Engineering Mechanics

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Module 5 Lecture 12
Application of Friction Part-3

Today, we will see some more applications of friction. For your reference, this is module 5, lecture 12, of the Engineering Mechanics course. In the previous lectures, we saw some applications like the screw jacks and thrust bearing. Today, we will see the application of friction in journal bearings and we will also see how rolling resistance and wheel friction are induced in the motion of wheel.

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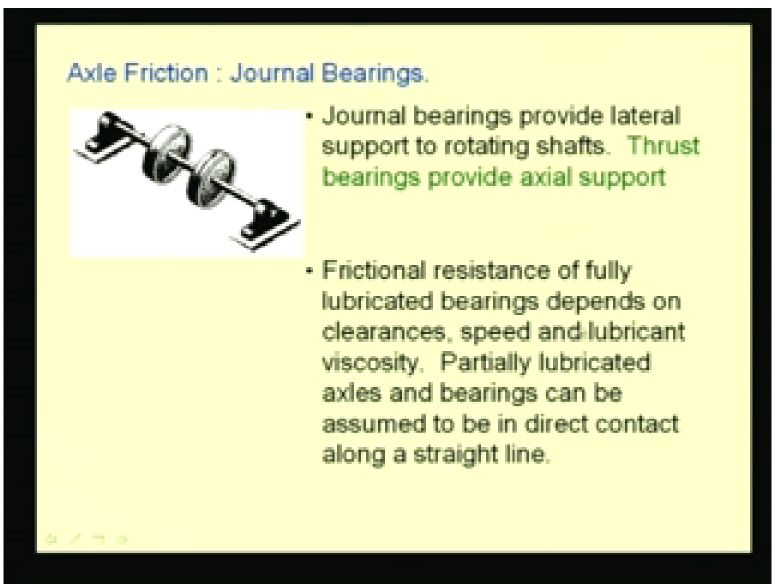

In this picture you see an axle, supporting pulleys which are driven or driving some machineries. This is the way commonly these pulleys are supported. These are the axles and the axles rotate about this axis, which has to be supported by the end bearings. The motion is in the horizontal, and so the radial forces, the weight of this pulley and other forces are radially transmitted to these bearings, which have to be supported by this bearing block at these two ends. We have already seen one kind of bearing, that is, the thrust bearing which provides axial support. So these bearings are known as journal bearings, which provide lateral support to the rotating shaft. To reduce friction, lubrication is generally applied between the axle and the bearing block.

The frictional resistance that occurs between the axle and the bearing block depends on many factors. Primarily, the clearances between the axle and journal block, the speed with which the shaft or the axle is rotating and the viscosity of the lubricant that is being used. Partially lubricated bearings can be assumed to be existing in dry condition and one can assume that the axle is having a point contact, or along the bearing it is having a line contact while in motion. So with this assumption, we will proceed to analyze this problem.

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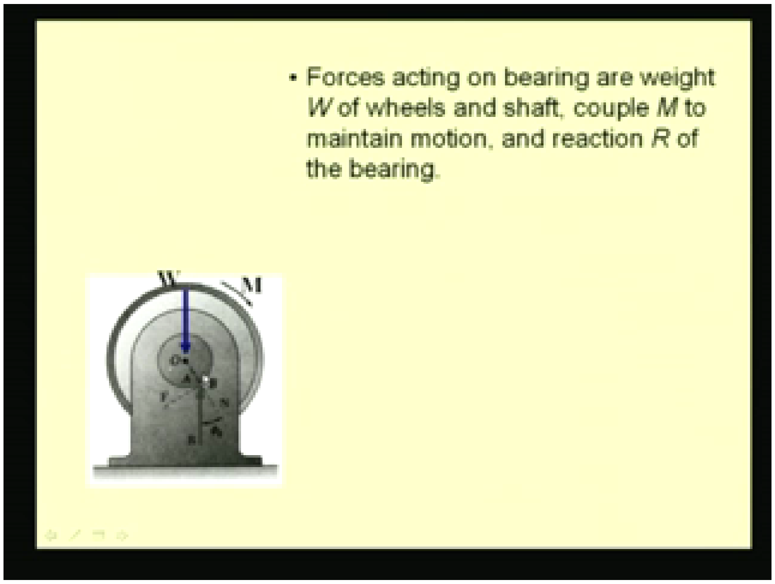

This figure shows the pulley which is being supported by this axle, which is supported by this bearing block. This circle shows the bearing surface where the axle is in contact with the bearing block. Let W be the weight of these pulleys or wheels that are being supported by this axle and a moment of M is being applied to keep this wheel in motion. So, the reaction from the bearing to the axle has been shown to be shifted to a location, which is not the point of contact, if the wheel is stationary. Let us see how this actually happens.

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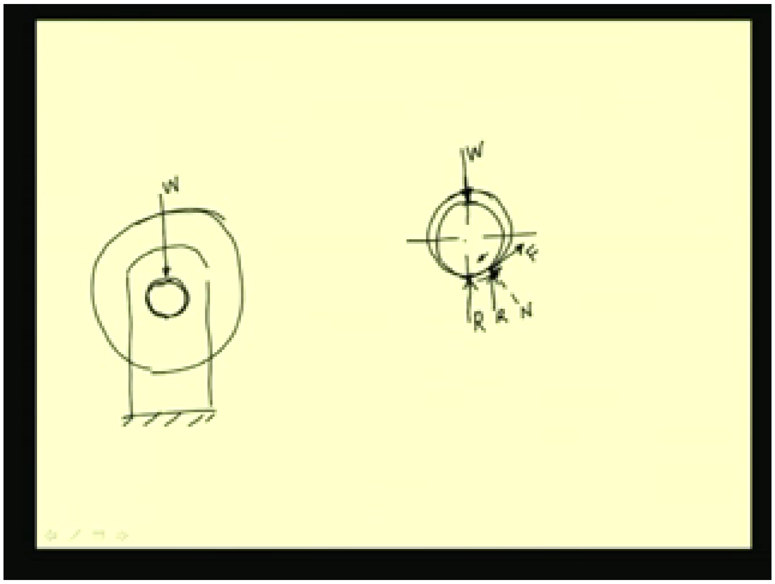

Let us consider this wheel and the axle being supported by a bearing block. For clarity, let this be the weight W that is being applied to this bearing block through this axle. When the wheel is stationary, the point of contact is here. If we exaggerate the situation that there is a clearance between the journal and the shaft or axle, this is the clearance that you see. When the shaft is idle or not in motion, the point of contact is here or along the journal, this is the line of contact between the axle and the journal. This is the weight that is being carried by this axle. So the support reaction from the bearing block acts at this point, let it be R.

