Engineering Mechanics

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Module 4 Lecture 11

Application of Friction Part-2

Today we will continue the lecture on various applications of friction. For your reference, this is module 4 lecture 11 of the engineering mechanics web-based course. In the last lecture, we saw an application of friction in square threads where they are used in clamps and screw jacks, in order to raise the load or to apply some force of clamping. Today, we will see some more applications of friction, what we call as the application of disc friction.

In order to analyze these problems again we will use the concept of deriving these frictional forces for elemental areas of contact. Disc friction particularly finds application in automobile brakes and clutches, and in machineries in the form of thrust bearing or collar bearing.

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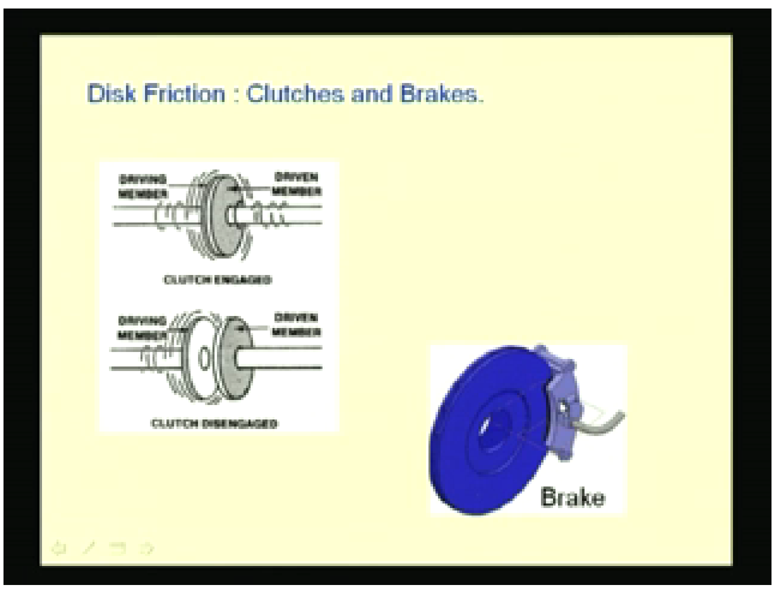

This picture shows the working of a clutch. There is a driving unit. A motor or an engine is connected to this shaft and it is running. You see a disc pad attached to the driving member and another disc pad attached to the driven member. When these two pads are in contact and there are sufficient normal forces in order to maintain this contact, then this disc also rotates along with the disc of the driving member. Thus, this shaft which may be connected to the machinery also rotates. Here, one should note that the torque is being transmitted from the driving member to the driven member through the friction that exists between these two pads. So, the normal force as well as the friction between the pads should be sufficiently large to transmit the required quantity of torque. When we distinguish, this member does not rotate. This is the principle of the clutch and we see the application of the friction in transmitting torque.

The other application is in braking. Here you see this disc which is attached to the wheel of an automobile and a braking pad or what we call as braking shoes, which when engaged will apply a normal force on this disc and the friction between this pad and this disc will impart its rotating motion thereby bringing this disc to a standstill. This is how a brake works. Here we see that this friction is used either to transmit the torque or to apply the braking torque.

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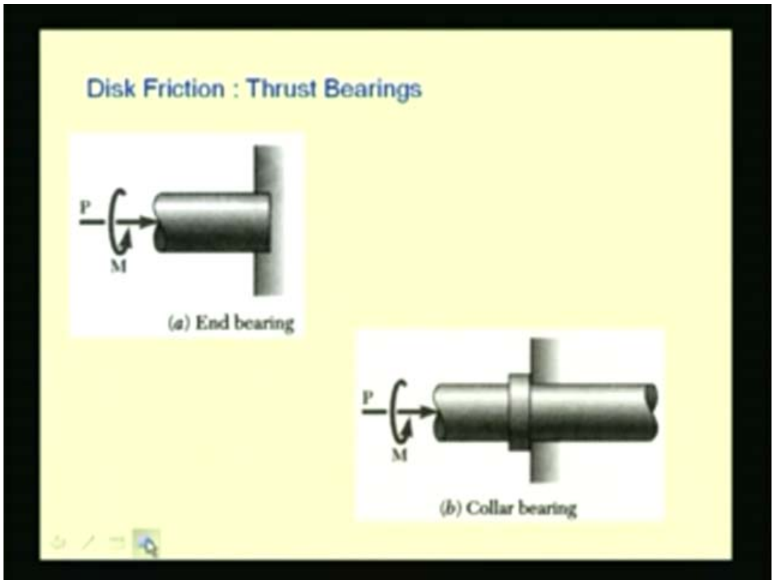

Other applications are in the case of bearings. Bearings are used to support shaft which are transmitting torque. So, if some axial force exists in a shaft, in order to prevent the axial moment of the shaft, as well as the shaft has to rotate in order to transmit the required motion; so, we have to provide a bearing face such as shown in this picture which supports this axial force and as well as this shaft is kept in motion because of this torque M. This torque M has to overcome the friction in this contacting face in order to keep the shaft in the rotary motion.

Here you see another type of a thrust bearing, which is a collar bearing where it is a through running shaft and we have a collar that supports this axial load P. So, the moment M has to overcome the friction that exists between the collar face and this support face. So, here in these bearings, we see that we are interested to minimize the friction, so that the required moment to keep the shafts in the running condition is less.

In order to analyze these kinds of problem, we have to develop the frictional moment equation for these disc faces that have to be overcome by the applied moment.

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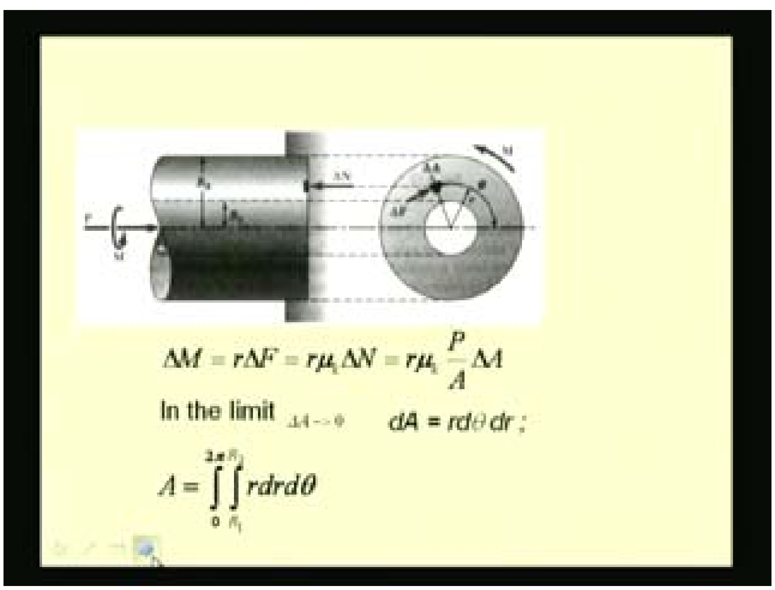

Let us consider a hollow shaft as shown in this picture. Let R_2 be the outside diameter and R_1 the inside diameter of this hollow shaft which bears onto this support in this N face. There is an axial force P. In order to overcome the friction in this N face, a moment M is applied in order to keep this shaft in the running condition. In order to solve, for this problem, we consider a small elemental area delta A on this face where we have a normal reaction which is delta N and a frictional force which is delta F.

