Engineering Mechanics

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Module 6 Lecture 15
Product of Inertia, Rotation of Axis and Principal Moments of Inertia

In today's lecture, we will see some more topics on the area moments of inertia which we were discussing in the last lecture. In the last lecture, we saw how to determine the second moment of the area with respect to a given axis. Also, we saw how to use the parallel axis theorem to compute these moments with respect to a centroidal axis and then with respect to an axis parallel to the same.

Today, we will see the concept of product of inertia which is useful when we consider the rotation of axes. As we saw, if we have a theorem that relates the second moment with respect to two parallel axes, it becomes convenient to determine the moments when the object is translated. If the object is rotated, its moment with respect to a given axis also changes, but it will become difficult if every time we have to determine the second moment by integration. We will see theorems that help us determine the second moment of a given area when an axis is rotated. In order to compute that we also need the concept of product of inertia.

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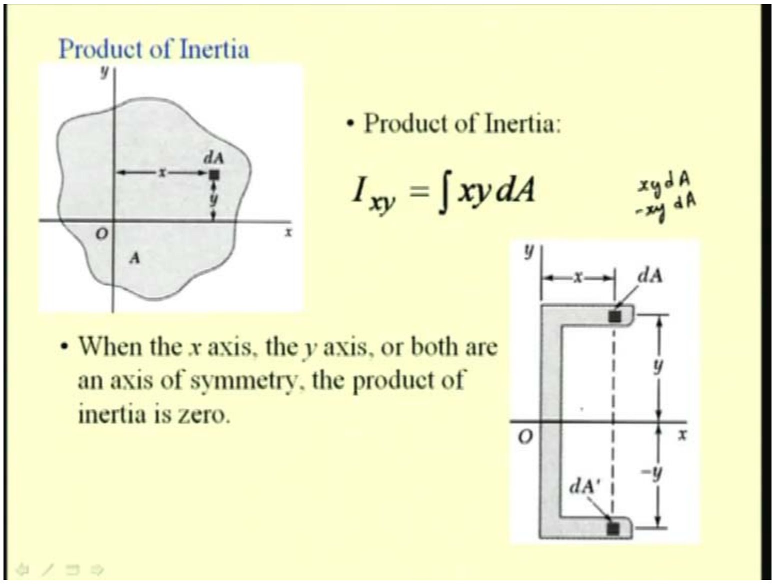

First, we define this product of inertia as for a given area we consider this small element dA, which is situated at a distance of x and y from the axes, x as well as y. We know that the second moment of this area dA with respect to the x axis is nothing but y square dA and the same way the second moment of this area with respect to y axis is x square dA.

The product of inertia of this elemental area dA with respect to this axis, that is, Oxy is given as xydA. If we integrate this quantity, that is, xydA for this domain, we get the product of inertia of the complete area that is $I_{xy.}$ So I_{xy} is determined as integration of xydA. If we consider areas which are having some axis of symmetry this area has an axis of symmetry Ox .

It is possible to find these differential areas dA such that for each of the area dA we have a corresponding area dA prime which is located at the same distance from this axis, but in the negative direction. The product of the inertia of this area is nothing but xydA and the product of inertia of this element is minus xydA which is equal to dA prime. So, it is possible to find a corresponding element for any differential element because we have this axis of symmetry Ox.

The summation of this is 0 and thus the product of inertia for this element is 0 with respect to this axis Oxy. This property can be conveniently used when computing the product of inertia for symmetric objects.

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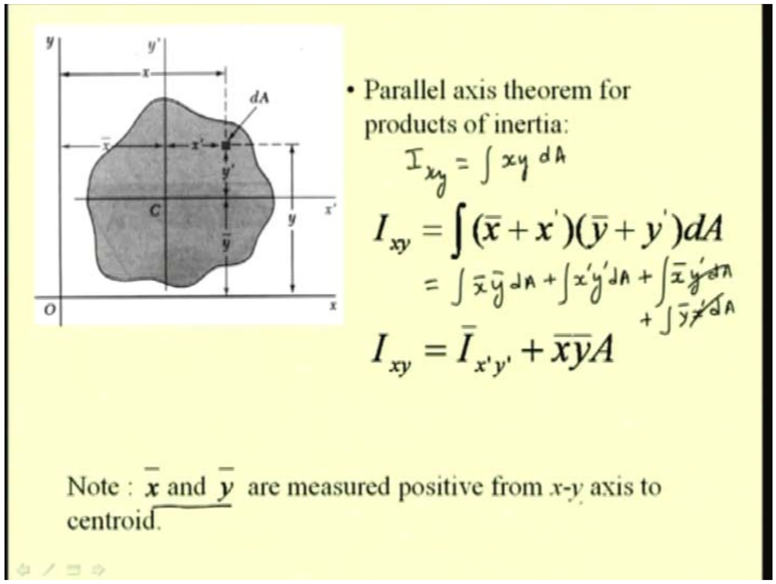

Let us see how to apply the parallel axis theorem for the product of inertia. Here, we have determined the product of this area with respect to an axis Oxy and then we are interested to find the product of inertia with respect to a centroidal axis. So C being the centroid of this area, we have the parallel axes, that is x prime y prime, passing through the centroid.

If the location of this differential element is x and y, they can be written as x bar, the location of this centroid, plus x prime, which is the location of this differential element dA, in this coordinate system, that is y prime Cx prime. Same way, the y coordinate can be written as y bar plus y prime. Thus, if we have Ixy which is nothing but integral of xydA, we know that Ixy is nothing but integral xydA and x is nothing but x bar plus x prime. If we simplify this, this is nothing but integral x bar y bar dA plus integral x prime y prime dA plus the quantity that is integral x bar y prime dA plus integral y bar x prime dA.

This quantity is nothing, but the first moment of this area with respect to this centroidal axis. So, this quantity is 0 and the same way, you will find that this quantity, which is again the first moment of the area with the respect to the centroidal axis, is again 0. These two quantities become 0. So, what we have is integral x bar y bar dA which is nothing but x bar y bar; integral dA is the area of this element. This quantity, that is x prime y prime dA integral is nothing, but the product of inertia of this area with respect to this axis; that is x prime y prime, that is the centroidal axis. The parallel axis for product of inertia is found in this way, but one has to be careful that this quantity can be positive or negative. We saw that in case of the second moments the quantity is always positive because we have y square or x square. So, whether the element is in the positive or negative direction of a given axis the quantity is always positive. In this case, the quantity is positive or negative depending upon the sign combination of the location of this centroid with respect to the axis that we have considered that is Oyx.

If the centroid is located in other quadrants then this value will have appropriate signs. So, one has to note this point that these values are measured positive from the xy axis to the centroid.

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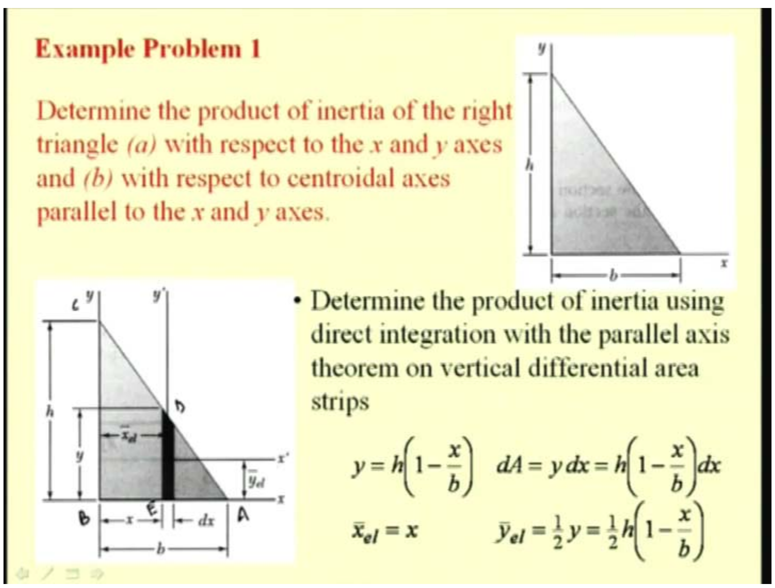

Let us take one example. Here, you see a triangular lamina for which we are interested to determine the product of inertia. It is of height h and base b. We are interested to find the product of inertia with respect to the xy-axis and with respect to the centroidal axis parallel to this xy-axis.

