


Theory and Practice of Rotor Dynamics
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Module - 8
Dynamic Balancing of Rotors
Lecture - 38
Rigid and Flexible Rotor Balancing

In the previous lecture we introduced a dynamic balancing of rotors. Now, we will be extending those procedures in more detail, a specially for rigid rotor case, we discussed in the last time that we can able to balance the rigid rotor using a 2 planes. And one practical procedure of balancing rigid rotor in a balancing machine described that we saw that method was heat and dry, you need to place the trial mass at several locations and of different size. And then finally we need to converge what should be the balancing required in a particular plane? Now, today we will be describing a procedure in which we require less number of measurement in the same machine.

And then we will go for some other advance technique of dynamic balancing of rigid rotors like influence coefficient method; in which we not only measure the amplitude of vibration but also we measure the phase reformation which is very important reformation in a rotor dynamics. So, with the help of that we will see that how rotor can be balanced on in field. So, that particular method is a field balancing method, we did not remove the rotor from the actual machine directly in the actual machine on at on operating a speed; we can able to balance that particular kind of a rotor. And then we will go for a flexible rotor balancing procedure, we will outline a basic theory of model balancing procedure.

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Overview of the Lecture


- Rigid rotor balancing
- Modal balancing of flexible rotor system

Basic Concepts would be covered of

- Influence coefficient method for rigid rotor balancing
- Modal balancing, Flexible rotors, Dynamic balancing
- Modal eccentricity, Orthogonality conditions

And so basically the outline of the a lecture is we will be briefing the rigid rotor balancing, a specially we will be seeing the method graphical method by which we can able to reduce the number of measurement required which is very important in actual machines. Then, the we will outline the model balancing procedure specially for flexible rotor balancing; various terminology and concepts like influenced coefficient a for rigid rotor balancing, model balancing, flexible rotor, dynamic balancing, model eccentricity, a orthogonality condition of the a Eigen values of rotor system. So, these are the things we will be a covering in this particular lecture.

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Systematic Balancing Method (Graphical)

Based on only four observations of amplitude :

- (i) without any addition of the trial mass to the rotor
- (ii) with a trial mass at $\theta = 0^\circ$, where θ is measured from a conveniently chosen location on the balancing plane
- (iii) with the same trial mass at 180° and
- (iv) with the same trial mass at $\theta = \pm 90^\circ$

All measurements have to be performed at the same speed.

So, now the same cradle balancing machine but we will be using more systematic balancing a procedure. And this procedure is after measurement we through graphical method; we can able to obtain what is the residual unbalance involved in that particular plane. So, in this basically we require only 4 observations of amplitude of vibration. So, first will be without any addition of the trial mass; we will rotate the rotor at part particular constant speed and we will measure the amplitude of vibration. So, this we will be doing in this same cradle machine as we discuss in the lecture previous lecture a then the. So, in this particular case because we want to obtain a correction mass in one of the plane in a discipline 2. So, we need to fix on the a rotor in the at f_1 and a . So, in this particular case we will be obtaining the correction mass in plane 2.

Then, the same trial mass a now we can able to introduce a trial mass at convenient location after the rotor; that convenient location we can call that as a angle θ . And this we this particular angle we have choosing conveniently on to the shaft; and that will be the reference for the angle measurement. So, once we keep the trial mass there will be change in the amplitude of the vibration; when we rotate the rotor at the same speed as the first case and that particular speed will be measuring it. So, this will be the second measurement; then the third measurement will be the same trial mass we will keep at 180 degree. So, just opposite to this location we will be keeping it on to the rotor and will be measuring the amplitude of vibration.

And, so a as we have done in the previous lecture for coming at f_1 and putting the correction mass in plane 2. So, these measurement this trial mass is we are keeping in plane 2 at the previous lecture cradle machine. And that fourth measurement will be same trial mass we need to keep a at 90 degree a the plus 90 or minus 90. And then again we will rotate the rotor at the operating the constant operating speed; and we will be measuring the amplitude of vibration. So, basically we will be getting 4 amplitude of vibration; based on these 4 a measurement we will be obtaining what should the correction mass required at a plane 2 when we are factor at the machine f_1 .

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Let \overline{OA} is the amplitude measured with trial run (1) without a trial mass,


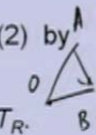
\overline{OB} is the amplitude measured in trial run (2) by addition of a trial mass T_R at 0°

Hence, \overline{AB} represents the effect of trial mass T_R .

\overline{OC} is the vibration measured in trial run (3), with the same trial mass at 180° .

So we will have $\overline{AB} = \overline{AC}$ with 180° phase difference between them.

From these information we need to construct and locate points O, A, B and C on a plane.



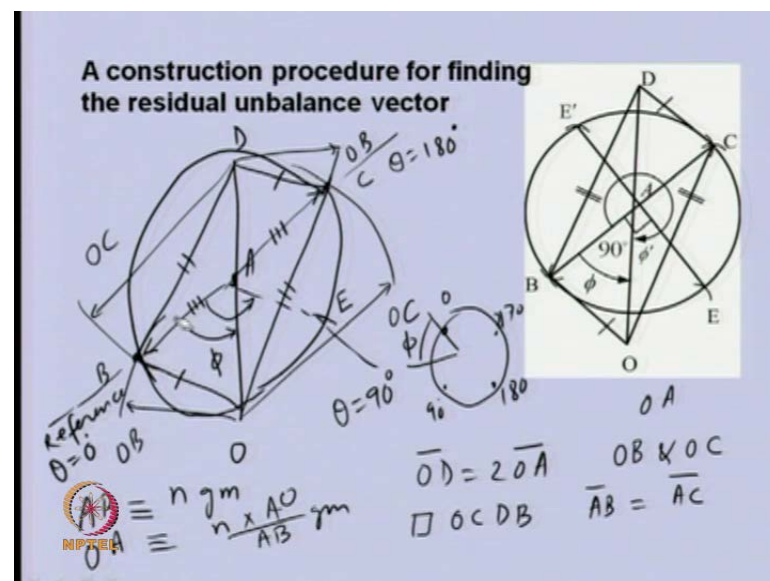
So, now let us see a, what are the measurement we have done, how to analyze? So, let O A is one vector a an the amplitude of this we know and that is the amplitude of vibration when we are riding the machine without any trial mass; that is the first measurement. So, this is basically first measurement; O B is the second measurement in which a we are keeping a trial mass at 0 in degree; trial mass is we know the amplitude and amplitude of this. So, we have kept that at 0 degree and because of this whatever measurement is coming amplitude of vibration. Let us say that is O B.

