Theory and Practice of Rotor Dynamics

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Module - 3
Rotor Mounted on Bearings
Lecture - 7
Rigid Rotor Mounted on Simple Anisotropic Springs as Bearings

So, today we have seen the analysis of a Jeffcott rotor with an disc is offset from the centre of the shaft or the mid span of the shaft. In this particular case we derived the equation of motion also we have seen the procedure to obtain the unbalance response. Now, because the degree of freedom of the system is four, because we have four equation of motion and we have obtained the response with varying speed, if we vary the speed and solve the unbalance response for various speeds. We will find that wherever there will be coincidence of the speed with the natural frequency of the system there will be critical speed.

These can be plotted, that means the amplitude of the response and the speed can be plotted. And we can able to see that we will observe four critical speed in this particular system. As a case in the previous case for the Jeffcott rotor only one was there and because the rotor is symmetric in two plane, so it is 1, but here we will find that now. Then it will be having four critical speeds. Till now we have considered simple rotor in which case, mainly we have considered the flexibility of the shaft, but we have considered the bearing as rigid or in some more machines. The rotors are relatively rigid, but bearings are more flexible.

In such cases the rotor on the shaft can be considered as a rigid body and is mounted on some kind of flexible support. For such analysis obviously we need to have a different analysis as compared to the Jeffcott rotor model. In the present lecture we will consider not only the flexibility of the bearing also the support or the pedestal flexibility. How we can able to study the effect of these flexibility on to the... Specially, we are interested in the critical speed of the shaft, how it is effected due to these flexibility?

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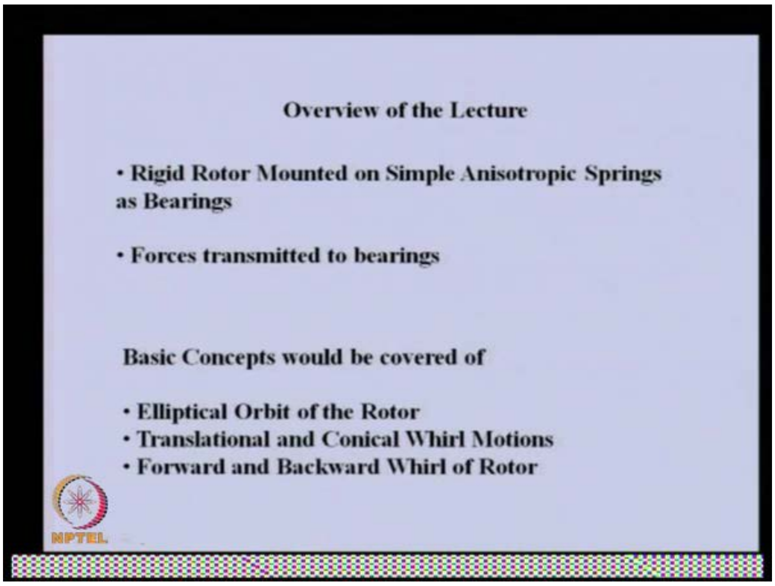

Now, let us see the overview of the presentation. So, we will be considering the rotor as rigid and we will be mounting the total on simple anisotropic spring as bearings. Anisotropy will be in this particular case of the spring will be there that in two orthogonal direction, the stiffness property will be different and even we will be analyzing apart from the critical speed of the system. How this particular motion gives forces to the bearing?

Now, how much forces are getting transmitted to the bearing that also we will be studying. Some of the concept which we will be seeing in this is elliptical orbit of the rotor, translatory and conical whirl motion of the rotor, even phenomena like forward and backward whirl of rotor. So, some new phenomena we will be studying in this particular rotor system.

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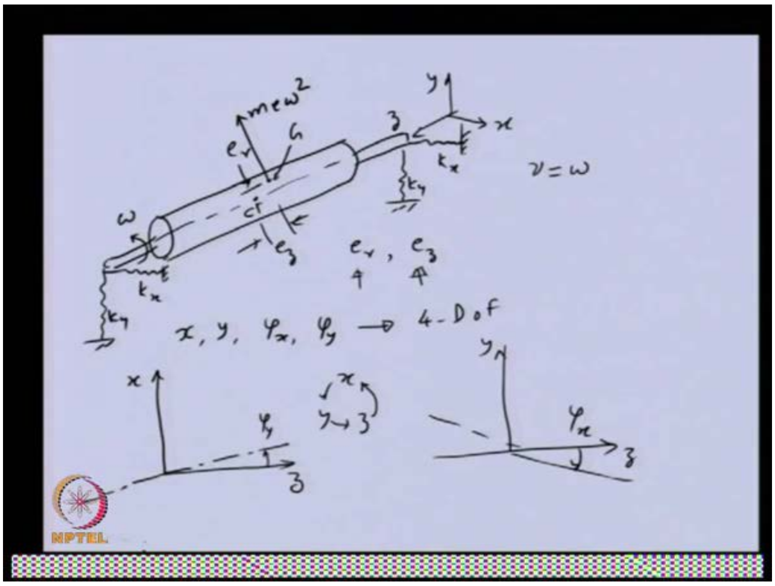

So, we will start with a rigid rotor. We know rigid rotor is having in general, a rigid body is having in general six degree of freedom, if we are not constraining it in any of the direction. But for this particular case we are constraining the motion of the rotor in the axial direction, as well as because this is spinning about its own axis of rotation. So, total degree of freedom will reduce by 2 for this particular case. So, this is a rigid rotor and it is supported, let us say by a spring at ends in two orthogonal directions.

