Theory and Practice of Rotor Dynamics
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Module - 8
Dynamic Balancing of Rotors: Influence
Lecture - 40
Coefficient Method for Flexible Rotor

Previous few lectures we have been discussing about the dynamic balancing of rotors. We started with balancing methods, which are available for rigid rotor case. Then we explained one of the method for balancing a flexible rotor; that is the modal balancing. In modal balancing, we saw that the method is quite cumbersome, in the sense that we need to balance the rotor mode by mode from fundamental mode to the higher mode. When we are balancing the higher mode, we need to ensure not only we are balancing that particular mode, but also we need to ensure that the lower modes should not get disturbed, because of the correction in the higher modes.

So, that makes that particular method quite tedious. In that particular case, we have seen that mainly the balancing, which we do is based on the observation of the response. When we balance a particular mode, we always look into the response and some kind of trial and error we need to do in selecting the correction mass. Now, in today’s lecture, we will be dealing with more advanced method, that is influence coefficient method. In this influence coefficient method, we already discussed in the rigid rotor case also.

But this method is slightly different, because once the rotor is having the bending and especially due to crossing the critical speed, the shaft deforms. Because, of that whatever eccentricity is there in the system, in that particular case whatever the centrifugal forces are there in the system. They either will be decreased or increased along the length of shaft depending upon the, which mode we are considering. Like, we have seen in the flexible mode case, there are nodes in between the shaft length and in those places. We will see the centrifugal force will be minimum, but at anti nodal point where maximum displacements are taking place, the centrifugal force will be more.

So, accordingly in this particular case, whatever the concept of the influence coefficient we described earlier will be slightly different. In this particular case, the influence coefficient will depend up on the speed of the rotor, especially at what mode we are operating. We need to obtain these influence coefficient for all the speed at which we want to operate the rotor. That means in a particular speed range, if you want to operate the rotor and we want to balance the rotor in that range.

We need to obtain first this kind of influence coefficient for those speeds. Then, we can be able to use them for finding the correction mass. In this particular case, as we described in the modal balancing case, that if you want to balance a particular rotor up to n th mode, so that many number of planes are required to balance that. So, that is still valid in the present case and we will see that through some case study, how this method can be illustrated. That will give more clarity into the method.

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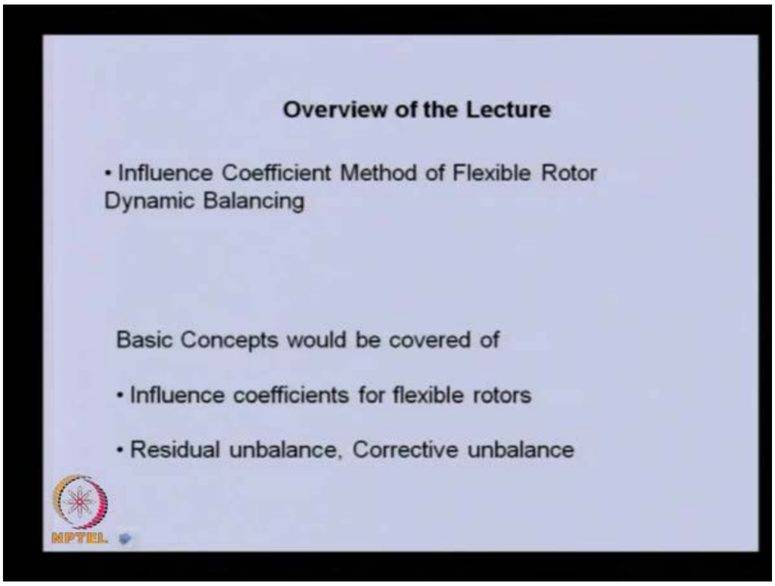

The overall overview of the lecture will be describing the influence coefficient method of for the flexible rotor balancing, especially the dynamics balancing of rotor. In this, apart from the concept of the coefficient for flexible rotor, which is slightly different as compared to the previous rigid rotor case, we described that some of these are the terminologies. We will be using also, that residual unbalance and corrective unbalance in the present analysis.

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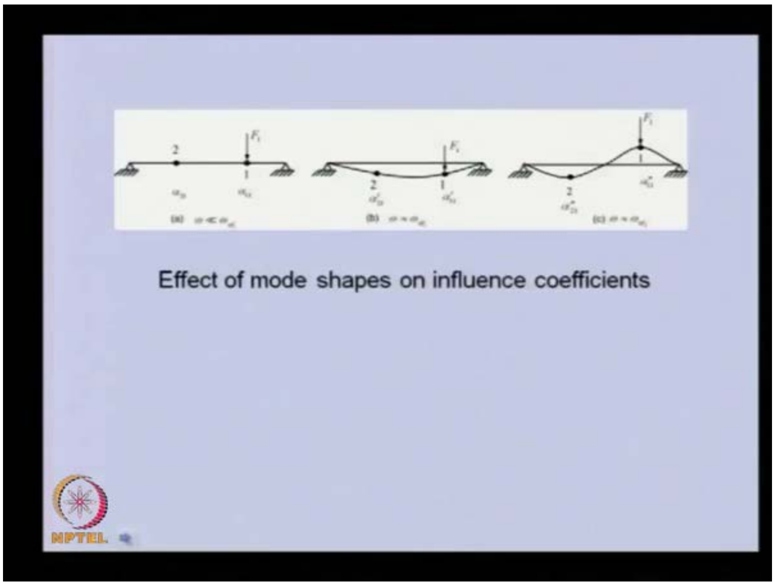

Now, coming to the influence coefficient, especially for the flexible rotor we can be able to see that earlier. We defined rigid rotor in which the deformation is less as such. The shaft is not getting bent. What were the forces we are applying for that case? The shaft is getting deformed from its static equilibrium position. So, this is valid for when we are operating the rotor much below the critical speed of the rotor. If you are near the first critical speed as for simply supported case, we expect the shaft will be having deformation like half sin wave.

Now, if we want to apply some force here, you can be able to see that whatever the deformation will further take place, because this is slightly different as compared to this. This will be more clear in this particular case, in which we are operating the rotor around the second critical speed in which we expect the full sin wave. Now, we can be able to see that there is a node here, where the displacement will be 0. Now, the same amount of force if we are applying here which we applied, here the deformation will be entirely different.

They will be entirely different. Because of this, the influence coefficient of the shaft of this particular mode, the speed will be different as compared to this. So, this figure illustrates that how the influence coefficient will be different for different speed, especially when we are at different modes. We need to calculate these influence coefficients for all such speed of operation or all such mode of operation in which we want to operate the rotor.

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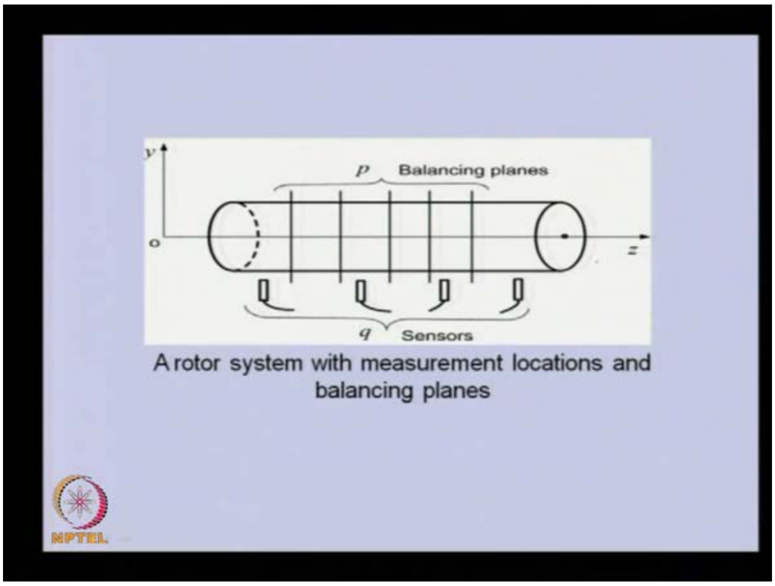

So, for the balancing purpose, let us say we have a route system in which we have p number of balancing planes. So, that means we need to we want to balance p number of modes. This balancing plane, we have chosen arbitrarily here, but even the choice of the balancing plane is important. Obviously, that should not fall in the place where the nodes are present. Otherwise, whatever the mass we will keep on this plan will be ineffective. We can be able to choose conveniently the measurements locations also. The measurement location will see that generally in the actual rotors the measurement will be available at the end of the shafts where the bearings are there.

So, wherever bearing is there, we will be having access to the rotor. There we can able to measure the response in very few cases. We could be able to measure the response in between the span of the rotor. But, this particular figure is more general in which we are assuming that there are q number of sensors by which we are measuring the response. In this analysis, we are showing one of the plane that is the y z plane, but by this analysis we can be able to on the same line, extend for the other plane, that is the x z plane.

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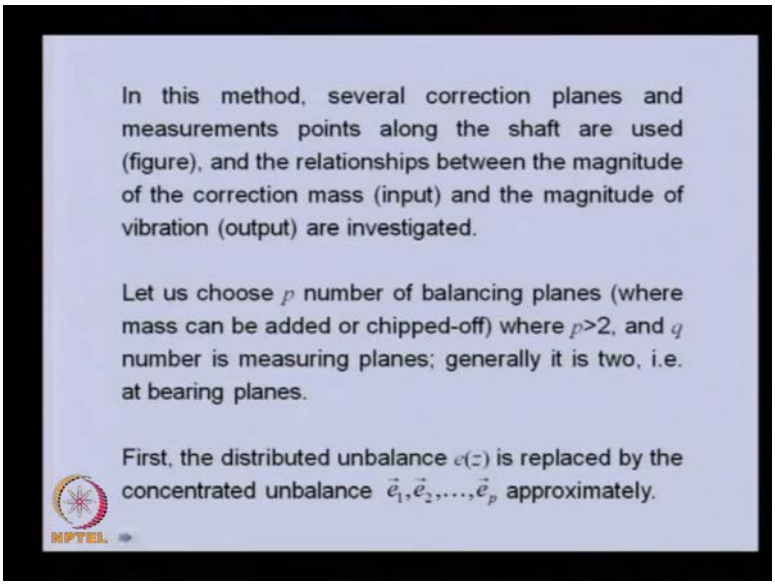

Now, once we have defined the balancing plane or the correction plane and the measurement locations along the shafts, now we need to relate the magnitude of the correction mass. So, once we are keeping the correction mass in these p number of planes, basically they are some kind of force. So, they are giving input to the system. The magnitude of the vibration, that means the output we will be measuring at the location of the sensor. So, whatever the response or the magnitude of the response is there, that is the output for a given input. Those correction mass will be keeping on to the balancing plane; that will be known to us.

So, basically we are trying to relate the input and output for this particular case, when we have p number of balancing plane and q number of measuring planes. Generally, this balancing plane when we are talking about the flexible mode should be greater than 2. Generally, we will be having access to the router at the ends where we can be able to remove the material or we can be able to add the material in the form of correction mass to balance the rotor.

The number of measuring plane, also as I mentioned, there is a subcritical limitation. Generally, that is also at the bearing location. That means generally it will be for a simply supported shaft, which will be having only 2 measuring planes to measure the vibration. But in general if there are multi support or of the rotor system, then we can have more number of access to the vibration response. Now, let us assume that rotor is having residual unbalance, which is distributed along the shaft as e z.

So, a continuous rotor will be having this kind of variation of unbalance. Now, we are replacing this particular unbalance to equivalent concentrated unbalance that is p number of unbalances. Basically, we want to find out corresponding to this distributed unbalance which is actually present in the rotor, what is the equivalent of that if you want to keep those in the correction planes which we have chosen, so that we are able to balance the rotor.

So, obviously this will be approximate because the flexible rotor cannot be balanced perfectly for all the modes. So, we need to limit ourselves to some few modes. Here, we have chosen up to p th mode. So, that many number of planes we have chosen. So, that many number of correction mass we need to find out. So, basically these concentrated unbalance will be unknown to us, which we will be finding through this procedure which we are describing now.

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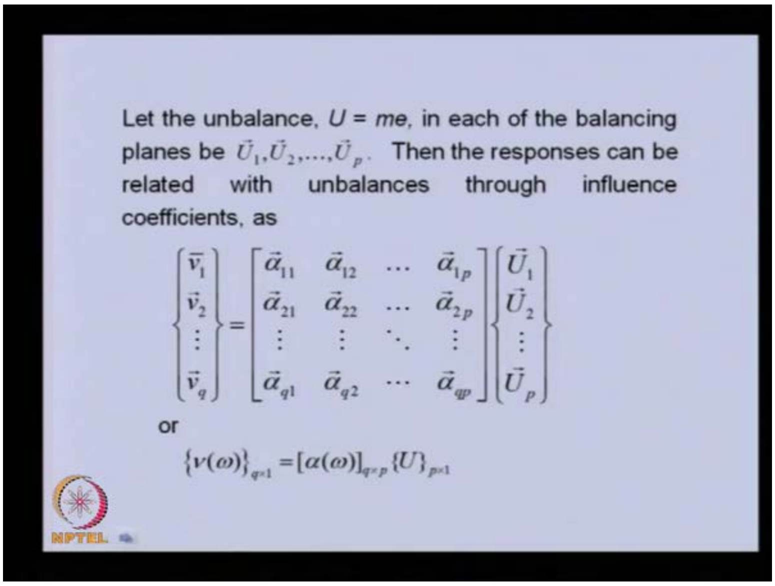

So, in this we have this is unbalance as we defined earlier which is nothing, but mass into eccentricity. In these planes, the balancing planes let us say U 1, U 2, U p, these are the unbalance which we want to find out. These are the corrective unbalances, which we need to find out. Now, these are the equivalent unbalances corresponding to the distributed unbalance present in the system. Now, we are relating the response, that means the vibration of the rotor with these unbalances through influence coefficients.

