Engineering Mechanics
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Module 7 Lecture 16
Second moment of Mass

Today, we will see some topics on mass moment of inertia. For your reference, this is module 7 lecture number 16 of the engineering mechanics course. The context in which we would like to study the second moment of mass is to understand the dynamic behaviour of the objects when subjected to forces and moments. For translation, one will see that the resistance to the motion is offered by the mass of a body; that means the body accelerates when subjected to a force or a set of forces and the response that is the acceleration, depends on its mass.

The same way, for the angular displacement, that is when an object or a rigid body is subjected to a moment or a couple, the response that is in terms of the angular acceleration will depend upon the mass moment of inertia of the body. When we will take up the topics on dynamics, you will see it in more detail, but right now we will see how to determine this quantity - that is mass moment of inertia or the second moment of the mass. So, the concepts are nothing but an extension to the mass, because we have seen in the earlier lecture how to determine the second moment of the area and so forth.

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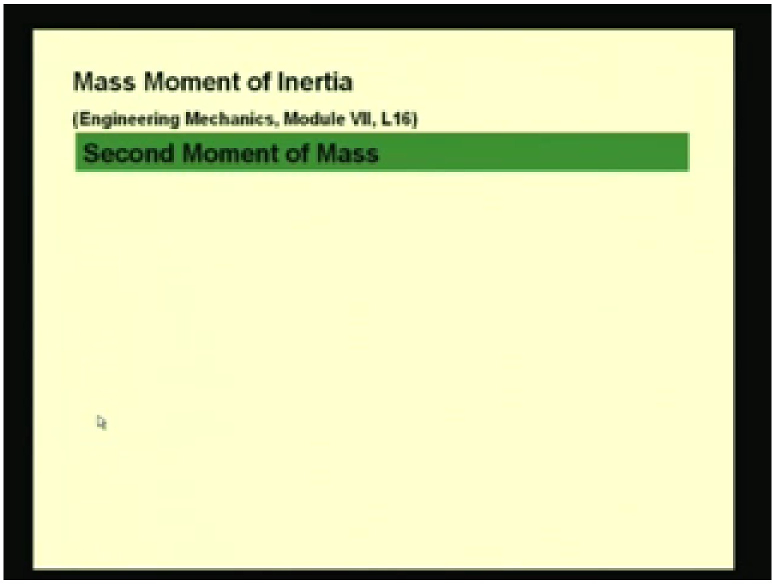

Today, we will see how to determine this second moment of mass or what we call as the moment of inertia of a mass.

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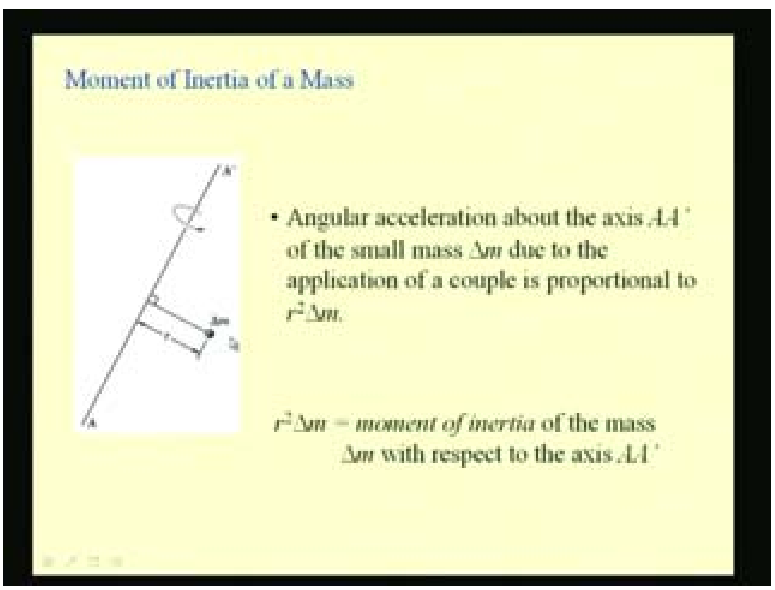

Let us consider one small volume or an elemental volume delta M, which is situated at a distance of r from an axis that is AA prime. The response, that means, if a torque is applied whose vector is along this AA prime will induce a rotation of this mass about this axis AA prime. The response in terms of the angular acceleration will depend upon the distance as well as the mass of this particle. That is why we are interested to compute this term, the moment of inertia of this mass about this axis say AA prime.

The angular acceleration is proportional to r square times the mass of the particle or an object. Since we are assuming that the mass is very small there is no difference between various particles of this unit with respect to this axis AA prime. That means, the distance of various particles of this delta m is assumed to be located at a distance of r from the axis.

In that case, the angular acceleration is proportional to this term that is r square delta m, when a torque is being applied. So, this term is the moment of inertia of the mass delta m with respect to the axis AA prime. If we know the moment of inertia of a small mass, then we can compute the moment of inertia for a rigid body by considering elemental masses.

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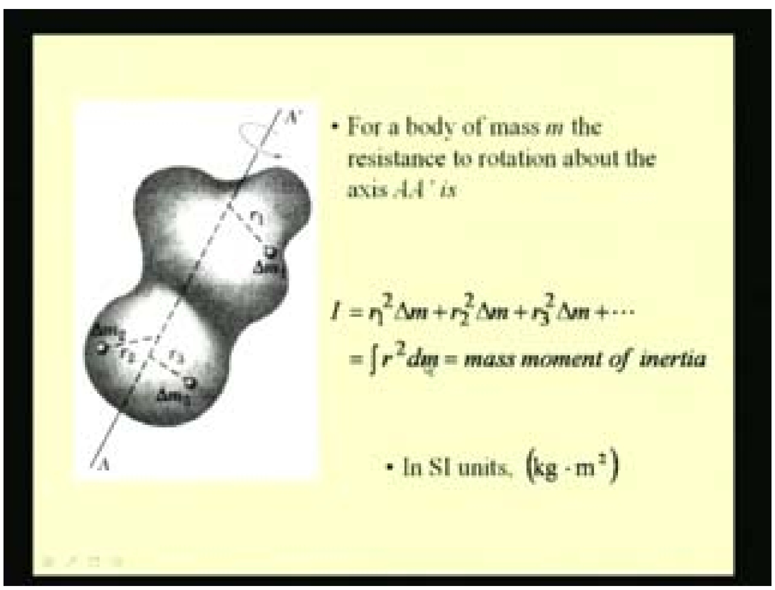

Here, we are seeing one rigid body and we are interested in computing the second moment of this mass or moment of inertia of the mass with respect to this axis AA prime. The moment of this entire mass with respect to this axis is nothing but summation of the second moments of the various particles or various elemental volumes that constitute this rigid body.

We have these masses delta m_1 m_2 m_3 and there are infinite such kind of masses that constitute this entire rigid body. This mass delta m_1 is situated at a distance of r_1 from this axis AA prime. Similarly, for other particles like delta m_2 and $m_{3,}$ we know their distance that is r_2 and $r_{3,}$ which are the distances from this axis AA prime. So, when we know these quantities then we can compute the second moment as the summation of the second moments of these particles.

We write, I, the second moment of this mass about this axis AA prime as summation of these second moments of the various constituent particles that is r_1 square delta m plus r_2 square delta m plus r_3 square delta m, etc. In the limit, this becomes dm and this is the integration over this complete volume. We have r square dm and this is the mass moment of inertia of this body. The unit if you will see is kg meter square because the unit of the mass is kg and r square is meter square. So, the unit of the second moment of the mass is kg meter square.