First, we determine the product of inertia using direct integration and then we use the parallel axis theorem. Let us take vertical differential area strips. Here, we are considering a thin vertical strip, whose centroid is located at this distance y bar element and x bar element which is nothing but the x coordinate of the location of this element. This distance can be found by knowing the height of this thin strip that is y. So, y element is y by 2. Then, we can integrate it between the limits, that is x from 0 to b. So, from the similar triangles we have this y equal to h times 1 minus x by b. So, we have considered this triangle say ABCDE. We consider these similar triangles, that is EAD and BAC and relate this distance y and h with respect to this x and b. Once we have this quantity, we can write the area of this differential element dA as y times dx, where y is this quantity h into one minus x by b. Knowing these two quantities that is x element is x and y element is half of y which is half of h into 1minus x by b.

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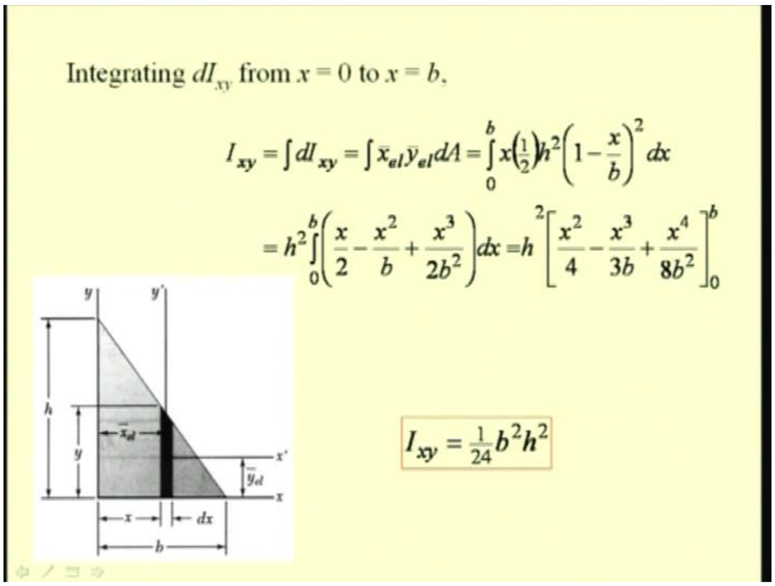

Now we can write the product of inertia of this thin strip with respect to the xy axis as dI_{xy}; then we integrate it between the limits that are 0 to b from this to this point. We have I_x as integral dIxy, which is equal to integral x bar element, y bar element times dA, x bar element is x itself, y bar element is y by 2, which is half of h square into 1 minus x by b square dx into this area.

We simplify this and apply the limits we have this quantity. After simplification, it is 1 by 24 times of b square h square. So, this is the product of inertia of the given triangle with respect to the Oxy axis. Now, we can determine the product of inertia with respect to the centroidal axis.

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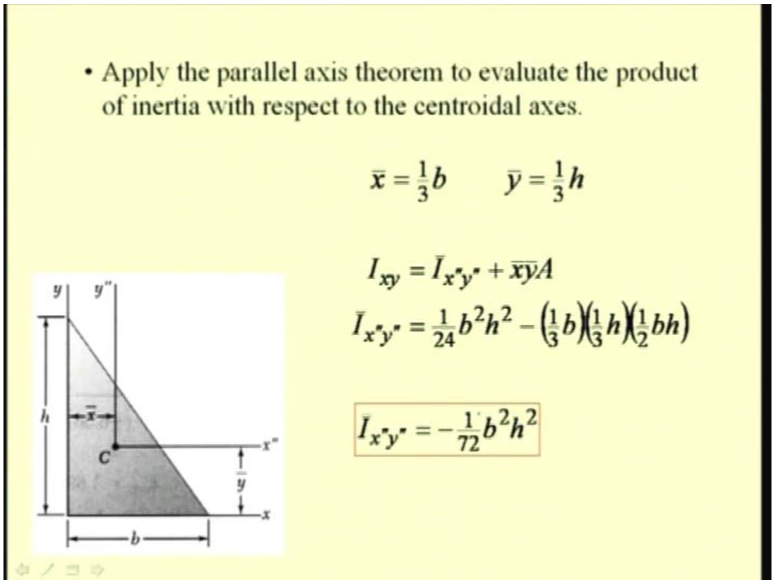

We know the location of the centroid of this triangle and now we want to determine the product of inertia with respect to an axis x double prime y double prime passing through the centroid C.

We know these values: that is the distance of this y double prime is one-third base and this is again one-third of h. These are available as standard values or one can find it by the first moments of this area. Now, we write the parallel axis theorem for the product of inertia. That is, I_{xy} product of inertia with respect to the axis xy is equal to the product of inertia with respect to the centroidal axis plus x bar y bar A. This value is to be determined and we know this value I_{xy}. So, we have I bar x double prime y double prime is equal to I_{xy} minus x bar y bar A. Ixy. We have just determined as 1 by 24 times b square h square minus x bar is 1 by b, y bar is 1 by 3 h times the area, which is half bh.

From this we determine the product of inertia with respect to this centroidal axis x double prime y double prime and that is equal to this quantity that is 1 by 72 b square h square. In this way, it is possible to apply this parallel axis to find the product of inertia of a given area.

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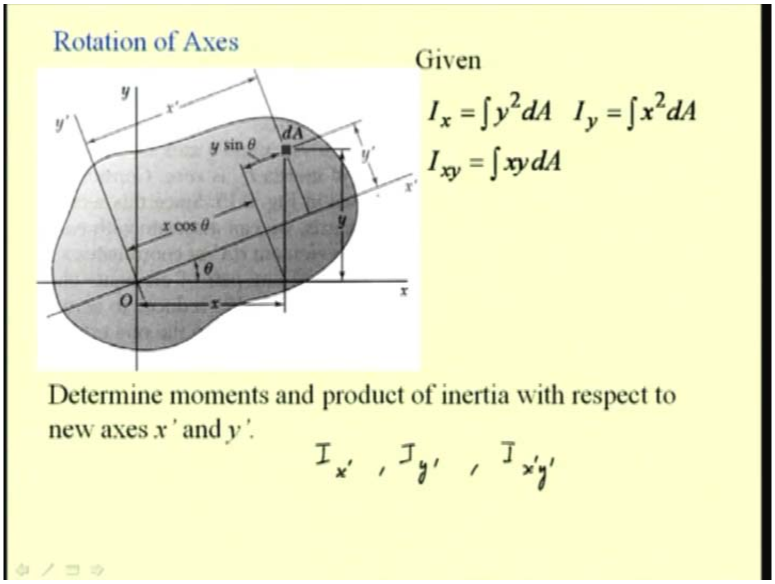

Let us discuss what happens to this product of inertias when the axis rotates. As we developed the theorem for the parallel axis, now we are interested to determine the relation between the product of inertia for two axes which are rotated with respect to a point. We will see that this product of inertia is useful when we determine this relation between the rotated axis and the original axis, for finding the second moment.

We have this axis Oxy and this differential element dA. The product of inertia and the second moments of inertia of this area with respect to this axis are known like we have the second moment of this area dA with respect to Ox as y square dA and when we integrate it, we get the second moment for this complete area.

