Theory and Practice of Rotor Dynamics
Prof. Rajiv Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module – 3
Rotor Mounted on Bearings
Lecture - 8
Rigid Rotor Mounted on Complex Anisotropic Bearings

In the previous lecture we studied a rigid rotor, which was mounted at ends by springs, and the spring property in two orthogonal directions were different. Such model is valid for the bearings in which rolling elements bearings are there. And generally it imparts the stiffness damping is relatively low and the stiffness property in two directions may be different. Now, if we are considering the hydrodynamic bearing. Then these property not only changes the stiffness in two directions in vertical and horizontal direction, but also it imparts damping in the two orthogonal direction. Apart from this, there is a cross coupled stiffness and damping in the such kind of bearing.

So, what are these we see in the present lecture, how this stiffness and damping can be derived either by a theoretical model or through experiment, that basic outline we will study. And then we will study, when a particular rigid rotor is mounted on such complex bearings how it is behaviour take place.

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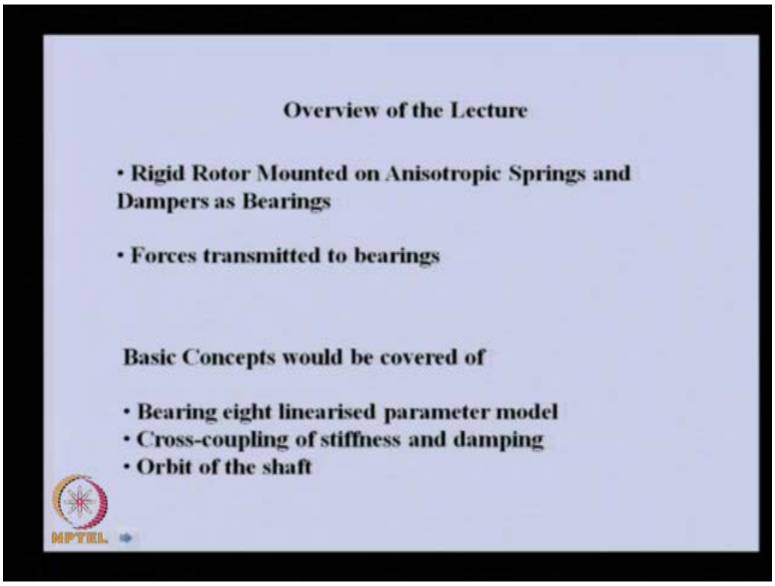

This is the overview of the lecture, this is rigid rotor mounted on anisotropic spring and damper as bearings. Then even we will study the force transmitted through the bearing from the rotor. And in this we will be having the following concept to be covered in this particular lecture, that is a bearing it linearize parameter model, cross coupling stiffness and damping and orbit of the shaft.

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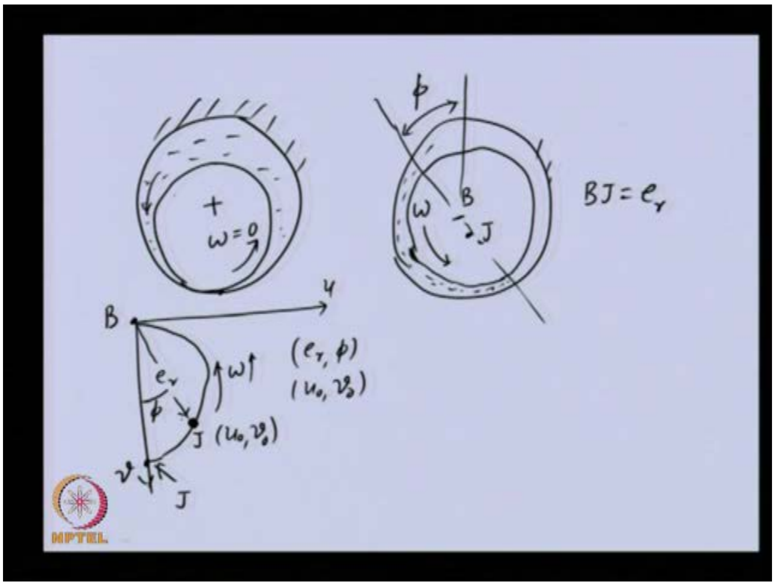

So, let us take a hydrodynamic bearing and this particular bearing. This is the bearing centre when the journal which is inside, which is in the bearing, when it is not rotating. Then we expect it will be resting at the bottom of the this bearing, where omega is 0 for this particular case. But when the this particular speed increases, now you can able to see whatever the fluid which is here, which is in the parents. It will try to go into this particular wage and because of this the pressure will be generated at the wage and this particular journal will lift on inside the bearing.

So, I am drawing a particular configuration in which the journal is rotating at particular speed and it has taken some equilibrium position inside the bearing. So, now, you can able to see that because of rotation of the journal. The fluids are getting pumped in this wage part and because of that the pressure is generated in this region and the weight of the journal is supported by the pressure generated at the fluid frame.

In this particular let us say, this is the bearing centre and this is the journal centre and bearing centre are offset by some amount. Let us say B J is the radial eccentricity and if, we draw a vertical from the bearing centre. This particular angle is called gratitude angle, which defines the position of the journal.

So, if we want to plot the journal motion how it changes it is location with respect to speed. Because, for a particular speed it occupies certain position inside the bearing and so you can able to see that at rest that is here it is the journal centre it is the beating centre. Now as speed increases it changes it is position and theoretically when speed is at infinity it reaches to the b. That means, both the rotor and journal centre journal and the bearing centre will be at the same place.

So, we can able to say when speed is increasing the shaft centre takes a particular path and for a one particular speed it will occupy. This particular position and this is the radial eccentricity, which we have drawn here the B J is the journal position. Let us say at equilibrium for a particular seed it is position is given by u not and v not. This is the altitude angle.

So, e r and phi define the position of the journal or even we can able to define with respect to v not and u not, now you can able to see that, when we are changing the speed this particular journal changing it position. And if we talk about how this the fluid fill is imparting the stiffness and damping to the rotor. So, obviously we expect that when we are changing the speed, when the journal position is changing. We except it will be having different stiffness and different damping property. So, now let us analyze those here.

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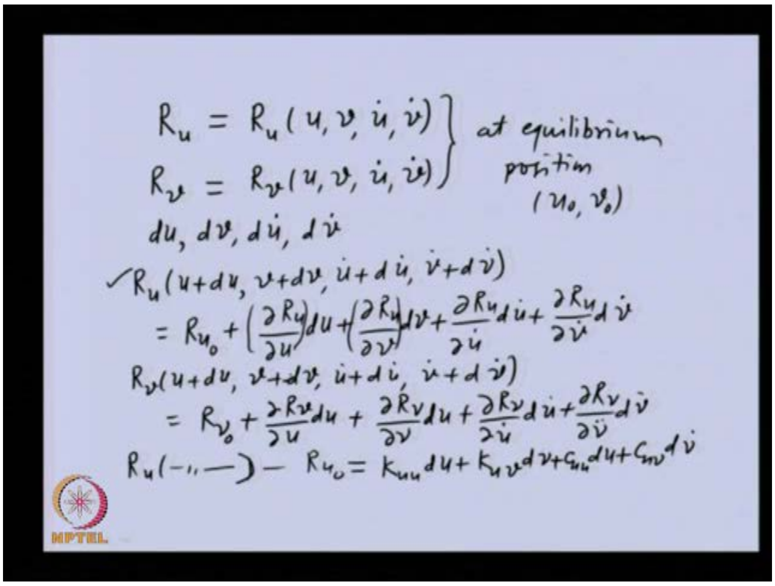

The fluid fill which is giving a reaction force on to the general is R u in the direction of horizontal direction and this reaction force is function of the position of the journal and the instantaneous velocity also. The fluid force in the vertical direction also is function of this quantity. So, these are the reaction forces, which is coming from the fluid fill on to the journal.

