Theory & practice of Rotor Dynamics
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Lecture - 3
Dynamic balancing of rotor flexible rotor balancing
Module – 8

In last few lecture, we have been discussing regarding the dynamic balancing of the rotors, especially we have described methods how to balance rigid rotors. Previously we discussed, how we can able to balance rigid rotor using the cradle balancing machine or using a influence coefficient method, which is more advance method. Today, we will be extending the balancing method of the flexible rotor, especially will be concentrating on the modal balancing method. Initially we try to distinguish what is the basic difference between the rigid rotor balancing and the flexible rotor balancing. Then will go more detail about the modal balancing method, especially the principle of that and then with we will try to see how we can able to balance a rotor up to second mode. What is the more detail analyses for this we will see in the today's lecture.

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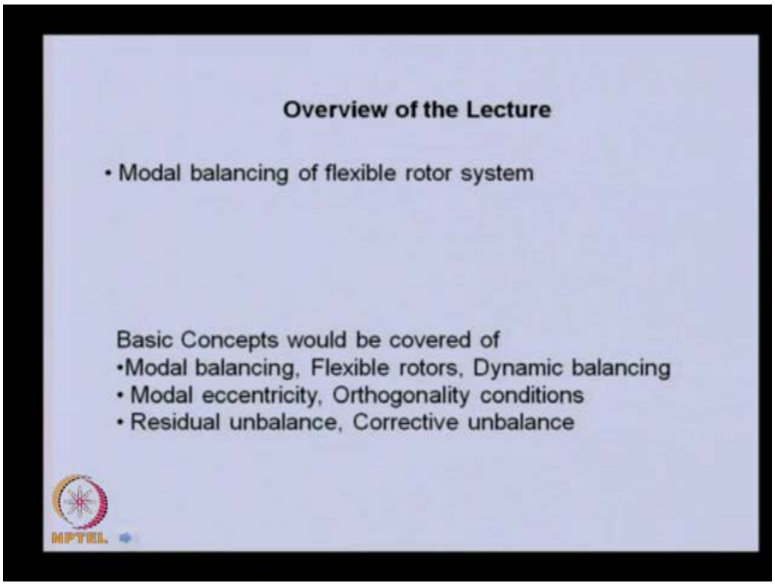

So, some of the terminology and so this is overview of the lecture in which various terminology will be introducing, like model balancing of flexible rotor, dynamic balancing and model eccentricity. This new concept which we will be introducing orthogonal condition of mode shape, this we concerned earlier, but we will be using them in a present formulation residual unbalance corrective unbalance. These are the various terminology, we will be introducing and using in the present presentation.

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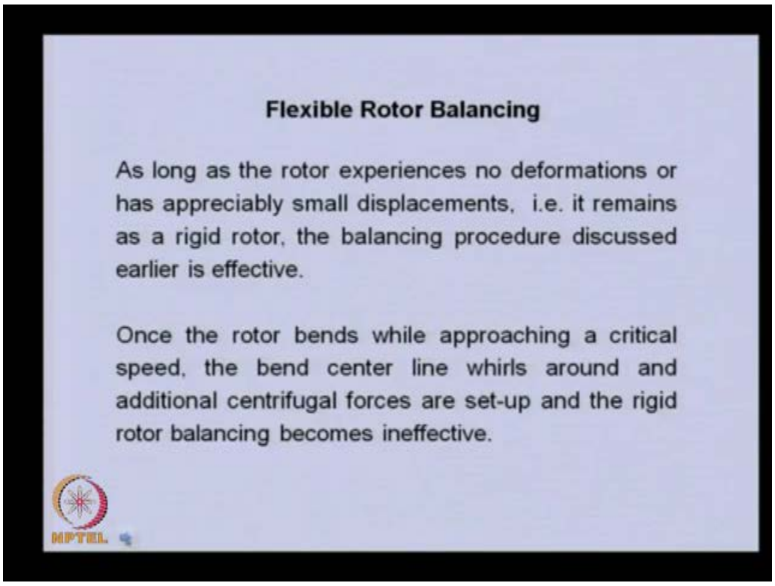

When we are taking about the flexible rotor balancing, we previously describe what is the different between the flexible rotor and rigid rotor. So, any long rotor when it is experiencing, not appreciate able deformation then it remains rigid and the same rotor. We can able to balance with whatever the rigid rotor balancing method we discussed earlier, but when the rotor speed increases and the rotor bends, while approaching a critical speed, the bend central line whirls around the whirls around about the bearing axis. The additional centrifugal forces are set up and the rigid balancing rotor becomes ineffective. So, in this particular case for flexible rotor balancing there are two methods, one is the modal balancing which was developed by these people, and then second is the influent coefficient method which was developed in 1980.

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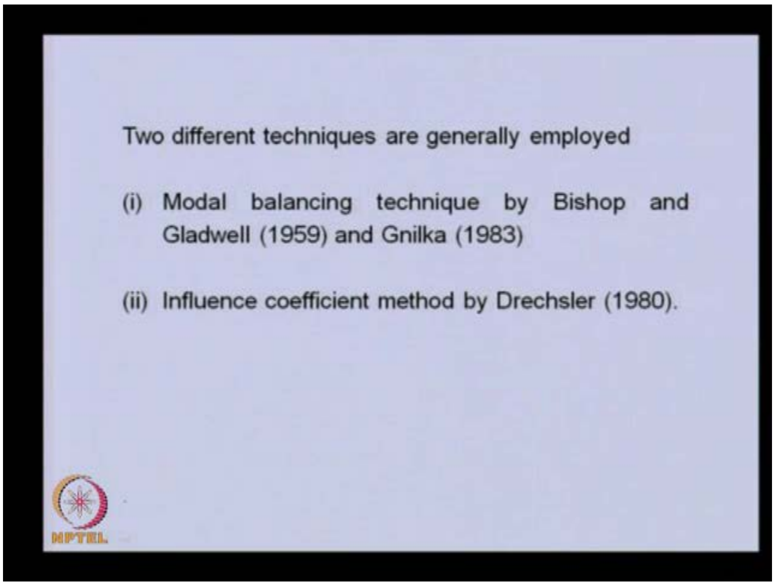

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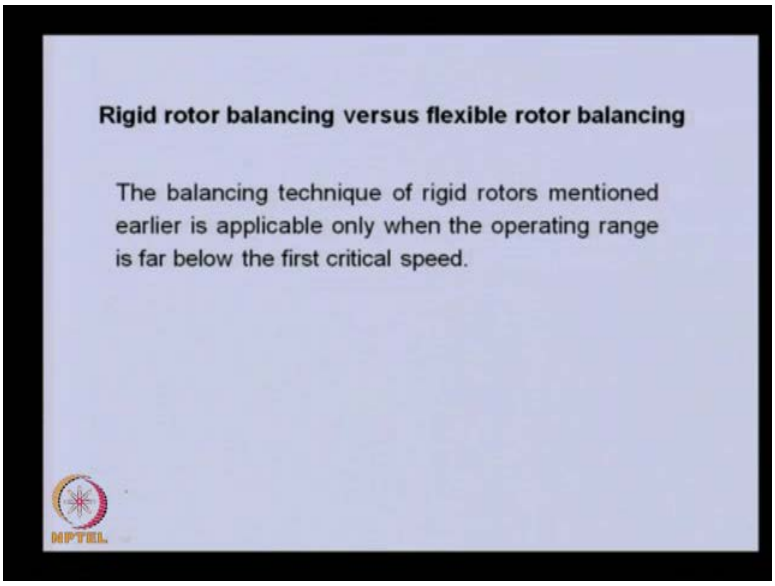

So, let us see in more detail how this two balancing method are different and how the rigid rotor balancing is not suitable for flexible rotor balancing.

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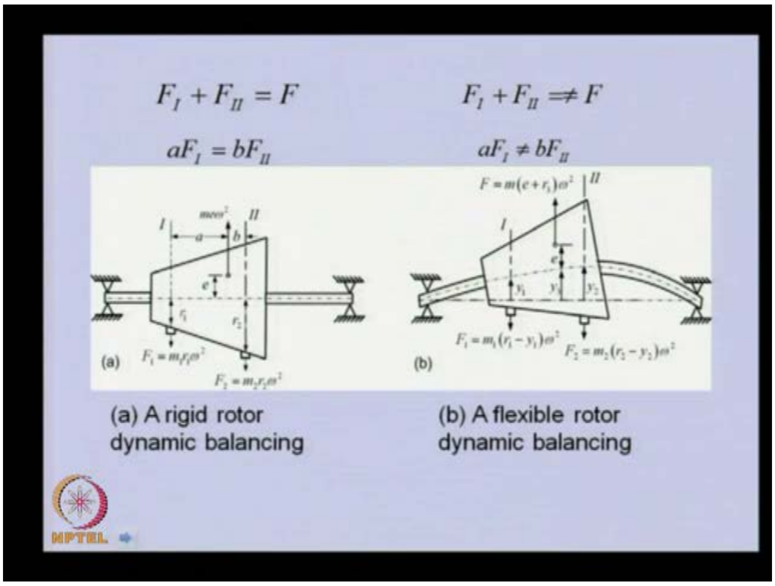

So, with this figure this is the rigid rotor balancing the rotor which we discussed earlier in which we tried to balance the eccentric rotor with two planes. So, for this we plane have F as the unbalance force and F 1 and F 2 we are putting the correction masses. So, centrifugal force these condition we need to satisfy to balance the rigid rotor, not only this one, but also the movement.

