Engineering Mechanics

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Module No. # 07

Mass Moment of Inertia

Lecture No. # 02

Principle Mass Moments of Inertia

An IITG person promises only what he can deliver, and an IITG person, delivers what he promises.

So, in the last lecture, we have seen how to determine the second moment of the mass; we will continue in this lecture, some discussion on the same topic. We will see initially some example problems, where we will compute the second moment of the mass, based on the concepts that we have seen in the last lecture.

Later on, we will go on to derive the expressions for principle moments of inertia of the mass; it will be similar to the way we determined the principle moments of the area.

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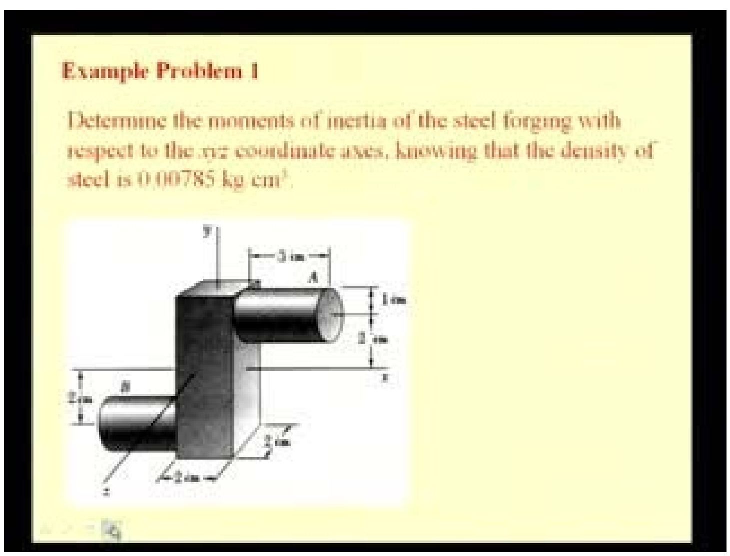

So, let us initially see one example problem to determine the moment of inertia. So, here you see a steel forging and we are interested to determine the moments of inertia of the steel forging with respect to this xyz axes.

The various dimensions have been given like, we know the radius of the cylinder, the lengths of the cylinder, the width, the height, etcetera of this feature. Knowing that this density of the steel is point 0.00 785 kg per centimeter cube, we need to determine the second moment of the mass with respect to the coordinate axes.

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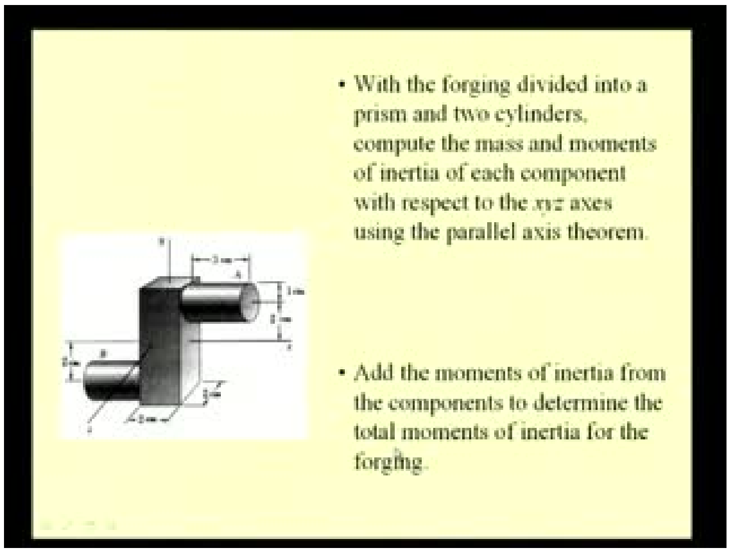

So, what will be our strategy? We will divide the shape into components for which the second moment of the mass is known. So, here we see that this component is made of two cylinders and one prism.

So, we will see that we can decompose this forging shape into two cylinders and a prism, and thus, we can find the moments of the mass of these individual component with respect to the xyz axes, and we will see that, we will be using this parallel axis theorem, because the tabulated values may be with respect to the centroidal axes of the individual components; that means, we may be knowing the second moment of the mass with respect to a centroidal axis of a cylinder.

So, we have to now find the second moment of this cylinder with respect to this axes xyz, for which we may use the parallel axis theorem; then, we will add all these moments to determine the total moment of inertia of the forging.

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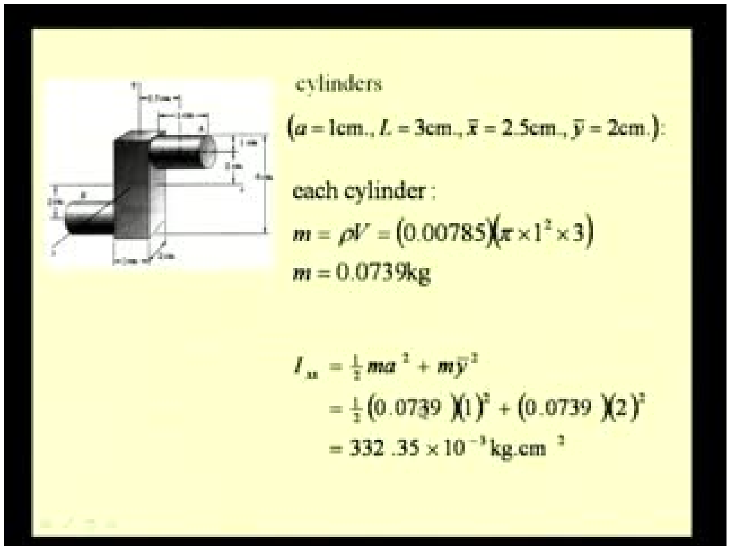

So, let us take first the cylinders; so the cylinder has a radius of 1 centimeter, the length of the cylinder is 3 centimeters and the location of the centroid is x bar, that is along the x-axis is equal to 2.5 centimeters, because we know the length of the cylinder is 3 centimeters; so this is located at a distance of 1.0 centimeters from this phase of the prism and this phase of the prism is located at 1 centimeter from this plane y-z, because we know that this total dimension is 2 centimeters.

So, from this, we know these various quantities say L, a, x bar and y bar from the geometry that is given. Let us find the mass of the cylinders; the mass of the cylinder is rho times V the volume of the cylinder; rho is known; the density of the steel is given to us as point 0.00785 times the volume is nothing but pi r square times h, that is, here in this case the length of the cylinder. So, from this, we get the mass of the cylinder as 0.0739 kg and this is the mass of this cylinder also.

So, now we can write the second moment of this cylinder with respect to this axis, that is, x-axis, I xx is equal to the second moment of this mass with respect to a centroidal axis, which is parallel to this x-axis, which is nothing but the axis of the cylinder itself. So, we know that the second moment of the mass with respect to the cylindrical axis is half ma square, where a is the radius of the cylinder.