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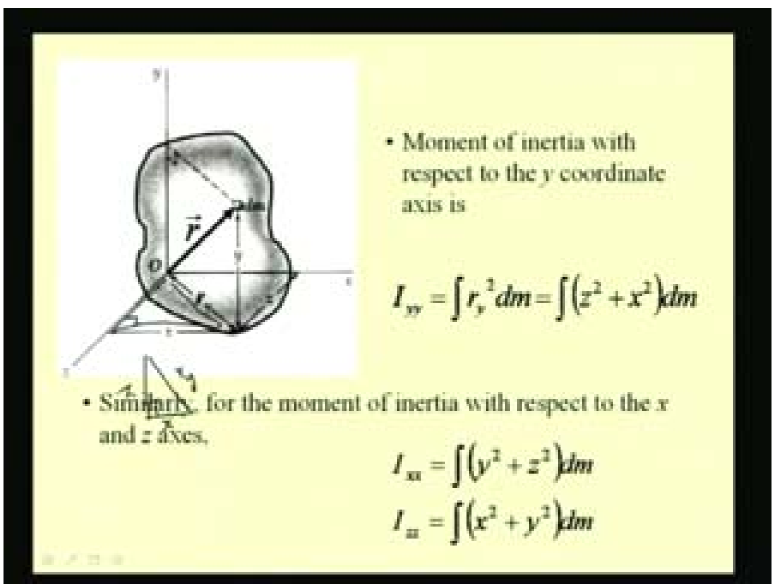

We are interested to know the second moment of a given body with respect to the principal axes with respect to a particular coordinate system. We are interested to find the second moment with respect to x, with respect to y, z, etc., because we have seen earlier that a motion or the angular velocity can be decomposed along their coordinates that is x,y and z.

If we know their second moment of a given mass with respect to the various principal axes, then one can also analyze the motion by considering these quantities along with the components of forces and the torque in those principal axes. So in that context, we are interested to find the second moment about the various principal axes x, y and z.

Let a mass of value dm or a particle whose mass is dm be situated at a distance of r from the point O which is our origin of the particular coordinate system. This vector r gives the position of this particle with respect to this axis. So, the coordinate of this point is x, y and z along the various principal axes. We have the second moment of this entire rigid body with respect to the axis y, which is I_{yy} is equal to integration the distance of this particle from this axis, that is y axis, which is this distance, which let me call the projection of this vector on the xz plane. This quantity, that is r_y, is the projection of this vector on this xz plane which gives the distance of this particle from the y-axis.

We have integral r_y square dm; this gives the second moment of this mass with respect to the y-axis. But from the geometry, we see that r_y is nothing but z square plus x square, so this comes from this right angle triangle. So, we have from this right angle triangle because this angle is 90 degrees. I can redraw it for your convenience; so this is your r_y, this is z and this is x in the xz plane. We have r_y square as z square plus x square dm which is nothing but the coordinate of the particle along the z-axis and along the x-axis. Similarly, we can define the moments with respect to the other two axes that is x and z as $I_{xx,}$ which is nothing but the second moment of this mass with respect to the x-axis. It is defined similarly as integral y square plus z square dm and I_{zz} or the second of moment of the mass with respect to the z-axis as integral x square plus y square dm. So, in this way, we can compute the second moment of this mass with respect to the principal axes.

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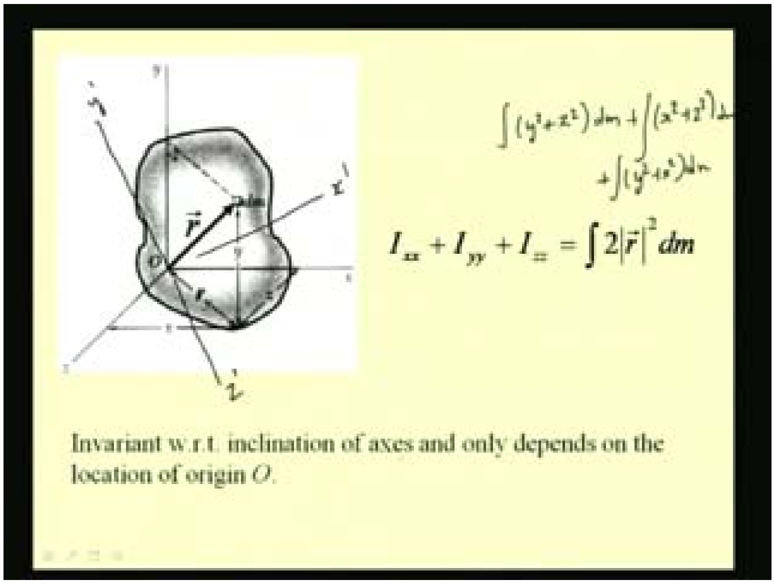

Let us sum these values. If we sum all these values, we will see that this I_{xx} is nothing but integral y square plus z square dm. The same way, we are adding up the other quantities that are integral x square plus z square dm plus integral y square plus x square dm, which is nothing but two times x square plus y square plus z square which is equal to this magnitude square of this vector r. So, we write it as integral two times r bars magnitude square.

Thus, we see that this quantity, that is the summation of the second moment with respect to the principal axis that is x y and z is invariant, irrespective of any coordinate system that we may take. Let us take another coordinate system, y prime x prime z prime which is a rotated coordinate system. Even if we will compute the corresponding second moments and sum it up, it will be equal to this quantity as far as this origin point O is considered. Only if we displace this point, the second moment, the summation of the second moments will change. We see that it is invariant with respect to the inclination of the axis and only depends on the location of this origin, O.

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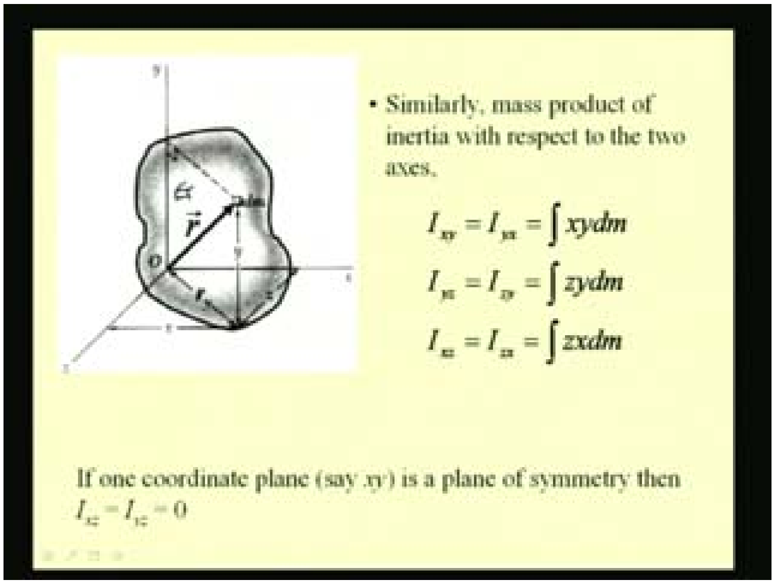

Similarly, we can define the product of inertia. So, we have seen when we computed the product of inertia of an area we take the distance of the elemental area from the corresponding two axes; that is, we are interested to find the second moment with respect to x and y, we take the distance from these two axes and compute the product. Same way, we have this I_{xy} that is the product of inertia of the mass with respect to the axis x and y, which is same as I_{yx} equal to integral xy dm. Same way, we have I_{yz} as integral zy dm and I_{xz} as integral zx dm.

