Theory and Practice of Rotor Dynamics

Prof. Rajiv Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module – 3
Rotor Mounted on Bearings
Lecture - 9
Flexible-Shaft with a Rigid Disc Mounted on Anisotropic Supports

In previous lectures we have seen rotors, which was of Rigid Supported on Flexible bearings, but sometimes both the rotor, the shaft as well as the bearings are flexible. So, today we will analyze such case in which not only the bearings, but also the shaft is rigid. And in this particular case, if we see effectively there is the stiffness of the shaft and the bearing they are connected in series and there we are providing some kind of effective stiffness and damping to the disk.

So, our aim in this particular lecture will be first to obtain some kind of effective stiffness which the disk is experiencing from the shaft as well as the bearing. And once we have that then we can able to do an unbalance analysis of such rotor system, in which we can able to calculate the critical speeds.

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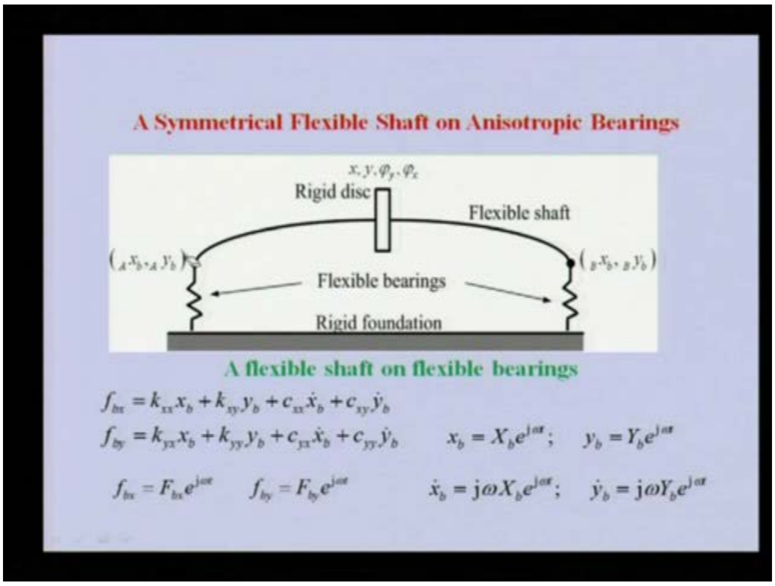

So, in this particular case because now we can able to see that there is a disk which it not necessary is at the center of this particular feasible shaft. This particular disk if it is not at the middle; obviously, we need 4 degree of freedom to defined it is position that is x, y phi y and phi x that 2 are translator motion and another 2 are the tilting of the disk. And these particular shafts at ends are supported by the spring flexible spring.

Not necessary these are on a spring, we can able to take the 8 coefficient various model of the bearing in this particular case, but just for simplicity, we have considered a spring here, but and in the analysis we will be considering as a 8 linearize coefficients of the bearing. And correspondingly at the shaft end we need to define what is the displacement because the fluid flowing forces of the bearing will be depending up on these 2 displacements that is the relative displacement between the shaft end and the housing.

And, so the left hand of the shaft here representing by the displacement were representing as x b and y b and that substitute is representing this is in the left bearing. Similarly, in the other side we have right bearing the displacements are different at this end because we are considering these left and right side bearings as they are having different property. In this particular case we are considering the foundation; that means, the pedestal on which this bearings are mounted as a rigid subsequently, we will take the flexibility of the rigid the foundation also.

So, in this particular case the analysis will be in several steps. So, first is let us try to see what is the bearing force, due to the displacement of this. So, you can able to see that the because of this displacement and the velocity the bearing force we can able to write as f b x this is the bearing force in x direction and this is the bearing force in the y direction. So, bearing force in x direction is k x x into x b k x y into y b. So, these are the from stiffness of the bearing and these are c x x x dot b, and c y c x y y b dot these are from the damping.

So, the force in the x direction at the bearing due to the motion of the shaft end one of the end let us say will be given by this expression in x direction similarly, in the y direction we will be having another coefficients that is k y x, k y y corresponding displacement if you multiply you will get the forces in that direction. And these are from the damping terms. So, these bearing forces they are representing for 1 of the bearing end we can able to write similar expression for other bearing end.

In this particular case, as we know whatever the forces which are coming on to this bearing from the shaft end they are coming from the basically from the unbalance force.

Because, unbalance force is the external force which is acting at the disk they are giving to the bearing of the shaft. So, these forces are; obviously, they are function of spin speed. So, these bearing forces we can able to write as F b x as capital F capital F b x which will give the magnitude and the phase in formation of the force.

And this is the angular velocity will be taken in the e j omega t similarly, the bearing force in the y direction will be given as F b y this is a complex quantity and e j omega t. So, corresponding to these forces in this equation of motion; obviously, the bearing displacements will also be harmonic in nature with this same as the spin speed of the shaft they will be having different phase. So, that will be taken care in this complex amplitude.

So, this is the we are writing the displacement in x direction at the bearing end as capital X b e j omega t and this is in the other direction of the same bearing. With this if we take the derivative we can able to get this 2 expressions of the velocity term because in our equation of motion for the bearing force we have velocity terms. So, these assume solution we can able to substitute in this equation of motion.

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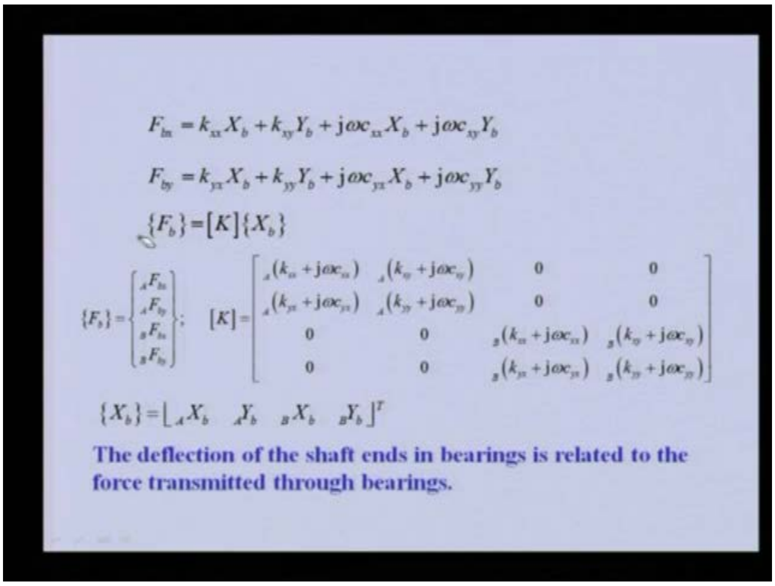

And if we substitute we will get this kind of expression, in which time dependency will go e j omega t will be common. So, that will go, and now you can able to see that this is the bearing force in x direction this is in y direction there in the complex form and they contain both the magnitude and the phase information. So, these 2 equations we can able to combine in a matrix form like this, in which the bearing, now these 2 equations we are writing not only for one bearing, but identical to this we can able to write for another bearing that is bearing b.

And, so we will be having basically 4 bearing equations at 1 bearing will be having force in x direction y direction, similarly for other bearing. So, this is the assembly of 4 bearing forces at bearing a that is left side and right side and correspondingly the X b because we have two bearings. So, X and Y direction displacements of the bearing this is the right hand side. In the bearing model generally we consider all the translatory motion whatever the bearing coefficients we have defined that is only for the translatory motion.

Therefore, tilting motion we have not defined any stiffness or damping coefficients. We have already for example, we have written k x x or k x y we have not written k phi x or phi x phi y because those coefficients will be negligibly small. So, we have neglected those coefficients, so already then the translatory motion coefficients we have defined and in these equations because of this you can able to see that this particular bearing displacements are linear in natural only.

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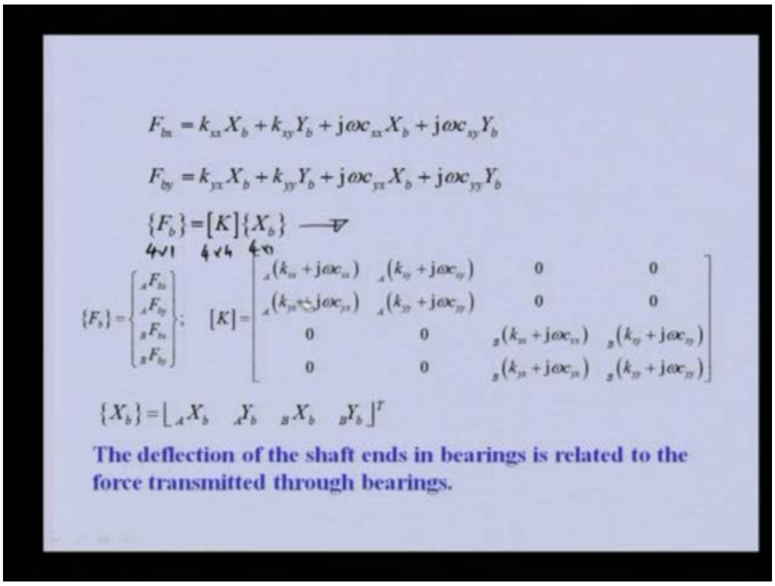

So, this particular equation which we have written in a compact form is having you can able to see for 4 into 1 size four into 4 size 4 into 1. So, this particular equation is relating the bearing displacement and the bearing force with the help of bearing property, dominate property that contain the stiffness as well as the damping terms. Now, will obtain the bearing force from the shaft equilibrium analysis because the bearing forces basically coming from the disk onto the shaft and shaft is transmitting the forces to the bearing, so now will analyze the shaft equilibrium equation to get the bearing forces.

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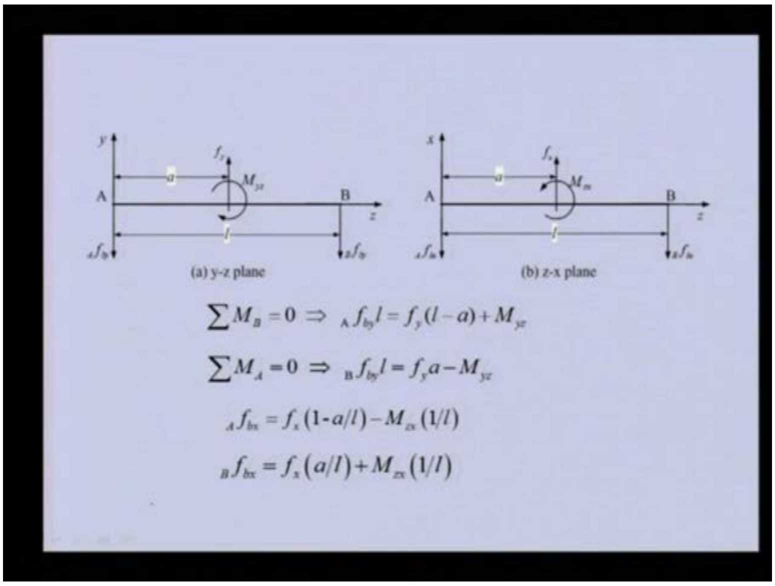

So, you can able to see this is the free body diagram of the shaft, we removed the disk from the shaft. So, the reaction forces M y and moment M y z from the disk on to the shaft is acting in the, positive in these directions, because these are reactions. So, they will acting in the positive y and positive displacement, angular displacement direction. Now because of this the 2 end of the shaft A and B were removed from the bearing also this particular shaft.