Now, because O B and A B these are a displacement basically they are vector quantity. So, we expect that the difference of this 2 vector because origin of these 2 are at O and from there we have O A and O B 2 vectors are originating. So, the difference of these 2 vector that means, A B will represent the effect of trial mass. So, if we have origin here and O A is this, O B is this. So, A B will be the basically a vector which will represent by this change is due to the a; when we are putting the trial mass into the rotor at 0 degree. So, this is the effect of trial mass A B. Similarly, in third measurement that is this one; when we are keeping the same trial mass at 180 degree let us say there is O C.

So, is the vibration measured and trial round 3 with the same trial mass capital 190 degree. So, now we can able to see that same trail mass we kept at 180 degree. So, we have another vector OC which is again terminated is starting from the O. So, basically A C vector A C which is a is without trial mass and O C is with trial mass. So, A C will be

But in one case it has been kept at 0 degree for A B and for a A C it has been kept at 180 degree a the effect should be equal but opposite. So, basically A B should be equal to A C. But they should have 180 degree phase difference because a they add the effect of trial mass and we are keeping the trial mass at 180 degree phase. So, basically A B should be equal to AC but minus of this. So, because they are having a 180 degree phase. Now, we have the following informations from the previous measurements. And from the above analysis that we have O A, A B, A C, O C 3 amplitudes are available; when we know that A B is equal to A C but they have 180 degree phase difference. So, these informations we will be using it to construct a diagram. And, from where we will try to obtain the what should be the correction mass and it is a location.

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So, basically this is the diagram that I will try to construct this diagram a fresh. So, that the procedure is more clear. So, first I am drawing a line OD which is basically OD is twice of OA ; because OA is the first measurement and any direction we can take. So, I have taken this as vertical at any inclination we can able to take on this. So, once we have drawn this then we have 2 measurements OB and OC . So, we will take these 2 arc OB and OC and skipping center here we will cut up to 2 arches. So, let us say this is OB

and another arc keeping at the same O A center let us say this is O C; we note down the position of B and C. But we know a they will lie on this arc somewhere.

Now, this same the same arc that means let us say O B and we will keep the center is D we will cut this line here; and we will keep at D and radius as O C. So, basically this is O B and this is O C. So, we got these points they represent B and C; basically we have obtained a parallelogram like this. So, we got a parallelogram O C D and B. So, basically this side should be equal to this one and this side will be equal to this one.

Now, once we have obtained this if we want to join this B and C because this is lying in the two opposite end of the parallelogram; if we draw it will pass through point A because A is the center of this parallelogram. So, if we join in this diagonal this will pass through A. Now, if we see what condition we obtained earlier that A B was equal to A C. But they were 90 degree, 180 degree phase difference was there. So, A B is from here to here; this is the vector and AC is from here to here.

So, and we know from the parallelogram property we know that these 2 sides will be equal. That means A B is equal to A C and vectorially they are 180 degree phase difference because they are in the opposite directions. So, that means we could able to construct a polynomial with the given property a the measurements. That means we utilize O A, O B and O C informations and this property to construct this. So, till now if we go back to the measurement we could able to utilize the first 3 measurement.

The fourth measurement we are not utilized; that we will be utilizing it. So, before that let us see we the construction what we have got. So, in this what we will do now; we will take A as centre and the C A C or B C because they are same as a radius a we will draw a circle. So, let us say we are drawing a circle with A B and A C. So, it will pass because the radius is A B; so it will pass through B also through C. So, this is a circle this circle need not pass through D or O; from this figure you can able to see is not passing through the D and O. But it should pass through B and C. Now, A now our construction is over; basically now if you want to analyze this diagram if we see A B is the effect of the trial mass when we kept at the 0 reference. So, that means A B direction is the reference.

The reference that is theta is equal to 0 reference; A B direction is that is zero difference. And AC is when we kept the mass at 180 degree so that means this direction is theta is equal to 180 degree because this is 0. So, this has to be 180 degree and 90 will come

either this side; if we want to measure the angle counter clock wise direction or it may come this side if you want to measure the direction from in the clock wise. But how to decide whether you should measure we should measure the angle from counter clock wise or clock wise direction? So, basically the fourth measurement this measurement will give us the location of the direction of the angle either we should use clock wise or counter clock wise. So, let us say because of this we are getting a measurement which is O E.

So, O is a vibration amplitude. So, we will take OE as the arc and if we are cutting the circle. So, we will take center as O and we will cut the circle if we are cutting circle approximately at the middle; then let us say this point is E. Now, we can able to see that this direction is 90 degree because we have kept the trial mass at 90 degree. And we are getting the intercept here and if system is linear we find that this particular angle will be approximately 90 degree in this particular here it is clear; that E is a intercept is a perfectly at the 90 degree.

So, this particular point E will give us how we should measure. So, we can able to see now the measurement of the 90 degree is that we need to. So, positive convention for the measurement of this will be counter clock wise because this is the zero line and this is the 90 degree line. So, this should be the counter clock wise direction will be the positive convention for measurement of the angle. So, if the arc is cutting on some other place; then the angle of direction will be changing.