We consider this element at a radius of R and at an angle of theta from the reference. So, if we see, we will have a similar element on the other side which will have an elemental frictional force, delta F, as shown. So if we consider the equilibrium, the sum of all these normal forces has to be equal to this force P. Since these two forces are equal and opposite, for all such elements we can find similar differential elements. Thus, these forces will sum to zero and their effect will be only to cause a moment about this point or the axis of the shaft. So, this moment, the sum of the moments of all these forces - the frictional forces, has to be overcome by this applied moment M.

Let us write the equations of equilibrium for this elemental area and then integrate it for the complete area in order to know the effect of the frictional force on this N face. Let us write the moment because of this frictional force delta F. The momentum is R, the radius at which this element has been chosen; so the moment is R times delta F, which for the running condition, can be related to the normal force by the coefficient of kinetic friction. We have this equal to r times mu_k delta N. So this force that is delta N on this elemental area is equal to the total load divided by the total area of this disc. If we consider that the contact is uniform then this is valid; that is P by A is equal to delta N divided by delta A.

So we have delta N equal to P by A times delta A. We can find the area of this disc by integrating this element delta A in the limits that is theta varying from 0 to 2 pi and R varying between R_1 and R_2. We have this differential element in the limit as dA which is equal to rd theta dr. If we integrate this in the limits that is 0 to 2 pi and R_1 to $R_{2,}$ we have the total area of the disc.

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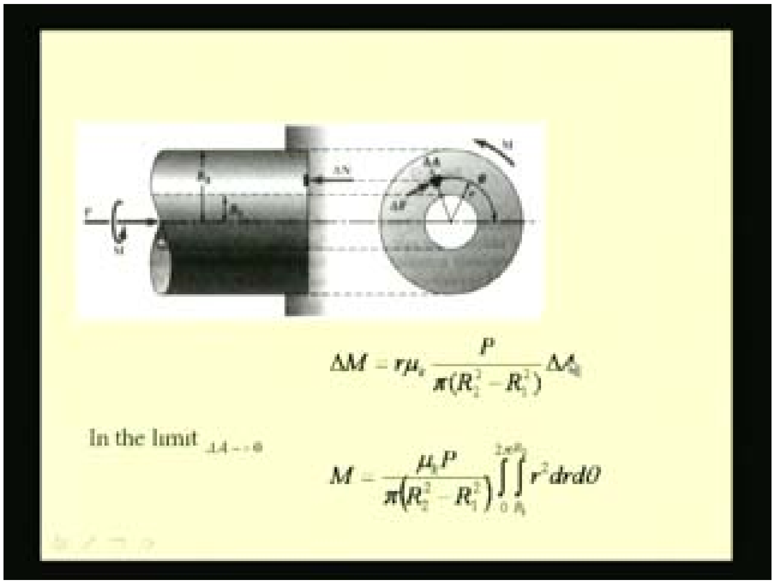

So now, we can write this moment equation. That is delta M is equal to R times mu$_k$ P by A, which we have just found as pi times R_2 square minus R_1 square. We very well know that this is the area of this annular disc times delta A. This is the moment that has to be resisted for this differential element delta A. Now we integrate to find the total moment. So the integration limit is 0 to 2 pi and R from R_1 to R_2. We have the total moment, as these things being constant can be pulled out, mu$_k$ P divided by pi R_2 square minus R_1 square, integral 0 to 2 pi, integral R_1 to R_2, R square dr d theta; because we know this delta A is rdr d theta.

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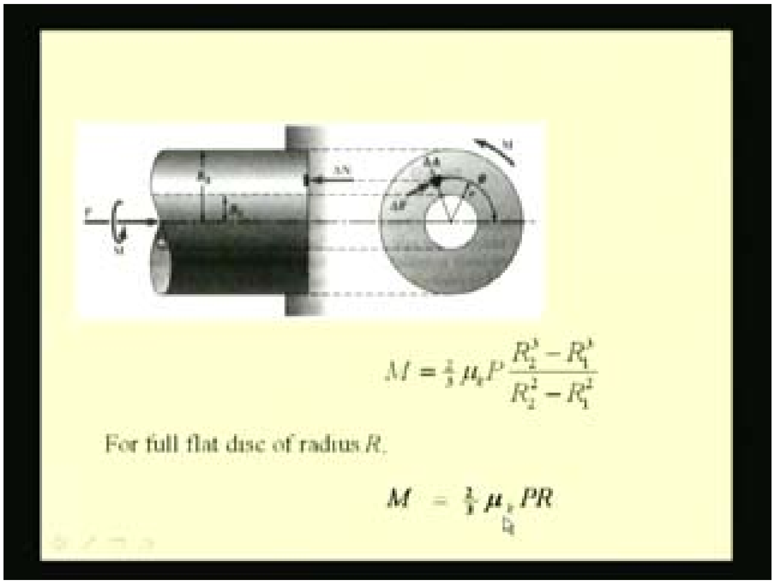

If we integrate it, we have the total moment as 2 by 3 mu_k times P the axial load into R_2 cube minus R_1 cube divided by R_2 square minus R_1 square. This is the moment that has to be overcome in order to keep this shaft in the rotating condition. If we consider the shaft to be a solid shaft; that is R_1 is 0, then we have the moment that has to be overcome as two-thirds mu_k P times R. This illustrates the method of integrating the frictional effects on differential areas, in order to predict the total behaviour of these frictional forces or the total effect of the frictional forces.

Let us consider one example.