So, this is the second moment of the mass with respect to the centroidal axis plus a mass time the distance; the distance is in this case 2 centimeters. So, the distance of the centroidal axis to the x-axis is 2 centimeters. So, we have half mass of the cylinder which has been computed times a square the radius square plus mass of the cylinder times y bar that is this distance square. So, we have this as 332.35 into 10 to the power of minus 3 kg centimeter square.

You would have noted that we are using this unit of length as centimeter; so, we have the unit of this I xx as kg centimeter square. So, we have determined the I xx or the second moment of the mass with respect to the x-axis for the given cylinder and it will the same for the another cylinder which is also located at the same distance from the x-axis.

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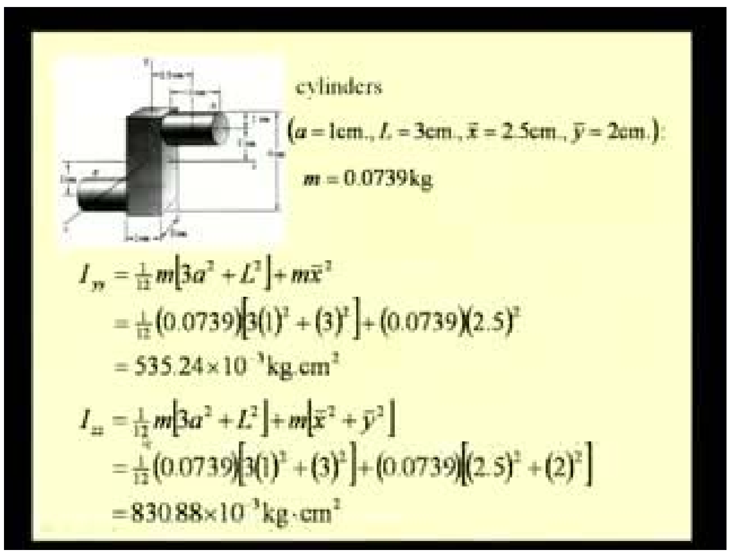

Now, for these cylinders, we can determine the other moments, that is, with respect to y-axis and with respect to z-axis; with respect to y-axis, we write it as the second moment of this mass with respect to its centroidal axis which is parallel to the y-axis. So, the value of this is 1 by 12 m 3a square plus L square, this if you remember in the last lecture, we have tabulated these values plus mass times x bar square, that is, the location of this centroid with respect to this y-z plane along the x-axis, which is nothing but in this case 2.5 centimeters.

So, we have 1 by 12 mass of the cylinder times 3a square is 1 centimeter plus length of the cylinder that is 3 centimeters square plus mass times the distance that is 2.5 centimeter which is the location of the centroid with respect to y-z plane or this is the x bar value. From this, we have the value as 535.24 into 10 to the power of minus 3 kg centimeter square. Now, we can determine the second moment with respect to the z-axis that is I zz and it is equal to 1 twelfth m 3a square plus L square plus m times x bar square plus y bar square.

So, we are interested to find the second moment about this axis; so, it is equal to the second moment with respect to a parallel axis to this z plus the distance of the centroid with respect to this axis which is equal to x bar square plus y bar square. So, substituting the various values, we have the value as 830.88 into 10 to the power of minus 3 kg centimeter square.

So, we have now determined the second moment of the cylinders with respect to x y and z-axis; these cylinders being located in a symmetrical way about the xyz axes. If we determine for one cylinder, the values are the same for the other cylinder also.

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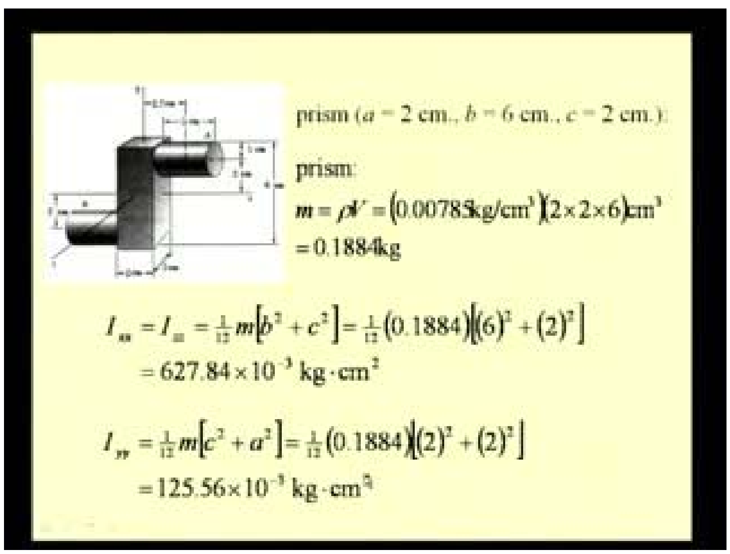

Now, let us move on to determine the second moment of the mass with respect to x y and z-axis for this prism. The various dimensions are that the prism is having a square base; so a is 2 centimeters and c is also 2 centimeters, the length of this prism is the total distance that is we have 3 plus 3 totaling to 6 centimeters; so, the length of this prism is 6 centimeters.

Let us determine the mass; mass is equal to rho times V rho the density of the material times the volume 2 into 2 into 6; so, we have the mass of this prism as 0.1884 kgs.

Now, let us consider one of the axes, say the x-axis and let us try to determine the second moment of the mass with respect to this axis. One can also note that, these axes, that is, x y and z are nothing but centroidal axes as far as the prism is concerned.

So, we have the direct formulas for the second moment with respect to the centroidal axis. And for this prism, the moment about the x-axis and z-axis is same and it is equal to one-twelfth mass times b square plus c square. We have already seen that, let us say if I am interested to compute the second moment with respect to the z-axis, then I have to consider the dimensions, that is, this b and c that is or in this case, this is b and this is c.

So, we have these values as b square, which is 6 square plus c square, which is 2 square times mass times 1 by 12, we have this value as 627.84 into 10 to the power of minus 3 kg centimeter square.

In the same way, now we can determine the other value, that is, I yy which is equal to 1 by 12 m, the dimensions are in the plane x z so that is c square plus a square. So, we have it as one-twelfth mass of the prism times both these dimension equal to 2 centimeters. So, we have this value as 125.56 into 10 to the power of minus 3 kg centimeter square.