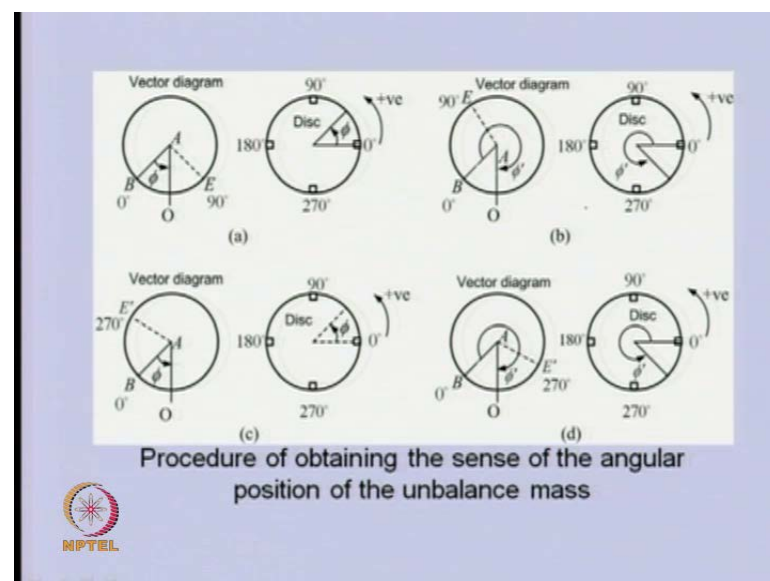
So, let us say if we arc is a cutting here like E prime; then what will happen? Then, this A E prime is the 90 degree direction; that means you need to measure clock wise in this particular case. So, measurement will be clock wise. So, a let us take that we got the intercept here and based on this we have decided that we have concluded that the measurement should be taken in the counter clock wise direction. Now, our aim is to obtain the that is the residual imbalance and its orientation.

So, now we have already seen that A B is the effect of the trial mass. So, if A B is because we know the trial mass so let us say we are having some number n grams. So, it is A B is equivalent to n grams so what is A O? A O we now that this is the measurement which we took without any trial mass that means this is the effect of residual un balance. So, that means we can able to measure O A from this diagram and we can able to find

out what will be the equivalent unbalance in the form of this will be O in the form of grams. So, A O length will give how much gram of unbalance we have? Because we know A B is equal to some finite some number gram. So, O A we can able to find from this relation. So, once we obtain the value of the magnitude the direction will be this one.

So, can able to see this is the phase of the unbalance. So, once we have decided this 90 degree counter clock wise. So, the phase of the residual unbalance which is A O from the reference axis; zero axis, zero theta angle; this phi will be the phase of the unbalance. So, in actual rotor this is zero angle, this is 180, this is 90, this is 270; then we know that the unbalance is near at phi angle that means is 0 phi. So, unbalance will be here and in this particular case we kept all the trial mass at the same radius. So, the radial position of the unbalance is known. Now, we will be keeping at the same radial position but magnitude will be here by this and the angle will be here by this. So, this is the same.

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So, you can able to see this the diagram which we have drawn which we obtain the position on the unbalance and this is the actual disc. So, in actual disc this is zero difference corresponding to this phi which is residual angle is this orientation will be given here. And for the different cases a given here the, if intercepts is intercept at the other location; how we can able to obtain is provided in this figures. Now, this graphical method in which we require only 4 measurement we will list out through one example. In this particular case we are taking a single rotor, a single plain rotor. And we will see

that how we can able to obtain the orientation and the magnitude of the residual unbalance a using only 4 measurements.

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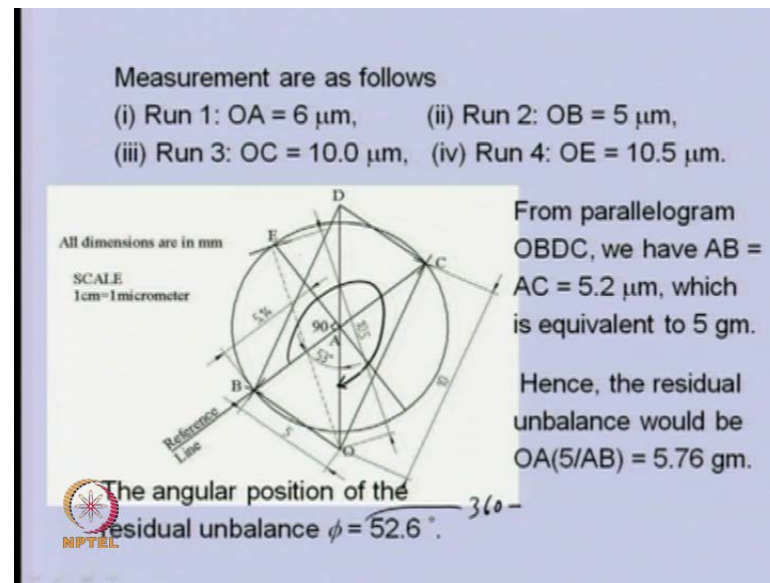
Question

A short rotor or flywheel has to be balanced. Observations of the vibration at one of the bearings are made in four runs as follows: (i) Run 1; rotor "as is" : amplitude $6.0\ \mu\text{m}$, (ii) Run 2; with 5gm. at 0 deg.: amplitude $5.0\ \mu\text{m}$, (iii) Run 3; with 5 gm. at 180 deg.: amplitude $10.0\ \mu\text{m}$, and (iv) Run 4; with 5gm. at 90 deg.: amplitude $10.5\ \mu\text{m}$. Find the weight and location of the correction. Take the trial and balancing masses at the same radius.



So, in this particular case we are considering a short flywheel and we need to balance this. And we have 4 measurement run one in which we are not writing any unbalance magnitude of the vibration is 6 micro millimeter; for run two 5 grams we are keeping at 0 degree. So, this is the amplitude of vibration third run a this is the amplitude of vibration and fourth run. So, all run amplitude of vibration is given a fourth run is at 90 degree. So, we need to obtain the weight and location of the correction mass.