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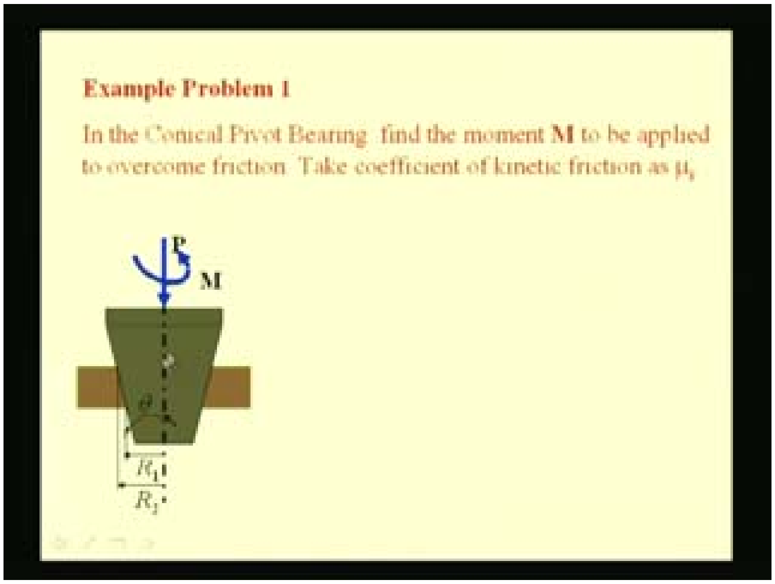

Here you see a conical pivot bearing. In the earlier discussion, we had flat end bearings. So here we have the conical pivot bearing. The axial force that has to be supported is P and a moment of M is applied in order to overcome the friction on these faces. The radiuses of contact are R_1 at the smaller end and R_2 at the larger end of this cone. Let us take the coefficient of kinetic friction as mu_k. In order to solve this, we again consider a differential element. Let us consider a differential element along the axis of the pivot bearing.

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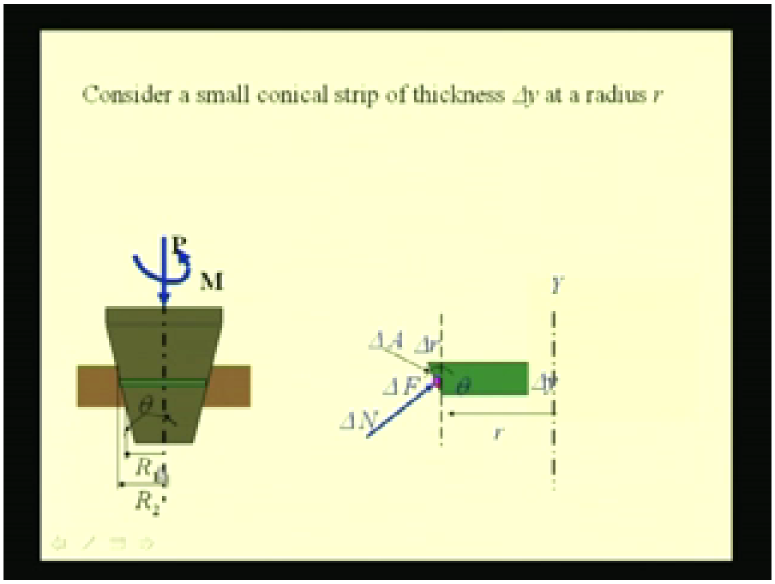

Let us say this axis is y. Then, we consider this differential element, which is a thin slice of this conical pivot bearing. Let us draw the free body diagram of this element. Here, this vector which is perpendicular to the plane of the paper is the frictional force delta F corresponding to this area; delta N is the normal force and since this moment M is in the clockwise or the counter clockwise direction to this force, del F is moving inside the plane of paper or this plane of the board.

Let delta A be the area of this differential element that bears with the conical face. Let this element be considered at a radius of R between R_1 and R_2. Let the thickness of this element be delta Y. Here theta is the semi conical angle of this pivot bearing. Now, based on this free body diagram, now we can write the differential moment that has to be resisted for this element.

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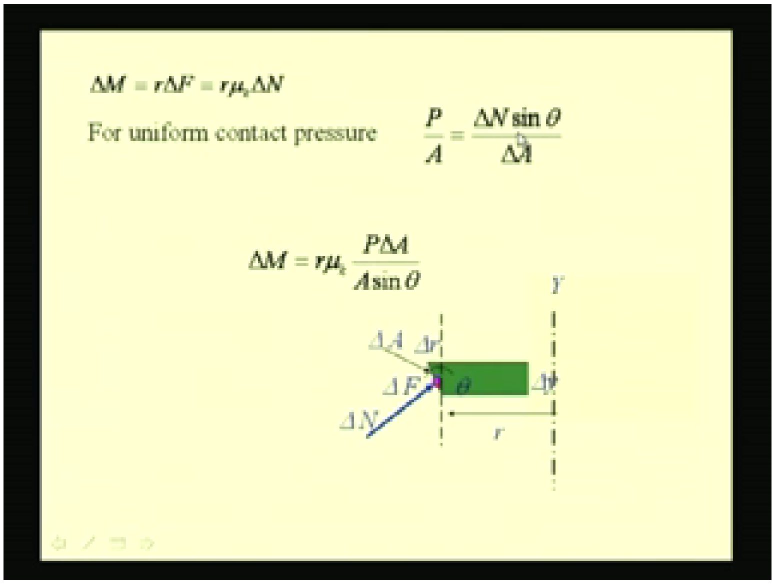

So, the frictional moment that has to be resisted for this differential element is delta M, which is equal to r times, which is the momentum, the frictional force that is delta F. For the constant motion case, this frictional force delta F can be related to this delta N by the coefficient of kinetic friction mu$_k$. We have delta F equal to mu$_k$ times delta N. So delta M is r mu$_k$ delta N.

Now, for uniform contact pressure, which means the conical face is having a uniform contact throughout from its minimum radius R_1 to maximum radius R_2, for that assumption, we have a uniform pressure which is given by P by A, the total area of contact, which is equal to delta N sin theta by delta A, because we are finding this uniform contact pressure with respect to the y axis. Now substituting this, we have delta M as r mu$_k$ P delta A by A sin theta, where A is the total area of contact and theta is the semi conical angle of the pivot bearing.

Now, we can integrate this in the limits in order to find the total moment. Before that, let us find the area.

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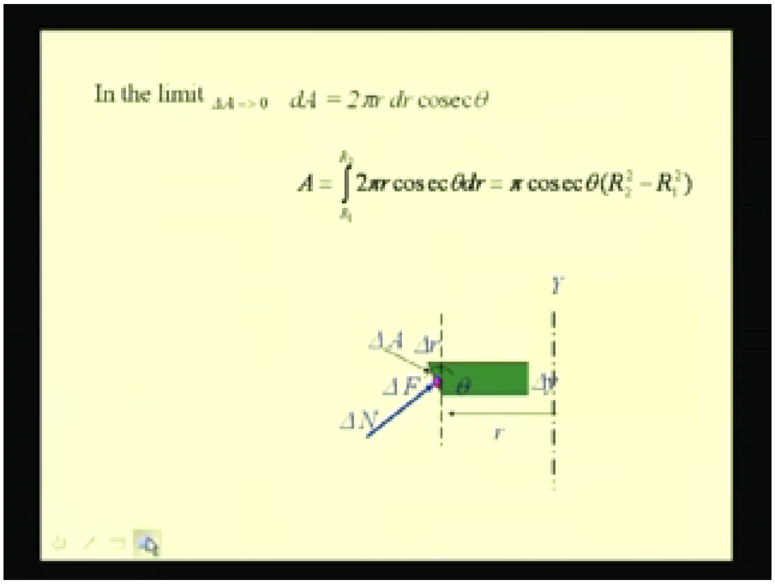