Same way, we can determine the second moment with respect to the axis Oy and also we can determine the product of inertia for this area for Oyx frame. We are interested to find the second moments for a rotated axis with respect to O, that is x prime y prime, which is rotated by an angle of theta, which is in the counter clockwise direction. We want to determine the second moments for this rotated axis, so these values are known. We want to determine the values I_x prime, I_y prime and I_x prime y prime, that is the second moments and product of inertia for the rotated axis x prime y prime.

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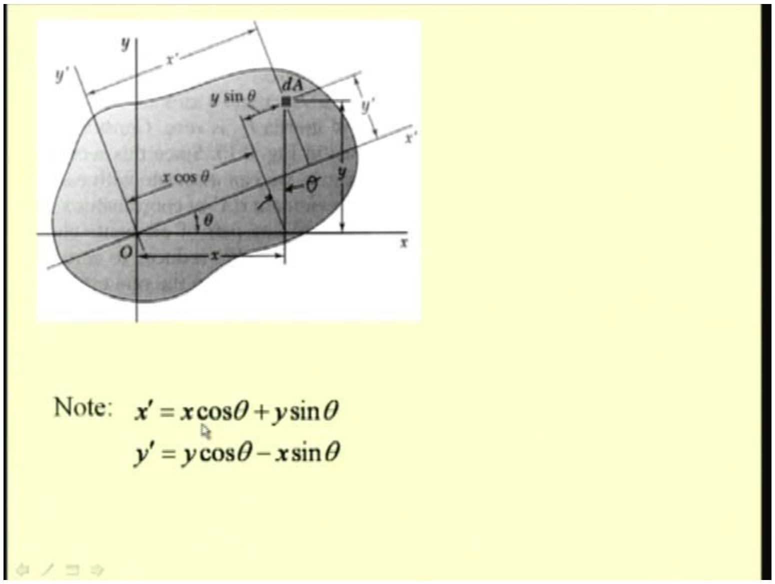

Let us try to relate the coordinate of this area dA in this reference frame that is Oyx and the reference frame y prime Ox prime. In the reference frame y prime Ox prime the location of this area dA is x prime from the Oy prime axis. This quantity x prime is nothing but x cos theta plus y sin theta. We find that this distance is nothing but x times cos theta and this distance is nothing but y sin theta because we have this as y and this angle is again theta.

This angle is also theta and so we have this distance as y sin theta. In the same way, we can write this y prime with respect to this x and y. We get this x prime as x cos theta plus y sin theta and y prime as y cos theta minus x sin theta which is nothing, but this distance minus this distance.

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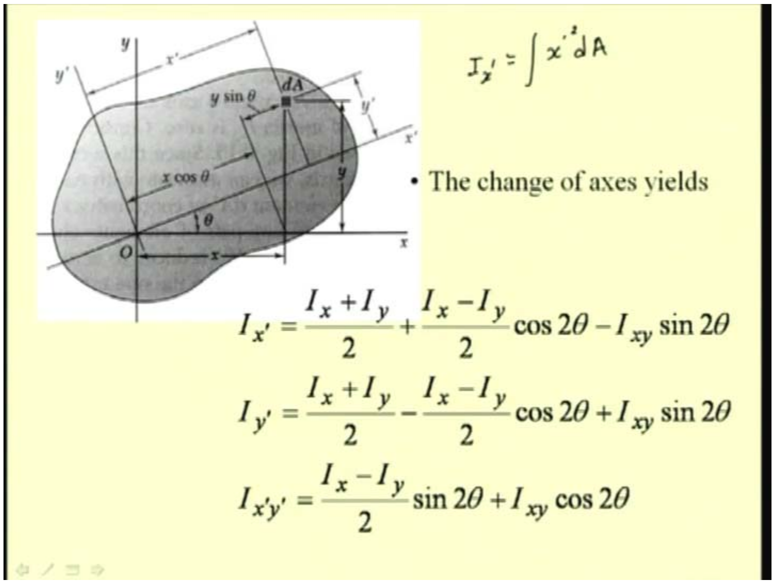

If we substitute in the equations, that is for the equation of this I_x prime, we have it as integral x prime square dA and we substitute this value of x prime as x cos theta plus y cos theta. If we simplify that, we get this expression. In same way, we can write the expression for I_y prime and I_x prime y prime. We have I_x prime as I_x plus I_y by 2 plus I_x minus I_y by 2 cos 2 theta minus I_{xy} sin 2 theta and same way we get the expression for I_y prime as I_x plus I_y by 2 minus I_x minus I_y by 2 cos 2 theta plus I_{xy} sin 2 theta. That means, we are writing the expression of the second moment of this area with respect to the new axis, as a combination or an equation that relates this I_x prime to the I_x, I_y, I_{xy} or the second moments of products of inertia of the original axis.

These equations relate the second moments of this area in the rotated axis to the second moment of the area in the original axis that is: I_x, I_y and I_{xy}. These are related to I_x prime, I_y prime and I_x prime y prime. These equations are nothing, but the equations of a circle, which we will see a little later.

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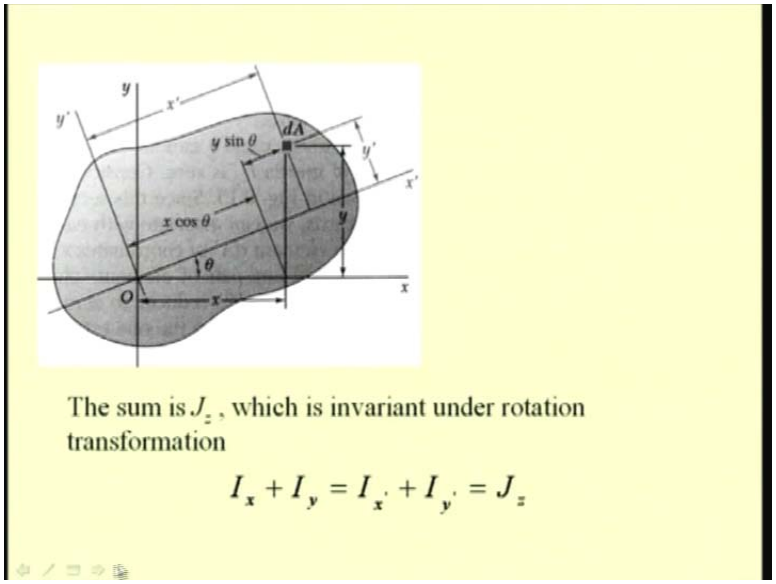

We know that the product of inertia is the product of inertia of the z axis that is the polar axis is invariant when we rotate the object or the axis, that is, the sum of the second moments that is I_x plus I_y is equal to I_x prime plus I_y prime, which is equal to the polar moment of inertia or the moment of inertia with respect to the z axis.

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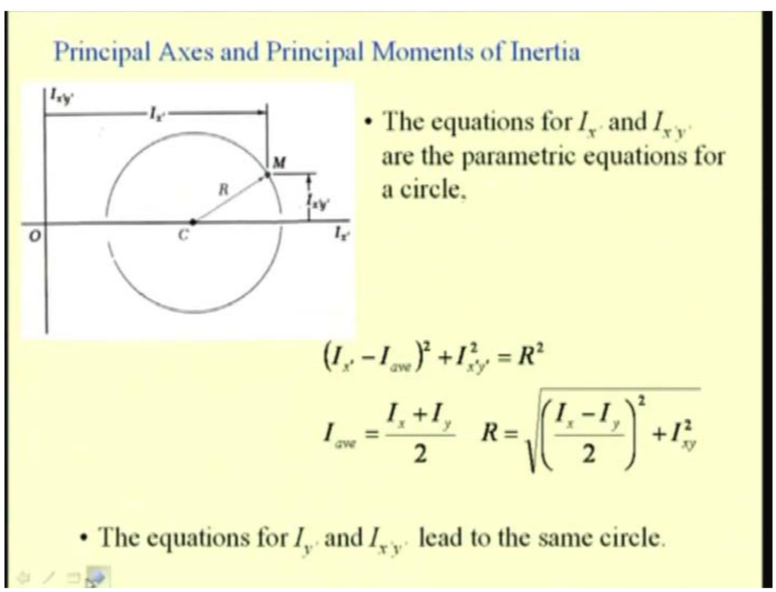

As we rotate these axes, the second moments keep on changing. That is, this I_x prime and I_y prime, keep changing. But, for a particular set of axis, we will see that these values become the extremum values, that means they reach a maximum or a minimum value because as we rotate these axes, their distances corresponding to these axes keep on changing and for a particular orientation we will see that the second moment with respect to this axis reaches the extremum. That is either they become maximum or minimum. These are known as the principal moments of inertia and the corresponding axis is known as the principal axis.