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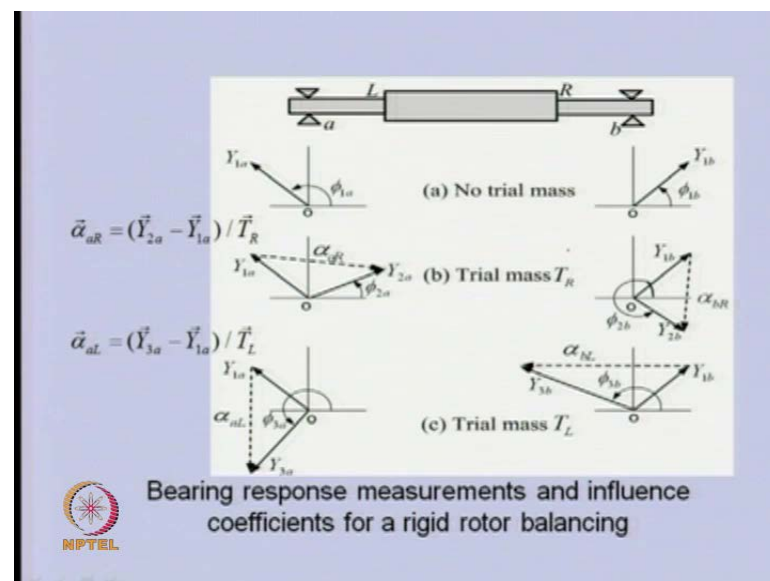


So, first one let us say there is O A is equivalent to 6 micro millimeter amplitude of vibration run two O B is equal to this much; O C is this one and OE is this much. Now, with this we can able to construct the parallelogram a that is O B D C as we described earlier. And we can able to see a once we have drawn the circle also the intercept which is coming is here. So, E is here corresponding to the 90 degree. So, now we can able to see the measurement which we need to take because this is the zero reference line. So, measurement is we need to take from the a counter clock wise direction. Because this is corresponding to the 90 degree because the fourth measurement is corresponding to the 90 degree and the intercept is coming here. So, we need to measure the angle clock wise rotation. So, the actual a position of the residual which is residual unbalance which is A O will be given by a this angle; so 360 minus 53.

So, that will be the a position of the angular orientation. So, basically this will be a 360 minus this. And for amplitude because we know what is O A B which is equivalent to the a given that is this weight that is weight is 5 gram A B is the measurement. So, AB we can able to measure it that is 5.2 and that is equivalent to 5 grams. And O A is the residual unbalance we can measure this also; and then we can able to obtain what should be the equivalent weight of the residual unbalance. So, it comes around 5.76 and orientation is 360 minus 53 degree.

So, this example is clear that how we can able to get not only the residual magnitude, but its orientation also. If we are getting intercept here or here that we will decide; how we should measure the angle either from this side or this side. Now, we will describe another method for rigid rotor balancing that is called influence coefficient method; this is also called field balancing. Because this in this particular case we do not require any balancing machine in actual machine itself; we can able to balance the rotor. Only thing in this particular case we require measurement of amplitude of vibration as well as the phase information; and a now let us see how we can able to....

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So, basic method procedure first I am explaining. So, basically it is a rotor which is mounted on ball bearings in this case generally rotors will be having access to add the mass or removal of the mass at a the ends only. So, let us see L and R are 2 balancing planes. So, here we can able to add or remove the mass and near the bearing we are measuring the vibration by some sensor may be approximate sensor or displacement sensor. So, a and b are the measuring plane; L and R are the balancing plane. So, first we will be rotating the rotor at some nominal speed at some speed; and we will measure the vibration. So, we will not add any trial mass in that front of the rotor what to whatever the residual unbalance is there in the system; based on that we will measure the vibration at plane a and plane b which are the measuring plane.

Now, let us because we are measuring not only the amplitude but also the phase. Phase measurement I will explain subsequently how we can able to measure it. So, let us we got it amplitude of vibration as y_{1a} and the phase is ϕ_{1a} . So, this is the measurement at plane a; when we are not kept in a trial mass. In plane b similarly we got a measurement of amplitude Y_{1b} and the angle the phase is ϕ_{1b} . So, these are the 2 vectors which we have got it that is due to the residual unbalance on the system of the rotor.

Now, in this the next time what we are keeping? We are keeping a trial mass in one of the plane let us say balancing plane R; this trial mass we will be knowing the magnitude as well as in the phase; what should be the orientation that also will be knowing the angular rotation. And that angular rotation a will be so in the rotor basically angular location will be measuring at some fixed reference; always we will be measuring with respect to some fixed reference.

So, once we kept the trial mass in the right plane so obviously we expect there will be change in the magnitude of the vibration in plane a and b; even the phase will change. And we are rotating the rotor at the same speed; we are not changing the a changing the speed; as we rotated in the first case and the same speed we will be rotating in. So, that we have a similar dynamic effect of the residual unbalance; and apart from that whatever the extra mass we have kept in the form of trial mass T R.

So, let us say because of this trial mass now we got an amplitude of Y_{2a} this is the vibration and the phase of that vibration is let us see ϕ_{2a} . So, both amplitude and phase we measured and these are the. So, this particular vector is representing the second measurement that plane a. Similarly, the this amplitude Y_{2b} is the amplitude of vibration and the phase is ϕ_{2b} . So, this particular vector is representing the vibration here for this particular trial mass. So, now we can able to see if we a put this vector here. And so from here to here this vector which is a difference of these 2 vector will be nothing but the effect of the trial mass which we kept it in but the effect we are measuring in the plane a.

So, here a we need to we need to take care in which plane we are keeping the unbalance and at which plane we are measuring. So, here the trial mass is kept in right plane but measurement is the a plane. So, this is the effect of that particular a trial mass on plane a.

