Engineering Mechanics

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Module 4 Lecture - 10
Application of Friction Part 1

Today, we will continue our lecture on friction.

In the earlier lectures, we saw simple contact friction, where the geometry of the contacting surfaces was simple. So, computing the friction and the normal forces were to some extent easy. For contact surfaces which have complex geometry, we have to consider small elemental areas and write the friction and the normal forces for those differential areas. Then integrate their effects, in order to arrive at the overall effect of friction.

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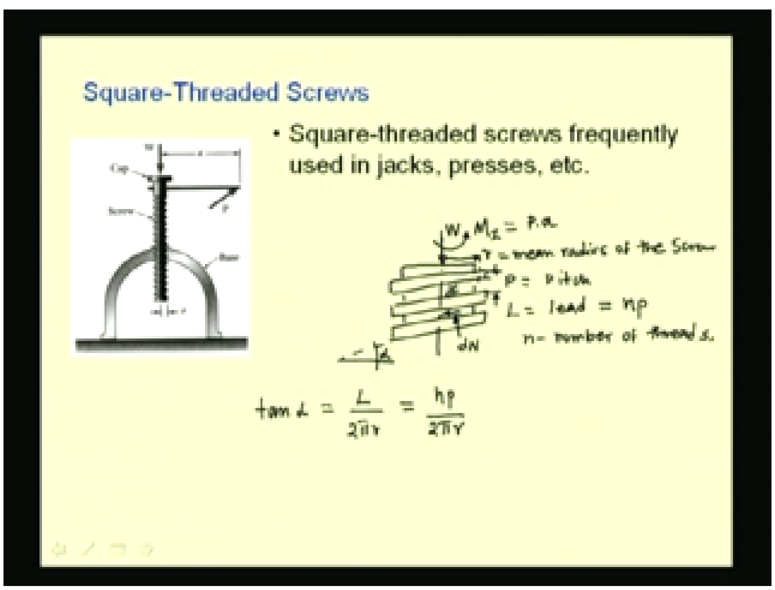

Today, our discussion will be on one such interesting problem on square threads.

Square threads are used for screws used in the jacks and in presses, and for clamps. The main purpose of these devices is to hold components, or in case of a jack, to raise the load. The friction between the threads is advantageously used to do the required function. In this picture, you see a jack which is typically used to raise the automobiles for doing repair work.

The automobile axle is supported on this cap. So, w represents the weight of the automobile that is carried by this cap of the screw. This is the square thread and this is the base in which there is a square thread.

The automobile workshop person applies force on this lever in order to rotate this screw and thereby to raise or lower the screw, and thus the car body. The friction between the thread and the meeting phase on the base has to be overcome when raising this load. If we want the raised load to be kept in place, then the friction here should be sufficiently large to prevent this screw from unwinding, if this force is removed from the lever.

The design of this screw jack is essentially based on the condition that the force required to rise should be minimal as well as the friction should be sufficient enough to prevent the screw from unwinding.

Let us see this geometry of the screw. Let us consider here a portion of this screw. These are the threads. These threads have an angle of inclination say alpha. The screw cap supports a load of w. The effect of this force P is to produce a moment required to rotate this screw. So, it can be replaced by a moment M_z, where M_z is equal to P times of a, the moment produced by this force about the axis of the screw.

Let us consider the mean radius of the threads as r; so here, r is the mean radius of the screw. The distance between two successive threads along the axis of the screw is known as the pitch of the thread. For a single threaded screw, the pitch is equal to the distance that a nut will travel when it completes one revolution; but there are threads which have multiple studs, where the advancement of the nut is equal to, let us say, if we have two studs thread then we have two times the pitch.

If we designate P as the pitch, and L the lead that means the amount the nut moves on the thread, this L is equal to np, where n is the number of threads. From the geometry, we see that the distance moved by the nut which is L, is related to this angle by this relation, because when we move by a distance of 2 pi r, the load is raised or lowered by a distance L. So, this angle is tan inverse of L by 2 pi r or it is equal to np divided by 2 pi r.

We have understood the geometry. Let us see where these frictional forces come into play. On these thread faces, the frictional forces act along this thread face. If we tend to move this screw in this direction, the force of friction offers resistance. Thus, the direction of the frictional force will be in the opposite sense and this will be the normal force. Let us say dN and df; df is this friction force and dN is the normal force. These forces can be assumed to be acting at a distance of r from the axis of the screw, because the width of the screw threads is very small. These forces are acting in the entire circumference of the thread and all these forces are having same direction cosines with respect to the axis of the screw.

We can consider an equivalent free body diagram, where we represent the sum of all these frictional forces and the normal forces by a single force f and n, but one should remember that this is only with respect to the axis of the screw.

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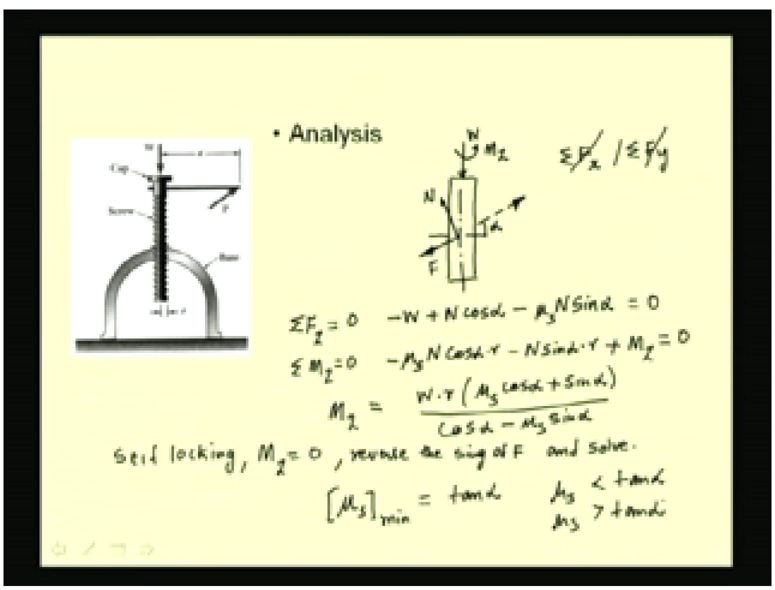

Let us consider that free body diagram. Let this be the screw. We have the weight and the applied moment, M_z. This is the sum total frictional force acting on the thread face and this is the sum total normal force acting on the thread face. This free body diagram is an equivalent free body diagram that can be considered for arriving at the relation between the applied moment and the frictional forces for the z direction. One should remember that we cannot use from this, the relation sigma F_x or sigma $F_{y.}$ These cannot be used.

