Engineering Mechanics

Prof. U. S. Dixit
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

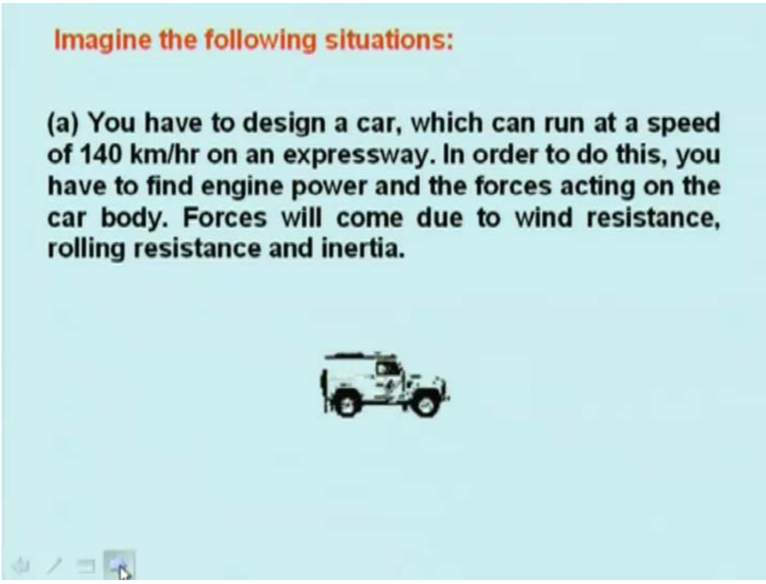
Module No. - 01 Basics of Statics

Lecture No. - 01

Fundamental of Engineering Mechanics

This is the first lecture in this course on engineering mechanics. In this lecture, we will be discussing fundamentals of engineering mechanics.

(Refer Slide Time: 01:16 min)

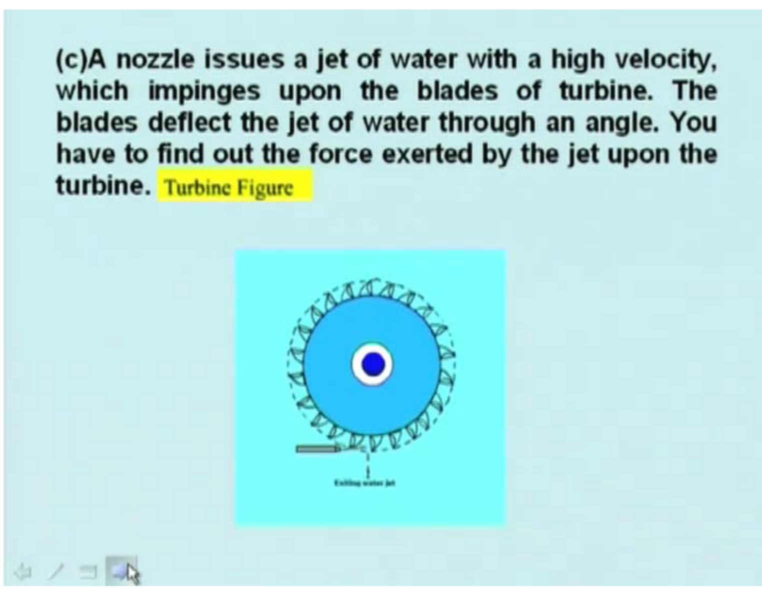


When we study a course, we must have some motivation for studying this. Therefore, now let us see some typical examples where engineering mechanics finds application. This slide shows that a car is moving on the road. If you have to design the car which runs at a say particular speed - let us say 140 kilometers per hour, on some expressway. Now, you have to find out the engine power, forces acting on the car body, the forces will come due to wind resistance rolling resistance and inertia; so, all these type of things you have to do. Then you must know engineering mechanics.

(Refer Slide Time: 02:00 min)

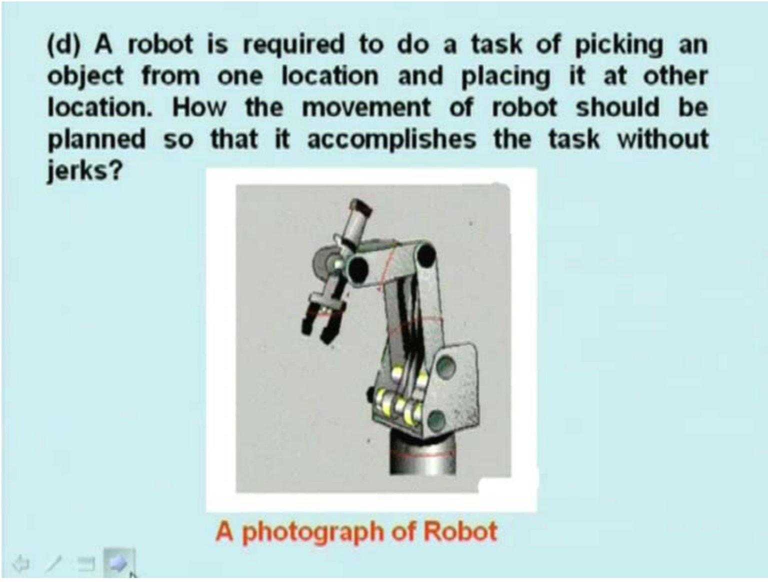
Then if you want to design a CD drive motor and you want to find out the power needed for a CD drive motor, then also engineering mechanics must be applied.

(Refer Slide Time: 02:14 min)



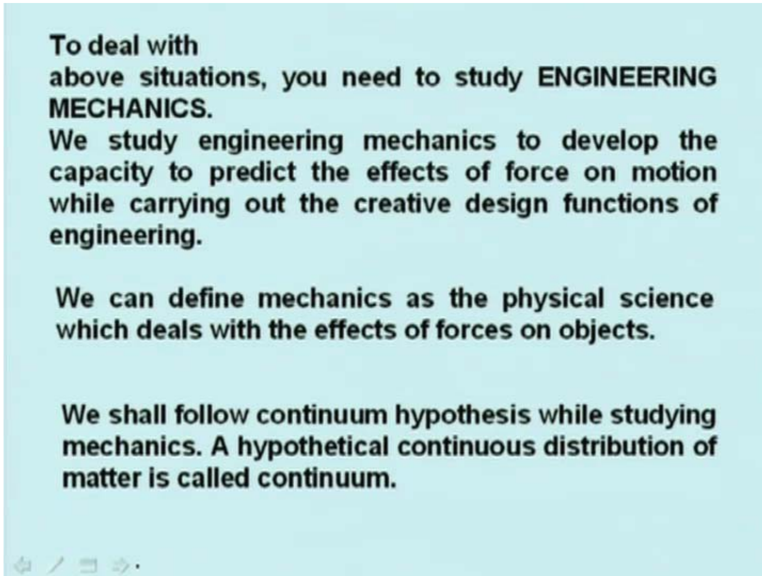
This is a figure of a turbine. A nozzle issues a jet of water with a high velocity which impinges upon the blade of the turbine. The blades deflect the jet of water through an angle. You have to find out the forces exerted by the jet upon the turbine. This problem also can be solved using engineering mechanics.

(Refer Slide Time: 02:41 min)



Similarly, if one has to design a robot for doing a task of picking an object from one location and placing it at other location. So, how should the movement of the robot be planned, so that it accomplishes the task without jerks? That is called path planning of the robot. Such type of problems also requires knowledge of engineering mechanics.

(Refer Slide Time: 03:05 min)

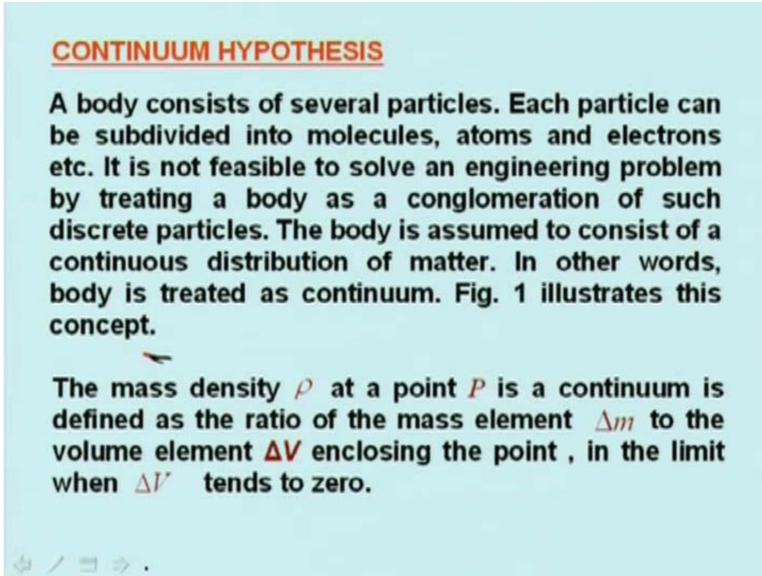


To deal with these types of situations we must need to study engineering mechanics. We study engineering mechanics to develop the capacity to predict the effect of force on motion while carrying out the creative design functions of engineering. Now, there can be two types of problems: one is the dynamic problem in which we study the motions; the other is the statics problem in which the motion is not studied, but only what is the effect or what type of forces are acting on various components - these type of things are studied.

We can define mechanics as the physical science which deals with the effect of forces on objects.

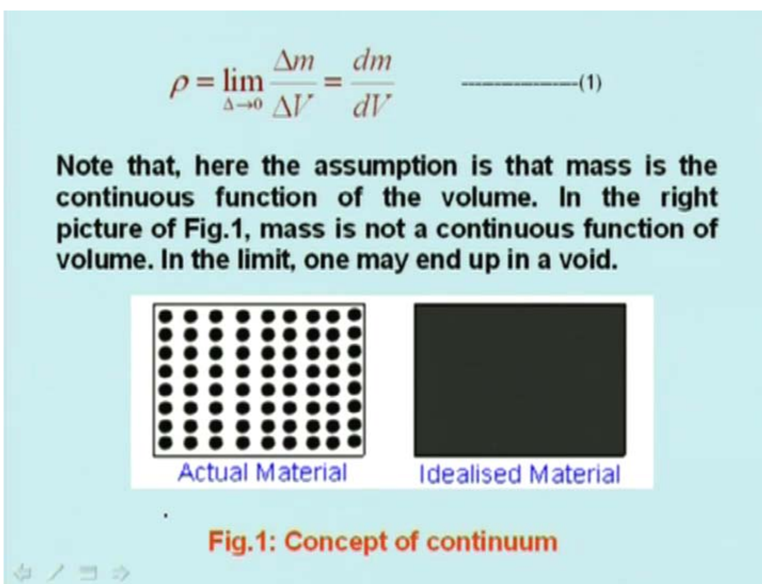
We shall follow continuum hypothesis while studying mechanics. A hypothetical continuous distribution of matter is called continuum.

(Refer Slide Time: 04:09 min)



A body actually consists of several particles. Each particle can be subdivided into molecules, atoms and electrons. It is not feasible to solve an engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter. In other words, body is treated as continuum. The next figure shows this concept.

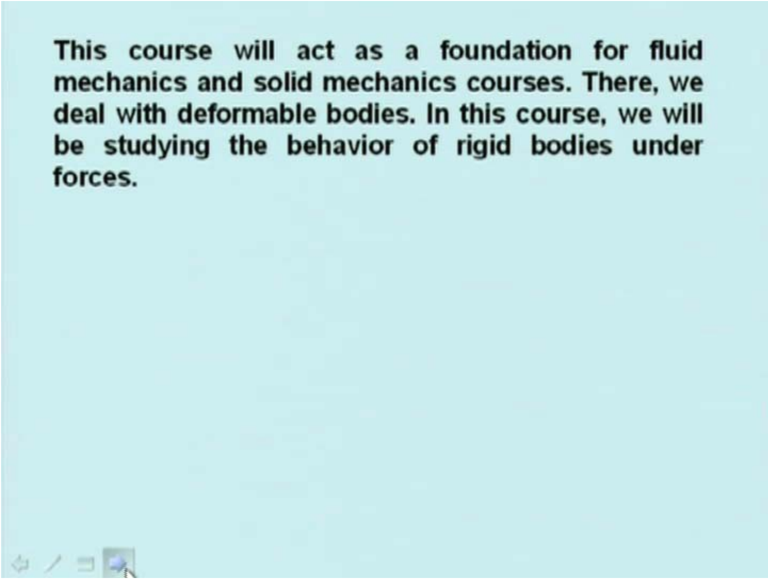
(Refer Slide Time: 04:42 min)



Here this is the actually material in which there are particles there maybe some voids in between, but the idealized material is considered continuous; there is a continuous distribution of matter here. Therefore, what happens if mass density rho at a point P in a continuum is defined as the ratio of the mass element delta m to the volume element delta V enclosing the point in the limit when delta V tends to zero. Then density rho becomes limit delta tends to zero delta m by delta V that is dm by dV.