Similarly, if we keep this trial this vector here so from here to here this is the difference of the 2 vectors; which we obtained without a trial mass with trial mass. And this will be the effect of trial mass which we kept in right plane when we are measuring in the b plane.


So, we now in the third case will keep will remove this particular trial mass from the right plane. And now we will keep another trial mass in the left plane; and again we will take the measurement. So, let us say for the case measurement is this vector this is $Y_3 a$ and phase is this one $\phi_3 a$. Similarly, in this particular plane is $Y_3 b$ this is the measurement amplitude and phase is $\phi_3 b$. And again if we keep this first vector here this vector here. So, difference of these 2 vectors basically will give. So, this will give effect of trial mass in left plane a after the measurement in plane a.

So, only this will be the that is difference of vector is the effect of transformers; which we are keeping in the left plane after the vibration of the b plane. So, basically with this now we can able to get the influence coefficients. So, we can able to see how we will define the influence coefficient in the previous a lectures specially in the transverse vibration case. So, this is the a influence coefficients a or b. So, a is representing the displacement where we are measuring; the second is representing the force where we are applying. So, in the influence coefficient the first subscript represent the displacement and the second represent the force. So, in this the force is applied at right plane because we have kept the trail mass there and we are measuring at plane a. So, this is defined as displacement.

So, you can able to see because this is the difference of these 2 is the effect of trial mass. So, this is the displacement due to trial mass T R. So, the ratio will give us this influence coefficient; generally if we come here this difference which is $Y_3 a$ minus $Y_1 a$ divided by T L will give you influence coefficient on plane a; when the force is applied in the left plane on the same line we can able to get 2 additional trial masses a the influence coefficient.

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Estimation of Influence Coefficient based on
Measurements

$$\vec{\alpha}_{aR} = (\vec{Y}_{2a} - \vec{Y}_{1a}) / \vec{T}_R$$
$$\vec{\alpha}_{aL} = (\vec{Y}_{3a} - \vec{Y}_{1a}) / \vec{T}_L$$
$$\vec{\alpha}_{bR} = (\vec{Y}_{2b} - \vec{Y}_{1b}) / \vec{T}_R$$
$$\vec{\alpha}_{bL} = (\vec{Y}_{3b} - \vec{Y}_{1b}) / \vec{T}_L$$


So, let us see the influence coefficient. So, these are the first 2 and in the right hand side we will get 2 additional trial masses then this is b R. So, this is the difference 2 vectors in plane b and this is the difference of a vector in plane b for that third case in which the trial mass in the left plane. So, we will get 4 influence coefficients.


So, and this influence coefficients now we are obtaining through measurements; by measuring the displacement and its phase and the trial mass as we know. So, this is the first step in the influence coefficient; that we need to estimate the influence coefficient through vibration measurement itself. So, once we got this; now we can able to obtain the correction mass in 2 planes.

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Let the correct balance masses be \vec{W}_R and \vec{W}_L in the right and left balancing planes, respectively.

Since the original unbalance responses due to residual unbalances are Y_{1b} and Y_{1a} as measured in the right and left planes, we can write

$$-\vec{Y}_{1b} = \vec{W}_R \vec{\alpha}_{bR} + \vec{W}_L \vec{\alpha}_{bL} \quad \text{and} \quad -\vec{Y}_{1a} = \vec{W}_R \vec{\alpha}_{aR} + \vec{W}_L \vec{\alpha}_{aL}$$

$$\begin{Bmatrix} \vec{Y}_{1b} \\ \vec{Y}_{1a} \end{Bmatrix} = - \begin{bmatrix} \vec{\alpha}_{bR} & \vec{\alpha}_{bL} \\ \vec{\alpha}_{aR} & \vec{\alpha}_{aL} \end{bmatrix} \begin{Bmatrix} \vec{W}_R \\ \vec{W}_L \end{Bmatrix}$$


So, let us assume that the correction balance masses in right plane is this one; this contain both amplitude and the phase information. Because we need to know a at what orientation we need to keep the this unbalance and this is in the left plane. So, this 2 are unknown to us.

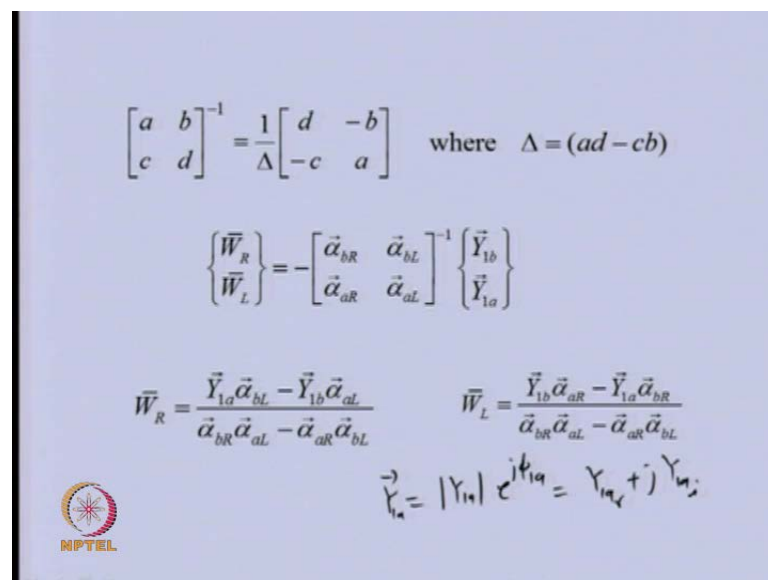
So, let us see these are the 2 variables which we want to find out. If you are coming here if you are multiplying this particular force with alpha b R and this unbalance with alpha b R. So, basically this quantity because if we are keeping this particular correction mass in right plane and this in the left plane with these influence coefficients; we will get a displacement in plane b. Because you can see in the influence coefficient the first subscript is b. So, basically we will get the effect of these 2 in plane b in the form of displacement. So, this will this quantity in the right hand side is basically is what is the effect of keeping this trial mass on to the peak plane b; because these are correction masses.