Let us write the equations that the sum of the forces in the z direction has to be 0. From this, we have minus W plus N cos alpha, this is also alpha minus, for the impending slippage case, we have this frictional force F related to this N by this coefficient of static friction mu_s N sin alpha equal to 0. Then we can sum the moments of these forces with respect to the z axis and equate it to 0.

We have minus mu_s N cos alpha r which is the moment of the component of this frictional force in the horizontal direction minus N times sin alpha dot r which is the moment of the component of this force N in this direction. This is the circumferential direction; the line that is shown in this figure is for taking the components of these forces that are tangential to the circumference of the thread. They cause a moment that has to be balanced by the applied moment Plus M_z which is the applied moment has to be 0. If we solve these two equations and eliminate this N which is the unknown, we have M_z equal to w r mu_s cos alpha plus sin alpha divided by cos alpha minus mu_s sin alpha. This is the moment that has to be applied, in order to raise the load.

If you are interested to know whether the screw jack will unwind if the force is removed, or whether a screw jack is self-locking; that means, once we remove the applied moment, if the raised load is maintained at the same position then we call it as a self-locking screw. If it unwinds then it is not a self-locking screw.

If you want to know the condition or the minimum frictional force that is required, then in this equation, we put for the self-locking condition, we put M_z equal to 0. Since it is unwinding, the direction of this frictional force has to be reversed; that is, it will be acting in this direction when the screw tends to unwind, and reverse the sin of F and solve.

If we do that, we will find that the minimum frictional force or the minimum coefficient of friction that is required is equal to tan alpha. We already know that mu_s can be represented by an equivalent frictional angle.

We have a relation between the angle of friction and the lead angle of the screw. If mu_s is less than tan alpha then it is not a self-locking screw. If mu_s is greater than tan alpha then it is self-locking.

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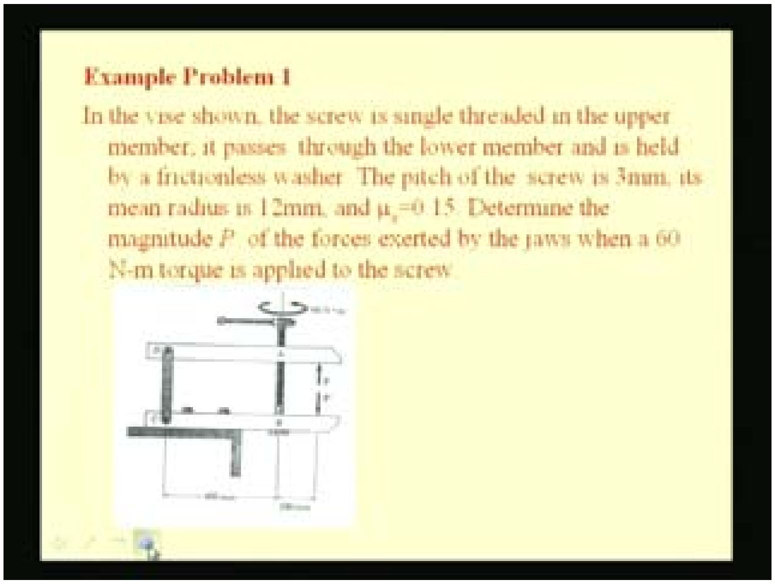

Let us see one example problem for solving the required friction in a square threaded screw. Here, you see a vise which is supported on to a platform. One of its jaws is supported and the other jaw can be tightened or loosened using this screw. The mean radius of this screw and the coefficient of friction is given. We are interested to find the force that is being exerted, when a 16 Newton meter torque is applied to the screw. Here, we see that these jaws are pinned at D and at B. This screw is free to rotate.

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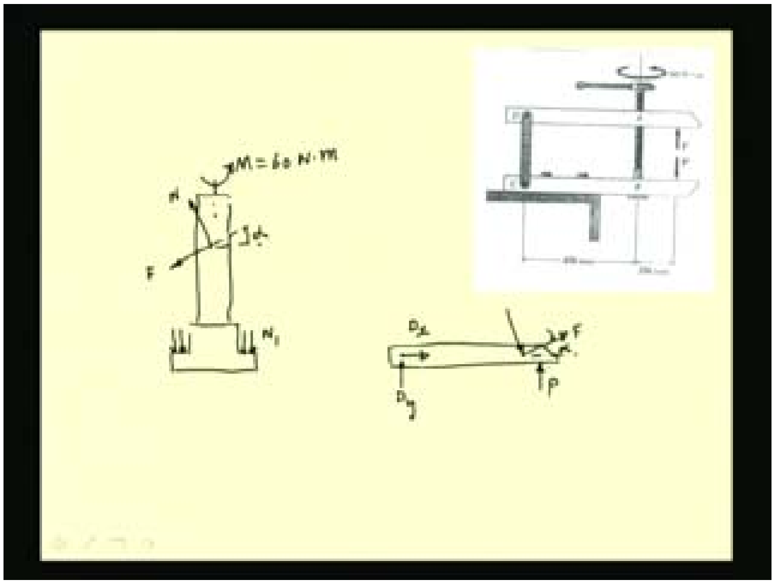

Let us consider the free body diagram of this screw and the jaw.