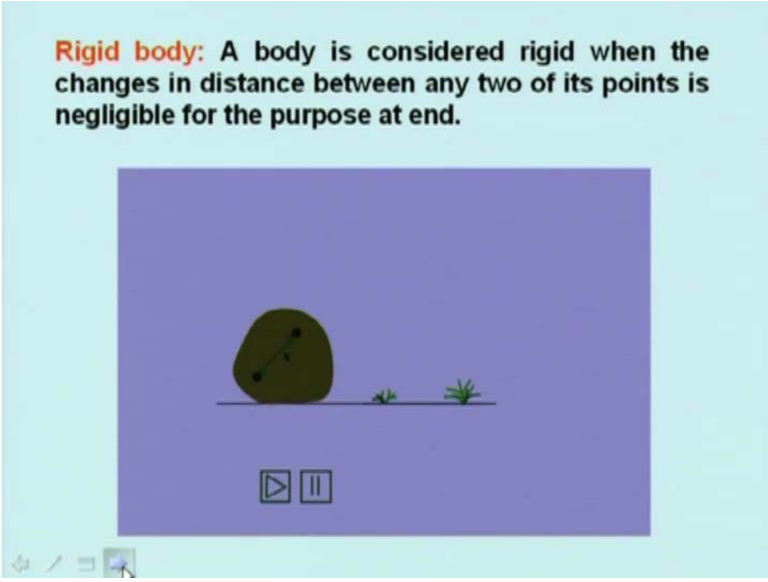
Now, here we have taken the assumption that the mass is continuous function of the volume; that is why we could take the limit. Now, if you know, we consider the actual material then in the limit, we may end up in a void. Therefore, in solving the day-to-day problems by engineering mechanics we usually apply continuum assumption.

(Refer Slide Time: 05:51 min)



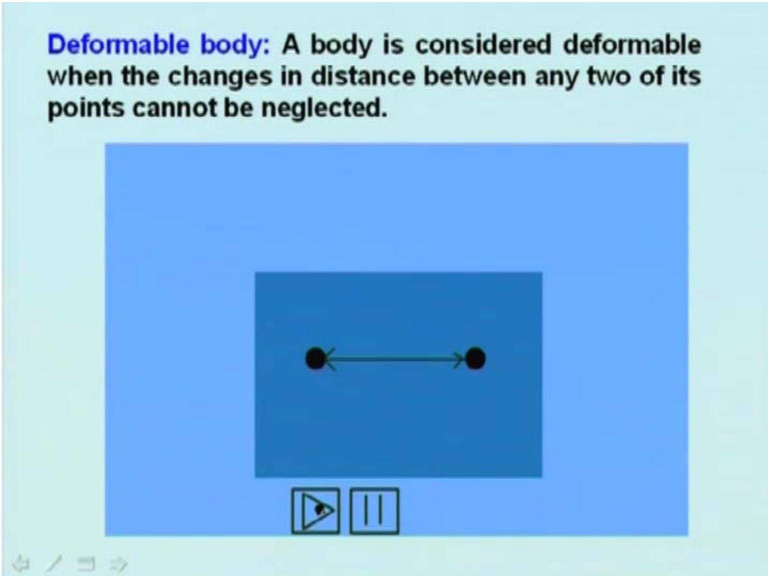
This course will act as a foundation for fluid mechanics and solid mechanics courses. There we deal with deformable bodies also. However, in this course, we will be studying the behavior of rigid bodies under force. No body is rigid; it is only a concept based on some assumptions.

(Refer Slide Time: 06:14 min)



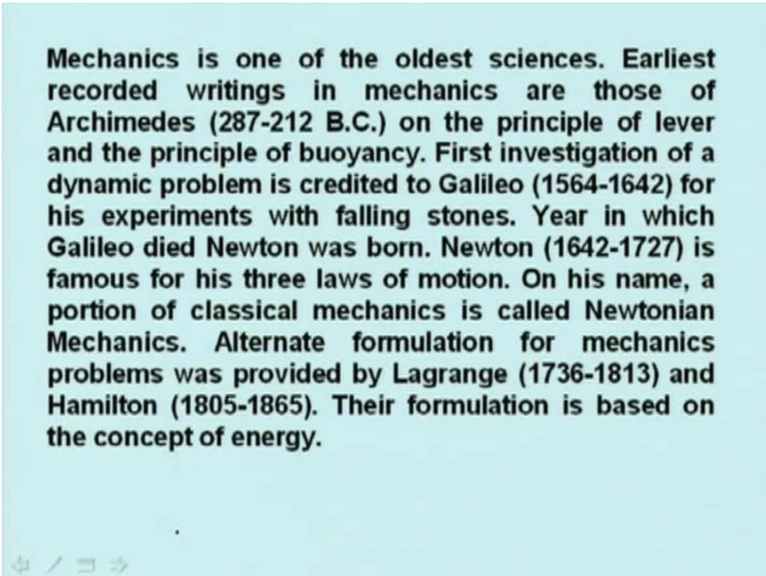
A rigid body is the body which can be considered rigid when the change in distance between any two of its points is negligible for the purpose at end. So in this suppose this is a body; if we apply the force then the distance between two particles remains same. Therefore, this body can be considered as a rigid body.

(Refer Slide Time: 06:49 min)



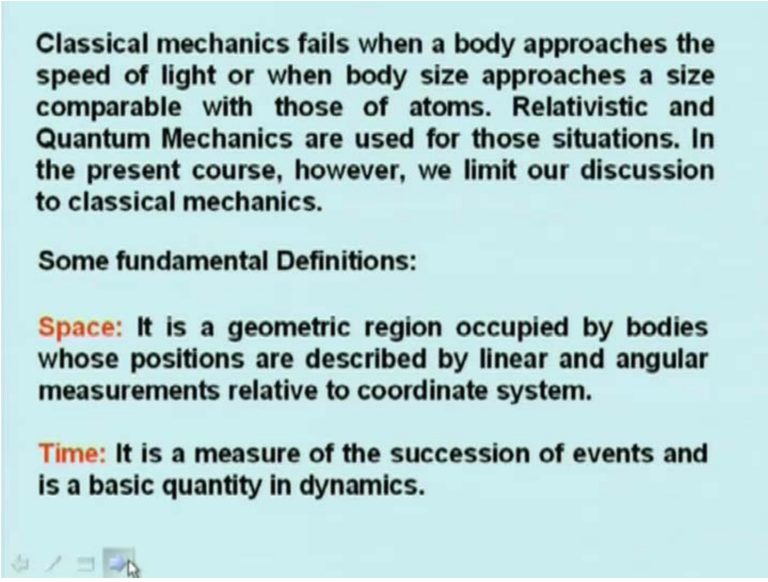
Next deformable body: a body is considered deformable when the changes in distance between any two of its points cannot be neglected. In this case, imagine that this is made of a rubber material and if there are two particles, if you apply the force then the distance between these two particles changes. Suppose, this particle is here after the application of the force, this can be here and therefore this body is a deformable body.

(Refer Slide Time: 07:23 min)



Now some notes on the history of mechanics. Mechanics is one of the oldest sciences. The earliest recorded writings in mechanics are those of Archimedes (287 to 212 BC), on the principle of lever and the principle of buoyancy. The first investigation of a dynamic problem is credited to Galileo (1564 to 1642) for his experiments with falling stones. The year in which Galileo died, Newton was born. Newton was born on 1642 and died on 1727. He has provided three laws of motion and for that, he is very famous. On his name, a portion of classical mechanics is called Newtonian mechanics. Later on, alternate formulations for mechanics problems were provided by Lagrange who was born on 1736 and died on 1813 and Hamilton from 1805 to 1865. Their formulations are based on the concept of energy.

(Refer Slide Time: 08:51 min)

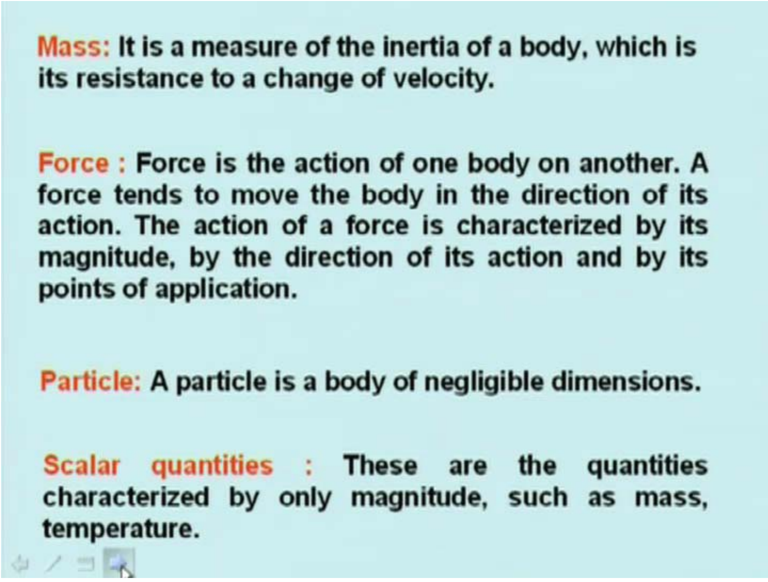


Now classical mechanics fails when a body approaches the speed of a light or when body size approaches a size comparable with those of atoms. Relativistic and quantum mechanics are used for those situations. In the present course, however, we limit our discussion to classical mechanics. Let us understand some fundamental definitions, because, these terms we may be using again and again.

Space - what is a space in mechanics? It is a geometric region occupied by bodies whose positions are described by linear and angular measurements relative to coordinate system. So there will be a space in which bodies are placed and position of the bodies is described by linear measurement from some coordinate, some origin and angular measurement.

Time: It is a measure of the succession of events and it is a basic quantity in dynamics.

(Refer Slide Time: 10:03 min)



Mass: Mass is a measure of the inertia of a body which is its resistance to change of velocity.

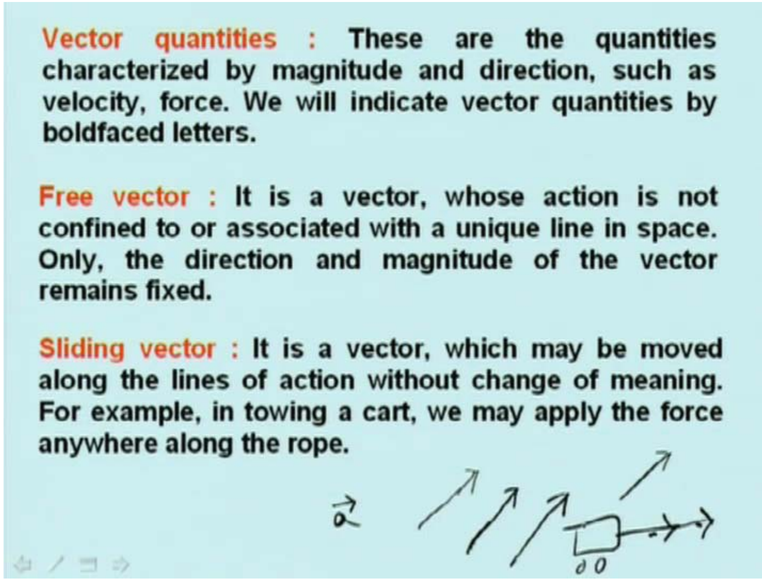
Force: Force is the action of one body on another. For force to apply there should be presence of another body. A force tends to move the body in the direction of its action. The action of a force is characterized by its magnitude, by the direction of its action and by its point of applications. So, three things are important: magnitude of the force, direction of action and point of application.