Let us see how we determine these principal axis and principal moments of inertia. In order to do that, we first see the equations of I_x prime and I_x prime y prime which are nothing but the parametric equations of a circle. That is, if we try to plot the values of I_x prime, y prime and $I_{x,}$ we will get a locus corresponding to a circle, whose center and radius are nothing but the average second moment of inertia, that is, I average is I_x plus I_y by 2, which is the location of this center and the radius of this circle is nothing but root of I_x minus I_y by 2 square plus I_{xy} square.

Once we plot this circle, we can find that the second moment of inertia with respect to the x-axis has extremum values at these two points and they correspond to the principal moments of inertia. Even if we use the other set of equations, that is I_y prime and I_x prime y prime, they also lead to the same circle.

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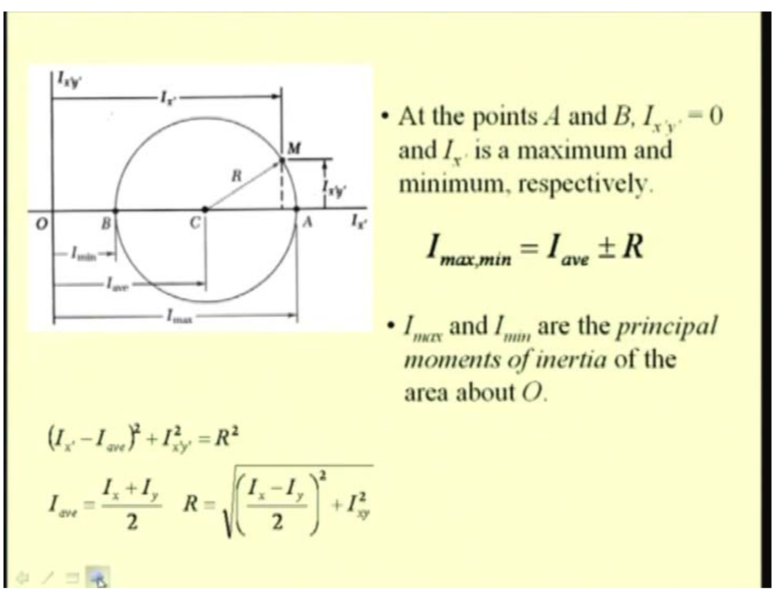

We will find that the center of this circle is nothing but I average, which is I_x plus I_y by 2 for a given axis and R the radius of this circle is I_x minus I_y by 2 square plus I_{xy} square. These two points that is A and B correspond to the extremum values of the second moment of inertia. We see that for those points the value of this I_x prime is maximum and here it is minimum and these are the principal moments of inertia. For these points, we find that this value I_x prime y prime or the product of inertia becomes 0.

For the set of principal axes the product of inertia is 0. What are these values? From this graph or this picture, we get that this maximum value is nothing but I average plus this radius R and this value B is nothing but I average minus this radius R. We have this I_{max} and I_{min} as I average plus or minus R and we have the expression for I average and R and these are the principal moments.

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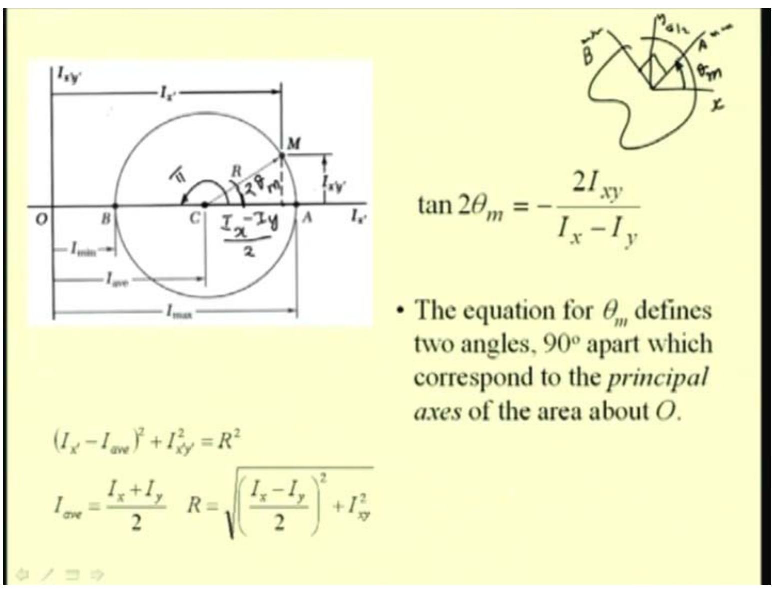

Let us see for a given axis this point M corresponds to the second moment of inertia I_x prime and product of inertia I_x prime y prime. So, this point M corresponds to the set of values that is the second moment of inertia and product of inertia for a given axis. As this axis keeps rotating about O, this point moves along the circle and for each point this x axis corresponds to the second moment of inertia I_x prime and the y coordinate corresponds to the I_x prime y prime or the product of inertia. Let us say for a particular axis, M corresponds to the point which is nothing but I_x prime and I_x prime y prime. For this point, we will see that, if we take this angle as

2 theta m - we will see later why we are taking this as 2 theta m - then, tan of this angle is nothing by I_x prime y prime divided by this distance, which is nothing but from the geometry, knowing these values of I average and R, it can be found that it is equal to I_x minus I_y by 2. The tan of this angle is nothing but I_{xy} divided by I_x minus I_y by 2. Why we are using this 2 is that the principal moments of inertia are located on axes which are 90 degrees apart. That means, for any body, let us say if I take this body, if this axis, this is the original axis x and y and these are the axes corresponds to the principal axis then they are at 90 degrees to each other.

If this is the axis corresponding to the maximum moment then this corresponds to the minimum moment, second moment, then they span 90 degrees, but on this diagram we see that, this axis corresponds to the point A and this for B, we see that it is spanned by an angle of pi. Though this here the axis is at pi by 2 on this diagram, this point A and B is located at an angular displacement of pi radians.

From this, we know that this is twice the angle between the principal axes. That is why we say that if the angle between the axes is theta, then the points on the circle are at an angular displacement of 2 theta. So, this equation gives a value of theta m that defines two angles 90 degrees apart, which corresponds to the principal axis of the area about O. One value corresponds to theta m; the other value corresponds to theta m plus 90 degrees.