This differential element has an area dA when delta A tends to 0 as 2 pi r dr cosecant theta. This is found by computing the area swept by this line when it is moved along the circumference that is 2 pi r distance and this length is equal to dr cosecant theta where dr is the change in the radius for a change in the vertical distance delta y. When we integrate this from R_1 to $R_{2,}$ we have the total area as integral R_1 to R_2 dA which is equal to 2 pi R cosecant theta dr. We perform this integration; we get this as pi times cosecant theta R_2 square minus R_1 square. This is the total area of the contact between these pivot bearing and the bearing surface. Now that we have found this total area, we can substitute this in the differential moment equation and integrate it to find the total moment.

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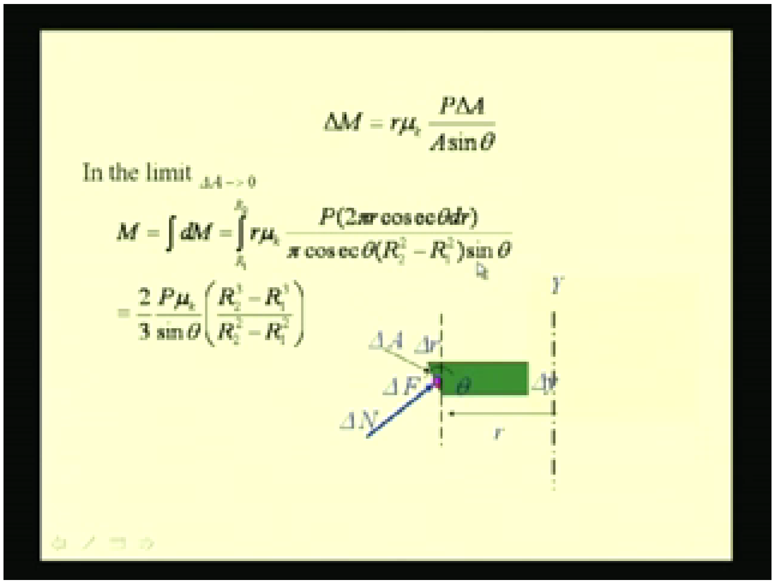

In this equation, we substitute for this A as well as delta A. In the limit, the total moment is equal to integral of this differential moments dM, which is equal to the limits being R_1 to R_2 r mu_k times P dA, which is equal to 2 pi r cosecant theta dr divided by the total area, which has been found as pi cosecant theta R_2 square minus R_1 square times sin theta. Now we can integrate between the limits R_1 and R_2 and that when simplified is equal to 2 P times mu_k divided by 3 sin theta into R_2 cube minus R_1 cube divided by R_2 square minus R_1 square. This is the moment that has to be overcome in order to keep this shaft in the rotating condition.

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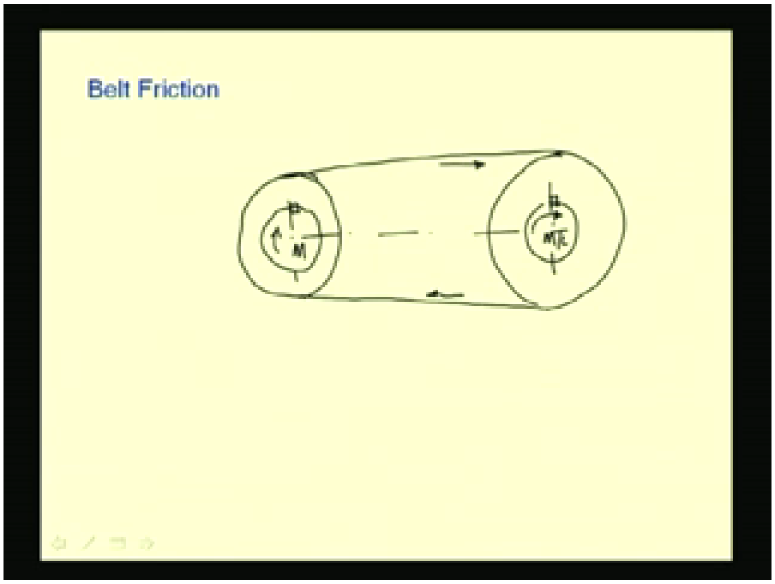

Let us see another application of friction that is employed in belts for transmitting power from one shaft to another. Let us consider two shafts, one which is connected to the motor and the other to a rotating machinery. The torque that is developed by this motor has to be transmitted to this machinery. This can be done by many ways.

One of the ways is to employ a pulley which is keyed to the shaft and a belt that runs over these pulleys to transmit the torque. The belt moves in this fashion and the torque developed by this motor is transmitted to the machinery. In this process we can increase the torque that is available or decrease the torque that is available by changing the ratio between the pulley diameters connected to the motor shaft and the machinery shaft. Also these center distances can be varied. That means, we can place the motor in a location where we have suitability for connections and we can place the machinery in its suitable position.

These are some of the advantages why we employ these belt drives; particularly you would have seen this in rice mills or in other industrial machinery also. In order to analyze this problem and to determine whether this drive is capable of transmitting the required torque, we need to find the conditions of friction that exist between the pulley and this belt, because that is the friction that enables this belt to be pulled by this pulley and thrown on the other side. Same way, the tension that is developed because of this process, can drive this pulley on the machine shaft. In order to analyze such problems, let us consider the free body diagram of one of these pulleys; let us say the pulley that is attached to the machine shaft.