So, these are the bearing forces in the y direction and other bearing in the y direction both are, but they are in the 1 plane. Similarly, we will be having bearing forces in the other plane also like this in the z x plane. So, these are the reaction force of x and z x in the moment which is coming from the disk, these are the bearing forces which is acting at the shaft when we remove the bearing. So, these are the reaction forces. Now, we can able to take the equilibrium of each of this free body diagram.

Like in the y z plane we can able to take the moment about point B here. So, this particular force will give a moment this is the moment which will be getting and that is balanced by moment from this particular reactions from the disk and this moment. So, you can able to see the total length is l from here to here is distance l. So, the moment on for f y will be l minus a plus this one because they are acting in the same direction. So, sign will be the same.

Similarly, we can able to take the in the same diagram their moment about point A then we will get this particular force, bearing force moment on is l for that and these are the from the disk reaction forces. The sign is opposite because they are producing on opposite direction moment you can see f y z is acting in the counter clockwise direction M y z is acting in the clockwise direction. So, it is sign is negative this 2 equation we can able to rearrange.

So, that we can write the bearing force at A in x direction and this is in the I am just repeating again here. So, these two are the equilibrium equation in y z plane on this based on similar analysis in the z x plane, we can able to write the equation of bearing forces in the x direction at bearing A that is the left side and bearing B at right side. So, basically we got 4 equilibrium equation from these two free body diagram. These relations which are relating the bearing forces with the reactions onto the shaft from disk can be put in a matrix form.

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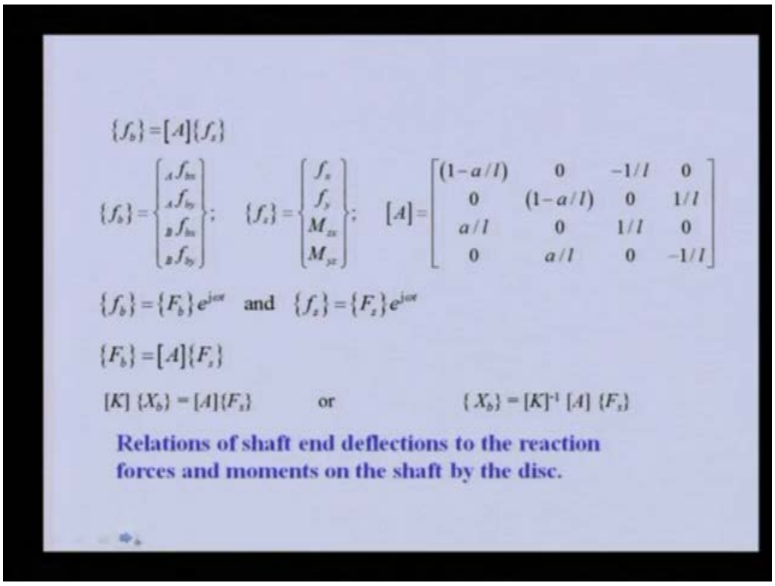

So, this is the bearing force, this is the shaft reaction forces and this is the A matrix. So, you can able to see the bearing force is having 4 components corresponding to bearing a in the x and y direction and bearing b similar 2 directions the shaft reaction force are force which contains not only the force in x and y direction, but also the moment acting in 2 planes that is z x and y z plane the sticking at this we can able to choose according to our convenience. So, in this particular case I have chosen force in the x and y direction first and then the moments. So, if you choose this the A will take these particular forms, which are nothing but the coefficients which was there in the previous 4 equations of f x f y and moments.

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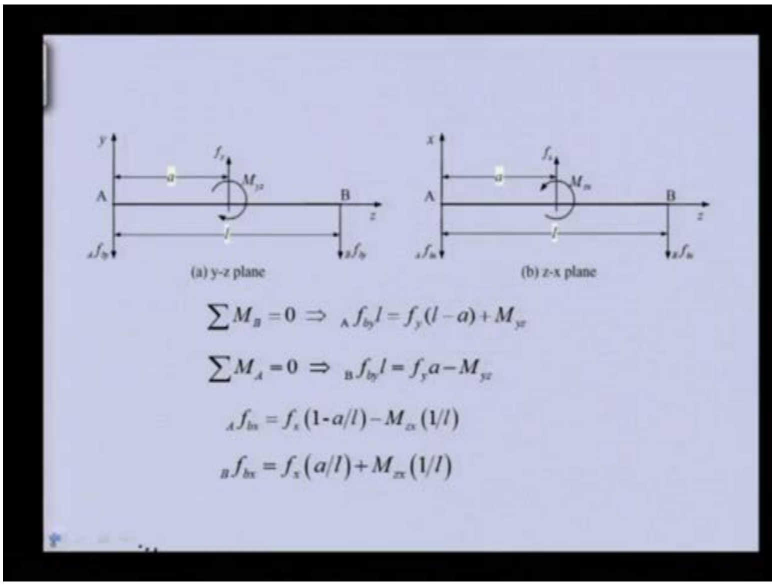

So, these are coming from the, these you can able to see this coefficients are coming in the matrices.

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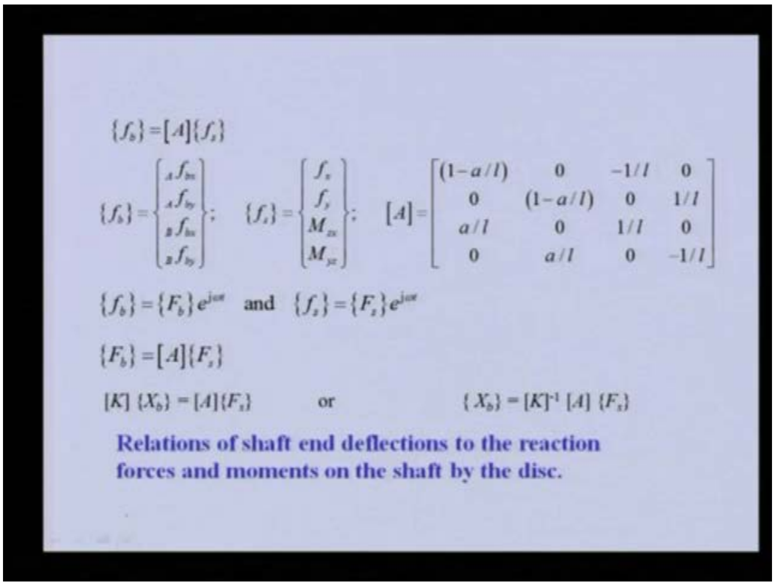

So, this relation which is relating the bearing force, with the shaft reaction force from the disk, again this bearing force we can able to write in terms of the amplitude and phase information and the harmonic term this shaft reaction force which is coming from the disk; obviously, that is having the frequency as the spin speed because it is coming from the unbalance and this is the amplitude and phase of the shaft reaction force. So, they if we substitute in this equation of motion, we will get in the frequency domain bearing force is equal to matrix A and shaft reaction forces from disk. Now, earlier we obtained the bearing force in terms of the bearing displacement. So, that relation was the stiffness of the k into x b.

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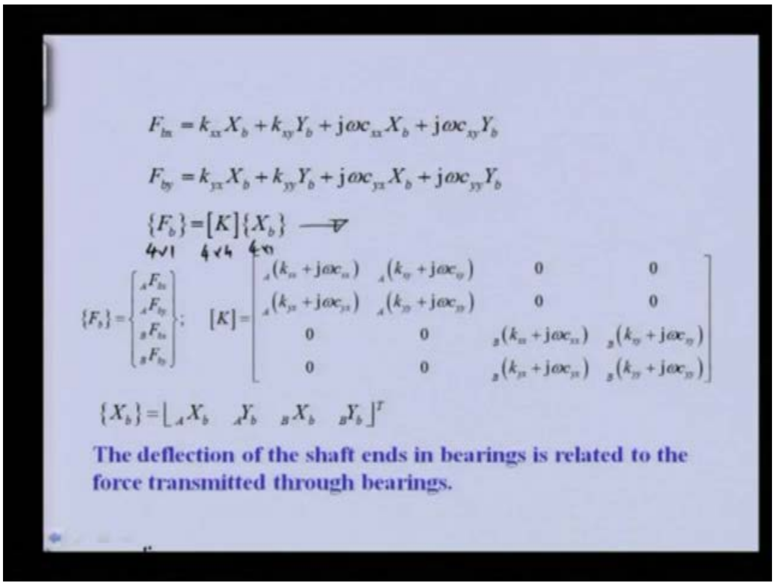

So, if you go back to the previous here we related the bearing force with the bearing displacement. So, this relation we can able to use here.

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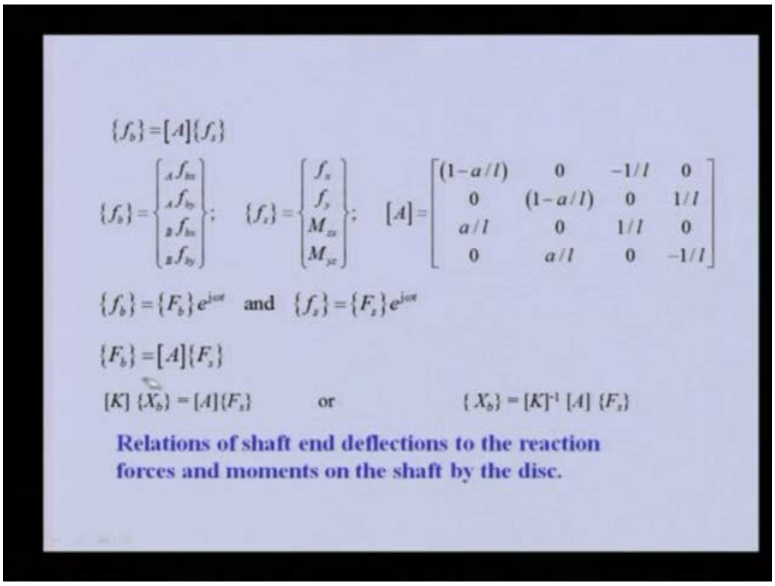

In place of F b we can able to write K into X b and with this we can able to see we can able to get the bearing displacement, like this by inverting K matrix multiplying with the A and this is the reactions on to the shaft from the disk. So, now you could able to relate the bearing displacement with the shaft reaction force. Next now, our we are planning is we are trying to obtain the, the disk displacement. Because, disk is place on the shaft and shaft is flexible.

So, we want the absolute displacement of the disk. So, we need to have determination of this particular displacement in 2 steps, 1 is when this particular shaft is rigid and once it is rigid disk is there at some location. So, whatever the displacements of the bearing is taking place, that will be giving to the two ends of the shaft when we are considering the shaft is rigid. So, because of whatever the displacements are there from the bearing, the disk will be having some displacement.

And in second step we will be considering the bearing as rigid and because of the deformation of the shaft because shaft is also flexible. So, it will deform in some shape because of that what is the displacement on the disk. So, these two displacements we need to add vector ally, so I am again repeating first is when we are considering the bearing as flexible shaft as rigid. So, because of the bearing displacement what is the displacement of the disk then next step bearing is rigid shaft is flexible.

So, what is the displacement of the disk because of the shaft flexibility this two displacements will add up to give the total displacement of the disk.