Let us say, we give a small rotation to this axle. Then, because of the friction that exists, the axle climbs the journal because no slippage occurs at this point, and with respect to this point this axle tends to move and so, new points of contacts happen. Slowly, the axle tries to climb in this direction. The point of contact actually shifts. This happens till some maximum point which corresponds to the impending slippage case, where the normal reaction is like this. Since the shaft or the axle is tending to slip in the downward direction, this is the direction in which the axle is trying to slip, so the friction will be acting in this direction. This will be the frictional force and this will be the normal reaction. The resultant of these two will be R. This value reaches the maximum limiting value, where this frictional force F is related by this N by the coefficient of static friction.

Once the journal starts rotating at some higher speeds, the point of contact slightly comes down because the kinetic friction is lower than the maximum static friction. So, the point of contact shifts from this, let us say the maximum point up to which the axle has climbed to an another position and it remains in equilibrium at this position.

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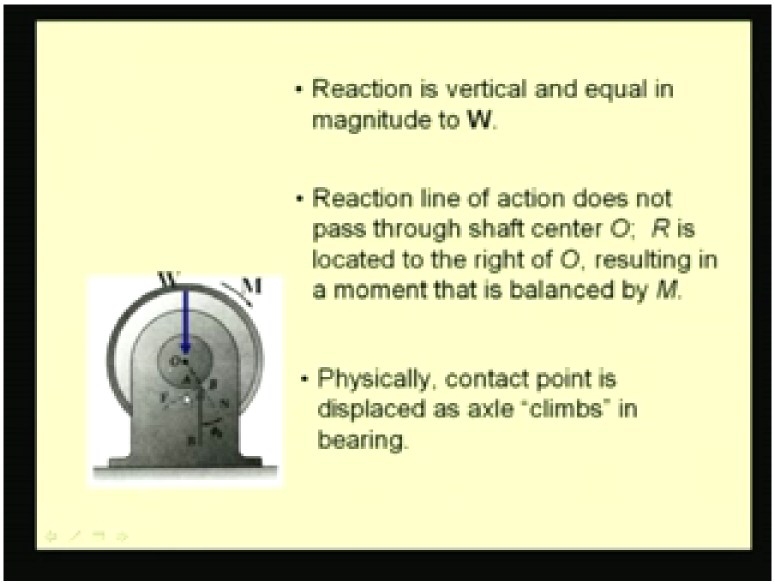

That is what is represented in this picture. For a constant motion of this axle, the point of contact has shifted from say the lowest point A to the point B. This is the frictional force, which is the kinetic frictional force, existing between the axle and the journal. This is the normal reaction along the radial line. The resultant of these two is R, which is vertical and equal in magnitude to that of W. The moment or the couple of these two forces, equal forces, that is W and R, has to be balanced by the applied moment. Else, this couple which is anticlockwise will retard the motion; that means, the wheel which is revolving or rotating will come to stop if this M is not applied.

To keep this wheel in constant motion, this moment M has to be applied. The clockwise moment of M has to be applied. So we call this as the frictional moment that has to be overcome in the journal bearings. For this equilibrium position, we know the angle between the resultant and the normal force is phi k, which is angle of kinetic friction. Certain things you can again observe; that reaction is vertical and is equal to the magnitude of the weight carried by the axle. The line of the reaction which is passing through this B does not pass through the shaft center that is O, which is the case when the shaft is idle. The reaction passes through O when the shaft is idle, but now it has been shifted to a location because of the motion of the axle.

Here for this case, R is located to the right of O, because the axle tends to climb in this direction. If the wheel is rotating in the counterclockwise direction that is the applied moment was counterclockwise, then the wheel will climb in the other direction. So this point of contact B will be to the left of O. But in this case, it is located to the right of O. This results in the moment and which is balanced by this applied moment. So this is what we call that physically the axle climbs in the bearing.

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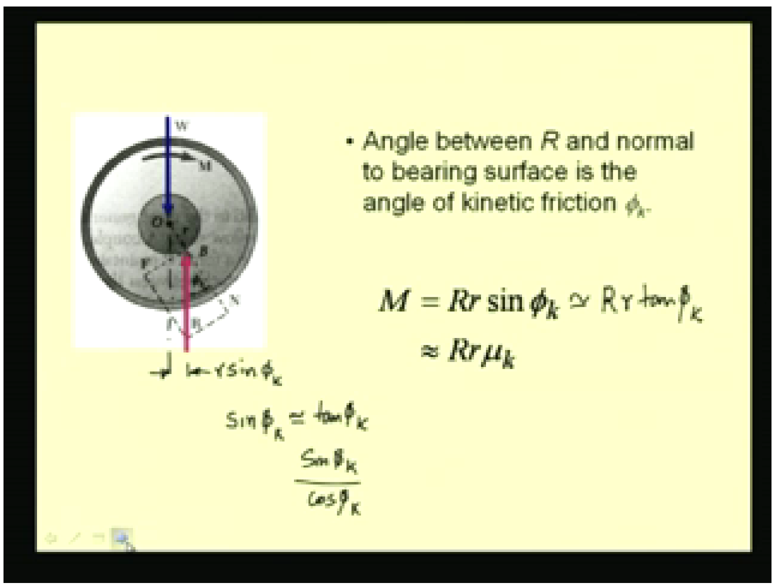

This picture just shows the various forces and the moments that are acting. The angle between R and the normal as we have seen is phi$_k$. So we can write the moment caused because of this resultant R as r times, r which is the radius of the axle times sin phi$_k$ which is nothing but the horizontal distance of displacement of this point of contact B. So this distance is r sin phi$_k$. For small angles of phi$_k$, we know that sin phi$_k$ is equal to tan phi$_k$. This is because tan phi$_k$ is nothing but sin phi$_k$ divided by cos phi$_k$ and for very small angles, this cos phi$_k$ is nearly one.

This can be approximated for small angles and that is why we write this is approximately equal to R times r tan phi$_k$. We know that tan phi$_k$ is nothing but the coefficient of kinetic friction. So we replace this by mu_k . We see that this moment M is equal to R times small r mu_k, where R is the reaction force, which is equal to the weight W and r the radius of the axle and mu_k is the coefficient of the kinetic friction that exist between the axle and the bearing block.