Particle: We use the term particles again and again in dynamics. Particle is a body of negligible dimensions. Particles dimension - particle is having a mass but it is not having a dimension.

Scalar quantities: These are the quantities characterized by only magnitude such as mass and temperature.

Now these quantities are invariant under coordinate transformation. If you know some object is weighing 1 kg in one coordinate system, its mass will be same in the other coordinate system. Similarly, the temperature also. If we rotate the coordinate axis, these quantities do not change.

(Refer Slide Time: 11:39 min)

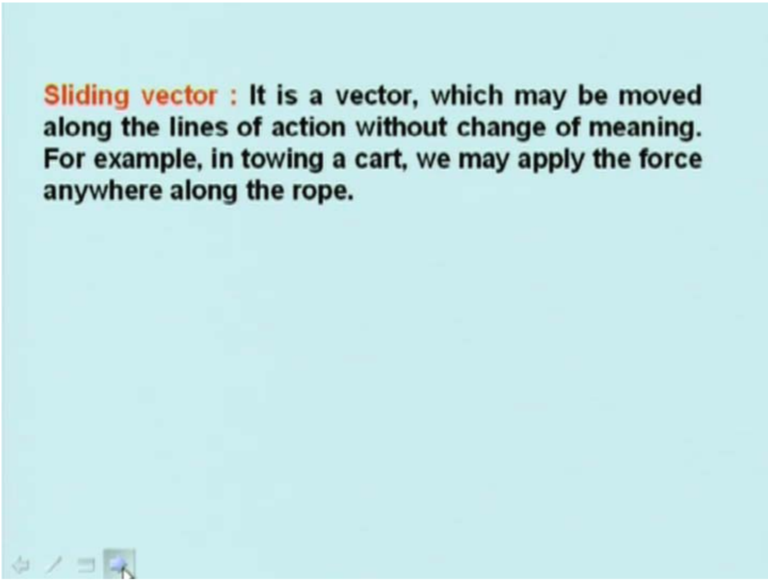


Vector quantities: These are the quantities characterized by magnitude and directions such as velocity and force. Velocity has a direction as well as magnitude; force has a direction as well as magnitude. We indicate vector quantities by boldfaced letters or put a sign of arrow on the top of alphabet like this; this is a vector (Refer Slide Time: 12:13 min). Now vectors will definitely change under the coordinate transformation. If a vector has some components in one coordinate system, in another coordinate system its components will be different.

Free vector: It is a vector whose action is not confined to or associated with a unique line in space. Only the direction and magnitude of the vector remains fixed. Suppose I make a vector like this (Refer Slide Time: 12:46 min) and if I make another vector which is parallel to it and same magnitude and it can be considered equal, then naturally this vector is a free vector. So the free vector is having fixed direction and magnitude; however, it can be placed at any point in the space.

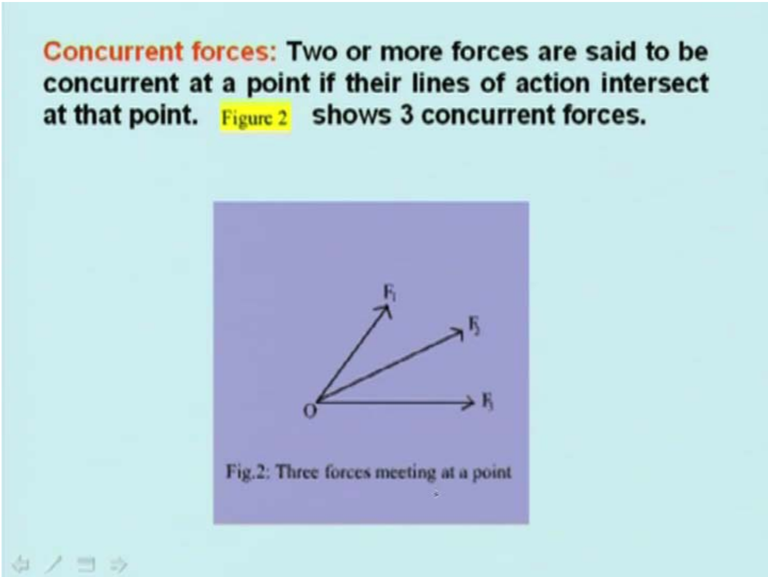
Sliding vector: It is vector, which maybe moved along the lines of action without change of meaning. For example, if we are towing a cart by rope, in this case whether we apply a force at this point or apply at this point, it is having usually the same effect (Refer Slide Time: 13:37 min). Therefore, it is case of a sliding vector in which line of action remains fixed. This is the line of action, but the vector can be placed anywhere on that line of action. Therefore, this is an example of sliding vector.

(Refer Slide Time: 13:59 min)



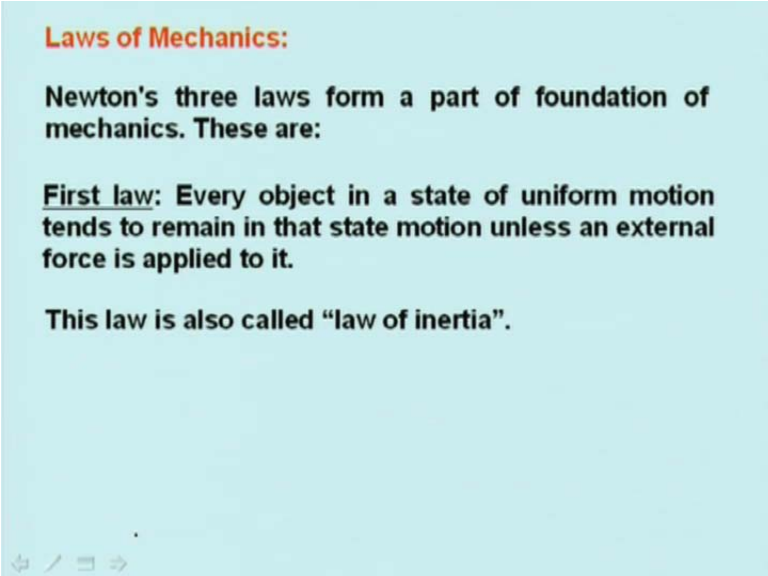
Sliding vector: It is a vector which maybe moved along the line of action without change of meaning.

(Refer Slide Time: 14:11 min)



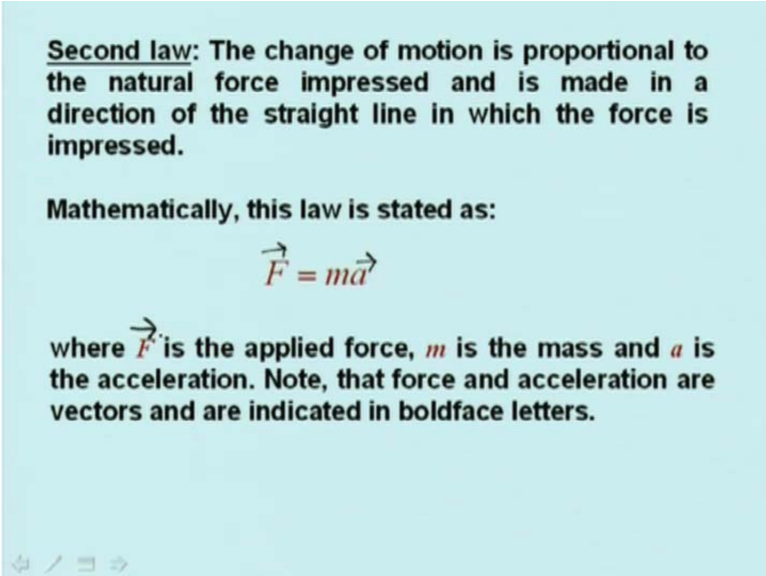
Now what are concurrent forces? Two or more forces are said to be concurrent at a point if their line of action intersect at that point. This figure shows that three concurrent forces meeting at a point. All these force are meeting at a point. Therefore, they are concurrent forces.

(Refer Slide Time: 14:40 min)



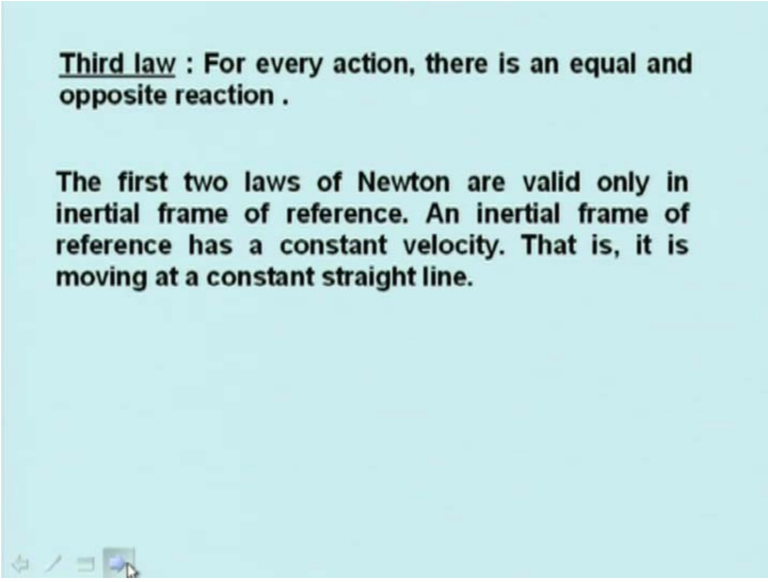
Now, we discuss Newton's laws of mechanics. Newton provided three laws which form the foundation of mechanics. These are: the first law states every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it. This law is also called law of inertia.

(Refer Slide Time: 15:11 min)



The second law states the change of motion is proportional to the natural force impressed in a direction of the straight line in which the force is impressed. Here this force is external force acting on the body. Mathematically this law is stated as F is equal to m times a. I will put arrow here to make, a vector whereas mass is scalar. F is the applied external force on the body; m is the mass and a is the acceleration. Since they are vectors therefore, they should be indicated in boldface letters or one should put arrow in the way I have put here.

(Refer Slide Time: 16:10 min)

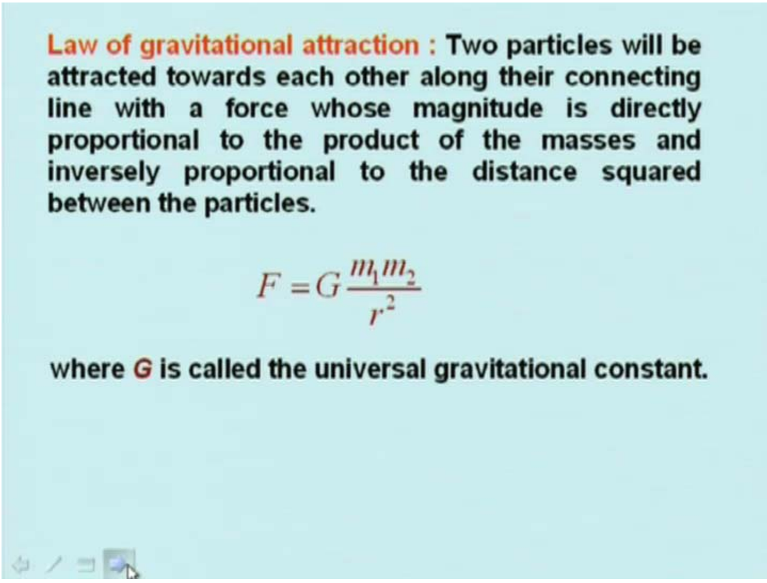


Third law is that for every action there is an equal and opposite reaction. Now the interesting point is that the first two laws of Newton are valid only in an inertial frame of reference. An inertial frame of reference has a constant velocity with respect to another inertial frame of reference. That is, it moves in a constant straight line. The question is that which frame of reference is inertial? Inertial frame of reference is that frame of reference which moves with respect to another inertial frame of reference with a constant velocity. However, we must find out one inertial reference frame. For example, on the earth, when we design some machines in our life we find out that Newton's laws, first two laws, are valid in it.

Force is applied on the body, then it accelerates and acceleration is proportional to mass times force. Therefore, here this law holds good. Therefore, we know the ((17:35 min)) inertial reference frame as far as our design of machine is concerned. Now any frame of reference, like a moving cart, which is moving with a straight line, that can be considered inertial frame of reference; it is having constant velocity. Therefore, this is inertial reference frame. However, when we study planetary motion, then that cannot be considered; earth cannot be considered inertial reference frame.