Now this is at equilibrium position let us see that means, it is at let us say u not and v not. If we are giving a some kind of perturbation or disturbance from it is equilibrium, then let us say we are giving perturbation to each of this by d u d v and it is velocity components to each of these variables.

So, the fluid film force now will be written as, so this fluid filling force we except that it will be changing. Because of this disturbance like this similarly, now this particular is in u direction similar variation will be there in the v direction also, this particular function we can able to expand using Taylor’s series expansion.

So, let us say the reaction force in u direction due to the at equilibrium position plus the variations components. So, this is the variation in the d u, then variation in the d v and the velocity components. These are the velocity components or similar variation we can able to express in v direction. So, this will be v plus d v, I am again repeating this. So, similar variation will be there in the v direction force velocity components.

So, this will be equal to force in vertical direction at equilibrium and variations, this we are doing with the help of Taylor's series expansion. These are the velocity components changing the force due to the velocity components in horizontal and vertical direction. Now we can able to see that this particular force is the force due to the initial equilibrium position and the disturbance. Now the change in the force in the at u direction and v direction, we can able to get by writing this particular u, which is function of all those functions minus at equilibrium.

So, this can be written as this particular expression that, I am writing as let us say K u u into d u, then this expression I am writing as a coefficient K u v d v. The first subscript, I am representing to the numerator and the second subscript to the denominator. So, numerator is having force and denominator is having displacement. So, this is representing the force direction and this is representing the displacement direction similarly, these are corresponding to the velocity.

So, I am writing this as C u u as a subscript d u plus C u v and d v dot. So, here also the first subscript is in the force direction u direction and second is in the displacement direction that is the velocity direction. So, the change in the fluid forces I am writing like this. Similarly, we can able to write in the other direction that is in the v direction.

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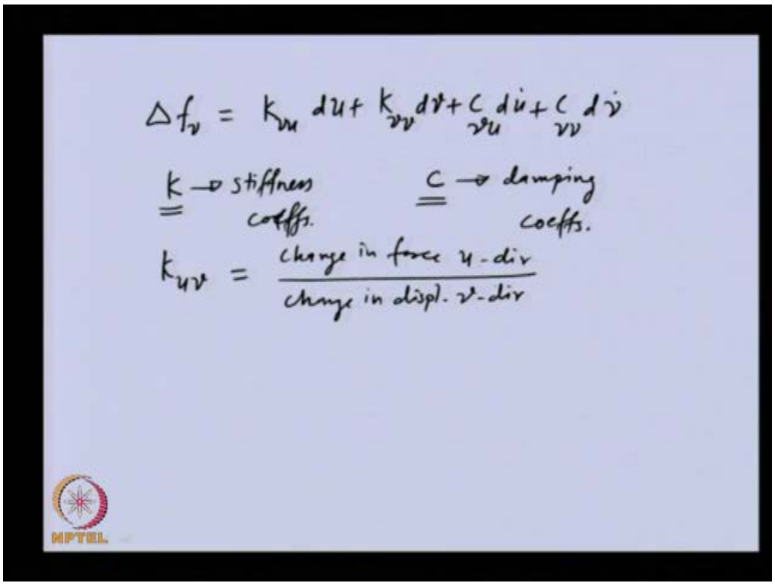

So, the change in the force in v direction will be K v v d u plus K sorry, this will be u v and d u then it will v v d v plus C v u d u dot plus C v v d v dot. So, you can able to see that, this K and C, which we are represented. This is representing the change in the force for a small perturbation in the displacement and this is representing the change in the force due to the change in the velocity.

So, if I actually this is nothing but the stiffness which is stiffness is defined as change in the force with respect to the displacement and this is the damping coefficient, which is defined as change in the force due to the velocity. So, these are stiffness coefficients and damping coefficients. So, if we take one particular coefficient let us say u v. So, this is representing change in force fluid filling force in u direction divided by change in displacement in the same direction, that is sorry in the v direction because I have taken the different subscript v direction, so this particular stiffness.

So, this particular stiffness is K u v, I have defined in which we are taking the force direction u, the change in the force in u and the displacement in the v direction. So, this is a cross coupled term, so in this particular case if we are considering a rotor, which is supported on such bearing hydrodynamic bearing, if you are giving a force in u direction that is horizontal direction.

There will be displacement in the horizontal vertical direction also so that means, if you are giving force in horizontal direction displacement will take place in that direction as well as in the vertical direction and when it is taking the displacement is taking in the other orthogonal direction. Then those terms are called as cross coupled terms and those terms, which are giving the displacement in the same plane they are called director terms, so in this particular case.

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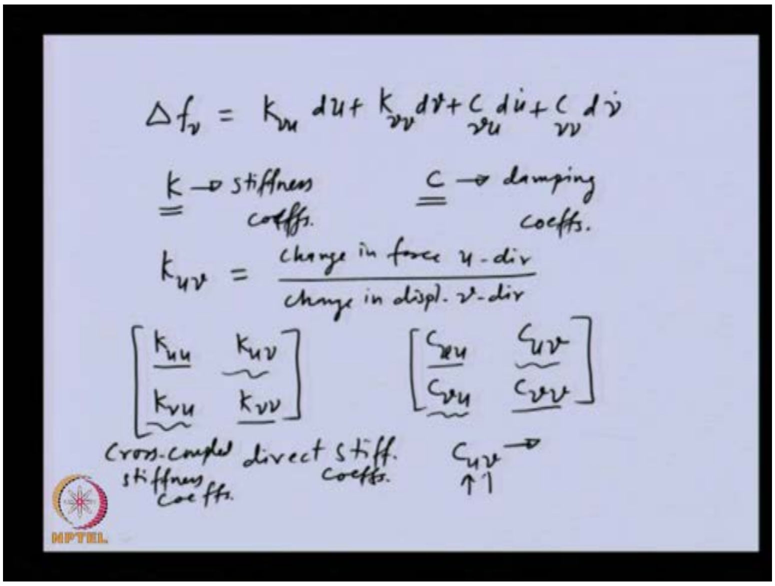

We can able to write these coefficients like this because we have defined four stiffness terms and four damping term. So, in this you can able to see this is u K u u K u v and this is K v u and this is K v v. So, this four stiffness term and similarly, we have four damping term u v and v u and v v. So, in this and this they are called direct stiffness term, direct stiffness coefficients and these two are called cross coupled stiffness coefficients, similarly here this two are direct damping term and these cross coupled damping terms. So, in this particular case, if we are want to define the C u v. So, this is the change in the force in v direction due to change in the velocity in the v direction.