In this particular case we can able to take let us say axis z along the direction of the rotor, vertical axis is y and we have x axis in this direction. This particular rotor is spinning with omega and during analysis we are considering, again the synchronous whirl condition that means the whirling frequency of this will be equal to this spin speed of the shaft and this is the bearing axis shaft axis. C is the geometrical centre of the shaft, in this particular case the G is here let us say that is offset in the radial direction. That is by e r this is offset in this axial direction by e z and offset in the radial direction is also there that is e r.

So, we have around the radial eccentricity, but also the axial eccentricity. And because of this model we will be getting the unbalance force, but also we will be getting the unbalance moment. So, in this particular case let us say a centrifugal force is acting at G is m e omega square. Now, as we can able to see the rotor can have up and down motion, so we will be describing that by x and y and also it can have tilting about x axis and y axis. So, we will be representing that by these angels.

So, basically we have four degree of freedom system and for the angular displacement let us see the sign convention. So, this is the bearing axis, this is x axis. So, according to the right hand rule, if we have x, y, z, so x to y, y to z, z to x, so in this particular plane we should take z to x as positive direction that means if we are interested in the shaft centre line, so this is the shaft centre line. So, tilt is phi y this is the positive direction z to y, z to x. The same shaft in z y plane, we taking this direction as positive this is the phi x angle.

This will be the positive direction of the angular displacement of the shaft in the y z plane. So, basically if we see in this particular rigid rotor, it is having linear displacement above vertical direction that is y direction also is having horizontal direction and apart from this it is having tilting about horizontal axis x and vertical axis y. So, these are four degree of freedom, which will be requiring for the motion of this particular shaft and in this particular case as we have considered the plane.

So, let us define the spring stiffness k x, k y, k x, k y, so these springs are spin in which if we are applying a this particular rotor. Let us say for this particular rotor if we are applying a vertical force at the centre of the shaft, we expect only the linear displacement in that particular plane. If we are applying a horizontal force then it will be again motion in that plane. If we are applying a couple at the centre, this will be having pure tilting, so it will tilt about its centre. Also, if we apply moment about the vertical axis, it will tilt about that in that particular plane.

So, as such the bearing property, which we have considered at present, they are such that all the four degree of freedom are uncoupled that means. We can give to this particular rotor the vertical motion, translatory motion, independent of other motion. Similarly, in horizontal plane and if we are giving a tilting, so there is no translatory motion, only pure tilting is taking place. Similarly, in the other plane, so that means in this particular case if we want to analyze the rotor, we can able to give various motion independent of each other. So, in this particular case we can able to give the motion independent to each other.

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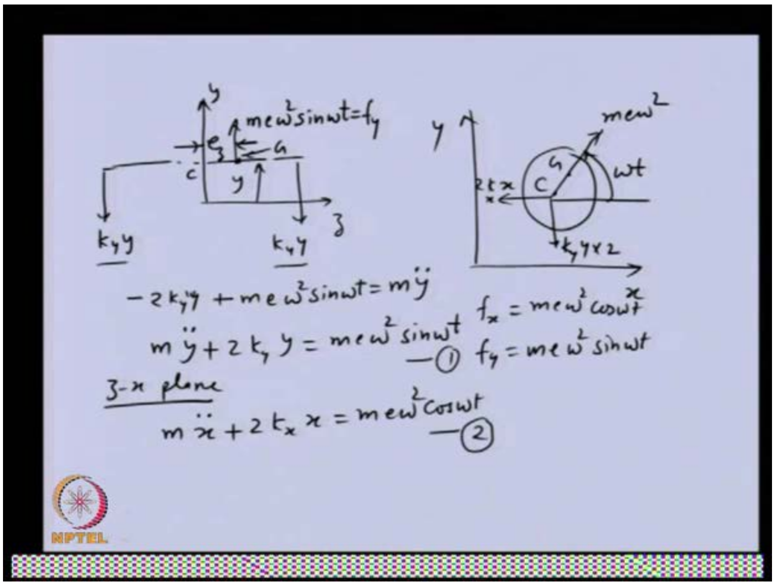

So, let us say, I am giving a motion to the rotor purely in the y direction. So, I am giving this particular displacement to the rotor and this particular displacement is y. So, because of this what will happen to the spring they will get stretched and because of that it will give a force at ends to the rotor and apart from this. This is the centre of gravity position. So, we will be having force in x direction as centrifugal force this one, this force we can able to see separately, how this unbalance force is coming? So, let us see in x y plane in the rotor, this is the centre of rotation, this is the centre of gravity and we have elastic force in y direction and x direction.