As we described earlier, we can be able to relate the displacement and force with the influence coefficient. These kinds of things we already discussed for the case of rigid rotor in which these are the locations. The measuring plane locations response, that is q e number. So, this response basically, we are writing in terms of the influence coefficients multiplied by the forces. So, you can be able to see that this particular influence coefficients matrix which we have obtained which is of size q into p is the number of balancing plane and q is the measuring plane.

So, this particular influence coefficient is the function of speed, because we have already seen that how the influence coefficients changes with speed. So, we are expressing this influence coefficients matrix as a function of speed. These responses, obviously if we change the speed, we expect that the response will change with the speed. This is the unbalance. Unbalance is defined as mass into eccentricity. So, basically they will not change with speed. Unbalance force will change, but this unbalance will not change with speed.

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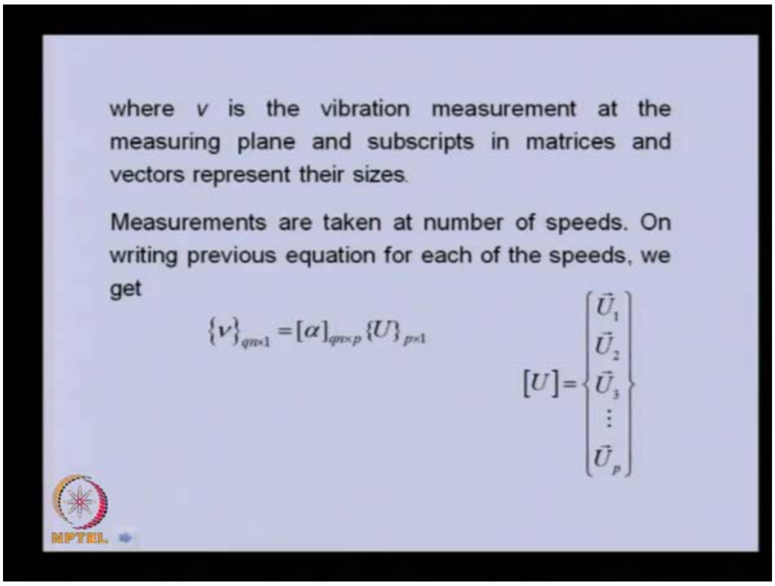

So, this is U. It is the vector which is not a function of speed. So, here we have response and influence coefficients as function of p, but this is independent of the speed, because this is the previous relation we have written for one speed. Now we can have the measurements at number of speeds. So, we can be able to vary the rotor speed from omega 1 to omega n, in the range of speed in which we want to operate the rotor.

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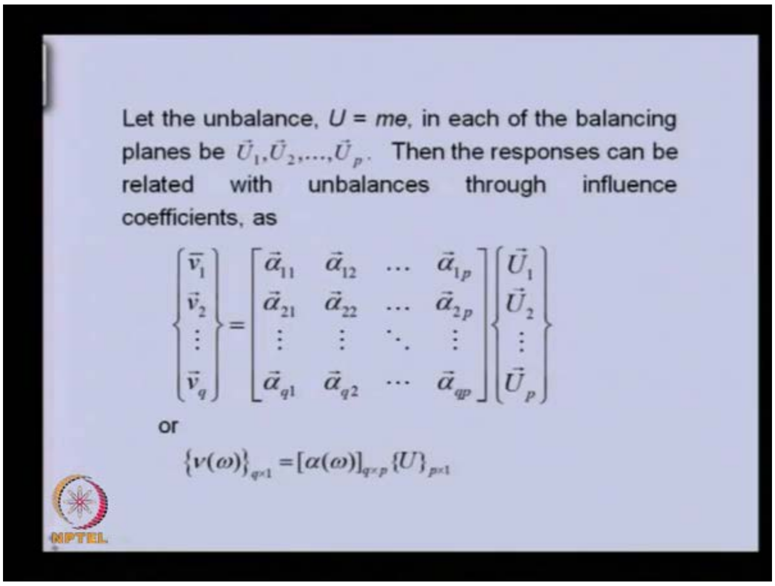

We can be able to write the previous equation for that many speeds. We will be having these equations for various speeds. So, that many number of the equations we will be having corresponding to these speeds. Those all equations we can be able to combine like this.

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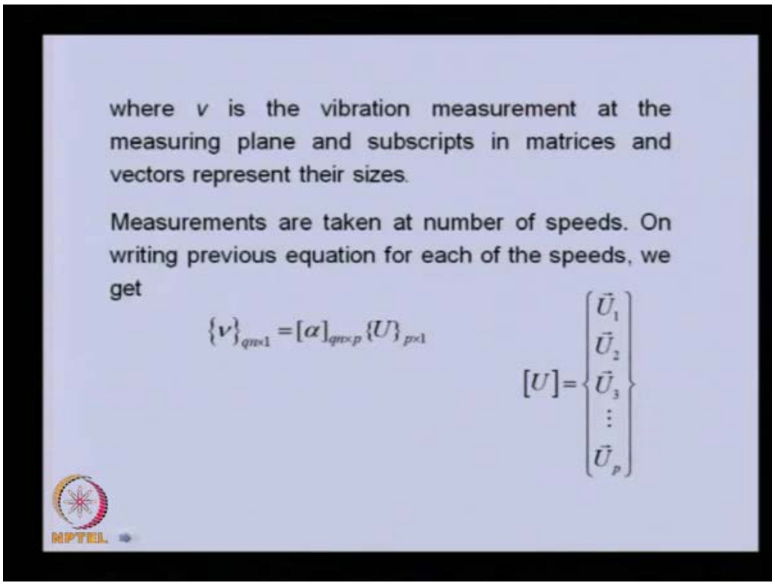

Now, this response is not the function of speed. Basically, you can be able to see here that it is not function of speed. This influence coefficient matrix, the size got changed. Now, I will be showing the expanded form of these here. So, basically this vector is the displacement vector. So, this is the displacement vector corresponding to speed 1 and this is on q number of measuring planes. This is called speed 2, speed n. So, that means first 2 responses are corresponding to speed 1.

Next is corresponding to speed 2 and last is corresponding to speed omega n. So, you can be able to see the size of this is now q, which is one of the set of the measurement for one particular speed into n cross 1. So, this is the total size of the v matrix. Similarly, the influence coefficients, because now they are changing with the speed, and earlier the size of the influence coefficients was p into q.

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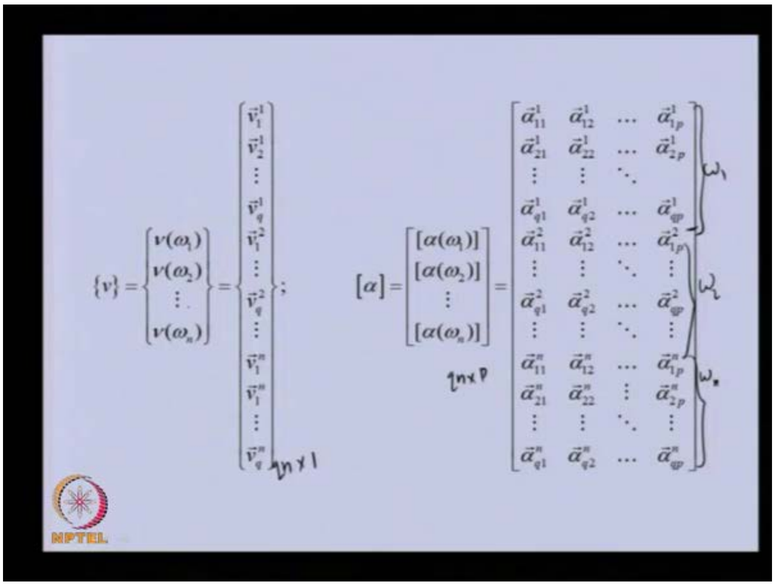

So, everyone is q into p. So, you can be able to see that the size will be q into n into p. So, that means q n times into p, because there are n such influence coefficients. In this the number of rows will be n times as one of the influence coefficients. Here, we have written in the expanded form. So, you can be able to see that the first is corresponding to this alpha omega 1 speed. Some of them I have shown here. So, these are corresponding to the n eth speed omega n and these are for omega 2 and this is for omega 1. So, the size of these influence coefficients and the arrangements of these displacement vectors are not clear.

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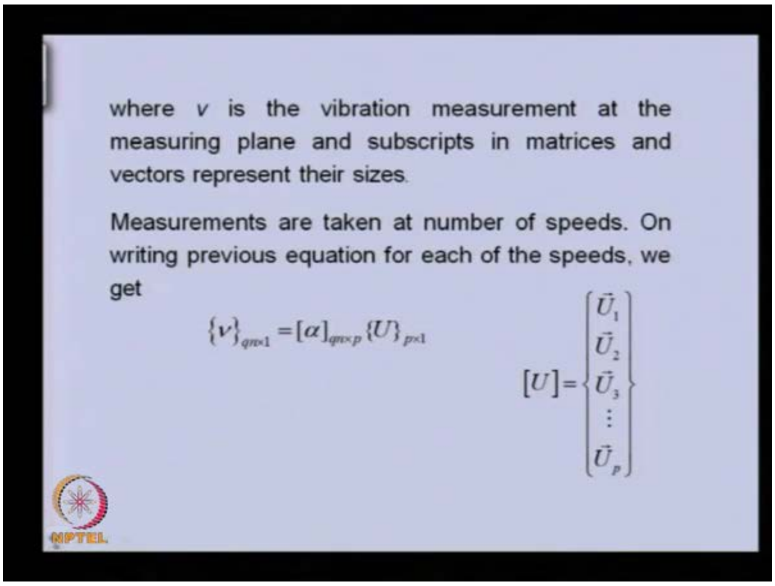

Now, we can be able to see that we could be able to write these equations for all such possible speed of operation of the rotor speed.

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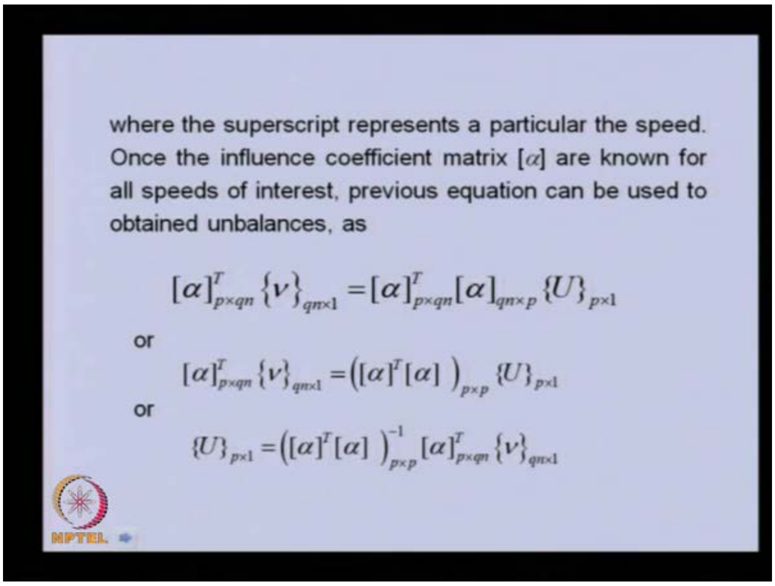

That particular equation, now we are using to obtain the unbalance. So, basically this particular equation, we can be able to use to obtain the residual unbalance which are there in the p number of planes.

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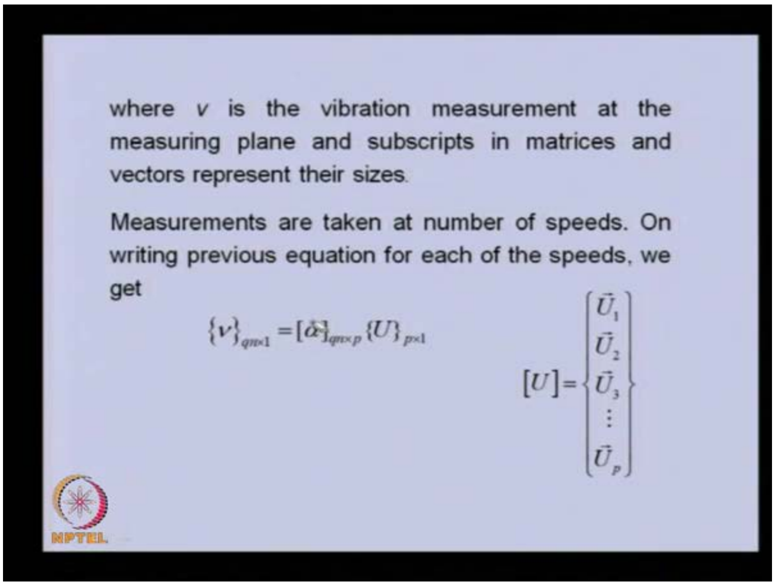

So, for this obviously we need to invert this matrix, but this is not a square matrix. This is the rectangle matrix.