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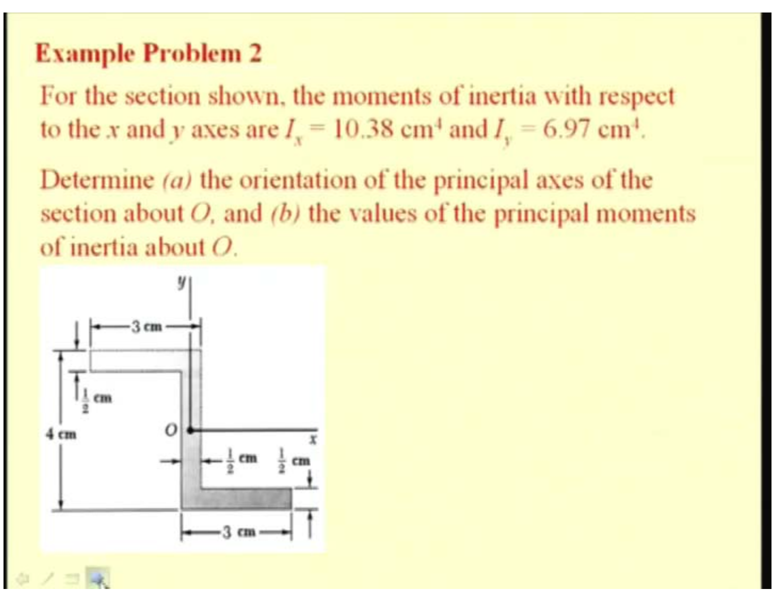

Let us take one example to illustrate how we find this principal moments and the axis of these principal moments. Here, you see an area which is symmetric about this Ox and Oy. The various dimensions are given; it is also known that the second moment of this area with respect to x and y are also given to you as 10.38 centimeter to the power of 4 and 6.97 centimeter to the power of 4. We are interested to determine the orientation of the principal axis and the values of the principal moments of inertia about O. In order to do that, first we have to determine the product of inertia that is I_{xy} for this axis Oyx.

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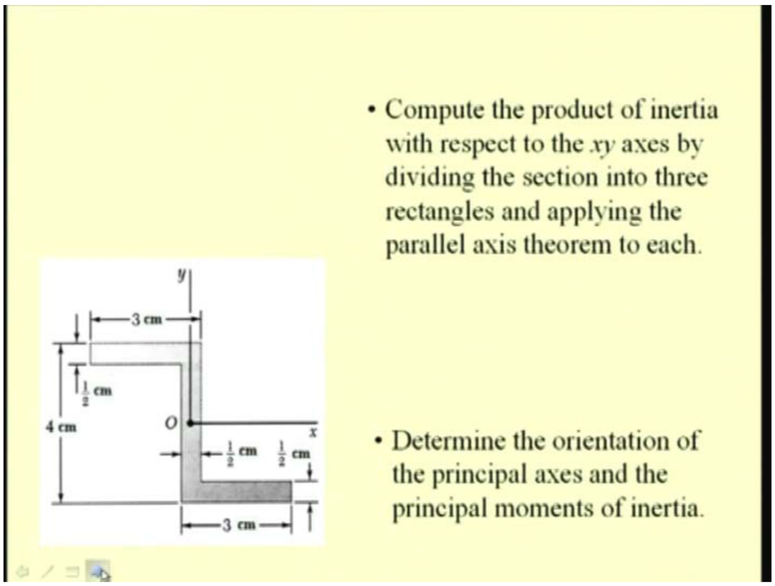

First, we determine the product of inertia with respect to this xy axis by dividing the section into three rectangles. That is, the rectangles corresponding to this area and this area and this area, then we can apply the parallel axis theorem to find the second moments of or the product of inertia of these areas with respect to this axis. Later on, we will determine the orientation of the principal axis and the principal moments. This will be our strategy.

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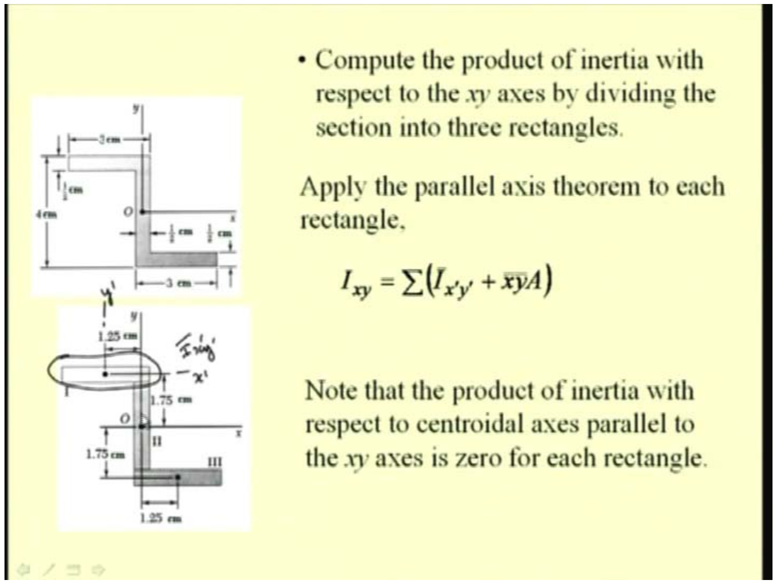

Let us first compute the product of inertia. So these are three areas that is one, two and three. The location of their centroids are given here. That is, for this area one, it is located at 1.25 centimeters in the negative x direction and it is located at a distance of 1.75 centimeters from x axis. That is, tabulate these values and apply this parallel axis theorem because we are now interested to find the product of inertia of these individual areas with respect to Oyx axis. But we know these values for each of the centroidal axis; that means for this area one we know the value I_{xy} prime with respect to x prime and y prime axis, I can say I know the product of inertia with respect to this axis. I want to determine the product of inertia of this area with respect to this Oxy axis and then I sum up all this, to determine the product of inertia of the total area.

Please note that the product of inertia with respect to centroidal axis parallel to the xy axis is 0 for each rectangle. So, the product of inertia is actually 0 with respect to the centroidal axes.

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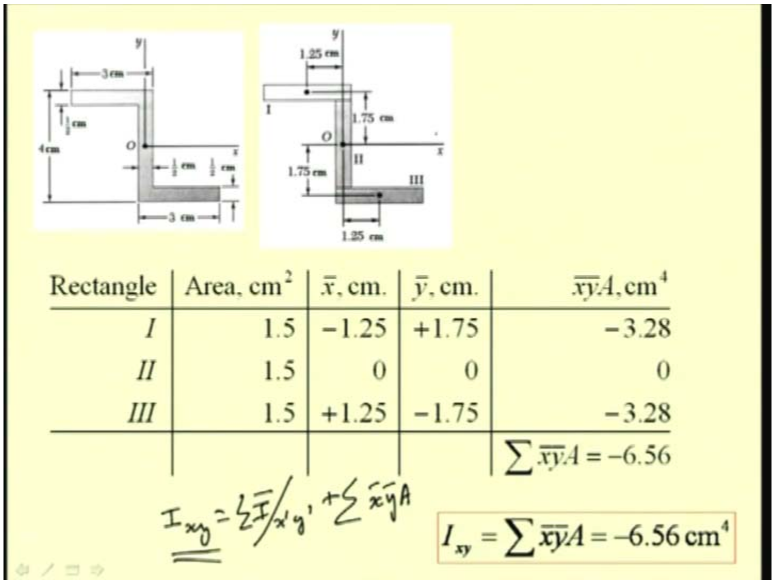

For the area one, we have the area as 1.5 centimeter square, the location of the centroid as minus 1.25 and plus 1.75 and the product of this value that is xy bar A is given here. For this area two, you can see that the location of the centroid is at O, that is 0 and that product becomes 0. The sum of this is minus 6.56.

Once we know this value, we can determine I_{xy} as, actually this I_{xy} is equal to I bar x prime y prime sigma that is sum of the products of inertia of all these individual areas with respect to their centroidal axis, which is 0. This quantity becomes 0 plus this sigma x bar y bar A and this is the quantity that we have just found, that is minus 6.5 A. That gives the product of inertia I_{xy} of this complete section or area with respect to this Oxy axis.

