any IC engines; so, this piston traverses a reciprocating motion length, then this crank undergoes rotation. So, the coordinate of this piston say with respect to some datum, say I call it as x can be defined, if I can define this angle theta.

So, for any displacement d theta of this link, the displacement dx is known; so, either by specifying theta or specifying x, I can completely define the configuration of the system; so, again this is a 1 degree of freedom system.

We can see an another example which is a four bar mechanism, what we call it as a four bar mechanism, where we have, say this position A is fixed, C and D, so these two points are fixed and this link that is A D also is fixed, because you know it is orientation as well as, this distance between these two points A and D is also fixed. So, if I say that, this is my crank or my input link, and its angle is theta, and these linked lengths are constant, because these links are rigid; so, if this undergoes a displacement to a new position B prime and this is D theta, then the new orientation say C prime; all the other angles say the angle between, this angle let me call it as theta 1 and the angle of this link theta 2. We can write these angles theta 1, theta 2 in terms of this angle theta and so any displacement D theta will result in the displacements B theta 1 and D theta 2, which is again possible to write, because these link lengths are constant. So, these two angles can be determined in terms of this angle theta; so, again this is a single degree order of freedom system.

So, such kinds of four bar mechanisms are used to traverse, say, some path for the rotation of this crank and they are used in many machineries to generate path; so, such mechanisms are known as path generating mechanisms. In the second course, in the mechanical engineering on the machines, you will study in more detail.

(Refer Slide Time: 11:58)

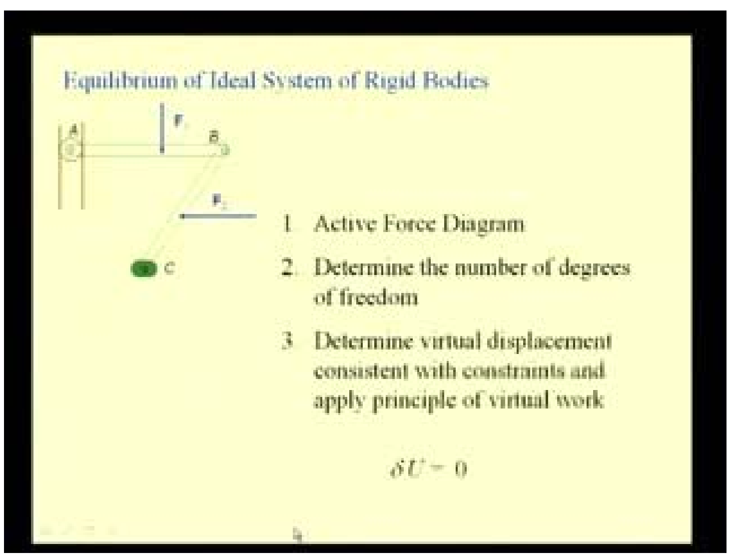


So, here our discussion is to understand the concept of this degree of freedom of a connected system of rigid bodies. So, let us see another example here, what you see here is a 2 degree of freedom system, so it is similar to an articulated arm, like our arm. So, the position of this end A or the end effector in case of a robot or our hand, in case of the human hand can be positioned by the rotation of this link and the rotation of this link, so that is what you see over here, the two links B C and B A, various forces are acting like the self-weight of these members and say some other forces. The configuration of this link system can be defined by these two coordinates, say, theta 1 and theta 2.

And so we call this as a two degree of freedom system, because you need two independent coordinates like theta 2 cannot be expressed in terms of theta 1 and so we need to define specific values of for theta 1 and theta 2, in order to obtain the particular position A. So, here, any virtual displacements, say, consistent with the constraint B prime and say A prime has to be defined by defining both this d theta 1 as well as the angle d theta 2. So, we see that the system can be uniquely defined, if we specify the two coordinates theta 1 and theta 2 and thus we call them as two degree of freedom system; can have an example of 5 bar mechanism, this way we defined the 4 bar mechanism; so, I can have and say this is the end effector, and this is trying to generate some path. In this case, if this is one input link, I say theta 1, these links lengths are fixed and this is l 5 this is also fixed both in orientation as well as length.

And in order to completely define this configuration, you have to define one more angle say this theta 4, then these 2 angles that is theta 2 and theta 3 can be determined in terms of theta 1 and theta 4. So, this is again a two degree of freedom system, where the configuration of this system is completely defined, if we define these angles theta 1 as well as this theta 4. So, these are some examples of two degree of freedom system and then we have multiple degree of freedom systems, where the degrees of freedom could be as large as n and the analysis of such systems becomes more and more complicated as we go on increasing the degrees of freedom of the system. So, initially we will limit our discussion to single degree of freedom system and maybe we will see some problems on 2 degrees of freedom system also.

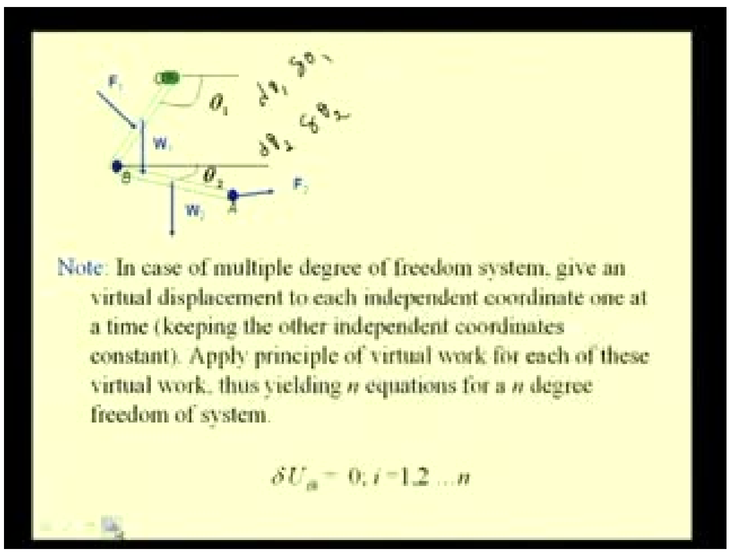
(Refer Slide Time: 16:23)



So, let us, now define this equilibrium of ideal system of rigid bodies; in order to solve such problems, the first step is to construct what we call as the active force diagram, where we depict only the forces that do the work, we do not show the reactive forces and the internal forces. The way we have constructed free body diagrams for solving the equilibrium problems, here we construct this active force diagram, where we only show those forces which constitute or which do work when a virtual displacement is undergone by a system.