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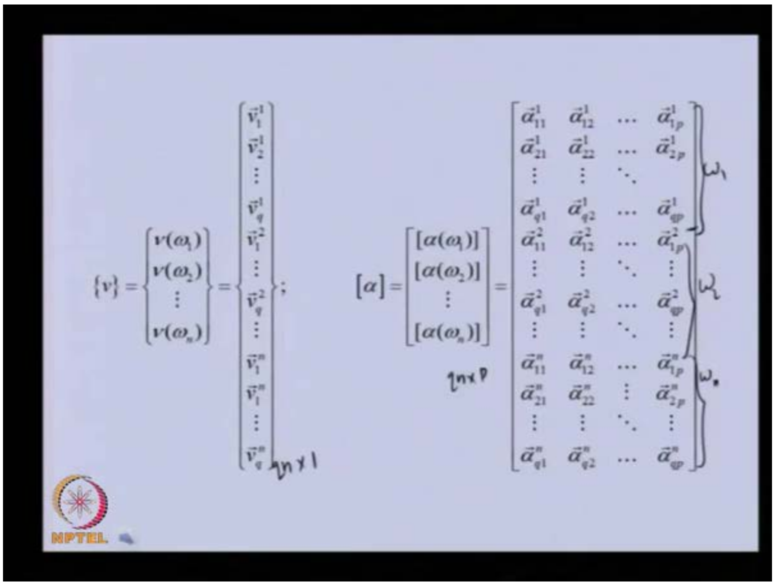

So, we are following this procedure to make it square. So, first to that particular equation we are multiplying transpose of the alpha matrix both sides. Now, we can be able to see. Earlier the size of this matrix was q n into p. So, transpose will be p into q. So, this will be size of p into p. Now, this is the square matrix and this is written here. This is the p into p. This can be inverted now. So, to get U we need to invert this particular matrix. So, we have inverted here and we multiplied by alpha this one and v.

So, we can be able to get what is the residual unbalances in p number of planes by this equation. But here at present only the response is known to us. But, these influence coefficients are not known. If we can find out these influence coefficients, the correction mass required in p planes we can be able to obtain it by using this. Now, let us see how we can be able to obtain these influence coefficients, because now our aim would be to obtain this first. Then, the correction mass can be obtained.

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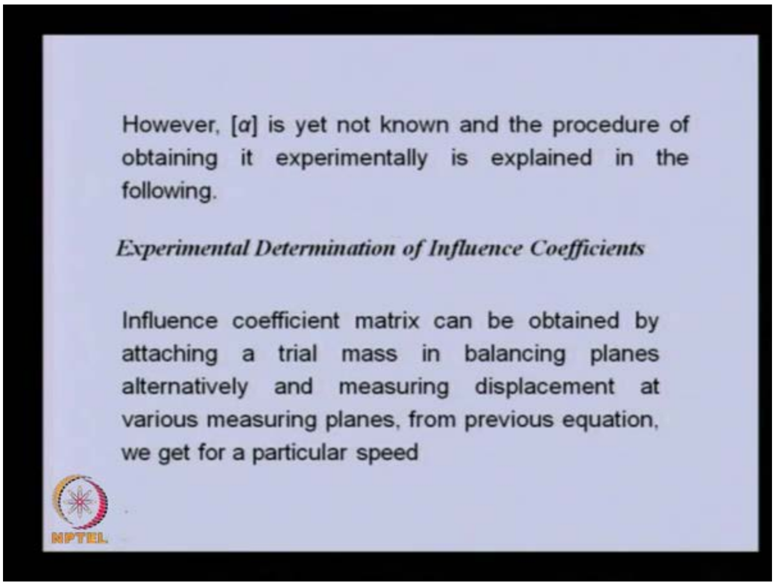

So, we will be obtaining these influence coefficients using the experimental method. That means, using the measurements only. So, this is the equation of the previous, there is a relation in which we related the influence of the response with the unbalanced force and the influence coefficients. The only difference here is we are keeping here the additional mass in the plane 1. So, these are the residual unbalances which we do not know. We want to find out these, but the procedure, which I am explaining here is to obtain the influence coefficients first.

If we can get this from the previous equation, we can be able to get these unbalanced responses. This is the residual response. So, this trial mass we know magnitude and orientation of that. We have kept in the first plane. Because, of that we expect and let us say we are operating the rotor at one of the speed. Let us say omega 1. So, corresponding to omega 1 we define the influence coefficients like this. So, they are belonging to the super script, representing these influence coefficients, which are corresponding to omega 1 speed.

These responses are also at speed 1, but when the second subscript is representing there is the change in the response due to the trial mass. So, earlier it was v 1 only when the residual unbalance was there. Now, we are keeping the additional trial mass. So, we expect that there will be some change in the response. That particular response we are differentiating here, as representing 2 subscripts 1 1 and 2 1. So, the second subscript is representing that we have kept the trial mass in the first plane. So, this is the relation which we had earlier for influence coefficients, but now the only thing is this trial mass is added here.

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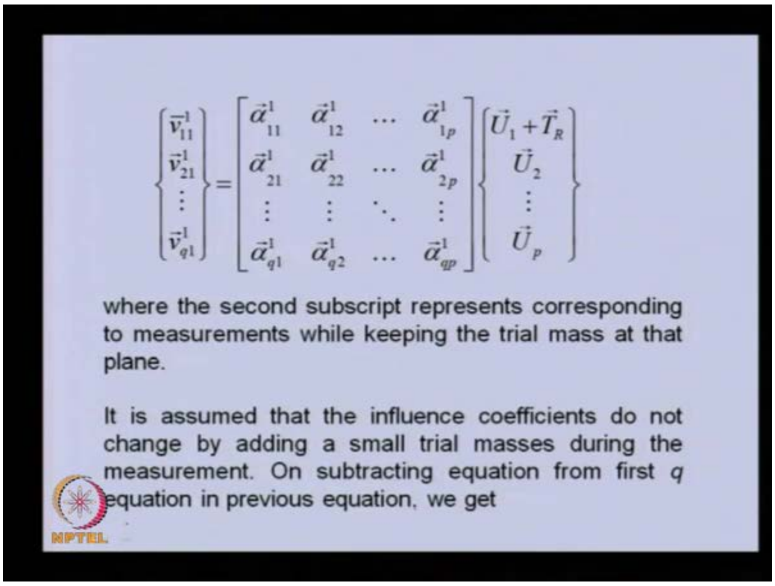

Now, you can be able to see we will go back to the previous equation.

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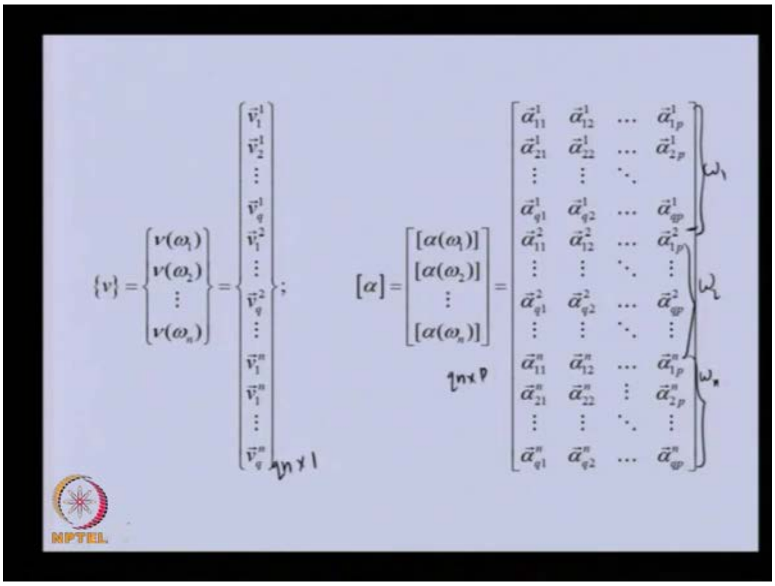

This equation basically, here v is W 1. Corresponding to this, these are the displacements and corresponding to this, these are the influence coefficients. These influence coefficients are same in the subsequent equation I showed. We expect that when we are keeping the trial mass, these influence coefficients will not change. They change only with the speed, but now if we keep some small trial mass, we showed that we do not expect that they will not change. So, this particular first set of q equation of this one in which we have only the residual unbalances, we are taking that expression and this expression.

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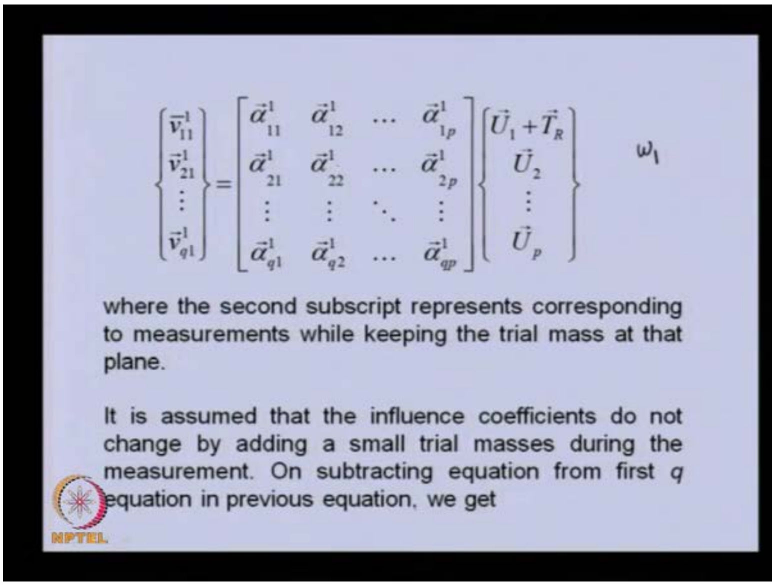

Basically, we are subtracting the first set of the previous equation with this.

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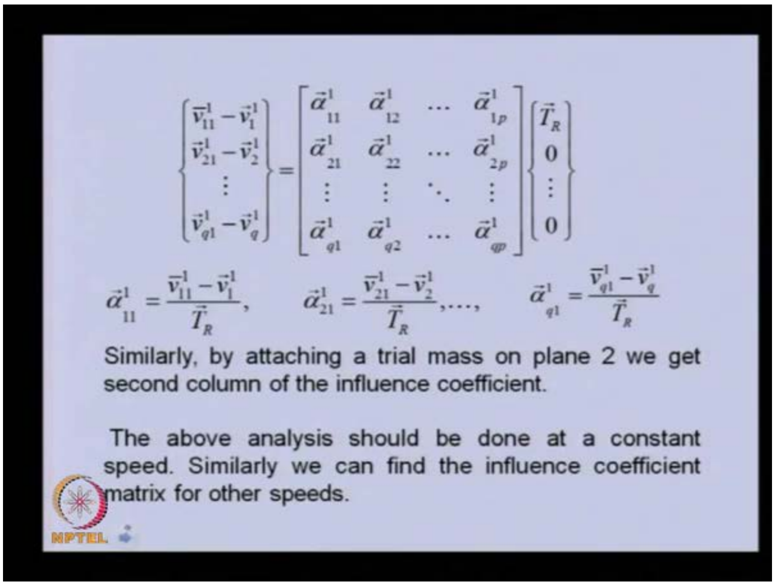

If we subtract it, we will see that this is the response corresponding to the trial mass. This is without trial mass. So, because we have taken only first q number of equation from that, so we will get these differences. This is common because influence coefficients is not changing with the trial mass. Here, all the residual unbalances will get cancelled because only the trial mass will left out. So, that means once we are taking the difference of these two equations; one with trial mass and another without trial mass, we will get this equation in which all the residual unbalance will be cancelled out. Now, if we expand this equation, let us see the first equation. So, this will give us the first influence coefficients alpha 1 1.

So, for script 1 this is divided by T r. So, these influence coefficients can be obtained using this expression. We have measured these responses, both the amplitude and phase of these responses. Also, the trial mass, we know the magnitude of that and at what orientation we have kept. These influence coefficients can be obtained from first equation. Similarly, these influence coefficients can be obtained from the second equation. If we expand the second equation, most of the term will get cancelled. Only this will be remaining. So, we can be able to get these influence coefficients.

Now similarly, we can be able to obtain all others up to this. So, when we kept the trial mass at the first plane, we saw that we could be able to get the first column of this particular influence coefficients for first speed omega 1. Now, in the second step what we will be doing is that we will be keeping the trial mass in the second plane here. Then, we expect that we will be getting the influence coefficients corresponding to the second column for the same speed omega 1. Similarly, we can be able to go up to n th plane. So, if we keep the trial mass in the n th plane, we will get the influence coefficients of the last column.

So, this is the procedure by which we can be able to get the influence coefficients for one particular speed. The similar procedure we need to follow for the other speed, that means omega 2, omega 3 up to omega n, that is the speed of operation of the rotor. So, in this whatever the influence coefficients we are obtaining, this is based on purely the measurements. So, based on measurements we can be able to get these influence coefficients.

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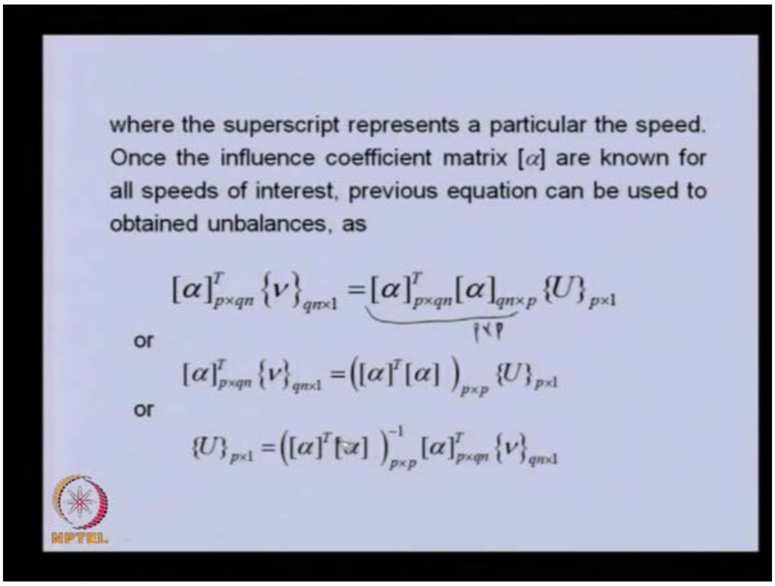

So, once we obtain these influence coefficients for all the speed, then we can be able to get the residual number of p number of planes by using this expression for all the speed, because this is independent of speed. Once we know, what is the residual unbalance in p number of plane, we can be able to balance the rotor by putting those correction masses just opposite to the residual unbalance in p number of plane. So, like this we can be able to do the balancing of the flexible rotor using influence coefficients method. So ok, how much time?