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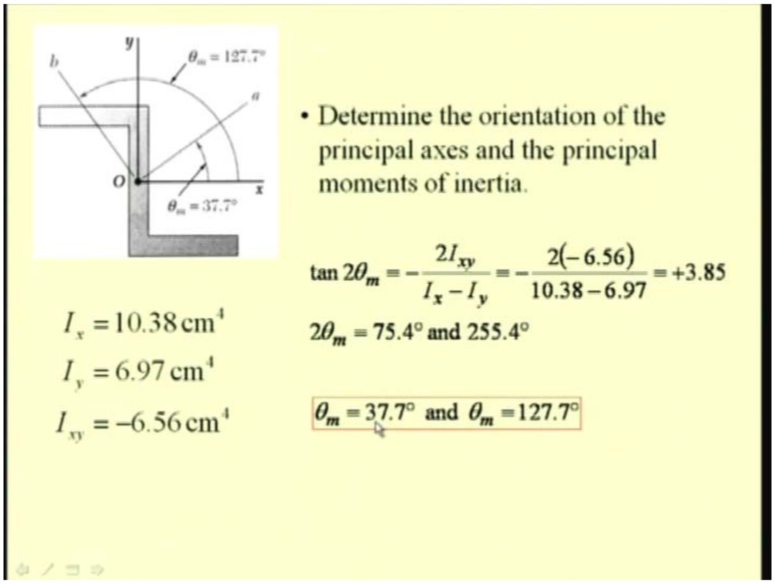

We can move on to find out the principal moments. We know this I_x, I_y and just now we determined this I_{xy} as minus 6.56 centimeter to the power of 4. We can determine the orientation of the principal axis and the principal moments of inertia. By considering this equation, that is tan 2 theta m, where this angle theta m is the angle of the principal axes.

Let us say this a and the axis which is at 90 degrees, that is b corresponds to the principal axis and if this axis that is bOa is rotated by an angle of this theta m, then the values of this principal or the values of the moments become the extremum and that angle is given by this equation, tan 2 theta m is equal to minus 2 I_{xy} by I_x minus I_y. Since we have determined these quantities, we can determine this value. So, one is 75.4 and other is theta m plus 90 which is 255.4 degrees. So these two angles correspond to the principal axes, that is, 37.7 degrees and 127.7 degrees. Because we know that the values that have been determined are 2 theta m, so it corresponds to 37.7 degrees and 37.7 degrees plus 90 degrees which is 127.7 degrees.

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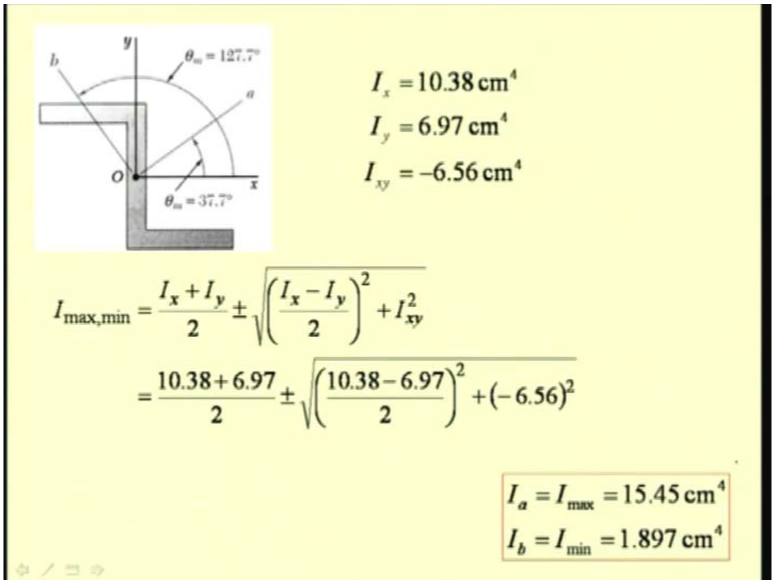

We can also determine the corresponding moments, that is the principal moments, from this equation that I_{max} comma I_{min} is equal to I average, which is I_x plus I_y by 2 plus or minus the radius of that circle that is I_x minus I_y by 2 square plus I_{xy} square. Knowing these values, we find that I_{max} and I_{min} are 15.45 centimeter to the power of 4 and 1.897 centimeter to the power of 4.

This problem illustrated how we use this product of inertia and the second moment of inertia with respect to an axis to determine the second moments and product of inertia with respect to a rotated axis and also the principal axis and the principal moments. Now, we will see another method which is a graphical procedure, to determine the second moments for a rotated axis. This method we call as constructing the Mohr's circle and determining the second moments and product of inertia for a rotated axis.

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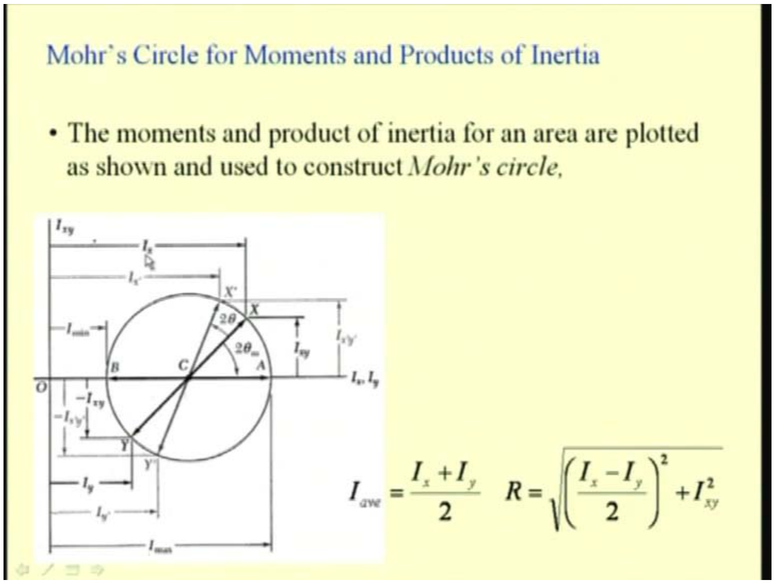

We construct this circle, which is the locus of the second moments and the product of inertia. You have already seen that this point x corresponds to the second moment with respect to the x-axis I_x and I_{xy}. Along the y-axis we have I_{xy} and along the x axis we have $I_{x.}$ If we plot $I_{y,}$ then that point corresponds to minus I_{xy} and this is the value of $I_{y.}$ So, this diametrical line xy corresponds to the second moment of inertia and product of inertia on this circle, which we call it as the Mohr's circle. This diametrical line ACB which corresponds to the maximum moments, that is the principal moments, lies along this x-axis.

We know that if we rotate the axis by theta then the point moves by 2 theta on the diagram. So for a given axis, if BCA corresponds to the principal axis, then this angle is 2 theta m or the theta by which we have to rotate to find the principal axis. It is now possible to use a graphical method by first constructing this circle and determining the required axes and values. If you are interested to find the second moments and product of inertia with respect to an axis, which is rotated by an angle of theta in the counter clockwise direction, then on this Mohr's circle, we have to find a diametrical line corresponding to a rotation of 2 theta, that is x prime y prime. This diametrical line that you see which is rotated by 2 theta in the counter clockwise direction and these points that is x prime and y prime corresponds to the second moments that is I_x prime and I_y prime for that axis. The product of inertia is given by the y coordinate that is either this value or this value; so that is I_x prime y prime. This circle can be constructed by first finding the average value that is I_x plus I_y by 2 for a given frame and then the radius of this circle is nothing but root of I_x minus I_y by 2 whole square plus I_{xy} square.