So, this is y direction because two springs are there, so it will be twice of this and in x direction 2 k, x, x and centrifugal force is acting m e omega square, our reference axis for time is x axis, so this is omega t. So, you can able to see the component of the force in x direction is m e omega square cos omega t and component of the unbalance force in y direction is m e omega square sin omega t. So, this particular component we have taken here this is unbalance force in the y direction and we define the axial eccentricity as e z, so this is the position of the...

So, c is here and G is this location. Now, you can able to see if we give the first balance in y direction, we will get the equation of motion like this two k y, y. Then we have these forces are opposite to the displacement direction. So, negative unbalance force is in the positive y direction, so there is... and then it should be equal to the inertia of the rotor, it should be equal to inertia of the rotor.

So, from this we got one equation of motion that is m y double dot plus 2 k y, y is equal to m e omega square sin omega t. On the same line if we draw the free body diagram of the rotor in other plane that is z x plane, it will be easy, it will be on the signal lines, you can able to obtain equation of motion like this. So, instead of sin omega t we have a force a force f x, m e omega square cos omega t, so that term will be come here. So, these are two equation of motion, which we obtained by giving two independent motion in y direction and x direction.

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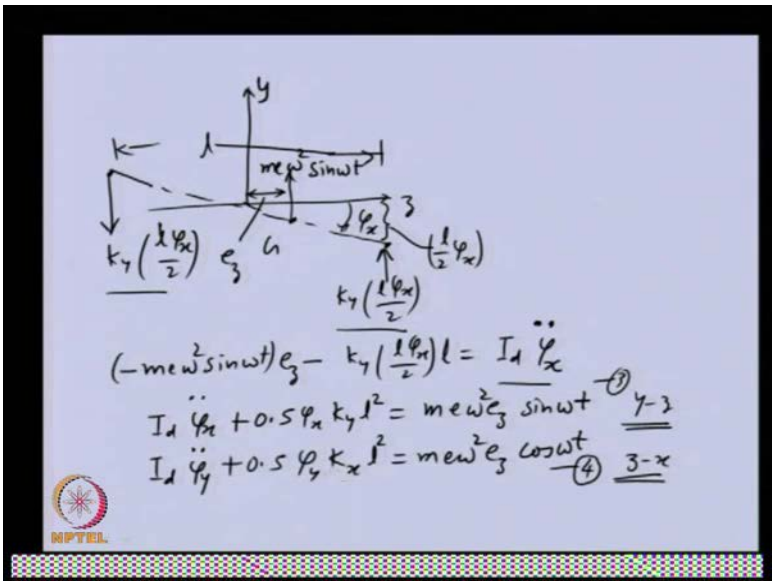

Now, we will see the tilting motion if we want to give tilting motion to the rotor. Let us say in y z plane, z is the bearing axis and y is the vertical direction, for this we need to have direction y to z for tilting of the rotor. So, let us see rotor is tilting about its centre, so this angle is phi x and the springs are at the ends. So, the spring which is attached here will give force in the downward direction because that will get stretched here it will get compressed.

So, it will give a force opposite to the compression and you can able to see that. This particular distance is the actual compression of the spring and if we take the total length of the rotor as l from here to here total length of the rotor. Then this angle will be l by 2, this distance will be l by 2 into phi x. So, compression of the spring will be this much.

So, you can able to calculate the force spring force into the, this deformation, here also same force will be acting, because rotor is symmetric l is the total length of the rotor.

Now, you can able to see there are two forces acting opposite to each other and apart from this if centre of gravity is here there will be centrifugal force, this will be same as the previous one m e omega square sin omega t. Now, you can able to do the moment balance of these this distance is e z radial eccentricity of the sorry axial eccentricity of the rotor. Now, we can able to write the equation of motion. So, now I am taking the moment balance.

So, we have unbalance force this is the force into the moment on that will be e z that is acting counter clock wise direction opposite to the deformation direction. So, it is negative then from the bearing support you can able to see this force and this force will give a couple in the counter clockwise direction that is also opposite to the deformation and the angular displacement, which is in the clockwise direction. So, this will also be negative, so we will be getting k y, l phi x this is the force into the moment term is l should be equal to I d y double dot x, because this is tilting about it is diameter about x axis.

So, this will be the rotor inertia, so this equation you can able to write it as y double dot x plus, let us say 0.5 phi x, k y l square is equal to m e omega square e z sin omega t. So, this particular moment balance we have obtained in y z plane. Now, similar expression you can able to get in the z x plane that we can able to write directly now. So, in that particular plane the angular displacement is about phi axis y axis and so these expressions can be written e z, then instead of cos omega t will come here.

So, this is the third and fourth equation of motion of this particular rotor system and now all these four equations this two and the previous two we can able to write in a matrix form. The four equations, which we have obtained if we see carefully all these equations have been obtained by giving displacements in x direction y direction and two angular direction, independent of each other. As such our assumption was there is no coupling between these motions, so these motions are also uncoupled.

So, if we are looking into the equation, let us say in one of the direction x direction, so this contain only the x variable y is not appearing. Similarly, if you see the second the equation of motion in y direction it does not contain y, apart from y it does not contain any other variable. Similarly, other two equations in they are not containing x and y, but they are containing a single variable. So, these equations can be solved independent of each other, so if we let us take one of the equation in the x direction first and try to solve it.