Student: 27 only, 27.

Now, we have described the method of influence coefficients for flexible balancing. Now, through some case study we will try to understand and illustrate this particular method. In this influence coefficients method we observed that the influence coefficients depends upon the speed. So, when we are trying to balance a particular rotor, we will be having some operating speed of the rotor. When we want to reach up to that particular operating speed, then we want to traverse several critical speed in between or sometimes may be we need to change the speed of the rotor continuously from one speed to another.

In that, in between we may encounter some critical speed. So, obviously in this particular case we can choose the speed at which we want to find out the influence coefficients and how we can be able to balance the rotor up to that particular mode. It is not necessary that we should try to balance the rotor for all the speed, but what was the bind of speed or range of speed in which we want to operate the rotor? We should try to balance them. In this, we have seen that all the calculation or the stimulation of the influence coefficients is purely based on the measurement. So, in this the measurement quality is very important. Once we ensure that, then we can be able to get the better balancing of the rotor.

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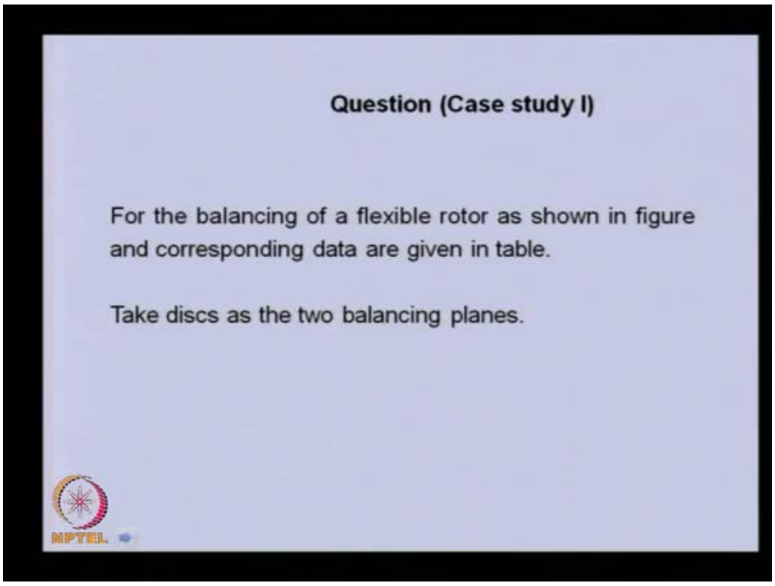

Now, through a simple case study, we will try to see this particular method in more detail. So, first let us see, what is the problem, which we are taking for balancing of the flexible rotor as we have?

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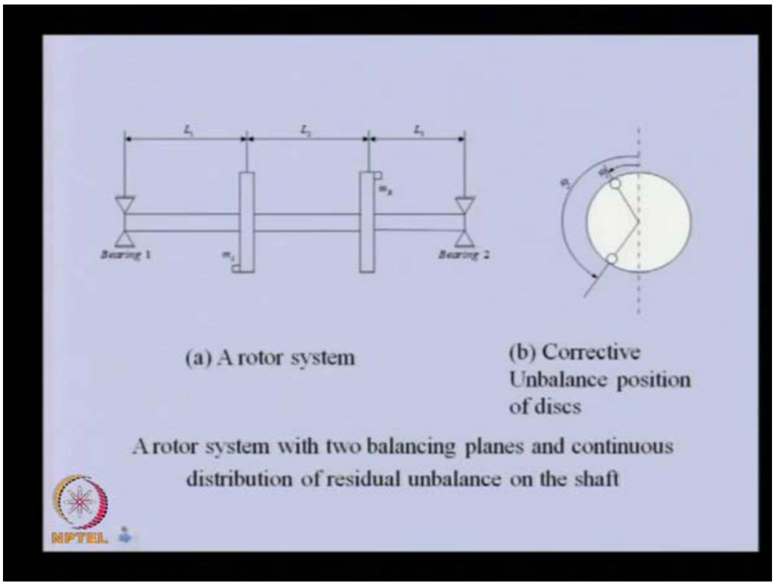

Shown in this particular figure, is the simple rotor in which the simply supported end conditions are there. In this, we are considering the shaft is having distributed mass property. Apart from that, there are 2 rigid disc mounted on to the rotor, on to the shaft. In this case, because the shaft is having the distributed mass property, so we will be modeling this particular motor using finite element method. In this case, basically because we are doing some kind of numerical simulation of the influence coefficients method, so what we will be doing is that we will be obtaining the response of this particular rotor system using the numerical study.

So, instead of the experimental set up, we want to illustrate the method using a numerical model. So, generally to test any particular method instead, sometimes we need to test that particular using numerical model itself as an initial trial. Once we get the testing of the method with the numerical responses, then we can be able to apply the thing for the actual element response. But, in this particular case we will be showing only using the numerically generated response. With that, we try to illustrate the method. As I mentioned, we will be modeling this particular rotor using finite element method.

All the responses which we require, that also will be generating with the numerical simulation. Because, we need some kind of residual to unbalance, some assumed residual unbalance we will be keeping in this disc and through response we will try to stimulate them. So, that we will be showing 2 case studies, 1 is only the residual unbalance will be keeping in this 2 disc. We will try to balance this particular rotor up to the second critical speed because we have chosen only two planes for balancing. So, this correction mass, this residual unbalance which we will be keeping, that will be used for generation of the response.

But, again re estimation of these we will be doing with the influence coefficients methods. We will try to see whether we can be able to balance the rotor to the second mode. In this second case study, we will be extending the method when we have residual unbalance in the shaft also, along with the residual unbalance in the disc. That means, we will be choosing some distributed, that is the residual unbalance of the shaft in the form of sin function. Then, we will try to balance that particular rotor up to second speed using these 2 as balancing plane. So, that will be the second case study. For illustration, we always will be using the numerical responses. So, in this particular rotor which is flexible and having distributed mass property, even we have tabulated the various rotor in a table. So, let us see the table.

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Data for the rotor shaft system with helical distribution of residual unbalance

Sl. No.	Property	Assumed value
1	Diameter of the shaft	10 mm
2	Young's modulus of the shaft and disc materials	2.1×10^{11} N/m^2
3	Length of the entire shaft	0.409 m
4	Distance between left bearing and left disc	0.1375 m (L_1)
5	Distance between the two discs L_2	0.157 m
6	Distance between the right disc and the right bearing L_3	0.1145 m

So, in table this is the diameter of the shaft, the property of the shaft, that is Young's modulus of the shaft and disc, the total length of the shaft, distance between the bearings, distance between two discs, and various other geometry.

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7	Mass of each of the balancing disc is	800 gm
8	Density of the material of the shaft	7800 kg/m^3
9	The trial mass kept on discs one by one	4 gm
10	The phase angle of the trial mass of the disc	40°
11	The eccentricity of the trial mass on the disc	37 mm
12	The outer diameter of the rigid disc	60 mm
13	The inner diameter of the rigid disc	10 mm
14	The speed range of operation	0 - 4000 rad/s
15	The eccentricity of the unbalance in the shaft	0.2 mm

Apart from that, we take the mass property of the disc and density of the shaft, because we are considering the distributed prop mass property of the shaft, so this will be requiring in finite element of the formulation of this. These are the trial unbalance. We have seen in the influence coefficients methods, we need to keep some trial mass in plane 1, in plane 2, plane 3 alternately. So, this trial mass will be keeping one by one, because we have only 2 balancing plane.

We are aiming at balancing up to second mode. So, we will be keeping the trial mass first in the first case in the first plane and in the second plane, the phase of that is given the unbalance. At what orientation we should keep this is? The is the eccentricity on the trial mass of the disc 2. This is basically the radial position of the trial mass, because we need to keep at certain radial position. So, that is the eccentricity we are calling. These are the other geometry of the shaft. The speed range of operation is up to 4000 radiation per second. In this, may be third or fourth critical speed of the rotor system may fall, but we aim only to balance the rotor of the second critical speed.

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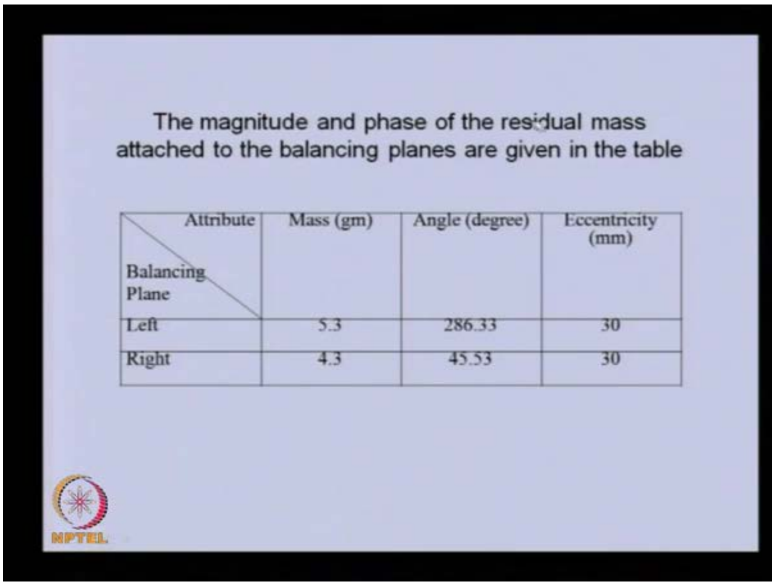

So, this is eccentricity of the unbalance in the shaft. So, this is regarding the residual unbalance and this is the trail mass. So, there are 2 different things. So, this is the residual unbalance basically and we have 2 planes, left plane and right plane. So, mass in grams of the residual unbalance are given here. The orientation is given. The eccentricity is given here. Now apart from this, some more information regarding the simulation we will be getting.

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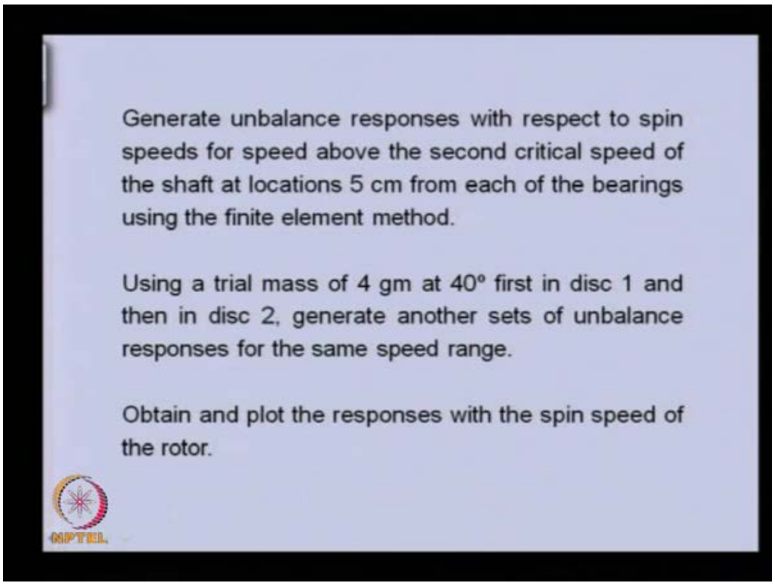

So, we will be generating the response above the second critical speed and we will be showing the response at the location on the shaft which is 5 centimeters away from the bearing. So, near the bearing the response we capturing from the finite element formulation, we will be plotting this to show, to illustrate the influence coefficient method. These are the trial masses, which we will be using, which is given in the table. Also, now let us come to the finite element formulation of this.

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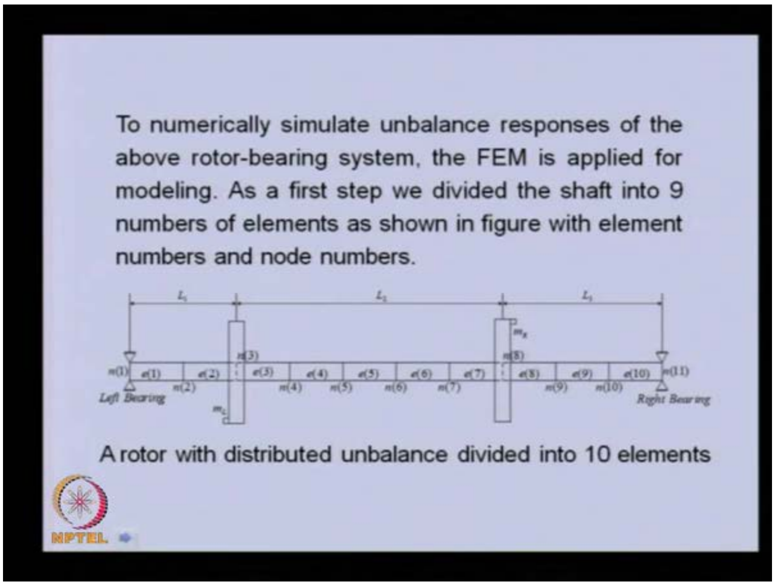

So, this is the rotor. So, we divided this into 10 elements and various nodes and the elements are mentioned here. All geometry of the rotor we know. So, we can be able to obtain the elemental equation for a rotor.