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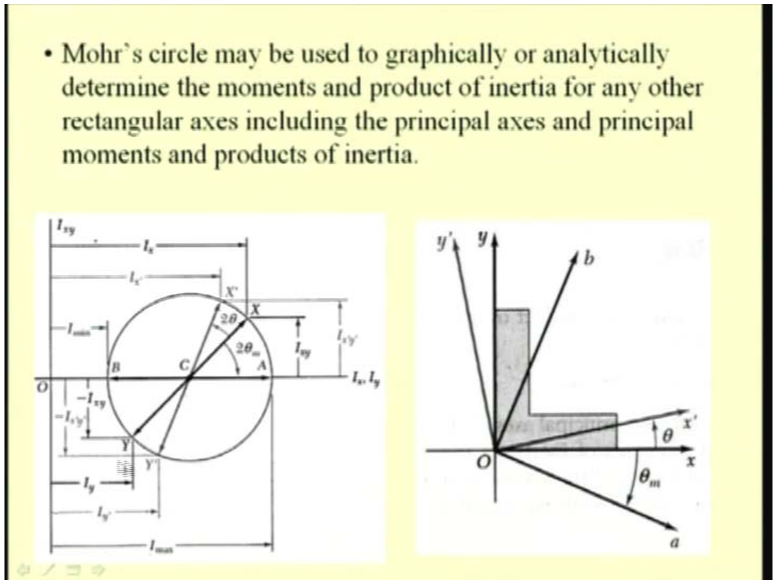

Let us see this method. Let us construct the Mohr's circle for this object. For this axis that is Ox Oy the corresponding moments are the points x and y; that is, this x corresponds to I_x, I_{xy} and this point y corresponds to I_y, minus I_{xy}, for this area and this axis Oxy. So, for an axis x prime y prime which is rotated by theta in the counter clockwise direction, if you want to know the principal moments and product of the moments, we move on this graph by 2 theta. We have these points x prime y prime giving the values. From this diagram, we determine that this diametrical line, that is YCX has to be rotated by this angle, to make it the principal axis.

We rotate the axis Oyx by half this angle, that is if this angle is 2 theta m, then we rotate it by an angle theta m in the same direction. That is, here we see that this axis YCX has to be rotated by 2 theta m in the clockwise direction in order to get this bCa axis. That is why we rotate clockwise direction by half this angle, that is theta m, and we get this axis Ob and a, the principal axis for this area. So, we see that a graphical procedure can be followed to solve this problem and we will see one example.

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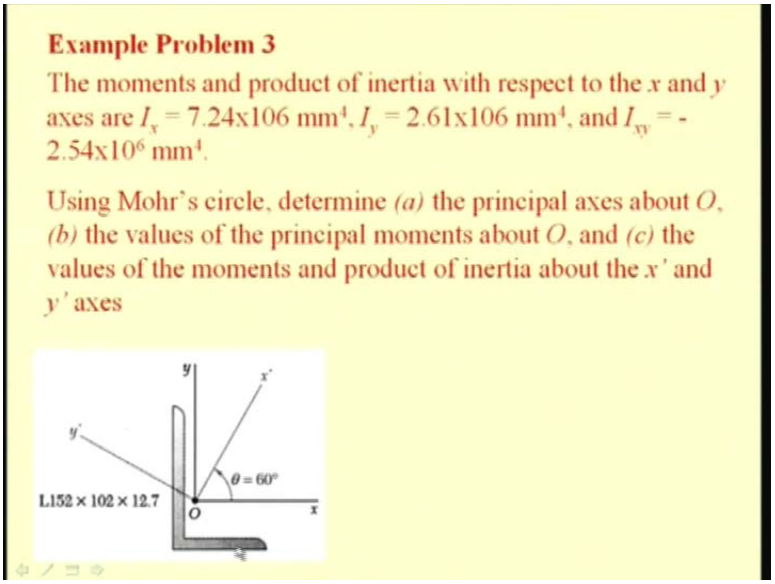

Let us take one typical engineering section, that is an L section and a given axis that is Oyx, about which the values of I_x I_y and I_{xy} are known. So, in engineering convention, we represent this section by this notation that is L 152 by 108 by 12.7, which tells the thickness, the width and length of this section.

We know these values - that is the second moment with respect to the x-axis, y-axis and the product of inertia. We are interested by this Mohr's circle method to determine the principal axis passing through this point O and the principal moments. Also, we are interested to determine the values of moments and product of inertia for this rotated axis that is x prime O y prime, which is rotated by 60 degrees in the counter clockwise direction. Let us see, how we solve this problem by the Mohr's circle method. First, let us construct the Mohr's circle by finding the center and radius of the Mohr's circle.

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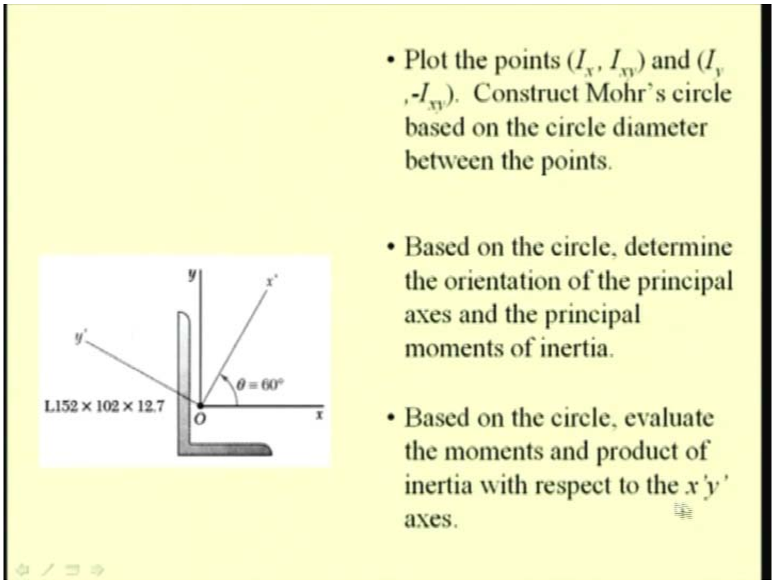

For this section, we plot this point I_x, I_{xy} and the point I_y, minus I_{xy} which are the diametrically opposite points on a Mohr's circle. Then, we can construct a circle passing through these two points graphically. Or else, we can determine these values, that is center and radius and also construct it but graphical construction is also possible. That means if we know two points which are diametrically opposite, then it is possible to construct a circle which passes through these two points. From this, let us determine this principal axis and principal moment of inertia. Then we will find the product of inertia with respect to the rotated axis. So, this is our strategy.

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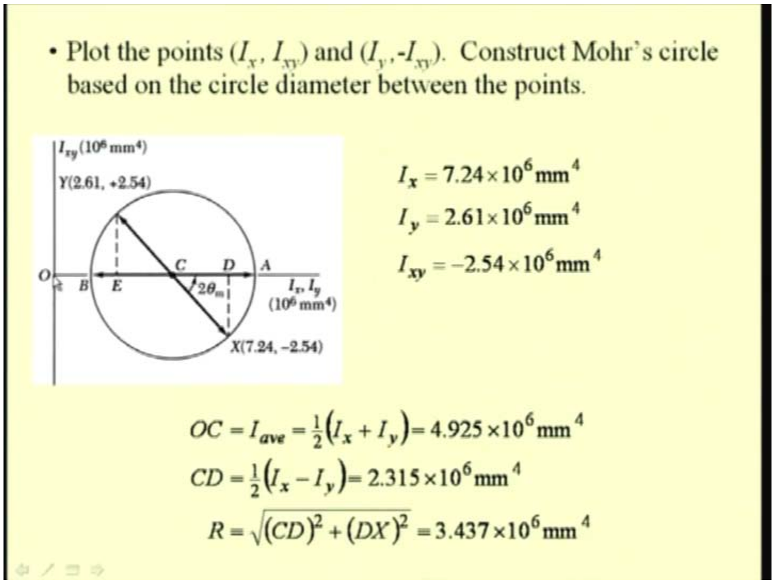

First, let us plot these points I_x, I_{xy} and I_y, minus I_{xy}. So, this picture shows the two points that is I_x which is 7.24 and I_{xy} corresponds to minus 2.54. So, x is the point corresponding to the second moment of inertia with respect to x axis and I_{xy} and the diametrical opposite point is I_y, minus I_{xy}. I_y is 2.61 and I_{xy} is minus 2.54 so minus I_{xy} is nothing but 2.54. This has been plotted in a scale of 10 to the power of 6 mm to the power of 4.