Let us draw the equivalent free body diagram for the screw, where the equivalence is valid for the axis of the screw. We have this screw and on this face, there is a uniform reaction because of this jaw CB. Let this total reaction be $N_{1.}$ The equivalent frictional force and the normal reaction on the face of the threads at this point A, can be represented by these single forces F and N which are inclined at an angle alpha which is the lead angle of the screw. We have the applied moment M which is 60 Newton meter.

If you consider the free body diagram of this jaw, D_A at D it is supported by a pin. So, we have D_y and $D_{x,}$ the two components of the reaction. At A, we have the equal and opposite forces. So, we have this normal reaction and the frictional force F, and we have this force P. The dimensions are known. From this free body diagram, we can now write the required equations to solve.

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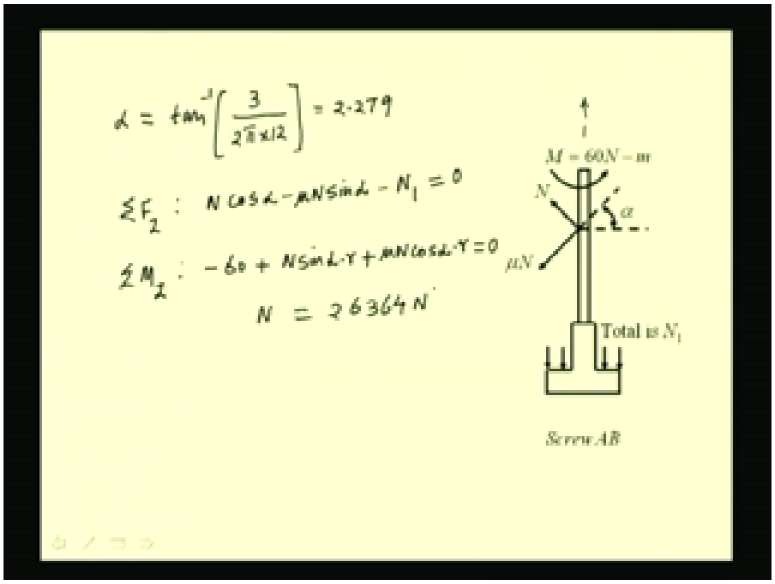

Let us consider, first the free body diagram of the screw which is shown here. We can write the moment summation equation and force summation equation along the screw. We have already seen that in this equivalent diagram, we cannot write the summation of the force in the x or y-direction. If this is the z-direction, then we can write the force summation along this axis, and moment summation of these forces about this axis. Let us write the equation.

Before computing, let us find the lead angle which in this case, is tan inverse of 3 by 2 pi r which is 12 mm. The lead is equal to pitch and it is equal to 3mm. The mean radius is 12mm. From this, we find the angle to be 2.279. Let us write, the force summation about this z-axis which is N cos alpha minus mu N sin alpha for the impeding case. This frictional force is equal to mu times N minus $N_{1,}$ the total reaction on this face. We equate it to 0.

Let us sum the moments. We have this applied moment which is minus 60 Newton meters plus the moment due to this frictional force and this normal force which is N sin alpha dot r the mean radius of the screw plus mu N cos alpha times r and this has to be 0. From this, we get to know the value of mu as well as alpha; we find N is 26364Newton.

Here, we are not using this equation, but if we are interested to know the reaction then we can use this equation to find N_1.

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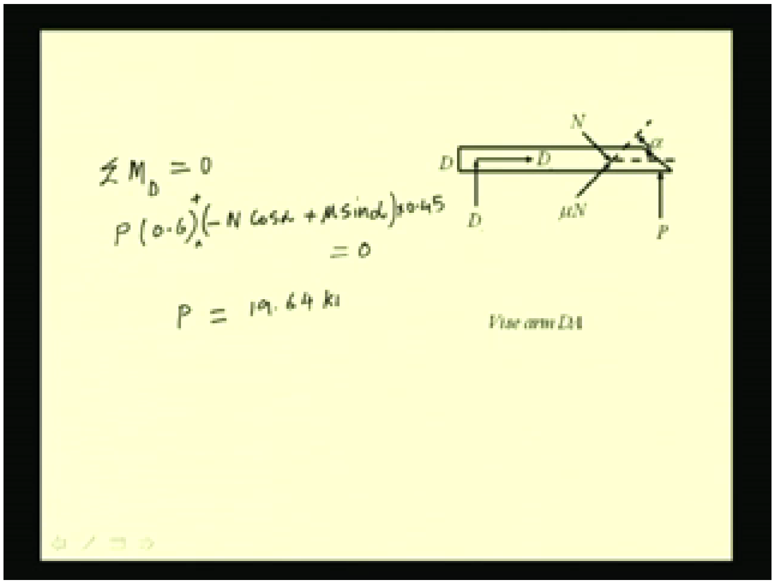

Let us move to the next free body diagram that is the y sum. This shows the free body diagram. Here, we have already computed N and we can compute force P, if we take a moment equation about this point D. Let us write that equation.

Summing the moments about D and equating it to 0, we have the load P times the momentum which is 600 mm. So, 0.6 meters minus force N cos alpha which is the vertical component and the vertical component of this force is plus mu sin alpha. Both of these have the momentum of 0.45 meters, times 0.45 equal to 0. From this, we obtain P as 19.64 kilo Newton.

This example illustrated the method used to solve for the square threads. We can determine the required unknowns by considering the equivalent free body diagram.