These three quantities define the product of inertia of mass with respect to the two concerned axes. We know the second moment of the mass with respect to the principal axis and as well as the product of inertia for a particular principal plane. If one of the coordinate planes, say xy is the plane of symmetry, if this plane that is xy is plane of symmetry, then it is possible to have two elements; one in the front of this plane xy and the other at same distances that is z_1 is on the positive z and other is on the negative z direction.

The sum of these two will become 0 and thus these two quantities that is I_{xz} as well as I_{yz} become 0. That is, when we compute this quantity I_{xz}, we have one element in the front of this plane xy and the other at the back situated at equal distances z and so their summation becomes 0. If a body has a plane of symmetry, then the product of the inertias, in this case if the plane of symmetry is xy then I_{xz} and I_{yz} will be 0. This can be conveniently used to evaluate the product of inertias.

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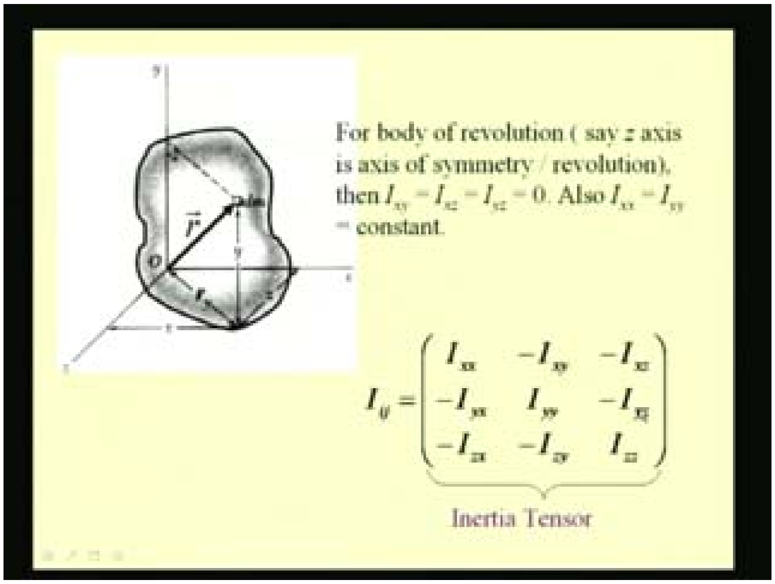

Now we extend the discussion that if the body is a body of revolution say it is about the z-axis, then it has two planes or all these three planes are symmetrical that is then we have I_{xy} is equal to I_{xz} equal to I_{yz} equal to 0. Also this I_{xx} equal to I_{yy} is constant because we have this z axis as the axis of symmetry, the quantities that are computed that is I_{xz} and I_{yz} becomes 0 and also in the plane xy the body has the symmetry and so this quantity that is I_{xy} also becomes 0.

We have seen the three principal moments or the moments of the inertia with respect to the principal coordinate axes that is x, y and z and also the product of inertias with respect to the planes. So these quantities can be conveniently written in a matrix form. We have this I_{xx}, I_{yy} and I_{zz} the second moment with respect to the axis xy and z and the product of inertia terms that is I_{xy} I_{xz} and I_{yz}; this matrix is symmetrical about the diagonal.

We have this I_{yx} and I_{zx} and I_{zy} terms in the lower triangle which is same as this upper triangle of this matrix. We have put this negative sign; we will see the reason for that when we will use this matrix for the computation in dynamics. We call this as an inertia tensor because it follows all the laws of tensor transformation and this is a very convenient way of representing the various moments in a concise form.

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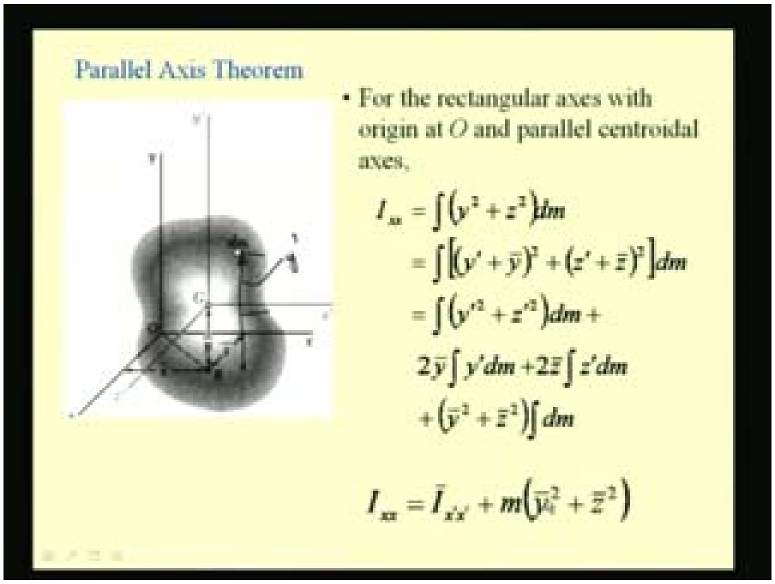

Let us see how we obtain the parallel axis theorem for the moment of inertia of mass. The motivation is again that for a given body if we have obtained the second moments with respect to a given axis, then we do not want to do the same procedure, that is, by the integration in order to find the second moment with respect to an axis system which is parallel. So, we would like to avoid that computation.

Here, in this picture, you see a rigid body for which we have determined by integration the second moment with respect to this axis system that is Oy Ox Oz. Now, we are interested to find the second moment with respect to an axis which is parallel to our original axis system. Let us consider this y coordinate or this plane xz and we have this z prime x prime plane in the parallel axis.

G is the centroid of this rigid body and the coordinate system that is y prime, z prime, x prime has its origin at this centroid G. This point G is located at a distance of x bar, y bar, z bar are nothing but the coordinates of the centroid with respect to our original axis that is Oxyz. Let us write these equations for computing the second moment with respect to the x-axis. We are interested to find this quantity $I_{xx.}$ As per our discussion earlier, this is equal to integral y square plus z square dm. We write this y square and z square terms in terms of the centroidal coordinate and the distance of an element mass in the axis that is centered at the centroid.

We are writing the coordinate of this location dm with respect to this transformed coordinate plus the distance by which it has been transformed. We have y as y bar or it is nothing but this is the distance y. This is equal to this y bar plus a distance y prime, which is the distance of this particle dm in this frame that is y prime G z prime x prime. So, we write this y square as y prime plus y bar square. In the same way, z square is z prime plus z bar square. Let us expand this and write. So, we have these two quantities, that is, y prime square plus z prime square dm.

So integral y prime square plus z prime square dm plus the two quantities that is 2y bar integral y prime dm, then we expand this and 2z bar integral z prime dm. Since these two quantities, that is y bar and z bar, are constant we are taking it out of the integration; then we have the last two terms that is y bar square and z bar square. Since these are constants, we take out of the integration and write plus y bar square plus, you know z bar square integral dm.

What are these two quantities that is integral y bar dm. It is nothing but the first moment of the mass with respect to a centroidal coordinate. So, this term becomes 0 because it is possible to find the mass element which is having a negative y value corresponding to this mass. So, this integral becomes 0, as well as this integral that is integral z prime dm also becomes 0. So, we have left with these two terms and that is what we write it as, what is this term? This is nothing but the second moment of the mass with respect to this axis that is x prime, because we see that this is of the similar form of this, we know that this is the second moment of the mass with respect to xx. So, this term is the second moment of the mass with respect to x prime axis which is nothing but a centroidal axis. So, we write it as I bar. This notation helps us to know that this is a second moment of the mass with respect to a centroidal axis. So, this x prime axis is passing through this centroid.