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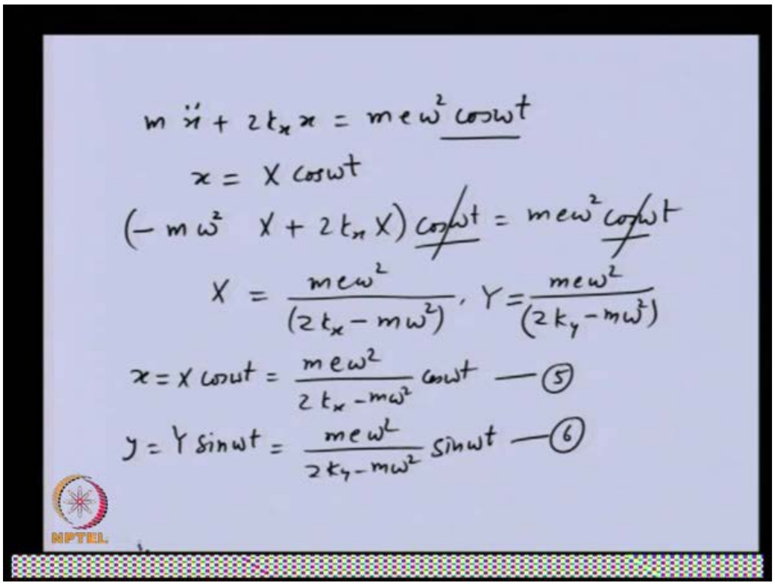

So, we have equation of motion, m x double dot plus two k x, x is equal to m e omega square cos omega t. A solution you can able to assume as cos omega t, because there is no damping we considered, so this can be with the same phase as the force this one. So, if we substitute this in equation of motion you will get capital X plus 2 k x capital X cos omega t term will be common. So, you can able to see this will get cancelled and you can able to solve for X that will give us m e omega square divided by 2 k x minus m omega square, I think here double m is coming only single m will be there.

So, this is one of the solution on the same line you can able to see you can able to write the equation for Y like this only the subscript of stiffness will change. Now, looking into these two equation the solution will be X cos omega t and that means we can able to write this as 2 k x, m e omega square cos omega t. Similarly, y you can able to write it as sin omega t, because there the forcing is sin omega t, so response also will be sin omega t. So, this is the response, so let us say this is fifth equation, this is sixth equation. Now, the fifth and sixth equation can be combined if we square the, these equations and add it we will get...

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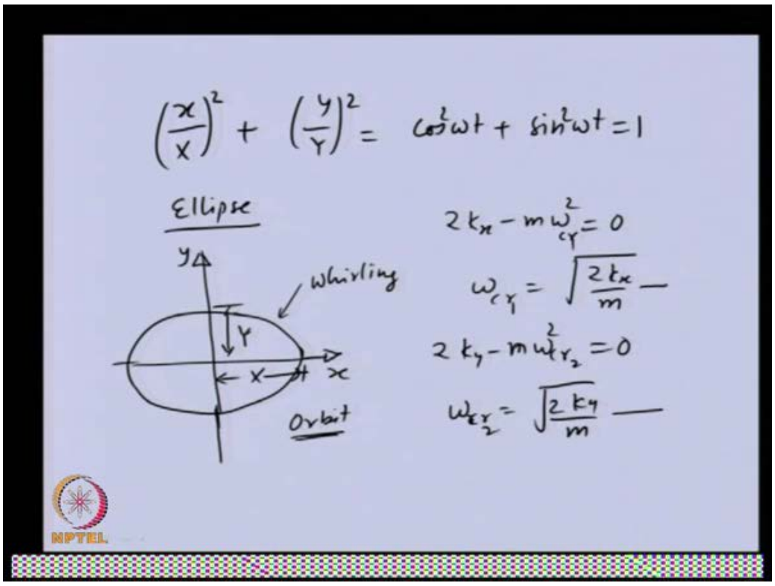

Let us say I am squaring this and this, so basically this will give me cos square omega t plus sin square omega t that will be 1 and this equation is nothing but equation of a ellipse. So, that means in this particular case you expect the rotor if we want the x and y if we want to plot it, rotor will be having elliptical path the rotor centre will be moving in a during the whirling. So, this is the during whirling this will be the orbit and where this distance is the capital X and this is the capital Y and this ellipse orientation will be along the x and y direction.

So, this ellipse is not inclined, because we have taken the spin system simple in this for this particular case. We will look into this orbit in more detail, let us first see the critical speed of the rotor. So, we have the response here, so you can able to see if this particular denominator becomes 0 for a particular value of omega, because here k x is a constant for a particular bearing m is constant and omega is a variable, which is the spin speed of the shaft that we can vary.

If we were increasing the speed continuously and if we are finding that this denominator becomes 0 that means we can able to write that separately 2 k x minus m omega square is equal to 0. So, that is a particular speed, which we call it as a critical speed and that critical speed will be given as root 2 k x by m, this is the one of the critical speed. If we put the denominator of the second expression corresponding to the y displacement we will get another critical speed that is second critical speed, the expression for that will be root 2 k y by m.