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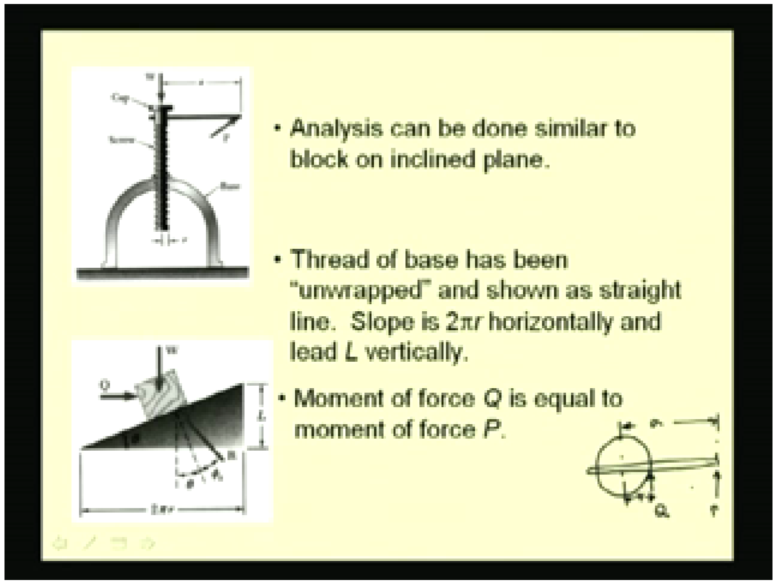

We will see another method of analysis, where the analysis of this thread is made as a problem of solving the inclined plane. Let us see that.

This is the screw jack that we have considered here earlier. This problem is now converted to problems similar to that of a block on an inclined plane. If we unwind the thread and put it on the plane of the screen then the problem can be considered as rising a load along this inclined. So, this picture shows the unwrapped thread. We know that when we move along the thread by the distance 2 pi r, the thread raises by the lead distance. If we complete one full rotation, this load is raised by a distance of L. So, it is equivalent to saying that we raise this load W, on this inclined plane. The force applied on this lever arm which is the moment that is required to rotate the screw, is represented by an equivalent force Q which is used to raise this W on this inclined plane. This theta is the lead angle of the screw.

This problem can be solved by using the free body diagrams that are derived from this equation. The problem of finding this Q is simple, because the moment of this force P is equal to the moment of this force Q which is applied on the circumference of the thread. If this is the mean radius r then Q is the force. So, we are considering the thread, as viewed from the z-direction and we have the applied moment of this force which is P times of a. The force Q should be large enough to resist the moment due to this lever which is $P_{a.}$ From this, we can find what is Q.

Once we know Q, we know the required quantities. We can solve the problem using the plane analogy. The problem of analyzing the self-locking behavior can be considered for the inclined plane analogy also.

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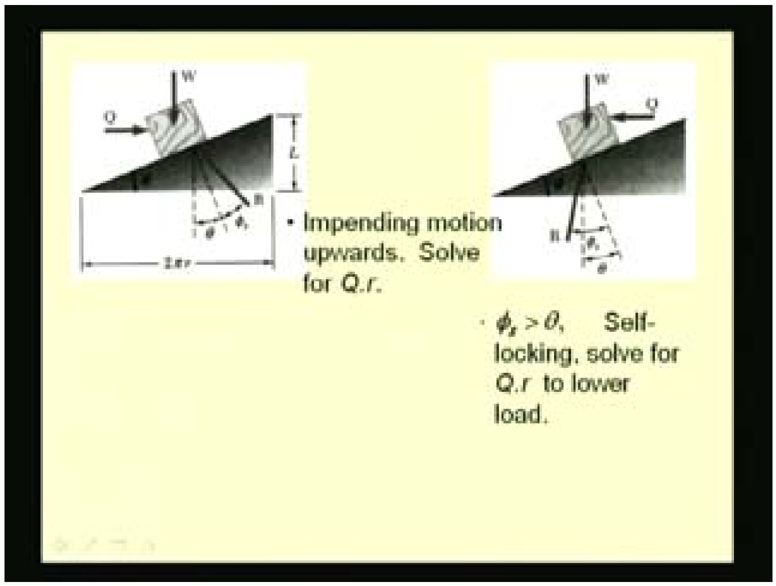

If we have the impending motion upward, it is equivalent to raising the load by applying the required moment on the screw jack. This is the equivalent free body diagram of the inclined plane. The reaction force which is the sum total of the normal reaction and the frictional force on this face, makes an angle of phi_s with respect to this normal. If theta is the lead angle then the normal of this inclined face, makes an angle of theta with this vertical.

If the thread is not a self-locking thread then a force or a moment has to be applied in order to lower the block.

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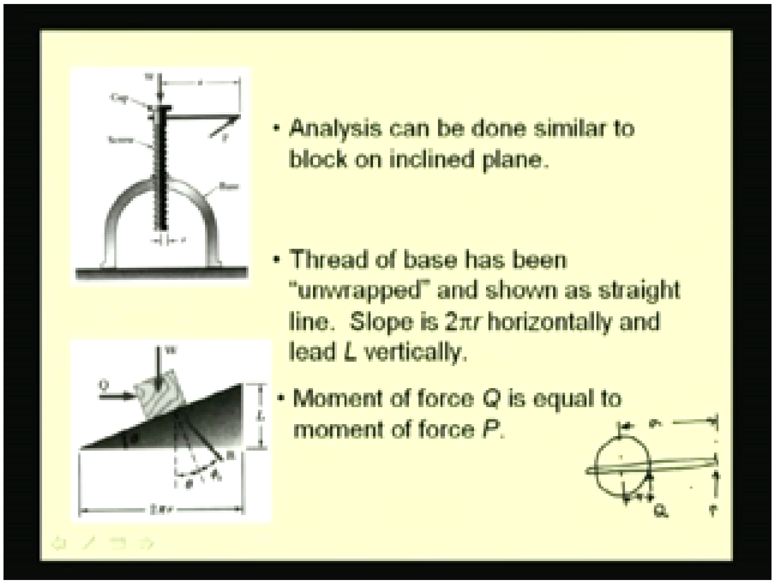

The analysis of the screws that we have seen can be done by another method, where we analogize the problem to that of a rising the load on an inclined plane. Let us consider this screw jack again, where we apply force P to cause a moment to raise the load W. We draw an equivalent inclined plane, by unwrapping the thread on the plane of paper. We know that if we rotate the screw by one revolution then this load W, is raised by a distance equal to the lead of this thread.