This we discussed in previous cases, but the steam rotor when the shaft is flexible or when same rotor is near the critical speed it will belt like this. And because of this we see that the effective eccentricity which was there earlier, because of the deformation it will increase and similarly, if the correction masses, their effective radial displacement from the centre of rotation decrease. So, earlier it was in R 1 position, now let us become R 1 minus y 1. Similarly, the second correction mass the radial position is change.

Now, you can see that the equations which we wrote earlier those will not be valued because this centrifugal force are not different. So, these equations will not be valued here unless they are not valued. So, we do not have the balancing of the whatever the residual balance in the term. So, basically we can able to see because of the deformation of the shaft due to the flexibility of the rotor, whatever the dynamic balancing we did for the rigid rotor is not valued for the flexible rotor case. In fact sometime it depurate the unbalance condition of the unbalance in the rotor system. Whatever the rigid rotor balancing we kept be it will be having the adverse effect on the rotor unbalance response. So, this was the difference between the rigid rotor and the flexible rotor balancing.

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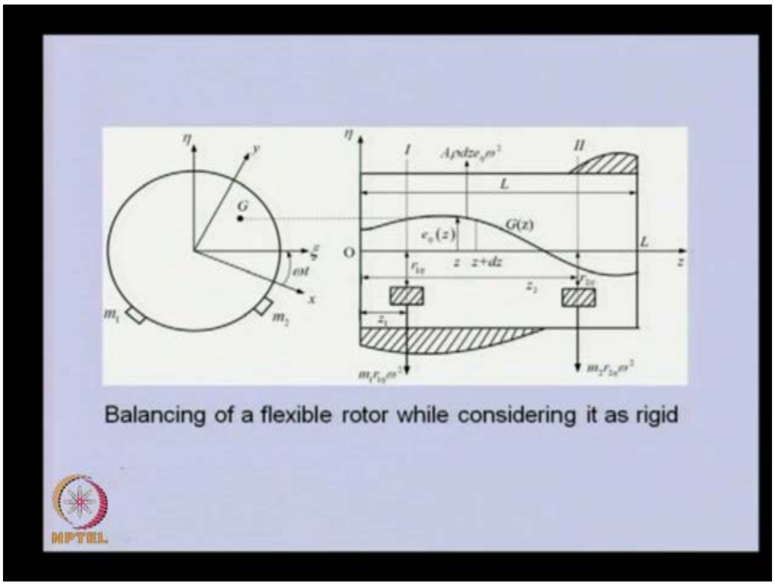

If we have a rotor like this in which let us say the rigid rotor and the extensity of the rigid rotor is continuously changing, this is the eccentricity variation. So, e eta z is the eccentricity variation in one of the plane, this eta z is rotating co-ordinate system. So, if body fixed coordinate rotating system, so with respect to that let us say to the variation of the eccentricity in similar length we can have extensity variation in the z i and z direction. So, in this particular case let us we are considering in one of the plane, so if we want to balance this particular eccentricity rotor which is rigid rotor, with the help of two masses which were keeping in plane 1 and plane 2.

So, they position may be on the rotor, the orientation may be different which is not visible here, but they are not only in the different plane, but also they are having different orientation. So, these are the correction masses to balance this particular eccentricity of the rotor or unbalance of the rotor. Now, if we want to balance this what are the condition we need to satisfy.

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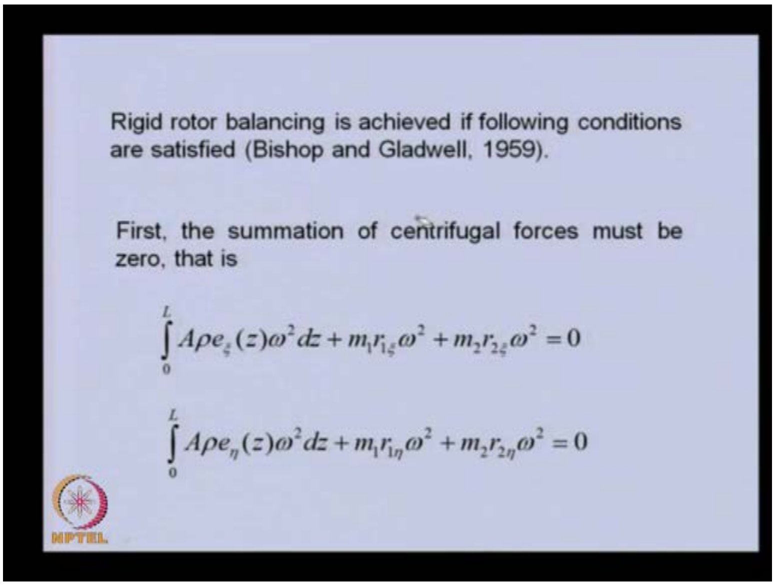

Let us say for rigid case, in this case whatever the summation of centrifugal force are there due to the variation of eccentricity of the rotor itself, this is the eccentricity, this is the mass on unit length. We multiply by z, that will be mass of the rotor of a slice and because this is eccentricity, so that is the total unbalance. If we multiply by omega square that will be the centrifugal force, this we are integrating over the whole length of the shaft.

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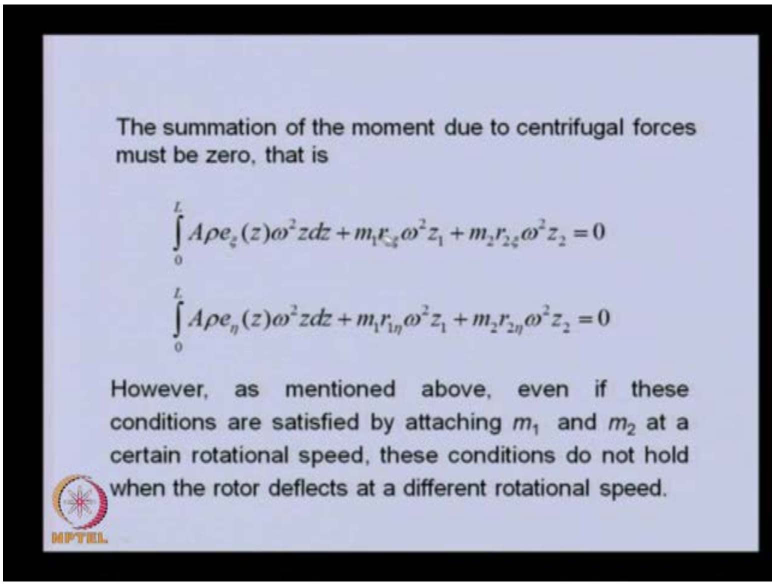

So, this is the total centrifugal force acting in the z i direction due to the rotor eccentricity and these are the correction mass. Correction mass component of the centrifugal force in this A n z i direction. So, this should be balanced, so whatever from the eccentricity rotor and correction masses should balance in this plane, not only in other plane. The eta direction also it should balance the centrifugal force.

So, apart from the centrifugal force, the movement if we take any point of the centrifugal forces, because of the continuous variation of eccentricity as well as the correction masses should also be balanced in both the plane. So, zi z and eta z plain the not only the force also the movement should be 0 for rigid rotor balancing case. So, you can able to see the location of the correction of the z 1, z 2. So, we have taken movement on this point, so that we can have movement of all not only the from the eccentricity of this, but also the correction masses.

So, these four condition we need to satisfy the rigid rotor balancing and this we need to do this particular, whatever the balancing we have done is for rigid rotor. If we are changing the speed and if we are nearing the critical speed deflection of the shaft will take place. These relation will not be valued because the correction masses, which we have obtained is for this particular configuration and because the deflection of the shaft will take place.

This equation will not be valued as we have seen in the previous slide that one shaft deforms this centrifugal force also changes. Now, to balance such a rotor in which continues variation of the eccentricity is there, one day we can able do it theoretically. If we put some kind of continuous mass here, you can see the shape of this should be same, may be the height should be different proportionately according to the position radial position of the correction mass.

So, this slash line is basically the shape of that similar to the eccentricity opposite. So, if we can put some kind of correction mass continues wiring like this and here then we can able to balance this rotor for each and every slice of the rotor. So, in that particular case what will happen if we rotate the rotor at any speed. If deformation take place then also we can able to have the. As soon we have the trail balance, this particular unbalance in the rotor.

So, putting this kind of variation, the continuous correction mass is not visible in practice. So, obviously we need to limit ourselves and in that particular case we do balancing up to certain speed. Now, will be describing two methods, one is the modal balancing method and another one is influence coefficient method, which will be describing the in the subsequent lecture. How we can able to balance particular mode, first mode, second mode, third mode using the correction masses.