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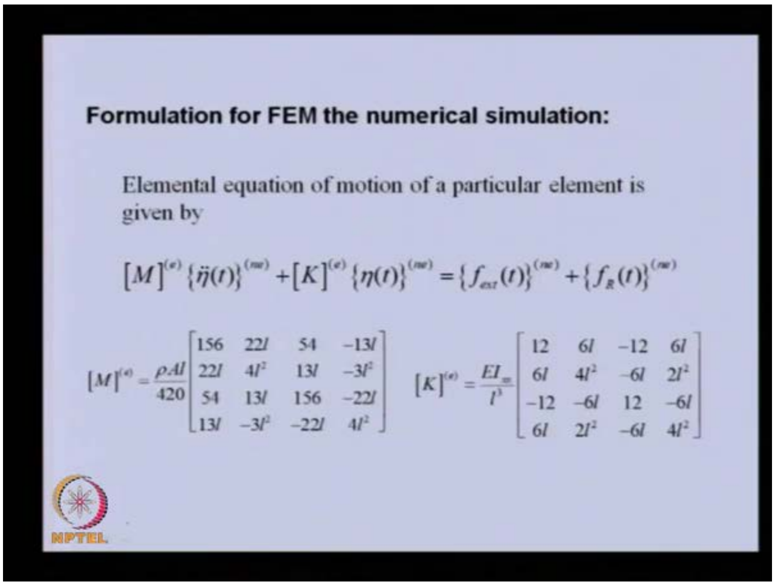

So, elemental equation of a motion for particular element is given by this. So, in general we already obtained earlier, discussed this kind of formulation. For one of the element, the typical elemental equation is like this, in which this is the external force, this is the reaction force and stiffness matrix, mass matrix and this is the placement vector. This mass and stiffness matrix, which we derived earlier was having this particular form. Now, with the help of this representative element matrix, we will be writing the actual mass and stiffness matrix for various elements.

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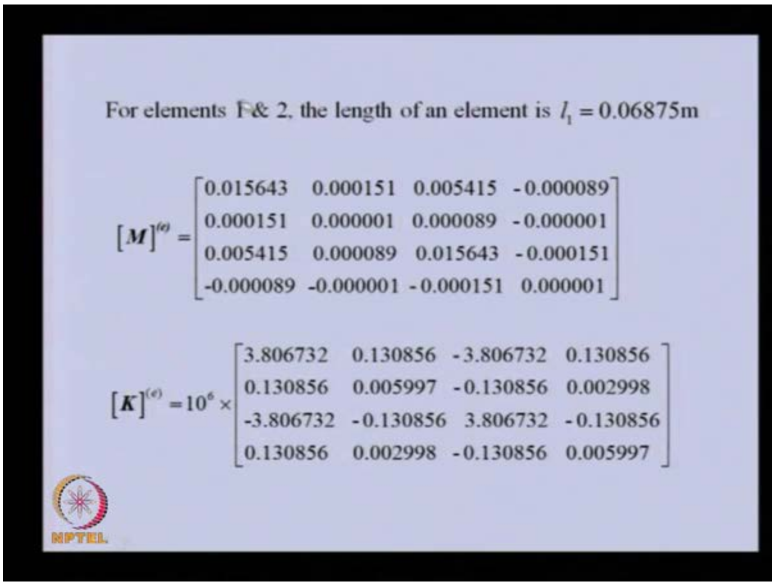

Like, for element 1 and 2, which are identical, the mass and stiffness matrix will be of this length of the element is this. One other property of the shaft element, we have already given in pre table. So, that can be used to obtain the mass and the stiffness matrix for element 1 and 2 of the shaft. So, element 1 and 2 mass stiffness matrixes are like this.

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For elements 3 to 7, the length of each element is $l_2 = 0.0314$ m

$$[M]^{(e)} = \begin{bmatrix} 0.007145 & 0.000032 & 0.002473 & -0.000019 \\ 0.000032 & 0.000000 & 0.000019 & -0.000000 \\ 0.002473 & 0.000019 & 0.007145 & -0.000032 \\ -0.000018 & -0.000000 & -0.000032 & 0.000000 \end{bmatrix}$$

$$[K]^{(e)} = 10^7 \times \begin{bmatrix} 0.007144 & 0.000031 & 0.002473 & -0.000018 \\ 0.000031 & 0.000000 & 0.000018 & -0.000000 \\ 0.002473 & 0.000018 & 0.007144 & -0.000031 \\ -0.000018 & -0.000000 & -0.000031 & 0.000000 \end{bmatrix}$$

Now, from the element 3 to 7, there is between 2 discs, the element is having this. The length, the mass and stiffness matrix will be like this. These are only corresponding to shaft and in this we are not considering this particular inertia. They are 0. Now, you can be able to see that this are corresponding to the element 3 to 7 and next is from 8 to 10. So, from second disc right hand side, there are 2 more elements. Corresponding to that, these are the mass and stiffness matrix and the disc mass; that we can be able to write either in one of the elements. So, as we have discussed earlier the disc mass is a lumped mass, so we can be able to write that with element 2 in this node or this particular element in this node.

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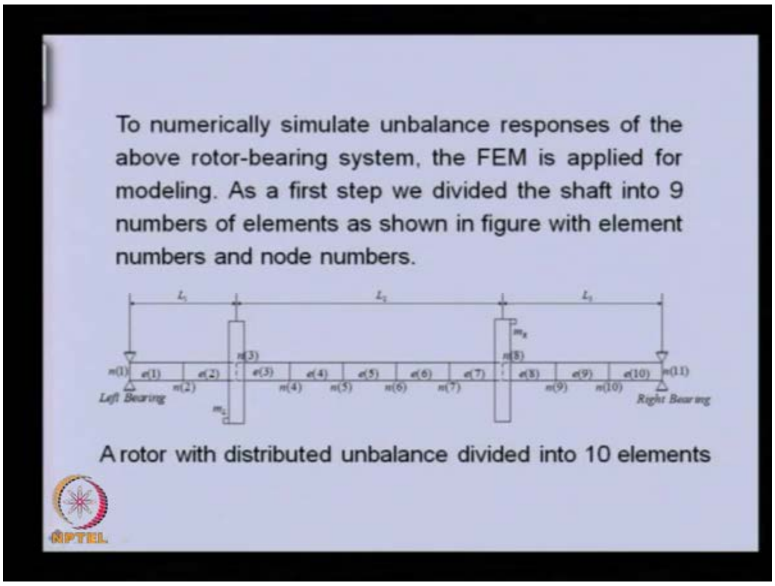

So, we need to add this one of the node. So, similarly this side also, once we have obtained the elemental mass and stiffness matrix of various elements, we can be able to assemble them.

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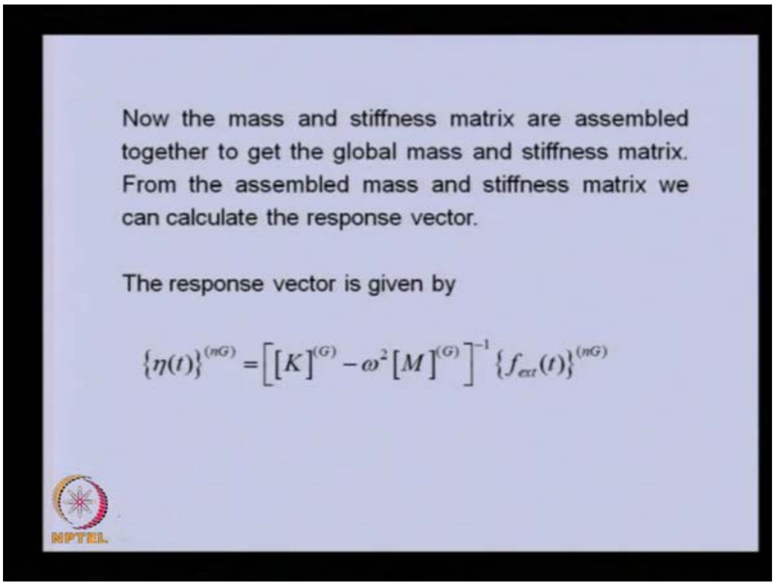

That assemble procedure, we have already discussed. Even, you can be able to apply the simply supported boundary conditions in this. Then, the force response can be obtained like this. This also we discussed earlier. Because, of the forced vibration and the main force is unbalance, so we expect this omega will be the spin speed of the rotor. This is the unbalanced force. So, this we have obtained by the global equation motion. After application of the boundary condition, this equation can be use to obtain the response.

So, that means whatever residual unbalance we have chosen corresponding to that, we can be able to a write the force. Then, we can be able to obtain the response corresponding to the various speed of the rotor system and forcing the vector we will be having, because we have 2 discs only where this unbalance will be there.

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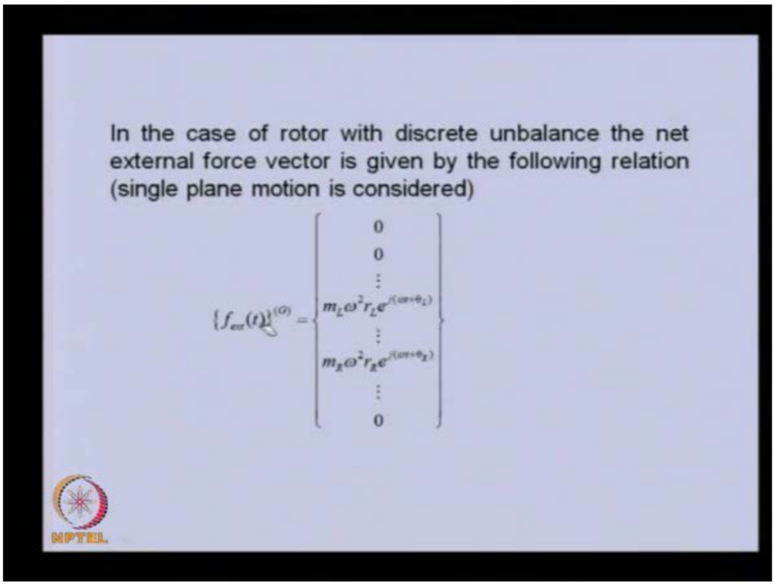

So, these are those corresponding forcing terms. Now, we are coming to the generation of the ... ((Refer Time: 40:42)).

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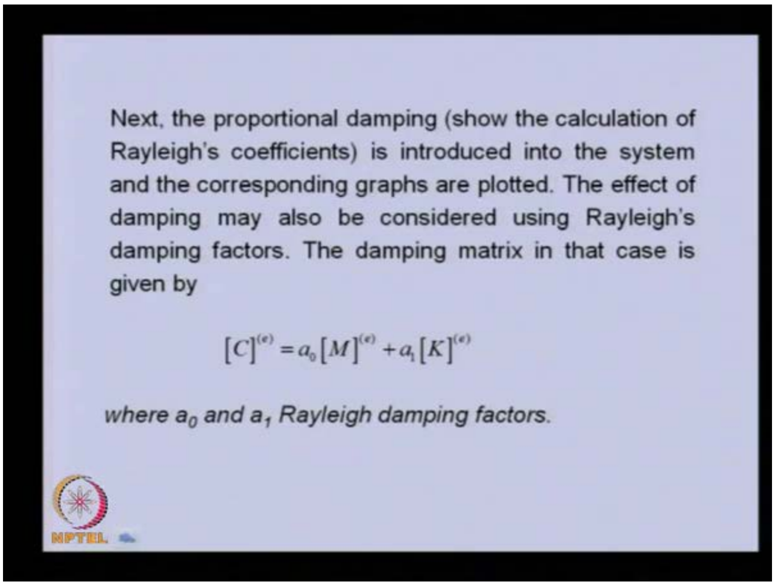

Apart from this, we are adding the damping in the system, so that the response are more realistic. So, here we added this proportional damping. So, the Rayleigh damping also we described, while discussing finite element and finite element analyses of the rotor system. So, this is the mass and stiffness element a naught and a 1 is the Rayleigh damping factor, which we can be able to obtain using this expression, in which this is the damping ratio and these are the natural frequency for particular mode,

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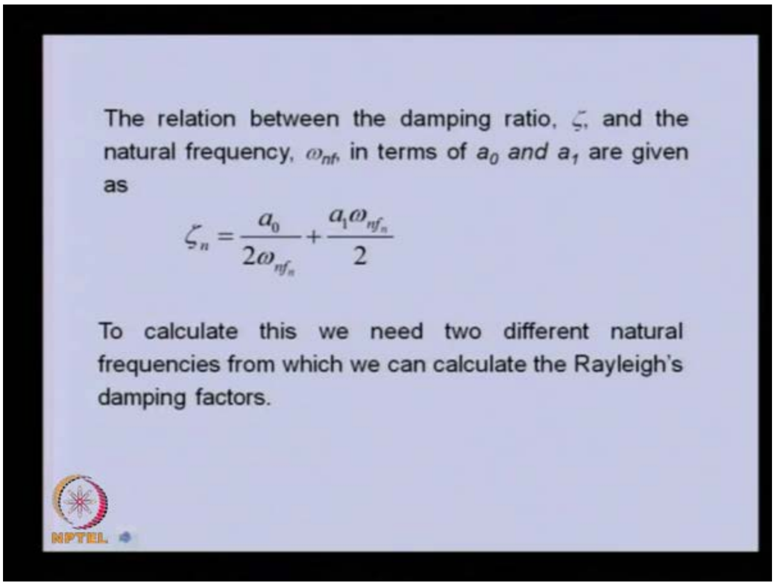

So, if we take two different modes, we can be able to get the these particular factors.