We have this quantity as I bar x prime x prime plus what is this quantity? This is integral dm is nothing, but the mass of the body. So, we have plus m times y bar square plus z bar square this gives the relation between the second moment of a mass when computed based on two parallel axes that is xx and x bar x bar. One has to again remember or keep it in note that this derivation is based on the fact that one of the axes is passing through the centroid.

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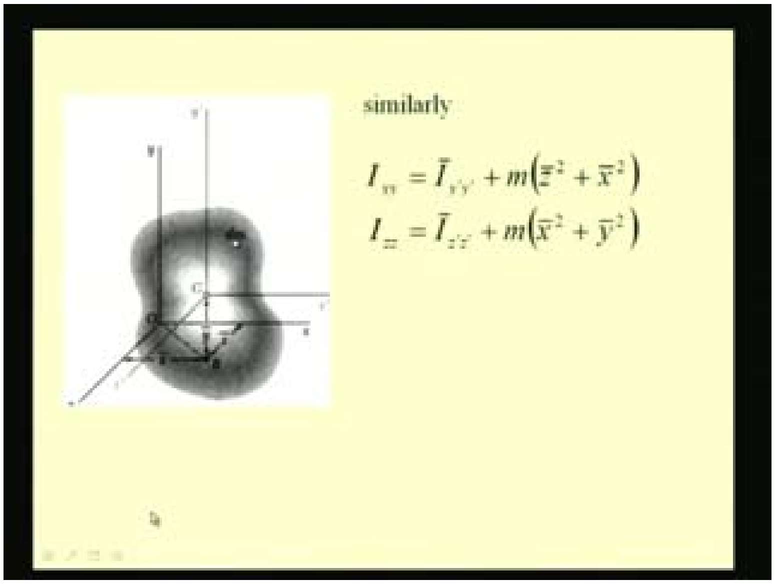

In the same way, we can write for the other two axes that is y prime as well as z prime. We have I_{yy} equal to I bar y prime y prime plus m, the mass of the body times z bar square plus x bar square. In the same way we have I_{zz} as I bar z prime z prime plus m times x bar square plus y bar square. In fact, it is possible to write this relation for any axis that passes through a centroid and a parallel axis.

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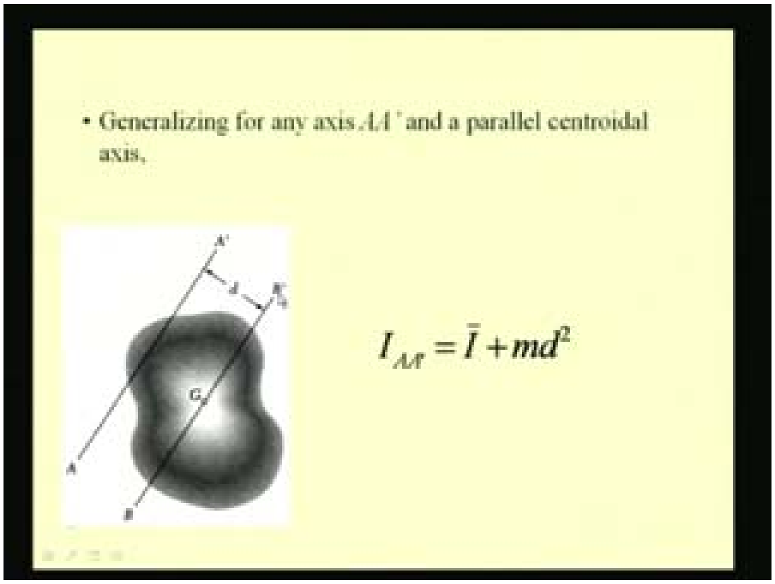

That is what we see here; let us say this is the rigid body and this is the centroid. So, this axis is the centroidal axis; that is BB prime. Let us say in the space we have another axis AA prime which is parallel to this centroidal axis BB prime and it is situated at a distance of D. Then we can write I_{AA} prime or the second moment of the mass with respect to this axis is equal to I bar that is the second moment of the mass with respect to the centroidal axis plus md square, mass into this distance square. Now, we have seen the parallel axis for the second moment of the mass with respect to an axis. Let us see how to derive the expression for the product of the inertia.

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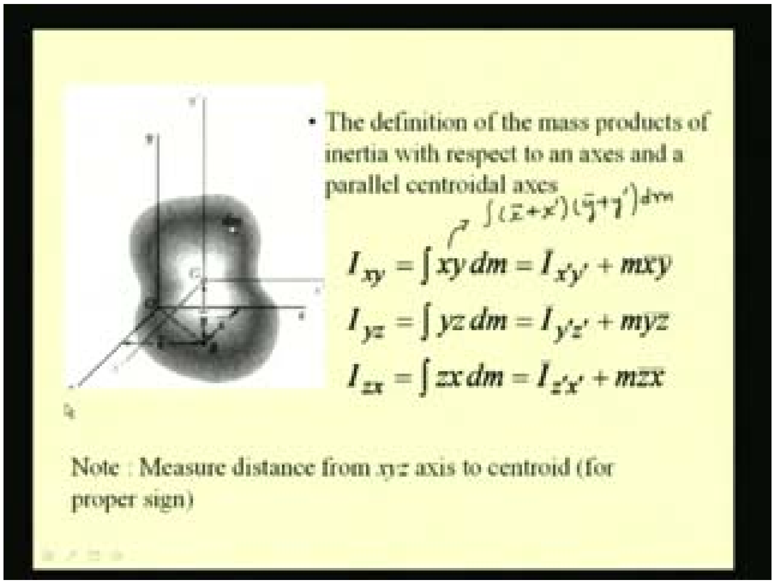

Again, we consider the same rigid body and the axis system; that is, we have this x prime y prime z prime passing through the centroid; we have another set of axis which is parallel to this centroidal axis that is Oyxz. We write I_{xy} as integral xy dm. So, if I write this term as integral, x becomes x bar plus x prime and this y becomes y bar plus y prime dm. If I take the products and see I have x bar y bar integral dm which is integral dm is mass, so we have m times x bar y bar; so we have this term m times x bar y bar. Now, if you take these two products that is x bar y bar, then it is same as this terminology and it is I bar x prime y prime. The other two products that is integral x bar y prime dm is nothing but the summation of the first moment of the mass with respect to a centroidal axis. Same way, this quantity that is x bar y bar dm integral of this term x bar y bar dm is again nothing but the first moment of the mass with respect to a centroidal axis.

These two terms will become 0 and we are left with only two terms that is I bar, the product of inertia with respect to a centroidal axis. So, I bar x prime y prime plus mass times the distance x bar y bar. Same way we can derive and get the expression for I_{yz} as I bar y prime z prime plus m y bar z bar. One has to again remember that these are x bar z bar are nothing but the coordinates of the centroid. The same way, we have the product of inertia I_{zx} as integral zx dm or when we expand it in this form and simplify, we have it as I bar z prime x prime plus m times z bar x bar. One has to note that these distances have to be measured from the coordinate system Oxyz. So, these quantities can be positive or negative depending upon the sign. So, this centroid measured positive in this axis that is Oxyz.