It means it is almost impossible to find out any absolute inertial frame of reference. Therefore, it is just a concept with which that you can find out. For practical purpose, for designing the machine, we consider the earth as inertial reference frame and we measure the other inertial or non-inertial frame of references with respect to this.

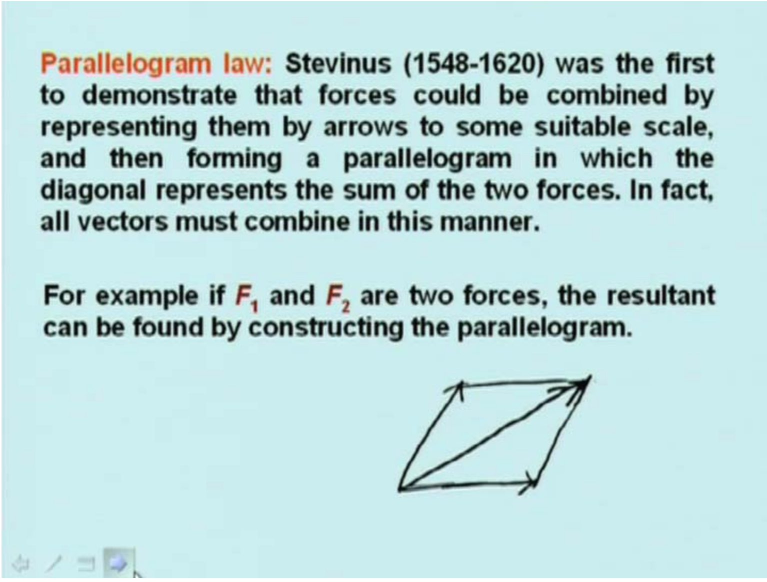
(Refer Slide Time: 18:45 min)



Newton also provided a law of gravitational attraction. Two particles will be attracted towards each other, along their connecting line, with a force whose magnitude is directly proportional to the product of the masses and inversely proportional to the distance squared between the particles. So F is equal to G times m_1 into m_2 divided by r square, where G is called the universal gravitational constant. By this you find out the value of the force and this force is given by G is equal to m_1 m_2 r square.

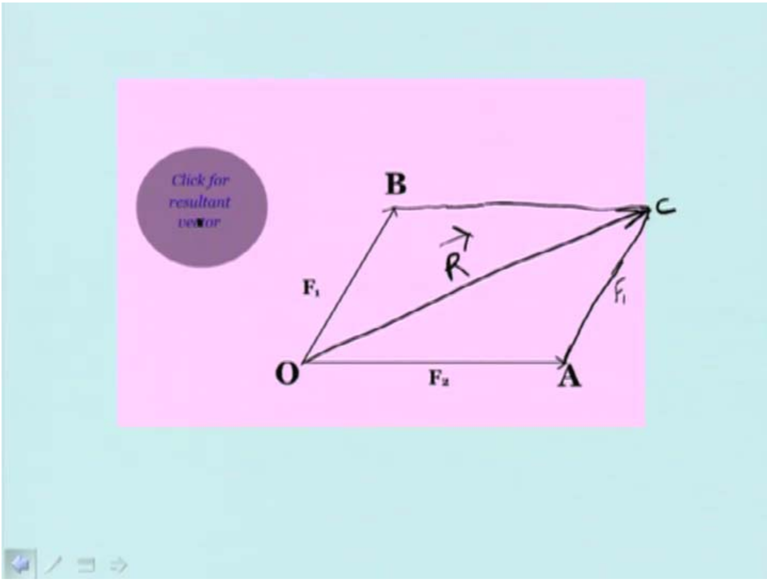
Then if you have got a particle, which is freely falling under gravity you can find out after applying the Newton's law, what is its acceleration. That will be called gravitational acceleration. So the force of the gravitational attraction is called a fundamental force.

(Refer Slide Time: 20:00 min)



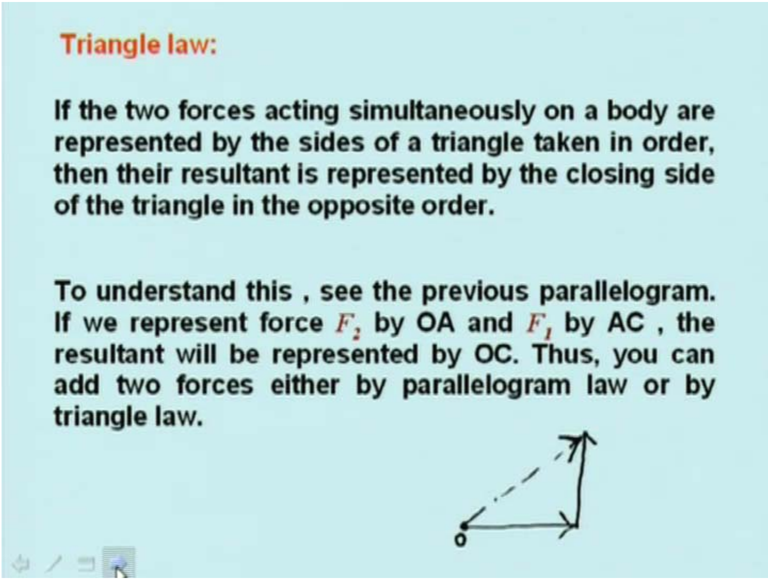
Now we discuss the parallelogram law which was given by Stevinus (1548 to 1620). He was the first to demonstrate that the forces could be combined by representing them by arrows to some suitable scale and then forming a parallelogram in which the diagonal represents the sum of the two forces. In fact, all vectors would combine in this manner. You have a force; two forces are there; they make the parallelogram and then the force is in that direction.

(Refer Slide Time: 20:54 min)



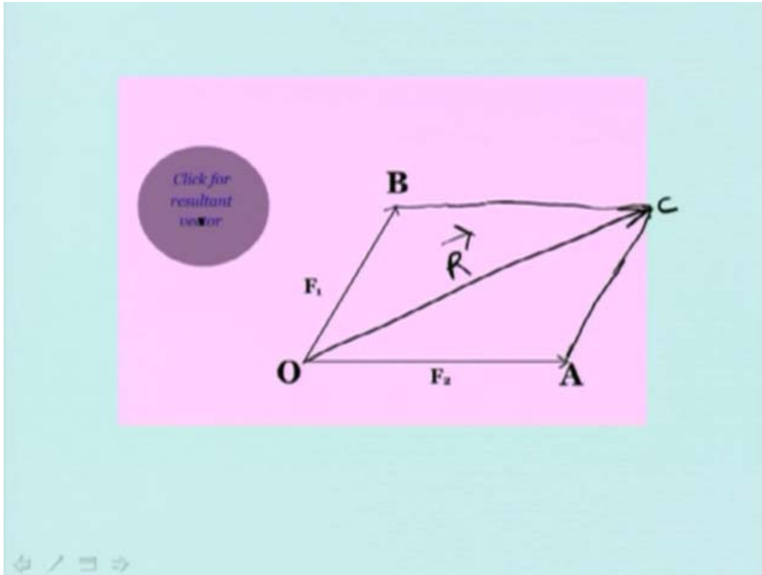
Now, here this is force OA this is F_2, OB is F_1. Now from B, I am drawing a line parallel to OA and of the same length as OA and from then joining A to this. This will be parallel to this OB. Then this is the diagonal; this is the resultant force R.

(Refer Slide Time: 21:38 min)



Now like the parallelogram law, there is a triangle law. If the two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, then their resultant is represented by the closing side of the triangle in the opposite direction. For example, here we understand parallelogram law; here one force is this; another force is this; these are two sides of the parallelogram. Complete the parallelogram, make the diagonal that is the resultant force.

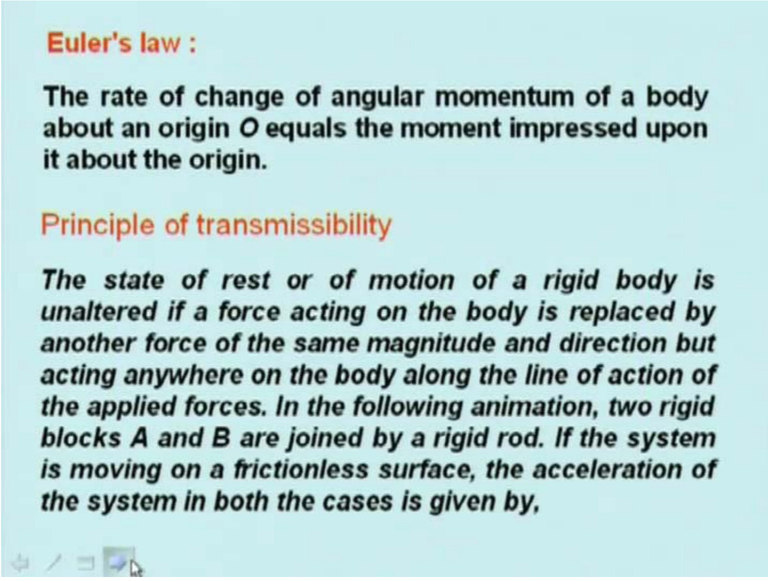
(Refer Slide Time: 22:32 min)



Now consider this triangle; this is OC; this is OA. Therefore, this is F_2; then you have got AC. Then after that, you have got OC. Therefore, what happens that if you consider in this triangle OAC, OA represents one force, AC represents another force, that is F_1 and the resultant is of course given by R. This R basically is represented by the third side, but if we move in a cyclic manner then this is OA, OC, the third side should be CO like this, but this force is of course in the opposite direction.

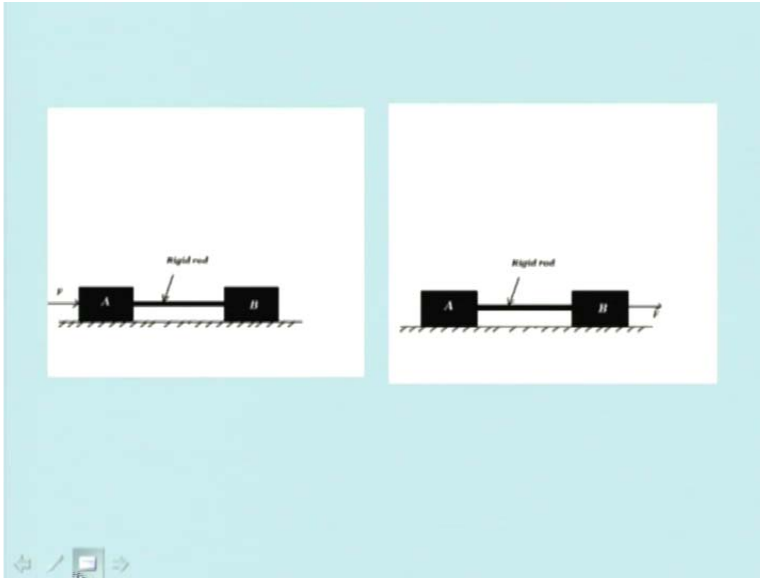
Therefore, it is written that if the two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, then their resultant is represented by the closing side of the triangle in the opposite order. If one force is represented like this, other is represented like this (Refer Slide Time: 23:43 min) then we close the triangle like this. However, you make the arrow in the opposite direction. So here, it is like this. Now this is a triangle law.