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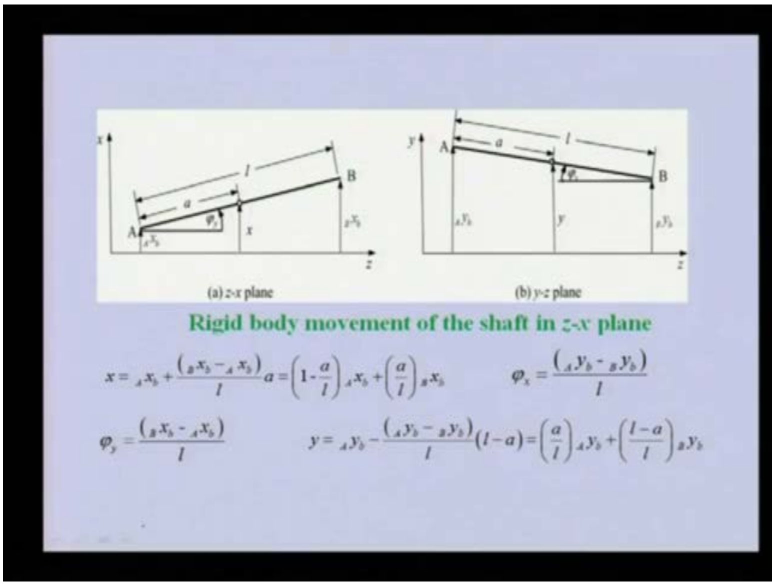

So, now we are considering in this particular figure the shaft as rigid and we are relating the, we are giving the bearing displacement to the 2 ends of the shaft. So, this is the A of the shaft we are giving the bearing displacement x b, and here we are giving bearing displacement x b which is at other end. Now because of this disk is here, so what is the displacement of the disk we need to find out also, what is the tilting of the disk because of these 2 displacements the bearing displacements.

So, you can able to see x we can able to write it as x a and the difference between the 2 displacements divided by l that will be the basically slope of this line l into a because a is the distance of the disk from left end. So, this will give us a. So, that is the value of this the slope of this into this distance. So, that is giving us the total displacement at the disk location due to the bearing displacements, this can be written we can able to rearrange in terms of the bearing displacement at plant a and plant b.

So, this is the translatory displacement of the disk due to bearing displacements. Similarly, the rotational displacement will be given as the slope of the line sorry this one is corresponding to this angle phi y. So, this is the slope of the line, so the disk will tilt by this amount in this particular plane, on the same line in the other plane that is y z plane we can able to give that the bearing displacement y b of the A end of the bearing shaft and B end of the shaft.

So, these 2 displacements we are giving at the end of the shaft, we are calculating the y what is the displacement, translatory displacement at the disk location also the tilting. So, this 2 we can able to see here y we are expressing as y b minus the slope this is the slope and the disk the location of the disk from the, this end. And this can be arrange again in terms of the bearing displacement at end a and at end b and this is the slope of the this particular shaft. So, these 4 relations which are giving the displacement of the disk due to bearing displacement when we are considering the shaft as rigid can be combined in a matrix form.

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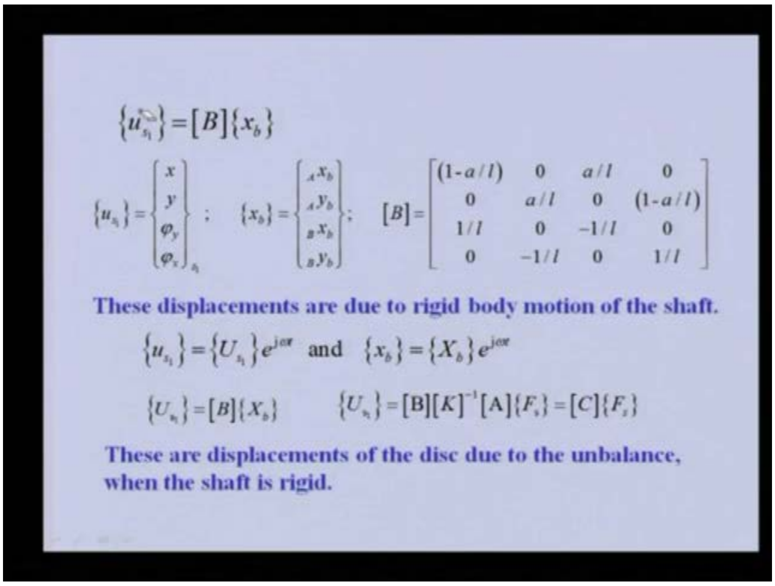

So, shaft displacements, bearing displacements, shaft displacements are x y phi y phi x bearing displacements at bearing a, and b their linear displacement only and this B matrix will contain all the coefficients related with the x b these coefficients.

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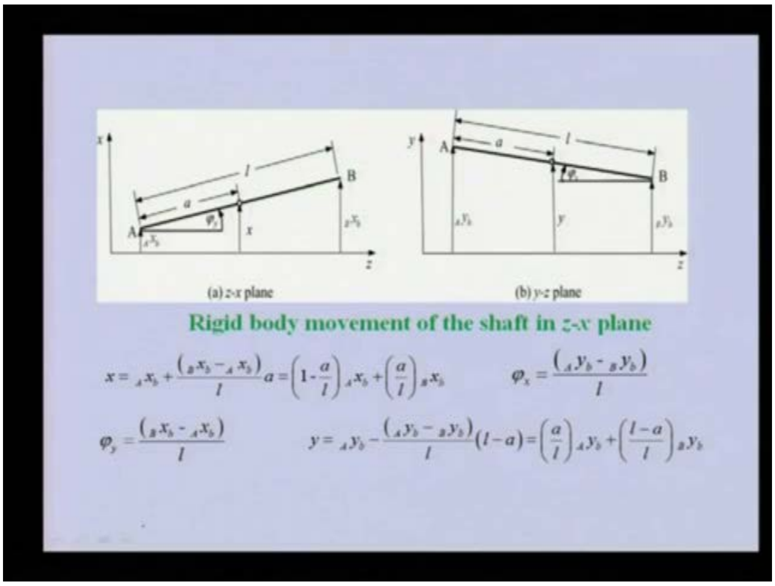

So, in previous equation if you see they are coming from these coefficients.

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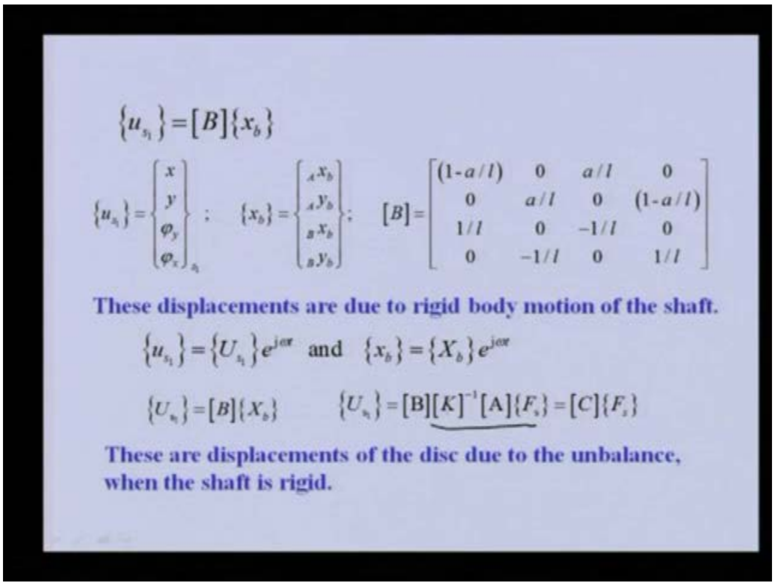

So, this particular displacement of the disk is due to the rigid shaft. Now, we will consider subsequently, flexibility of the shaft also, but before that let us write the this particular displacement in the frequency domain by writing in this particular form in which these displacements will be having harmonic motion same as the spin speed. This is the complex quantity similarly, bearing we have already written earlier like this. So, if you substitute this here you can get the shaft displacement in complex form in terms of the bearing displacement. And earlier we had X b bearing displacement we wrote these terms.

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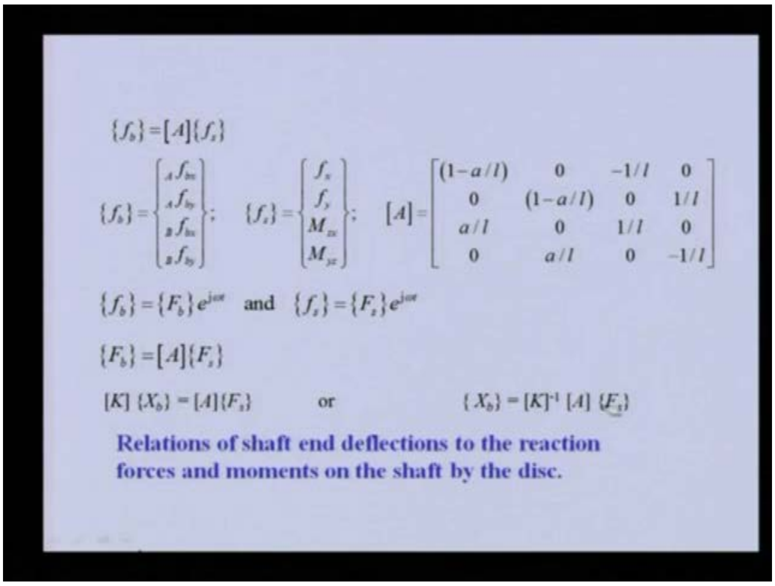

So, that we can able to substitute if we go back you can able to see X b how we defined X b we wrote like this. So, this we are substituting in place of x b and I am defining.

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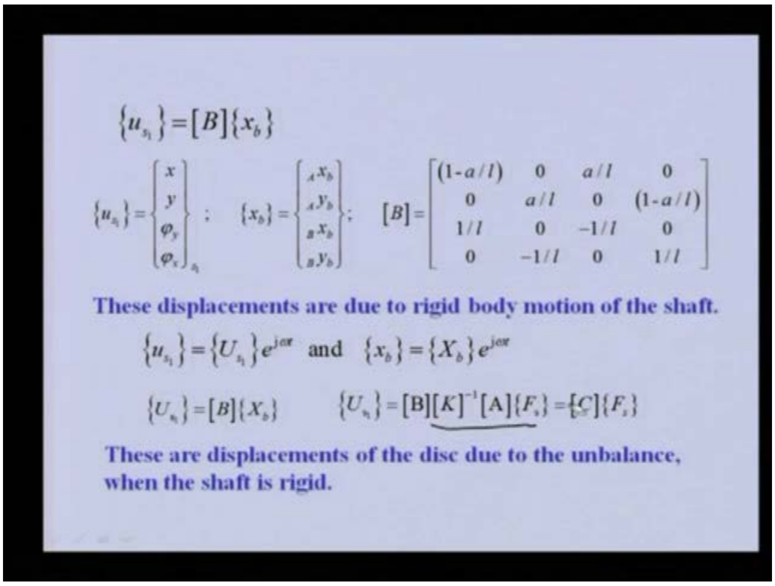

So, B my inverse of K and A I am defining as C now, and this is the shaft reaction force which is coming from the disk. Now, we will go for the flexibility of the shaft.