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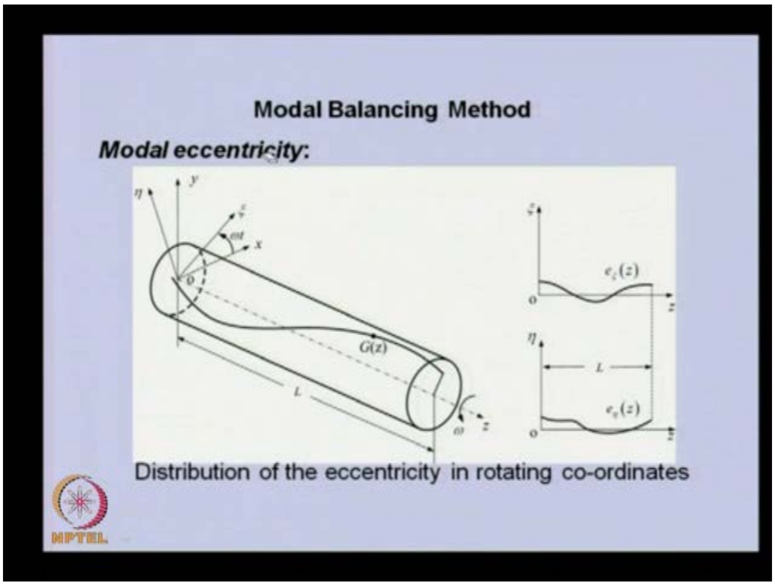

So, before going to the modal balancing method, let us see the concept of the modal extensity because earlier we use the concept of the model mass modal stiffness modal forces. Now, this is the modal eccentricity, so you can able to see this is a flexible rotor in which continues variation of the eccentricity is taking place. So, this has been execrated, but the shape we would like to highlight here that if we take one particular slice of the rotor here the centre of gravity is not only radial position is changing, but also the orientation is changing as we are going along the length of the shaft.

In this particular case, we have fixed system action y, another is the rotating coordinate system zi n eta which is rotating along with the rotor and because it is fixed with the rotor zi n eta. We can have the projection of this eccentricity in this two plane and you can able to see the variation of the eccentricity in zi direction, along the length of the rotor z is varying like this. This also, these variations are continuously changing in magnitude and also in the direction.

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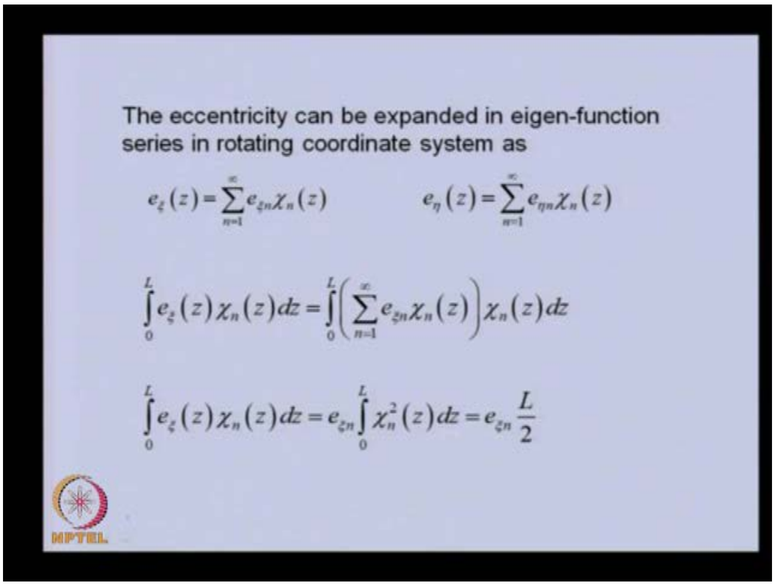

So, this particular eccentricity we can able to expand in the form of Eigen function series in rotating coordinate system, earlier we expanded the response of the system in terms of the Eigen because response comes from force. So, if we can express the response in terms of Eigen function then why not the force. So, basically here the eccentricity gives the centrifugal force because of that we are getting the response and that particular response earlier we expand in the form of Eigen function. Here, we are expressing the eccentricity in the form of Eigen function of the system.

So, it will depend upon what kind of rotor is there and what kind of bonding condition is there. We can able to get Eigen function from that and this is some constant which will determine which mode will be contributing how much and critically it will be having infinite terms, because if continuous rotor infinite of number of Eigen function. This is z i plane eccentricity variation, as we saw in the previous slide this is in the eta plain and in both plain we have express this eccentricity variation in terms of the Eigen function multiplied by some constant.

If we take one of the equation like this, one we multiply this by Eigen function, both side and then we integrate over the domain. So, generally we multiply the Eigen function which is different as compared to previous one, let us say multiplying by m x m. Then you can able to see that this is same expression, here we can use the autocratically condition of the Eigen function and when m is equal to n then it will be non zero. If they are not equal then it will be 0.

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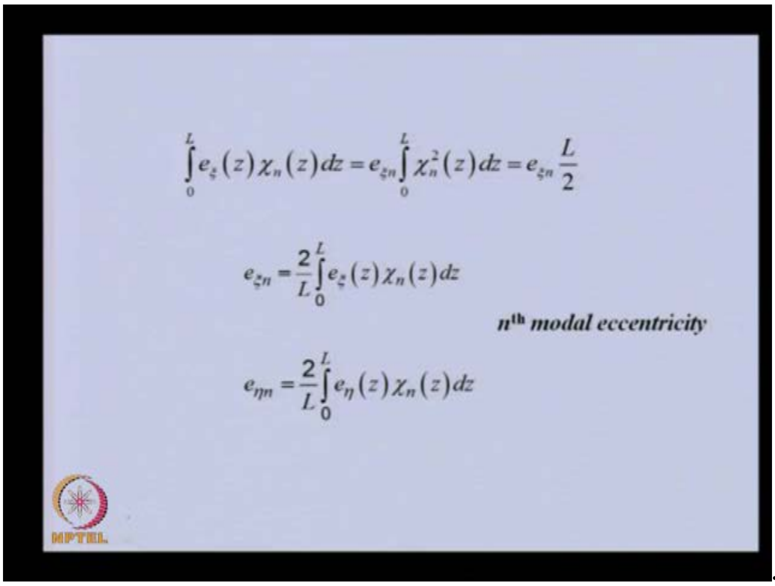

So, we will see that autocratically condition in the subsequent slide. So, this is one of the this is the previous slide on the extension of the previous slide. Here we operate the autocratically condition, I will be showing the autocratically condition simply supported case, how we can able to evaluate this particular integral also for simply supported case this integral becomes n by 2.

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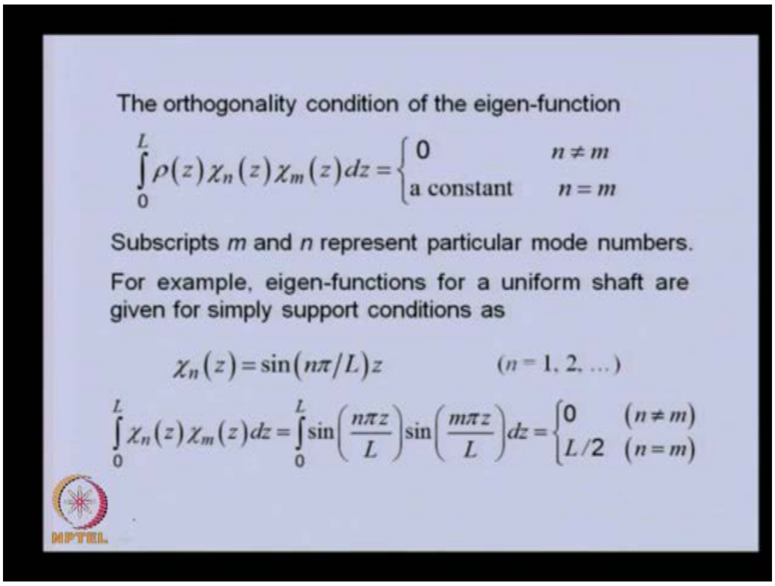

So, this is the autocratically condition in general. So, this we already discuss earlier that if m is not equal to n, then it becomes 0, n is equal to same then it is constant quantity. For simply supported rotor case we have this as the Eigen function. Earlier we obtain this also and we substitute this Eigen function. In this integral we see that when both are not equal, this quantity will be 0 if m is equal to m, this will be L by 2.

So, this property we used here to calculate this. Now, we can able to see, we can able to get the constant e z i n from this, like this z i n. So, 2 by L this, this is the eccentricity variation in the direction of z, in the direction of z i along the z direction. So, similarly we can able to take the second, this one and you can able to obtain the this constant.

So, this constant will be given as e eta n is equal to this much. So, these are we call it as model eccentricity and the subscript n the (Refer time: 18:00) the second subscript is representing which mode this eccentricity we are attaching. So, these are the n model eccentricity in z i n eta direction. So, now we know what are the basic difference between the flexible rotor and the rigid rotor balancing, even now we introduced the model eccentricity concept. Now, we will be focusing on the basic concept of the how we can able to use the model balancing for a rotor.