So, we can able to see that we are getting one critical speed here, another critical speed here, we got two critical speed. Till now we have consider only the equation of motion in the x and y direction, till now we have consider the equation of motion in the x and y direction, but we are not considering the equation of motion in the phi x direction and phi y direction. So, we expect another two critical speed corresponding to that direction. So, let us obtain them.

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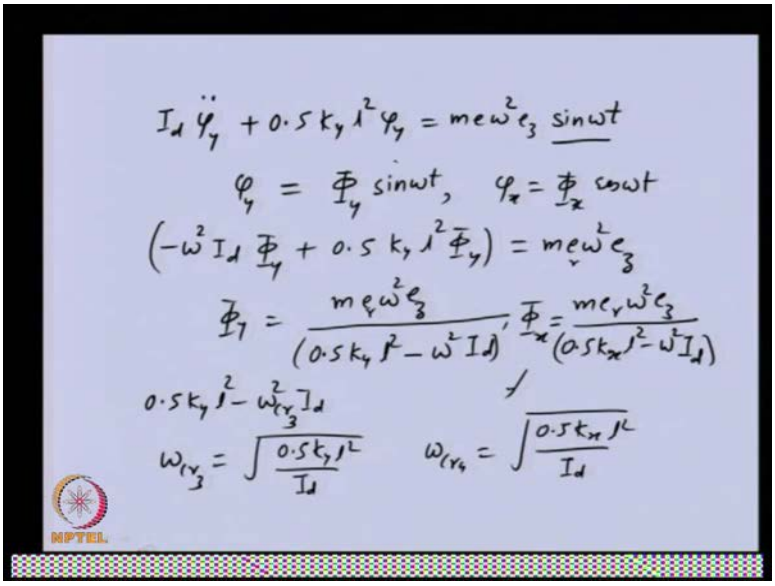

So, let us take equation of motion in one of the plane corresponding to the angular displacement. This was one of the equation of motion and if you take the solution of this, let us say capital phi x, e j omega and the sin omega t, because this is no damping. So, response will be in the same phase as the force and this we can able to substitute in the equation of motion to get this equation this is capital phi X capital phi sorry this is phi y, another phi y sin omega t will go from both sides.

Now, we can able to write the phi y equal to m e omega square e z, e z is the axial eccentricity, e is the radial eccentricity, which we wrote as e r. So, we can able to write this as 0.5 k y, l square minus omega square I d. So, we can able to see on the same line we can able write the amplitude in other direction by just changing the subscript of the stiffness coefficient in x direction I d is the di-neutral mass moment of inertia of the rotor.

Now, again we can able to see if this denominators are 0, we will get critical speeds, so I am equating the first one equal to 0. So, I am calling this as third critical speed, so this will give me critical speed in third one equal to 0.5 k y, l square divided by I d. Similarly, from here we can able to get the fourth critical speed that will be 0.5 k x, l square by I d. So, we could able to get four critical speed, apart from this if we see the solution which we assumed here for k epsilon sorry for phi y the for phi x this solution will be at this form. They can be combined this two expression can be combined on the similar way as we can combine in the x and y direction by squaring them and adding them like this.

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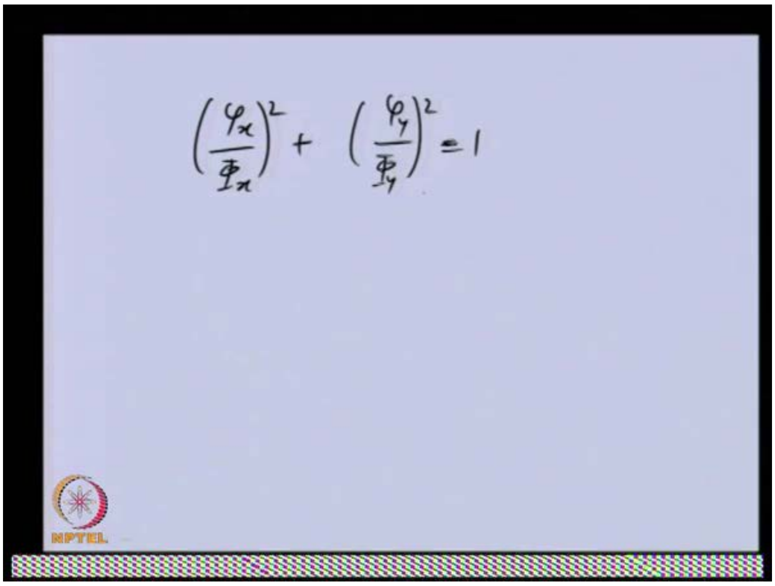

So, this will give us this capital phi is the amplitude and small phi is the time dependent term. So, that means in this particular case also we are getting the equation of an ellipse we will analyze this motion carefully how this will be. So, in both the case, in the linear case and the angular displacement case we got that this displacement is giving you some kind of a electrical motion. So, let us see try to see this motion how they would be how it will be elliptical. So, first is the translational motion, so both translational motion we can able to consider simultaneously.