Then we determine the number of degrees of the freedom, define the datum and determine the virtual displacement consistent with the constraints, and then we write the principle of virtual equation, that is, we define the virtual work and then equate the sum of all the virtual work to 0.

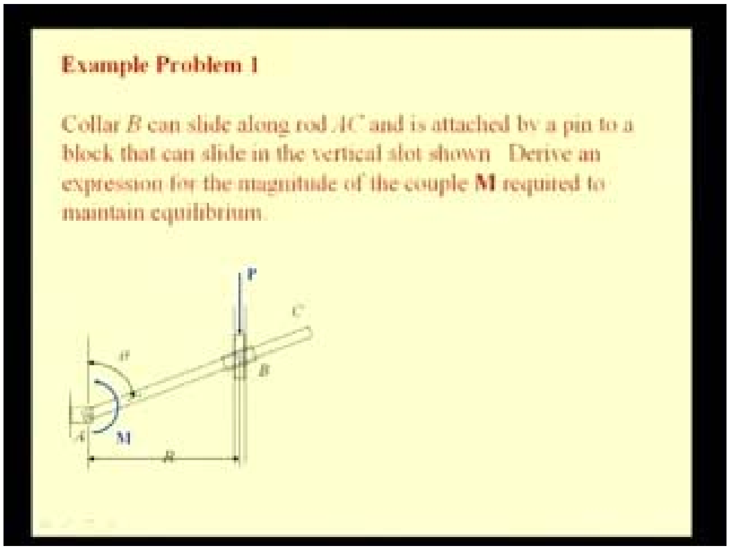
(Refer Slide Time: 17:47)



That is this equation, that is delta U the virtual work has to be 0. So, for such multiple degree of freedom systems, a virtual displacement is given to each individual coordinate, that means, we have to give one virtual displacement d theta 1 to this coordinate theta 1 and we have to give and another virtual displacement d theta 2, for this coordinate theta 2 and we apply the principle of virtual work for each of these virtual work; so, for each virtual displacements, say, for a small angular displacement delta theta 1, the virtual displacement is delta theta 1 and this is delta theta 2, for each of these, we define the virtual work and equate it to 0, there by resulting in n equations, for an n degree of freedom system and it is possible to solve for n unknowns in such a system, but initially let us limit the discussion to single degree of freedom system.

So, these are the various equations delta U theta i, where i is 1, 2 etcetera up to n, the degree of freedom of the system. So, each of this results in one equation, we have n equations for the n degree of freedom system.

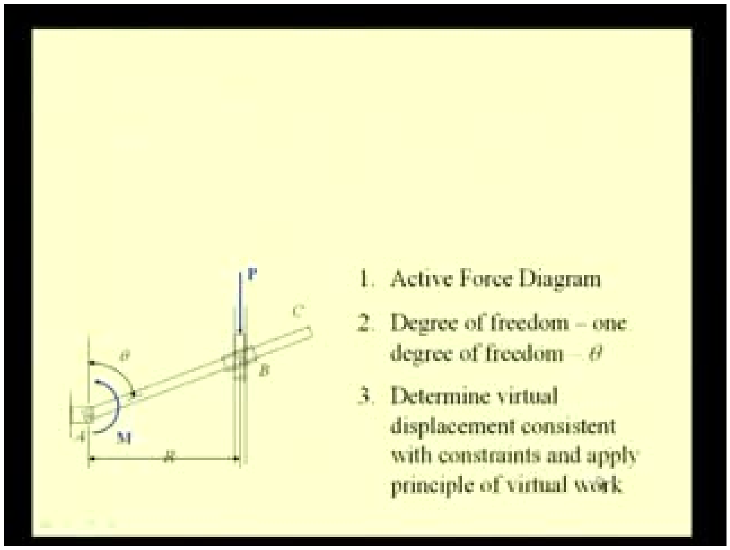
(Refer Slide Time: 19:33)



Let us consider one example and see how we apply this method of virtual work, to the system of connected rigid bodies; in this case, we are taking ideal system of rigid bodies.

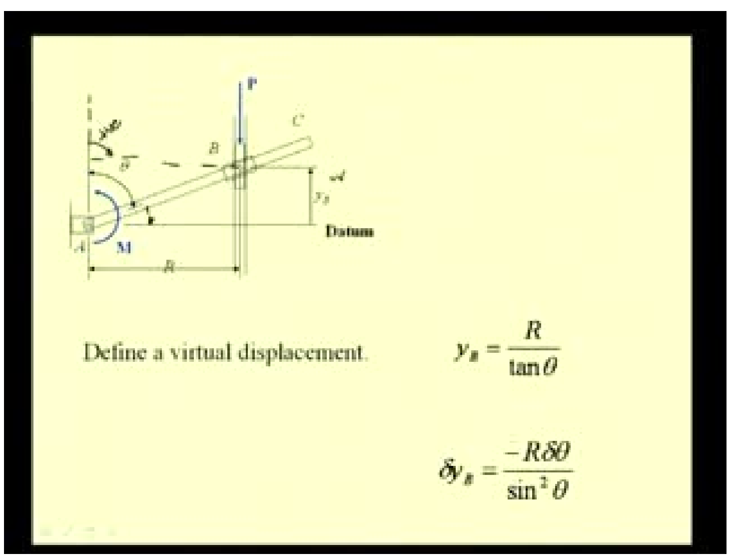
So, here you see a collar B, that can slide on this rod A C; this collar B can slide on this rod A C and it is attached to a pin here and a block which can slide in the vertical slot; so, this block can slide in the vertical slot, and this block and this collar is attached by a pin at B. We are interested to derive an expression for the magnitude of this couple M which is applied at A, required to maintain the equilibrium position when some force P is acting on this block attached to the pin B.