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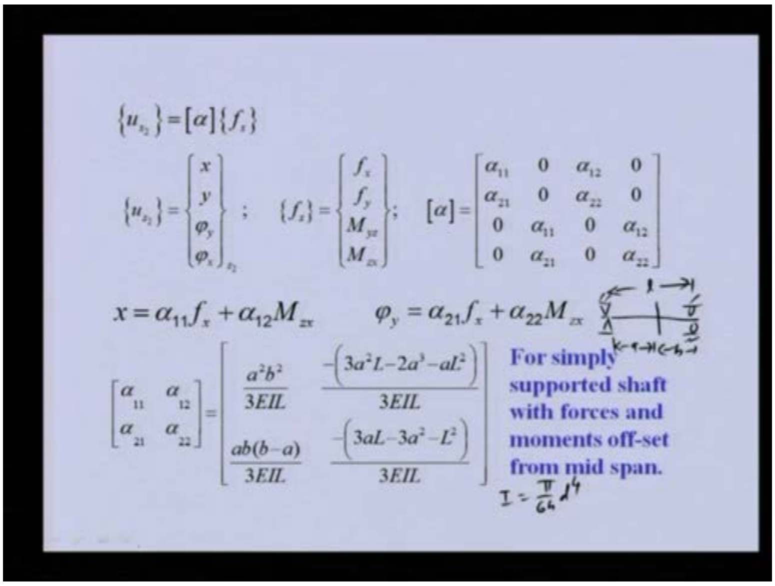

When we are considering the Jeffcott rotor model with offset over disk, we related the displacement of the disk location, with the shaft reactions using the influence coefficient matrix. So, this relation the relationship of the disk location displacement and the reactions at the disk location we already related earlier. So, we are using those equation directly here. So, you can able to see these are the shaft displacement due to, this is corresponding to the when we are considering bearing as rigid and shaft as flexible.

So, these are the reactions at the disk location and these are the influence coefficients which relate these 2 quantity. Now like for example, x displacement can be given as alpha 1 1 into this f x plus alpha 1 2 into M y z because linear displacement is not only obtained using the force but also due to the moment. So, this x displacement is obtained not only due to the force, also due to the moment and similarly, their angular displacement not only it has obtained with the force also due to the moment.

Now, you can able to see that this particular expression which is for simply supported shaft, the influence coefficients are defined in terms of various variable where, if you see in a simply supported shaft the disk is here. So, we have a distance b distance and total length is l. So, in this particular case a and b are defined like this l is the total length E is the young’s modulus I is the areal moment of second moment of area that is pi by 64 d 4 if shaft is there. So, this is the influence coefficient which we can able to use for other boundary condition we can use the different influence coefficients.

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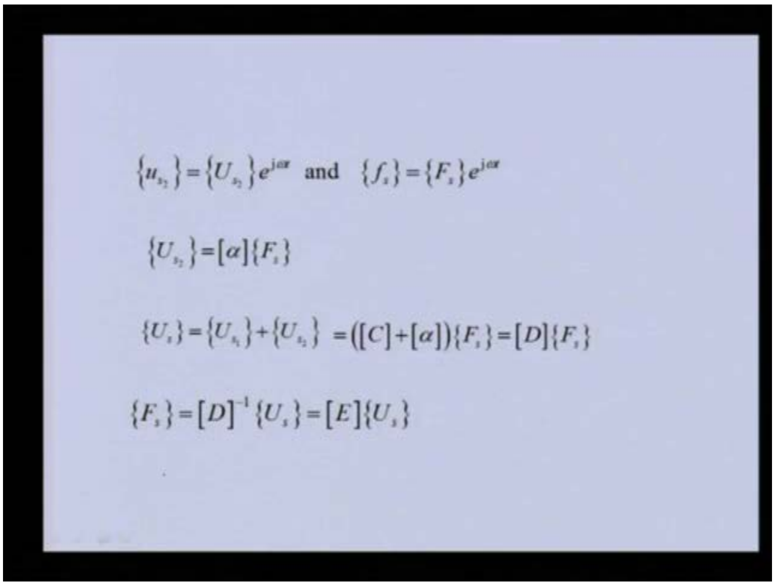

So, now we can able to write this displacement which is due to the this flexibility of the shaft in harmonic terms and complex term the force we already express like this. So, if you substitute this in the previous equation you will get the shaft flexibility displacement at the disk with the reactions at the disk. You can able to rewrite this is the previous equation only this equation.

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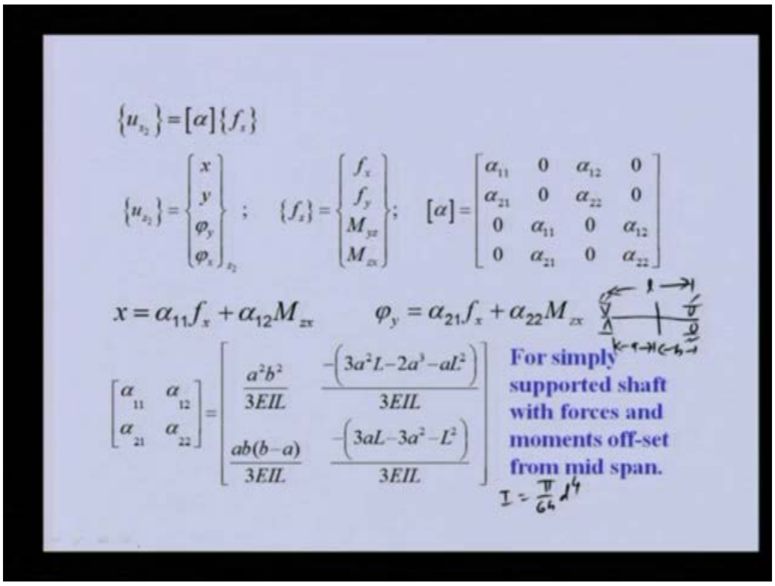

So, now we are writing in the frequency domain.

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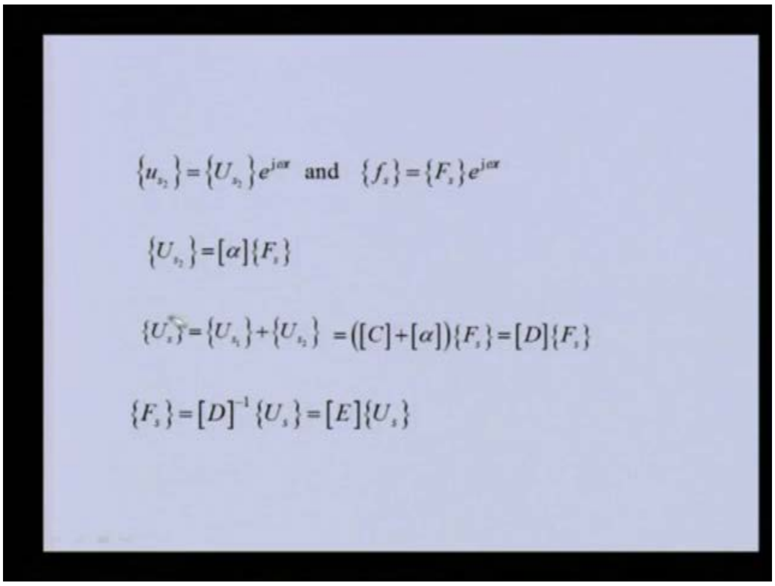

Now, these two displacements, which we obtained for bearing as flexible and shaft as rigid and this is bearing is rigid and shaft is flexible can be added and if you see earlier we wrote this as C into F s. And now, U s two we have written as alpha into F s. So, they can be written here, and where now, I am writing C plus alpha matrix as capital D matrix and F s. So, this particular equation is now, relating the shaft reactions or disk with the disk displacements at the disk location and if you want this reactions we can able to note this matrix to get this reactions. And let us say D inverse I am writing as capital E and these are the displacements at disk location. So, till now, we have considered the equilibrium of the bearing, equilibrium of the shaft now, we will consider the equilibrium of the disk to get the relationship between the unbalance force and the displacements at the disk location.

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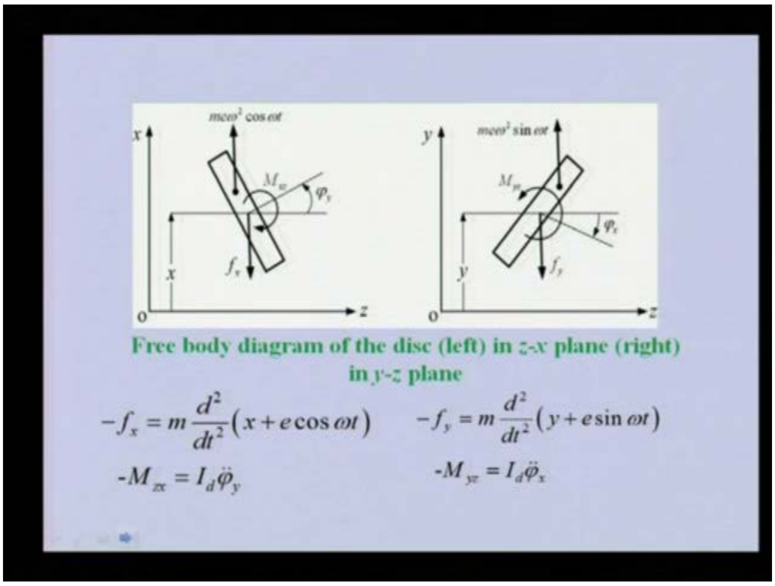

This is the equilibrium of the disk in y z plane z x plane and y z plane these, equilibrium we already seen, when we were analyzing the Jeffcott rotor with offset disk. So, this expression the free body diagram is similar to that one. So, here we have reaction force from the shaft as f x this is moment from the shaft, this is the centrifugal force. So, you can able to see there is a linear the translatory motion and a tilting motion. So, we can able to write the equation of motion these 2 plane like this which we already done earlier. So, you can able to see this is the force balance in x direction, this is force balance in y direction, this is moment balance is z x plane, this is moment balance in y z plane. So, we have 4 equations they can be combined in a matrix form.

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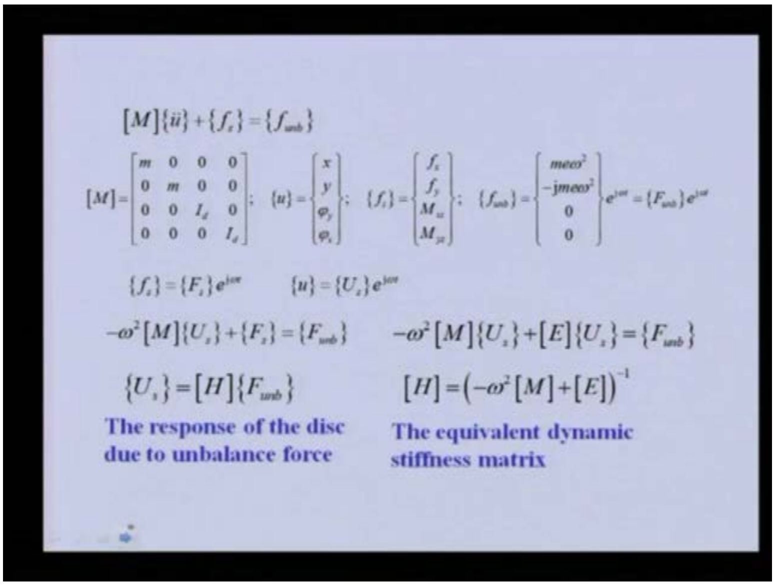

Like this, where this is the mass matrices define like this the u is a displacement vector you can able to see the you like this. Now, this is the shaft reaction force which is defined by the force and moments and this is the unbalance force. Now, you can able to define again the forces on displacement in terms of the amplitude phase and harmonic component like this. So, if you substitute this in this equation of motion, we will get this equation of motion has minus omega square and U s this 1 and F s which is coming from the shaft reactions equal to the unbalance force. And you can able to see that now; we can able to write the F s in terms of the U s from the previous expression.