So, change in the force in u direction due to change in the force in the v direction. Generally these stiffness and damping coefficients, which are defined for fluid filling bearing we have seen that, they are eight in number four are for stiffness term and four are for damping term.

They can be calculated by either calculating change in the fluid filling force for a given displacement in particular direction. This change in the fluid filling force either we can able to calculate using the lubrication theory, using the Reylonds equation and or sometimes people perform the experiments and they give this particular displacements or velocity.

And they measured the fluid filling force, how much they are changing and by with the help of this ratio, we can able to get the stiffness and damping coefficients. Now once we have the idea of the this particular model, we will analyze the rigid rotor which is mounted on these kind of bearings having eight linearized stiffness coefficients. In this particular case, you have seen that when we expanded the using the Taylor's series, these expressions we written only the first derivatives higher derivatives we neglected. So, basically we have linearized the fluid filling forces, which is actually non-linear in nature. So, that we can able to do some kind of analysis by which we can able to understand the critical speed calculation of the rotor or even we can able to do the instability analysis in subsequent lectures.

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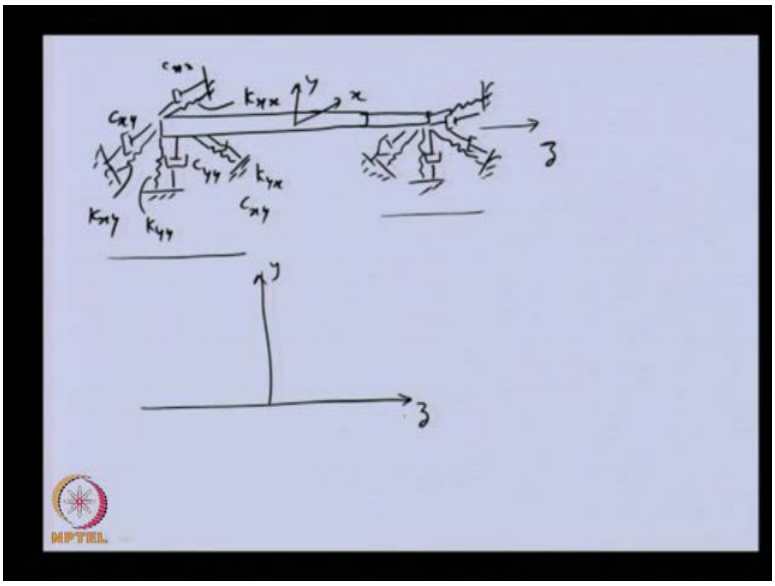

Now, will analyze a rigid rotor which is mounted on this kind of fluid filling bearing having damping and stiffness terms not only in one direction, but in the orthogonal direction also. And apart from these, we have cross coupled stiffness in damping terms. So, let me write this as C y y this is K y y this is C x x and this is K x x this is cross coupled term, let us say this is C x y and this is K x y similarly you will be having another cross coupled term. So, these are K y x and C x y.

So, and this is one end of the bearing similarly other end of the bearing will be having these terms. So, this particular rotor is mounted on two identical bearing, let us say for this particular analysis and is having both direct as well as cross coupled terms. So, you can able to see there are springs and dashpots, which are 8 in number at each end of the bearing of the shaft.

So, basically in this particular case, I am representing because now I am using x y z co-ordinate system. So, and this is z axis direction and we have y and x in this direction. So, these stiffness and damping, they are giving the forces fluid filling forces in the x and y direction, that is why I have changed the subscript of the coefficients to x and y.

So, basically the we can able to see that now, these this particular rotor which is mounted on to the two bearings is identical to the previous analysis only thing is now we are having so many spring and damper coefficients attached with this. So, if when we draw the free body diagram of this particular rotor in the similar lines of the previous lecture. So, let us say I am drawing the free body diagram in the y z plane and in this particular case also, we are assuming that we do not have any coupling of the translatory motion and the rotary motion.

They are uncoupled that means, if you are giving a shaft a vertical motion or a vertical force. There will not be any tilting there can be displacement in other direction or are not coupled and if on the same line, we can if we are giving a tilting motion to the velocity in other direction. That will be translatory in nature, but translatory and rotary motions rotor it will give tilting in two planes, but it will not give any linear displacements or translatory displacement. So, with that assumption we are analyzing this and.

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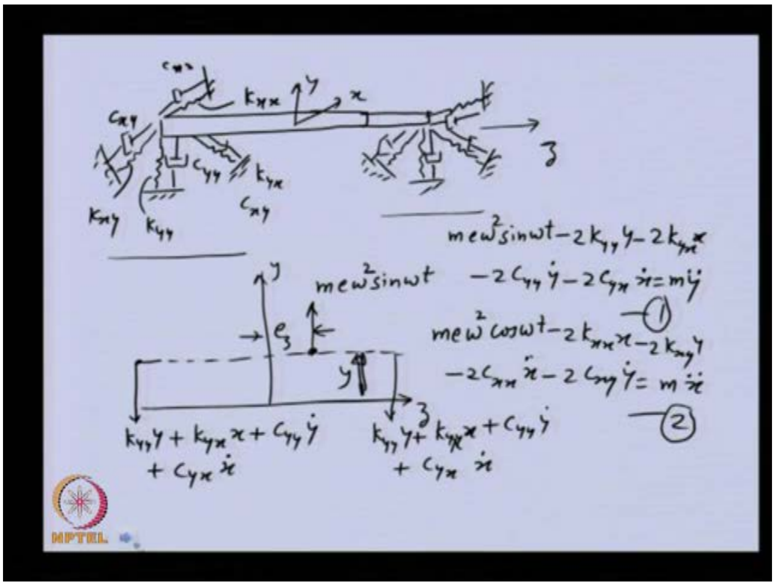

So, I am giving a pure translatory motion. So, I am giving a displacement in the y direction. So, this is the displacement in y direction and we except because of this there will be displacement in x direction also in the other orthogonal plane and earlier we had both the radial and the axial eccentricity of the rotor.

So, this is the axial eccentricity in the z direction and because it is rotating we have this centrifugal force, which is offset from the centre of the shaft. Now at ends bearing are there they will give forces. This time the forces will be not only from the spring, this was the term which we used in the previous analysis, but not because of the cross coupled term, we will be having some more terms like due to the displacement in x direction. Because, cross coupling is there now between two displacement linear displacement direction apart from that we have damping term also and we have the cross couple due to the damping.

So, these are the forces which are acting at the left hand same force because I am considering in this particular analysis same bearing. So, the same force will act in the other end also due to the damping and stiffness. So, this is the free body diagram of the shaft this dotted line represent, the shaft centre line and is acted up on by a centrifugal force and bearing reaction forces because of displacement are in the x and y direction. Because, now this x and y displacement we cannot able to separate it out. So, both the forces will be coming in this particular model.