So, if this is the rotor rigid rotor, what is happening? This particular rotor is having very poor translational motion in x and y direction start tilting and it is whirling. So, not only it is spring about its own axis, but during that is whirling. So, whirling path is basically this particular path if you see from this side this particular path is elliptical. So, this is the motion which we obtain as a ellipse path x and y direction, so this will be the elliptical path. So, not only it is spinning, but also it is whirling and whirling the motion is in the ellipse.

The second motion of the tilting, pure tilting is tilting about its centre and so it will be something like this the centre remains stationary and ends are moving and ends are moving in a ellipse. So, if you see this particular path of the end this will be moving in ellipse. So, if you see from the side again it will be having elliptical path not only it is spinning about its own axis, but also it is having conical motion about its centre of the shaft.

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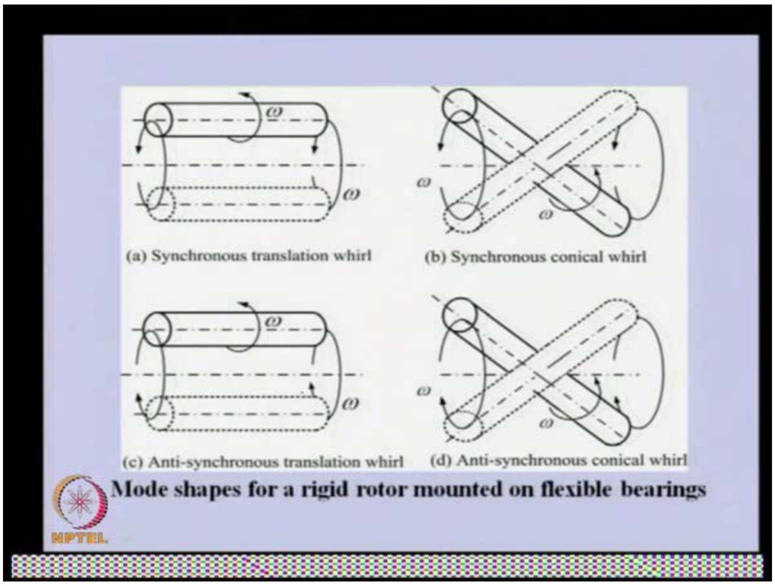

So let us see this particular motion. So, here the first cone the first graph we can able to see first picture that we have a translatory whirl, the rotor is spinning let us say in the counter clock direction. And it is whirling in ellipse the direction of that is also I have shown as counter clock direction that means I have shown both whirling direction and the spin direction is same, but not necessary they will be same. But here initially I have shown that is they are same we will see for some condition this whirling direction and this the spin direction may be different.

So, that means if you see the second, the third case in which the spin direction is still in the same counter clock wise direction, but whirling direction is clockwise. So, that means the shaft is spinning in one direction and in the first case is also whirling in the same direction, second case shaft is whirling in the same previous direction, but now it is whirling in the opposite directions. So, this two cases we can able to call as a synchronous translation whirl or anti synchronous translation whirl.

Similarly, if you see the conical whirl, in conical whirl the shaft let us see it is rotating in the counter clock wise direction and the ends of that if we see they are having conical elliptical path, but they are having the same counter clock wise direction rotation. So, this is synchronous conical model motion. So, basically if you see each and every particle of the shaft, will be having same motion elliptical may be different magnitude, but since of the rotation will also be same. This is the another case in which this conical whirl is there the spinning is in the counter clock wise direction, but whirling is in the clock wise direction.

So, this particular case we call it as a anti synchronous conical whirl, in this particular case the spin speed direction and the whirling direction are different. Now, we will see for this two kind of translatory and conical whirl, when it will be there is synchronous whirl, when it will anti-synchronous whirl, those analysis will see in detail.

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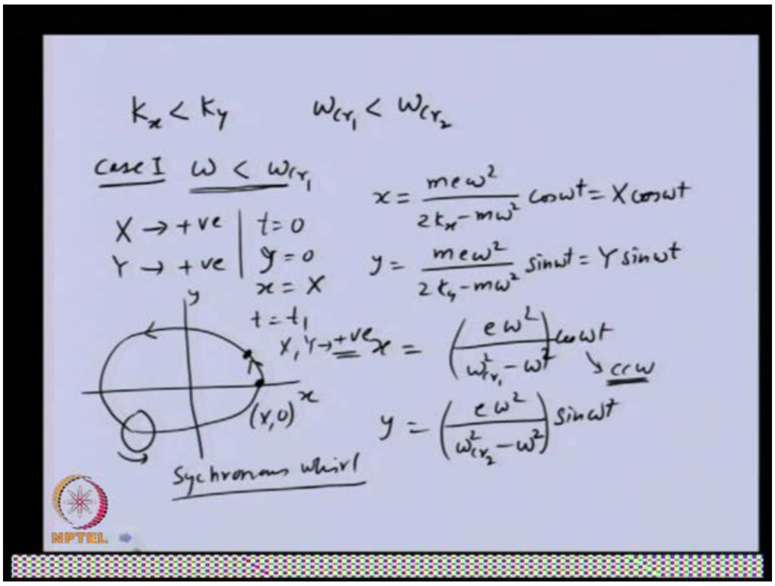

So, for that we let us we assuming some quantities like we are assuming the stiffness of the spring in x direction is less than the y direction. Because of this will find that we define the critical speed corresponding to the x direction is first critical speed. So, we will be having first critical speed less than the second critical speed, which we defined for the y direction whirl. Now, I am taking a one case in which the rotor speed which is omega, because this critical speed are fixed for this particular rotor system, the spin speed are rotor speed we can able to vary.