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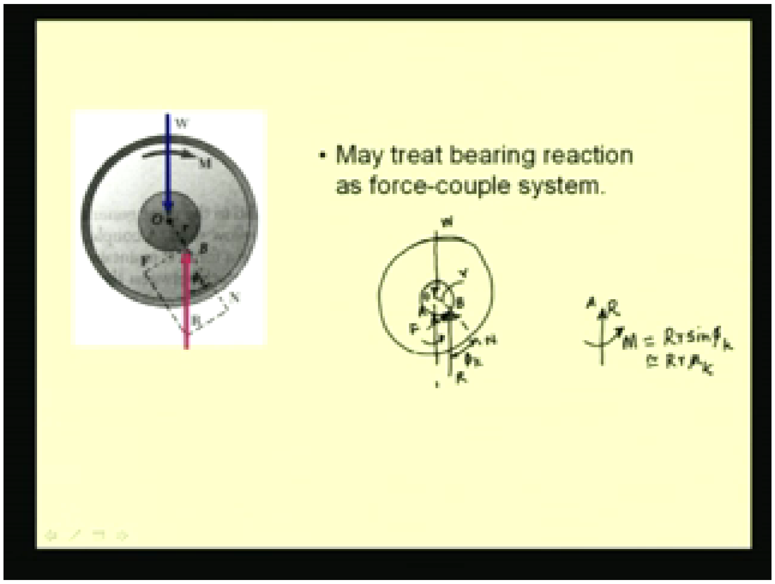

We can treat this bearing reaction R at this point B as a force and a couple system. Let us see how to do that. We have this wheel and the axle, the weight passing through O and the point of contact has shifted to this point B. This is the normal reaction and we have the frictional force and the resultant R. This is small r the radius of the axle.

Now I can replace this resultant force R, by a force that passes through point A which is the lowest point of contact or which is just below O, the center of the axle and replace this by a force R and the moment due to this force being acting at this point B. This can be replaced by this force R and the moment. So this is the force R acting at A and a moment M which is equal to R r sin phi_k, which we just found which is approximately equal to R times r mu_k. We can treat this reaction R at B as a reaction R at A and a moment.

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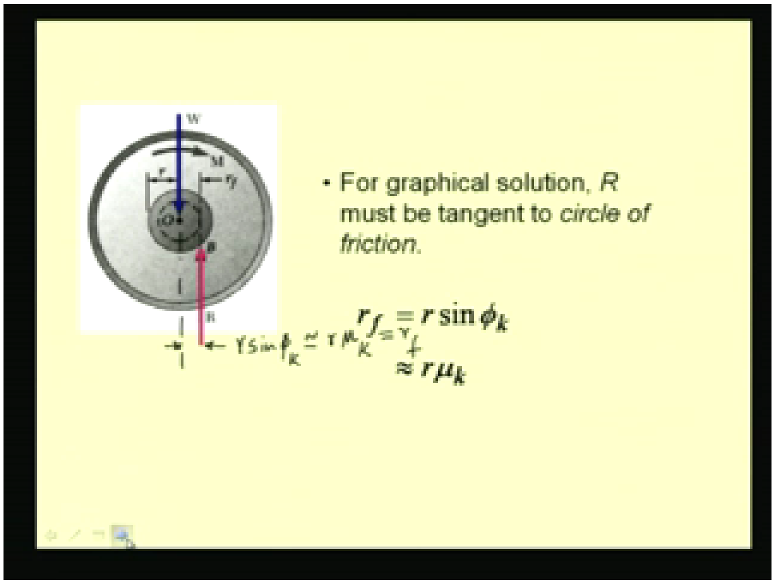

Graphical solutions are possible where we can place this reaction force R tangential to the circle, which we call it as the circle of friction, because we have seen that this R is displaced by a distance equal to r sin phi_k. This vertical displacement is equal to r sin phi_k which is approximately equal to r times mu_k. So I can draw the free body diagram with this reaction R, such that it passes tangentially to the circle of friction, whose radius is rf, which is given as r mu_k.

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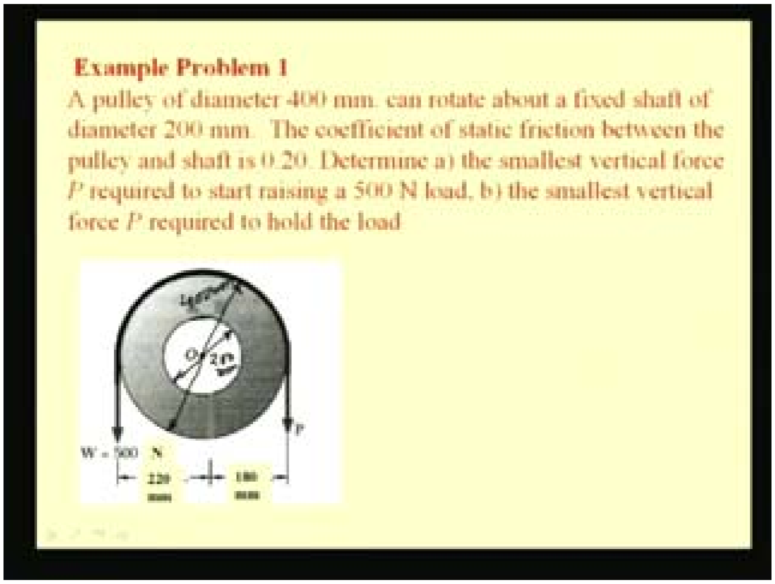

Let us take one example problem and see how we solve the problem of this journal bearings. Here you see a pulley supported on a shaft at O. The diameter of the pulley is given as 400 mm and it can rotate about the fixed axis shaft of diameter 200 mm. This axle or the shaft is having a diameter of 200 mm; this is 200 mm and this has been given; as this is the radius, we have it as 200 mm and the diameter of the pulley is given as 400 mm.

The coefficient of static friction between the pulley and the shaft is known as 0.2. We are interested to determine the smallest vertical force P required to start raising a 500 Newton load. We have this 500 Newton load attached to a belt passing over this pulley and this force P is being applied in order to raise this. We are interested to find the smallest vertical force that will be required to raise this load.