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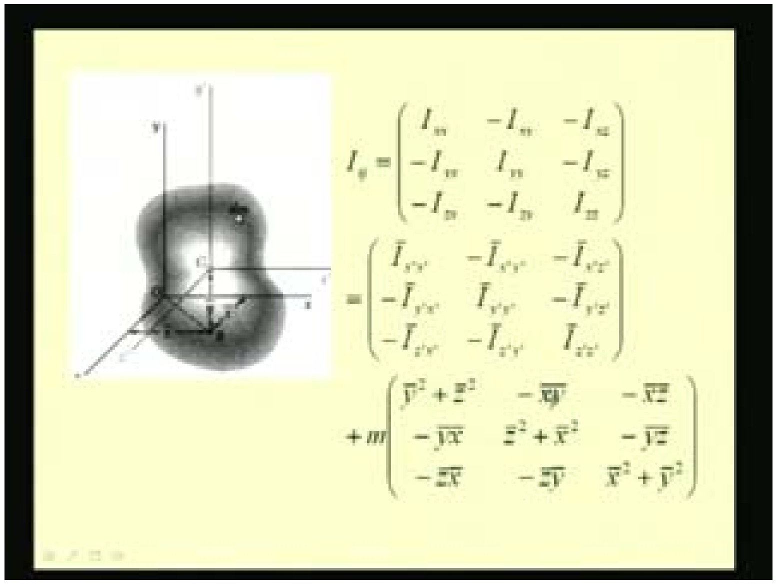

Let us try to write this in the matrix form that is the inertia tensor. So, we have this quantity I_{xx} minus I_{xy} minus $I_{xz.}$ This vector or this tensor is the inertia tensor in the coordinate frame Oxyz. Each of these terms can be written with respect to the term in the centroidal axis by the parallel axis theorem. So, we apply that for each of these terms and when we expand it and write, we have it as the inertia tensor in the centroidal axis, that is I bar x prime x prime I bar x prime y prime. This is nothing but product of inertia in this plane x prime y prime.

Same way, this I bar x prime x prime is nothing but second moment of this rigid body with respect to the centroidal axis x prime. When we write all these individual quantities in terms of their centroidal axis and the location of the centroid that is x bar y bar z bar in this coordinate frame Oxyz, we have this as the inertia tensor in the centroidal frame of reference plus m times y bar square plus z square minus x bar y bar minus x bar z bar etcetera. We already know that these are the coordinates that we have obtained from the parallel axis theorem. So again, writing it in this tensor notation helps in solving the problem and also this gives concise way of representing the various second moments of the mass.

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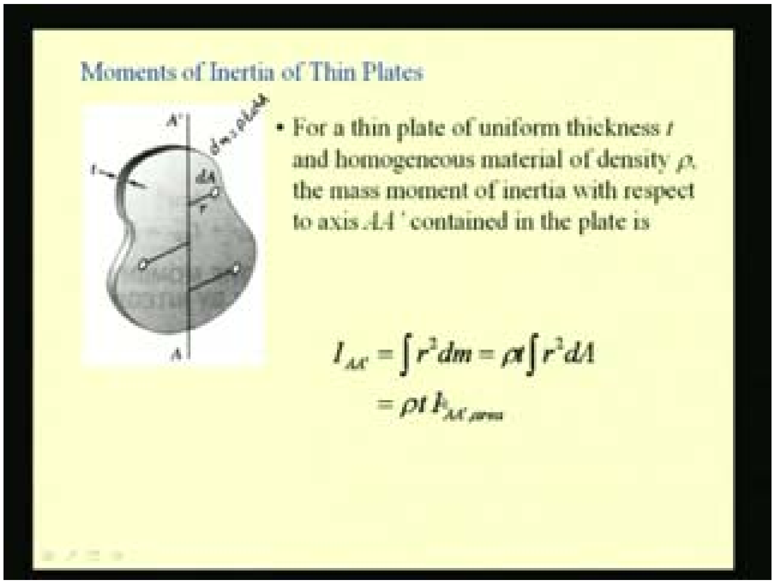

Let us see how to derive this mass moment of inertia with respect to thin plates because they are predominantly used in construction; many of the engineering objects involve these thin plates. So, if we know how to compute the second moment of these thin plates, we can then go on to determine the dynamic behaviour of these plates when subjected to some torque or some forces. Here, in this picture, you see a thin plate whose thickness is t and it is constituted of homogeneous material; that means the density of the material does not vary from one point to another point in the given object. Let us consider this axis AA prime about which we are interested to find the second moment of this plate.

This axis obviously lies in the plane of the object. If we take this as an area element, then we have various elemental areas say dA, which are situated at distance of r from this axis AA prime. The elemental mass is nothing but dA times t, the thickness, which gives the volume of that element times the density of the element. So, rho t dA will be nothing but the corresponding mass element dm will be equal to rho the density times the thickness times this differential area dA. Since we are assuming that this t is constant and this rho is constant, if we use this area element to determine the second moment, we have already seen how to determine the second moment of the area with respect to given axis, then we can also find the second moment of the mass with respect to the same axis.

That is what we are going to do now. So, we have this I_{AA} prime which is nothing but the second moment of this mass element with respect to this axis AA prime. Let us say r is the distance at which this is located. We have integral r square dm and dm we know it is rho times t times dA and that is rho and t being constant, we take it out of the integration and we have this as rho t integral r square dA. What is this quantity? Integral r square dA is nothing but the second moment of this area with respect to the axis AA prime.

We write it as rho times t times I_{AA} prime. So we see the significance of finding the second moment of the area because we can find the second moment of the mass which has direct relevance in terms of the dynamics of the given rigid body.

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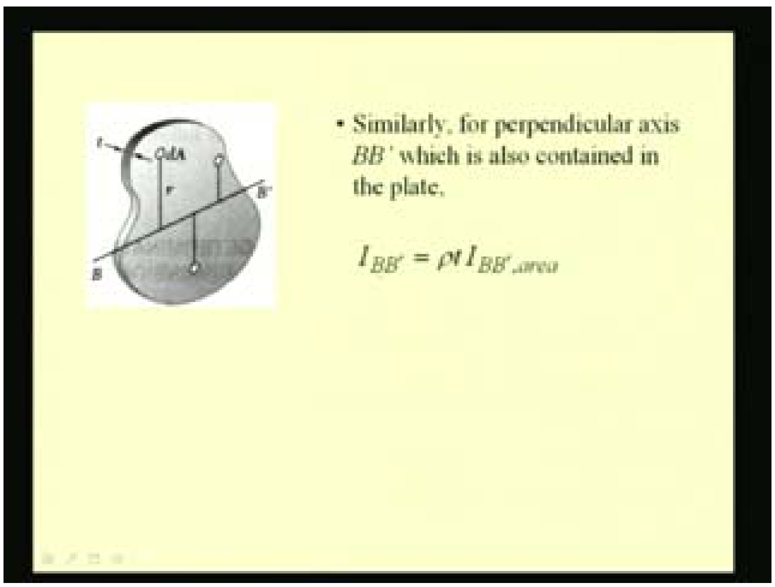

Let us take another perpendicular axis BB prime. Again, we can derive this and prove that this quantity I_{BB} prime is nothing but the second moment of this plate with respect to this axis. BB prime is equal to rho t which is density times the thickness times I_{BB} prime area; that means second moment of this area with respect to this axis BB prime.