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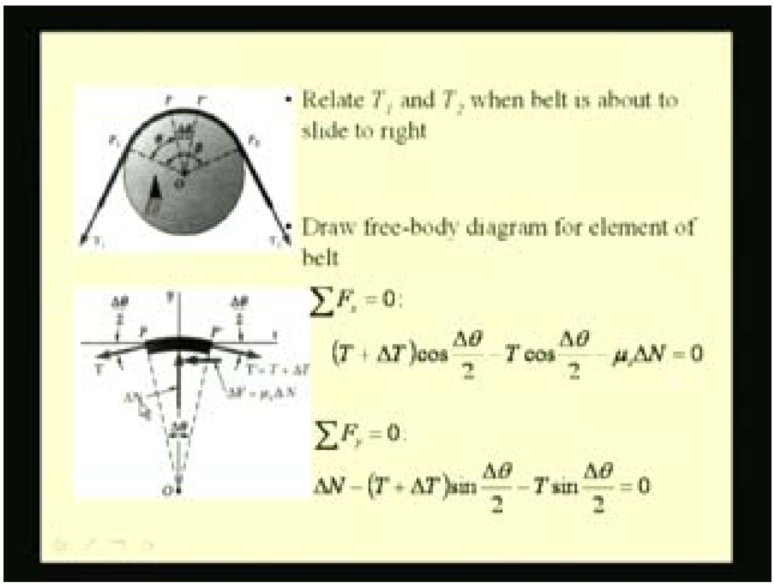

Because, the motor drives this pulley, the tension that is developed is larger on this side, which is depicted on this picture. Let us say this is the pulley. We have the belt element passing over the pulley and the tensions on the two sides being T_1 and T_2. Let us also assume that the belt tends to slip to the right hand side. That means the tension T_2 is larger than this tension T_1 and which is pulling and trying to rotate this pulley about this point O.

Let us consider a small element P, P prime, which is subtending an angle of delta theta. If P_1 and P_2 are the points of contact of this shaft, then beta is the total angle subtended by this belt on this pulley. Let us consider this element P, P prime and draw the free body diagram of that differential element. Let us assign these coordinates x and y; y along the radial direction and x tangential to the midpoint. O is the center of the pulley and this element subtends an angle delta theta. This force N is the normal reaction of the pulley onto this belt element. Let us say we have the tension T at this point P. Because this pulley develops a torque and applies the same to the belt, we have an additional tension in the right hand side.

Let that be delta T. So we have the tension on the right hand side as T plus delta T. A frictional force exists between this belt and the pulley. For the impeding slippage case, this frictional force can be related to the normal force by the coefficient of friction. So, delta F is equal to mu s times delta N for the impeding slippage case. Here, for the analysis, we assume this drum to be stationary. Later on we will extend the results obtained to the drums or pulleys in rotation also. Let us write based on this free body diagram, the equations.

If we sum the forces along the x axis and equate it to 0 for equilibrium, we have T plus delta T cos of delta theta by 2, which is the component of this force along the positive x direction minus T times cos delta theta by 2 in the negative direction and we have this frictional force which is minus mu s delta N equal to 0.

If we sum these forces along the y direction, then we have the normal force delta N for this differential element in the upward direction minus the component of the tension T and T prime in the negative y direction; that we have it as minus T plus delta t sin delta theta by 2 and the vertical component of this is, minus T sin delta theta by 2. So the sum of these forces has to be 0.

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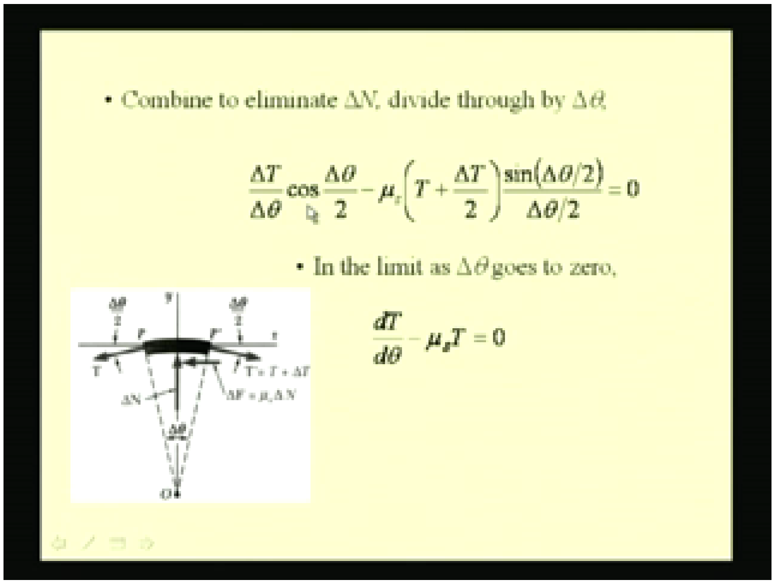

Let us eliminate this delta N from these equations by dividing it throughout by delta theta. So we get delta T by delta theta cos delta theta by 2 minus mu_s times T plus delta T by 2 times sin delta theta by 2 divided by delta theta by 2 equal to 0. If in the limit, that is delta theta tends to 0, this equation becomes dT by d theta minus mu $_s$ times T equal to 0, because this quantity, the product that is delta T times sin delta theta by 2 is negligible and this becomes 0.

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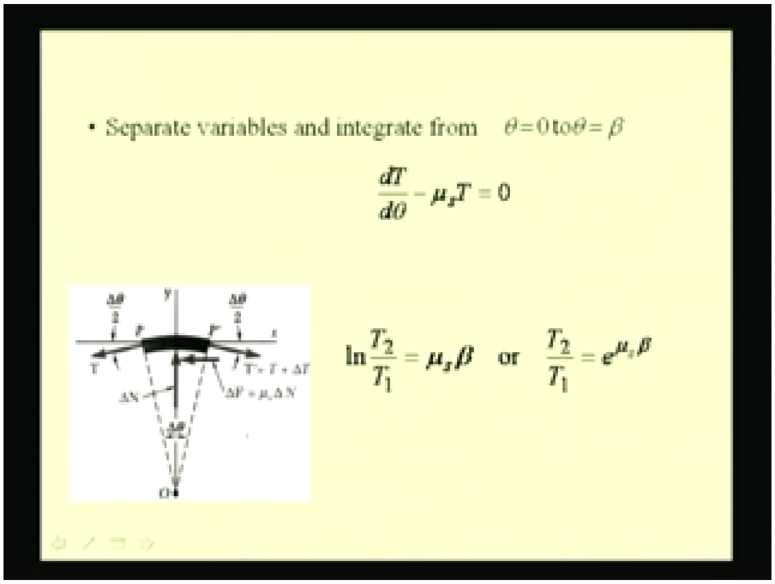

Now we integrate this between the limits 0 to beta, the total angle subtended by this belt. So we have logarithm to the base e T_2 by T_1 equal to mu_s beta or rearranging this, we have T_2 by T_1 as e power mu_s beta. So, if the belt subtends an angle of beta and if the coefficient of friction is mu_s, then the ratios between the tension in the tight side and the tension in the slack side are given by this equation. So now this equation can be additionally used to solve the problems.