So, now we can able to write the equation of motion by putting the equilibrium equation. So, we can able to write the equation of motion like this, so because centrifugal force is acting in positive direction. So, that will be positive quantity then these bearing reactions there, we can able to add them because 2 bearing forces are there and from damping and that should be equal to mass into acceleration in the y direction. So, this is one of the equation of motion, now we can able to write similar equation of motion, if we give a motion in the x direction or the equilibrium in the x z x plane.

So, that will be the centrifugal force will be m e omega square cos omega t minus the subscripts would change here. And here, it will be 2 x y and for damping we will be having C x x x dot because of velocity and C x y due to velocity in y direction this should be equal to mass into acceleration due in x direction. So, this is the second equation of motion these 2 equations, you can able to see that there, we have in both equation x and y terms. But the tilt motion is not there, because we are not considering this particular model the coupling between the translatory and the linear or the angular motion. Now on the same line we can able to go for the tilting of the this particular rotor.

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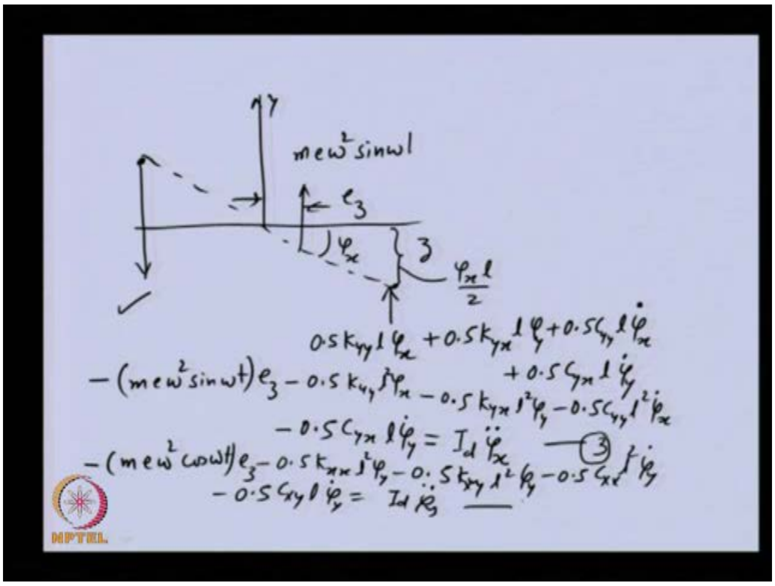

So, I am drawing the free body diagram in y in y and z direction in y and z plane. So, the tilting positive tilting direction is from y to z and this is the tilting angle phi x. Because, that tilting above x axis. Now, because of this tilt, we have this much deformation of the shaft. So, that is phi x, if length of the shaft total is l. So, this will be l by 2 and this side also. So, you can able to see that here the spring is getting compressed.

So, force will be upward here it is getting extended. So, this will be downward and values of these the forces will be the displacement. This we wrote in the previous analysis in the previous lecture. Now apart from this force there will be forces due to the cross coupled stiffness, this is due to the cross coupled stiffness. So, this is x and this is y because this particular force will come due to the displacement and a tilt in the about the y axis similarly, we will be having damping terms.

So, in this particular case we will be having instead of displacement derivative of this and additionally due to coupling we will be having another force term that will be due to velocity in the about the y axis this one. A similar force will be there at this end also because we have considered this particular bearing same as other end apart from that we will be having the centrifugal force which is e z.

So, now, we can able to see, we can able to write the normal equation. This particular force is acting this side also and the moment due to the centrifugal force, we can able to balance to the end that is rotary inertia. So, we can able to write let us say a moment m e omega square sin omega t into e z, this is the moment. This is giving moment in the counter clockwise direction, that is opposite to the angular displacement, that will be negative and these force will give a moment again opposite to the displacement angular displacement direction.

So, they will also be negative. So, this will give us 0.5 K y y l is the momentum. So, we will be having l square. So, these are the moments now they are negative because directing opposite to the angular displacements. So, here we are getting 4 terms from the fluid filling forces earlier. It was only single term, this should be equal to the diameter mass momentum of inertia of the rotor into acceleration angular acceleration in the y direction, that is in the this particular plane the tilting is about x axis.

So, this is another equation of motion, on the same line we can able to write another equation of motion in other plane. That is in x z plane where cos omega t will come and where subscript would change angle subscript will also be changing x y l square phi y 0.5 C x x, this will be x x l x square phi dot y and minus 0.5 C x y and phi y not should be equal to I d this will in the above the y axis.

So, this is the fourth equation of motion. Now we have seen that, we could able to get the equation of motion in four direction, that is in the x y phi x and phi y direction and these form of the equations are similar to the previous one in now we can put these equations in a matrix form and we can able to analyze for the unbalance response.

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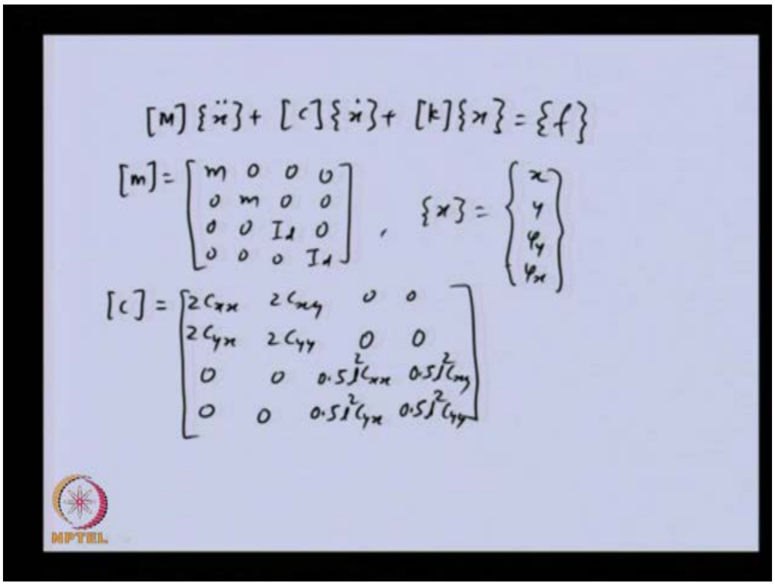

So, these equations we can able to put in a matrix form like this. Mass matrix, stiffness matrix, damping matrix and then stiffness matrix is equal to the external force, which is due to the unbalance force and the form of the mass matrix, for this case will be like this it depends upon how you have stacked the displacement vector.

So, in this particular case the displacement vector x, we have a step like x y phi y and phi x. The ordering of this stacking, we can able to change without any problem, but only thing is these coefficients will change it is position, if you are changing the ordering on this. Similarly, the damping stiffness damping coefficients will be now because of the cross coupled terms, now we are getting four terms earlier, one single only this direct damping terms were there earlier damping term was not there totally. But now we are having this all four components and also due to the velocity of corresponding to the moment balance. So, we have 0.5 l square C x x 0.5 l square C x y and that is C y x and this is C y y, now the stiffness matrix will be in similar form I will give you that in the next slide.