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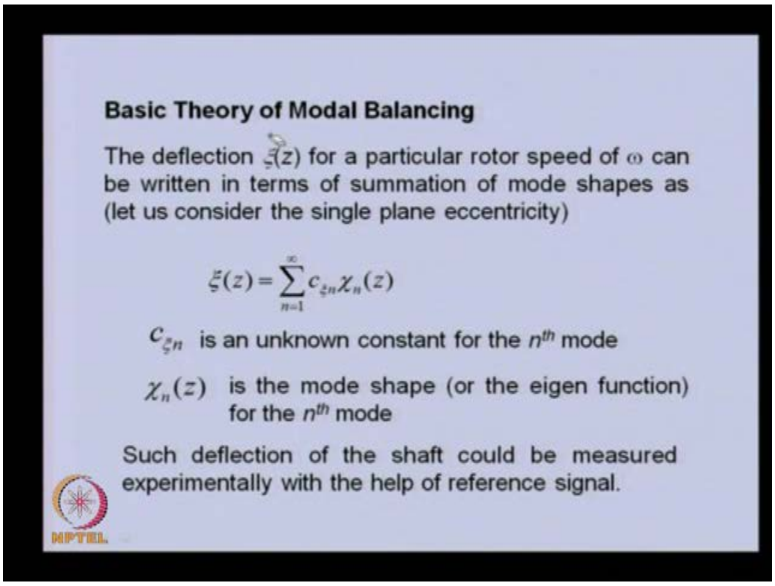

So, this particular case the deflection of the shaft in zai direction which is, which will be in the function of z and when we are talking about the flexible rotor then we need to talk about at what speed we are operating. So, this deformation deflection ((Refer Time:

19:00)) is there at certain speed and this deflection we can able to write in the form of series using the Eigen function. So, basically any response we can able to write in the form of summation series, in terms of the Eigen function multiplied by some constant. This unknown constant is again is attached with the particular mode.

So, this particular constant as we had earlier in for the eccentricity case here, it is c. Earlier it was e in terms of the we should not confuse this with the previous one. So, this particular constant is different, this is corresponding to the response and the previous one was corresponding to the eccentricity. This Eigen function is same as the previous one. So, for simply supported case we have seen that this is sign function and the deflection, this deflection generally in practice. We measured when the shaft is rotating at some rpm.

We can able to measure what is the response of this system. We can able to measure it either using proximately pro or any other pro with help of some kind preference signal to take care of the phase information of the response. Now, that particular expression again we are multiplying both side by the different Eigen function.

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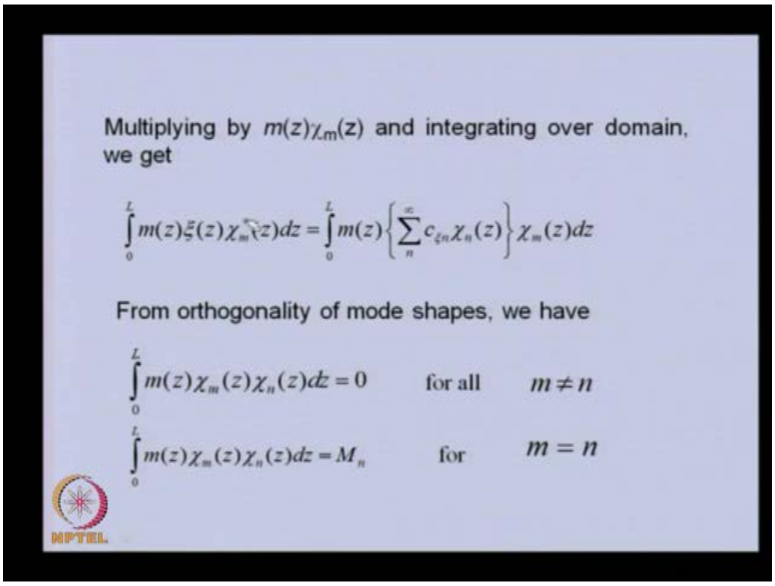

So, Eigen function and also mass per unit length and we are integrating over the domain. So, in previous expression e are multiplied by two terms and we are integrating over the domain. Both sides we have multiplied and here you can able to see. Again we can able to use the orthogonality condition to simplify this. So, whenever these two functions are having same subscript, we will be having this particular term as some constant as m n which we call as generalize mass and if they are not same then it will be 0. So, using this property this expression can be simplified.

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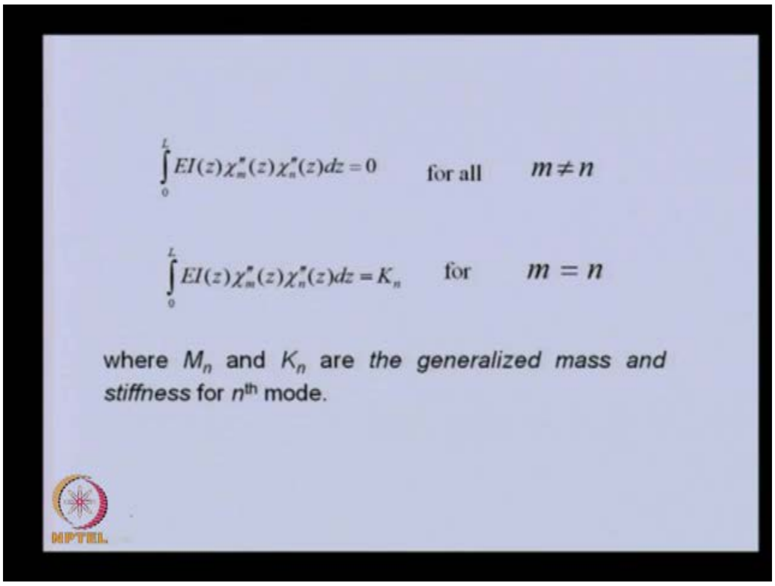

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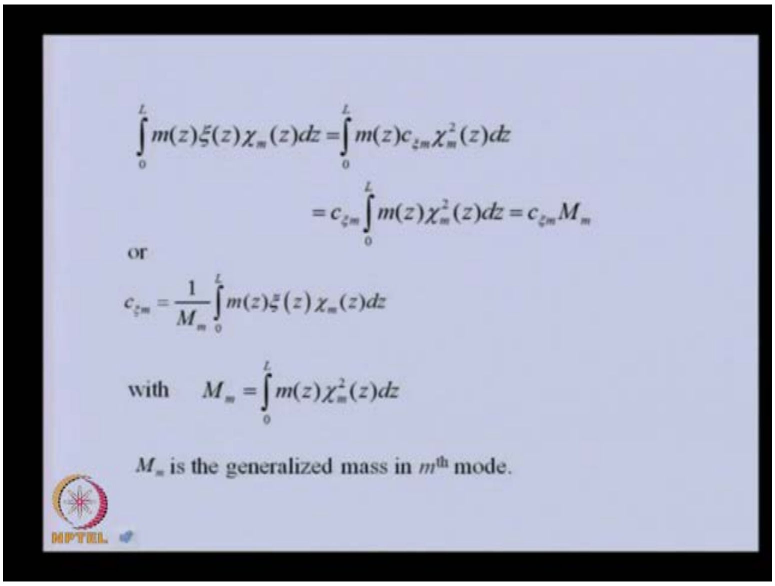

Similarly, the orthogonal condition for when we are the double derivative of the Eigen functions. Then also we have in both subscript mm mn are different. This will be 0, but if is same then I have some other constant, that is k n and we defined this mn and kn as generalized mass and generalized stiffness for nth mode. So, this is in the same line as we described earlier the eccentricity model eccentricity. So, here the model mass and model stiffness mean defined for vibration is called generalized mass and generalized stiffness of the that one particular mode.

So, using this orthogonality condition the previous equation which we multiplied this, we can able to multiply in this expression. So, this equation is as it is, but now we can able to see for n is equal to n only terms, that term is there remaining terms are 0. This can be because this is constant, this will come out and this quantity we defined as model or generalized mass. From this expression we can bale to solve the constant, unknown constant like this and m we have taken this side ((Refer Time: 23:00)). So, this is the unknown constant where model mass we defined like this, this integral. Now, we can able to see that we could able to get this constant, this was there in the chosen series of the ex response. This constant we could be able to get from using this analysis, okay?