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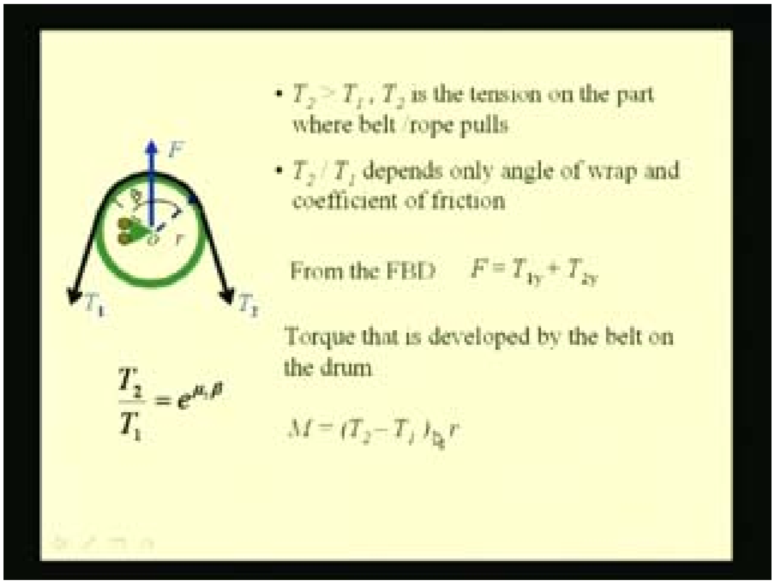

Let us consider, here the pulley and the tensions in the tight side which is T_2 and the slack side which is T_1 and the force F that is available as the normal reaction at this point O. Generally, in the pulley drives, one of the pulleys is movable and it can be fixed so that the required tension in the belts can be developed. So that is why this picture shows that the support in this pulley is on a roller and can be fixed at this point. Let us say r being the radius of the pulley, then for the impeding slippage case, we have the relation between the tension in the tight side to the tension in this slack side as e power mu_s beta; beta being the angle of overlap.

Let us say this. We see that this ratio only depends on this angle of wrap and the coefficient of friction. If you see the torque that is being transmitted, we can find by using this free body diagram, the total normal force that is available is F, which is equal to these forces; the components of the force T_1 and T_2 in the y direction and this force is a limited force. So sum of these two components of the tension cannot be greater than the available reaction at O.

So this limits the maximum tension that is possible and the torque that can be transmitted is T_2 minus T_1 times the radius r. We see that, we can increase the torque that can be transmitted by increasing the coefficient of friction; also we can increase this by increasing the angle of the overlap. But we see from this equation that the maximum tension is limited both because of the available reaction as well as because of the belt material which can only take to certain maximum tension.

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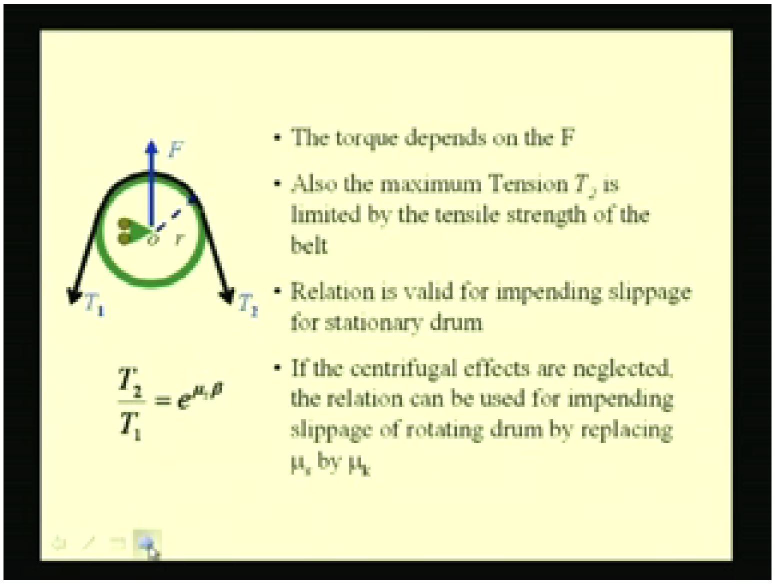

We see that the torque depends on F and also depends on this maximum tension $T_{2,}$ which is limited by the tensile strength of the belt. This relation we have found for the impending slippage for stationary drum. If we discard the centrifugal effects, then this relation can be extended to impeding slippage in the running condition by replacing the coefficient of static friction by the coefficient of kinetic friction.

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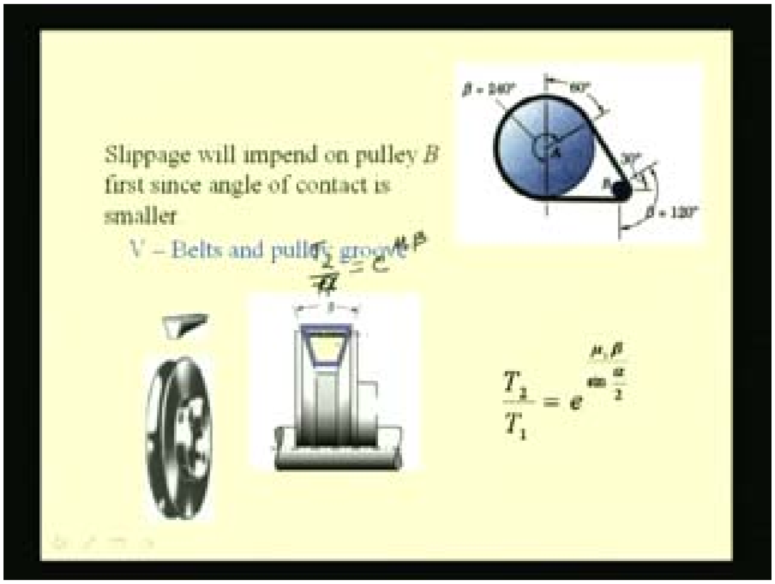

Let us see this picture where we have this large pulley and the small pulley at B. The angles of wraps are 240 degrees for the pulley at A and 120 degrees for the pulley at B. If we write this equation for the tensions, that is, let us say the tension in the tight side to the tension in the slack side is related by this relation; then this angle beta that one has to use, should be the angle of overlap for the smallest pulley, that is here in this case the pulley B. Because slippage will first occur in the small pulley for the given pair.

We have additionally another kind of belts that we call as V belts. This picture shows the V belt along with the pulley which has a groove to accommodate this V belt. These kinds of belts and pulley system are used to transmit larger torques. These kinds of belts can transmit more torque than the flat belts that we have just considered. Here, if we take this as the angle of the V belt, then the tension relation that we have just now derived can be found as T_2 by $T_{1;}$ the ratio between the tight side tensions to the slack side tension as e power mu_s beta by sin alpha by 2, where alpha is the angle of this groove.