(Refer Slide Time: 20:55)



So, let us try to solve this problem by the method of virtual work; so, what will be our strategy? We will first create the active force diagram; here, in this case, we have these two forces, that is P and the applied moment members which do work, for any consist virtual displacement consistent with the constrains.

(Refer Slide Time: 22:22)



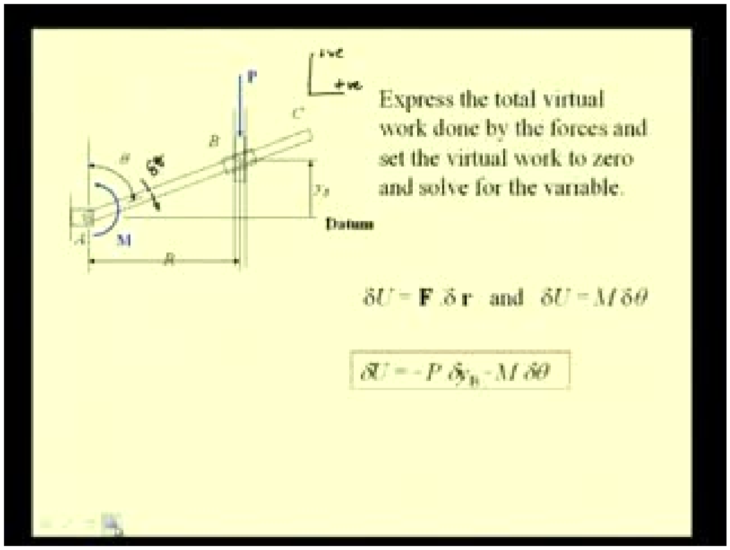
Now, let us define the degrees of freedom of this system, it is a single degree of freedom system, that means, if we define this configuration by this angle theta, then the position of this point B, where the other force P is being applied is uniquely defined. As this angle theta changes, this point B moves along the slide and its new position is uniquely defined if this distance R is known, which is a fixed distance, that is the point A and the slot are located at a distance of R and which is fixed. So, by uniquely defining the angle theta, it is possible to find the position of this point B and thus it is a single degree of freedom system.

Now, we will define the virtual displacement consistent with the constraints and then apply the principle of virtual work. So, let us first construct the active force diagram, showing also the datum and the coordinates; so, here, you see the two active forces, that is the force P applied to the block and the moment M that is applied to the link pinned at A, and we have this theta measured positively from this point or this axis; so, this is the positive measurement. And we have the distance at which this point B or the pin B is located with respect to the horizontal axis passing through A, which is taken as the datum for this position, let us mark this as y B. So, these arrows also mark the positive displacements, that means, when this block B moves in the positive y direction, we say that it is undergoing a positive virtual displacement and if this link takes the positive motion in this direction, we call it as it is undergoing a positive virtual displacement.

Now, let us try to relate the coordinate of this B to this coordinate theta which defines the configuration of the system. So, we have this y B as R by tan theta, where R is this distance divided by tan of this angle, because we know that tan theta is equal to R by y, because we have this R as this distance and y B is this distance and from this right angle triangle, we can right this relation.

Now, let us define the virtual displacement by differentiating this equation; so, we have delta y B equal to the differentiation of this tan theta is minus 1 by sin square theta delta theta, so that is what we have written here, delta y B is equal to minus r times delta theta divided by sin square theta. So, this negative sign indicates that if the angular displacement theta moves in the, or the virtual displacement is in the positive sense; that means, delta theta is positive the link is taking a new position in this direction. This B moves down or the distance y B shrinks, so the virtual displacement of this point B is in the negative sense; so, this negative sign indicates that the virtual displacements of delta theta and delta y B are negative and which is also seen from this picture.

(Refer Slide Time: 26:12)

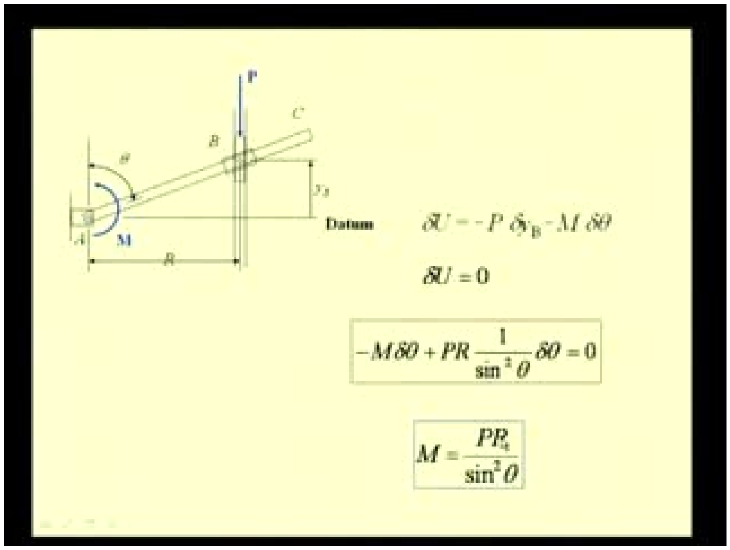


Now, let us express the total virtual work done by these forces and set this virtual work to 0; so, what is the virtual work, that is delta U equal to 0, we have this F dot delta r and the work done by the moment delta U is M delta theta; this work is positive, if both the momentum as well as delta theta are in the same sense and this is negative, if they are in the opposite sense.

So, we have the work done by this force P, which is in the negative direction, because this is in the negative y direction, because we have taken this as the positive y and this as the positive x, and the sign of this is minus P, because the force vector is in the negative y direction, minus P times delta y B is the work done by this force P, then let us see what is the work done by the moment.