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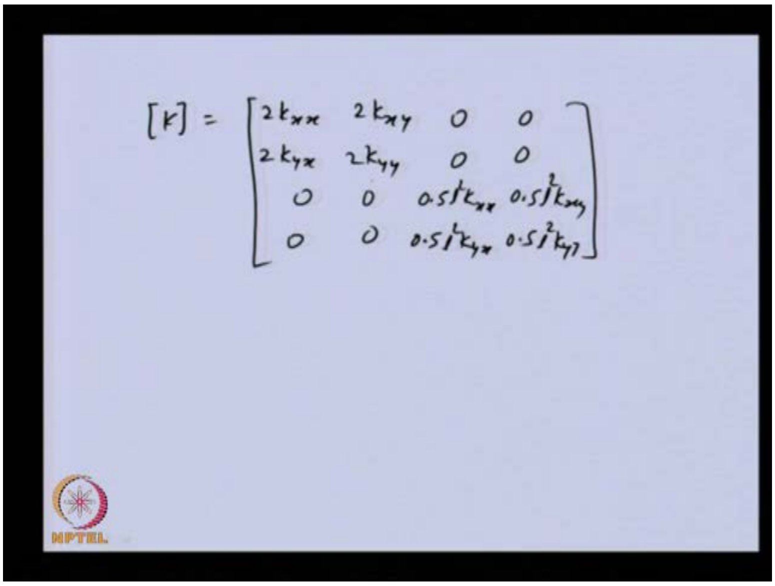

Earlier this stiffness matrix had only diagonal terms. Now because of cross coupled terms we have off diagonal terms also and the these terms are corresponding to the moment balance. These cross coupled terms were not there earlier only direct stiffness terms were there. So, in the previous analysis previous lecture we had these diagonal terms, now these of turning terms are coming into the picture. Basically, if you see these off diagonal terms because they are getting multiplied by the linear displacement, so a translatory displacement so they are coupling the motion in two directions.

And these are coupling the motion tilting motion in two directions. But these terms are 0, that represent that there is no coupling between the linear motion and angular the motion. So, here you can able to see these are 0. So, the linear motion and the angular motions are not coupled for this particular analysis.

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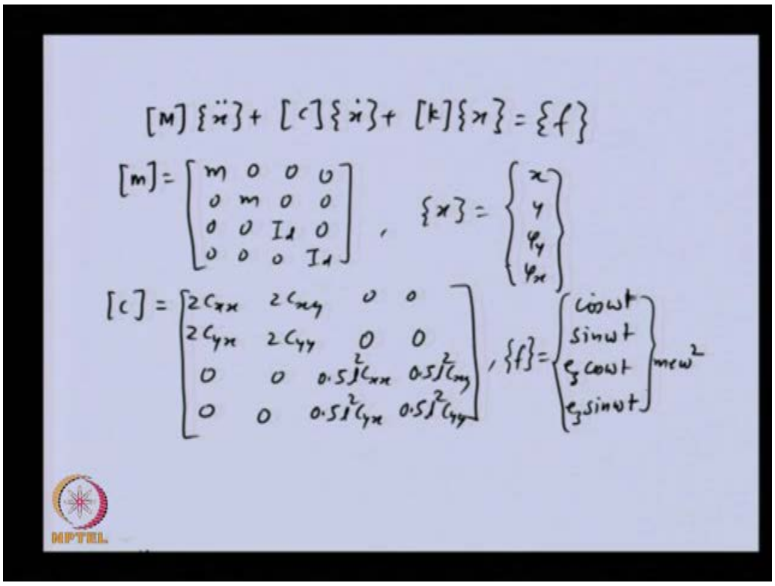

So, once we have defined all the matrices only the force is left out the force, we can able to write here the force will take the shape of because we have I am taking something common here, m e omega square that is common. So, apart from that we have sin omega t cos omega t in the at that is sorry, this should be cos omega t and sin omega t in x and y direction. These are the forces and then moment will contain e z also apart from cos omega t and sin omega t. So, this is the force external force vector.

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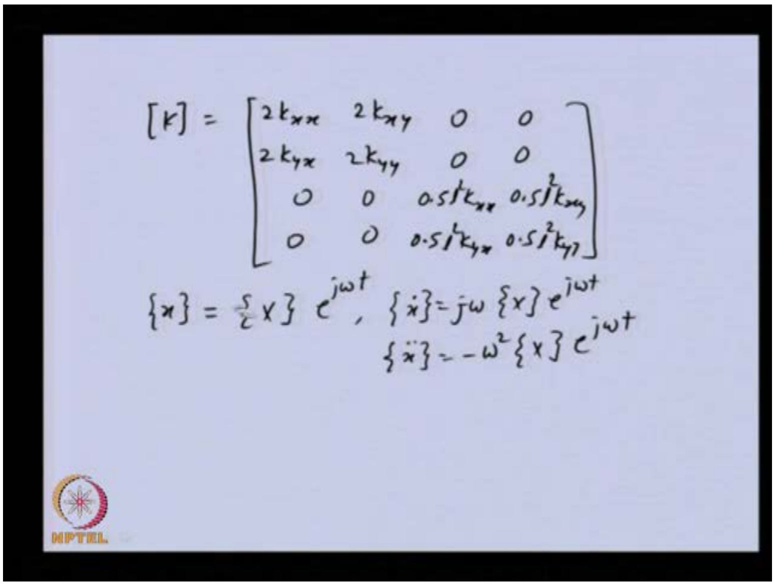

Now this can be solved using the procedure described earlier, we can able to assume the solution of the response as complex amplitude and j omega t. So, this will give as x dot as j omega capital X e j omega t and acceleration double dot of this will give minus omega square X e j omega t. This can be substituted this solution can be substituted in the equation...

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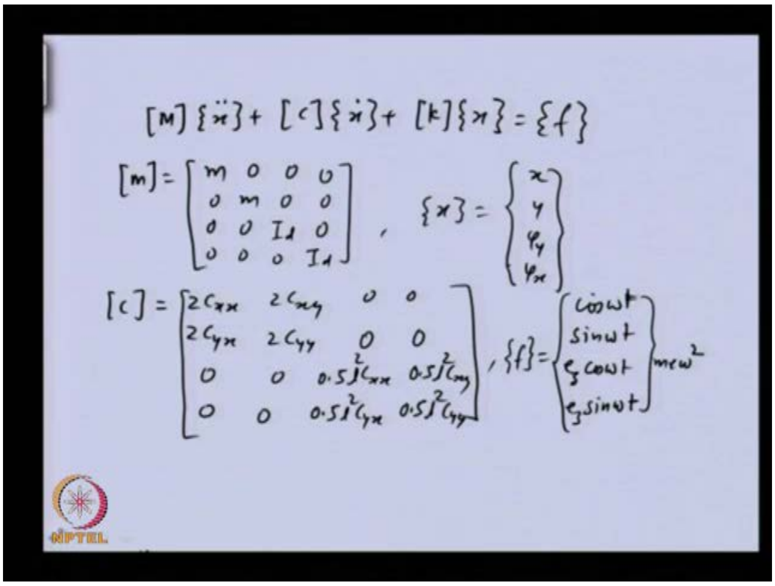