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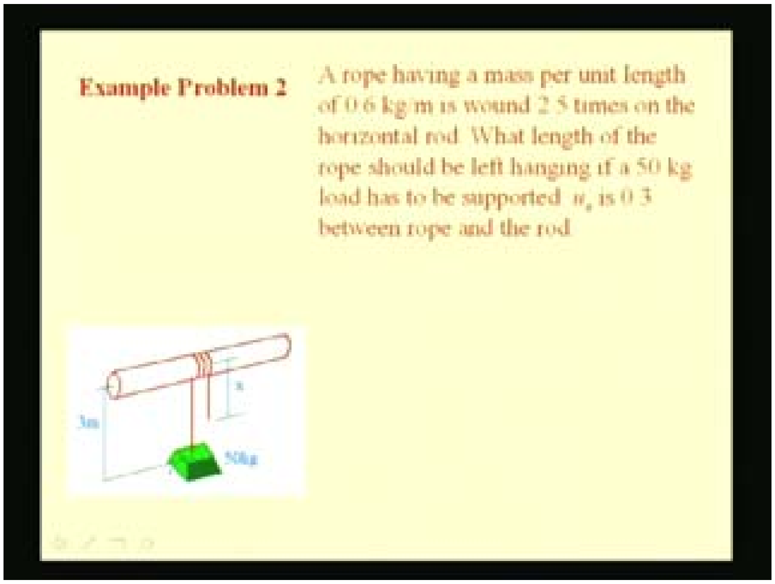

Let us take one example. Here you see a rope having a mass per unit length of 0.6kg per meter and is wound two and half times on the horizontal rod. So this rope has a self-weight of 0. 6kg per meter. One side of the rope is connected to the 50kg load and the other side, it is loosely hanging. The coefficient of static friction between the shaft and this rope is given as 0.3. So, for equilibrium, we are interested to find what length of the rope should hang in this free end side so that this load of 50kg can be supported.

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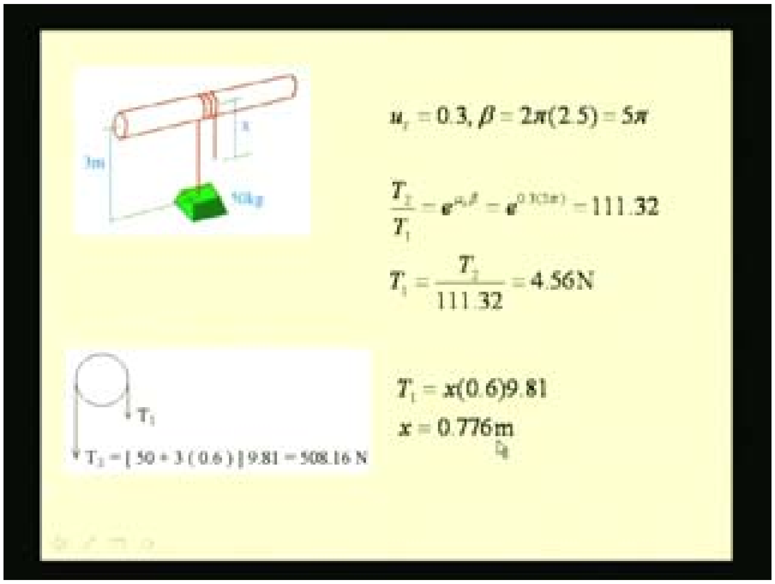

Let us consider this diagram. For the equilibrium, the tension in this side of the rope and the tension in this side of the rope has to be related by the coefficient of friction and angle of wrap that is occurring, in order that the equilibrium is maintained. We find the angle of wrap is 2pi theta; the theta is here, two and half times of wrapping, so it becomes 5pi. So for this case, the angle of wrap is 5pi. This diagram shows the tension on the side where 50kg block is attached. It is equal to 50 plus the self weight of the rope, which is hanging for 3 meters. So we have it as three times the unit weight of the rope which is 0.6kg per meter times the gravity, that is 9.81, which is equal to 508.16 Newton, if T_1 is the tension on the side where the rope is suspended for a distance of x, which has to be found.

We know this relation between the tight side tension and the slack side tension. For equilibrium it has to be e to the power mu_s beta which is 111.32 for this case. From this, we find T_1 the tension on the slack side has to be equal to 4.56 Newtons. Once we know the force, we can determine the distance of this rope, because we know the mass per unit length and thus x becomes 0.776 meters.

We will see one more example problem of this belt and pulleys.

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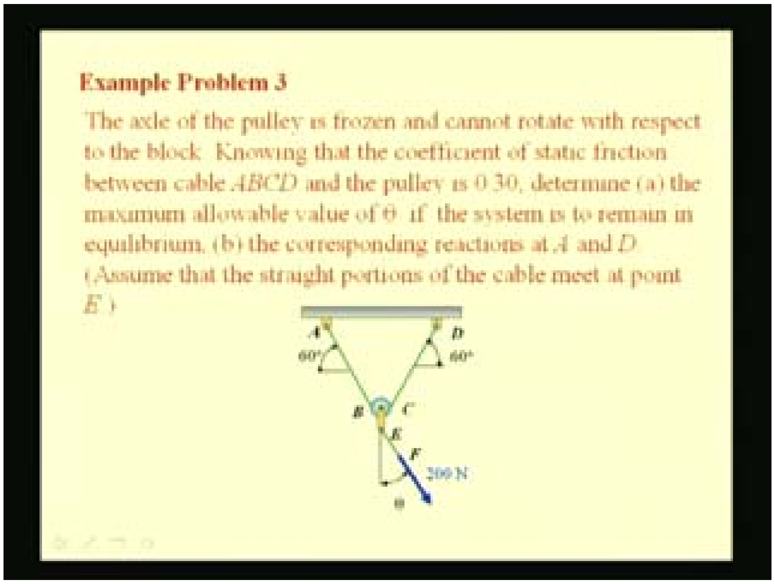

Here, in this problem, we have a pulley whose axle has been frozen; in the sense, this pulley cannot rotate about this pin and thus the short element if we call it as B, will also rotate along with this pulley. So, the block and this pulley are frozen at this pin. It is given in this problem that the coefficient of friction between the cable ABCD and the pulley is 0.3. We are interested to determine first the maximum allowable value of theta, that is, the angle of this applied force which is 200 Newtons, if the system is to remain in equilibrium. On the other hand, we are also interested to find the corresponding reactions at A and D.

We assume that the cables meet at this point E for this configuration. This is just an assumption for solving this problem.