Secondly, we are interested in finding the smallest vertical force P that will be required to hold the load. That means if we we leave this, the weight will fall. So, we need to keep some resisting force P, so that this load does not drop. Here, we assume that the impending slippage does not happen between the belt and the pulley, but rather the slippage occurs between the pulley and the shaft. So the slippage is on this surface and not on this surface. For the first case, let us draw the free body diagram.

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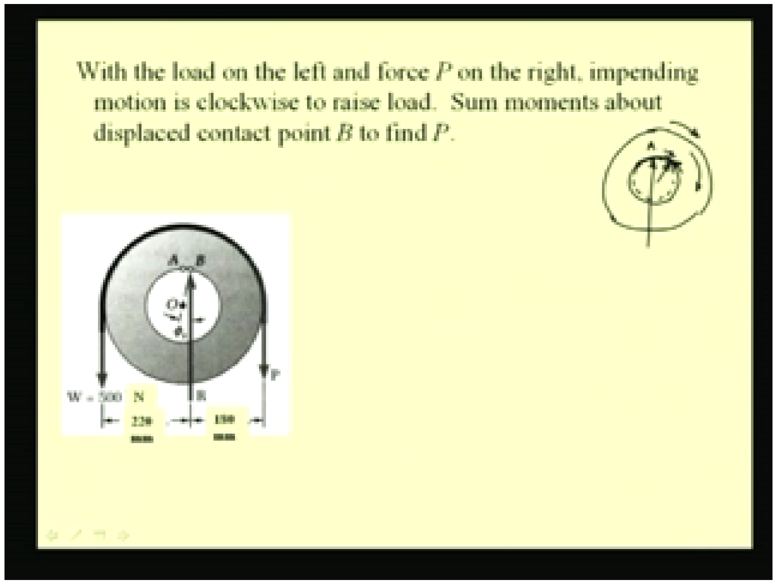

When this P tends to lift this weight W, the impending motion is clockwise; that means this pulley tends to move in the clockwise direction over the axle or the shaft. The point of contact which is originally at A, now shifts to B and it tends to slip at this point B. This can be found by considering the analogy that we have the axle and the pulley; originally, the point of contact is at A and as the pulley tends to move in this direction, for exaggeration, let us keep that this is the diameter of the shaft; the point of contact slowly shifts in this direction. Once it reaches some maximum value, it starts to slip and the pulley starts to rotate with respect to the axle.

Now we can represent the resultant R which is nothing but the resultant of the normal force and the frictional force. So this R is the resultant of the frictional force and the normal force acting at this point. Since the slippage is in this direction, the force will be acting in the opposite direction. If you want to solve this problem by a graphical method, we can place this reaction R at a distance which corresponds to the radius of static friction. The angle made by this R with respect to the normal is phi_s.

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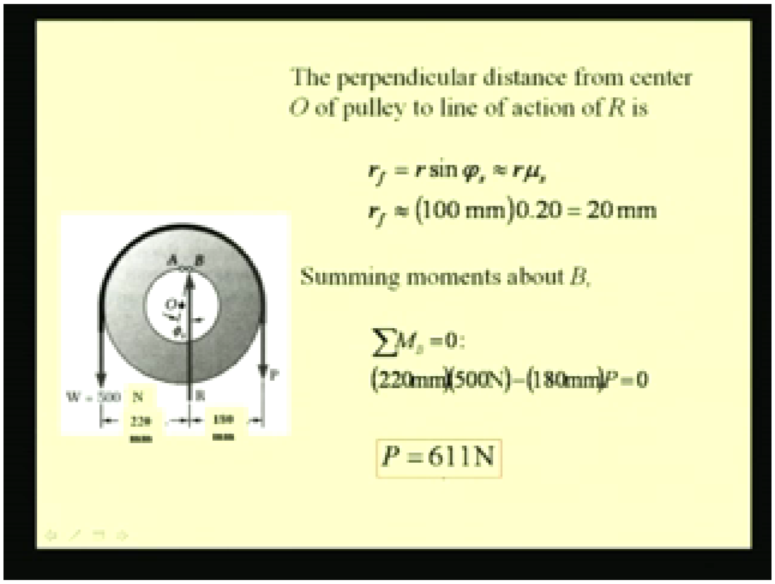

We find the radius of friction as r sin phi_s because here it is an impending slippage case and so we take the coefficient of static friction. The radius of the circle of friction will be r sin phi_s for small angle it is equal to r tan phi_s or r mu_s. For the particular case, we find this as 20 mm. The reaction R is shifted by 20 mm to the right from O. We have these distances as 200 minus 20 which is 180 mm and this distance between this vertical force W and R is 200 plus radius of friction which is 220 mm.

Now we can sum the moments about B and equate it to zero for equilibrium. So we have, the momentum for this 500 Newton force as 220 mm and the momentum for the force P that is used to raise as 180 mm. We have 220 into 500 minus 180 into P equal to 0, from which we find P as 611 Newtons.

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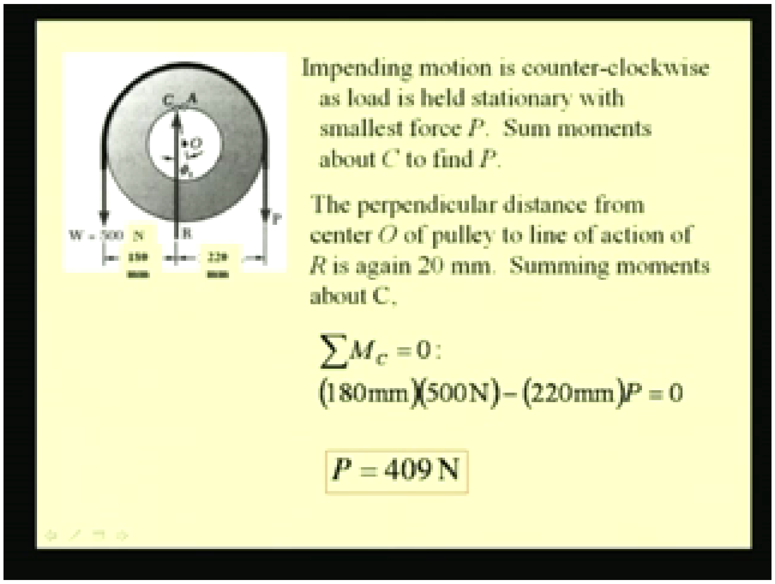

For case B, we are interested to find the force P that is required to just hold the weight. So that means, now the impending slippage is that, the pulley starts to move in the counter clockwise direction. The point of contact which is originally at A, now shifts in the counter clockwise direction to the point C, where the horizontal displacement of this reaction R is equal to the radius of the static friction circle, which is again 20mm. So, we see that momentum for this weight W is 180 mm and for this force P as 220 mm.