(Refer Slide Time: 23:59 min)



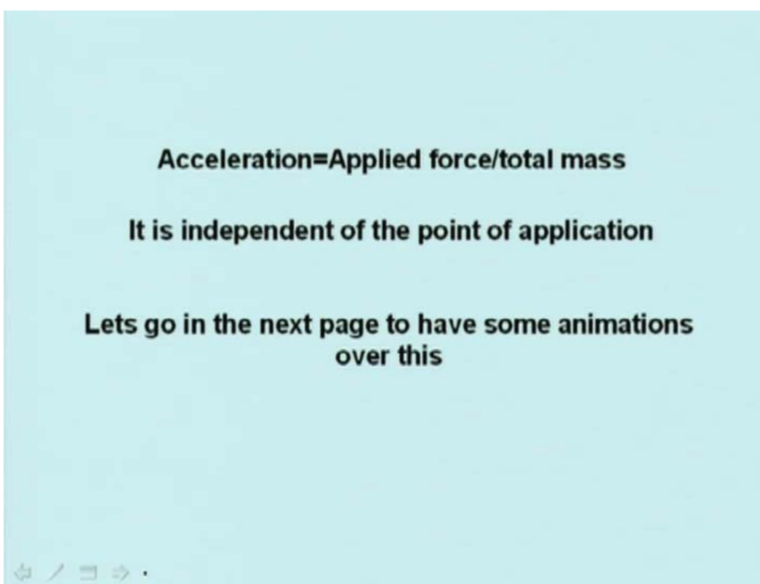
We have Newton's law which relates the force and acceleration or maybe change of linear momentum. We have Euler's law which states the rate of change of angular momentum of a body about an origin O equals the moment impressed upon it about the origin. Rate of change of linear momentum gives us force. Rate of change of angular momentum of a body about an origin O equals the moment impressed upon it about the origin. For the time being, we considered O to be a fixed point in the inertial frame of reference. Now, there is a principle of transmissibility which states that the state of rest or of motion of a rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction, but acting anywhere on the body along the line of action of the applied forces. That means the force behaves like a sliding vector.

(Refer Slide Time: 25:35 min)



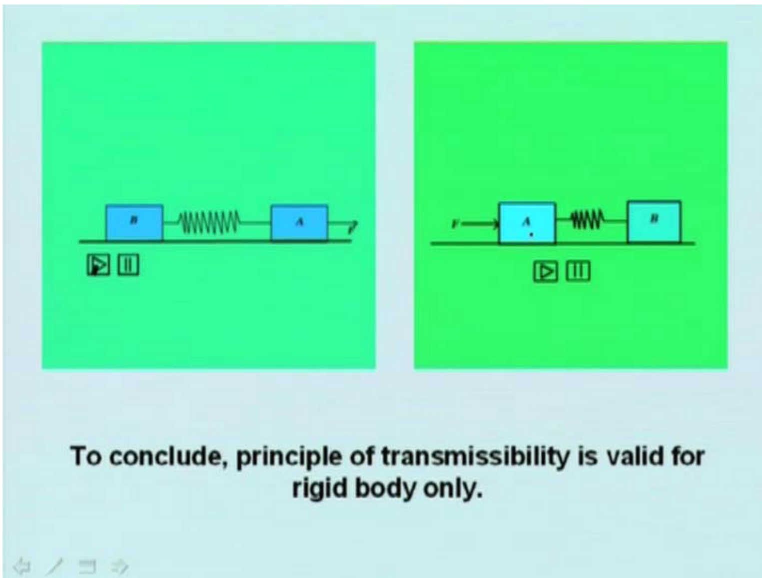
In the following animation, suppose two blocks are joined by a rigid rod. Here, I am applying a force on block A, then both the blocks are moving, because they are connected by rigid rod. Then I am applying the same force F, but now I am applying on block B; it is being pulled here. Then also the motion is same; it is unaffected by this.

(Refer Slide Time: 26:18 min)



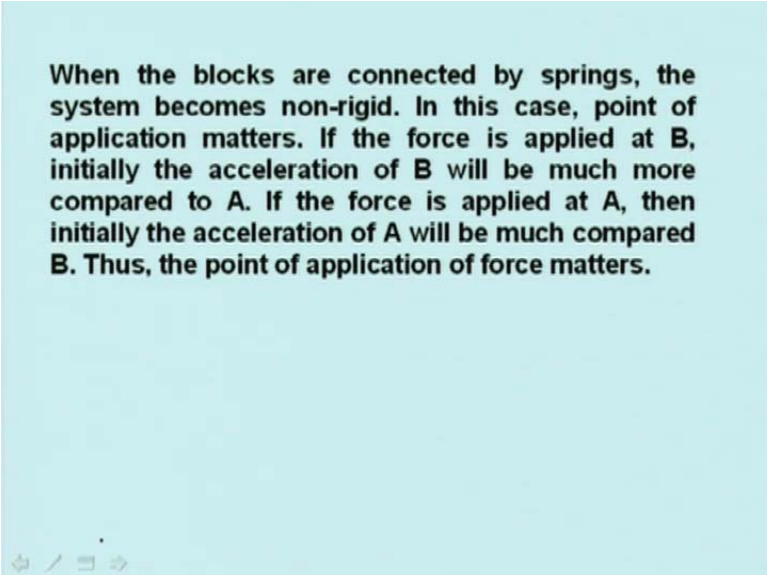
So if the system is moving on a frictionless surface, then the acceleration of the system in both the cases is given acceleration is equal to applied force divided by total mass. We do not ask that where the applied force is acting; it is independent of the point of application. Now let us see the other things.

(Refer Slide Time: 26:48 min)



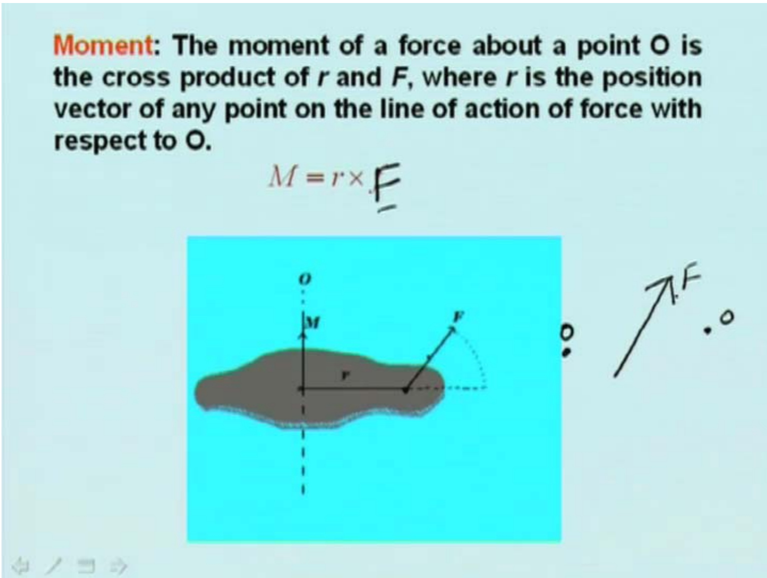
If the blocks are connected by springs, then the system becomes non-rigid. In this case if I apply a force F, then first the block A will start moving and after the spring has stretched sufficiently then the block B will also move. On the other hand, if we apply the force on this object like this, then first this block will move, then after that the another block.

(Refer Slide Time: 27:24 min)



Therefore, if the force is applied initially the acceleration of B will be much more compared to A. If the force is applied on A, then initially the acceleration of A will be much compared to B; that is the point of application of force matters. To conclude, principle of transmissibility is valid only for rigid bodies.

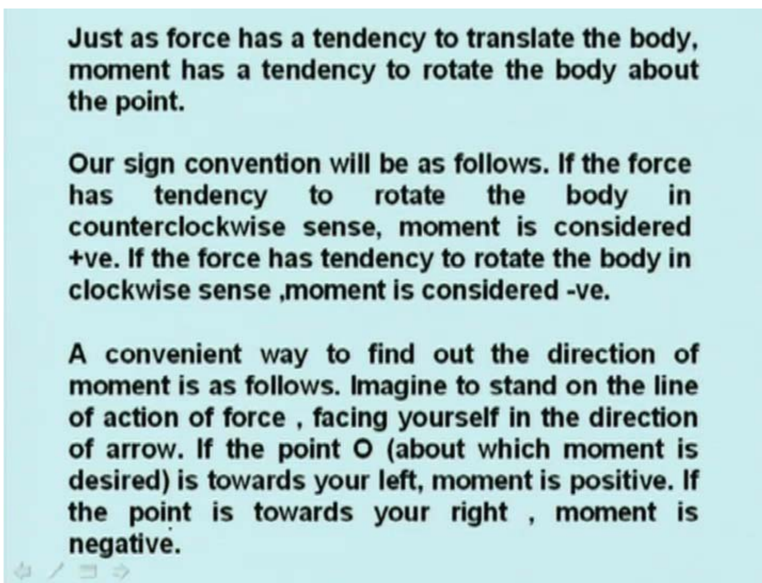
(Refer Slide Time: 27:46 min)



We define the moment. The moment of a force about a point O is the cross product of r and F where r is the position vector of a point on the line of action of force with respect to O. Therefore, M is equal to r cross F.

Now this one here, the force is acting and this is the line of action. On this line of action, I have taken a point here; this point has been taken. Suppose this is my origin point. From here, I join this point, then this becomes my position vector. I could have taken any other point also; in that case the position vector will change. However, when we find out the cross product r cross F, then we should get the same vector. So this is M is equal to r cross F. The direction if we represent F as a vector, r is a vector; then cross product is a vector; that will provide the direction of the moment. Therefore, moment is a vector like the force.

(Refer Slide Time: 29:30 min)

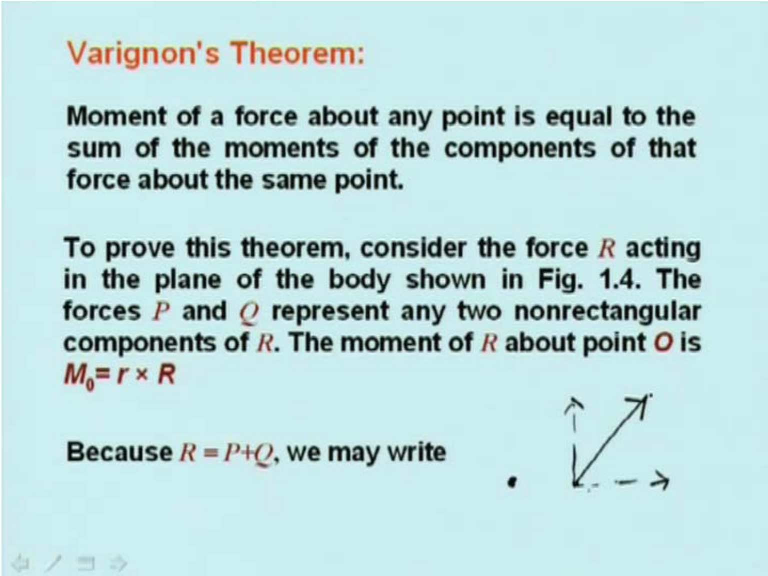


Just as force has a tendency to translate the body, moment has tendency to rotate the body about the point. If we apply a force on this body, it may rotate about this axis. It has a tendency to rotate the body about this axis. So moment has a tendency to rotate the body. Now we have to follow certain sign conventions. If the force has tendency to rotate the body in counterclockwise sense, then the moment is considered if positive. If the force has tendency to rotate body in clockwise sense, moment is considered negative. A convenient way to find out the direction of the moment is as follows: imagine to stand on the line of action of the force, if you just stand on the line of action of the force, look towards arrow. Then if the point about which you are interested find out the moment falls towards your left-hand side then the moment will be considered positive and if the point falls about the right-hand side then the moment will be considered negative.

For example, if I take a force like this, there is a point O here. If I stand on this force and I look towards the arrow, this point O is towards the left-hand side. Therefore, the moment is positive and it has a tendency to move the body about point O in the counter clockwise direction; whereas, if the point O is this side then the moment of F will be negative.

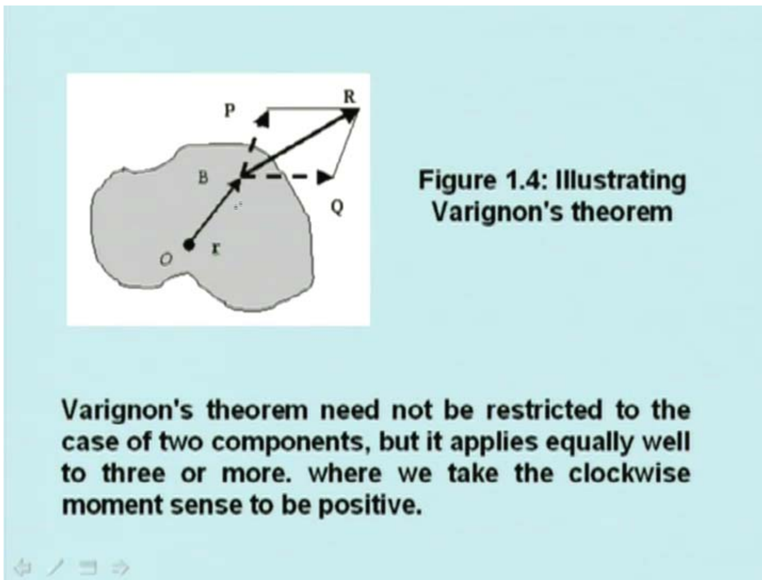
So we stand on the whole line of action of the force, look towards the arrow if the point falls towards left-hand side, then the moment is positive; otherwise, it is negative. However, this is just the sign convention which is followed by most of the people. If you want to say, a force which produces clockwise rotation that can also be considered positive. So it is just a matter of sign convention, but one should follow for solving one problem he should follow the same convention.