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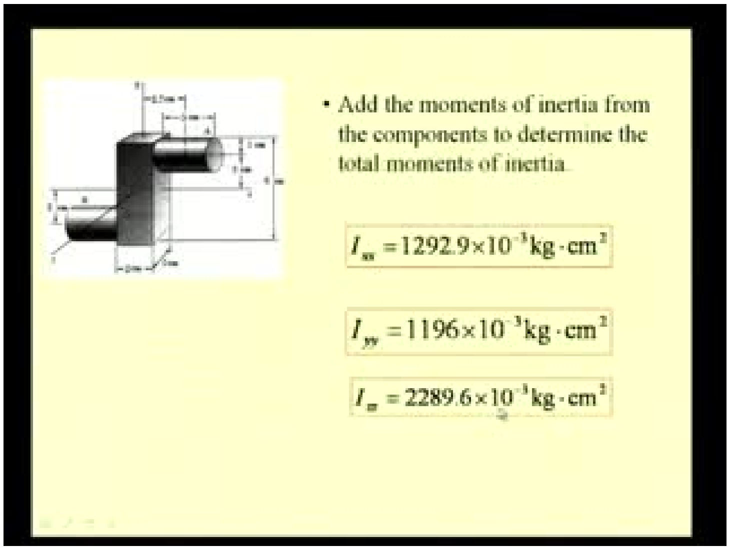

Now, we have determined the second moment with respect to the three axis for the prism. So, in order to now determine the total, we have to sum these values; so, we add all these moment of inertia from the components. So, we have I xx as sum of the second moment of these two cylinders and this prism with respect to this x-axis, which in this case has been determined to be 12929.9 into 10 to the power of minus 3 kg centimeter square. Same way we can add the moment components about y and z-axis and we have them as I yy as 1196 into 10 to the power of minus 3 kg centimeter square and I zz as 2289.6 into 10 to the power of minus 3 kg centimeter square.

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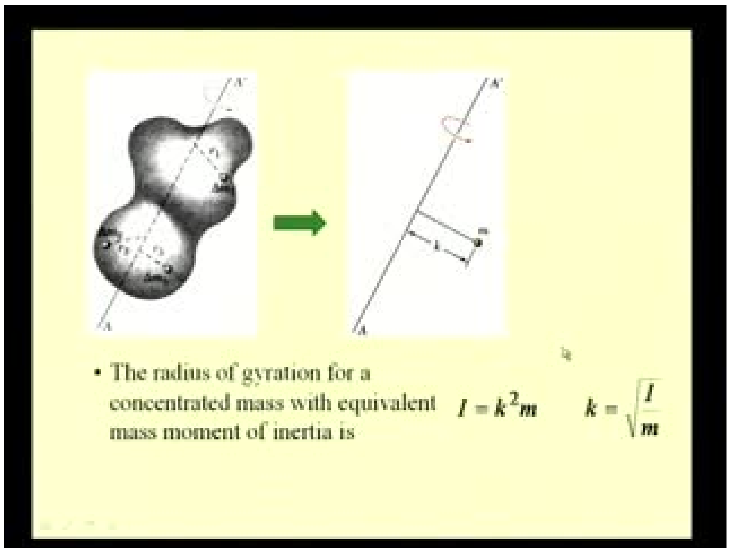

So, this example just illustrated how we can compute the second moment of complex shape by computing the second moments of the component shapes. Let us now see the concept of the gyration or the radius of gyration. We have already seen for the area, how to compute the radius of gyration; same way, let us extend the discussion for the mass moment. So, if we have this rigid body, and these various particles, and we have determined this second moment of all these elements with respect to say this axis A A prime.

So, now, if we can find an equivalent mass - a concentrated mass m - which is equal to the mass of this entire rigid body, and if that is situated at a distance of say a distance k, and the second moment of this element is same as the second moment of the rigid body with respect to the given axis say A A prime, then we can call this distance k as the radius of gyration.

So, we are trying to find an equivalent concentrated mass which has the same second moment of the mass with respect to the given axis. So that is how we defined this radius of gyration. And we have this second moment of this mass with respect to this axis is k square m, which should be equal to the second moment of the mass, this mass of this entire rigid body with respect to this axis A A prime which is say I, then I is equal k square m, and from this, we can find this radiation of gyration k as root of I by m. So, the derivation is similar to the way we derived the radius of gyration for the second moment of an area.

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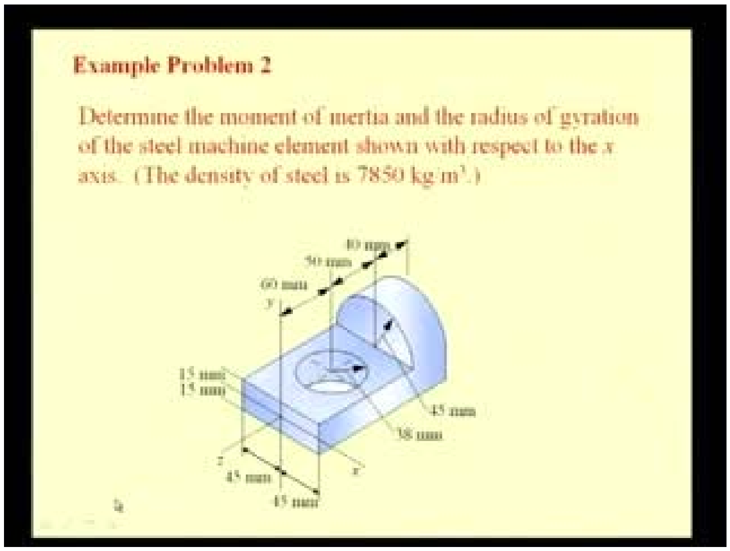

Let us see one example problem. So, here we have a machine element whose shape and let us say the various dimensions are shown; we are fixing the reference frame that is x y z to the front face of this base. We are interested to find the radius of gyration of this element with respect to the x-axis, that is, with respect to this axis the density of the material is given.

So, how do we go about it? First, we have to determine the second moment of the mass of this entire machine element with respect to the given axis. We have to compute obviously the mass of the element, and from these two, we can determine the radius of gyration. In order to determine the second moment of the mass for this shape, we have to decompose into simpler shapes for which the second moment is known. So that is our first step.

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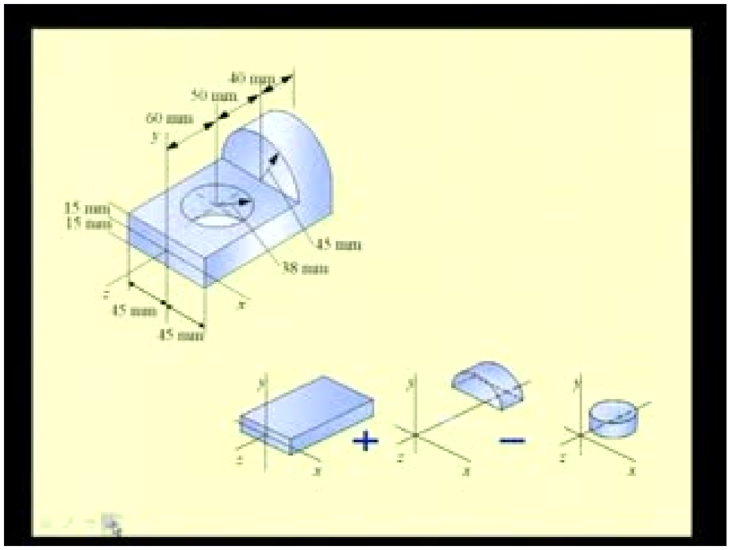

So, we see from the shape, that this element is constituted of prism which constitutes the base, and then it has this semicircular or semi cylindrical feature which has to be added to this volume, and then we have a through hole, and so this volume has to be negative. So, we see that these shapes are simpler shapes, for which the values of the second moments are available in the form of tables and now we can use that to determine the second moment of this machine element with respect to this axis, that is the x-axis.