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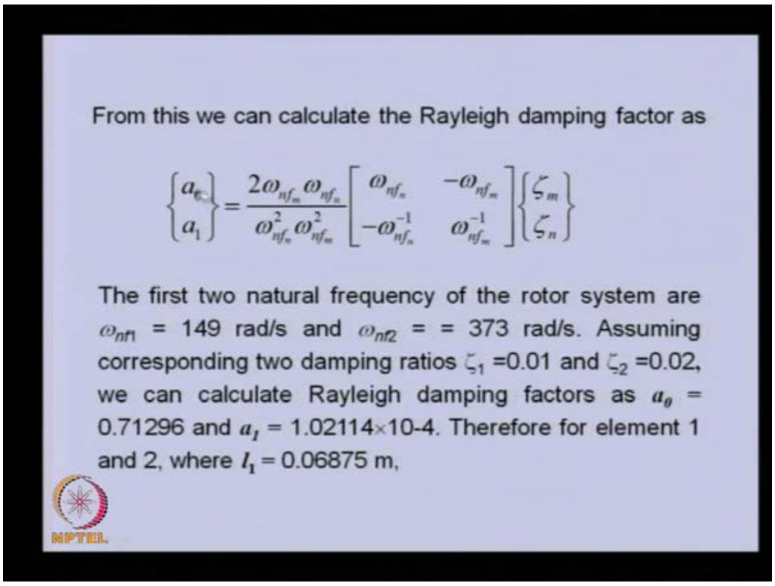

Like this, we already described. So, I am not describing this in more detail. So, when we are discussing the finite element formulation for the transfer vibration, we will describe this particular kind of damping. So, using this in this particular case for free vibration, if we want to solve this we found the natural frequency of system was 149 and 373 radians per second. So, corresponding to this, because we are trying to balance this particular rotor for 2 critical speeds, so will be having that.

This factor will be calculated corresponding to those particular modes. So, we have chosen this damping ratio for first mode and second mode. Like this, correspondingly we can be able to get these two factors. So, once we get this factor, we can be able to generate this damping matrix in terms of the mass and stiffness matrix. So, this also we can plug with this. So, this is damping matrix for 1 and 2 elements. This is for 4 to 6 and this for 8 to 10. So, these are the damping matrix.

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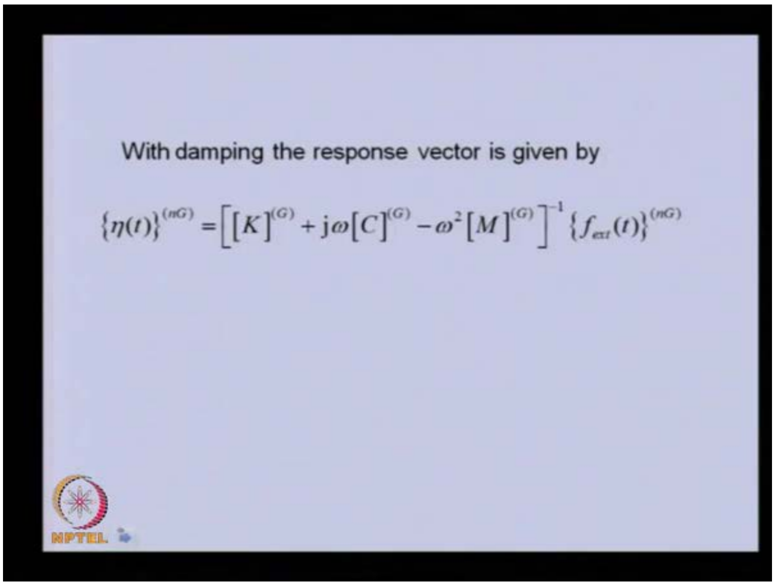

That also we include in the equation of motion, for this forced vibration response, we will change like this. So, this will be the additional term which will be coming, because of the damping. The other analyses will remain same.

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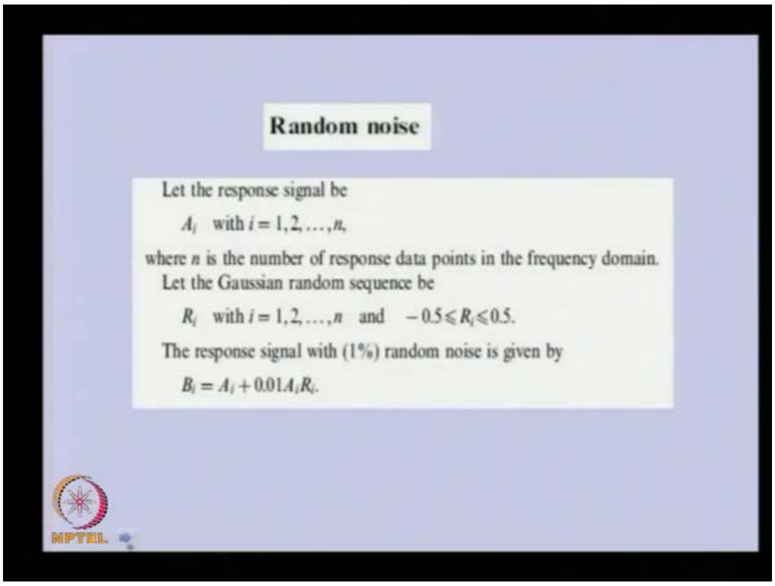

Now, apart from this, because whatever the response we are simulating from the numerical model, they will not having any noise. But, in actual test rate when we take measurement, we find that measurement noise to limit that we are adding random noise into the numerically generated response. This we will be using for estimation of the residual unbalance. So, this is the procedure in which we have added the random noise. So, if response is let us say A i, n number of response are there and R i is a random sequence which is also n number. The value of this, we have scaled down from minus 0.5 to plus 0.5, because we can be able to generate any random number. We can scale that into this range. So, the noisy response we can be able to get B as A i plus 0.01 A i into R i. So, this 0.01 is corresponding to 1 percent random noise into the system. If we are adding 2 percent or 3 percent noise, this factor will change. Otherwise, this expression will be same.

So, just to limit the actual experiment, we added some noise also into the system. Now, we have to describe the problem completely and also the finite element formation of that. Now, I will be showing various responses, how we can be able to use that for calculating the influence coefficient? How we can be able to reach to make the residual unbalances, which we have used for generating the numerical responses?

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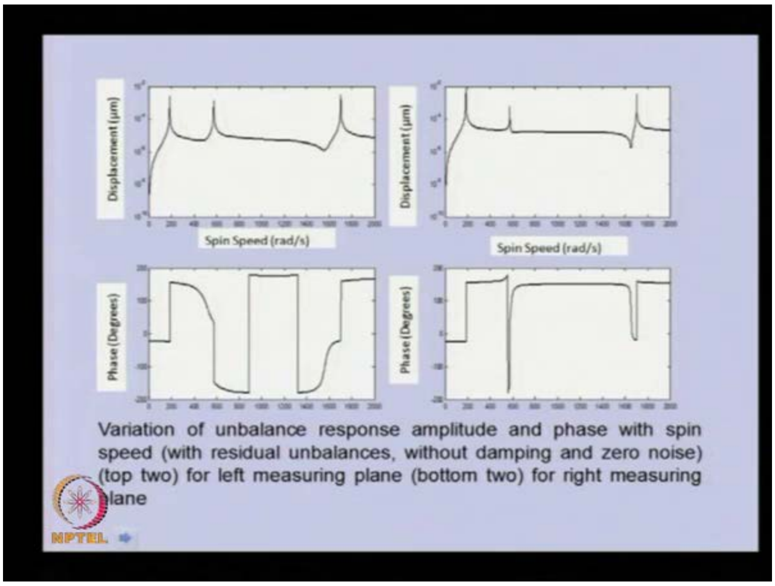

So, this is the typical plot of the unbalanced response in which we have shown this is the amplitude versus speed and this is the phase verses speed. In this, we changed the speed of the rotor beyond the third critical speed, so this is the third critical speed, first, second and third and in the phase plot you can be able to see there is a change in the phase wherever there is a change in the phase. Here also, these changes are basically because the phase is cyclic.

So, if it is 0, here the 361 be here. So, basically this line should have been here itself. As such there is no phase change here. The phase changes are only at the critical speed locations. So, one of that is a left side and this is the other side of it. So, this is the rotor response and we expect the similar critical speed in all the responses. So, in this particular case, we have not considered damping. Also, noise is added in this particular response. You can able to see this one is for the left measuring plane and another is for the right measuring plane. At two bearing locations, we have shown the responses.

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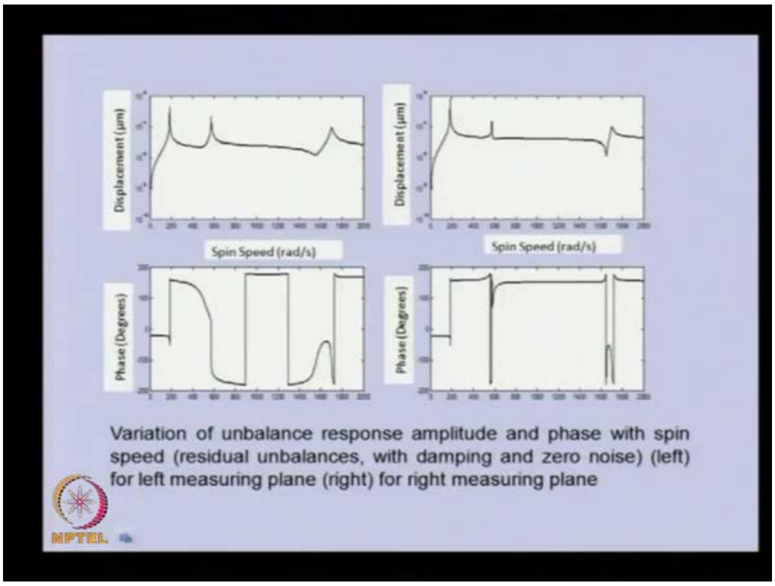

Variation of unbalance response amplitude and phase with spin speed (residual unbalances, with damping and zero noise) (left) for left measuring plane (right) for right measuring plane

Similarly, this is a similar response, but we have considered the damping. We can able to see that, because of damping these peaks are getting blunted slightly, but noise is not there in these particular responses. So, initially I am just showing typical response of the system. This is with the damping and some noise also we added. So, you can be able to see some haziness of the response, which is because of the 3 percent noise added to the system.

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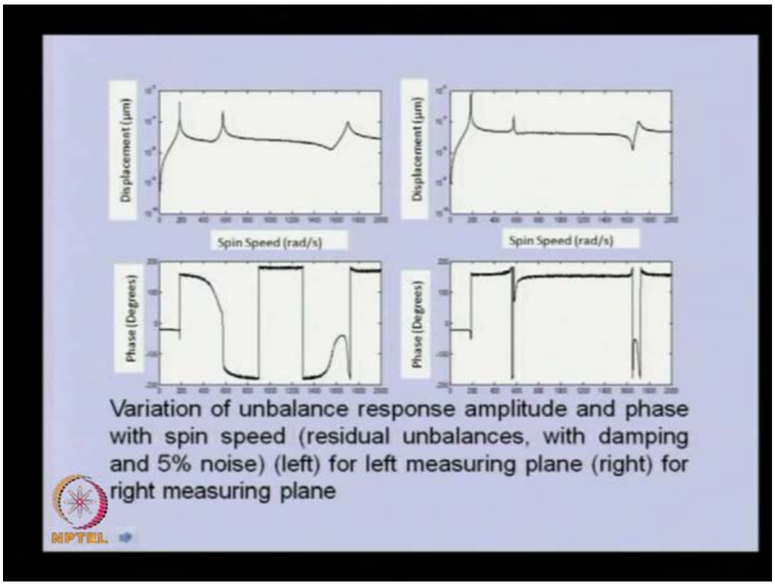

Variation of unbalance response amplitude and phase with spin speed (residual unbalances, with damping and 5% noise) (left) for left measuring plane (right) for right measuring plane

In this, an addition of the 5 percent noise; you can be able to see the change of the signal because of the more noise on the system. So, basically this addition of the noise will show, what is the effect of this particular noise on to the estimation of the residual unbalance or to balancing of the rotor?

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Comparison of different correction mass and its phase with different noise level

Balancing Plane \ Attribute		Correction mass (gm)	Phase of correction mass (deg)	Eccentricity (mm)
0% noise No damping	Left Plane	0.53	286.3	30
	Right Plane	0.43	45.52	30
3% noise	Left Plane	0.54	286.02	30
	Right Plane	0.43	45.86	30
5% noise	Left Plane	0.54	285.82	30
	Right Plane	0.44	46.08	30

So, then we estimated for all these cases, what is the correction mass in left plane, not only the magnitude, but also the phase. Eccentricity is same for all. So, this we estimated. So, you can be able to see this estimation for 0 noise. When we are adding the noise, slight change estimation are there, but not much. This estimation, if we compare with the assumed one which is in the next line, so you can be able to see for 5.3, 286 around. it is 6 around, so 5.53. So, basically these are all 10 raise to minus one is there So, all are 5.3, 4.3, like this. So, they are matching quite well with the assumed value.