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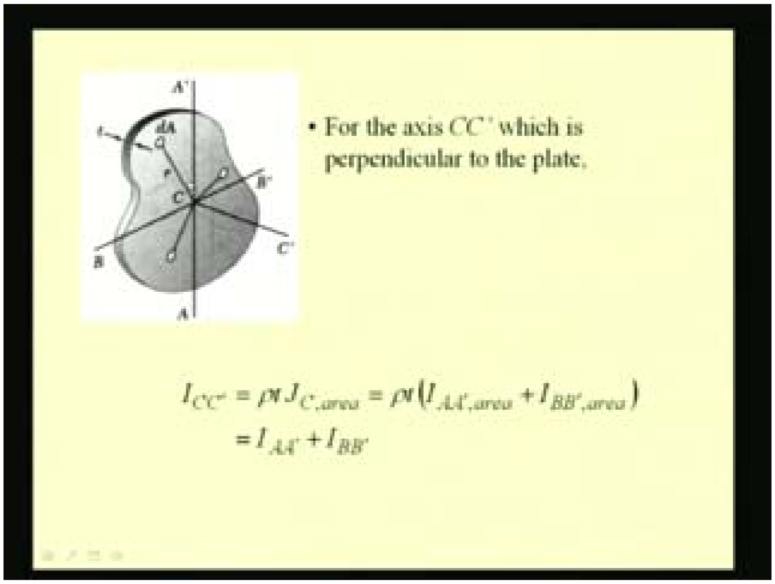

Let us take the perpendicular axis that is CC prime which is perpendicular to the given thin plate. We have already seen how to determine the polar moment of inertia, that means, the second moment of this area with respect to an axis perpendicular to the given plane of the area. Let us now extend the discussion, to the second moment of the mass. Let us say, we are interested to find this second moment of the mass with respect to this axis CC prime.

We have these various area elements in this thin plate situated at these distances r. Again, we can write the element dm as rho times t times the area element dA, then we can find that this I_{CC} prime or the second moment of the mass of this plate with respect to this axis CC prime is equal to rho t J_C, that means the polar moment of this area with respect to this axis CC prime. We already have the relation between the polar moment of the area with respect to the rectangular second moments of the area.

That is this $J_{C,area}$ is equal to I_{AA} prime plus I_{BB} prime, the second moment of the area with respect to two perpendicular axes. So, we thus have the polar moment of inertia of this thin plate, the polar mass moment of the inertia with respect to this axis CC prime as sum of the rectangular second moments that is I_{AA} prime plus I_{BB} prime. So, these discussions clearly follow our derivations for the area moments.

For thin plates, if we have found the second moment of the area with respect to given axis it is possible to find the second moment of the mass without any additional lengthy computation. We do not need to go in for the integration again with respect to mass elements. Already we have seen that, for many of the areas which are typically used in engineering construction, we have these values tabulated and thus can be directly taken for our computation.

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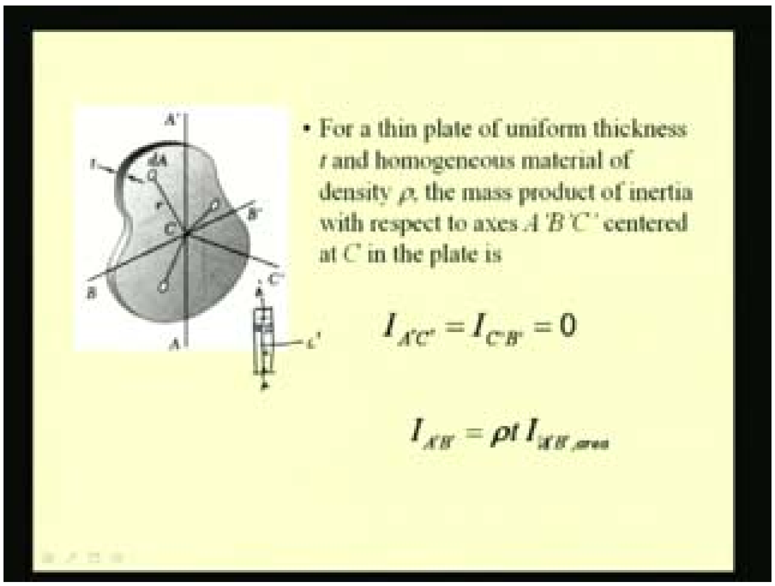

Let us discuss how we go about finding the product of inertia. So, here we are taking again the same plate again. It is having a homogeneous material, so this density rho does not vary and thickness is also constant. Let us try to find the product of inertia with respect to this axis A prime B prime C prime, which is the centroidal axis for this plate. Let us say C is the centroid and we are interested to find the product of inertia with respect to the centroidal reference frame that is A prime B prime C prime.

Let us write these two quantities: I_A prime C prime and I_C prime B prime. We will see that this quantity is 0. Why? Because if we view this plate along this axis BB prime, this is the axis BB prime, we have this CC prime axis and A prime A axis, then we see that we have the mass elements which are situated at equal distances from this axis. So, in the integration, they will cancel out and since this thickness t is uniform throughout and this frame that is A prime B prime C prime is a centroidal frame, you will see that this I_A prime C prime as well as I_C prime B prime, where we involve this axis C prime about which, the body has symmetry. So, these quantities become 0 and the product of inertia of this mass element in this plane; that is the plane A prime B prime is equal to rho t times the product of inertia of the area that is I_A prime B prime area.

We see that for a thin plate, we can determine all the quantities that are involved in the inertia tensor, those nine elements of the tensor can be found by finding the area moments first and then finding mass moments.

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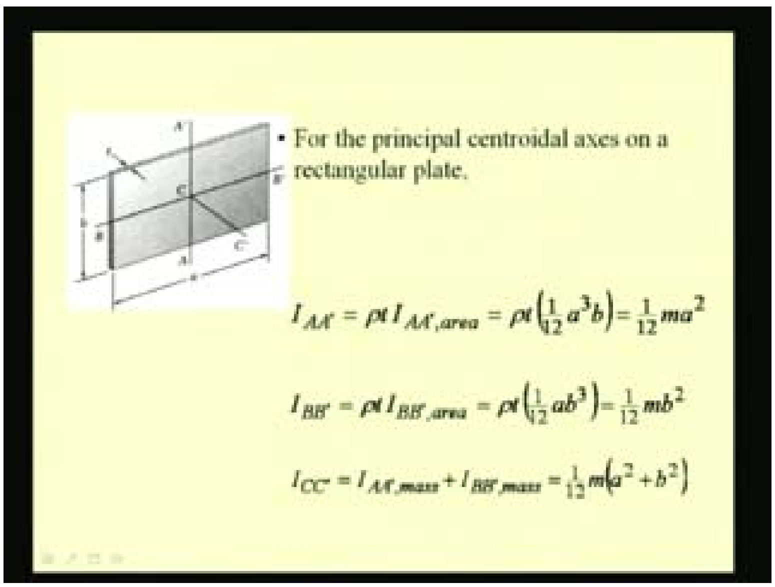

For example, let us take this rectangular plate. We have C, the centroid and this centroidal frame A prime B prime C prime, thickness of this rectangular plate is t and the other two dimensions are A and B. So, we have the second moment of the mass with respect to this axis AA prime as rho times t times the second moment of this area with respect to the same axis that is AA prime that is I_{AA} prime area. Already in our earlier discussion, we have seen I_{AA} prime for area as 1 by 12 a cube b.

The mass moment of inertia of this plate with respect to this axis AA prime becomes 1 by 12 mass times a square. Same way, for the other axis that is BB prime it is rho times t times the second moment of the area with respect to the same axis, that is BB prime and this becomes 1 by

12 m b square. For this axis perpendicular to the plane of this rectangular plate that is CC prime, we have this being the polar axis we can sum the moments with respect to A and B. So, we have the summation of these two leading to 1 by 12 m times a square plus b square.