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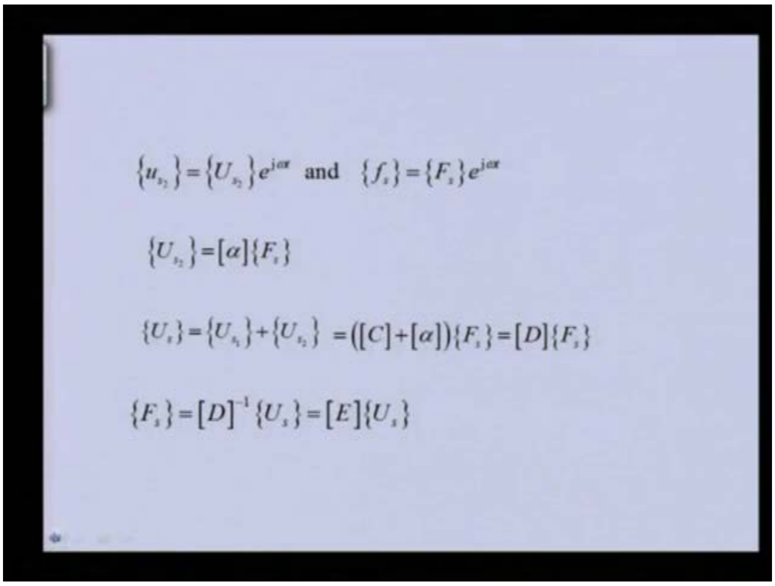

So, we define the F s shaft reaction force in terms of the U s with E matrix. So, that we can able to substitute there.

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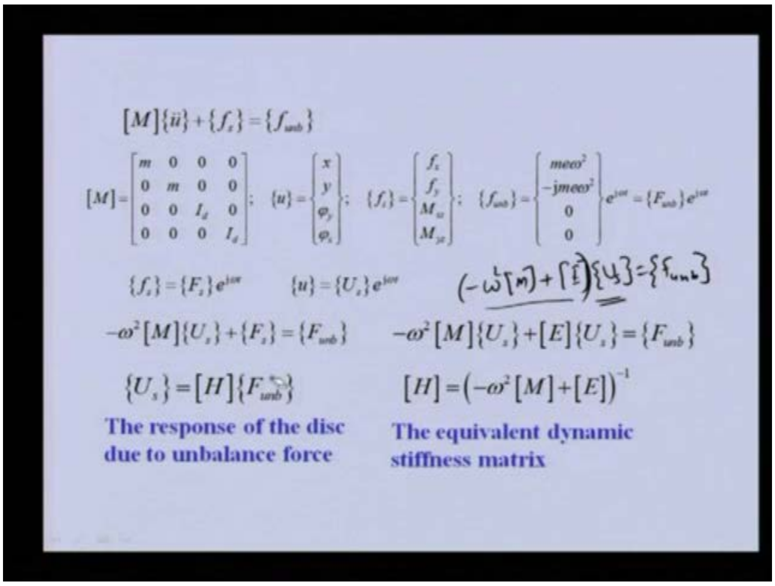

So, I am substituting for F s E into U s, now you can able to see U vector is here also here also we take that as common. And so basically we can able to write this as minus omega m plus E U, we can take as common U s is equal to F m balance and to get the U s you need to invert this matrix that we have done here, H is the inverse of this matrix you can able to see. So, U s is the displacement at disk location is the unbalance force and there is the equation we need because this is a unbalance force which is known and what is the displacement of the disk.

So, if you see basically H is giving some kind of effective stiffness, effective influence coefficient of the rotor; that means, not only from the shaft, but also from the bearing. So, we can able to get the unbalance response using this relationship. So, we have seen that when we are including the flexibility of the bearing, then we need to basically obtain the effective the either in reference coefficient or the stiffness which particular disk is experiencing not only from the shaft, but also from the bearing.

And in some cases not only the bearing, but the bearing the pedestals also flexible so; that means, bearings are mounted on some kind of columns and they are also flexible in that is the case then will be having a third level of flexibility from the pedestals. So, if we want to analyze that; obviously, we need to find the effective stiffness are the influence coefficient from not only from the shaft from bearing as well as the pedestal. So, let us see that particular analysis. In which we have three level of flexibility.

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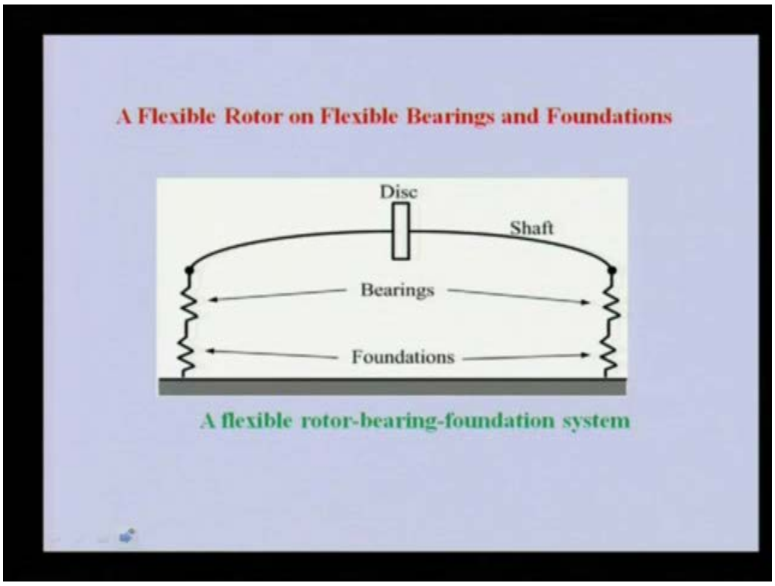

So, here you can able to see that we have disc, shaft is flexible bearing is there and then the pedestal or foundation is there. So, we have basically we need to obtain the effective stiffness, how this particular disc is experiencing at the bottom of due to the shaft bearing and foundation.

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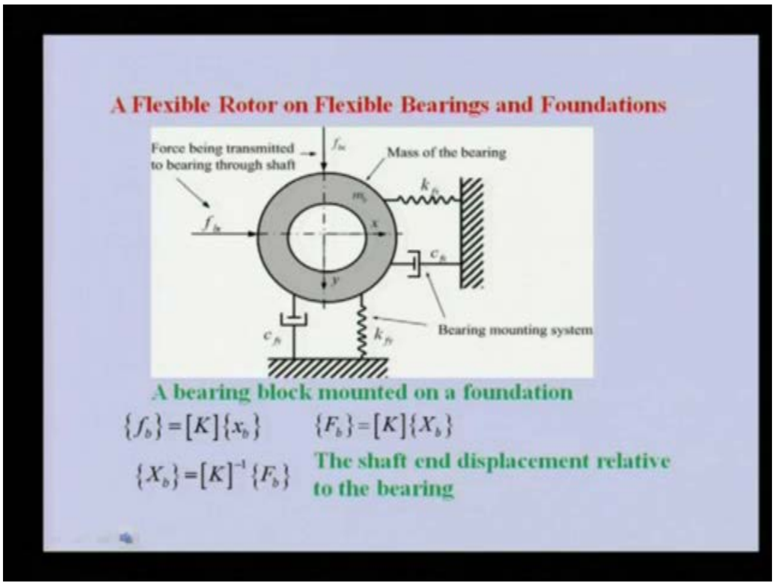

So, I am coming to the free body diagram of the let us see the bearing. So, this is the bearing mass in the previous analysis we did not consider the bearing mass now, this is the bearing and general we have removed from the bearing. So, obviously, the fluid fill forces which is coming what are the bearing mass is f b x in x direction f b y in the y direction these are the fluid fill forces which are coming on to the bearing mass.

These are the foundation stiffness and damping in x direction and in y direction in this particular case we are not consider the cross coupling comes in this for simplicity. Now, if we see, we can able to relate the earlier related the bearing force, with the bearing displacements, with the help of K matrix and if we convert this in the frequency domain, we will be having this expression in which time dependent c terms are not there and the bearing displacement we can able to get by inverting the K matrix and multiply with the bearing force in the frequency domain.

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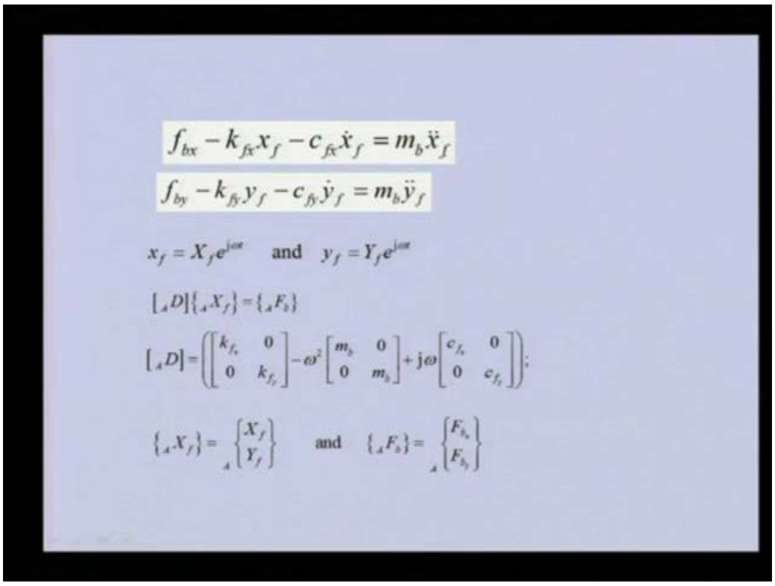

Now, we are writing the equilibrium of the bearing mass, on the previous figure. So, this is the fluid film force, these are the elastic force from the foundation in x direction, this is the damping force from the foundation, this should be equal to inertia of the bearing.

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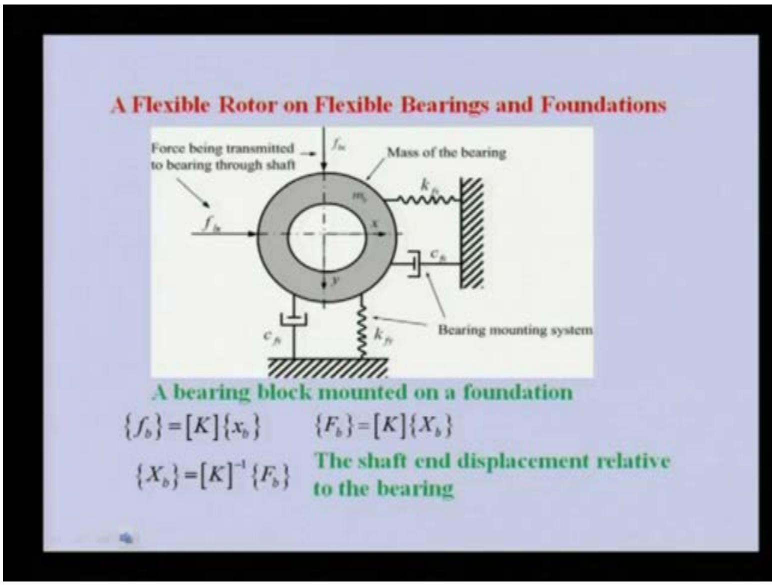

In the previous figure we have seen that the bearing mass is having in x and y direction.