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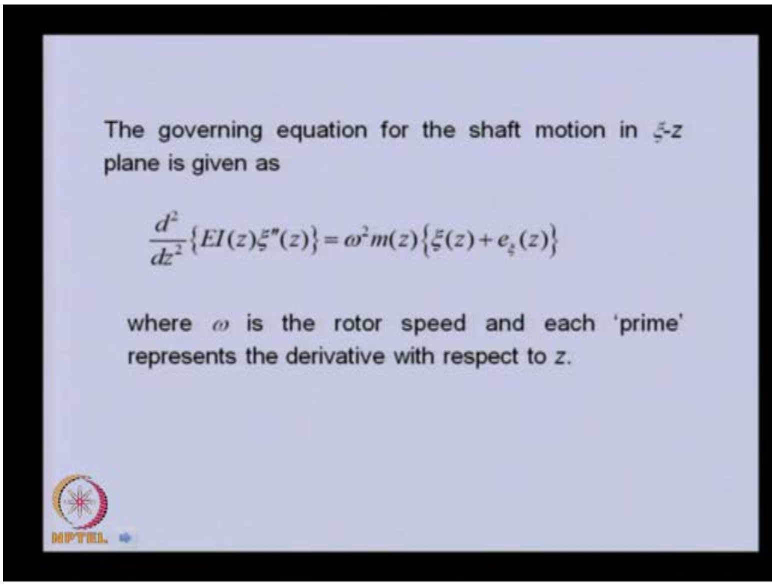

Now, we will take up the governing equation of the shaft in continuous system approach and we have to king this in the zai z plane. So, this is the equation of motion which we derived during the transverse vibration of vibration using continuous system approach. So, you can able to see this prime is basically is the derivative with respect to z to prime is double derivative with respect to z, similar to this derivative.

So, this is the elastic component and this is the inertial component. So, in this apart from the response we have the eccentricity, also we have included in this and the time derivative because the rotor is rotating at omega speed. So, that derivative come in the form of omega square. So, basically this comes in the form of inertial force and elastic force in the continuous system approach. Now, will be substituting the eccentricity which we expressed earlier in the form of series, also the response in the form of series is substituting in the equation of motion.

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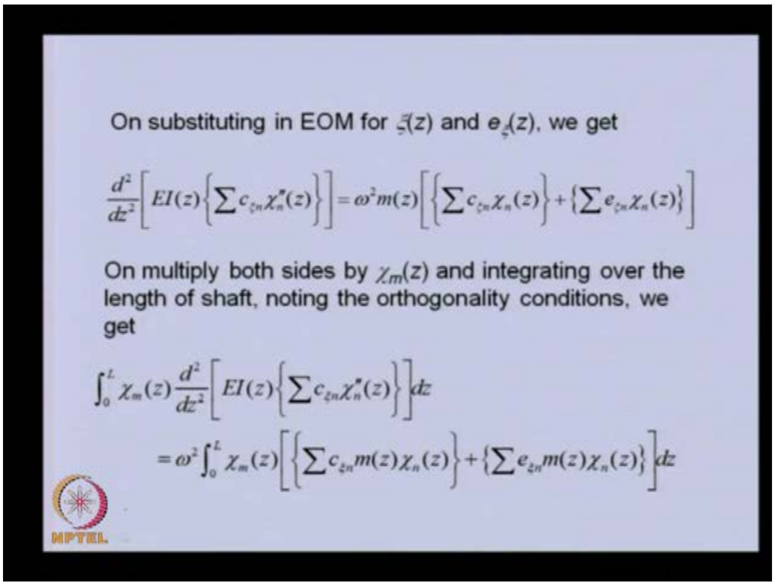

So, here you can able to see substituted the response, because it contains double derivative, this is constant. So, double derivative will come in the Eigen function and here this is the response and this is the eccentricity expansion and this some, this all the summation is varying from 1 to infinity. So, for all the mode of the continuous system, now will be multiplying both sides of this equation by Eigen function of nth mode. Then will be integrating over the length of the shaft and will be putting out the orthogonality condition to simplify the expression. So, once we multiply this by x n, this term will give us this and here again we are multiplying by this and integrating over the length of the shaft.

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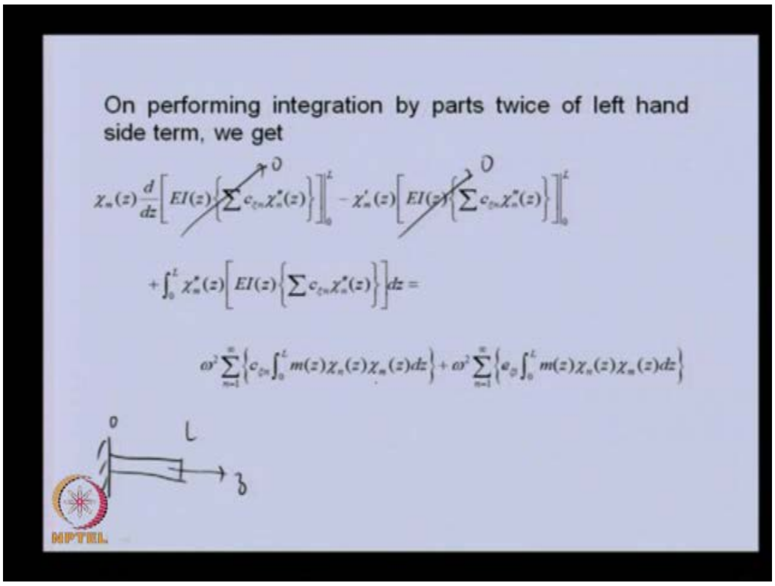

Now, in this here we are doing this, this particular term we are integrating the part twice, but I am taking this as first function and this is the second function so that this derivative comes here. So, basically this is the first term, different integration of the second term in it and then minus differentiation of the first term and integration of the second term 0 to n. So, in two steps we are doing in one shaft to the integration by part and then the third term will be double derivative of this and the integration of the second term twice. So, this term we have split into two parts. So, this is corresponding to the response and this is corresponding to the eccentricity.

In this we will see that this particular will be 0 for all possible bounding conditions in a rotor system like a simple condition of cantilever case. We have here displacement is 0. So, for if it is our axis system is z 0 and l, so for z is equal to l we will be having the share force and the banding moment is 0, this term is banding moment. So, this we can see z is equal to l, this term will be 0 for x is equal to 0. This term will be 0, for both the term x will be 0.

Similarly, here for x is equal to l the shear, this particular term will be 0, sorry this is a shear force because derivative, third derivative is there. So, this is binding moment, so by binding moment and shear term are 0 at x is equal to l and at x is equal to 0, z is equal to 0. This slope is 0, so you can able to see this two terms, they are 0 force for all possible boundary conditions. Now, whatever the left out terms are there we will apply the orthogonality conditions here.

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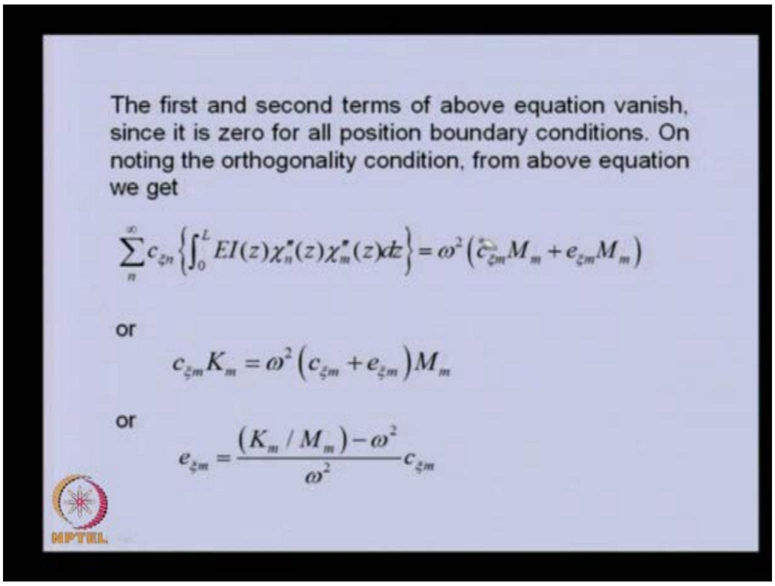

So, from the first term here that the term corresponding to x n will be non zero and other terms will be 2. So, this expression will come like this and in the right hand side we have already this as generalized mass. This is also generalized mass, this is constant R outside this integral.

So, these are the generalized mass and these are the constants and the response corresponding to the eccentricity. This we earlier saw this particular equation as generalized stiffness. So, that can be written like this and there will be basically 0 for if m is equal to m is not equal to n. So, that means only one term is left out corresponding to the m is equal to m, other will be 0.

So, corresponding generalized equations is written here. Now, you can able to see that this particular expression we could able to get this. Now, in this basically our aim is to obtain the what is the eccentricity variation in the shaft. So, if we can find out this constant, then we can able to obtain the distribution of the eccentricity along the length of the shaft. So, this equation we can able to solve for e zai n like this.

So, in this basically now we know all the terms in the right hand side like this. This is the ratio of the generalized mass and generalized stiffness by generalized mass, which is nothing but the natural frequency for the nth mode. This is the spin speed of the shaft, this is the constant from the response. This we already seen how we can able to get this, if we know the response of the system if because the response can be measured and from then we can able to get this constant.