Now, we sum the moments with respect to C in order to find this force P required to hold this weight. So summing the moments, we have the momentum for the 500 Newton force as 180 mm. So we have 180 times 500 Newton minus 220, the momentum for this force P, which is a clockwise moment, so negative equal to zero. From this, we find the value of P as 409 Newtons.

So, this problem illustrated how we can solve the problems involving journal friction, by first finding the radius of the circle of friction and then displacing the reaction tangential to the circle of friction and then solving the problem.

Now, we will move on to discuss wheel friction or the kinds of friction that occurs when wheels move on platforms.

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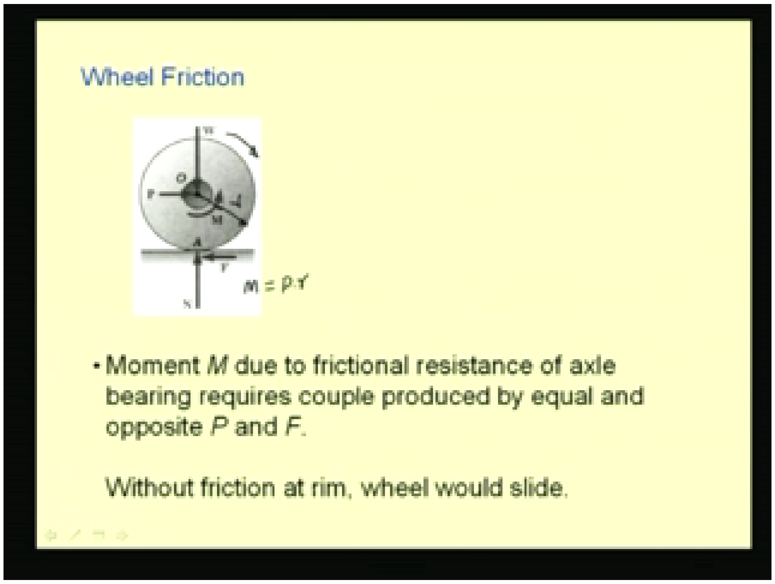

In this picture, you see a wheel supported in an axle at O. Let W be the weight of the wheel acting through the axis of this axle O. When the wheel is idle, the normal reaction from the ground N passes through this point A and the wheel is in equilibrium. When this wheel starts moving in forward direction, we know that there exists a friction between this axle and the wheel which resist the motion. If the wheel tends to move by rotating in the clockwise direction, then we have the resisting moment acting in the counter clockwise direction at the axle. So that is what is marked here as M. So, this frictional force or the frictional moment existing in the axle and wheel prevents this motion in the forward direction.

In order to sustain this motion or the forward motion, we have to apply a force on the axle. So that is what is marked as P. In order to overcome this momentum M such that, if r is the radius of the wheel, then we find that, if we apply a moment which is equal to P times of r, then we can sustain this motion.

From the free body diagram, we see that if this force P acts at the axle, then we need a force F at this point A. Only then, the system will be in equilibrium. You can see that W is balanced by N, the moment M is balanced by the moment due to this force P, that is P into r, but when you sum the horizontal forces, if this force F is not there, then this force P is unbalanced. In that case, what will happen is rather than rolling this wheel, it starts sliding and this point of contact will start moving.

In order that this wheel is able to roll, we need to have this force F. That is how this frictional force helps in the rolling motion. Even though we see that there is a point contact, the presence of friction is necessitated for the rolling action to take place. So that is what we say that, the moment M due to the frictional resistance of the axle bearing requires the couple produced by the equal and opposite P and F. Thus, the presence of F is necessitated. Without the friction on the rim, which is at this point A, the wheel would slide.

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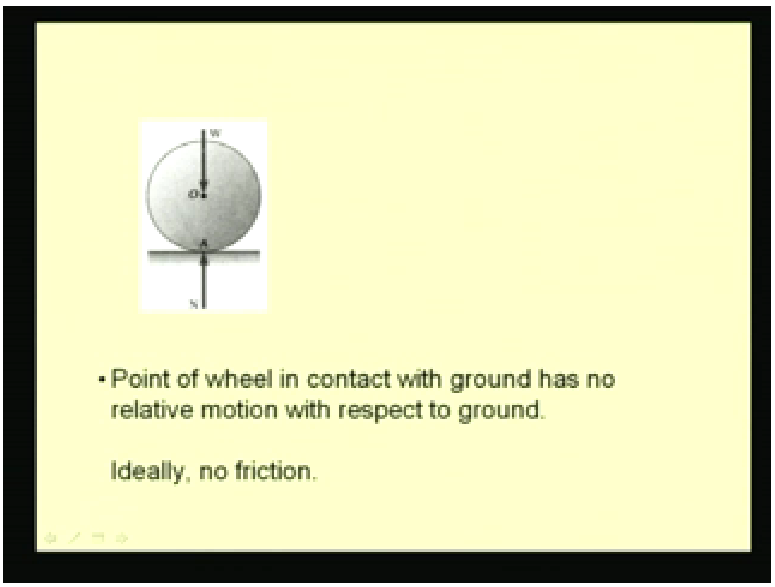

Let us consider a free wheel which does not have an axle. Here, you see a free wheel which does not have an axle, which is pending to role. Assume that the contact at A between the wheel and the ground has no relative motion for the wheel to roll; because for this rolling action, the point A will be idle and the other parts of the wheel will rotate with respect to this point of contact A. If the surfaces are hard, then the point of contact or this contact is a point or line along the width of the wheel. So ideally, we see that there is no friction.

When a free wheel, that is a wheel which is not supported by an axle and which is hard, the ground on which it rolls is also hard, so that a point or a line contact can be maintained, should ideally roll without stopping, if a small rolling motion is given to the wheel. But, we see that this generally does not happen for real systems; that means, we have a real material constituting the wheel and the ground, then we see that when wheel is rolled on the ground, after sometime it comes to rest.