So, let see this rotor speed first we are operating below the first critical speed. For this particular case let us see what will the whirl direction. In this particular case let us again see the expression for the displacements, which we derived earlier. So, this was 2 k x minus m omega square cos omega t, which was we wrote as capital X cos omega t and y is m e omega square 2 k y m omega square sin omega t, which we wrote as capital Y sin omega t. Now, because now we are rotating at below the first critical speed, so that means we can able to see that if we substitute we can able to rearrange this equations, such that we can able to write this as m e if you take out, let us say m from denominator.

So, we will get this as first critical speed square minus omega square cos omega t and y as we take out m from the denominator, so it will cancel from the numerator, so will be having these as second critical speed where as this. So, with this expression we can able to see that when the omega is below first critical speed this capital X which is this quantity and capital Y which is this quantity, both are positive. So, for this particular case x is positive also y is positive, if we see the orbit that is elliptical orbit, let say it is the elliptical orbit.

So, we at time t is equal to 0, we can able to see if we are substituting the time t is equal to 0, y is 0 that means an x is having maximum value that is equal to x. So, at t is equal to 0 y is 0, but x is capital X value, so that means this position here y is 0 and x is having maximum value, so at t time we are here. Now, after sometime t, if t is some finite value t 1 what will happen both quantity this x and y will be positive. Because for small variation in the time this will some finite value positive, this will also be finite value positive, capital N at capital Y we already seen their positive.

So, will be having both x and y for sometime t is equal to t 1 this both are positive that means from here we from here we can go this side where both x and y are positive, because this a positive quadrant. So, we can able to see that in this particular case with time we have this particular direction of the whirl and in this particular case we have consider the direction of the omega also as counter clock wise. So, that we can able see that the whirling direction is also counter clockwise and the spin speed is also counter clock direction.

Basically, here we have a rotor which is moving, so this is the path of the rotor during whirling and at time t is equal 0 it is here and some finite time it becomes which is here. So, not only it is spinning about it own axis in the counter clock wise direction, but also its whirling. So, we saw that for this case when we are rotting rotating the rotor in below the first critical speed. This for the this particular condition in which I think, the second critical speed is more than the first one we have the forward or the synchronous whirl condition, synchronous whirl condition both are having same direction of whirling.

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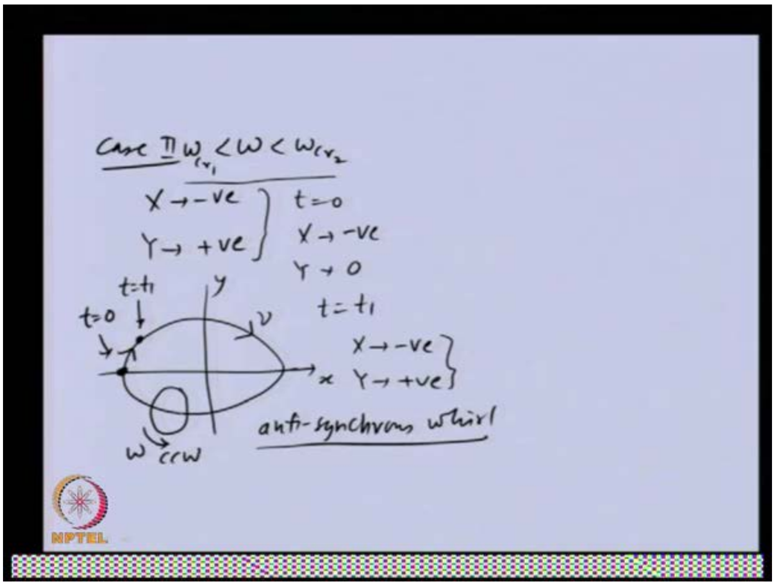

Now, let us see in the second case in which now we are operating the rotor between two critical speeds. In this particular case we will see that in the previous slide when we are here, when omega is more than the first critical speed this quantity, which is capital X will be negative and this quantity. Because, this is more than the first critical more than the omega, because omega is in between the two critical speed, so this quantity will be positive. So, in this particular case X is negative and Y is positive. Again if you see at time t is equal to 0, this is 0 y is 0 and x is this becomes one at time t is equal to 0, but x is negative.

So, when we want to plot the orbit of the shaft, let us say this is x direction y direction at t is equal to 0, x is negative and y is 0. So, y is 0 and x is negative that means it is only possible when we are at time t is equal to at this position, because now that position the at X is equal to negative, Y is 0 except this position. So, at t is equal to 0 we are here. Now, at time t is equal to t 1, again we will go to the equation we will see that this two quantity, this become positive, this remains negative, but they will become some finite, so they will not be 0. For that particular case we will see that X remain negative, Y becomes positive and this condition is will prevail if we are moving from here in upward direction.