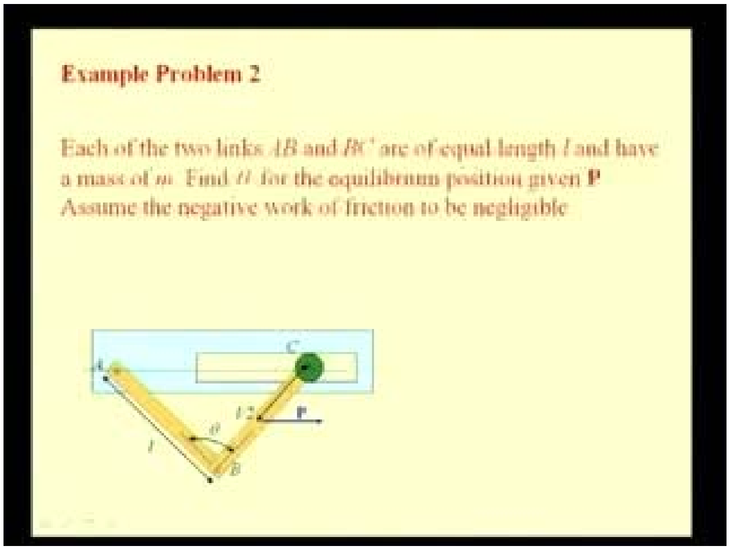
Here, the positive displacement delta theta is in this direction and the moment is in the other direction; so, the sign of this is negative, so we have minus M delta theta, because the moment on the positive virtual displacement or on the opposite sense; so, the angle between these two vectors is pi superclass cos pi is minus 1, so we have minus M delta theta. So, this gives the expression for the total virtual work done by the force as well as the moment that is occurring in this system.

(Refer Slide Time: 28:14)



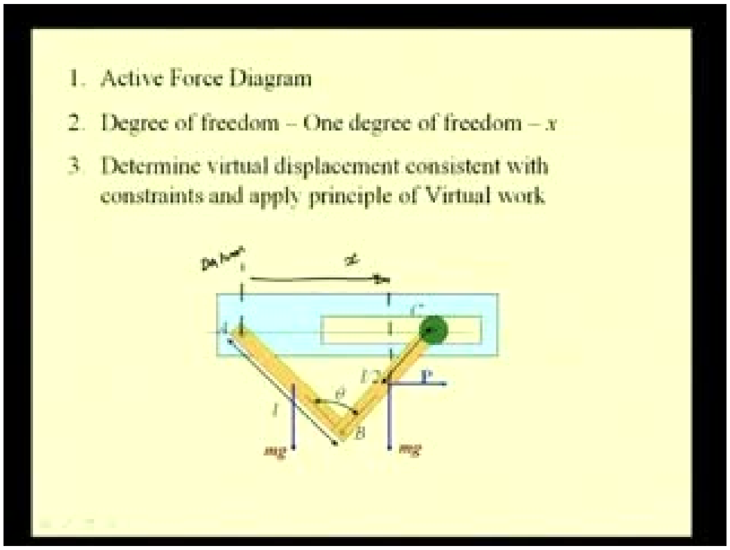
So, let us expand this by substituting the values of this delta y B and equate this term to 0, so we have this force P is, this force P times delta y B which is R by sin square theta delta theta, and this negative signs and the negative sign of this delta y B gets cancelled, and so we have plus P R divided by sin square theta delta theta plus the work done by the moment, which is minus M delta theta and which has to be 0. So, from this equation, we can now remove this delta theta, because virtual displacement is not 0, so the other term has to be 0 and from that we get the relation between the applied moment and the force. So, if we know the distance R and this configuration theta, we can find this value of m with respect to this force P.

(Refer Slide Time: 29:30)



Let us take an another example; so, here you see two links, that is AB and BC which are of equal lengths say l and have mass of m. We are interested to find this equilibrium position theta, which is the angle between these two links, for a given value of this applied force P. So, we are interested to find this position, which is defined by this angle theta for a given load P; we are assuming that the negative work of friction is negligible in this case; so, we are neglecting the work done by the friction.

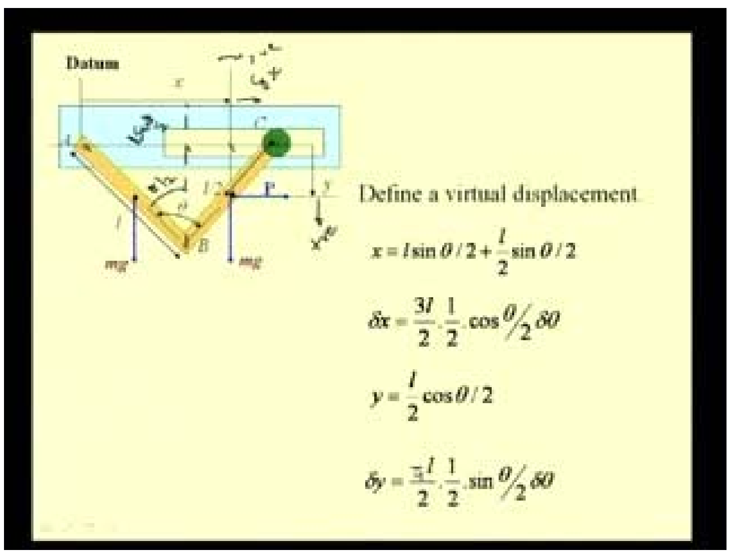
(Refer Slide Time: 30:26)



So, again our strategy will be to construct the active force diagram; here, what are the various forces that are active? It is the self-weight of these links that is mg, the weight of this link A B and the weight of this link B C, which are acting through their center of mass, which is nothing but at a distance of l by 2 from say point A, along this link A B, and same way it is l by 2 along this link B C from this point C, and we have this applied force P, which is acting through the center of mass or the center of gravity of this link C B.

So, these are the various active forces, which are doing, which are doing the virtual work if the system undergoes a virtual displacement and this system is clearly a 1 degree of freedom system, where I can define the position of this point P with respect to a datum, say I fix this as my datum; so, either I can have the configuration specified by this distance x or I can specify the configuration by this angle theta.

(Refer Slide Time: 32:12)



Then we determine the virtual displacement consistent with, the, constrain and apply the principle of virtual work. So, let us first construct the diagram showing the active forces and then coordinate that specifies the configuration, that is x, which is nothing but the center of mass of this link BC with respect to this datum passing through the pin connection at A. So, let us, now write this distance x with respect to this angle theta, we also would like to know the position of the center of mass of this link AB as the system takes new configuration, because we are interested in computing the work done by these active forces.