(Refer Slide Time: 32:03 min)



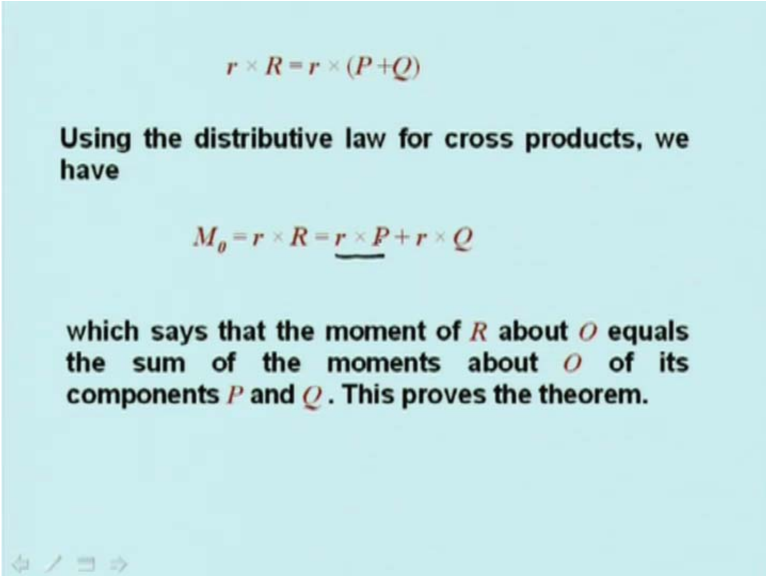
There is theorem of Varignon - it states that moment of a force about any point is equal to the sum of the moments of the components of that force about the same point. This theorem helps us to find out the moments easily. In that case, that means, if there is a force and I am interested to find out its moment about this point, I can resolve this force into two components. Then I can find out the moment of these two components about this point O and add them. Then it will be equal to the moment of that force. This theorem can be easily proved. To prove this theorem consider the force are acting in the plane of the body as shown in this slide.

(Refer Slide Time: 33:13 min)



In this figure, suppose we have got this point is there. Now in that a force R is acting. Now resolve the force into two components, that is, Q and P and you have to find about point O.

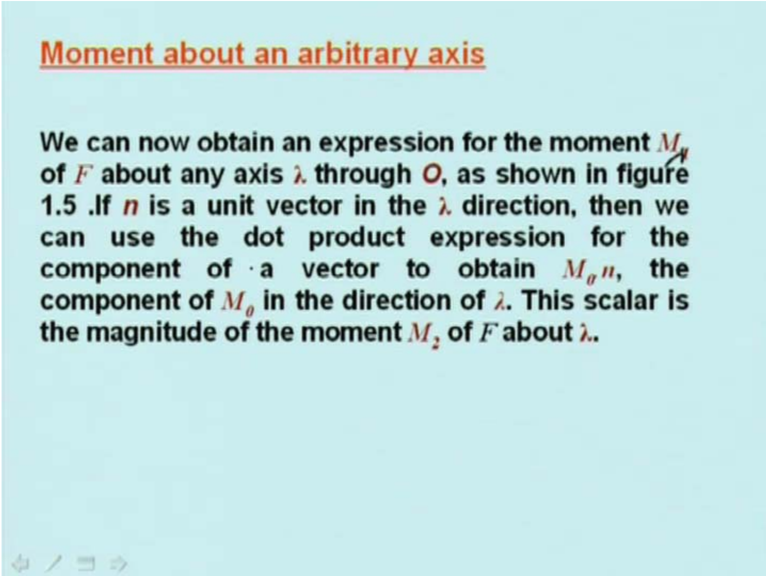
(Refer Slide Time: 33:33 min)



Now we can see that the moment will be r cross R, but R can be written as P plus Q into r. So using the distributive law for cross products, we have M_0 is equal to r cross R, that is r cross P plus r cross Q. Now r cross P is the moment of force P about r, whereas r cross Q is the moment of force about Q. Therefore, it says that the moment of R about O equals the sum of the moments about O of its components P and Q. This has proved the theorem.

Varignon’s theorem need not be restricted to the case of two components, but it applies equally well to three or more components. We have to be consistent in taking the sign of the moments.

(Refer Slide Time: 34:31 min)



Now we have defined the moment about a point. Let us discuss moment about an arbitrary axis. We can obtain expression for the moment M lambda of F about any axis lambda through O.

(Refer Slide Time: 34:59 min)

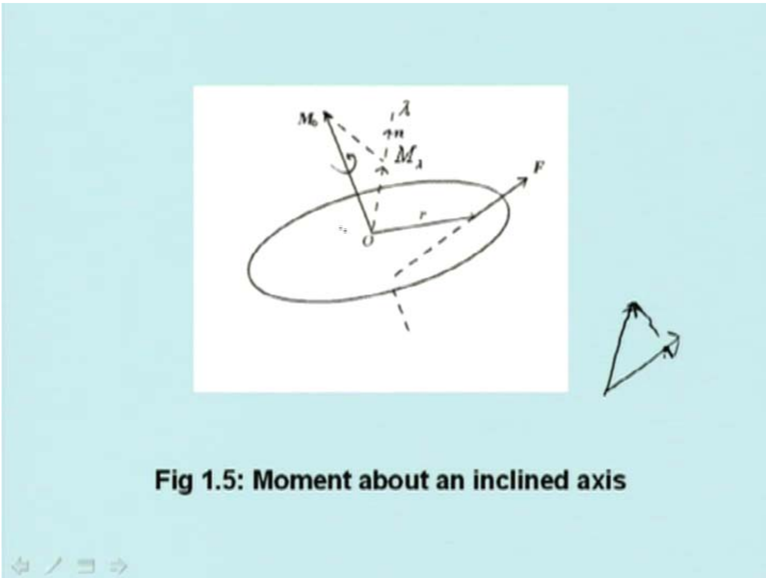


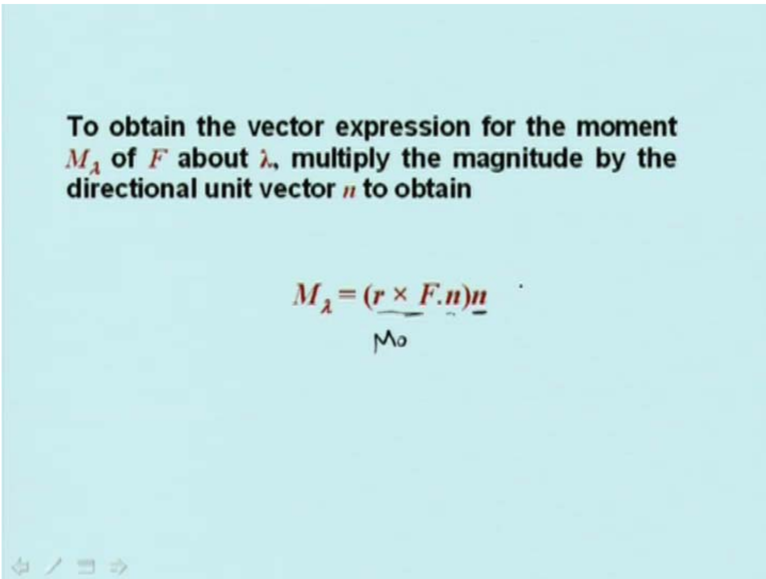
Fig 1.5: Moment about an inclined axis

This is the point O and this is the force F. Therefore, take any point on the line of action of the force, join it by O. This provides you position vector r. Take the cross product r cross F; it will give some vector; that vector is represented by M_0. M_O is the moment of the force F about point

O. Now if you have got any arbitrary axis in the lambda direction and this unit vector is indicated by n - this vector. In this direction, you take a vector whose magnitude is 1 and direction is O. Then we have to find out M lambda; that means moment of the force F about the axis lambda. So if n is a unit vector in the lambda direction, then we can use the dot product expression for the component of a vector to obtain M_0 dot n. We can have the dot product; we can take the dot product of these two vectors M_0 n which will basically be the projection of M zero on this line. Suppose you have a vector here; another vector here; take the dot product. Therefore this is just the projection of this vector and this vector, because, this vector is a unit vector; so this is like that M_0 cos theta.

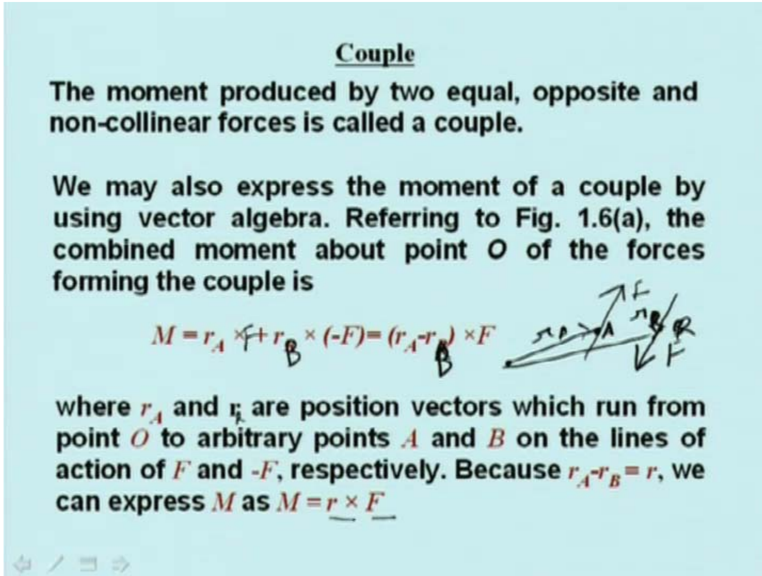
If n is a unit vector for the component to obtain the component of M_0 in the direction of lambda, this scalar is of the magnitude of moment of F about lambda.

(Refer Slide Time: 37:12 min)



Now to obtain the vector expression for the moment M_{lambda} of F about $lambda_a$, multiply the magnitude by the directional unit vector n to obtain: M lambda is equal to r cross F; r cross F is a vector that is basically M_0. Then you take the dot product with n; that is M_0 dot n and this is the magnitude. The direction is same as the direction of that lambda. So they both are attached in here. This r cross F dot n is a scalar quantity. So this is a moment about that arbitrary axis.

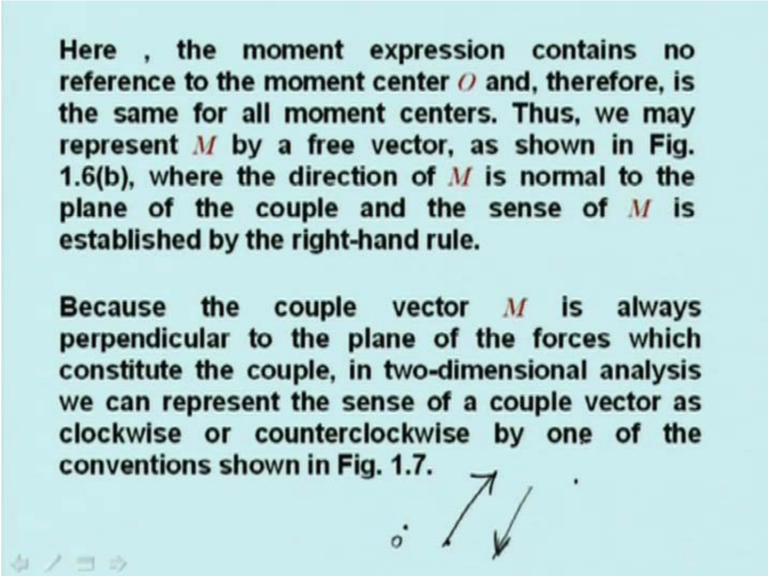
(Refer Slide Time: 37:57 min)



Now we discussed about couple. The moment produced by two equal, opposite and non-collinear forces is called a couple. We have two equal this is opposite but there are non-collinear, they are not acting in the same line; therefore, they will produce couple. They will have tendency to rotate the body. Their net resultant is zero, because, if we add them we will get zero. We may express the moment of a couple by using vector algebra. These are the two forces acting here (Refer Slide Time: 38:11 min) and then if you have got a point O, with that you join this point A, this point B. So this will give you r_A this will give r_B. So M will be equal to r_A cross F plus r_R cross minus F. That is it will give r_A minus r, this is if we indicate it by R, this is r_A cross minus r_R into F where r_A and r_R are position vectors, which run from point O to arbitrary points A and B on the line of action.