Because, here X is negative and Y is positive, here it is not possible because both x and y are negative. So, only possibility that after time t 1 this will reach here. So, that means you can see now is whirling in the clockwise direction, the spin direction we have not change, the spin direction is the previous one counter clockwise direction this is spin direction this is the whirling direction. So, you can able to see that for this particular case when we are operating the rotor between two critical speed, we have anti synchronous whirl condition. In this particular case the rotor is spinning in one direction, but is whirling in opposite direction. Now, consider a third case when we are operating the rotor at above the second critical speed for that what happen?

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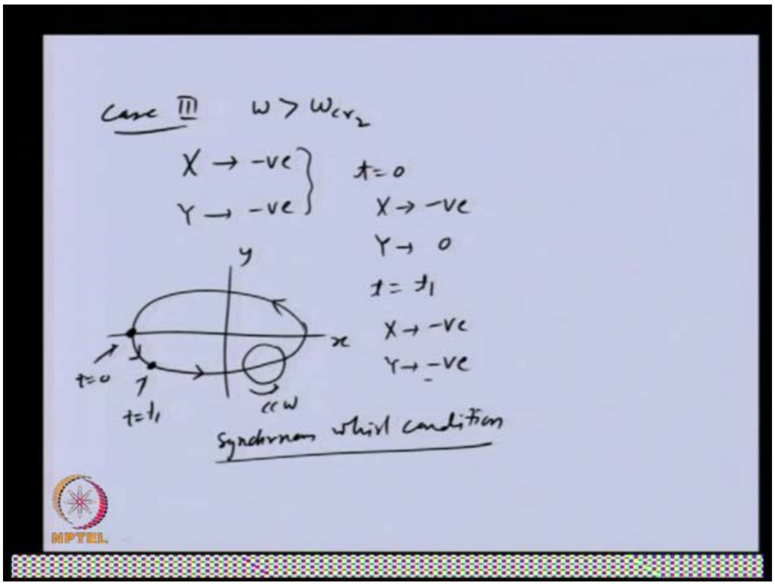

So, this third case is when rotor is above the second critical speed in this particular case x and y both will be negative that we can able to see. Because now, omega is greater than both, so capital X and capital Y both will be negative. So, at time t is equal to 0, X is negative and Y is also negative, so if you see the orbit both are negative. So, this is the position sorry Y is 0 again I am repeating. So, at time t is equal to 0, X is negative, Y is 0, because this it contain sin omega t term. So, that means we are starting with position, for time t is equal to t 1, X remain negative, Y is negative now.

So, the only possibility is it will occupy this position at t is equal to 0 here and at t is equal to t 1, t is equal to t 1 here. So, that means you can able to see now the rotor start whirling in the same direction as spin speed that is counter clock direction. So, in this particular case again rotor will start whirling in the same direction, so we will be having synchronous whirl condition, but here we will be having because of line negative sign, we will be having this 180 degree phase between the response and between the response and the force. Now, we have seen that how a rigid rotor, which is supported on anisotropic bearing having stiffness property different in two direction can give rise to four critical speeds.

Two related with the translation motion of the rotor and two related with the titling or the that is the rotational displacements. In this particular case we have not considered the axial vibration of the rotor, also we have not considered the torsional vibration of the rotor, only the transverse vibration we have considered. In this we have observed that the path of the shaft during whirling, when we consider let us say pure translation it is an ellipse. Also, when we are considering pure rotating this titling motion that is also ellipse, but that is in conical whirl.

So, first corresponding to the translational whirl and another is the conical whirl and in both cases each and every particle of the shaft as an ellipse. And we have seen that as we change the speed or linear case, we have seen that when we are changing the speed the direction of the whirl changes when we are below the critical speed or in between the two critical speed and after the third critical speed the whirl direction changes. The same analysis is valid for the conical whirl also, there also in the same line we can able to say that when we are operating the rotor below third critical speed we will be having synchronous whirl.

Between third and fourth we will be having anti synchronous whirl and after fourth critical speed again it will become synchronous whirl. So, in this particular we have seen very interesting concept of the whirl, especially how it changes it direction when we change the speed. So, in the today's lecture we have seen analysis of a Jeffcott rotor, but the disc was offset from the mid span. In the Jeffcott rotor case we had two degree of freedom system that is because there was no titling of the disc. So, we had equation of motion in two in numbers. So, we expect two critical speed in that particular case, but the because shaft is symmetric.

So, effectively we get one single critical speed in that particular case, in the present case when disc is offset we have seen that it is a four degree of freedom system. We expect four critical speed from this, but because of the symmetry in two plane that means in the vertical plane and horizontal plane, effectively we will get two critical speeds only. This particular wobbling motion in this, again I am repeating the last sentence. So, in this lecture even we have seen the wobbling motion or the titling motion on the disc. And in subsequent lecture, we will see that this wobbling or the titling motion along with the spinning gives gyroscopic couple. Also, in the subsequent lectures we will we will analyze those motions also.