So, we would like to know their position of action of these forces as this system undergoes displacement. So, we are interested to know, how this point from where this force mg acts, undergoes displacement as this x changes, and also this point where this mg as well as P is acting, how this point undergoes the virtual displacement consistent with these constrains; so, first we define these virtual displacement.

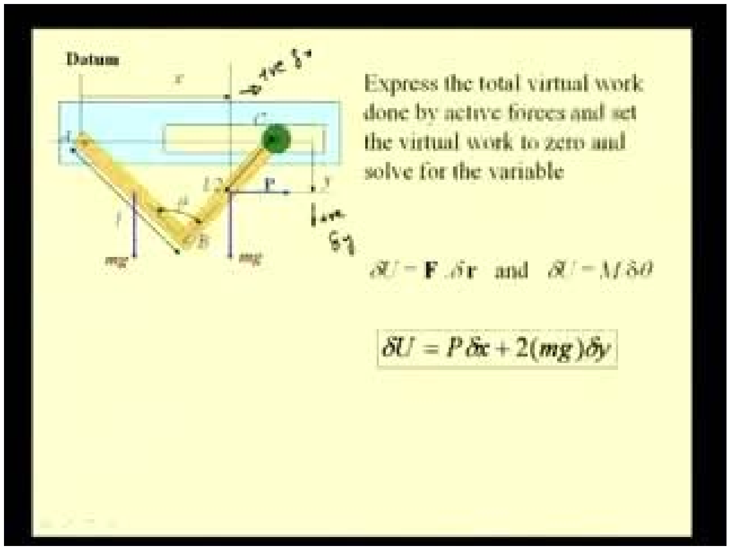
So, first let us relate this x with this theta, so we have this x as this distance plus this distance, so this distance is nothing but l sin theta by 2, because if this is angle theta and these two links are having equal length, then this angle is theta by 2; so, we have this distance as l sin theta by 2 and this distance will be l by 2 half times sin theta by 2.

So, now, we can define this virtual displacement delta x, that this point undergoes in the x direction, as delta x which is the differentiation of this is equal to 3 l by 2 times half cos theta by 2 delta theta; so, this is the differentiation of these quantities. Now, let us see how this vertical distance of this point moves as this virtual displacement delta x is undergone; so, we define from this datum y and the positive sense, so here this is the positive virtual displacement for x and here this is the positive virtual displacement for y direction.

So, this distance y is equal to l by 2 cos theta by 2 cos, this is l by 2 and this angle is theta by 2 and so we have y as l by 2 cos theta by 2 and from this we get this virtual displacement delta y as minus l by 2 times half sin theta by 2 delta theta.

So, here these two relation states that as this angle increases, this x also increases, but as this angle increases, the virtual displacement of this point in the y direction is in the negative sense, that means, this point moves upward as this link expands or this theta expands the point of center of mass of these two links actually moves upward or in the negative sense of the positive virtual displacement that we have considered; so, that is why we get this negative sign and that is the physical significance of this negative signs, from this differentiation to this problem.

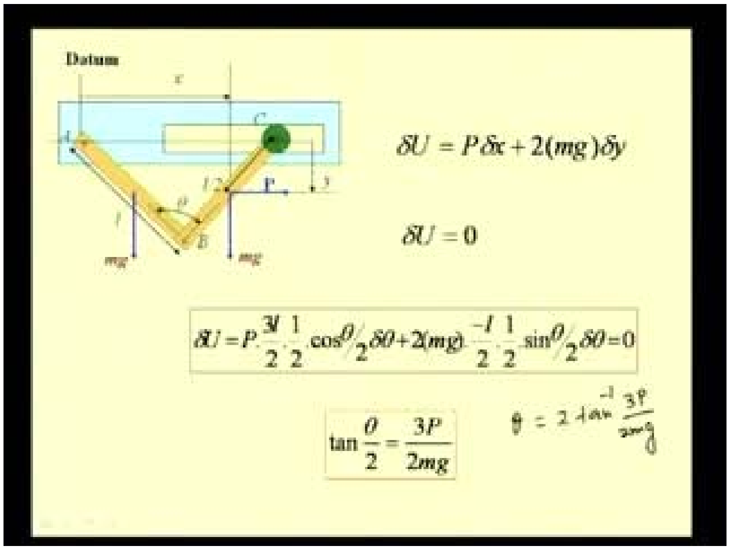
(Refer Slide Time: 36:52)



Now, let us express the total virtual work done; so, delta U is f dot delta r and the virtual work done by the moment; so, in this case, there are no applied moments, so we have only the work done by these various forces, which is equal to the work done by this force p, which is in the positive x direction times delta x plus these two forces which are also in the same direction, as the positive sense that we have taken; this is the positive sense and this is the positive sense, for these virtual displacement delta x and delta y.

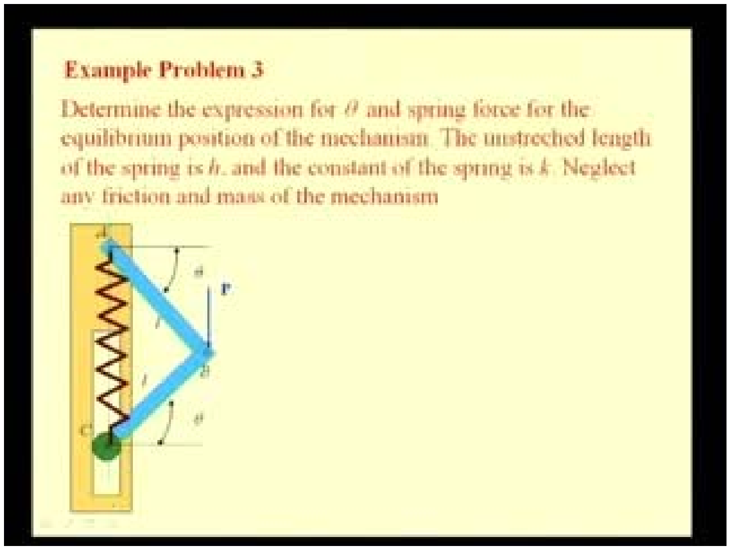
And since these forces are in the same direction, that is mg is in the same positive sense of delta y and P is in the positive sense of delta x, we have both these work as positive P times delta x plus 2 times mg delta y, the work done by these two forces.