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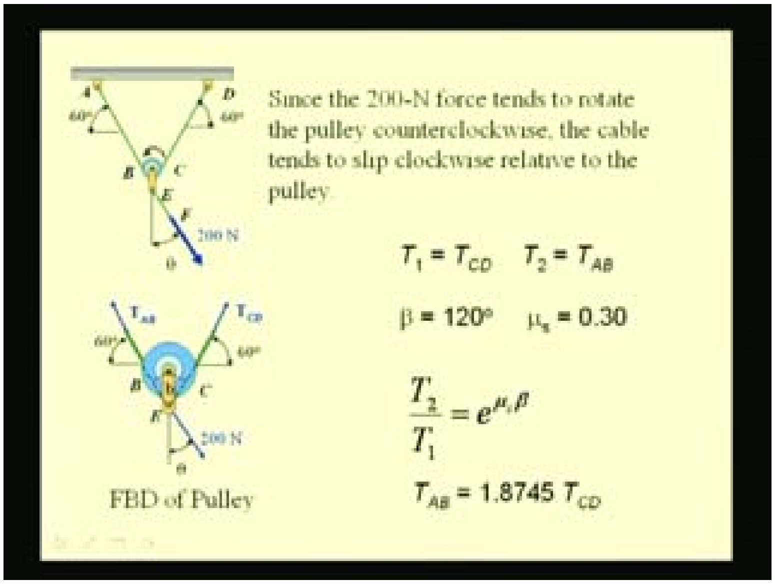

Let us proceed by considering this diagram, where we have this force of 200 Newtons that is being applied at an angle of theta. We are interested to know this angle for the equilibrium position. This 200 Newton force tends to rotate this pulley in the counter clockwise direction. So, this force tends to rotate this pulley in the counter clockwise direction. Since the cable that is fixed at A, passes over the pulley and goes to D, the cable tends to slip clockwise relative to the pulley.

Let us consider the forces that are acting on this pulley block. We have the tension on the cable CD marked as TCD and from geometry this is inclined at 60 degrees. We have the tension on the cable AB as TAB again inclined at 60 degrees at the left hand side. We have these dimensions of these blocks say B and this force which is 200 Newton, which is being applied at an angle of theta.

Let us take the tension T_{CD} as T_1 and the tension T_2 as T_{AB}. Since we have found that this cable tends to slip in the clockwise direction, this force has to be greater than this force T_{CD}. This tension in T_{AB} is greater than tension in T_{CD}. The angle of wrap from the geometry can be found as 120 degrees because these two being 60, 60 degrees; we have this angle of warp as 120 degrees and the coefficient of friction is 0.3.

For the impeding slippage case, we have the ratio between the tight side and the slack side of this cable related to the angle of wrap and the coefficient of static friction which is equal to e power mu_s beta.

We write this as tension in the portion AB is equal to e power mu_s beta which has found to be 1.8745 times the tension in the cable CD.

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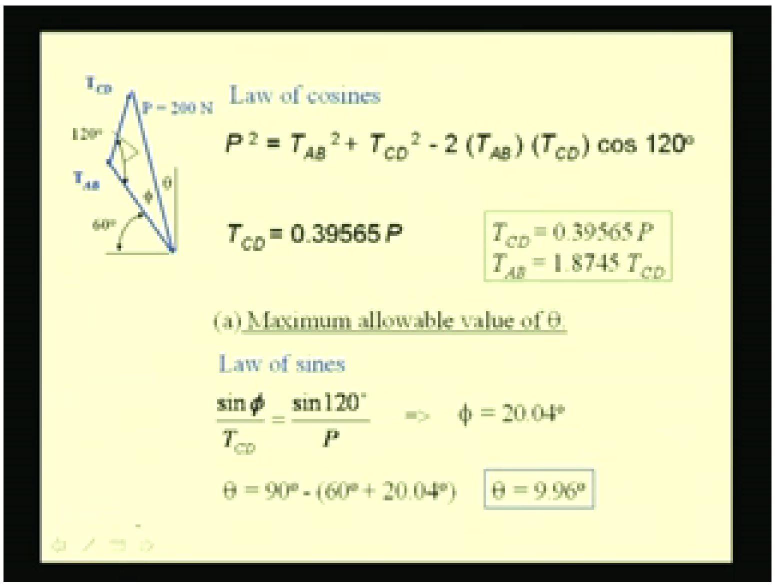

Let us consider this force triangle, where we have marked these forces; that is T_{CD}, T_{AB} and this 200 Newton force by these vectors. So we have this 200 Newton force which is having an angle of inclination of theta, with the vertical, the force in this cable CD and the force in the cable AB. The angle of inclination of these two vectors is known from the geometry.

T_{AB} is inclined at 60 degrees and T_{CD} is inclined to T_{AB} by 120 degrees or the angle of wrap. So, from this force triangle, by using the law of cosines, we can write P square equal to T_{AB} square plus T_{CD} square minus 2 times T_{AB} T_{CD} cos of this angle, that is 120 degrees. From this, we have T_{CD} as 0.39565 times of P, because in this equation, we can substitute for the force in terms of T_{CD}. This force T_{AB} can be substituted in terms of T_{CD} from our earlier equation.

We have T_{AB} as 1.8745 times of T_{CD}, which is coming from our earlier equation. So, in order to find this maximum value of theta, we use this law of science. It states that sin of this angle divided by this edge length that is T_{CD}, should be equal to sin of this angle divided by this edge length which is P. Since we know this phi, we get another relation between T_{CD} and P. We can solve this to find the force T_{CD}. We have this phi as 20.04 and theta as 9.96 degrees, from this diagram.

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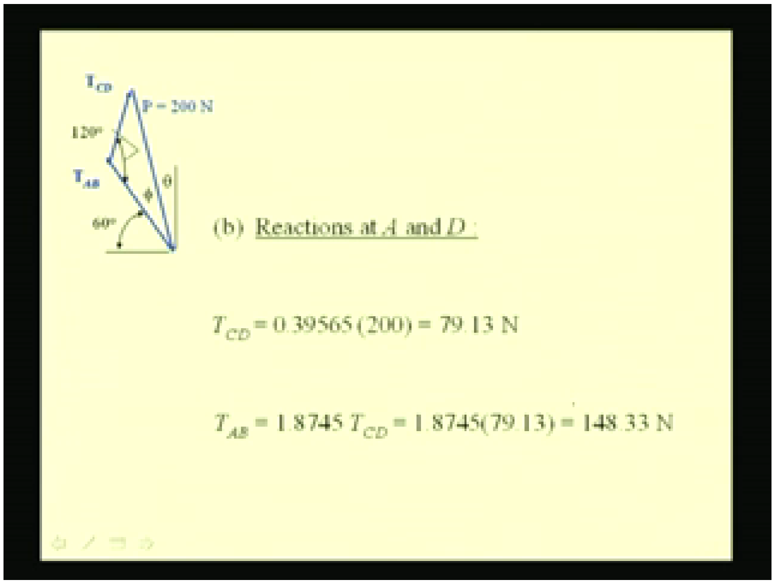

Now, we can determine these two tensions T_{CD} and T_{AB}, which are nothing but the reactions at A and D. They are 79.13 Newton and 148.33 Newtons. So this example illustrates solving problems on belts and finding the required friction or the tendency of slippage between the belt and pulleys.

We will see some more applications in the next lecture.