Now in this case, r_A minus r_B is equal to r; therefore, we can express M as M is equal to r cross F.

(Refer Slide Time: 40:16 min)



Here the moment expression contains no reference to the moment center O and therefore is the same for all moment centers. Thus, we may represent M by a free vector. So M is a free vector; that means, suppose if this is a couple, these are the two forces, their moment M is a free vector; it is not acting at any particular location. Even if we take any point O about that, we find out the moment that is same. Another point we take this place then also the moment is same. So it is a free vector. Direction of M is normal to the plane of the couple and the sense of image established by the right-hand rule; the right-hand side rule, maybe anticlockwise moment couple can be considered positive, because, the couple vector M is always perpendicular to the plane of the forces, which constitute the couple; in two-dimensional analysis we can obtain the sense of a couple vector as clockwise or counterclockwise by one of the conventions.

(Refer Slide Time: 41:33 min)

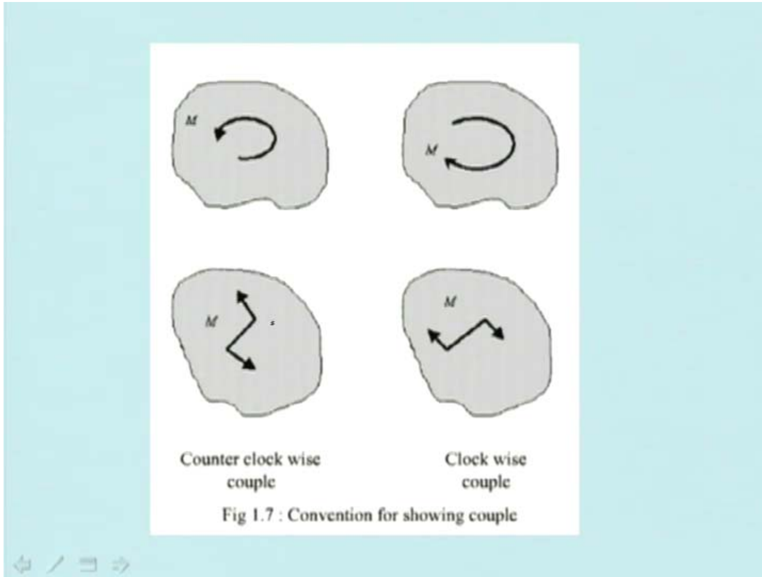
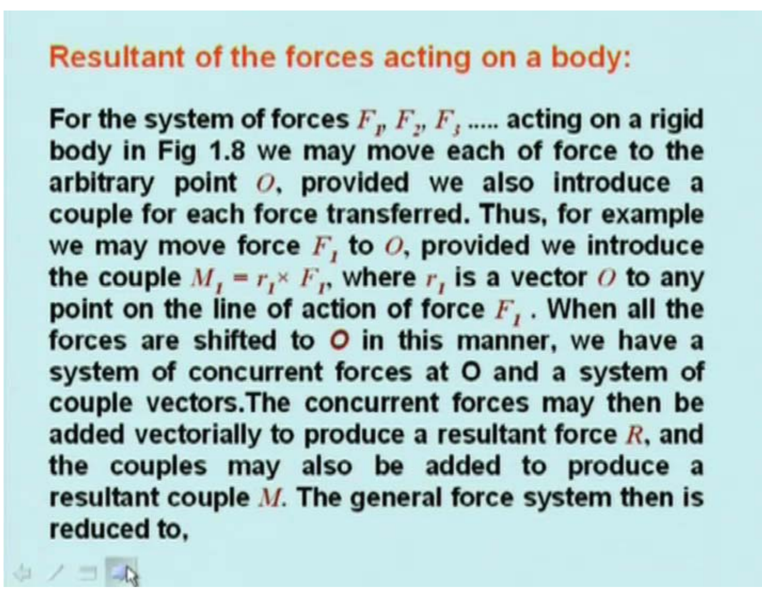


Fig 1.7 : Convention for showing couple

Now this is a counter clockwise couple. We have just made this type of convention. This is a clockwise type. We can also represent it like this - counterclockwise couple like this; clockwise couple this. It is a free vector it can be shown at any location; that means, even here I can show like this.

(Refer Slide Time: 42:00 min)



Let us discuss the resultant of the forces acting on the body.

(Refer Slide Time: 42:14 min)

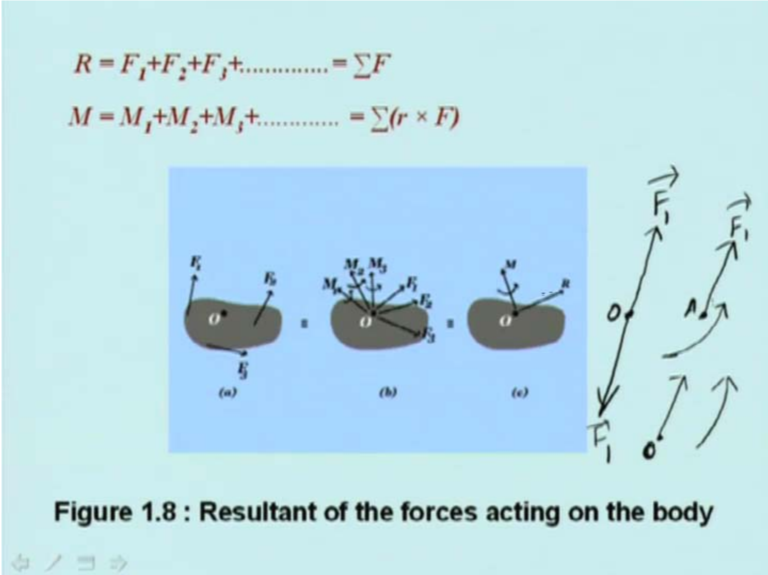


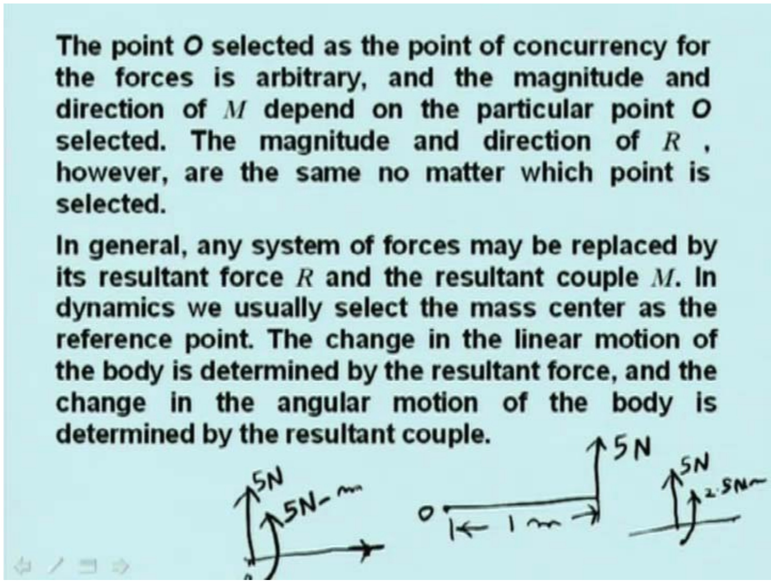
Figure 1.8 : Resultant of the forces acting on the body

Now for the system of forces F_1, $F_{2,}$ F_3 acting on a body like this. This is a body in which number of forces are acting; here I have shown F_1, $F_{2,}$ F_3 and this a point P. What can be done? You can shift these forces from here to here, then you have to apply some couple to balance it. Therefore, we name each of the force to be arbitrary point O and we if we can introduce a couple for each force transferred. Here suppose we move a force F_1 to that position O, then if we introduce a couple, we also have to introduce a couple for that. Suppose this is a force P at this force acting here, now what I am doing that this can be transferred at this point. I apply two equal and opposite force at this point.

Then if I consider that these two forces are constituting the couple like this which was F_1 force acting here; this is another F_1 force acting here; this is the force $F_{1.}$ This F_1 and this F_1 is constituting a couple and that can be moved in this direction. Therefore, this is a couple; therefore, net effect is that there you have another force whose line of action now passes through O instead of this point say A. At the same time there is a couple acting here, because of these and these forces. So in this way we can transfer all the forces to this one, point O. Then we have a system of concurrent forces at O and a system of couple vectors.

The concurrent forces may then be added vectorially to produce a resultant force R and the couple may also be added to produce a resultant couple M. Therefore, now this couple has been taken with respect to point O. Therefore, now you have that this is R and this can be called the moment.

(Refer Slide Time: 45:01 min)



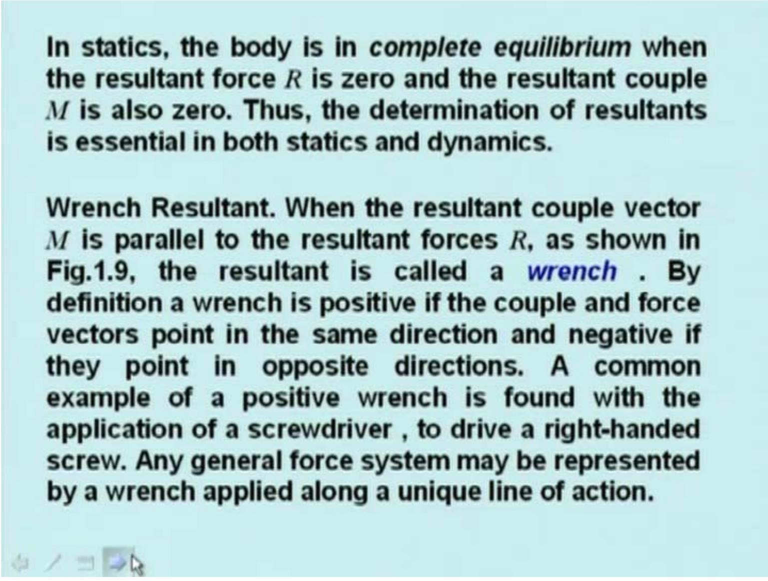
Now the point O selected as the point of concurrency for the force is arbitrary and the magnitude and the direction of M depends on the particular point O selected. The magnitude and direction of R however, are the same no matter which point is selected. Therefore, if you know, like giving a simple example, if you have a force F_5 Newton force acting at a distance of 1 meter from O, that can be replaced by a force O; force is now acting at O itself, but there is a corresponding couple also or moment, that is 5 Newton meter.

I could also have transferred this 5 Newton here at the middle. Therefore, what happens I could have shown it like 5 Newton and this distance was 2.5. Therefore, this is 0.5; so this is 2.5 Newton meter. You see that in these two cases, the magnitude of and direction of force is the same 5 Newton is same as 5 Newton, although we transferred the force.

However, this magnitude of the moment has changed. Similarly, the direction can also change; in this case, the direction has not changed.

In general, any system of forces maybe replaced by its resultant force R and the resultant couple M. In dynamics, we usually select the mass center as the reference point. We try to find out the resultant forces acting at the center of mass and we find out the moment also about that point. The change in the linear motion of the body is determined by the resultant force. The change in the angular motion of the body is determined by the resultant couple or moment.

(Refer Slide Time: 47:40 min)



In statics, the body is in complete equilibrium when the resultant force R is zero and the resultant couple M is also 0. If there is a resultant couple acting then the body will have tendency to rotate. Thus the determination of resultants is essential in both statics and dynamics.