(Refer Slide Time: 38:07)



Now, let us substitute the values of delta x and delta y in terms of delta theta, we have this equation which we equate it to 0, so we have P times this quantity delta x plus 2 mg times delta y, which has a negative sense sign with respect to delta theta. So, if we simplify this equation, we know that delta theta is not 0, because the virtual displacement is not 0, so these other component that is P times 3l by 2 half cos theta by 2 plus 2 mg times minus l by 2 sign theta by 2 into half should be 0 and from that when we simplify it, we have tan theta by 2 is equal to 3 P by 2 mg; so, if we know this force P and the mass of these links, then we can determine this angle theta as 2 times tan inverse 3 P by 2 m g.

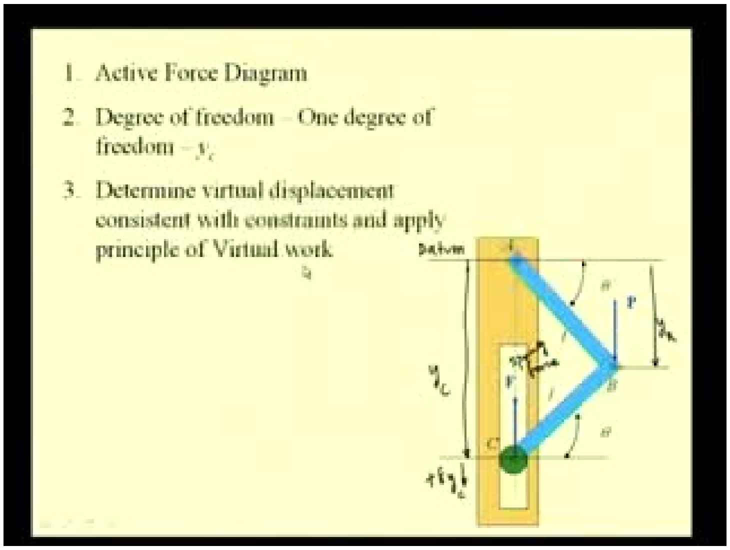
(Refer Slide Time: 39:24)



Now, let us take an example where we have some elements which can absorb energy; in all the previous examples, we saw that we are neglecting friction and also we are not having any elements that can consume energy by absorbing them either by elongation or compression. So, here in this example, we see a spring element which is an elastic element which undergoes deformation when some forces are applied and it absorbs energy, because when the forces are removed, it regains its original shape by releasing the energy.

So, in this example, you see two links AB, BC which are pinned at A and B and it is connected to a roller at C and this roller is free to slide in this slot, that is, we neglect, the, we neglect this friction that is there and also let us neglect the mass of these links for simplicity. So, we are interested that to determine this position theta for which the system is in equilibrium, if we know that the spring constant is k and the un-stretched length of this spring is h. So, we are interested to find this equilibrium configuration theta and the spring force for this equilibrium position.

(Refer Slide Time: 41:18)



So, what will be our strategy? We will first construct the active force diagram, and then, we will find the degree of freedom of this system; this is a 1 degree of freedom system, because the position of these links can be specified uniquely, if we specify the y coordinate of this point c; so, if we take this as our datum and this as the positive delta y c the direction of the positive virtual displacement.

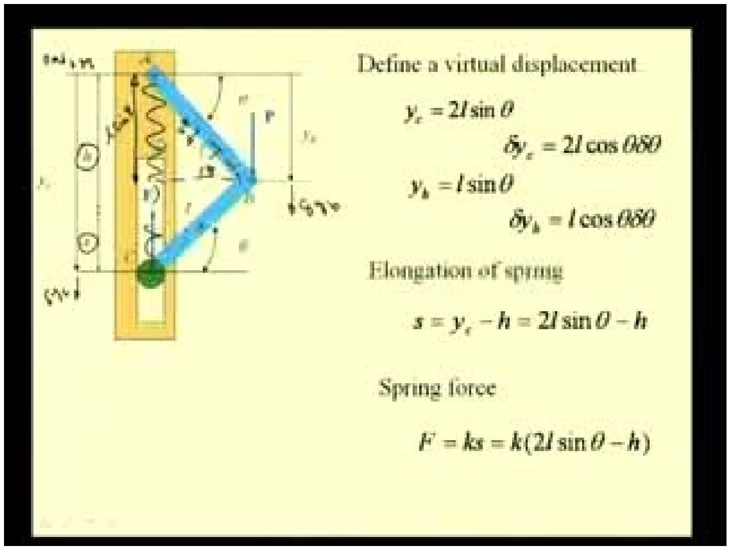
And if you specify this value y c, then the system configuration is uniquely defined and since we have seen that the spring element can absorb energy, when the system undergoes virtual displacement; we remove that from the element to idealize the system. So, we represent the spring by its force on this mechanism, at this point C; so, this force F is the spring force.

So, if the spring is elongated because of the supplied force P, then it will tend to pull this point C towards A and thus this direction of the force is for the condition when the spring is elongated, if it is compressed then this force will be downward.

If we define this coordinate y c, then the other coordinates that is say the coordinate of this point, I can say this is y b is defined in terms of y c and this angle also is defined in terms of y c. So, this is clearly a single degree of freedom system, where I can specify the coordinate y c in order to completely define the configuration of the system.

Then I determine the virtual displacement consistent with the constrains, that is, it is pinned at A, here this point C can move in the vertical direction along this line AC and then I apply the principle of virtual work.

(Refer Slide Time: 44:04)



So, let us proceed; first we have defined these datum and we have defined this coordinate of this point as y c; this coordinate y c equal to the un-stretched length, because this force F is representing this spring which was there and if this is the un-stretched length of the spring then this y c is equal to the un-stretched length of the spring plus the stretch of the spring that is S. So, the length of the spring for this configuration which is y c is equal to this un-stretched length plus this stretch of the spring.