If we move on this inclined plane by a distance 2 pi r which is equal to one revolution of the thread then the load is raised by a distance of the lead of the screw. So, this inclined plane represents an equivalent of the thread. This moment that is Pa, is applied in order to raise this load W. We have to represent that by an equivalent horizontal load Q which is used to raise this load W. This load Q can be found by considering the top view of this screw and the lever. This force P is applied at a distance of a. If this circle represents the circle corresponding to the mean radius of the screw then this force Q is being applied at a distance of r.

Since we are equivalent diagram we have considered where the moment of this force Q has to be equivalent to the moment of this force P. We have this Q dot r equal to P dot a. This angle theta is the lead angle of the screw. From this equilibrium diagram, we see that this reaction force r which is equivalent to this frictional force and the normal force, is inclined at an angle of phi_s for the impending slippage case, where the block impends to move upward. This angle theta is nothing, but the angle between the normal to the inclined plane to the vertical axis.

The problem of analysis can be converted to a problem of analyzing this inclined plane to find the required quantities.

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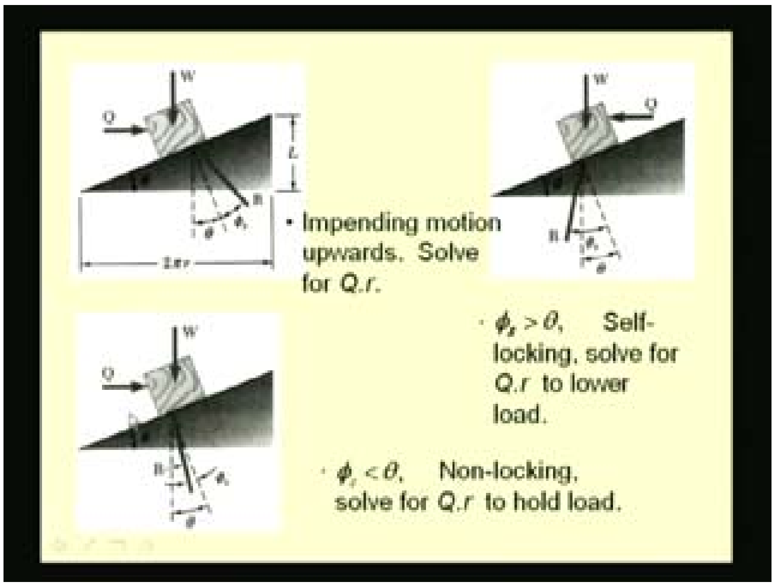

This diagram represents, for the impending motion upward, where the force Q pushes the weight against the slope. If the thread is a self-locking thread then if we leave the load on the inclined plane, it will not slide down. So, that is the equivalence to a self locking thread; that means, we need to apply a force Q in order to push this load down the inclined. We can consider this free body diagram and find the required force Q; Thus, in turn, the required moment in order to lower the load.

Here, we see that this angle of friction phi_s is greater than this angle theta which is nothing but the lead angle of the screw. If the thread is not a self-locking thread then we need a moment that has to be applied to the screw so as to keep the load in the position. If that moment is removed then the screw will unwind and the load will be lowered.

We have to find the corresponding load Q, for this inclined plane analogy that will resist the load from moving down the incline. Here, we see that this angle ϕ_s is less than the lead angle. Once we know this Q, we can find the required moment to keep the load in the position.

Let us use this analogy of the inclined plane to solve some problems on square threaded screws.

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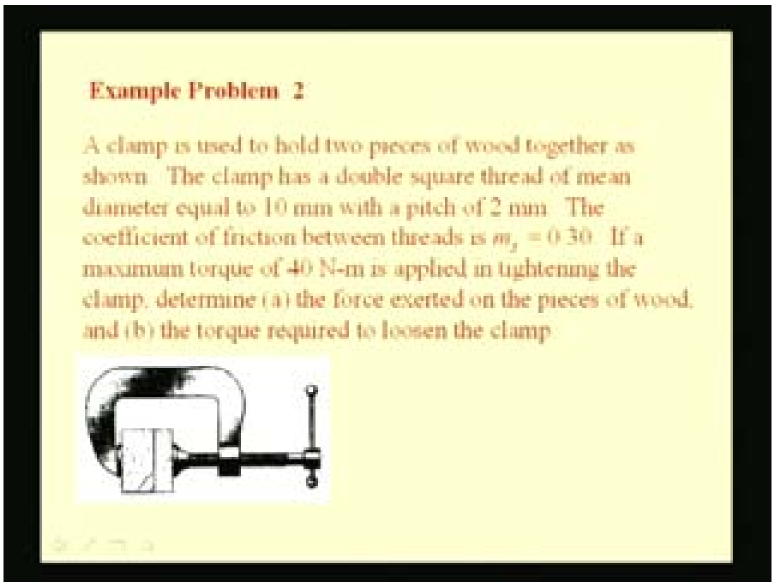

Here, you see an example of a clamp that is used to hold blocks. When some work is being done on those blocks, the diameter of the screw is given as 10 mm, the pitch is given as 2 mm, and the coefficient of friction is given as 0.3. If a torque of 40 Newton's is applied in tightening the clamp, we are interested to determine the force that is being exerted on these blocks. In the second case, we are interested, the torque that is required to loosen this clamp. Before we proceed, let us find the lead angle and the angle of friction so as to construct the analogical inclined plane for this screw thread.