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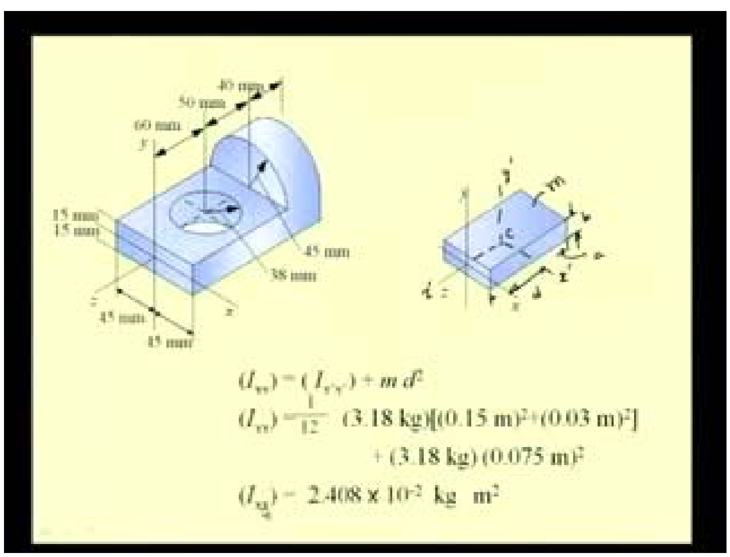

So, let us take one by one the components. First, let us take this base prism and compute its second moment with respect to this axis, that is, the x-axis. So, we write this I xx as the second moment of this prism mass with respect to its centroidal axis, which obviously is a parallel axis and which is passing through the centroid; so this is the x prime axis, y prime and z prime, the centroidal axis. So, the second moment of this mass with respect to x-axis is equal to second moment of the mass with respect to its centroidal axis plus m times d square, where d is this distance and m is the mass of this element.

For this prism, we know the second moment with respect to the centroidal axis as 1 by twelfth mass time this dimension square plus this dimension square. So, if I say this to be a and b, so it is 1 by 12 mass time a square plus b square. So, what are these values? This a is nothing but this total length of the machine element which is from this, we see 60 plus 40 plus 50 that is 150 mm or in meters it is 0.15 meters; we have this, the dimension a, and b the thickness is nothing but 15 plus 15 that is 30. So, in terms of meters it is 0.03 meters.

So, this quantity is the second moment of the mass of this element with respect to its centroidal axis plus mass times the distance d that is 0.075 meters. This mass value that is 3.18 kg has been computed from its volume; so, here I am not showing the computation for the mass.

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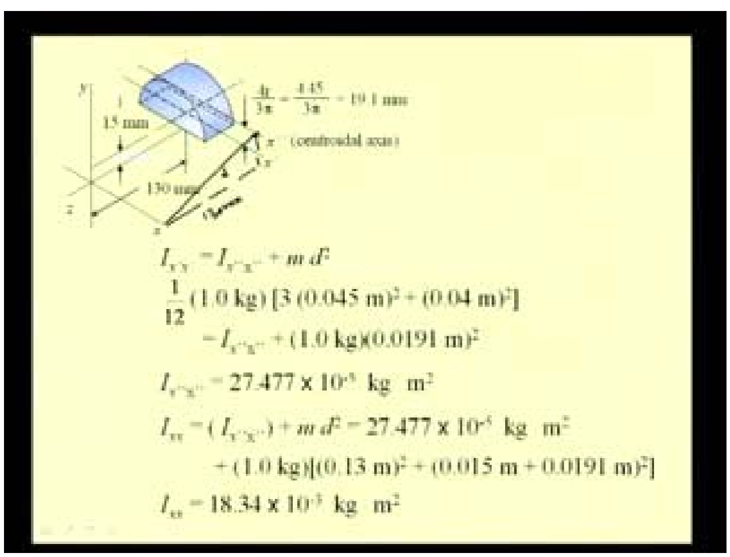

So, we have this I xx as 2.408 times 10 to the power of minus 2 kg meter square. So, let us now find similarly, the second moment of the other components. Let us take this semi cylindrical feature, the centroidal axis is x double prime and this axis, that is, x prime is an axis which is parallel to this centroidal axis, but it is lying on the base of this semi cylindrical feature or I can say that diametrical plane of this cylindrical feature.

And we have the axis x about which we are interested to find the second moment. So, all these axis, that is x, x prime and x double prime are all parallel axes and so let us compute their second moments. The second moment with respect to x prime is known; so from that, we determine the second moment about the centroidal axis using the parallel axis theorem, and then, we again apply the parallel axis theorem to compute the second moment with respect to x-axis, when we know the second moment about the x double prime axis.

So, this distance, we know it is 4r by 3 pi which is 19.1 mm; the other dimensions come from the geometry of the machine block or the machine element. So, we first determine the second moment with respect to the centroidal axis. So, we write this I x prime x prime or the second moment with respect to this x prime axis is equal to the second moment about the centroidal axis centroidal axis which is parallel to this x prime plus m times d square, where d is this distance.

So, from this, we get this value from the table as 1 by 12 mass times 3a square or in this case, r square, where r is the radius which is 0.045 meters plus the length square. So, length is in this case it is 40 mm or this distance; so we have it as 0.0 4 meter square.

So, this quantity is the second moment of this semi cylindrical feature with respect to this x prime x-axis , that is, equal to I x double prime plus mass times the distance which is 19.1 mm or 0.0191 meter square.

So, from this we get the second moment with respect to the centroidal axis that is x double prime and we get this I x double prime as 27.477 into 10 to the power of minus 5 kg meter square. Now that we have determined the second moment with respect to this axis, we again apply the parallel axis theorem to determine the second moment with respect to this axis x.

So, now we write I xx is equal to the second moment with respect to the centroidal axis. You can note at this point that we write always the parallel axis theorem by considering one axis as the centroidal axis.

So, I x double prime plus m d square; in this case, this distance d is the distance between this axis x and x double prime. So, it is the distance between this and this; this is the distance which is equal to this distance square is equal to this distance, which is 130 mm and the vertical distance that is 15 mm.

So, we have this as 27.477 into 10 to the power of minus 5 kg meter square, and which is the quantity, that is 1 kg the mass of the element times 0.13 meter square, this 130 mm plus this distance, that is 0.015 plus 19.1 mm or 0.0191 whole square. So, from this, we get the value of I xx as 18.34 into 10 to the power of minus 3 kg meter square.