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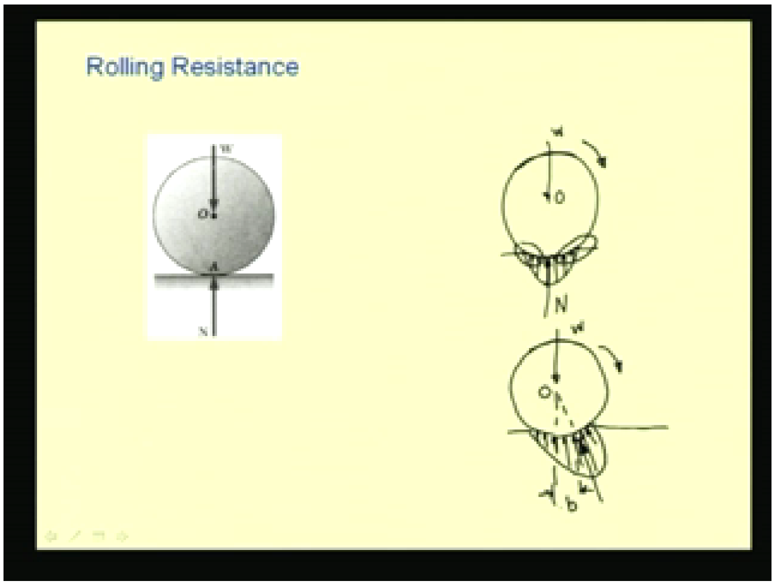

We can see why this happens. This happens because of the presence of rolling resistance. Let us see, how this rolling resistance comes into picture. In the ideal contact, both these bodies are deformable and so the point of contact is no longer a line. But rather, if this is the weight of the wheel passing through its center, we have a region of contact or in otherwise a surface of contact. Thus, the reaction is no longer a point force but rather a distributed force. The resultant of all these forces is equal to this R or N. So the resultant of all these forces is equal to N, which is equal to the weight of the wheel when this wheel is idle or which is not in the rolling motion.

But, when the wheel tends to roll, the pattern of these uniformly distributed forces changes. This can be explained by considering the same wheel. Now let us say, the wheel tends to move in the clockwise direction. So what happens, the contact that is there in this vicinity, experiences a compression because of this wheel. We are assuming the ground as well as the wheel to be constituting of materials, which are deformable under the action of these forces. The material here compresses and thereby offers more reaction; the material here is relieved because the wheel is moving away from this portion. So, what happens the distribution here, the material gets relieved and here it gets compressed because of this motion. So the reactions here are more than the reactions that are in this place. So, the distribution becomes something like this. We see that if we take the resultant of all these forces, now it will no longer be this vertical force N, but rather a force which acts at some distance away from the original point under consideration below, say O. Now this reaction force still passing through O, but having a displacement in the direction, in which the wheel tends to roll. So this point of contact, now, would have shifted by a distance, b.

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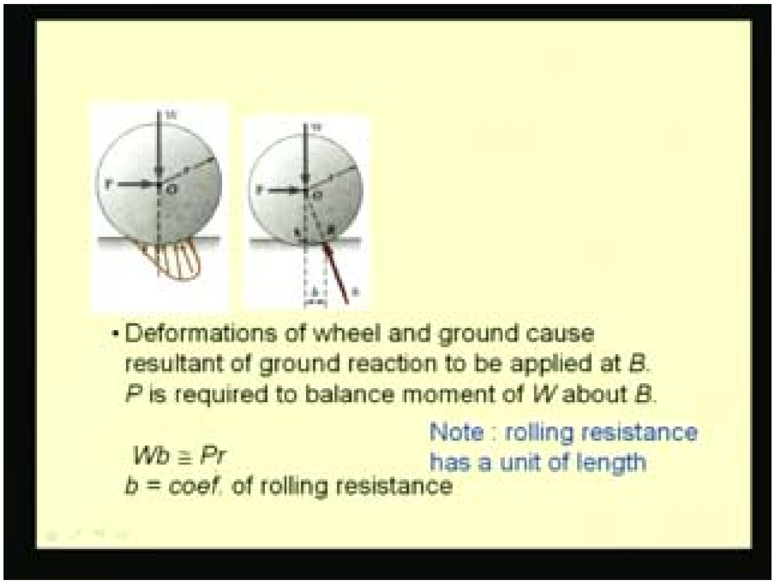

That is what you see in this picture that, the resultants of these forces have shifted from the point vertically below the axle or the center of the wheel, to this point b, whose horizontal displacement is b. Now, the moment due to this force, the vertical component of this force r is balanced by W but, the horizontal component of this r is balanced by this applied force P, which keeps this wheel in rolling motion. The moment of this force balances the moment of this reaction. So we see that the resistance through rolling comes because of the moment of this reaction force r, which has to be balanced by an applied moment. If this force does not exist, then the wheel comes to a rest because of this resisting moment, the moment due to this force r. We see that, there is a reaction applied at B or b, and P is required to balance the moment of W about b or r with respect to this O or the vertical axis.

We see that, the moment of this weight is W times b, the horizontal displacement of this point, which has to be equal to the moment due to this force P times the momentum, which is this point A, so OA. But for small deformation, we can say that this distance OA is equal to the radius of the wheel itself. This is approximately equal to P times of r.

If the surfaces of contact are deformable then we need to have this force P in order to keep this wheel in the rolling condition. This distance b, to which this reaction r has displaced from the vertical, is known as the coefficient of rolling resistance. We can easily see that this has a unit same as the unit of length. We know that the coefficient of friction does not have any unit but the coefficient of rolling resistance has the unit of length.