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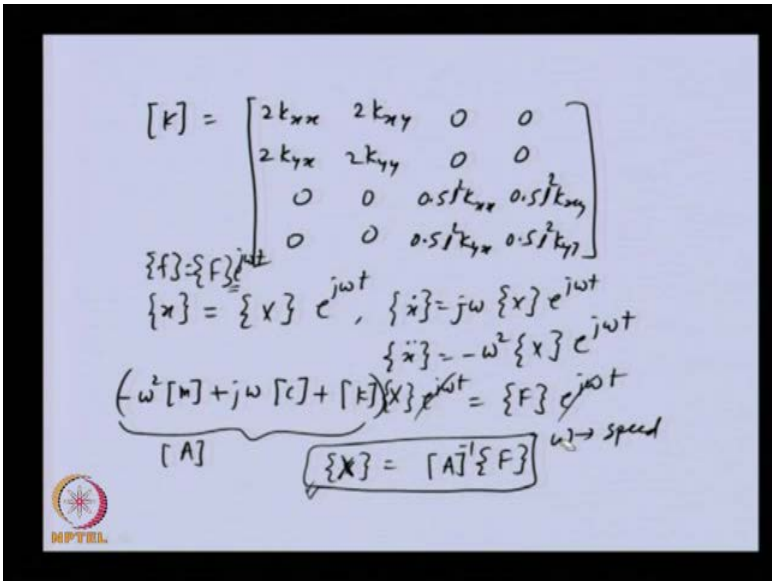

Minus M minus omega square M x i will be taking common then j omega C plus K, I am taking x common e j omega t. The force vector we can able to write the this particular force vector, which for unbalance was there we can able to write amplitude and the this time dependent term, which we earlier defined how this cos omega t and sin omega t can be written in this form.

So, this will give us j omega t. So, this will get cancelled. So, we have if we define this particular matrix as A. So, the unbalance we can able to get as inverse of A into force. So, this is the solution for unbalance we can able to solve this unbalance response X which is having four components in linear displacement and angular displacement in two directions.

So, you can able to see here omega spin speed of the shaft is variable, we can able to solve this particular response for various spin speed and we can able to plot how this, these are varying with speed. So, wherever we coincide the natural frequency with the speed, the resonance will take place, those are the particular speeds. Now this particular problem of rotor supported on anisotropic bearing, we will see through one numerical example, how this unbalance response can be obtained for various speeds for this.

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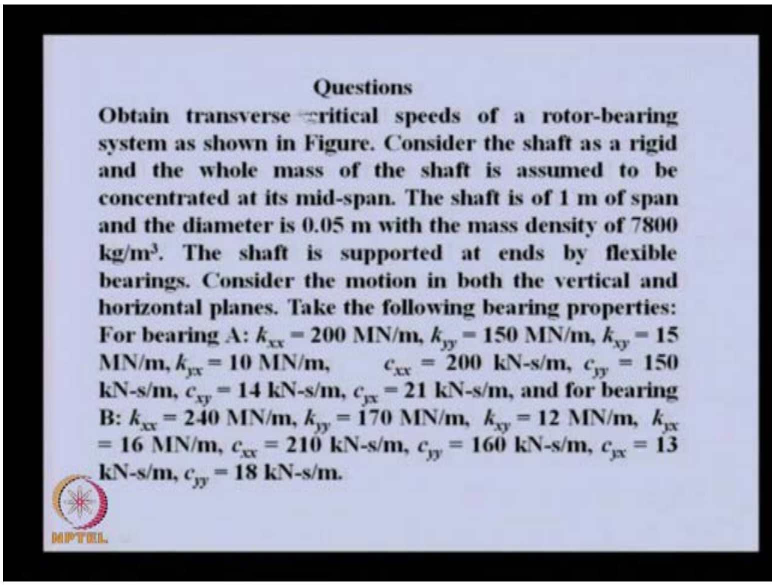

I have taken one problem in which we are obviously, and interested in the transverse critical speed of a rotor bearing system, which is, this is a rigid shaft which is supported on 2 anisotropic bearing the shaft is symmetric. So, and the length of the shaft is one meter the diameter of the shaft is 0.05 meter mass density is given. This the shaft is supported by a flexible bearings and we are considering motion both in the vertical and horizontal plane, because we are considering the bearing property with 8 coefficients.

That is four for damping and four for stiffness. So, these are the stiffness property K x x K y y, these are direct stiffness, these are cross coupled stiffness. They are generally little to be smaller in magnitude as compared to a direct stiffness and similarly the damping terms, when cross coupled damping terms are there and in this particular case not only. We are considering the bearings, which are there at the left hand and right hand we are considering the different bearings, so that we can able to generalise in this particular method. So, this is the other end bearing the right hand bearing. So, it is having another four coefficients of damping and stiffness.

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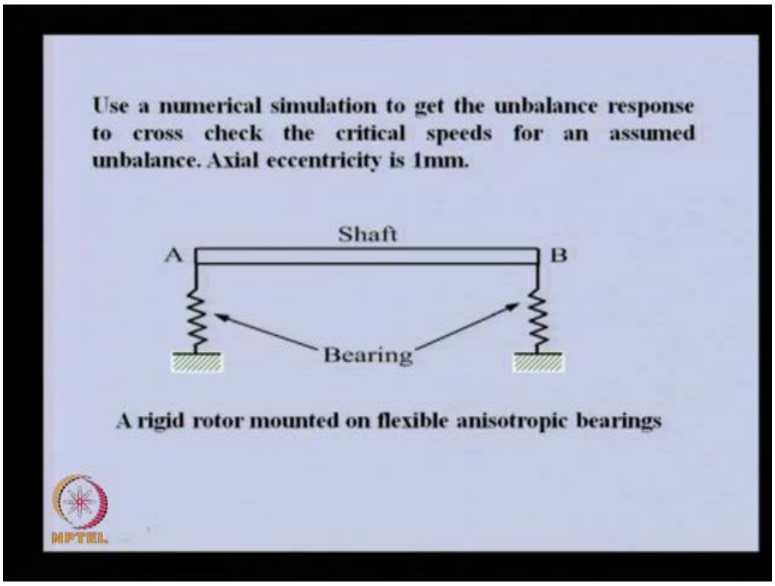

So, this is the shaft rigid shaft supported on two different bearings and some more details are there we need to get the unbalance response. So, what we are doing, we are obtaining the natural frequency of the system. This system and also through unbalance we have obtained the critical speed and we are cross verifying whether, we are getting the same values through free vibration and force vibration here. So, in this particular unbalance response, we will be having this particular eccentricity of the unbalance.