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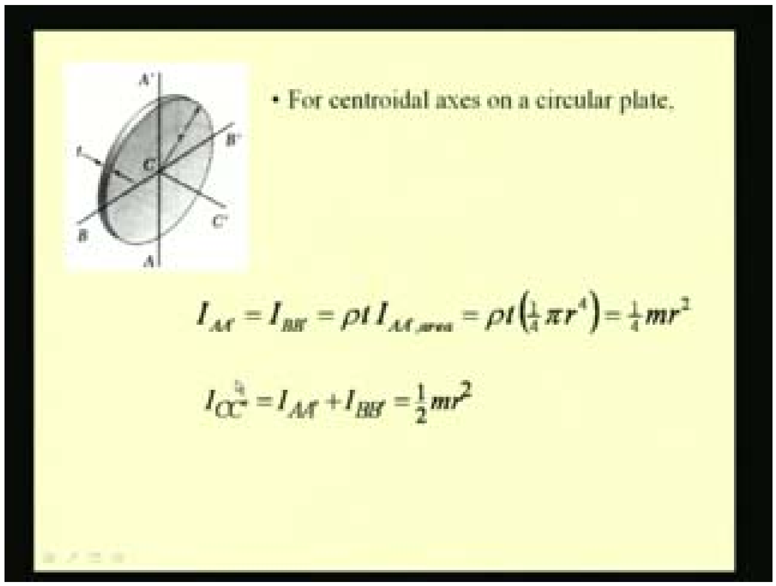

Let us take another example. Here you see a circular plate. Again we will see how we extend our findings for the second moment of the area in order to compute the second moment of the mass for this thin circular disc. Let us say we have a disc of radius r and thickness t and we have the centroidal frame of reference A prime C B prime C prime. We have the second moment with respect to the axis AA prime which also equal to BB prime, because we have seen that this is a symmetrical body and these two quantities are the same.

We earlier computed the polar moment of inertia and from that, we computed these values for the area moments that is I_{AA} prime area. We found it to be 1 by 4 pi r to the power of 4 and to compute the mass moment we multiply it by rho times t. So, this becomes 1 by 4 mass times r square. Mass is nothing but rho times the volume. Volume is pi r square times t; t is the volume of this disc and so from this, we get this expression that is 1 by 4 m r square. The polar moment of this inertia with respect to the perpendicular axis that is CC prime is nothing but sum of these two quantities which becomes half m r square. These examples just illustrated how we can extend our discussion for finding the mass moment of area and relating it to the area of the thin plate.

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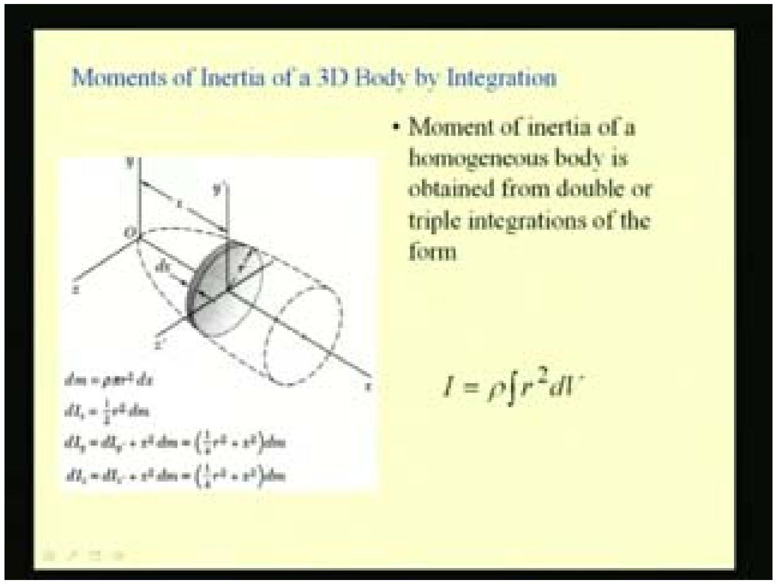

For three-dimensional objects, when we perform this integration in order to determine these moments, as we discussed for finding the moments of the area, it is convenient to choose appropriate elements in order to avoid the double integral or triple integral. So, here you see one object which is having two axes of symmetry or it is an object of revolution with respect to this axis Ox. So, about this axis Ox it is symmetrical. It is possible to constitute a thin disc as the elemental volume for the integration. We consider the mass of this thin plate and the second moment of this small disc element and then integrate the same in order to obtain the second moment of the mass of this complete object with respect to any given axis. So, here we can write I as rho the density times integral r square dV. We have taken out rho because we are assuming the body to be homogeneous that means its density does not vary in the object. So, we can write it in this way.

We can see for this thin element dm or the mass of this element is nothing but density times pi r square the area of this element times the thickness of this element that is dx or the small distance that we are traversing along dx. So, if we integrate this value we can find the results. We have these quantities dI_x which is nothing but the second moment of this mass with respect to the axis x and the other two axes, that is y and z is also defined in this way. We can integrate these quantities to find the values.

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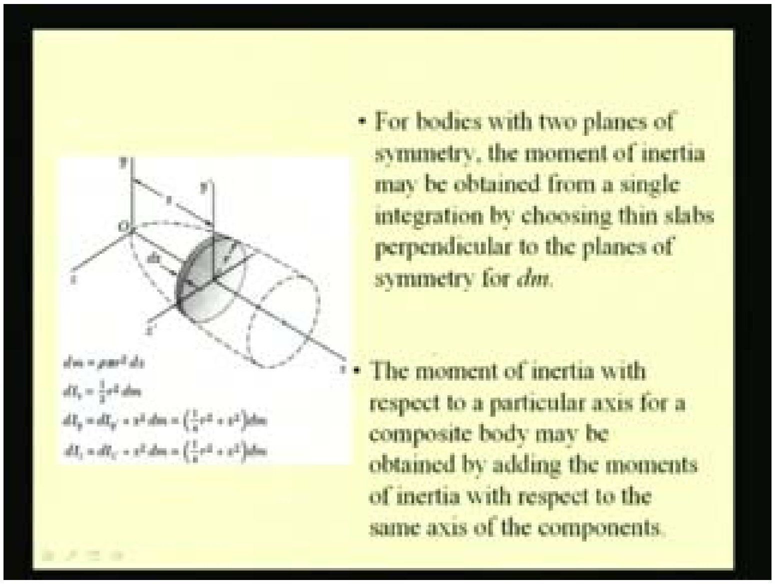

The moment of inertia with respect to a particular axis for a composite body maybe obtained by the adding the moments of inertia with respect to the same axis of the components. Till now in the discussions we have seen that, we are able to determine the second moment of the mass for individual objects whose geometry is known. But for objects which are complex, we can still determine the second moment by decomposing into simpler elements, for which the second moments are known or it can be computed by integration. By the method of integration, we can determine the second moment of the mass for various simpler shapes; it can be tabulated and it can be used for determining the second moment of the mass for any complex object or objects that are predominantly used for our engineering structures either machines or in civil structures.