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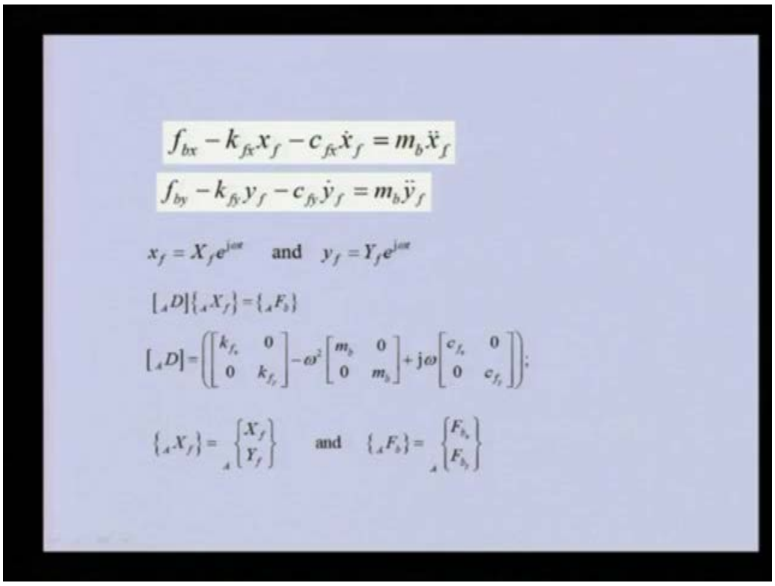

X f and y f displacements. In this particular case the bearing if you are considering the bearing forces, bearing forces are coming from the displacement that is the relative displacement between the bearings in the shaft. So, that is generally is inside the bearing. So, how much relative displacement between, the bearing mass and the shaft is taking place that is the displacement which will be governing the bearing force. Now, we are considering the bearing mass itself is having some displacement and that is related with the foundation force in by this expression.

So, these are the inertia of the bearing mass. Now, with again in the we can able to write the displacements of the bearing mass in terms of the amplitude and phase at harmonic components, we can able to convert these equations in the frequency domain and they, can be clubbed like this. So, you can able to see, this is the bearing mass displacement D matrix which will contain these coefficients and even the mass is define like this.

And this is the component of the bearing mass, these are the components and similarly, the force which is bearing forces coming from the fluid film on to the bearing mass. So, this particular expression is relating the bearing mass displacement with this particular bearing fluid film bearing forces which is coming on to the mass, bearing mass. We can able to this previous equation again I will be putting this one.

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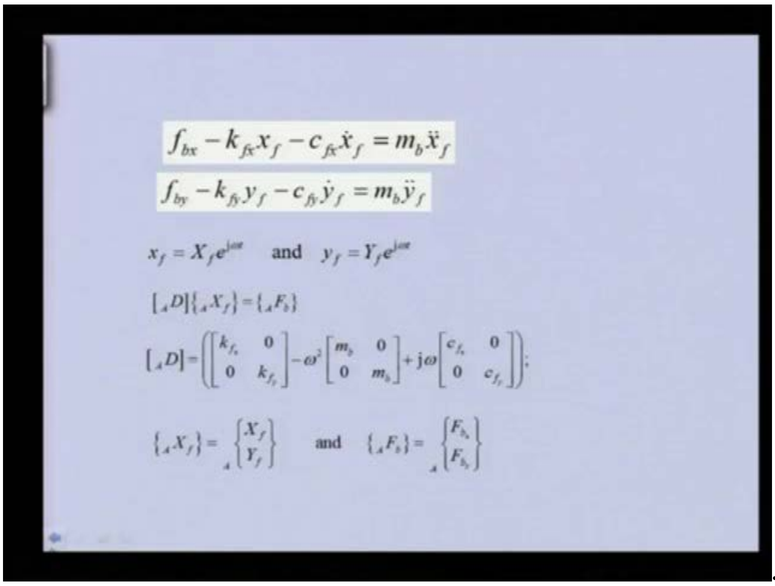

Now, this particular expression we have written for one and up the pedestal, we can able to write similar expression for the other pedestal that is b side or the right side.

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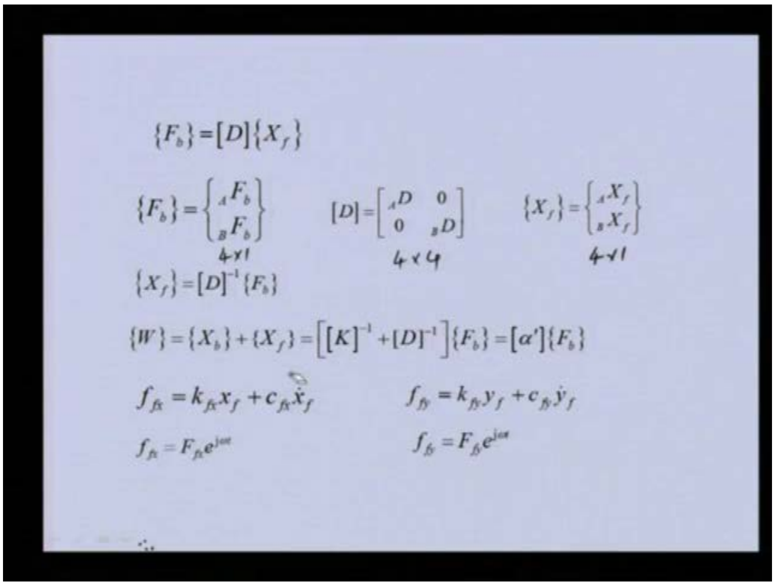

And now, I am combining this bearing forces and pedestal displacement of 2 bearings. So, this D matrix contain now, not only for the bearing on A also for bearing on B and similarly, these are the displacements in x and y direction and bearing A and bearing on B. Now, in the size of these matrices are basically 4 into 1 4 into 4. So, this is 4 into 1 and this equation you can able to write X f by inverting this D matrix in other side in and multiplying by the baring force to get this relation.

Now, this particular displacement is of the pedestal, earlier we obtain the bearing displacement. So, we can able to add that bearing displacement with this pedestal displacement. So, we will get the total displacement not only between because of the pedestal also due to the bearing and earlier we related this as K inverse F b and here we related X f s D inverse of F b, so that we can able to substitute. So, basically you can able to see.

Now, this is a effective influence coefficient, which we obtained and this can be used in the previous analysis instead of the bearing only now, this particular influence coefficient not only content the information of the flexibility of the bearing as well as the foundation. So, now, we can able to consider the shaft flexibility and then this can be added. So, from here onwards the analysis will be identical as the previous one this particular expression is representing now, both the flexibility of the foundation or the pedestal as well as the bearing.

So, this equation can be club with the previous equations to get the final unbalance response of the disk. So, in this particular analysis where we are considering the pedestal or foundation flexibility as well as the bearing flexibility basically, we obtain what is the effective flexibility they are in parting and then we can able to at that with the shaft flexibility, as we did in the previous analysis. Now, we will see how much pedestals are getting forces because of the unbalance.

So, the pedestal force expression will be given by, this elastic force and the inertia force. So, what are the force which is coming from the bearing on to the foundation, some force will be used to are the inertia of the bearing mass. So, that will not come here in this expressions. So, these forces will be smaller than the fluid film forces which is coming on to the baring mass. So, this can be written in the frequency domain.

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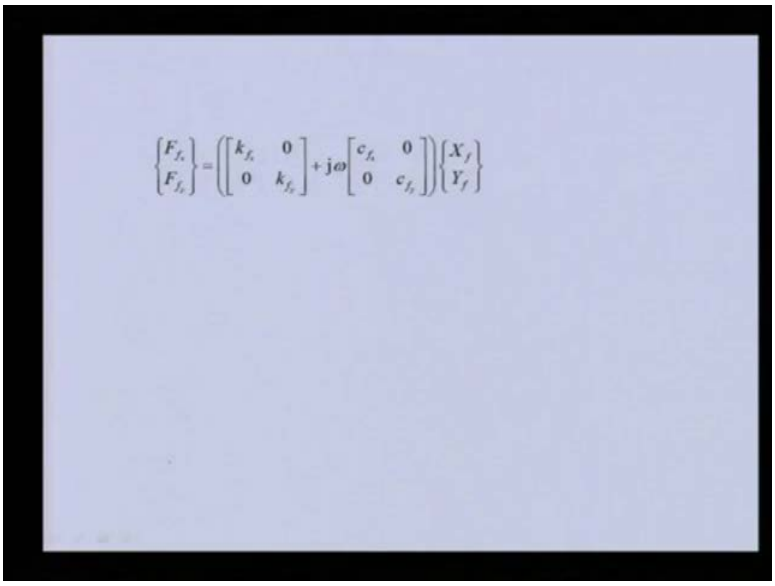

$$\begin{Bmatrix} F_{f_x} \\ F_{f_y} \end{Bmatrix} = \left(\begin{bmatrix} k_{f_x} & 0 \\ 0 & k_{f_y} \end{bmatrix} + j\omega \begin{bmatrix} c_{f_x} & 0 \\ 0 & c_{f_y} \end{bmatrix} \right) \begin{Bmatrix} X_f \\ Y_f \end{Bmatrix}$$

Like this. So, this is the foundation forces which are coming on to the bearing through one numeric on to the foundation.

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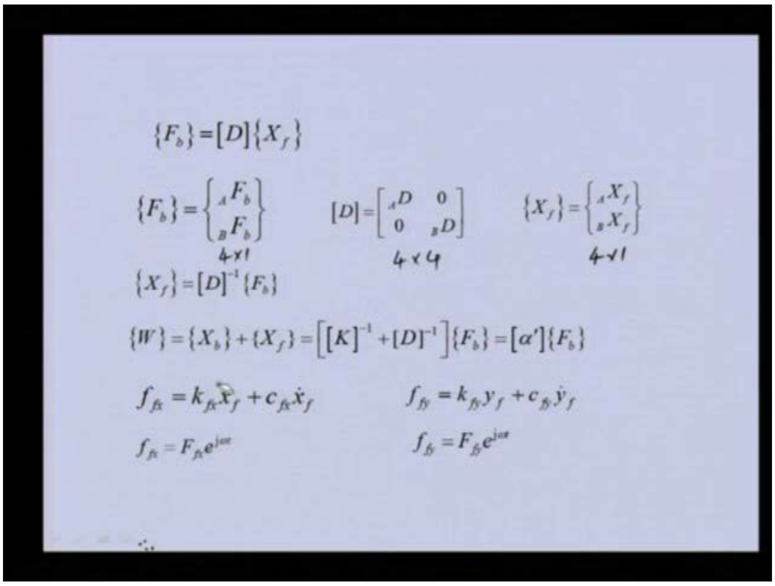

Some force will be used towards the inertia of the bearing mass. So, that will not come here these expressions. So, these forces will be smaller than the fluid film forces which is coming on to the bearing mass. So, this can be written in the frequency domain.

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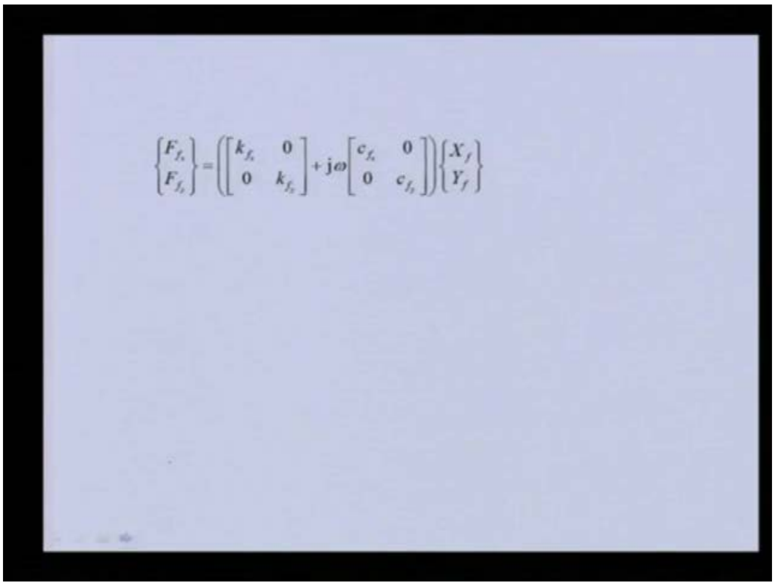

Like this. So, this is the foundation forces which are coming on to the bearing through one numerical example, will try to see how the flexibility of the shaft and bearing if it is there it can be analyze to get the unbalance response of a this disk. So, in this particular case.