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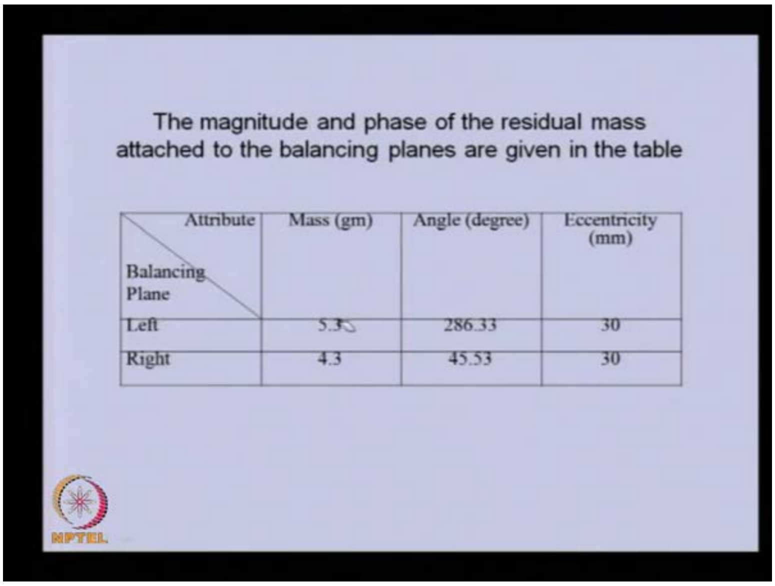

Balancing Plane \ Attribute	Mass (gm)	Angle (degree)	Eccentricity (mm)
Left	5.3	286.33	30
Right	4.3	45.53	30

So, these are the assumed. These are residual unbalance left plane and the right plane of the disc.

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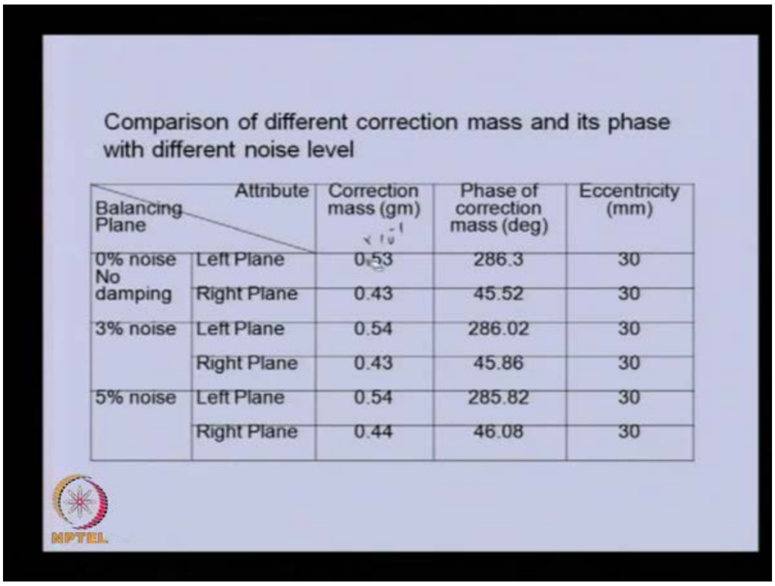

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	Right Plane	0.43	45.86	30
5% noise	Left Plane	0.54	285.82	30
	Right Plane	0.44	46.08	30

These are estimated using the influence coefficient method directly from the responses. So, they are telling quite true value.

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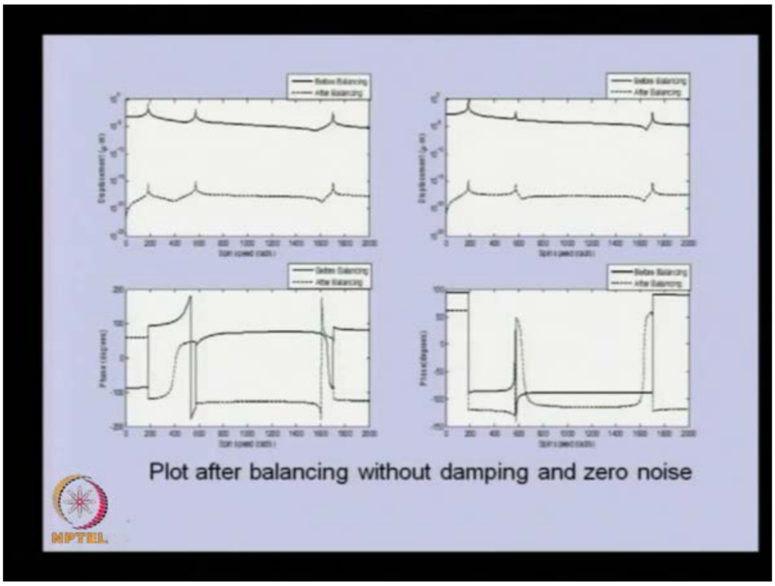

Now, when we are comparing the responses, this is without a correction mass and this is without balancing. So, this is after balancing. So, whatever this estimated correction mass we calculated, we kept into the balancing plane. Then we found that the responses are drastically reducing in both the plane. Here, also you can be able to see the response drastically reducing.

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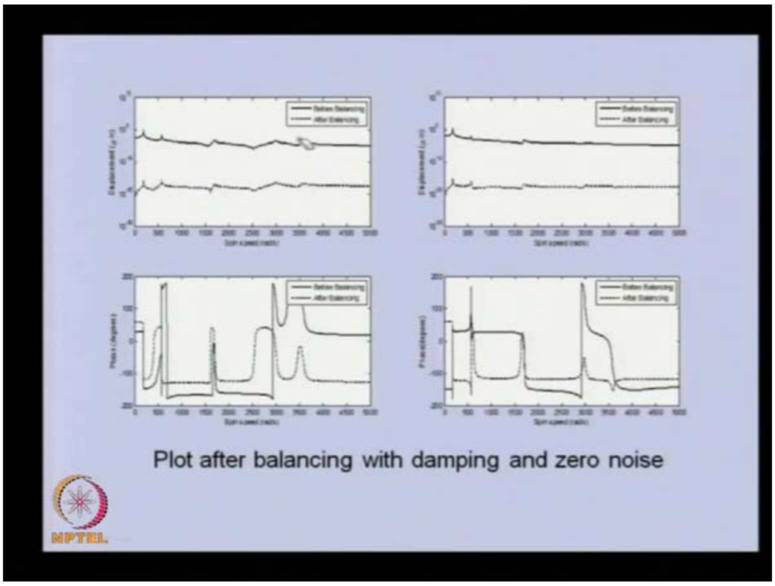

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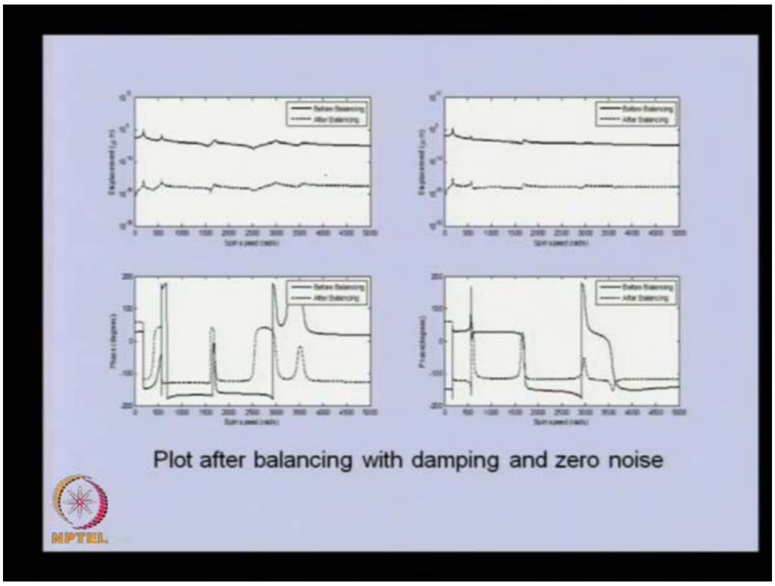

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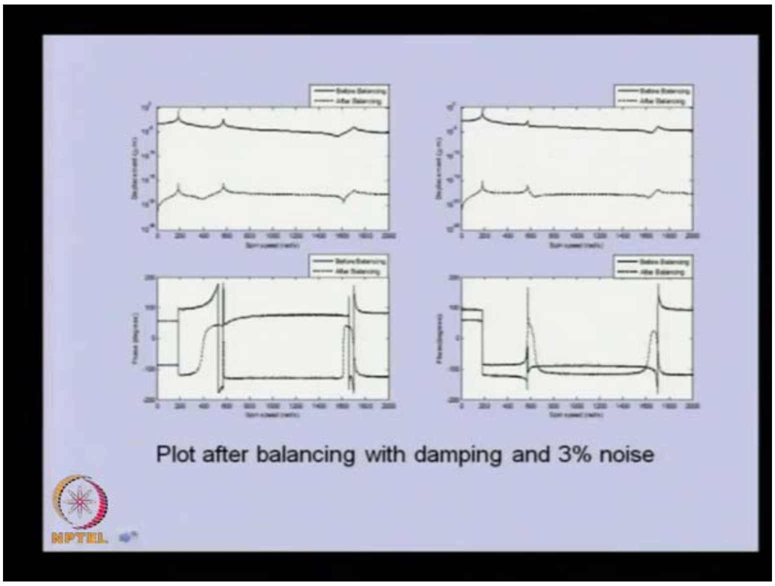

In all the cases, the response are drastically reducing with the noise.

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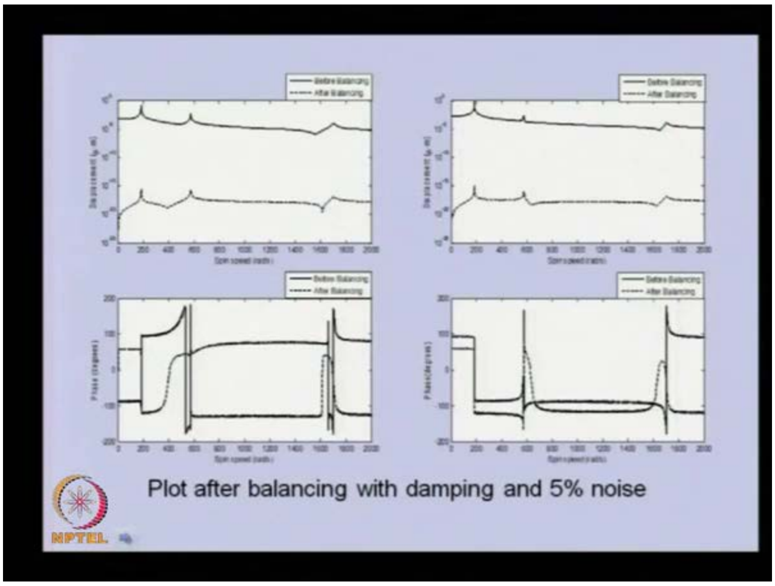

We can be able to reduce the response at the third critical speed, because in this particular case the residual unbalance was discrete in nature only as the disc was there.

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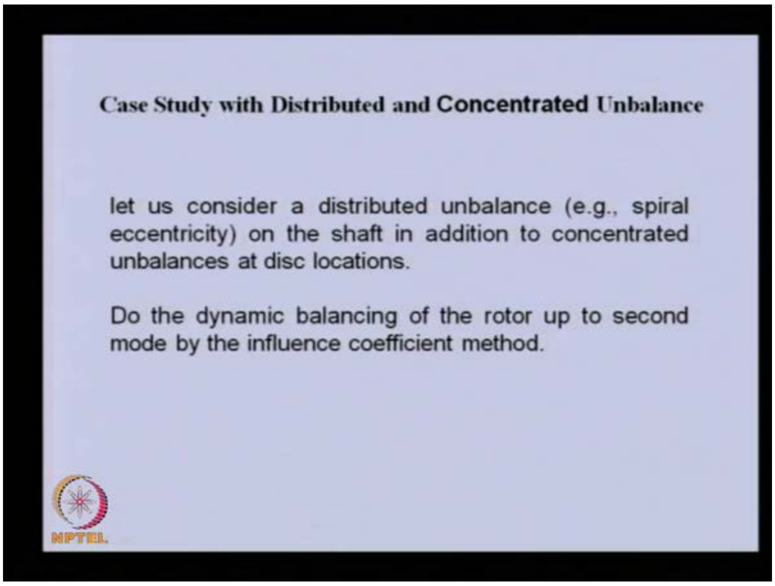

In the second case study, we took not only the concentrated unbalance, but also distributor unbalance was there.

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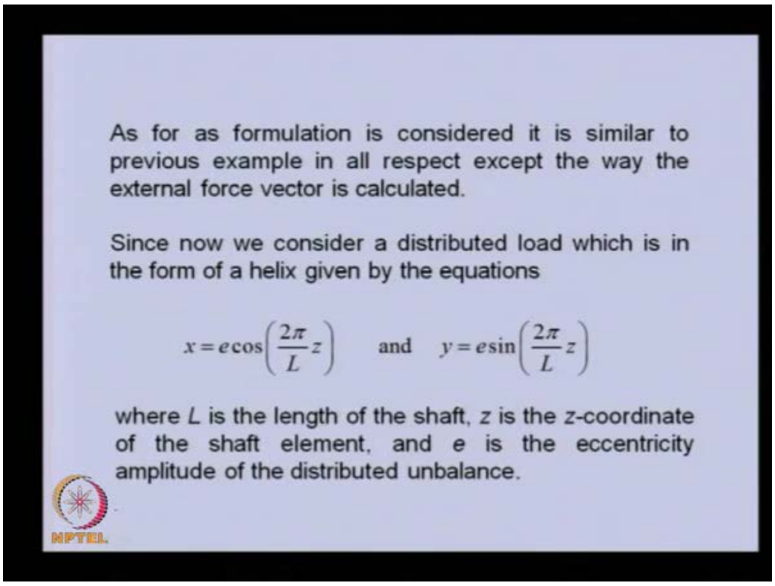

In this, we took the eccentricity variation like this of the unbalance and we assumed that, because the shaft was having unbalanced variation something like this.