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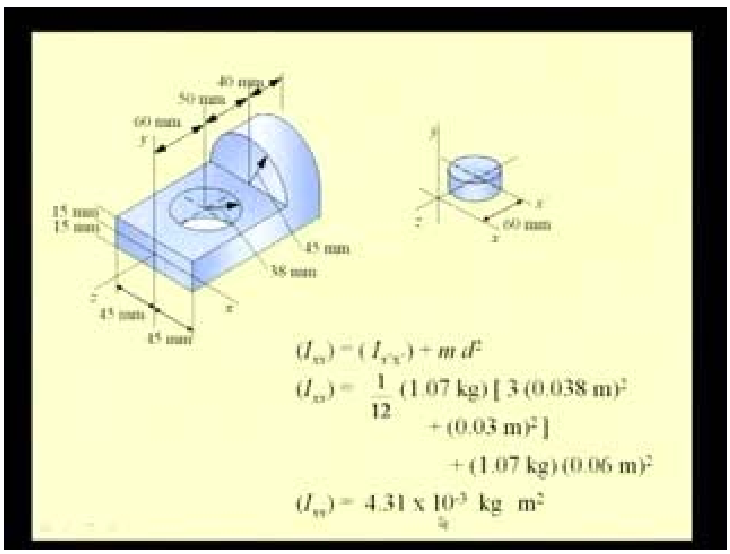

Now, we can move on to the third element, that is, the cylindrical feature that has to be negated; that is, this cylindrical feature which when negated gives this machine block.

The centroidal axis is situated at a distance of 60 mm from this x-axis and also we see that this x prime lies in the same plane as z x, because this cylindrical feature has the same thickness as the prism feature that we have considered.

So, we again write the parallel axis theorem, that is, I xx is equal to I x prime x prime the centroidal axis plus m d square. The second moment with respect to this centroidal axis is 1 by twelfth mass times 3 radius square; in this case, the radius of this cylinder is 38 mm. So, we have 0.038 meter square plus the length square in this case, the length is 30 mm, so we have 0.03 meter square plus mass times d square or this distance square which is 60 mm or 0.06 meter square and from this we get the I xx as 4.31 into 10 to the power of minus 3 kg meter square.

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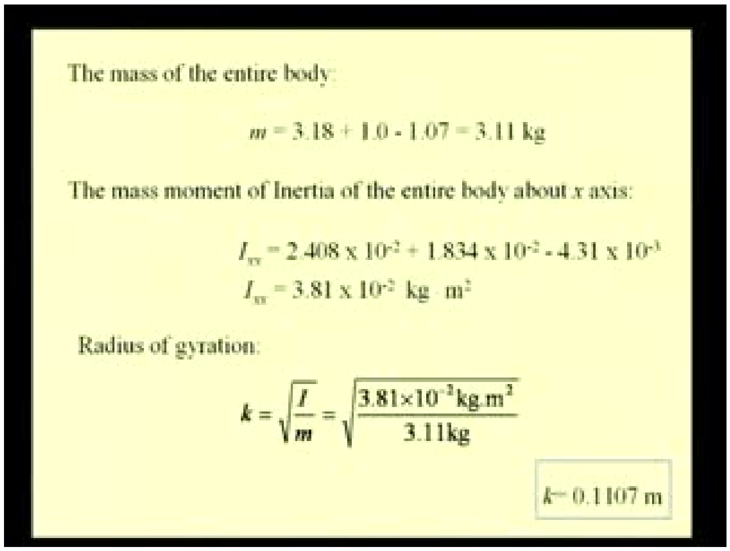

Now, we have determined the second moment of the various components with respect to the x-axis. Now, we can do the required algebra, that is, we sum the second moments of the prism plus the second moment of the semi cylindrical feature and we negate the component corresponding to the hole or the cylindrical feature.

So, first we determine the mass, the mass is 3.11 kg, and next, we determined the second moment of inertia; we sum the components corresponding to the prism base and the semi cylinder feature and negate the second moment corresponding to the whole feature. So, we get I xx as 3.81 into 10 to the power of minus 2.

So, we know the total mass of this machine element; we know the total second moment of this machine element with respect to the x-axis. Now, we can determine the radius of gyration as root of I by m which is this quantity; we find this to be 0.1107 meter. So, this example again illustrated how to compute the second moments and the application of the parallel axis theorem, and also we determined the radius of gyration.

So, we will continue the discussion; we are now interested to find the second moment of the mass with respect to, let us say an arbitrary axis; we have seen how to compute with respect to the principle axes that is x y and z.

Now, we are interested to compute with respect to an arbitrary axis passing through the same origin. This motivation comes from the fact that if we have determine the second moment of the mass with respect to a given axis system, by let us say the method of integration or by any other method, like you know summing up the component moments, we are not interested again to do this computation when the axis is getting rotated and also we are interested to find the principle moments of inertia; that means, the axis system for which the second moments become extremum.

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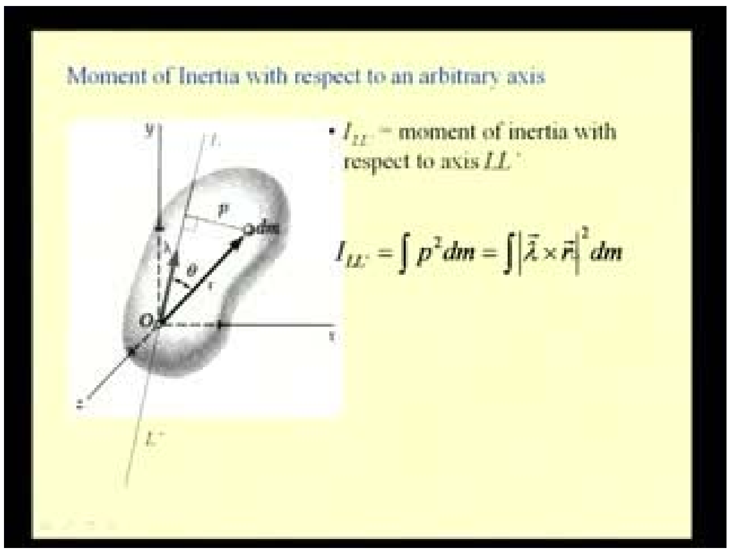

So, first let us see how to determine the second moment of the mass with respect to an axis passing through the origin of a given axis system. So, here in this picture, you see the axis xyz passing through this origin O, for which the second moment of this mass as well as product of inertia of this mass have been computed r is known.

Now, we are interested to find the second moment of the mass with respect to an arbitrary axis say LL prime passing through this origin O. Let us take any element dm, which is having a distance of r from the origin. So, we have already seen that the second moment of this body with respect to the origin is invariant, that means, we saw the sum of the individual moments of inertia, that is, I xx plus I yy plus I zz is invariant with respect to rotation.