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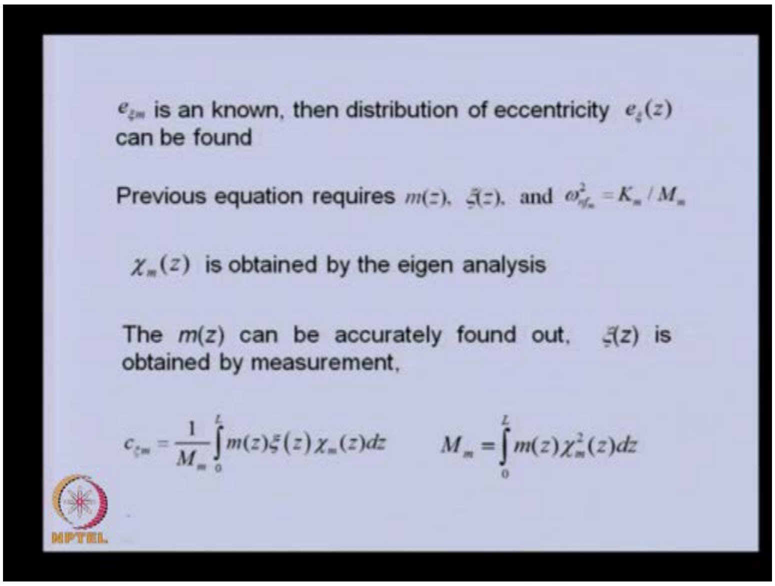

So, once we know this eccentricity we can able to get the how the dist the eccentricity will bearing along the length of the shaft. So, this distribution will be known because we expressed this eccentricity variation in terms of the Eigen function and these constants earlier for calculating the C we require these terms. So, the distributions of the mass that is per unit length of the shaft that we can able to measure from geometry, we can able to obtain and from mass property of the shaft. This Eigen function we can able to obtain from the system for the chosen boundary conditions and this again generalized mass is expressed in the terms of mass distribution of Eigen function. So, we can able to see that with the help of these quantity we can able to get the how the distribution of the eccentricity will be there.

Once we know this eccentricity variation, then we can able to balance the rotor because this is the main aim to know what is the eccentricity variation? If we know this we can able to balance the rotor. So, we outlined basic method of rotor balancing, no we will use this particular method or balancing a simply supported rotor. In this particular case we will try to balance the rotor for two modes and will be balancing the rotor, one first by first mode and the second mode. Now, let us see the example here.

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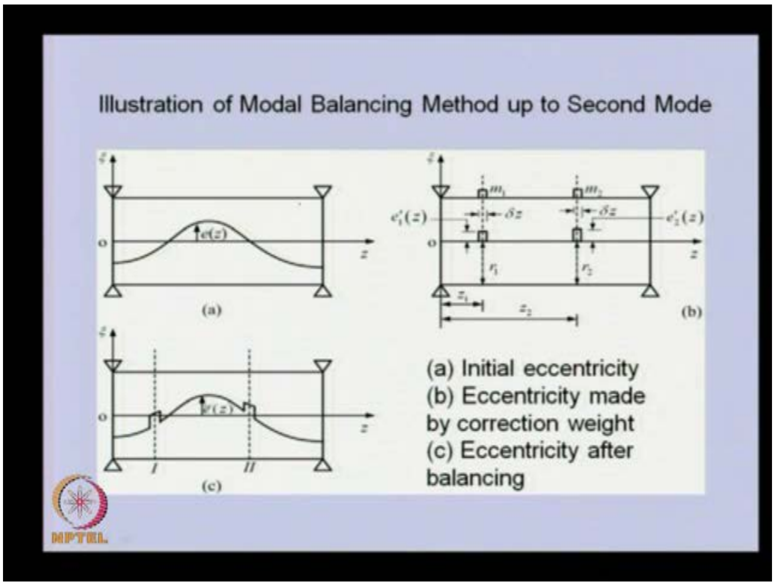

So, we have a shaft which is basically continuous shaft and this is flexible shaft. That means we will be operating on the rotor around up to the second particular speed of this particular rotor. In this, this is the variation of the eccentricity in which it will be continuously varying along the shaft of the along the shaft and we do not know what is the variation of this. Actually, we need to find out and based on this actually we need to balance the particular to second critical speed, because finding exactly what will be the eccentricity variation will be very difficult. So, will be focusing mainly because the first few critical speeds are important and they are dangerous because normally the rotors are operating at that range.

So, we will be taking two planes, so here basically this is the shaft and we will be taking one plane and two plane here and one end and two end are two eccentric correction masses which we are keeping in two planes. In this particular rotor we are not upon the this particular eccentricity. So, this is actually this particular step kind of thing is there. That is the eccentricity corresponding to the mass which we have put at the outside of the rotor, because rotor is having R 1, R 2. Generally it will be uniform, even if it is non uniform they we can able to choose different radius in our problems, in our and this mass will be having some finite length.

So, these are the basically eccentricity corresponding to these masses, will see how these eccentricity are in the subsequent slide and basically now this is the super positioning of this two. So, that means to balance this particular eccentricity which is residual eccentricity in the system. We have put two masses, how the eccentricity will vary because this add up here, because this side is negative direction, this eccentricity is there.

So, that will adopt to give this particular shape, so here it will adopt to give the shape. So, basically this steps are corresponding to the concentrate mass, correction masses which we kept. So, we are expecting that this two masses will balance the rotor up to second speed, critical speed. The meaning of that not only balance the first mode near the first critical speed, also it will balance the second critical speed and our aim would to obtain the m 1 and m 2. So, now let us see how we can able to do this.

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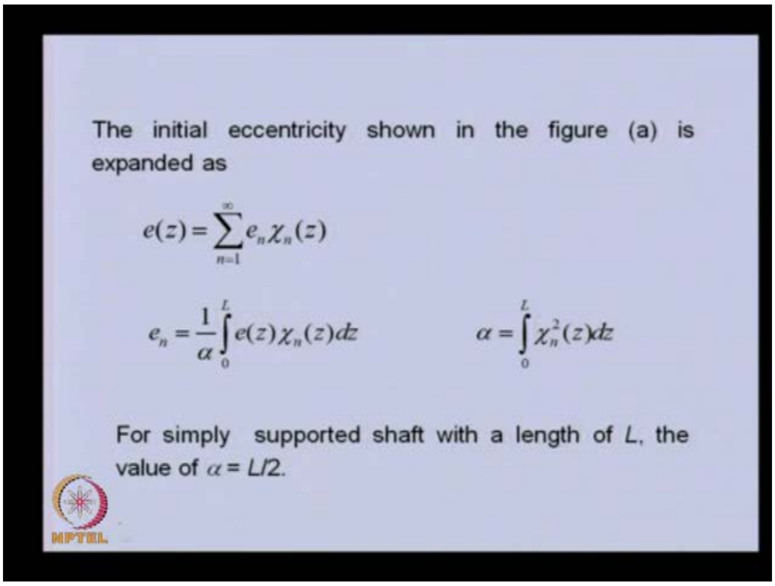

So, coming to the initial eccentricity which we are shown in the figure, a first figure we can able to express that in terms of the inherent series in form of some constant and diagram function. This constant as we already seen earlier can be express like this, where alpha will be given by this. Now, first simply supported case, we already know this alpha will be l bear to because this is sign function. If we evolutes this value, this will be to first simply supported mounded condition, but for other bounding condition again function will changed and corresponding this alpha will be changing.

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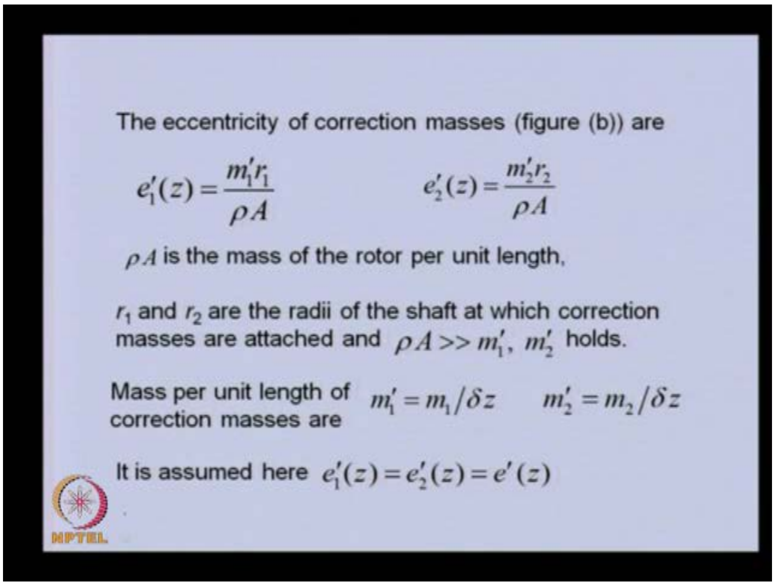

Now, the eccentricity of the correction mass which we kept they are given like this. So, in this you can able to see that the correction masses are at r 1 location and the eccentricity which we are calculating. So, basically we are equating the force which this particular correction mass is giving. So, basically m 1 r 1 omega square will be the force corresponding to the that correction mass and row is the mass of the mass moment length of the shaft. So, corresponding to the shaft eccentricity mass of the shaft and this eccentricity to omega square will be the equivalent force, which the shaft eccentricity will accept. So, we can able to calculate the shaft eccentricity equivalent to the correction mass like this.