(Refer Slide Time: 48:09 min)

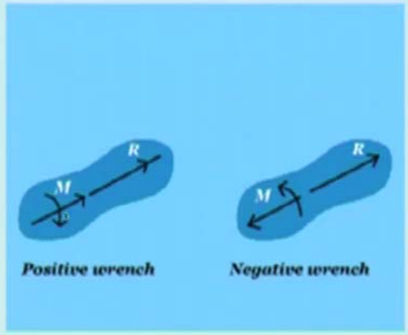
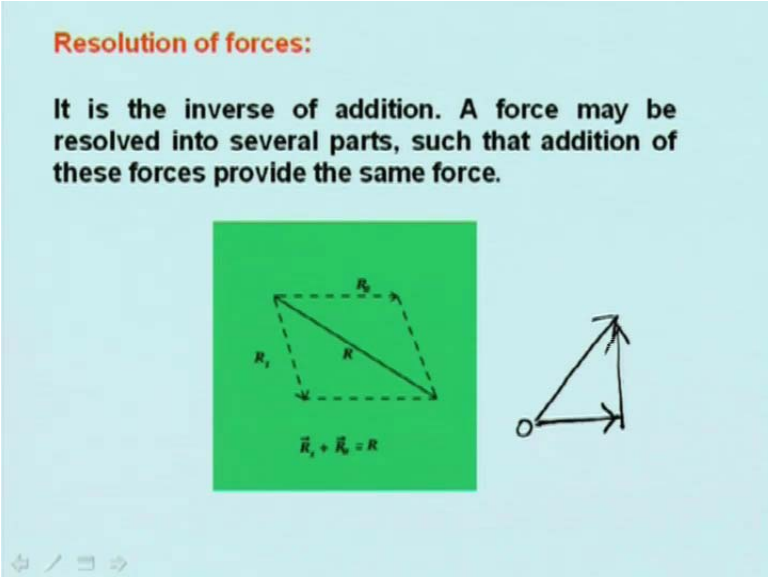


Fig 1.9: Wrench Resultant

We discuss about Wrench Resultant. When the resultant R, is in the same direction as the resultant moment M, then this is called Wrench Resultant. In this case, it is a positive wrench, that means direction of R and direction of M is same. Negative wrench means direction of R and direction of M are opposite. So when the resultant couple vector M is parallel to the resultant force R as shown in the figure, the resultant is called a wrench. By definition, a wrench is positive if the couple and force vectors point in the same direction and negative if they point in opposite directions.

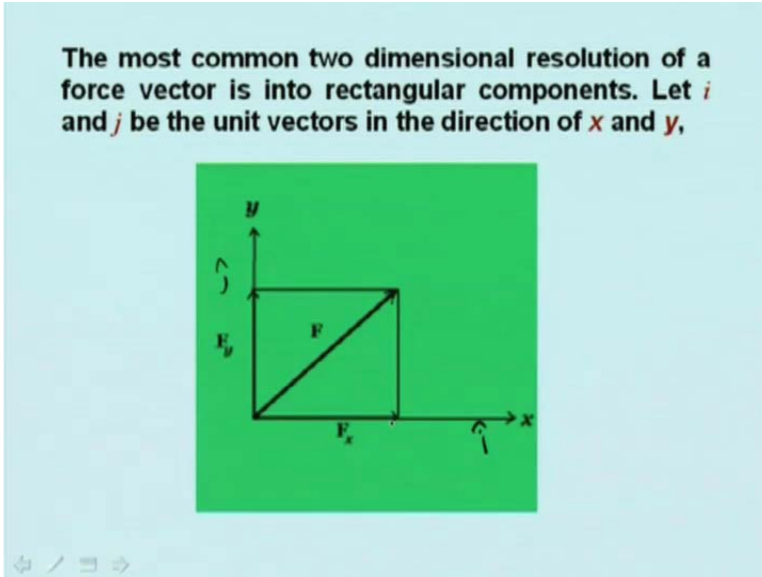
A common example of a positive wrench is found with the application of a screwdriver. When we move a screwdriver, it applies a moment which is along the axis of the screw and there is a thrust force which is also in the direction of a screw. Any general force system maybe represented by a wrench applied along a unique line of action.

(Refer Slide Time: 49:26 min)



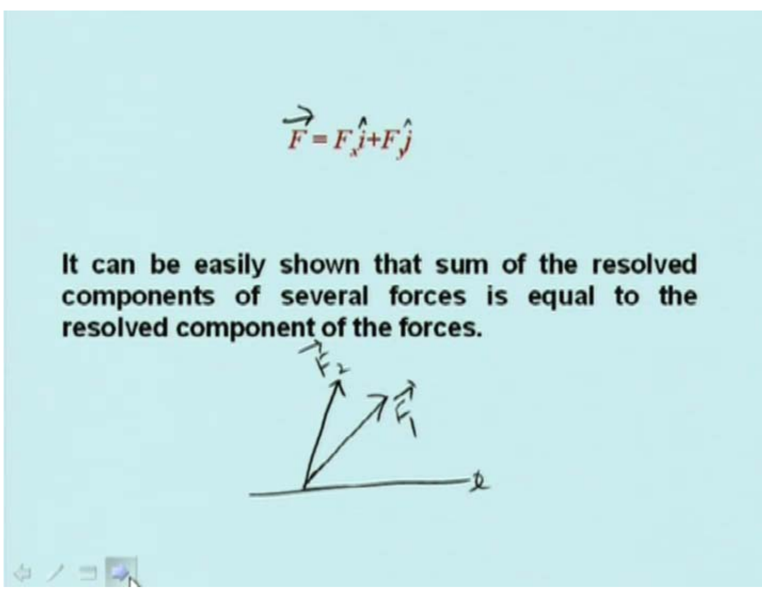
Then we discuss the resolution of forces. The resolution is the inverse of addition. Two forces can be added to make the resultant. Similarly, the force can be resolved in two components or several components, such that the addition of these forces, provide the same force. Whereas, the resultant of the force is a unique vector, resolution of force may give many solutions. However, if we fix up the directions, then you may get unique. Say for example, now if this is a force F, it can be written as the sum of this as well as this force. The force has been resolved into two components.

(Refer Slide Time: 50:33 min)



Therefore, the most common two-dimensional resolution of a force vector is into rectangular components. Let i and j be the unit vectors in the direction of x and y. This is i, this is j, this is jth component. The force F has been resolved into two components - this is F_x and this will be F_y and the resultant force is shown by the force F here. Therefore, this has been resolved.

(Refer Slide Time: 51:07 min)

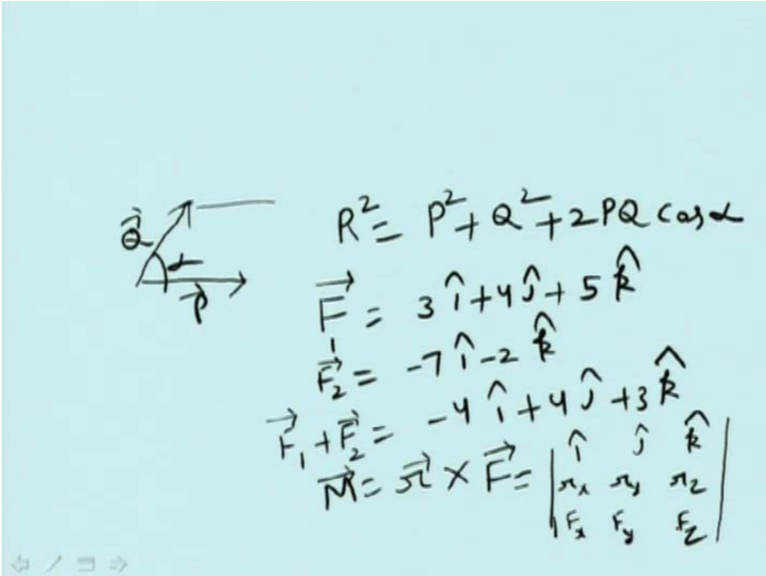


So we can write F is equal to F_{xi} plus F_{yj}, where F is the force. F_x is the component of the force; this maybe indicated by boldfaced but F_x is just a component in the direction of unit vector i then F_y and j. It can be easily shown that the sum of the resolved components of several forces is equal to the resolved component of the forces. That means if you have a one force and another force is acting - this is F_1 and this is F_2; if you combine these two forces, you get the resultant force. If you take the resolved component along a particular direction, let us say x, that is same as if you take the resolved component of F_1 and then take the resolved component of F_2 and then join them. Therefore, this can be done.

So this concludes the basic concepts about engineering mechanics. In this lecture, we have discussed basically the means definition of engineering mechanics. We have discussed about the continuum hypothesis which will be applied while studying the rigid body. The rigid body is basically a hypothetical concept. When we apply the force, if the distance between two particles does not change significantly, then the body can be considered as a rigid body. Actually, always there is some change between the two particles when you apply a force, but it can be considered.

Then we discussed Newton’s laws. Newton’s first law and second law are valid only in inertial reference frame. Then we discussed about the gravitational force acting between the two particles. We also have told that the forces can be resolved and the resolution of the forces offers many advantages, because the resultant can be easily found.

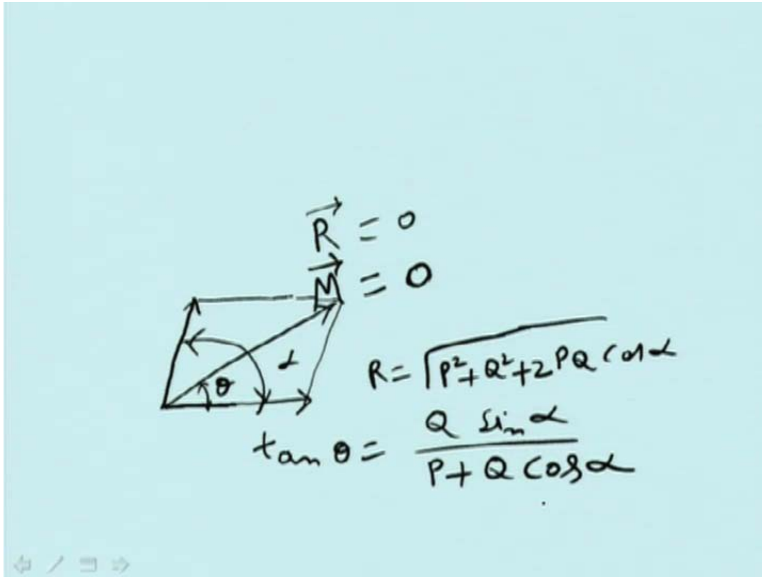
(Refer Slide Time: 53:54 min)



Suppose there are two vectors, you cannot construct graphically again and again that parallelogram and although there is a formula, by which you can find out resultant. That formula is R square magnitude P square plus Q square; this is this force is P; this is Q; P square plus Q square plus two PQ. If this angle is alpha; this is cos alpha. Similarly, you can find out the direction also. However, if one can resolve the forces in the components form and express the force as a vector quantity, let us say one force is like this - F_1 is equal to 3i plus 4j plus 5k. Another force is F_2 is equal minus 7i minus 2k.

Then these two forces can be joined and this becomes F_1 plus F_2 is equal to minus 4i plus 4j plus 3k. Then you can find out the magnitude of the forces. Similarly, if you have expressed the force in this form of the components – i, j, k - are the unit vectors along say three orthogonal direction that is x direction, y direction and z direction. Then you can find out the moments also - r cross F is the moment. This can also easily be found out. You know how to find out the cross products. If we can have r cross F is equal to basically i j k in the form of a determinant. We can write this is r_x component; this is r_y, r_z and this is F_x, F_y, F_z. Then we have just introduced about the Euler's law, which is related to the angular momentum.

(Refer Slide Time: 56:52 min)



Then we have told the concept of the couple. For complete equilibrium of the body, the resultant force must also be 0 and the couple acting on the body should be 0. These are the necessary and sufficient conditions for a body to be in equilibrium.

In the next lecture, we will elaborate on these conditions in detail. Here I will just recapitulate one formula which is already taught at the plus two level. This R is equal to under root P square plus Q square plus 2PQ cos alpha. Then this makes angle theta from here; therefore, tan theta this angle is alpha; this angle theta tan alpha; tan theta is equal to Q sin alpha divided by P plus Q cos alpha. Here theta is the direction. That we will be discussing in the next lecture.