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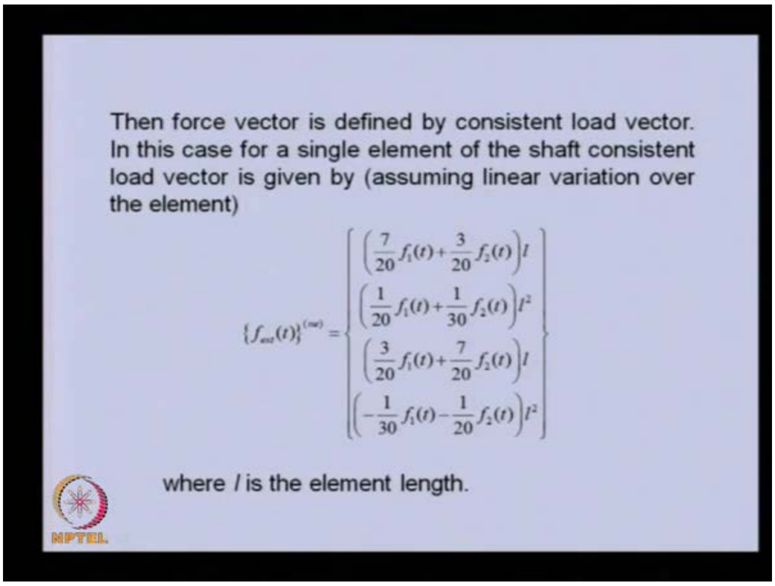

When we are taking the elements, so we assumed that in between the element, the variation of the eccentricity is linear and the amplitude of the that particular eccentricity is at two ends of the elements which are known, which are f 1 and f 2. They are based on the linear variation. We could be able to get the consistence force factor. We have shown the calculation of the consistence force factor when we discussed this FEM analyses of the transfer vibration. So, I am not repeating this. So, this directly we used in the formulation for calculation of the unbalanced force, because of the distributed eccentricity.

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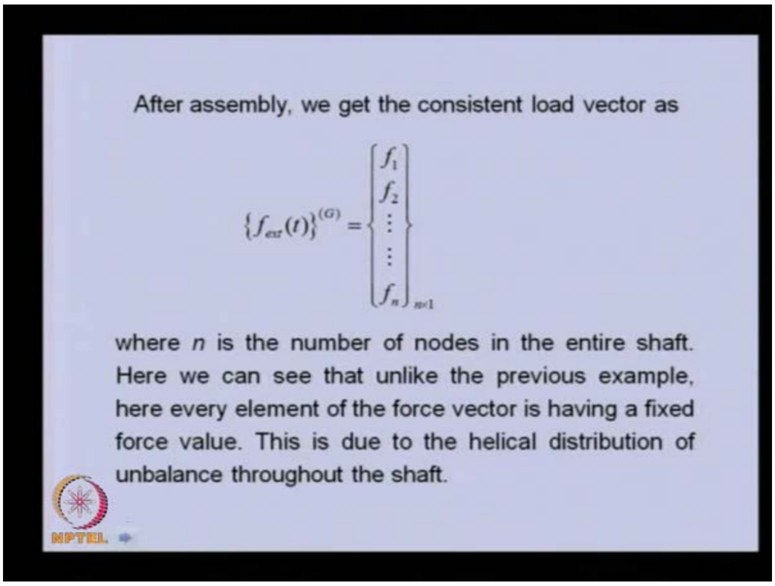

So, in this particular case, we will see that now the distributed and unbalance will be there in all the nodes because of this one.

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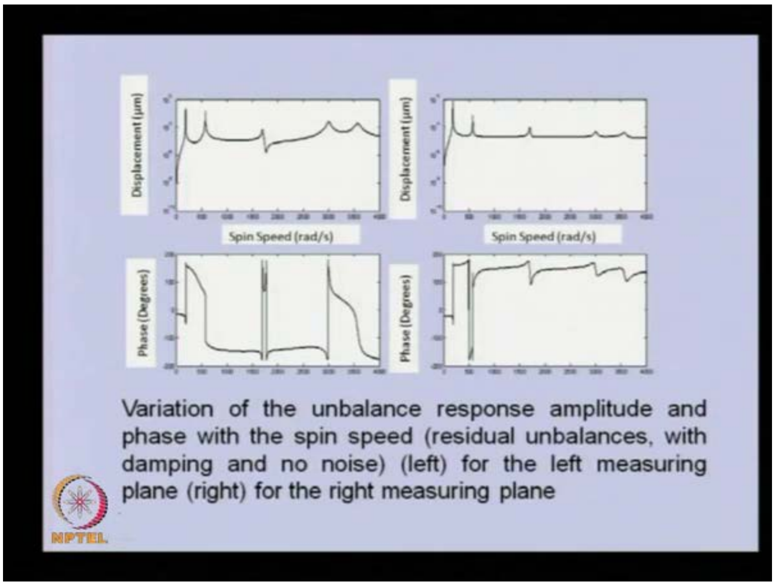

This is the typical response. Now, in this particular case, we have shown higher mode also. Similarly, as we discuss various cases of the previous one, so here there is no damping. But, no noise is there. This is the response generated with some noise in the system with the damping.

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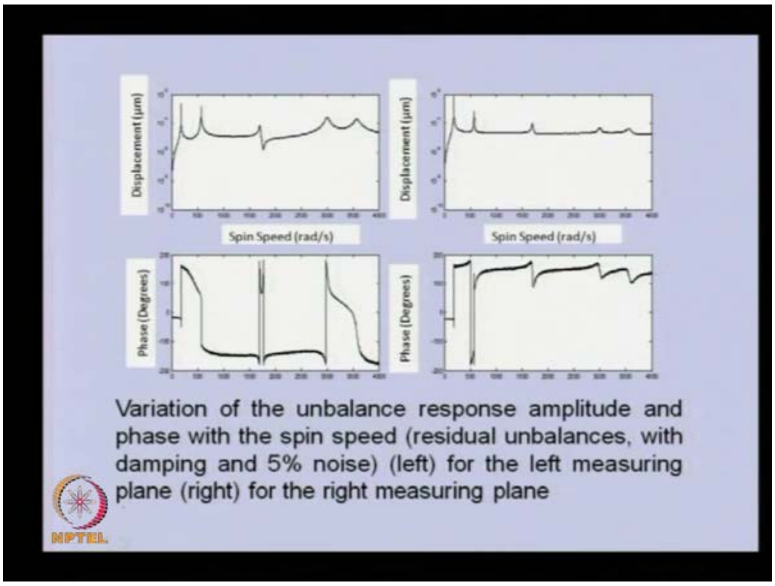

These are the responses due to the residual unbalance, which is there in the system.

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Estimated residual unbalance mass and phase

Noise	Balancing Plane	Estimated residual unbalance mass (gm)	Estimated residual phase angle (deg)
0% noise	Left plane	4.15	-18.1
	Right plane	5.84	28.5
3% noise	Left plane	4.15	-17.9
	Right plane	5.83	28.5
5% noise	Left plane	4.16	-17.7
	Right plane	5.83	28.5

Then, the estimation of this using these responses, using influence coefficient we did the residual balance calculation. So, these are various estimation in for various cases. There, the consistence we are obtaining this, basically plane in left and right. So, basically this is the effective correction mass, we are obtaining normally for the concentrated residual mass, but also distributed residual mass over the length of the shaft. There consistently similar values are there even at the higher value at the noise.

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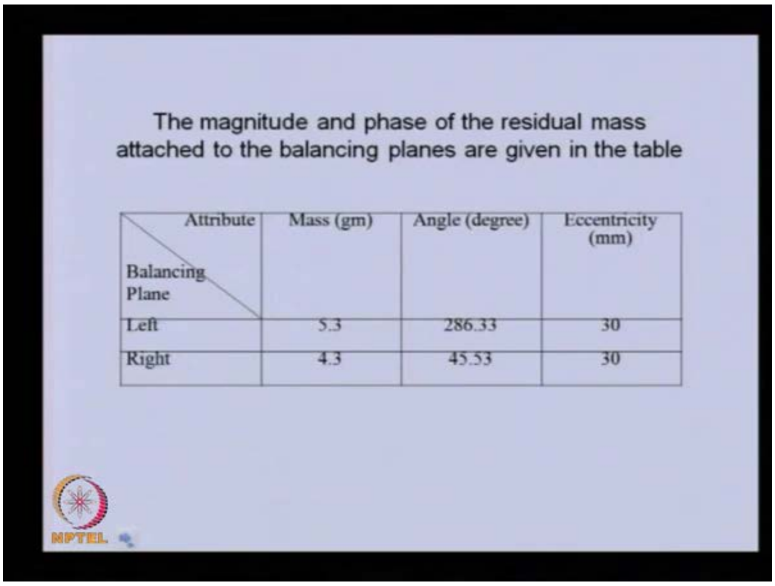

Balancing Plane \ Attribute	Mass (gm)	Angle (degree)	Eccentricity (mm)
Left	5.3	286.33	30
Right	4.3	45.53	30

Now, when this is the concentrated one, but apart from this we will be having distributed one. But, the value of that is less, so they are deviating this only.

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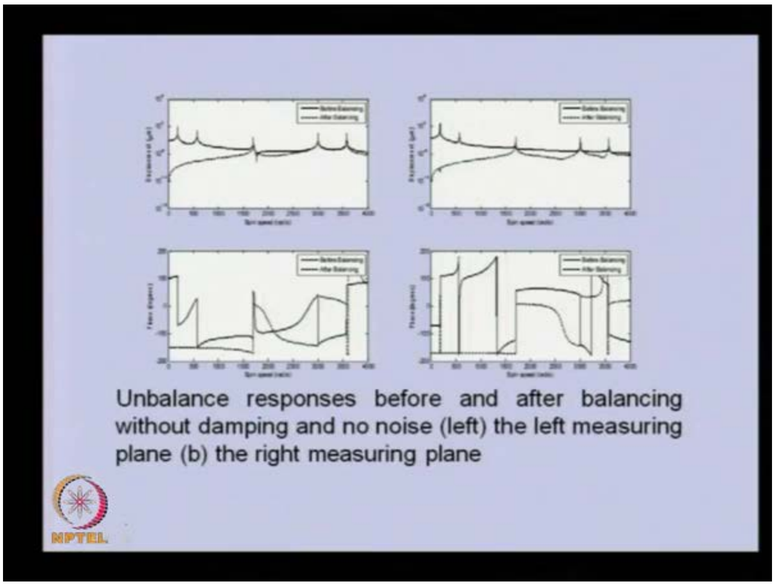

This is the comparison of the main important plot that you can be able to see. We focus on balancing of the 2 modes. You can able to see in the distributor case, the third mode and higher mode are not getting balanced. Only the first and second modes are getting balanced. These are not getting balanced. So, this with unbalance, this is without balancing. So, first two modes we could be able to balance, but higher modes are not getting balanced, because we have used the distributed residual unbalance in the system in the form of spiral.

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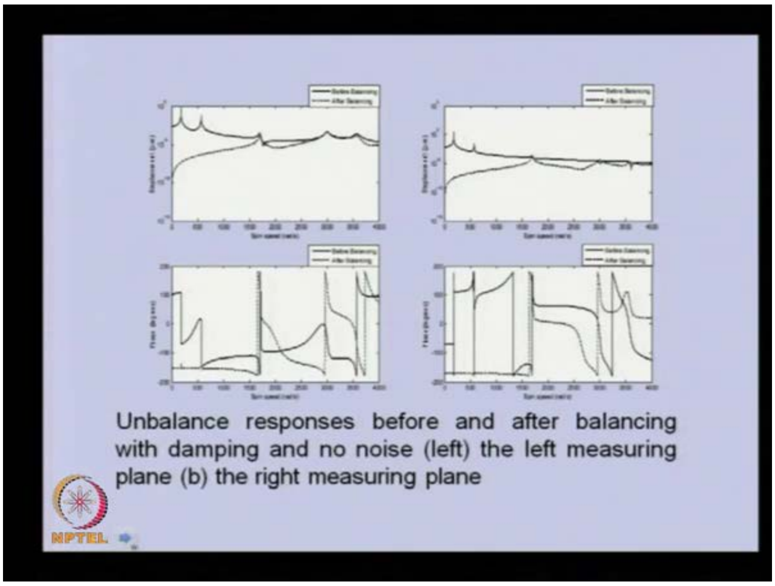

Similar plots we will see for different cases; that is with noise and without noise. So, in every case you can see that we could be able to balance the second mode, but third and higher modes are not getting balanced. Today, we describe one very advanced flexible rotor balancing method, that is a influence coefficient method. In this particular case, we have seen that influence coefficient changes with speed. Especially when we are operating at different modes, then influence coefficient changes.

For this case, we need to calculate the influence coefficient for all the speed of the range in which we want to balance the rotor. We saw that this influence coefficient can be simulated purely based on the measurement. The only thing is we need to keep the trial masses in various balancing planes, which we have chosen from that, without a trail mass responses, if we take the difference these two, we could be able to estimate the influence coefficient. Then, this influence coefficient can be used to obtain the residual balance in the descript plane.

So, in this we illustrated the method with two case studies. In one case study, we chose 2 balancing plane and only discrete unbalance we kept. We could be able to balance the rotor perfectly in this particular case. In the second case, we considered not only the discrete residual and unbalance, but also distributed residual and balance in the shaft. In this, we focused on balancing up to the second mode. We saw that other modes were not balanced, because of the distributed eccentricity in the shaft. So, this is very important observation that, if we have discrete residual unbalance, then we can able to balance all the modes of the shaft.

But, if you distributed unbalance we could be able to balance to finite number of modes, up to which we are trying to balance the rotor. In subsequent class, we will see other kind of faults apart from the unbalance. How they can be identified? How they can be corrected? Most common fault is unbalance, we will see other kinds of fault? They are difficult to remove. It is not like unbalance, in which we could be able to balance the rotor, so that we will see in the next class.