Let us consider this lambda as the unit vector along the direction of our interest, that is, LL prime passing through O, and let theta be the angle between the vector r and this unit vector lambda, and let p be the perpendicular distance of this elemental mass d m from this axis LL prime. So, the moment of inertia of this mass with respect to this axis LL prime is nothing but p square d m and if we integrate this value, we will get the moment of inertia of this mass with respect to this axis LL prime; so we have it as integral p square dm.

Now, let us write this p with respect to the known vectors. So, we can write this p square as the square of the magnitude of this vector lambda cross r. So, lambda vector is the unit vector along this LL prime axis and r vector is the position vector of this mass element with respect to this origin. So, if we try to expand this term and write in terms of its components, then we can get a relation between this second moment with respect to LL prime to the second moment with respect to the principle axes.

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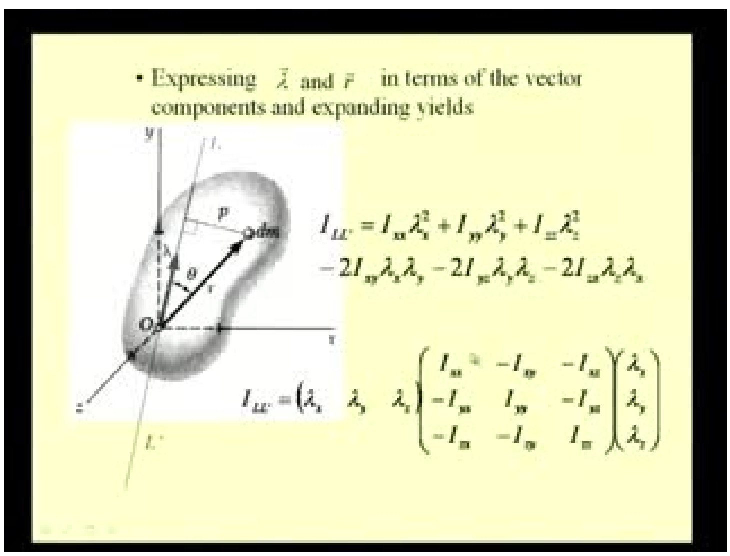

So, let us write these vectors that is lambda and r in terms of their components, that means, lambda x, lambda y, lambda z, etcetera and also expand the square of the cross product term; we will get upon simplification I xx times lambda x square plus I yy times lambda y square plus I zz times lambda z square; here this lambda x lambda y and lambda z are the direction cosines of this unit vector lambda minus 2 times I xy the product of inertia times lambda x lambda y minus 2 I yz lambda y lambda z minus 2 I zx lambda z lambda x.

So, these quantities that is I xx, I yy, I zz, I xy, I yz and I zx are nothing but the inertias that are known with respect to the x y and z-axis. So, if we know the inertia tensor, all these six values are known to us and if we know the direction cosine of the axis for which we are interested to find the second moment, then we can find this value, that is, I LL in terms of these products of inertia.

So, we can write this again in the tensor notation; this is the inertia tensor or the inertia values with respect to this xyz axes and this is multiplied by the direction cosines of the axis for which we are interested to know. So, I LL is equal to lambda x lambda y lambda z multiplied by the tensor matrix and multiplied by the transpose of this direction cosine vector, that is, the column vector lambda x lambda y lambda z.

If you multiply this and simplify it, you will get this same equation. Now, this is a more compact form; we know this inertia tensor for a particular axis, then if we know the direction cosine, then we can determine the second moment with respect to any axis passing through the given origin or the origin of the reference frame for which this inertia tensor has been computed.

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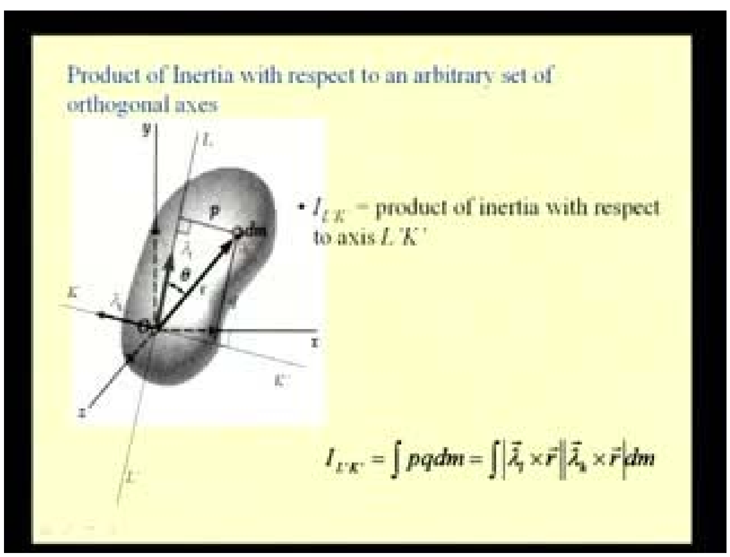

Now, let us try to determine the product of inertia with respect to two perpendicular arbitrary axes. So, here again, we take the same object; now, we consider one more arbitrary axis say KK prime such that this LL prime and KK prime are orthogonal axes or perpendicular axes and let this lambda l be the unit vector along this axis, that is, LL prime and lambda k be the unit vector along the axis KK prime, and say theta is the angle between this LL prime axis to the position vector r of this elemental mass dm and we have this perpendicular distance that is p from LL prime axis and q from the KK prime axis. So, for this elemental mass dm, p and q are the perpendicular distances from these two orthogonal arbitrary axes.

So, now, we are interested to find the product of inertia with respect to these two orthogonal axes that is L and K. So, we are interested to find this I L prime K prime which is equal to the product of inertia with respect to these two axes

So, what is the product of inertia with respect to these two axes it is nothing but the product of the distance of this elemental mass from these two axes that is p and q. So, we have integral p q dm as the product of inertia of this body with respect to these two orthogonal axes. So, again we write this p and q in terms of the vectors that is lambda L lambda K and r; p is nothing but the magnitude of lambda l cross r vector, and q is nothing but the magnitude of lambda k cross r vector.