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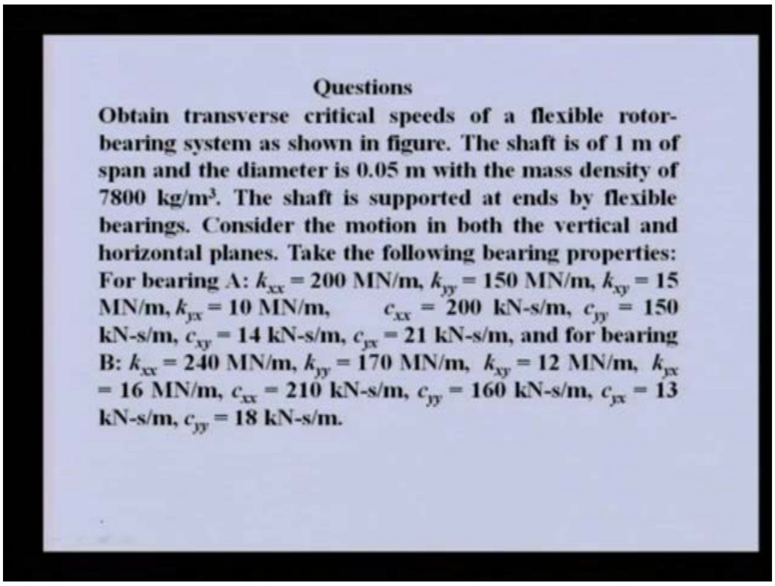

This particular example is similar to what we did in the previous lecture. Now, we are considering the flexibility of the shaft also earlier, we consider that as rigid. So, data are as it the same as the previous lecture numerical example for bearings, as well as for shaft. But, now we are considering the flexibility of the shaft also.

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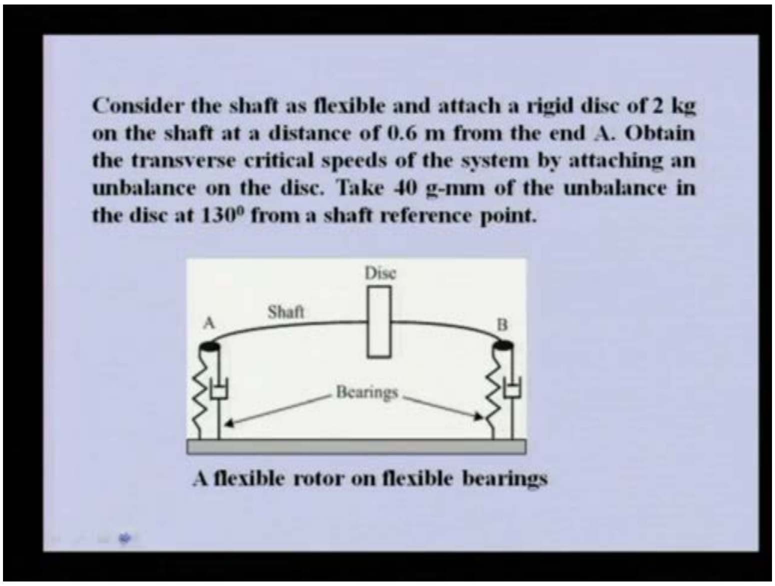

So, our model is like this, in which not only the flexibility of the bearing is there, but also the flexibility of the shaft is also there. Now, we want the unbalance response of this basically, we are interested in finding the critical speed of the system. And in this particular case the disk is not at the midpoint is because total length is 1 meter. So, is from one end A and A is 0.6 meter and we are having some unbalance response unbalance in into the disk, at certain angle is a reference point. So, now let us write various matrices.

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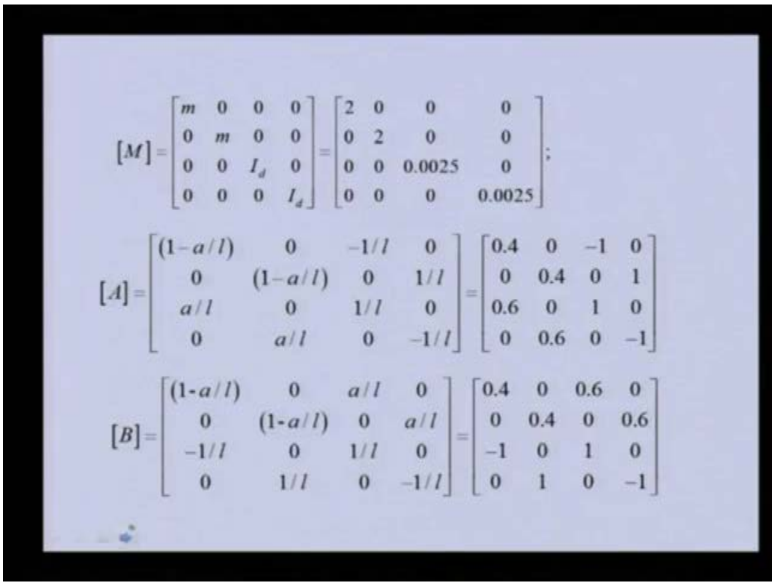

So, mass matrix for this particular which we define earlier will be this, various matrices which we defined in this analysis like A matrix, B matrices which we relating let us go back and see what are these matrices just for recapitalization.

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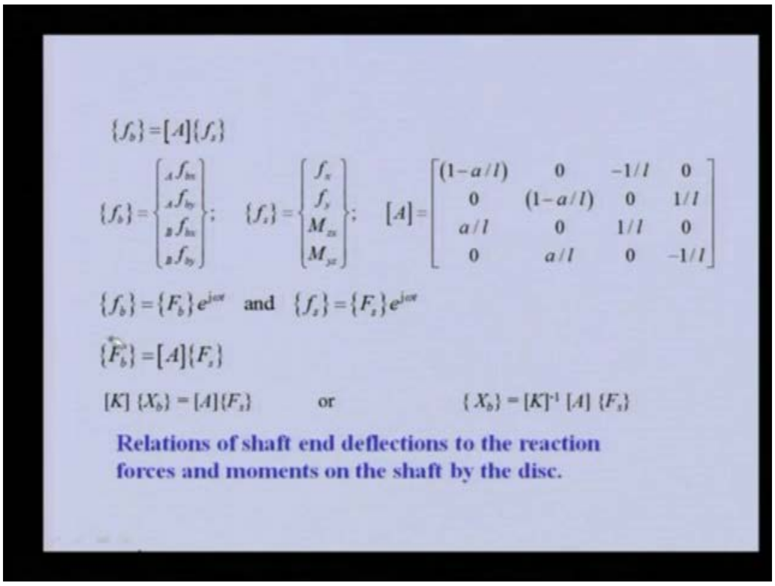

Like A matrix it was bearing force and shaft reaction force we related this be we related.

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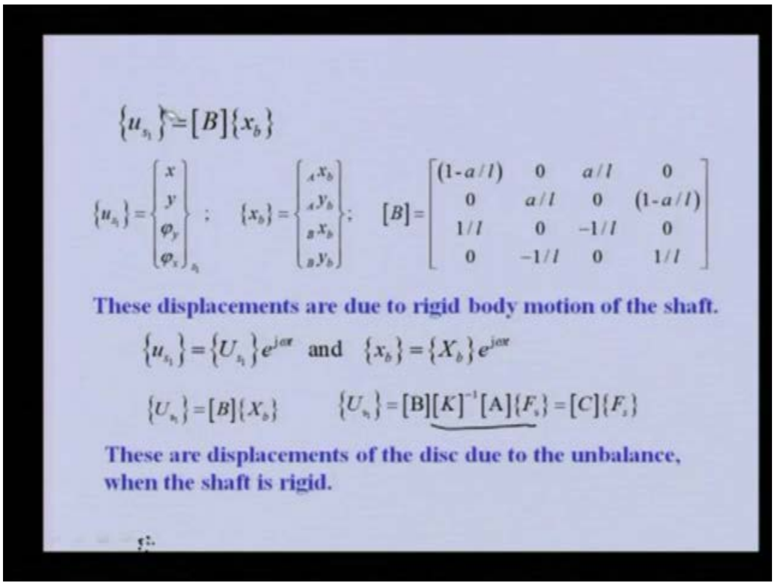

The when we are considering the shaft as rigid, this is B matrix.

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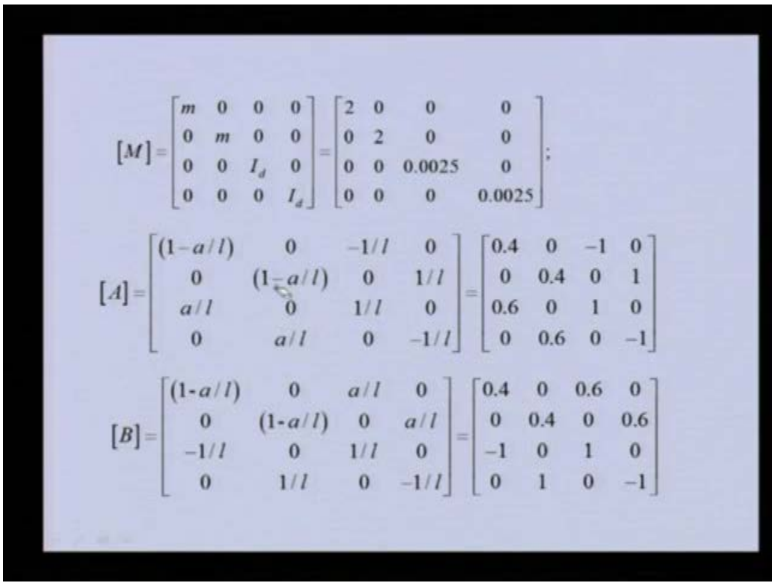

So, these are the A matrix and B matrix for your numerical problem.

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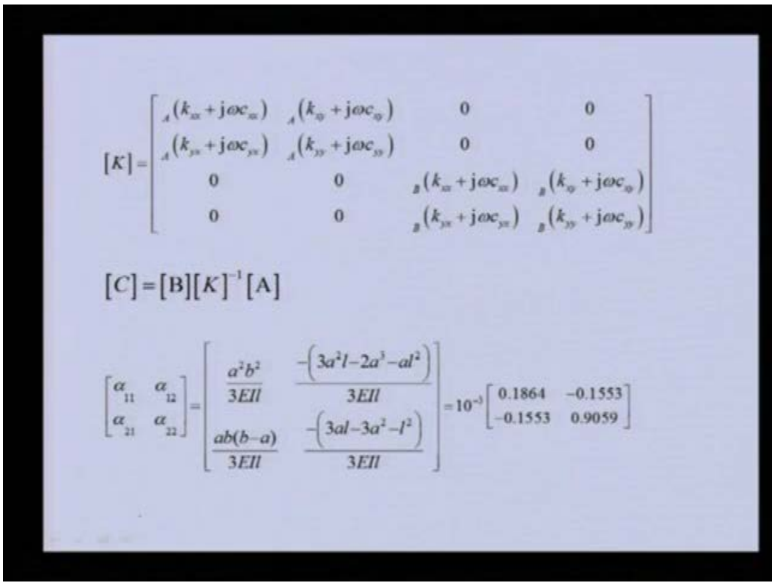

K matrix is like this and we can able to substitute the value of this also C matrix which we in turn defined as B a into K inverse of K into A where, and this is the influence coefficients matrix. We are considering simply supported, this is for simply supported and un condition for given value of this parameter can be written like this.