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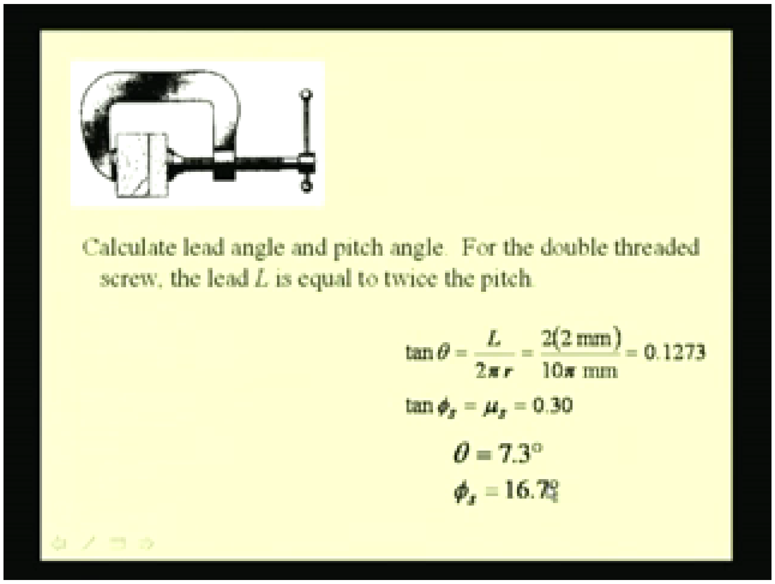

We know the lead is twice the pitch, because it is a double thread.

The angle theta which is the lead angle is equal to L by 2 pi r which is equal to 2 times the pitch which is 2mm in this case, and r is 5 mm. We have, tan theta as 0.1273 and the coefficient of static friction is given as 0.3, from which we can find this phi_s. We find theta as 7.3 degrees and phi_s as 16.7 degrees.

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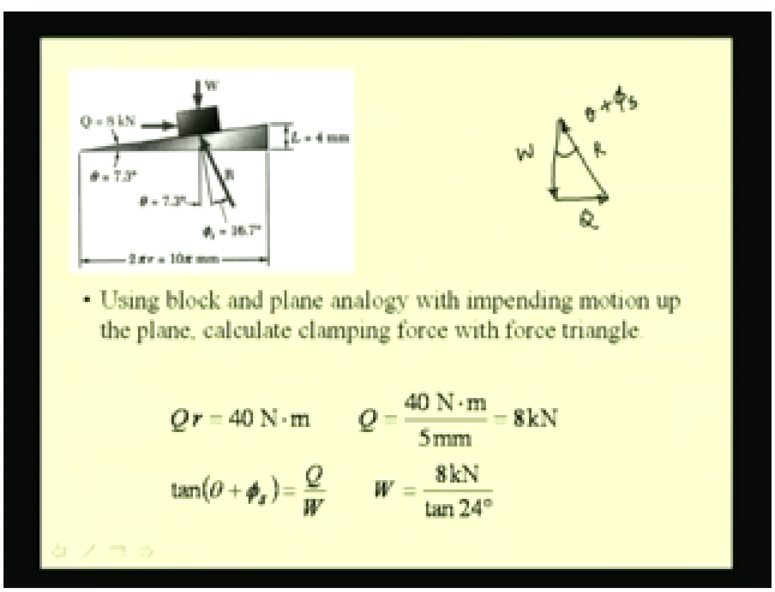

Let us construct an analogical inclined plane. This plane has a lead of 4 mm. This is the lead angle which we have found as 7.3 and this is the angle of friction phi_s which we have found as 16.7 degrees. When we apply the moment, it is represented by an equivalent force Q on this inclined plane which is used to raise this weight. In this case, this W is the force that is being applied on the blocks.

Knowing the moment that means we know what is Q, we are interested in finding the force on the block, or the clamping force on the blocks. We have already seen that this phi_s is greater than theta. So, the free body diagram of the inclined plane will have the reaction force R, inclined to the normal face by this angle 16.7 degrees.

We have Qr as 40 Newton meter which is the applied torque. Knowing the radius of the thread, we find the equivalent load Q on the inclined plane as 8 kilo Newton's. Now, from this diagram, we relate this W and Q. We know that tan of this angle that is tan of theta plus phi_s is equal to Q, the horizontal load, divided by the vertical load W from this force triangle. We have this W, we have the horizontal force Q, the resultant of the normal reaction and the frictional force that is R and this angle is equal to theta plus $phi_{s,}$ from the diagram.

Now, we know Q, we know theta and we know $phi_{s.}$ So, we can determine the load that can be raised, or in this case the clamping force which is equal to 17.97 kilo Newton's in this case.

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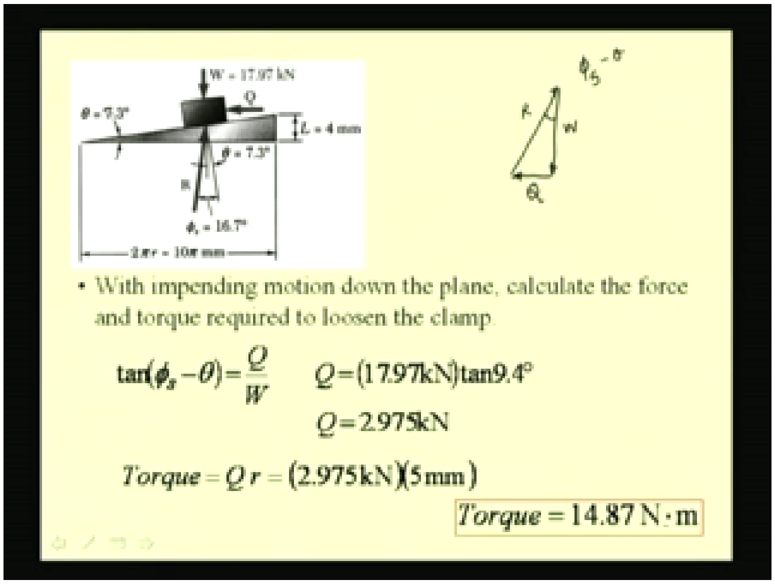

Next, we are interested to find the moment that is required to loosen the clamp. So, this is equivalent to lowering the load in the inclined plane. This diagram shows the load being lowered on the inclined plane. This angle theta is the lead angle of the screw. In this case, the resultant of the frictional force and the normal force is this R which is acting on the left side of the normal of this inclined plane. This angle, again for the impending case, is the angle of friction which is equal to 16.7 degrees and this angle is the lead angle which is 7.3 degrees.