So, they should give equal and opposite displacement as we obtain in the case of residual unbalance. That means they should give us same displacement at this is in the plane b. So, they should give a this circular amplitude and but it should give the opposite displacement. So, that this vector can get nullified because these are the correction masses. So, that means if we are call if we are keeping the correction masses we expect that they will be low vibration at all. So, that is why we equated this equal to Y_{1d}

which is response due to residual unbalance in plane b but minus. So, that these corrections are these corrections numbers are giving exactly same a response as that of previous one but negative attack. Similarly in the pane a so in plane a we have.

So, if you multiply these 2 correction mass a unbalance with the influence coefficient alpha a R and alpha a L; we will see that this should give a vibration equal and opposite to Y 1 a which was corresponding to residual unbalance, so just equal and opposite. So, that they should nullify each other and we should have net a displacements as 0. So, now we have a 2 equations and in this we have 2 unknowns basically w R and w L. So, these 2 equation we can able to write in the matrix form like this; we can able to rearrange them and now we can able to invert this matrix.


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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\Delta} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \text{where } \Delta = (ad - cb)$$

$$\begin{Bmatrix} \vec{W}_R \\ \vec{W}_L \end{Bmatrix} = - \begin{bmatrix} \vec{\alpha}_{bR} & \vec{\alpha}_{bL} \\ \vec{\alpha}_{aR} & \vec{\alpha}_{aL} \end{bmatrix}^{-1} \begin{Bmatrix} \vec{Y}_{1b} \\ \vec{Y}_{1a} \end{Bmatrix}$$

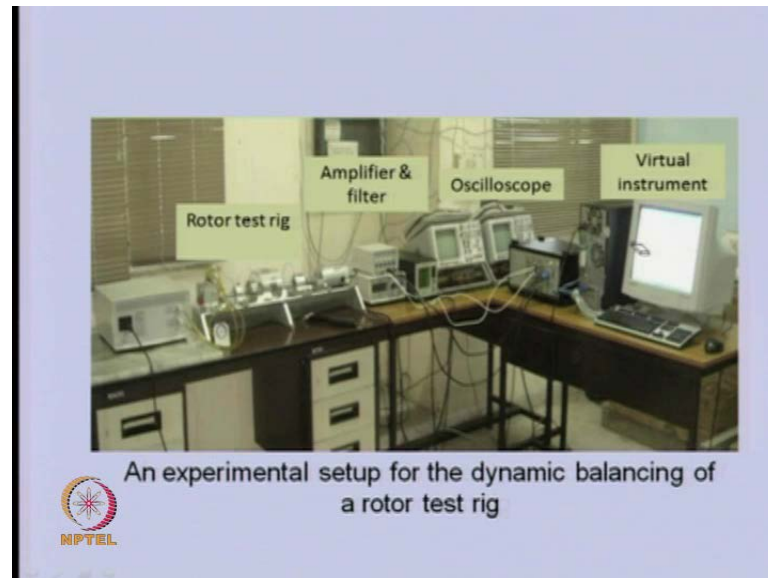
$$\vec{W}_R = \frac{\vec{Y}_{1a} \vec{\alpha}_{bL} - \vec{Y}_{1b} \vec{\alpha}_{aL}}{\vec{\alpha}_{bR} \vec{\alpha}_{aL} - \vec{\alpha}_{aR} \vec{\alpha}_{bL}} \quad \vec{W}_L = \frac{\vec{Y}_{1b} \vec{\alpha}_{aR} - \vec{Y}_{1a} \vec{\alpha}_{bR}}{\vec{\alpha}_{bR} \vec{\alpha}_{aL} - \vec{\alpha}_{aR} \vec{\alpha}_{bL}}$$

$$\vec{Y}_a = |Y_a| e^{j\phi_{1a}} = Y_{1a} \angle \phi_{1a}$$


So, if we have 2 by 2 matrix it is like this is the inversion of that in the closed form where so lambda is nothing but the determinant of that matrix. So, with that if we invert the previous matrix; this matrix we can able to get the W R and W L or like this where if we take the inverse of this using this formula; basically w R and W L will be given by this 2 quantities. You can able to see these are vector quantity generally we will be working in this in the complex form. So, that we have the phase information and the amplitude information, so like one of the vector if it is Y y a. So, this will be Y 1 A e j phi 1 a. So, if we expand we will get the complex quantity in the form of 1 a real part plus j imaginary part. So, once we have in this form we can able to calculate a what will

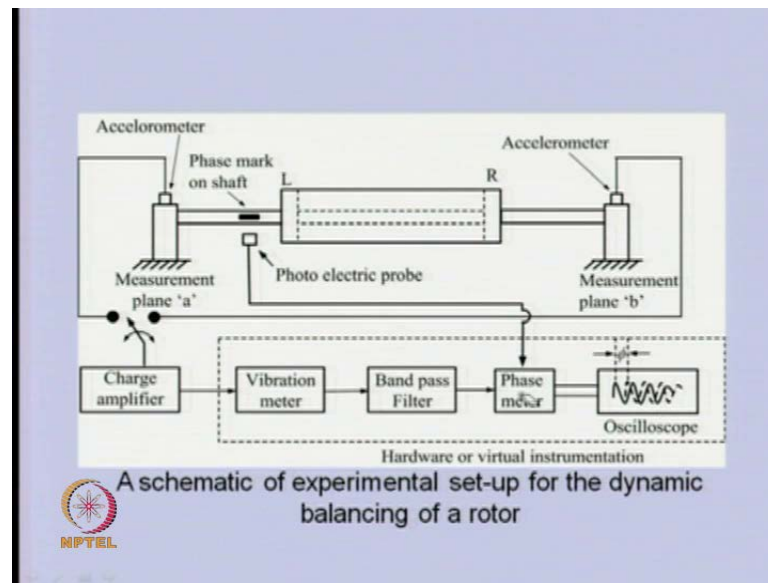
be the orientation a . So, once we have got the W_R and W_L in the complex form again; we can able to calculate what will be the magnitude and the phase of the that particular quantity?