So, this is in plane 1 and this is in plane 2 and this we expect that the correction masses are negligible small as compare to the mass length of the shaft. So, basically m 1 prime and m 2 prime are mass permanent length of the correction mass. So, here you can able to see this is the mass of the correction. Now, mass in plane 1 and this is the length of the correction mass, that length is some finite length. So, what over the length is there that we are calling the delta z.

So, m 1 prime is the mass length of the correction masses and plane 1, this is plane 2 and we are assuming here this particular eccentricity which we are calculating because mass which we are putting the correction planes, they will not be much different. So, we are assuming that this eccentricity which we are calculating there equal and we are representing that as a e prime z. Now, you can able to see this is the basically eccentricity due to the correction masses which are given here. So, this height is eccentricity due to the correction masses.

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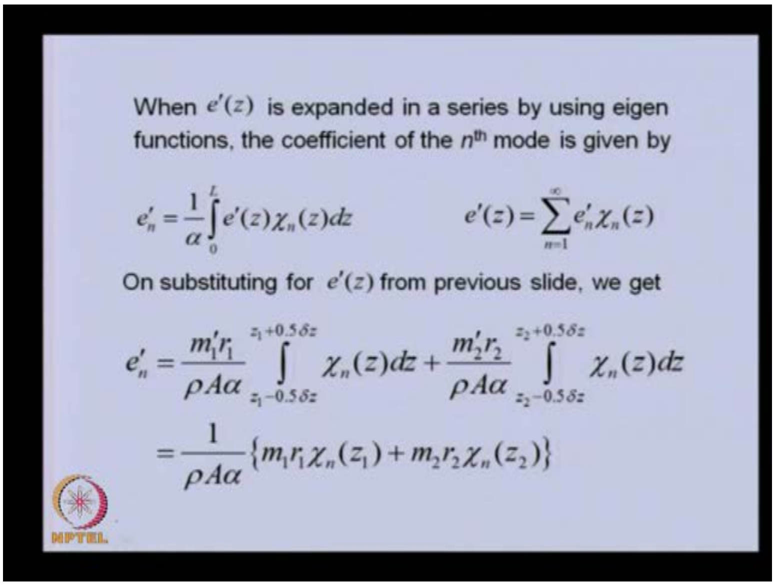

Now, we are expanding this eccentricity due to correction masses in similar form as with the help of again function. So, this is the expansive function and this is the constant for particular mode. So, this is the distribution of the eccentricity, this is the constant and this constant I will be define like this. The similar form as we define for the eccentricity of the shaft. Now, in this we substituting the eccentricity because we have over the length of the shaft only at two places. This eccentricity is there, one is here and here the length of the shaft and we are assuming that this distance is z 1 and z 2 and around this the length the length of the shaft or which this eccentricity is distributes delta z.

So, you can able to see that will be having integration from z 1 minus half of delta z to plus and in that we have the other is first eccentricity distribution, that is corresponding to e 1. This one and in the second demining in the remaining reason, this eccentricity is 0. So, in the second case where the second plane is there this will be the eccentricity constant. Now, you can able to see that we can able to express this like this and because this particular integration is there at the particular very small location, the length of the shaft. So, we can able to now this next slide will show how we can able to get this, from this centrifugal.

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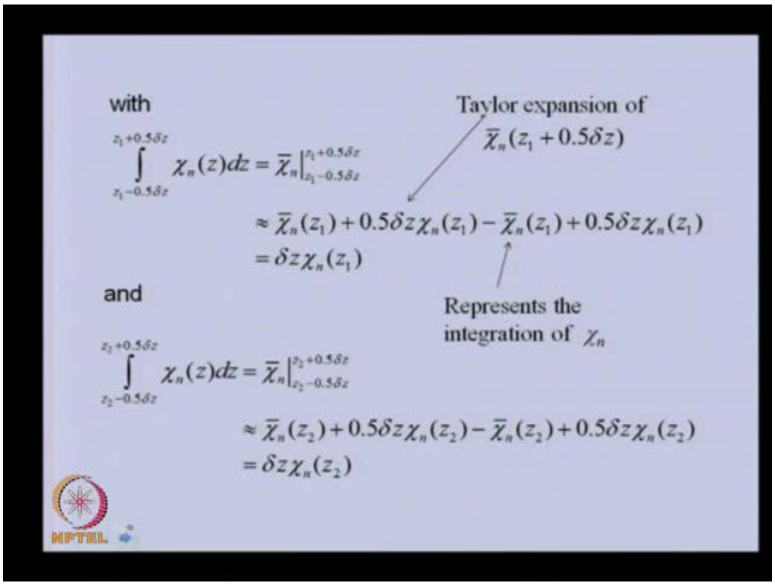

So, first centrifugal is like this, so let us say this zi and bar is the integration of this function and if we are keeping the limits of this. We will get like this for the first limit and for second limit another terms will be there. If we expand that using Taylor expansion will see at this is the first term and this is the second term or the different session of this term will be there.

So, the integration term will go away, so once we expand this function for this two cases up to the first derivative, two will get this two terms and this will get cancelled and second and fourth term will give this. Similarly, for the second integration we will get this which we substitute in the previous here. So, basically this are the quantity which we obtain along with the from here.

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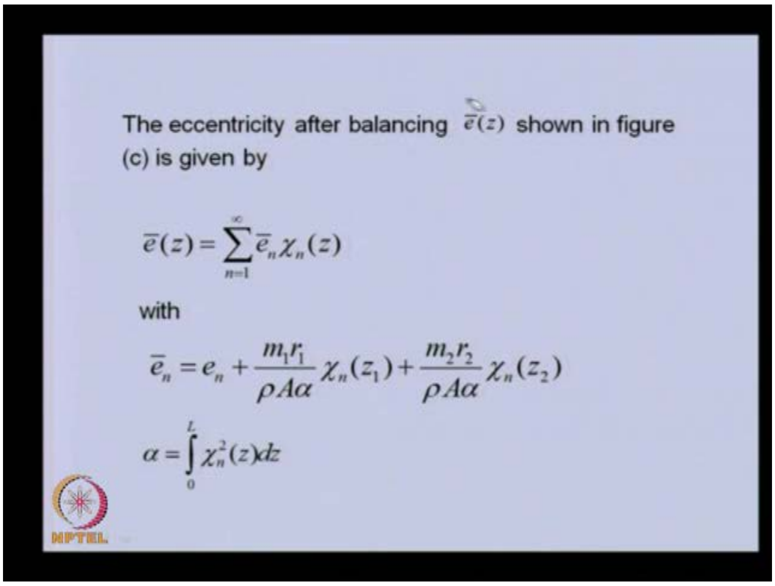

Now, once we obtain this the eccentricity after balancing this. We already have seen in the third figure in which along with the eccentricity of the shaft eccentricity of the correction mass was also shown. So, that distribution we can able to express like this again in the form of again function and this constant will be given as the constant of the shaft, eccentricity and of the correction masses eccentricity. So, these are this two we obtain here this eccentricity. So, this is due to the shaft eccentricity this are due to the eccentricity due to the correction mass where alpha is given by this.

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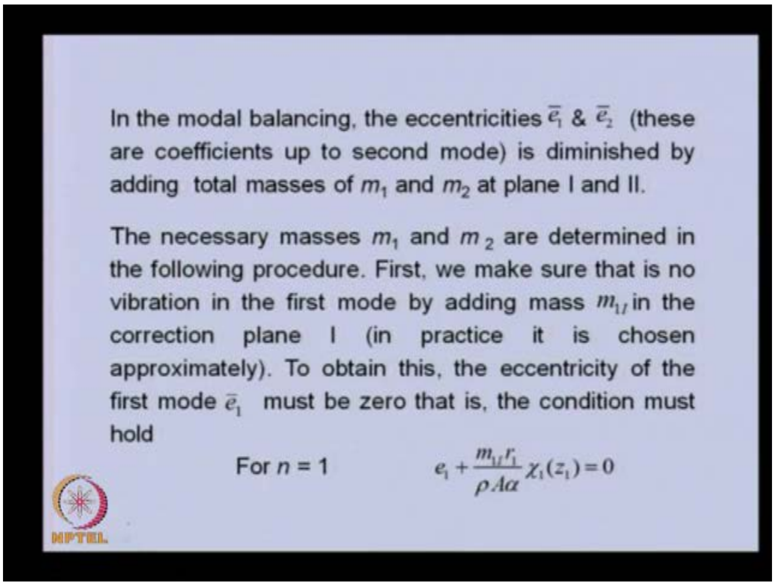

Now, in the model balancing the eccentricities e 1 and e 2, these are coefficients up to the second mode. So, in the here basically we need to balance this up to the n is equal to 1, 2. So, correspondingly we need to this eccentricity e 1 bar and e 2 bar, we need to vanish up to the second mode. So, we diminish by adding total mass m 1 and m 2 at plane 1 and 2. So, basically we are removing these two eccentricity by addition of the correction mass m 1 and m 2 in plane1 and 2 respectively. This is total mass where we will be calculating first balancing the first mode and then for the second mode.

So, the necessary masses m 1 and m 2 are determined in following procedure. The following first we make sure that there is no vibration in the first mode by adding the mass m 1 1 in the correction plane 1. So, m 1 is the first mass in plane 1, the second subscript is representing plane 1 in practice. This is chosen approximately to obtain this the eccentricity of the first mode m 1 first be 0, that is the condition this following condition must be satisfied.