Again, we construct this force triangle. This is R, this is W, the clamping force and this is the force required to lower the load. This angle, from the diagram is phis minus theta. We can relate this horizontal load Q to the clamping force or the weight of the block in this inclined plane analogy as tan of this angle phi_s minus theta is equal to Q by W.

We know the clamping force that is existing. From this, we can find the horizontal force required to lower the load, or in other words, the required moment to loosen the clamp. This force Q is found as 2.975 kilo Newton's. The moment that is required is equal to Q times the radius of the thread which is 5 mm, in this case.

We find this torque as 14.87 Newton meter. So, this is the torque that is required to loosen the clamp. In the problems, when we use this inclined plane analogy, one has to be careful in drawing the equivalent inclined plane diagram and representing the forces.

Let us see one more problem so that you become conversant with this method of using the analogical inclined plane.

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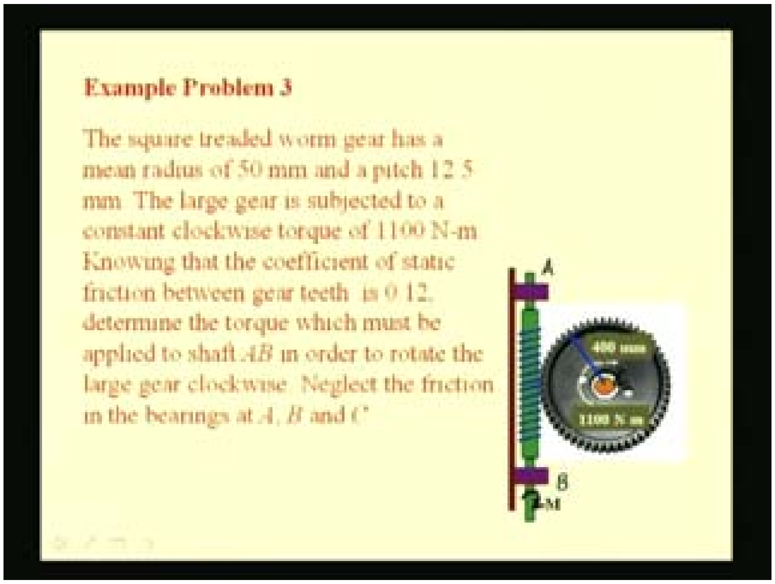

Here, you see a worm gear which resists the rotation of this big gear that you see in the picture. This worm gear is being supported by the bearings and this big gear is supported on this shaft. This large gear is subjected to torque of say 1100 Newton meter and this worm gear prevents the free rotation of this large gear. The geometry is given; that is, you know the mean radius of this worm gear. Worm gears have these square threads which match with the threads of this large gear and the coefficient of friction is given as 0.12.

We are interested to determine the torque or moment that has to be applied to this worm gear shaft so that the large gear can rotate in the clockwise direction; that means, in the same direction of this 1100 Newton meter.

We can neglect the friction in the bearings. Let us say, this A and B are the bearings supporting the wormgear shaft, and C is the bearing on this shaft. We see that even though this 1100

Newton meter torque acts on this large gear, because of the friction existing in the worm gear and the large gear contact, the large gear is not rotating. So, in order to make it rotate in the clockwise direction, we need to provide additional moment to the worm gear shaft.

Let us see, how to determine this moment.

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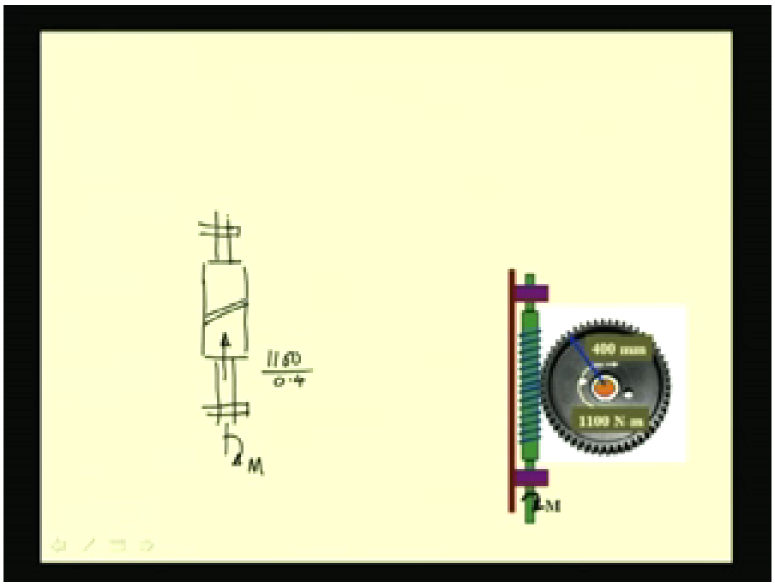

In order to construct an equivalent inclined plane for this problem, let us see this diagram carefully.

This torque that is 1100 Newton meter torque that is being applied to this large gear, applies a vertically upward load on this shaft, because of this gearing. So, It is equivalent to represent, let us say, this is the applied moment M to the worm gear shaft. This 1100 Newton meter torque applies a vertically upward load on this worm gear. This force can be found, by knowing this momentum which is nothing but the radius of this large gear which is 400 mm. We have this force as 1100 divided by 0.4. This problem is equivalent to an inclined plane with this, as the load to be raised or lowered on the worm gear face. So, this is the worm gear tooth face.

Let us draw the equivalent inclined plane.