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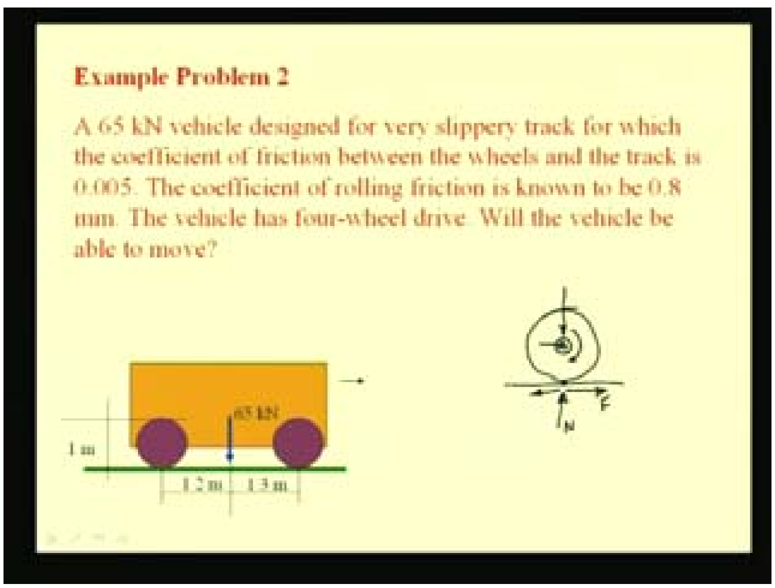

Let us see one example problem. Here, you see a vehicle which is having a weight of 65 kilo Newton and it is designed for slippery tracks; it could be a vehicle that moves on ice or it could move on steel tracks, where the coefficient of friction between the wheels and the track is very low, say, 0.005. The coefficient of rolling friction is known to be 0.8 mm. The vehicle has four-wheel drives, which mean all these wheels propel this vehicle forward. We have two wheels in the front and two wheels in the rear. So, all these wheels are propelling this vehicle forward and thrust is developed on all these four wheels. We are interested to find whether the vehicle will be able to move or not. In order that the vehicle moves, the thrust developed by this wheel should be less than the maximum friction that is available between the wheel and the track. This can be seen by considering this picture of the wheel.

Let us say this is the axle on which this vehicle is supported, and we have the track. This wheel is developing a moment or the thrust in order to propel this vehicle forward. So, this will in turn, will be experienced as a force at this point and this point will tend to slip in this direction. But to prevent this point from slipping the friction acts in this direction; we have the normal force. Let us say, some component of this weight is being transmitted by this axle through this wheel.

We see that we have four wheels. So this load will be distributed by those four wheels. If we have this as the normal reaction, then this frictional force F prevents the wheel from slipping at this point. How large this thrust could be? It could be maximal equal to the limiting value of friction at this point, because beyond which, this point will tend to slip. If this point does not slip then because of this, the axle will experience a forward force and thus the vehicle will propel. The forward motion of the vehicle depends on the fact that the developed thrust force is less than the maximum available friction. This developed force for traction should overcome this rolling resistance.

We have to ultimately see that the rolling resistance has to be less than the maximum available friction on the track. If this condition is satisfied, the wheel will be able to propel forward. If the rolling resistance is more than the available friction, then the vehicle will not be able to propel, but rather the wheels will keep revolving and it will be slipping from the track and the vehicle will be idle and it will not get propelled.

Let us go to the problem.

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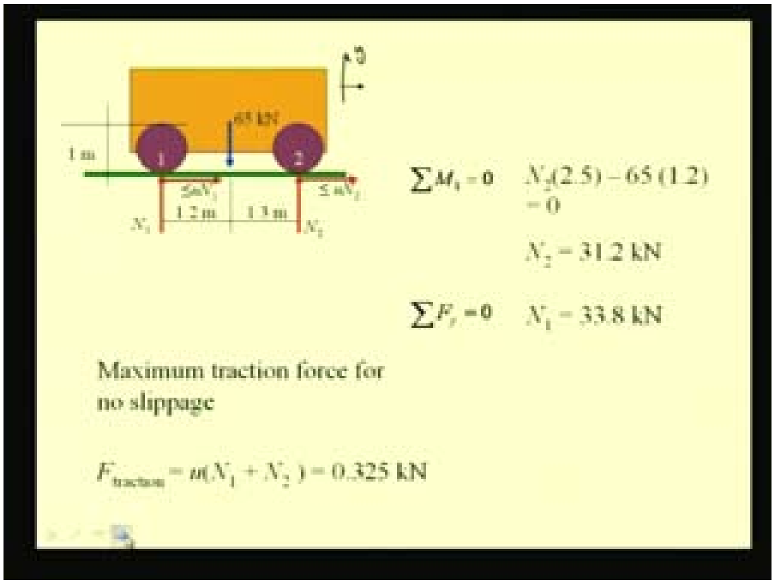

This shows the various forces that are acting. Let N_1 be the total of the reactions, because of the two wheels in the rear and N_2 be the sum total of the two wheels that are situated in the front. The frictional force that will exist or that can exist to the maximum is mu times of N_1. This is the maximum value of frictional force that can exist at this point and same way, the maximum value of the frictional force that can exist between the wheel in the front and the track is limited by this value, mu times N_2. We know this geometry that is the location of the CG of this vehicle and it is located at 1.2 meters from the rear axle and 1.3 meters from the front axle and 1 meter from the ground.

We write the equilibrium equation. Let us say, we take the moment summation about this point. One, the momentum for this reaction force N_2 is 1.2 plus 1.3 which is 2.5 meters minus the moment due to this weight 65 kilo Newton force, which has the momentum of 1.2 meters. From this equation, we find this normal reaction N_2. From force summation equation, we can find N_1. So we have this N_2 as 31.2 kilo Newton and we write the force summation in the y direction. We have N_1 plus N_2 should be equal to 65 kilo Newtons and from that, we have N_1 as 33.8 Kilo Newtons.

Once we have found these normal reactions $N_1$and N_2, we can find the limiting value of frictional forces that are available at the points of contact between the wheels and the track. We find the total traction force that can exist without slipping is mu N_1 plus mu N_2; that is mu times N_1 plus N_2 which is equal to 0.325 kilo Newtons. This is the maximum frictional force that is available and so the traction force cannot exceed this value. Let us see what is the rolling resistance and see whether that is less than this maximum available traction force.