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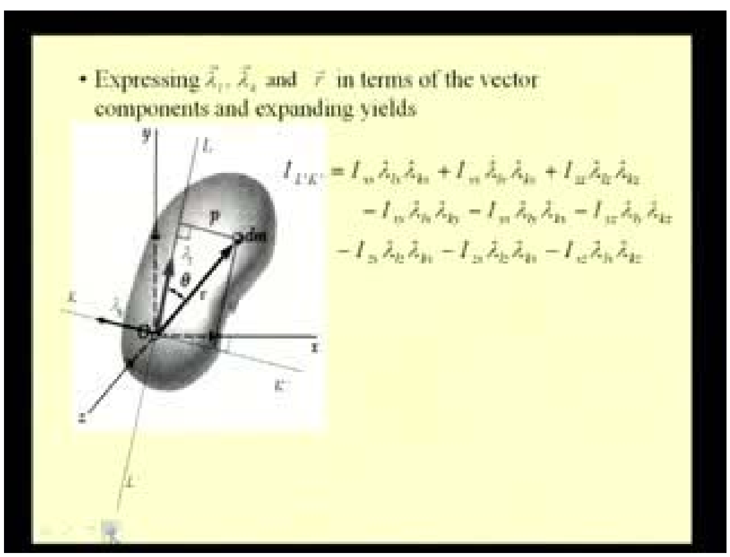

So, let us now expand this term and write it in the vector notation that is the x y and z component of these various vectors, that is lambda L lambda k and r are written in terms of their x y and z components and now we expand it and write it. So, we have this I LK or the product of inertia of this mass with respect to this axes is equal to I xx times lambda l x lambda kx plus I yy times lambda ly lambda ky plus I zz times lambda lz lambda kz. Then, we have the product of product terms that is minus I xy lambda lx lambda ky minus I yx lambda ly lambda kx minus I yz lambda ly lambda kz and again, we have the three more product of inertia terms that is minus I zy lambda lz lambda ky minus I zx lambda lz lambda kx minus I xz lambda lx lambda kz.

So, these are the components of these vectors along the x direction and these quantities that is I xx, I yy, I xy, I yx, etcetera are nothing but the inertia tensor associated with the x y z axes.

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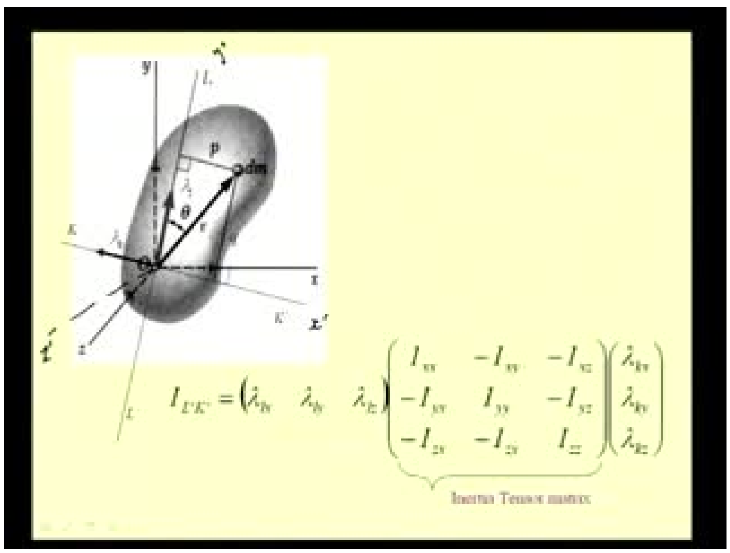

So, this can again be rewritten as lambda lx, lambda ly, lambda lz are the direction cosines of this vector LL prime, direction cosines of this unit vector along this LL prime axis times the inertia tensor for the xyz axes times the transpose of the direction cosine vector of the KK prime axis that is lambda kx, lambda ky, lambda kz.

So, now, we see that the product of inertia again can be written in terms of the inertia tensor matrix. Now, that we have the expressions for the product of inertia as well as the second moment of inertia with respect to an arbitrary axis, it is possible to find the inertia tensor associated with an arbitrary axis orthogonal system which is rotated with O, that is, now I can define an another vector say z prime I can call this vector as x prime and I call this as y prime, all these vector that is Oy prime, Ox prime and Oz prime are orthogonal and thus define a new set of principle axes.

So, if we know the second moment of this mass with respect to the original axis that is xyz, then it is possible to determine this inertia tensor matrix for an arbitrary orthogonal set of axes that is Oy prime, Ox prime, Oz prime passing through the same point.

So, now, it is possible for us to rotate this axis and find these values of second moment for all the set of orthogonal axes. And as our discussion for the second moment of the area, these second moments of the mass also reach some extremum value for a given orthogonal set of axes and those we call it as the principle moments of inertia of the mass.

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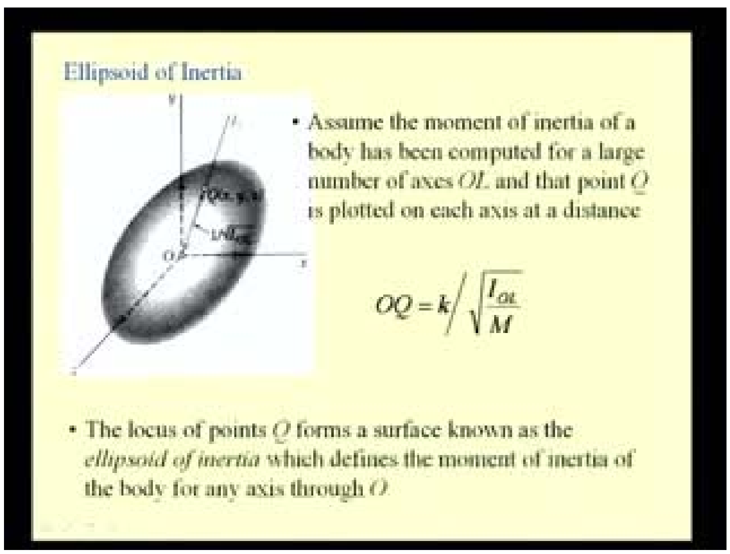

So, we represent these principle moments of inertia of mass by constructing what we call it as an ellipsoid of inertia. So, let us see how we do that let us assume that the moment of inertia of a body has been computed for large set of axes, say OL. So, we have computed the inertia tensor matrix for a given axis say Oy, Ox, Oz; now, using this inertia tensor, we can compute the second moment of mass with respect to arbitrary axes, for let us say infinite number of such axes we have computed and the value of the second moment for a particular corresponding axis is, say the value Q and we plot it by taking this value Q along that axis at a distance of 1 by root of I OL, that is, the second moment of this mass with respect to that axis OL.

So, if we try to plot all such points, we will see that they will lie on an ellipsoid surface. So, the distance at which we are plotting is equal to k, the radius gyration divided by root of I OL divided by M, where M is the mass of this rigid body I, OL is nothing but the second moment of the mass with respect to that axis and k is the radius of gyration of the mass or the rigid body.

So, the locus of all such points Q forms the surface which we call it as the ellipsoid of inertia and this reaches extremum values. As we will see that, the distance between the surface of this ellipsoid and this point O reaches extremum say somewhere here and it may also reach extremum somewhere here and also in the vector that is perpendicular to this plane of the paper. So, it has three extremum values like, we have seen in case of the second moment of inertia of area, we have seen that it reaches two extremum values corresponding to the two points on the diametrical axis of the inertia circle or the Moore's circle that we called.