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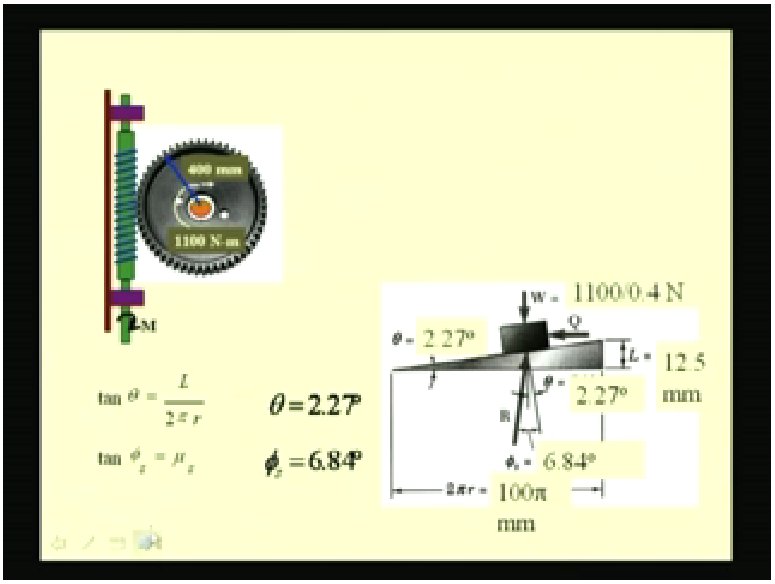

This is the lead angle which can be found from the lead of this worm gear. This is the load that has to be raised or lowered, which in this case is equal to 1100 divided by 0.4. If you see, the effect of this moment is to rotate this gear in the clockwise direction itself; that means, we are interested to move in the same direction of the load that is being applied on this worm gear. So, it is equivalent to lowering the load on the inclined plane. This force Q, represents the equivalent force that causes this moment M. From the coefficient of friction, we know this angle of friction for the impending motion which is 6.84 degrees which can be found from these equations that is tan theta equal to lead divided by 2 pi r and tan of the angle of static friction is equal to coefficient of static friction.

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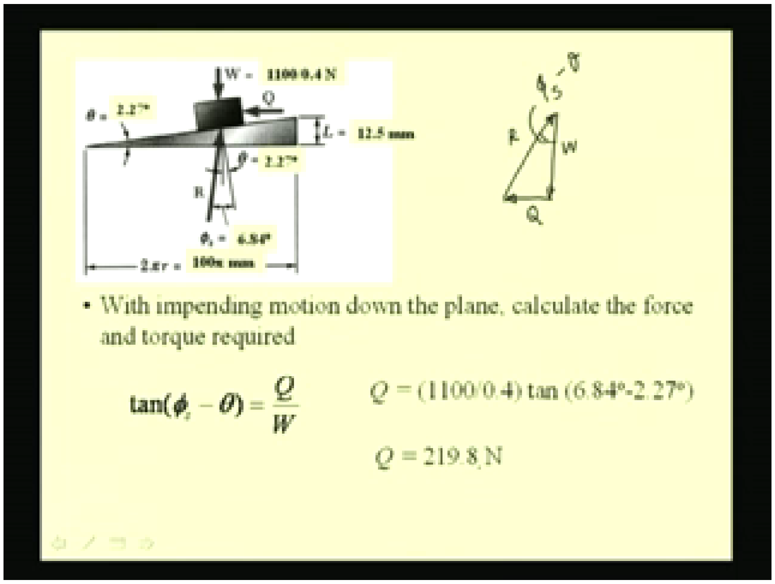

Once we have created this equivalent diagram, we can write the force equation.

This equation comes from this force triangle, where R is the resultant force, W is the weight that is being raised or lowered, and this is the force Q that it is being applied to the block in order to lower the weight, and this angle is phi_s minus theta. We have seen that when this moment is not applied, the larger gear does not rotate; that means, it is a self-locking gearing pair. So, we need to apply this load Q in order to lower this weight, or in order to rotate the large gear. From this, we get the value of Q as W times tan of phi_s minus theta which is found as 219.8 Newton's.

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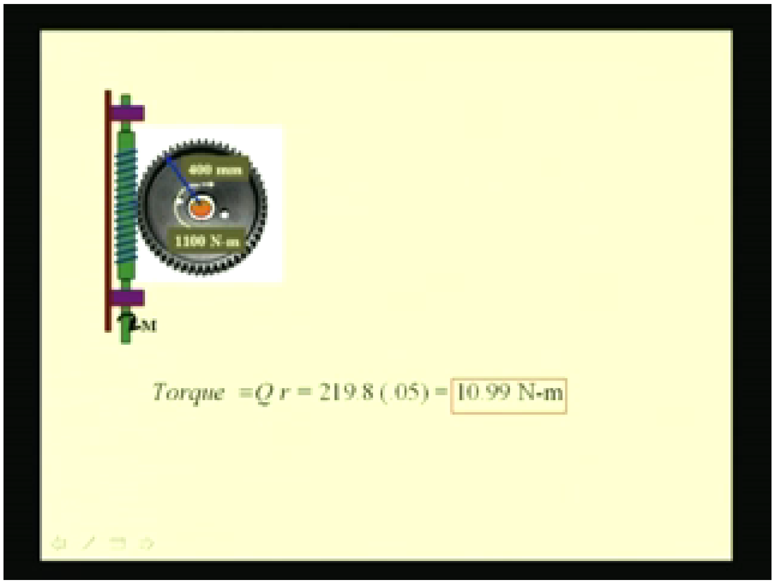

This force is equivalent to a torque of Q times r which is the mean radius of this worm gear which is 0.05 meters, in this case. We have the torque calculated as 10.99 Newton meter. This is the torque that is required to make the large gear rotate in the clockwise direction. These problems illustrate the method of using the inclined plane analogy for solving the problems of square threads.