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This is the basically experimental set up of a test ray rotor test ray. So, we can able to see 2 disc rotor system in and in this various instrumentation like amplifier, filter, oscilloscope and virtual instruments for data processing all will be required. In fact, with oscilloscope itself we can able to balance it. But if we want to automate we can use the virtual instrumentation for this purpose.

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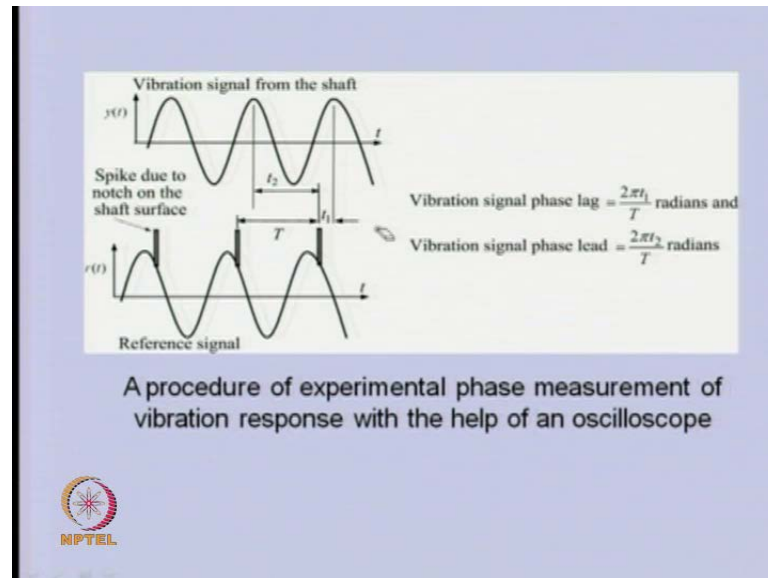
The line diagram of the instrumentation is like that. So, this is a rotor in this; this is the bearings. So, left plane and right plane of the a rotor is the this location we can able to keep the correction masses or trial masses; there is a reference signal here this could be photoelectric probe we can paste a tape on the shaft; that will give us a some kind of reference signal. And on the phase measurement we will be doing with respect to this one; the subsequent slide I will be showing how can able to measure the phase. Here on the bearing or either we can use approximate probe or oscillometer to take out the measurements at these 2 locations.

So, in this particular case this signals first we need to charge amplifier to or conditioning amplifier, to condition the signal. Then, from there either we can send it to vibration meter to get the amplitude or even a this bend pass filter we can able to use if we want to use; if there is a noise we can able to use the filter. So, before this vibration meter we can able to put the bend pass filter. And we can tune this to rotational speed of the rotor or through this photo electric probe reference signal; we can able to tune automatically a this particular filter.

And, even we have phase meter which requires the vibration signal and the reference signal. So, this will give the phase difference between the vibration measure at the bearing. And with respect to this particular reference mark onto the shaft or a alternately we can able to use oscilloscope in which we can able to depict 2 signal; one is vibration

signal from bearings near bearings and one the reference signal. So, this I will be showing in the next slide; how we can able to use oscilloscope to measure the phase if you do not have the phase meter.

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So, basically it is the vibration amplitude with respect to time. So, if we are filtering we will get a nice sinusoidal curve corresponding to the unbalance response and this is the reference signal. So, corresponding to the tape we were getting spike. So, we have kept tape out of the rotor because of that we are getting a spike; now with respect to we can take this as reference. So, we can able to see from here one peak to one peak is that time period of that signal; for both the signal it will be the same. Now, if this is zero reference this spike then we can able to see a corresponding to this we have; so this is the reference.


So, next peak of the vibration signal is here. So, you can able to see the time duration from this peak to the next peak of the signal is t_1 . So, that phase difference we can able to write as $\phi = \frac{2\pi t_1}{T}$ the phase like a of the vibration signal. So, this by that way we can able to measure the phase because phase is a relative flow. So, we can able to represent as a lag or lead; if you want to lead then maybe we need to take another time but consistently we need to take a fixed procedure. So, if you are taking from this spike to another spike; what is the time duration that is the phase; in all signal we should follow the same procedure than the measure of measurement of the phase will be

consistent. Now, we will take up one example for influence coefficient method. And we will see how we can able to obtain the correction mass and correction mass in 2 planes not only the magnitude but also the of orientation of information of the unbalance.

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Question

A rigid rotor machine is exhibiting vibration problems caused by unbalance. The machine is symmetric about its center-line. A trial balance mass of 0.3 kg is sited at end 1 at an angle of 30° relative to some reference position; this causes changes in vibration vectors of $50\ \mu\text{m}$ at 61° at end 1 and $42\ \mu\text{m}$ at 130° at end 2. Determine the influence coefficients for use in balancing the machine, and calculate the balance mass required at each end of the machine if the measured unbalance vibrations are $-30\ \mu\text{m}$ at 230° at end 1 and $-70\ \mu\text{m}$ at 330° at end 2.



So, in this particular case we are considering the rigid rotor and this the machine is symmetric. That means the rotor is symmetric about its center line. So, a trial balance of 0.3 kg when we are keeping at one of the end at an angle 30 degree relative to some reference position; this causes change in the vibration vector of 50 microns at a 61 degree. So, this is the phase we measured and this is at end 1 and 42 microns at 130 degree at end 2.