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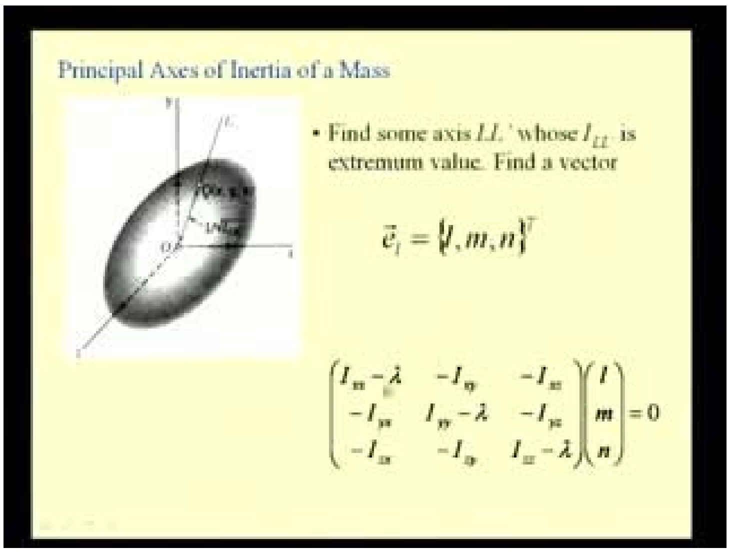

Here in this case, we describe a similar kind of a graphical analogy that is the inertia ellipsoid, where we have three extremum values; let us see how to compute them. So, for some axis, let us say LL prime whose second moment is the extremum value; we are interested to find the direction cosine of that vector. So, if we assume that the second moment of the mass reaches a extremum for given axis say LL prime.

Then we are interested to find the direction cosine and if let us say l m and n are the direction cosines of that vector, we write the inertia matrix times this direction cosine that is l m n should be 0 for some values of this lambda.

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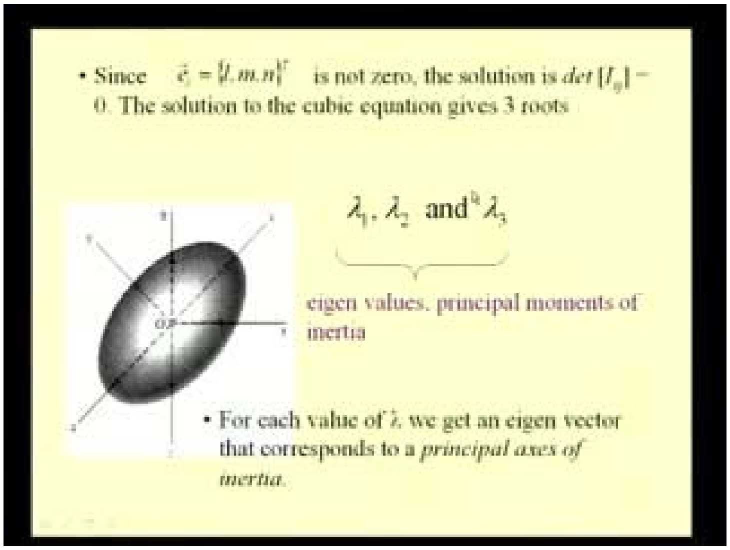

So, we are interested to find the value of this lambda for which this equation is satisfied. We know that the direction cosines do not become 0 all together, because you know l square plus m square plus n square is 1. So, this matrix has to be 0 and if we solve for this, it will give three roots, because we will see that the determinant of that matrix, that is, determinant I ij or determinant of this inertia tensor matrix equal to 0 gives a cubic equation for which there are three roots, that means, there are three values of lambda.

Say we are getting the values lambda 1, lambda 2 and lambda 3, which are known as the Eigen values in your first curves on mathematics, you would have studied about computing this Eigen values, and these Eigen values are nothing but the principle moments of inertia of the given rigid body and for each of these values, we will get 1 direction cosine say if we substitute lambda 1 in the equation, we will get 1 solution that is l 1, m 1, n 1 and same way if we substitute the other two Eigen values, we will get the other two vectors. So, these three solutions will give the three vectors which are the corresponding principle axes.

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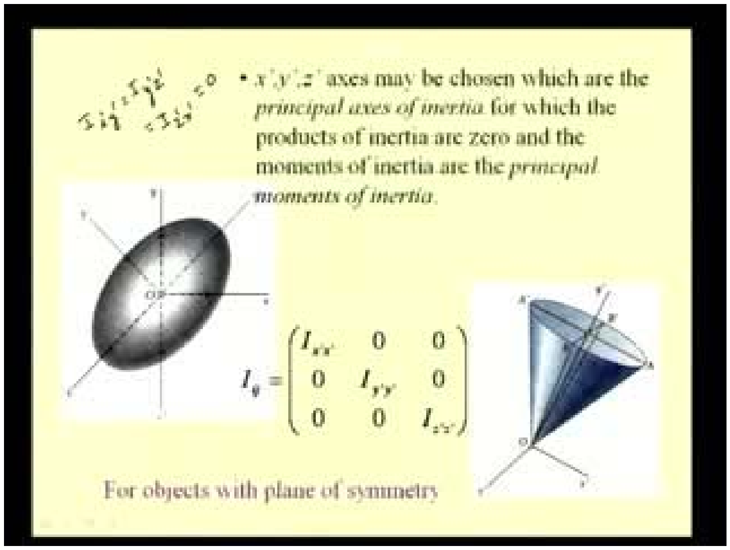

So, if x prime, y prime, z prime are the principle axes of inertia for a given rigid body, then the product of inertias are 0. Let us say the product of inertia, that is, I x prime y prime equal to I y prime z prime is equal to I z prime x prime is equal to 0 and the moments of inertia are the principle moments of inertia, that means, it becomes a this inertia tensor matrix becomes a diagonal matrix. So, all these off diagonal elements are 0, because these products of inertia are 0. So, the inertia tensor matrix for this set of orthogonal matrix that is x prime, y prime, z prime becomes a diagonal matrix with off diagonal elements as 0.

So, for shapes which have the planes of symmetry, say in this case, you see a cone which has two planes of symmetry say the plane containing A prime, A and O and the plane containing B, B prime and O are the two symmetrical planes. So, these two planes define the principle axis of inertia. So, we have this axis that is B prime, A prime and y prime are the axis of the cone defining a set of orthogonal axis for which these moments of inertia are the extremum values and the product of inertias are 0. So, if in a shape, we can find planes of symmetry, then computing these principle axes also becomes easy.

So, in this lecture, we saw how to determine the second moment of the mass with respect to say given axis and then, we also used the parallel axis theorem to compute the second moments; we also saw the concept of the radius of gyration and we solved some examples and then, we discussed some aspects of finding the second moment with respect to an rotated axis, which we used to determine the principle moments of inertia for a given solid.

So, these concepts will be useful for computing the dynamics of the rigid bodies and we are interested to find the response or the angular acceleration and acceleration of a body when subjected to a given force and a couple.