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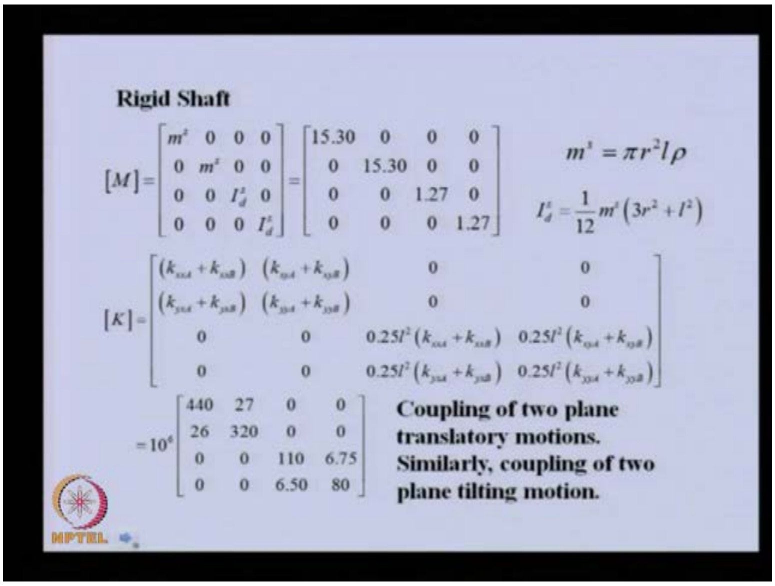

So, we already defined earlier the mass matrix. So, we can able to calculate the because we have all the geometry and material property of the shaft, we can able to get the mass diameter, mass moment of inertia of a cylinder, which is a rotor is given by this, where r is the radius l is the length of the rotor. So, m is the mass of the rotor, which we can able to get from here rho is the density of material l is the length of shaft.

So, from this we can able to form the mass matrix, the stiffness coefficients which we have which is given can be this is the stiffness matrix. So, we can able to see this two is stiffness's at two different bearings are getting added up here. So, there is small change in the stiffness matrix, which we have obtained earlier it was symmetric. So, we had two K x x now they are different.

So, we are adding them separately similarly this. So, if we substitute those given value of the stiffness in the length. We can able to get the stiffness coefficient like this stiffness coefficient matrix like this in this particular case as I told there is no coupling, only the coupling is there in the translatory motion or in the tilting motion.

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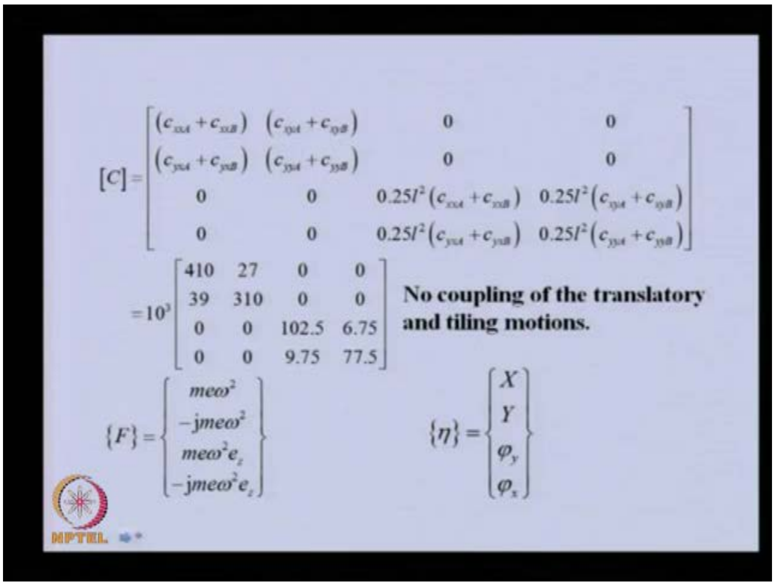

So, and this is the damping coefficient and if we substitute the value we can get this particular damping matrix. This force vector is given by this in which force and moments are there this is the stacking of the response vector.

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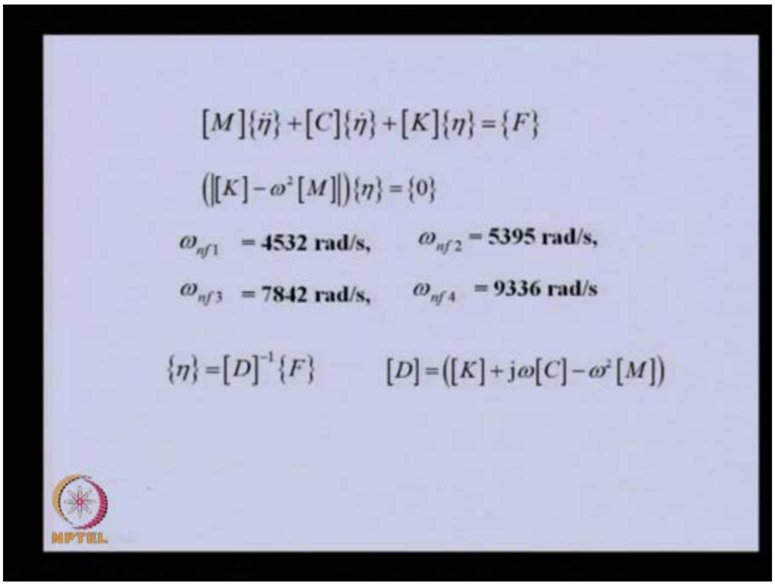

So, the equation of motion, which is like this, for free vibration an un damped system, we can neglect the force and damping, and so we will left with this particular equation. And we can able to solve the Eigen value problem of this and this will give us the critical speed, these are the four critical speed, we will get from the Eigen values basically, these are natural frequency.

So, the Eigen value problem will give this natural frequency. Now for force response we can able to use this directly without damping with damping. So, this is the unbalance response where d is this whole matrix, we can able to solve this particular for various values of speed and we can plot the response like this.

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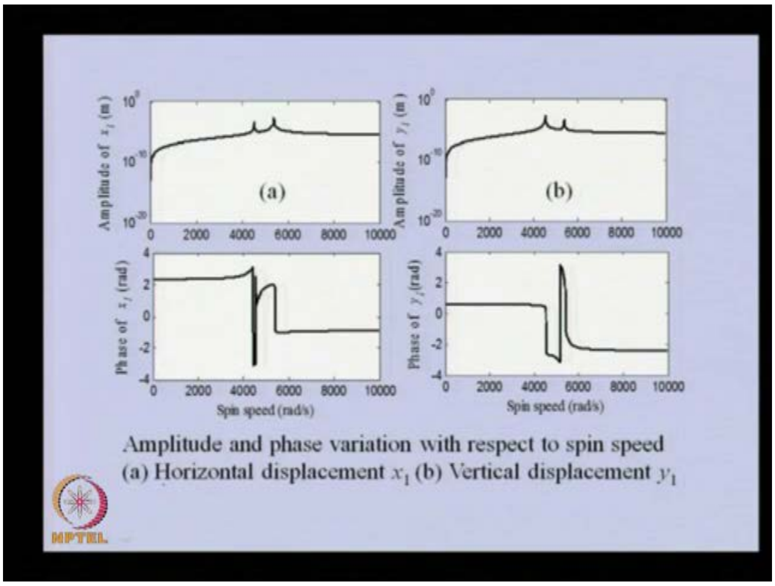

So, this is the speed variation at one of the amplitude, we have taken here of the displacement. And this is the phase of the displacement, these are the linear plot in the x direction and y direction, you can able to see we can able to get two critical speeds here, corresponding to the resonance and there is a change in the phase also in the other page also we are getting the same category speed. Because these two motions are coupled and these are corresponding to you can able to see they are around or between 4000 to 6000.