We also defined this coordinate of B y b with respect to the datum, where this force b is applied. Since, we are assuming mass less links, we do not need to consider the work done by the weight of these links, if we consider then we have to also find the coordinate of the mass center of these links with respect to the datum, because as the system undergoes virtual displacement, these points will also move and we have to find the work done by the self-weight of these links; in this case, we are assuming that the masses are negligible.

Since, we have now defined the coordinate of this C and coordinate of this point B with respect to the datum and we have seen that all these forces are vertical; so, we need to only define these vertical coordinates and these are the angles from symmetry and this the lengths of AB and CB being equal these two angles will be the same and which is equal to theta.

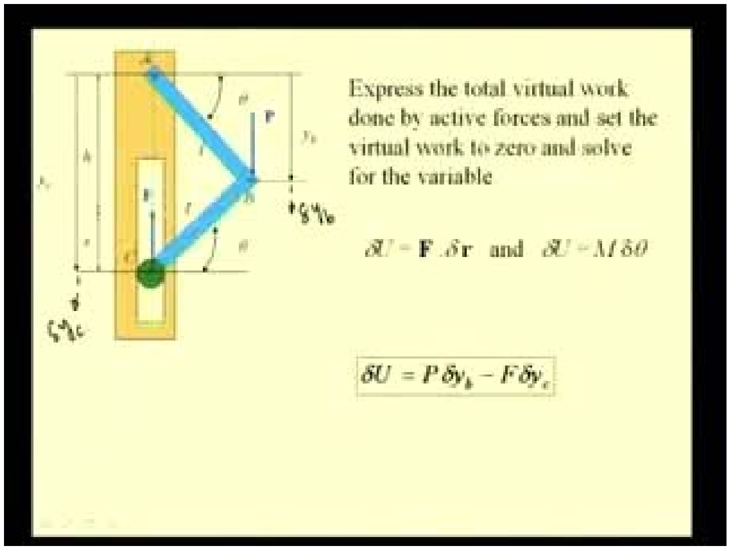
So, let us now write these distances in terms of this angle theta. So, we have this y c as 2 times this distance which is l sin theta this distance is l sin theta; this being the length of the link A B and this angle is theta, so we have this angle also as theta; so, we have this as l sin theta, so the y c is equal to 2 times l sin theta. Now, we can differentiate this to get the virtual displacements delta y c, is the virtual displacement in the positive sense is equal to 2 l differentiation of sin theta is cos theta delta theta, that means, as this angle theta increases this distance delta y c also increases; so, we have a positive correlation that is delta y c as well as delta theta are positively related.

So, we can see both mathematically as well as physically to verify this fact. Now, let us define this coordinate y b, so we have y b is equal to l sin theta this length times sin theta which is this distance and so to differentiate it we have delta y b is equal to l cos theta delta theta; again, we see that this positive virtual displacement delta y b and the positive virtual displacement delta theta are positively correlated, that is, as this theta increases this point moves down or this delta y b and delta theta are positively correlated.

So, now, we have to define the required virtual displacements for this point B as well as point C. Now, let us see what will be the elongation of this spring, that is, this distance s for this particular configuration, we know that s is equal to y c minus h or the un-stretched length of the string. So, we have s is equal to y c minus h and y c has been found to be 2 l sin theta so we have s, the un-stretched length as 2 l sin theta minus h.

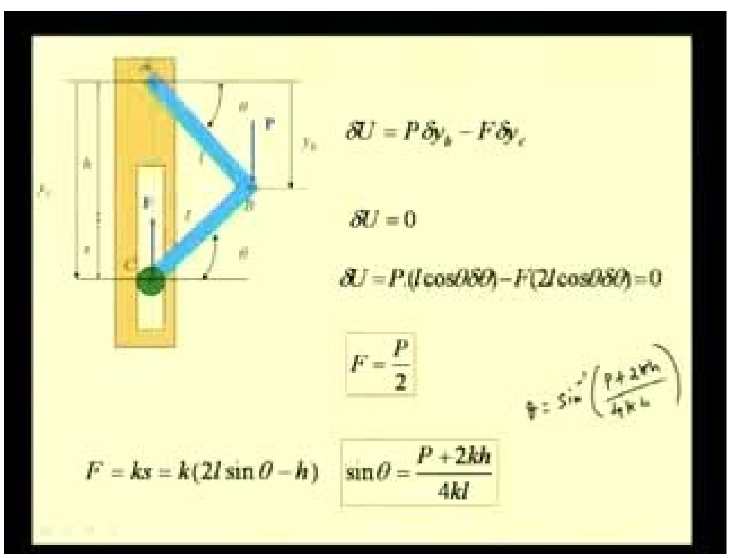
And from this, we can now determine this force F that will act at this point c, because we know this spring force is equal to F times k, where f which is the force of the spring is equal to k times s, where k is the spring constant and s is the elongation of the spring and so we get it as k times 2 l sin theta minus h; so, this is the force that acts at this point C for this particular configuration.

(Refer Slide Time: 49:59)



So, now, we can express the total virtual work done by these active forces, that is, this force F and P and set this virtual work to 0. So, we have the virtual work as delta U is F dot delta r and the work done by the moment as delta M delta theta and here, we do not have any active moments; so, this virtual work term is not involved and only we have the virtual work done by the forces.