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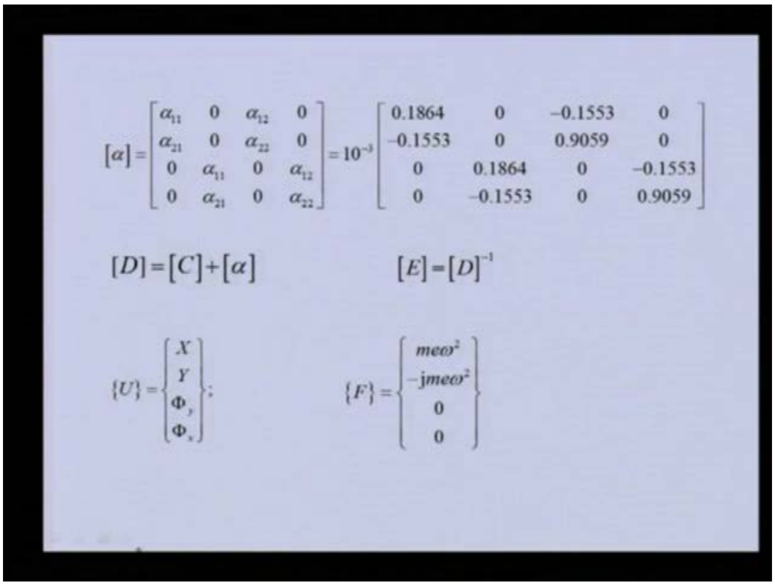

Now, the D matrix is C plus alpha. So, various numerical values of these matrices we already obtain to get subsequent matrices D matrix, E matrix is inverse of this they can be calculated, this is the stacking of the displacement vector this is the force, because, now it is a disk. So, there is no moment will come from the unbalance.

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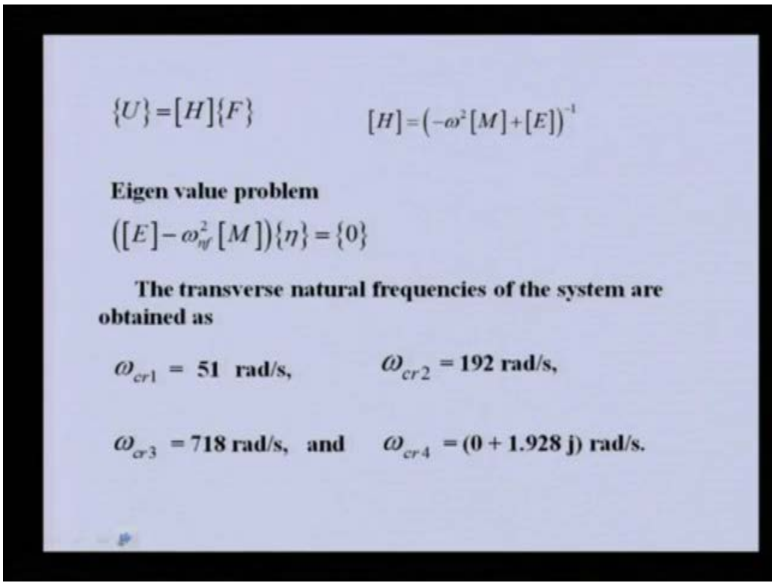

And this is was the final unbalance response from the unbalance force, and this was the total effective support flexibility which was define like this. So, you can able to see that if we the Eigen value problem if we take the inverse of this will be this into u; that means, this has to come this side and this particular response and because free vibrations. So, we are keeping the force 0. So, the Eigen value problem of this, if we take the determine and solve for omega n f will get these as critically speed.

In this particular case, we are finding one of the particular speed as not feasible; that means, the system is going into the honest able reason. So, in a 3 feasible critical speed we are obtaining in this particular, this is due to the bearing property which we are chosen. So, the previous equation we can able to simulate for various values of omega for various spin speed. So, will get the various unbalance responses you plotted these.

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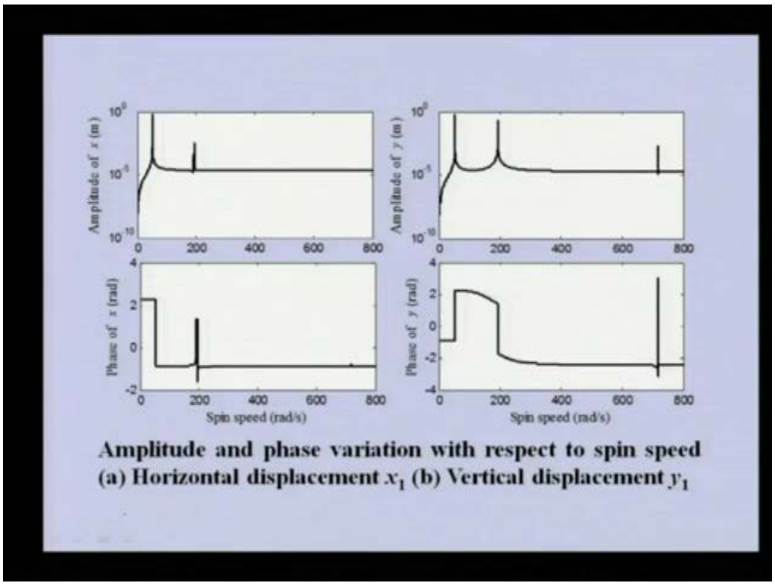

So, you can able to see this is the response in x direction and the phase of that. So, there are 2 critical speeds we can able to see in this because linear motions. So, here we have getting 2 critical speed third is also there these not seen here, but in y direction where we can able to see the third critically speed also, and there is phase change where there is these critical speeds are there here also, three critical speed. The fourth one was not visible for values we are chosen.

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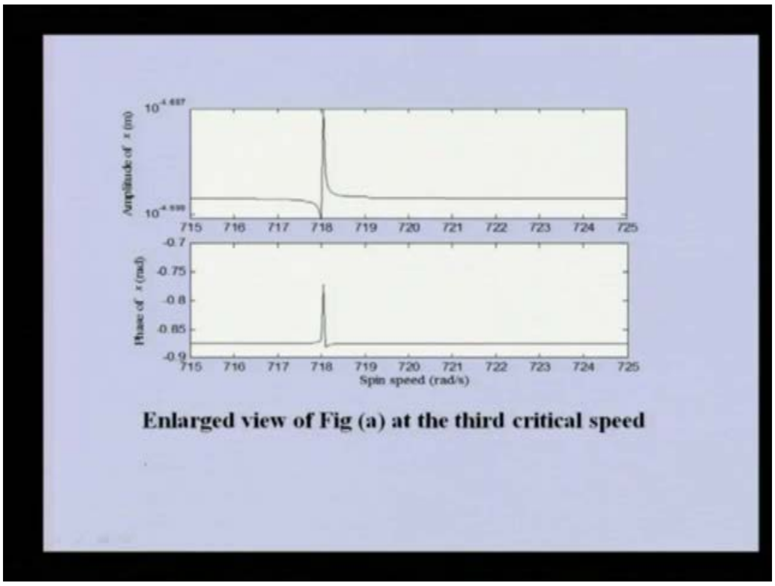

At this location we try to zoom the plot and we found, the displacement and the phase corresponding like this. So, but in this particular case, we could able to get only the three critical speeds, fourth was not feasible. Present lecture we have seen the feasibility of the disk in the bearing and the foundation also, how it can be analyzed the method which we used was very basic method we were doing the free body diagram of each and every component and relating the equations to get the final response of the system in terms of the unbalance force.

The procedure which we followed is having difficulty that when the system is having more and more components. Because, in the present case we have consider on the single either long rotor or single disk, but in general in actual practice. We will find that there will be large number of masses which will be there; that means, large number of disks will be there or the shafts will be coupled, two shafts will be coupled with coupling multiple bearings supports will be there.

So, complexity with the present analysis method will be more, it will be very difficult to obtain the equation of motion of and we get system in which let us say if we want to analyze, a turbine and generator model which is having multiple bearings. Let us say four bearings for whole system then in between there, is coupling for such system will be having, more difficulty in solving the unbalance response using the present method.

So, here we feel that there should be some methods, which should give us the analysis of either the finding the critical speed or finding the unbalance response that should be more systematic. So, that there is did not be derived the equation of motion each time for a particular system that should be more systematic. So, that once we write the govern equation for one particular sub system that we should able to use it for larger system.

And this requires based on this researches they develop various methods in analyzing such systems and two sub systems will be see in future where, is the transfer matrix methods, another is the finite element method which are more systematical especially they can be program and even a biggest system like 100 degree of freedom system or more than that, can be analyze easily. In the subsequent lecture will not go in to such complexity, but again I will try to introduce some more complexity in the single mass rotor system.

Especially the gyroscopic effect, I will analyze for a simple rotor model and then we will go for the more complex multi degree of freedom rotor system analysis.

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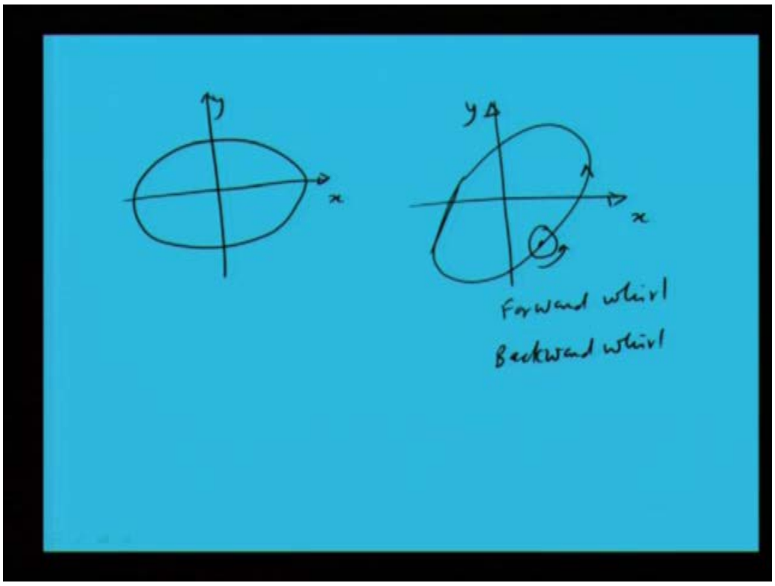

So, once we obtain the unbalance response, even we could able to plot the orbit of this particular shaft. So, once we obtain. The unbalance response for a particular speed you can able to plot the orbit of the shaft also. In this if you remember previous case when we have the springs, we had the orbit something, like this ellipse with and the principle axis of ellipse was aligned with the x and y axis. Now, in the present analysis we have consider the coupling of the stiffness and because of the orientation of the ellipse will be not aligning with the main x and y axis.

But it will be tilted among depending up on the flexibility of the cross couple terms. So, now, in this particular case also we will be having similar phenomena like, if we are having rotor, center which is rotating clock wise or counter clock wise the whirling will be same direction for forward whirl. But, as will cross the critical speeds this whirling directions will change speed will remain same, but this will change, that will be calling as backward whirl. So, wherever there is a crossing of the speed that change in the whirl it would take place.