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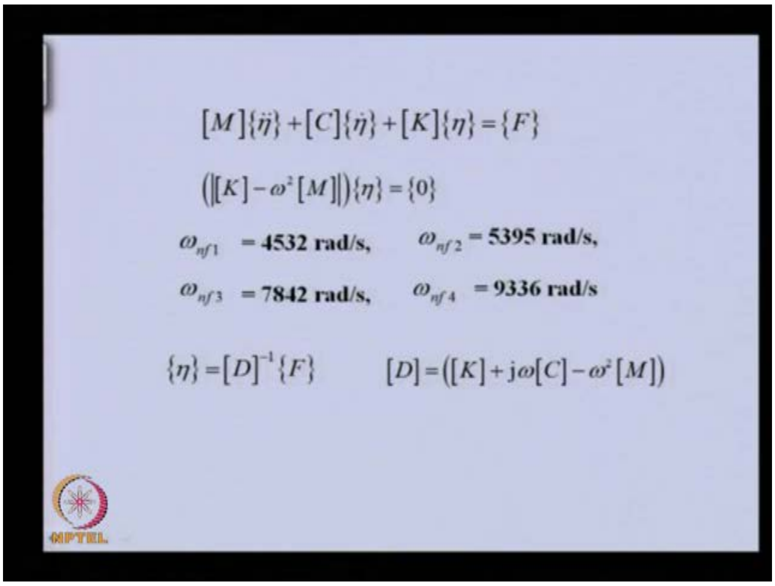

They are corresponding to the this first two.

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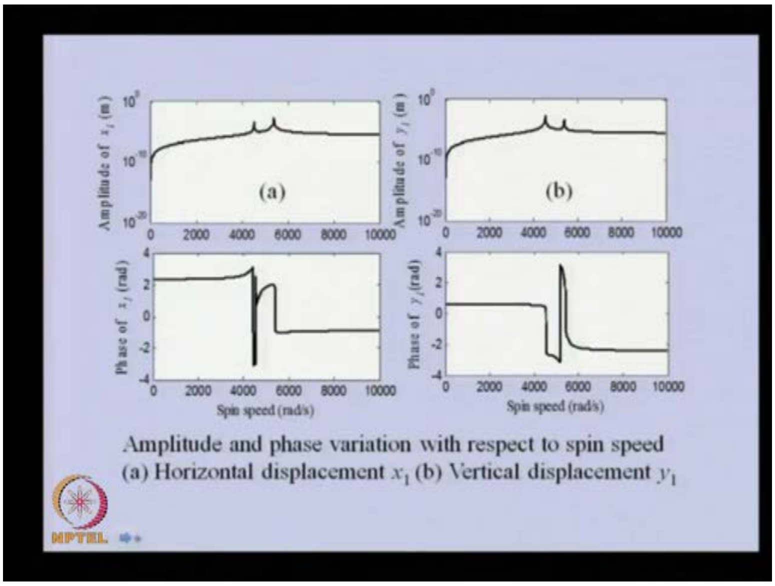

Similarly, we can able to plot the corresponding to the tilting motion in one plane and other plane here also we will be getting two critical speeds. These two are same and we can able to see they are, between around about 7000 and below 10000.

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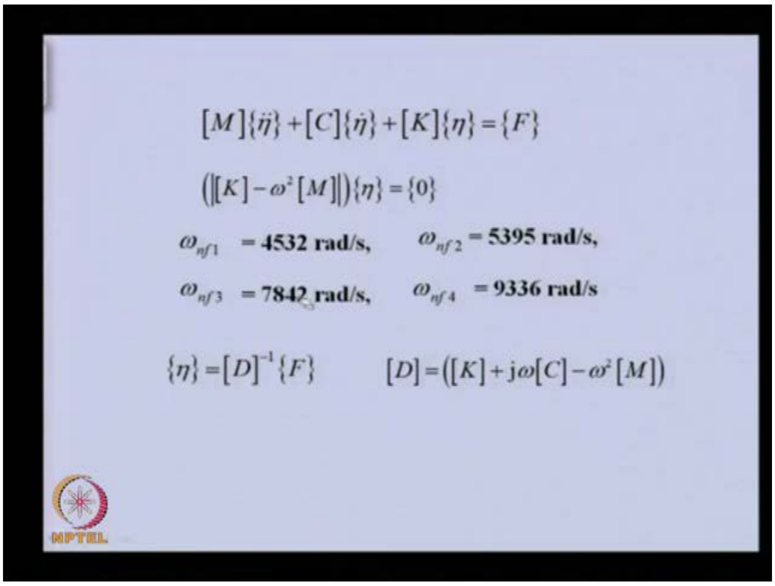

They are corresponding to the this one, this to... So, you can able to see that through free vibration analysis or Eigen value problem and through unbalance response, we are getting the similar.

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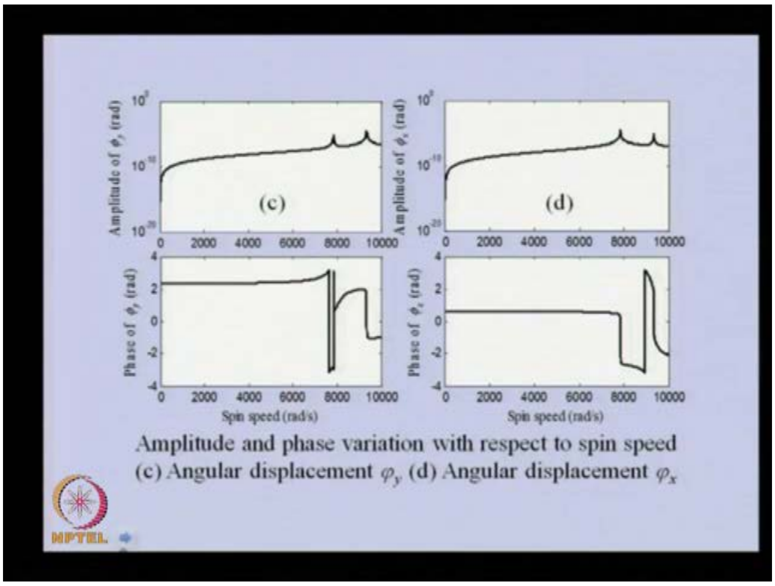

Critical speeds, today's lecture we have seen initially the basic concept of the hydrodynamic bearing. How this particular bearing fluid filling forces imparts stiffness and damping and we have defined the stiffness and damping both. The direct stiffness and cross coupled terms, which coupled the motion in two planes with this concept. We have then analyzed one rigid rotor which was mounted on this kind of fluid filling bearing and we derived the equation of motion. Even we solved a numerical example and we have seen that the free vibration analysis, un damped analysis and the unbalance response, they give critical speed calculations nearly same.