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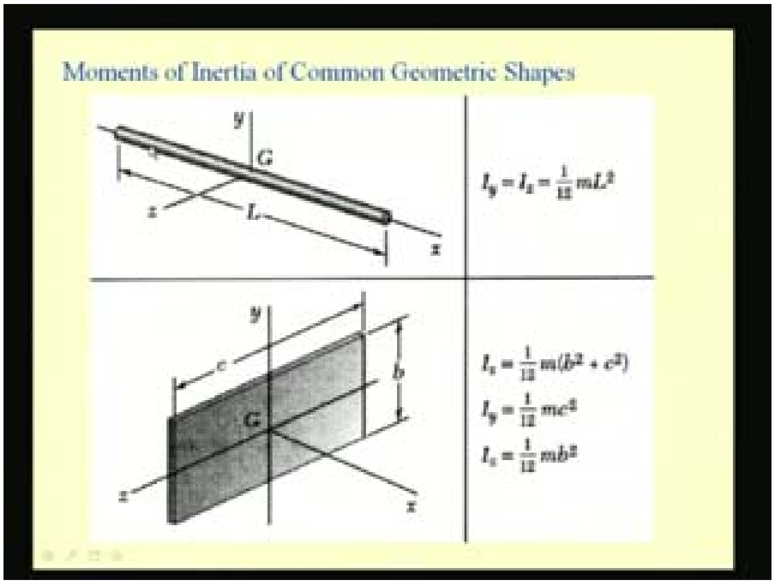

Let us see some common geometric shapes like here we see a shaft element or a rod element. The second moment of this element with respect to the y-axis and the z-axis is the same and it is equal to 1 by 12th of mass times length square. Because its radius is assumed to be small, the second moment of this shaft or the rod with respect to this x-axis is almost negligible. Let us see for this plate, we have already solved this example in a more detailed way. If we have these as the dimensions, c and b as the width and the length of the plate, all these frames of references that are shown are the centroidal frames of references.

If we know the second moments with respect to the centroidal frames, then it is possible to compute with respect to any other parallel planes or axis. We have, I_x the axis perpendicular to the plate as 1 by 12th m b square plus c square, the second moment of this plate with respect to the y axis is 1 by 12th m c square and with respect to the z axis is 1 by 12th m b square; these things we derived in our in earlier example.

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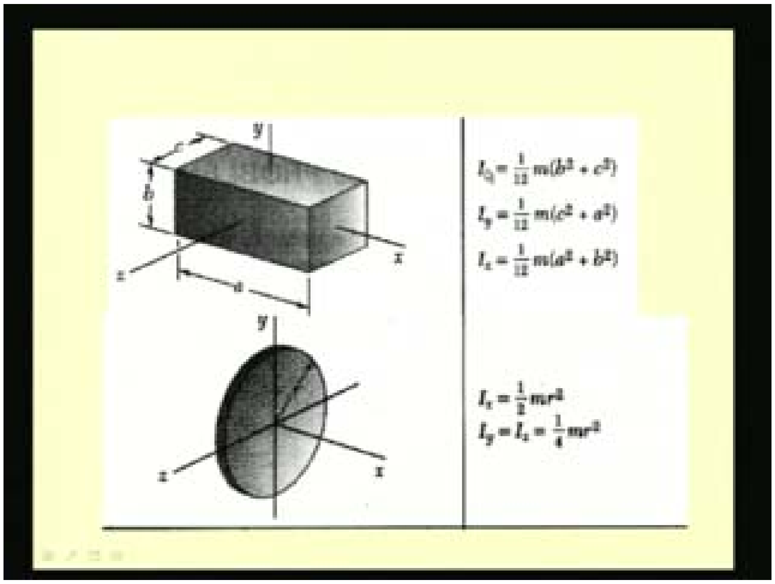

Let us take this rectangular or a cuboidal solid. Again, we are considering this centroidal axis. We have the dimensions as a and b and c. Then, we have this I_x or the second moment of mass with respect to the x axis as 1 by 12th mass times B square plus C square. So, you can try to quickly remember that these dimensions that is b and c are nothing but the dimensions in the plane yz. In the same way, when we compute this I_y or the second moment with respect to y, we have to take the dimensions in the plane zx; we have c and a, so we have this 1 by 12m c square plus a square, and same way $I_{z.}$ For the disc, we have already found with respect to the perpendicular axis it is 1 by 2 m r square and I_y which is same as $I_{z,}$; it is equal to 1 by 4 m r square.

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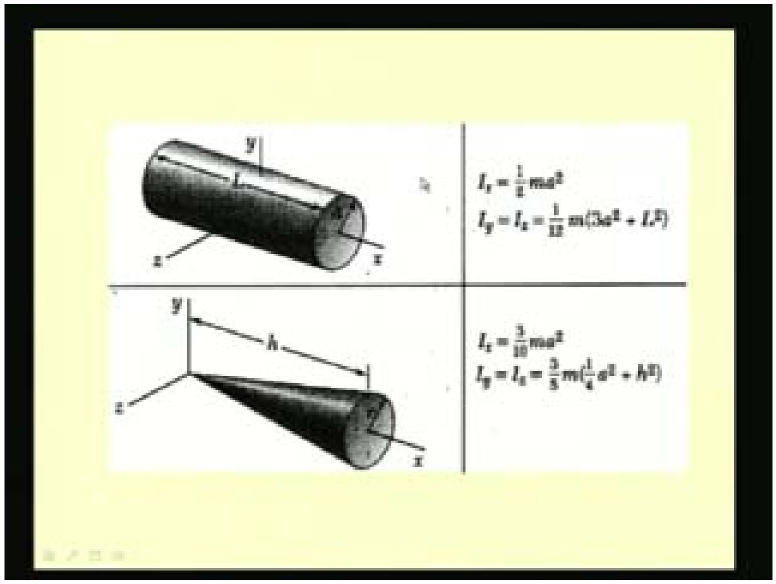

For cylinders, we have, with respect to the axis of the cylinder that is I_x it is 1 by 2 m a square. In this case, a is the radius of this cylinder and 1 is the length of the cylinder and we have other the two second moments that is I_y and I_z as 1 by 12 m 3 a square plus l square. Here, you see a cone. Again for the axis I_x it is 3 by 10 m a square and for the other two axes that is y and z, it is 3 by 5 m times 1 by 4 a square plus h square.

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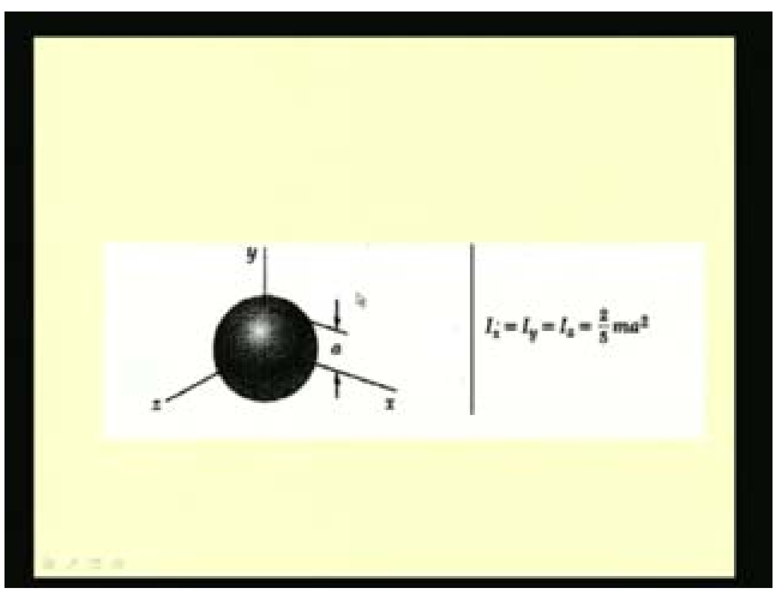

The other common interesting element is the sphere. The radius is a. Then we have all the second moments, that is the second moments with respect to the principal axes y, z, x, all being equal to 2 by 5 m a square. All these values can be proved by method of integration and these values, once available in the tabular form can be conveniently used for determining the second moment of the mass with respect to various axis for any parallel axes. In this lecture, we saw how to determine the second moment of the mass and in the next lecture we will see some problems in this topic.