That means e n is equal to 1, the previous expression this expression e 1 bar should be 0 because we want to balance the first mode and here even and other terms are there. So, that means we are trying to balance the first mode with m 1 1 mass, which we are keeping in the plane 1, because for first mode only 1 1 mass to plane balancing is enough. So, that is why for first mode we are using only single plane which is plane 1.

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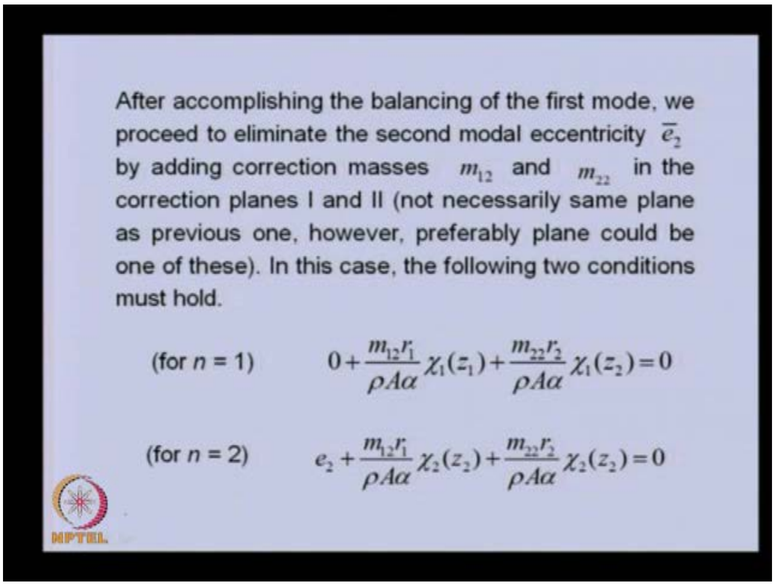

Now, once we accomplished, so basically this mass be chosen. So, we will be rotating the rotor around first critical speed will choose this particular mass such that the response are diminishing. That means this even we expect will be 0 corresponding to this particular mass. Now, after we have accomplish the balancing of the first mode. We will proceed for the second mode balancing, that means e 2 and this particular case will keeping the two masses in two planes. So, this is the first mass in first plane, this second mass in the second plane, these are the additional mass as compare to the previous mass which we kept. Now, for now we will be writing this the same equation. The previous equation, this equation for m is equal to 1 and n is equal to 2 because we are keeping two masses and when we n is equal to 1.

So, the same expression, so when we are keeping this two masses we are ensuring that here e 1 should be here, but we are ensuring the this two masses should a naught. This should not disturb the even because we already balance the even by the previous and m 1 1 mass.

So, that means this condition will be ensure that what over the choice of this two masses m 1 2 and m 2 2 is there. That should ensure the first mode is not disturbed, it is balance this is for m is equal to 2 in which e 2 is there and this are the two additional masses. So, you can able to see that the choice of this two masses will be such that the ratio from this we can able to get the ratio because other quantity are known. So, what should be the ratio of this two, we will ensure that the first mode is balance.

So, if we satisfied this equation by choosing this two masses will be ensuring that the first mode is balance and simultaneously we need to see that the e 2 is getting balance. So, that is means we will be choosing the m 1 and m 2 2 and we are rotating the near the second critical speed and we will try to change that masses, but ratio will keep in the same such that the responses is getting minimize corresponding to the second mode. So, that will ensure not only the second mode is balance but also the first mode is not disturbed.

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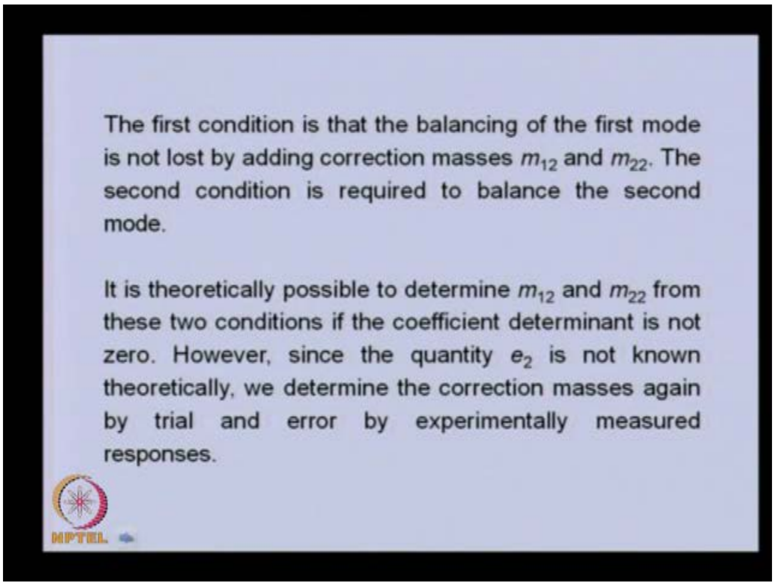

So, this is the discussion which I explain.

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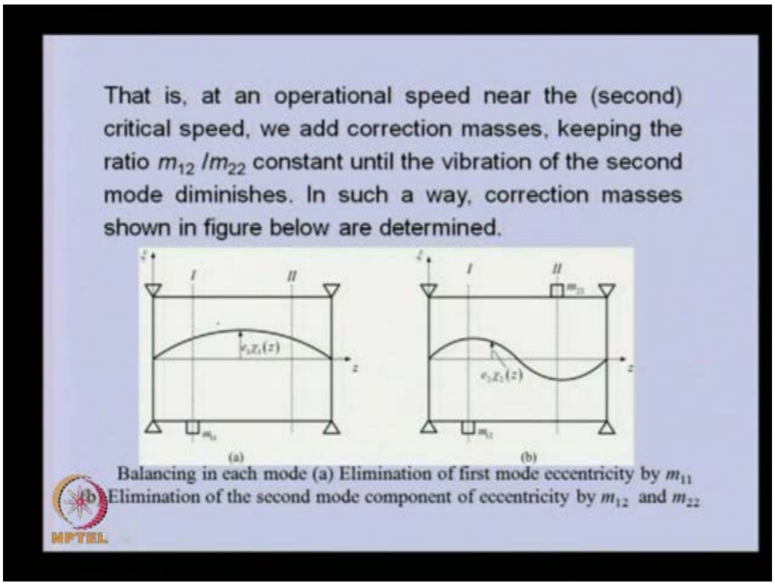

So, basically now this is summary, we balance the first mode by one mass. Then the second mode by two masses, like this you can able to do the second mode and the total balancing is the submission of this two.

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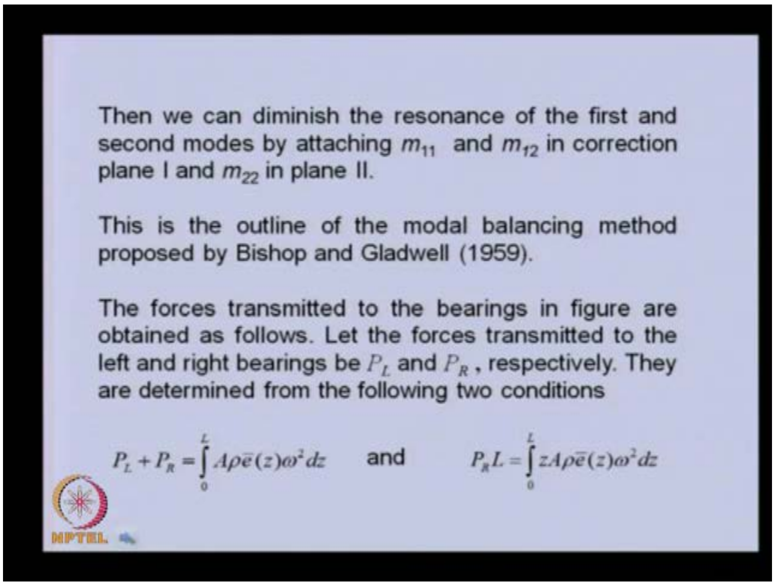

So, we will be having in plane 1, then you can able to see here in plane 1 submission of this and this will be the total mass which will be keeping it to balance the first mode and second mode and in the plane 2. This mass will be there, today we have seen how the flexible rotor balancing is different as compare to the rigid rotor balancing and once we differentiate this we then introduce the concept of model eccentricity. The line with the model mass and model stiffness, which we introduce in the previous lecture, then we focus on a particular method of balancing flexible rotor that is model balancing.

In this particular case, we balance the rotor mode by mode. That means we start with the mode balancing of the rotor and then gradually we go up to the mode up to which we want to balance the rotor. And with the very simple case of simply supported rotor and we try to show the analyses, how this model balancing can be use to balance the rotor of second mode, by using two planes which will which will balance. Not only the first mode as well as it will balance the second mode in the subsequent lecture. Now, we will be of taking another method which is more advance and more powerful. There is a influence coefficient method for flexible rotor balancing.