(Refer Slide Time: 51:42)

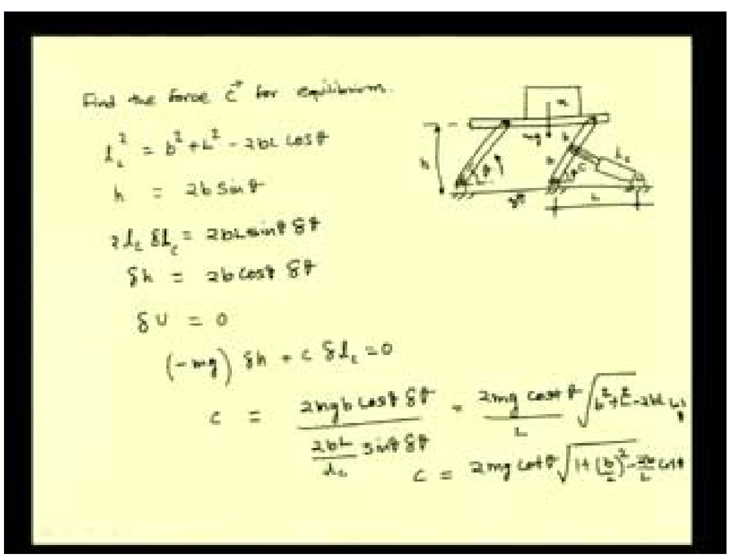


So, we have two forces F and P and we write them P times delta y b both are in the same direction, that is, its positive virtual displacement of this point B and this force P are in the same direction; so, their work is positive, so P times delta y b. Here, the positive virtual displacement is delta y c and the force is f which is in the opposite direction. So, the work done by this force F for this virtual displacement delta y c is negative and so we have it as minus F times delta y c. Now, we can equate this to 0, this quantity, we equate it to 0 for equilibrium, so we have P times delta y b which is l cos theta delta theta minus F times delta y c which has been found to be 2 l cos theta delta theta and this we equate it to 0.

And from this, we get that the force of this spring is equal to P by 2 and we already have this relation, that is, the spring force is equal to the spring constant times 2 l sin theta minus h and from this we can find the position sin theta, which gives the unique configuration of this link, that is, theta which is equal to sin inverse of P plus 2 k h by 4 k l. So, these examples illustrated how we apply this method of virtual work to the system of ideal rigid bodies, where we neglect friction and if there are any members that can absorb energy by elongation or compression, the same can be removed from the system and replaced by an equivalent force and we can apply these principle of virtual work to determine the equilibrium condition or any unknown forces that are involved.

So, we will see one more example on the method of virtual work as applied to connected rigid bodies, in this example, you will clearly see the advantage of using this method in solving these equilibrium problems.

(Refer Slide Time: 54:02)



Here, in this picture, you see an elevator platform which is supporting a mass and there is a hydraulic cylinder that exerts a force at this joint, in order to keep this platform, at say this height of h. The various other dimensions are given to you, if you would like to solve this problem by the conventional method of force and moment equilibrium equations, then you will see that, it is not possible to find this force C, from this given data, because we do not know the position of this mass in this platform, that means, its horizontal position with respect to these joints or with respect to the edges of this platform is not known. So, in this case, we see that information is needed in order to find this force C, if you would like to use the force and moment equilibrium equation.

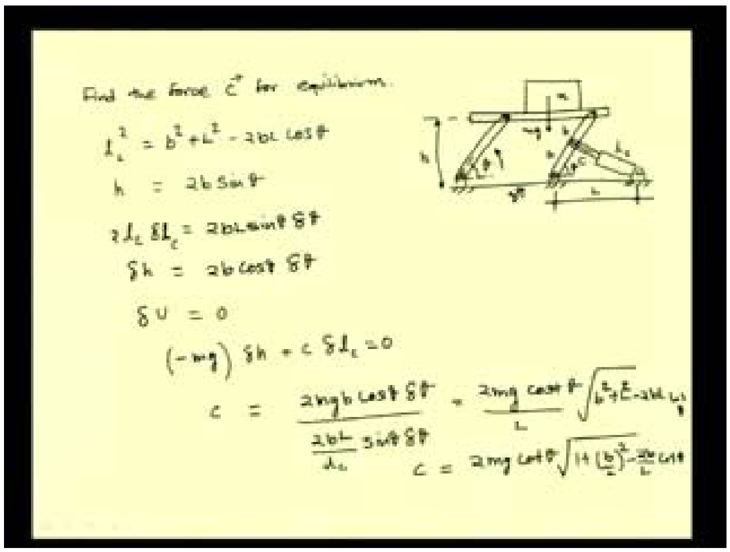
But using this method of virtual work, it is possible to compute this force C, even without knowing these dimensions. So, let us first write the equation, that is, for the virtual displacement; so, let us consider this length l c of this hydraulic link, that is, exerting the force C on to this link at this joint. The dimension l c can be written as b square plus L square minus 2 b L cos theta; this distance l c square is equal to b square plus this distance l square plus the 2 b L cos theta. So, now we can write for this height of this platform in terms of this angle theta, again we see that this system is a single degree of freedom system, which can be defined uniquely by this position theta, so the height of the platform is equal to 2 b sin theta.

So, these measurements are positive from this axis, so it is positive in this way. So, let us now define the virtual displacement by differentiating these two equation; from the first equation, we have 2 l c delta L c is equal to 2 b L sin theta delta theta, because these two things are constant that is b square and L square, so we have from this differentiation of this first equation, the expression for the virtual displacement delta L c. Let us differentiate the equation for the height, so we have delta h is equal to 2 b cos theta delta theta.

Now, the work done by the weight and the work done by this force can be computed, when the link is displaced by an virtual displacement delta theta, this mass is raised or lowered by this amount that is delta h and the length of this link changes by this quantity delta L c. So, these terms will help us derive the virtual work for the weight and the hydraulic force at this joint, and we will define this virtual work and equate it to 0. So, the first work is the work done by the weight, since the platform is raised against the weight it is a negative work minus mg delta h and this is a positive work done by this hydraulic link so we have plus c times delta l c equal to 0.

So, let us substitute these values for delta h and delta c delta l c from these equations. So, we have c equal to 21 m g b cos theta delta theta divided by 2 b L by l c sin theta delta theta or in other words, this is 2 m g cost theta by L root of b square plus L square minus 2 b L cos theta; if we simplify this, further we have it as 2 m g cot theta root of 1 plus b by L square minus 2 b by L cos theta.

(Refer Slide Time: 54:02)



So, this expression now gives the value of the force that has to be supplied by the hydraulic cylinder at this joint, in order to support this weight at this height. So, we see that the method can be conveniently applied to such systems, where it is difficult or not possible to provide detailed information about say the location of this body etcetera and with certain minimal information, it is possible to go ahead with solving for the equilibrium or solving for the unknown forces.

So, in the next lecture, we will see some systems which have also friction, all these analyses we have discussed is for systems without friction; so, we will take up in the forth coming lecture systems which have friction.