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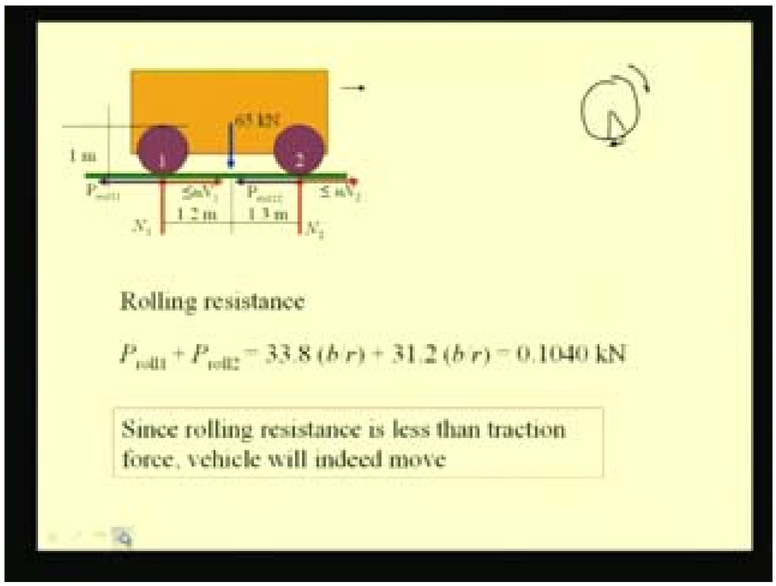

Now these vectors represent the rolling resistance that has to be overcome. Because when this wheel tends to move in this direction, the point of contact shifts to this side and this wheel tends to move in this direction. This is the rolling resistance or the force, which is acting from right to left, which has to be overcome. We will find this to be equal to the total weight or the normal reaction, because this is equal to W times of b by r, where W is the load that is being carried by each set of wheels. For this wheel one, we have the weight that is being carried as 33.8 and for wheel two we have 31.2. So we have the rolling resistance as 33.8 times b by r plus 31.2 times b by r.

We know, the radius of the wheel as well as the coefficient of rolling resistance, from which we find the total rolling resistance as 0.1040 kilo Newton's. Since this value is less than the traction force, which we have found to be 0.325 Newtons, the vehicle will indeed move. We will see one more example.

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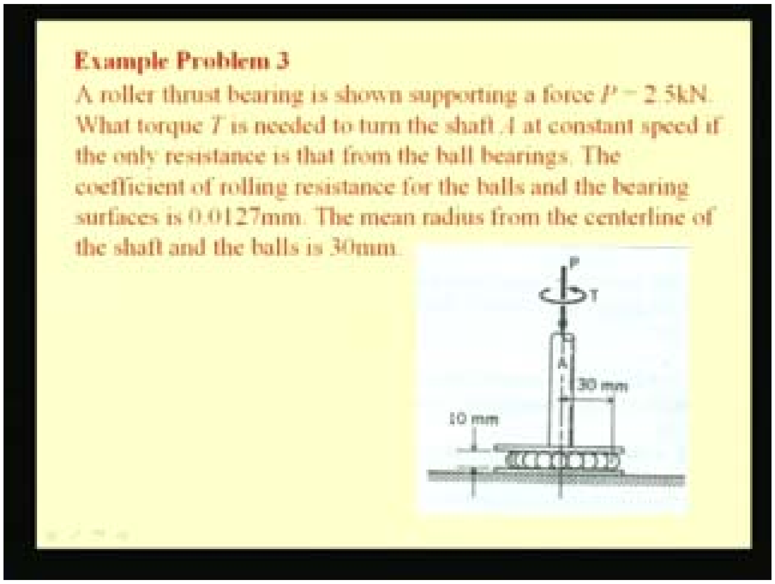

Here you see a roller thrust bearing, where between these two discs; we have a series of rollers which are used to support. The rolling resistance is generally less than the bearing resistance in thrust bearing without rollers and so they are generally preferred. Let us see in this problem, we have the mean radius of these balls as 30 mm and the diameter of these balls are 10 mm. It has to support a load of P. In order to keep this shaft in rotation, we need to apply this torque T. If the coefficient of rolling resistance is 0.127mm and the various other dimensions are given, we are interested to find this value of the torque required to keep the shaft in rolling motion.

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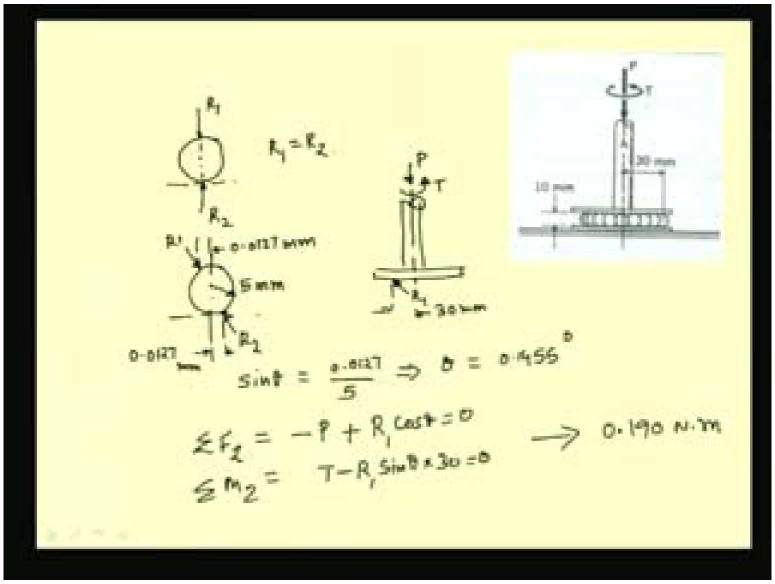

When the shaft is idle, let us consider one of the balls. So, we have the reaction coming from the ground as R_2 and the reaction coming from this top phase as R_1. They lie in the same line and the total of all this R_1 or the vertical reactions of these various balls should be equal to this load P. In a limited equivalence because all these forces are having the same direction cosine with respect to W, we can sum all these vertical reactions and say that, that is R_1 and sum of all the vertical reactions from this ground as R_2 and they will be like this. But, when the shaft starts to rotate in the counter clockwise direction, this point of contact for the ball, shifts to a new location in the forward direction. This is for the bottom surface.

This reaction shifts to a point, the distance to which it moves is equal to the rolling resistance that is 0.0127 mm in this case. The force coming from the top phase, which is R_1, is also displaced by a distance of 0.0127 mm. The radius of these ball bearings is 5 mm. If we draw the free body diagram for the top portion, this is R_1, we have P, the applied torque T. This R acts at a distance of 30 mm. This is a limited equivalent diagram, because all the forces R_1 of the various balls have the same direction cosine with respect to the vertical.

From this free body diagram, we can write the equations. We have sin theta as the rolling resistance, divided by the radius of the balls, from which we can find theta as 0.1455 degrees. When we write the force summation equation for this shaft, we have, sum of the vertical forces as minus P plus R_1cos theta equal to zero and when we sum the moments, it is equal to T minus R_1 sin theta times 30 equal to zero. From this, we can find that the torque T that is required is 0.190 Newton meter. So, this example illustrated how we solve for rolling resistance.