This diametrical line corresponds to the second moments and product of inertia for the given axis. We can draw a circle passing through these two points. So, these are the values and we used it to plot this. Either we can use a graphical procedure or we can find this value of the center and the radius of the circle.

The center is located at a distance of I average from this O, that is half of I_x plus I_y, which is equal to 4.925 into 10 to the power of 6 in this case. Then, let us find this radius which is equal to this distance CD square plus DX square, which is the square of this radius, where CD is nothing but I_x minus I_y by 2 and this d_x is nothing but, I_{xy} that is the product of inertia. We first find this value of CD, which is 2.315 into 10 to the power of 6mm to the power of 4 and this value XD is nothing but 2.54 into 10 to the power of 6 mm to the power of 4. From this we determine this radius as 3.437 into 10 to the power of 6 mm to the power of 4. On this diagram, we construct a circle with center at this distance, that is 4.925 and with a radius of 3.437. So, this is the corresponding circle.

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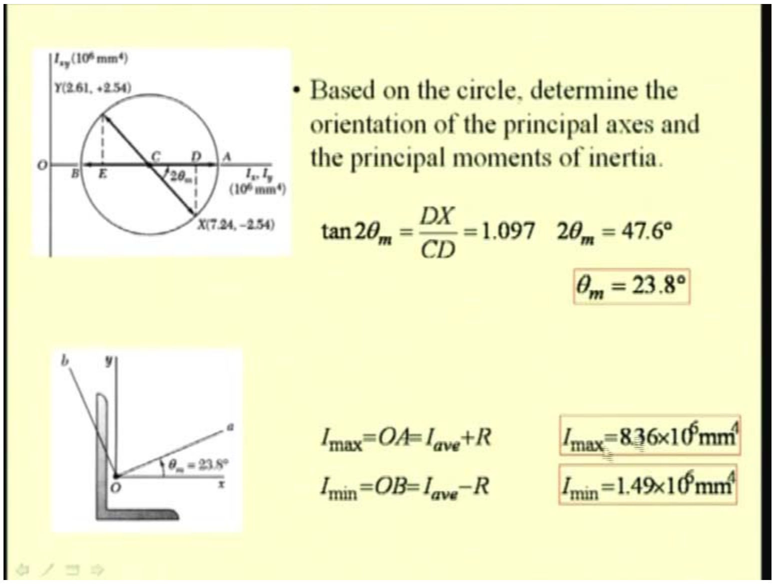

Based on the circle we find that these points A and B, are the points corresponding to the principal moments and the axis has to be rotated by this angle. That is let us designate it as 2 theta m in the counter clockwise direction in order to get the principal moments. So, that means if this is the axis Oxy, then we have to rotate half of this angle in the counter clockwise direction that is theta m, in the counter clockwise direction.

Let us determine first, this 2 theta m from this diagram in order to find this theta m. So, tan 2 theta m is nothing but DX divided by CD. We have just now computed these values. From this, we get 2 theta m as 47.6 degrees and theta m as 23.8 degrees. So, this axis OA is rotated by 23.8 degrees in the counter clockwise direction and this axis OB is 23.8 degrees plus 19 degrees. So, these two axes correspond to the principal moments of inertia of the given area. We can also determine their values that are the value of A and B by measuring this distance OA and OB. From the measurement, it becomes 8.36 into 10 to the power of 6 mm to the power of 4. This is also equal to I average plus R and this I minimum is nothing but this distance OB. So, you can graphically measure this or you can also compute it as I average minus R which is 1.49 into 10 to the power of 6. We have determined the principal moments, as well as the axis through which these principal moments have been computed.

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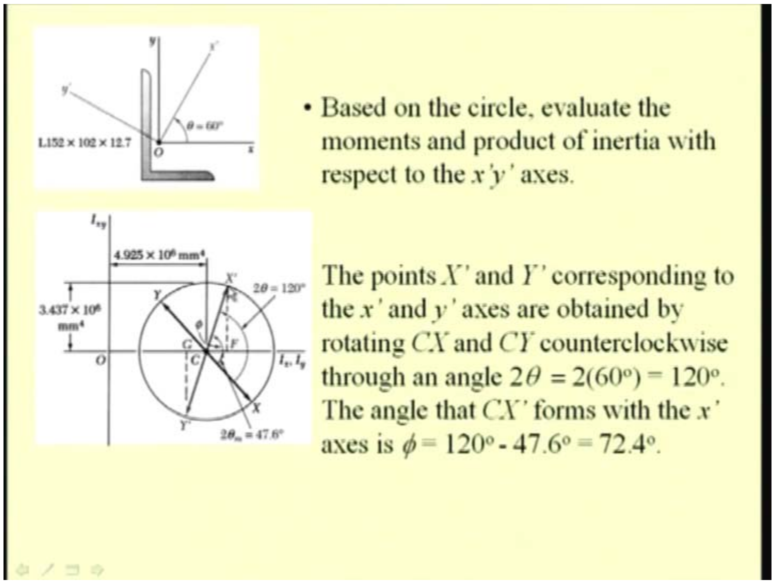

We can also determine the product of inertia with respect to a rotated axis. We are interested to find the product of inertia with respect to an axis Ox prime y prime which is rotated by 60 degrees in the counter clockwise direction. That means, in our original diagram, where we have this XCX as the diametrical line corresponding to Oyx we have to rotate counter clockwise by 2 theta that is 120 degrees and we get this axis. That is x prime C y prime. Now, by measuring graphically the value of OF which is nothing but the projection of this point x prime on the x axis and projection of this point y prime on the axis is G.

We can measure these values of OF and OG, which will give the second moment of inertia and we can measure this height, that is x prime F or y prime G, which is nothing but the product of inertia, that is I_{xy}. We have this value as 3.437 into 10 to the power of 6 from this diagram.

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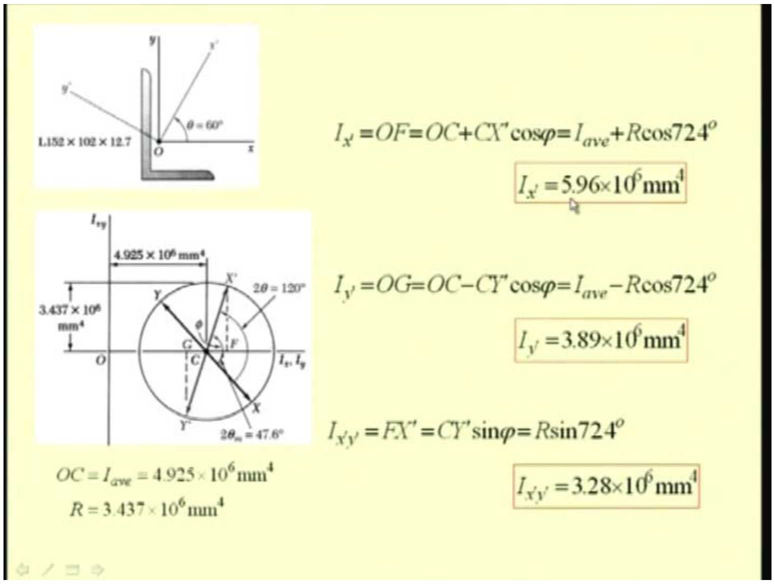

We know that, this is the I_{xy} for the rotated axis. We find this I_x prime, that is this value OF as 5.96 into 10 to the power of 6 mm to the power of 4 and I_y prime which is OG as 3.89 into 10 to the power of 6, by measuring it in this diagram, which has been constructed to the given scale and I_{xy}, which is nothing but FX prime or GY prime, which is equal to 3.28 into 10 to the power of 6 mm to the power of 4. So, these procedures illustrate how we can use this Mohr's circle diagram to evaluate the product of inertia.

In this class, we saw how to determine the product of inertia and the second moment of inertia and product of inertia for rotated axis and also we saw the concept of determining the principal moments of inertia and the corresponding axes.