So, what we have done we have kept the unbalance the trial mass in and one at certain location and we measured the vibration in end 1 and end 2. So, plane 1 and plane 2 we measure the vibration corresponding to trial mass kept at 1. Now, we need to obtain the influence coefficient and as well as the correction masses in each of the plane. And these are the measurement without trial mass. Now, because this is shocked to symmetric. So, only a we have one set of measurement dividing the trial mass because this influence coefficients are also symmetric.


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Measured responses due to residual imbalances are

In plane 1: $R_1 = -30\mu\text{m}$ at 230° phase
 $\equiv (19.2836 + j 22.98) \mu\text{m}$

In plane 2: $L_1 = -70\mu\text{m}$ at 330° phase
 $\equiv (-60.621 + j 35) \mu\text{m}$

$|R_1| e^{j\theta}$
 $= |R_1| (\cos\theta + j \sin\theta)$



So, symmetry we will be using in this particular case. So, measure responses due to residual unbalance are. So, plane 1 these are the measurement this is due to the residual unbalance without adding any trial mass. So, plane 1 this was the measurement which we can going to convert into the complex form like this.

Similarly, in the plane 2 that is we can able to convert the this particular information in the complex form; as I described we can able to use amplitude of this. And check theta angle this will give us the complex quantity because this is nothing but $R \cos\theta$ plus $j \sin\theta$. So, once we have converted this to information in the complex form.

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Trial mass in plane 1: $T_{R_1} = 0.3 \text{ kg}$ at 30° phase

$$T_{R_1} = 0.3(\cos 30 + j \sin 30) = (0.2598 + j 0.15) \text{ kg}$$

Displacement in plane 1: $R_2 = 50 \mu\text{m}$ at 61° phase


$$R_2 = (24.2404 + j 43.73) \mu\text{m}$$

Displacement in plane 2: $L_2 = 42 \mu\text{m}$ at 130° phase

$$L_2 = (-26.997 + j 32.173) \mu\text{m}$$


Now, our trial mass when we kept in plane 1 this is the trial mass information; this we can able to convert again in the complex form. The displacement corresponding to this is plane 1 this much and we can able to convert that in a complex form. Then, displacement in the plane 2 this much this we can able to convert into the complex form.

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$$\alpha_{11} = \alpha_{bR} = \frac{R_2 - R_1}{T_{R_1}} = (48.8919 + j 51.63766) \times 10^{-6} \mu\text{m/kg}$$
$$\alpha_{12} = \alpha_{aR} = \frac{L_2 - L_1}{T_{R_1}} = (92.3559 - j 69.1995) \times 10^{-6} \mu\text{m/kg}$$
$$\alpha_{21} = \alpha_{12} \quad \text{and} \quad \alpha_{22} = \alpha_{11}$$



Now, influence coefficients are defined. So, basically we are keeping the trial mass in dry plane and in the dry plane and these are the measurements in plane b. So, this is the influence. So, we can able to put R_2 , R_1 , T_R or there in a complex form; we can

substitute and simplify. So, we will get the influence coefficient this one. Similarly, α_{12} . So, trial mass we are kept in the right plane only measurement in the other plane that is the difference. So, we can able to see a if you substitute this to which we calculated earlier. And T_R in the complex form after simplifying we will get this. And because rotor is symmetry symmetric we will be having like α_{21} is equal to α_{12} ; α_{22} is equal to α_{11} . So, this symmetry will hold. So, there is no need to calculate the other 2 influence coefficient again.

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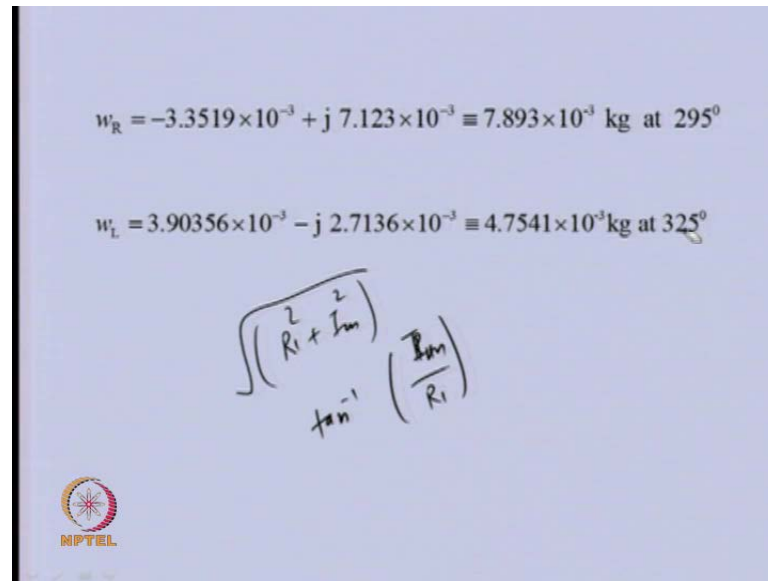
$$\begin{bmatrix} -R_1 \\ -L_1 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} w_R \\ w_L \end{bmatrix}$$

$$w_R = \frac{1}{\Delta} \{ L_1 \alpha_{12} - R_1 \alpha_{22} \} \quad \text{and} \quad w_L = \frac{1}{\Delta} \{ R_1 \alpha_{21} - L_1 \alpha_{11} \}$$

$$\Delta = \alpha_{11} \alpha_{22} - \alpha_{12} \alpha_{21} = \alpha_{11}^2 - \alpha_{12}^2 = (-468.066 + j 16907.73) \times 10^{-6} \text{ (}\mu\text{m/kg)}^2$$


So, once we obtain the influence coefficient we know how the correction mass are related with the influence coefficient and the residual unbalance information. So, we can able to substitute in this expression and this expression a to get the W_R and W_L ; this is determined. And this if we substitute this influence coefficient we will get this information this complex quantity; basically this is within bracket we can able to solve for W_L and W_R .

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The slide displays two complex mass calculations and handwritten formulas for magnitude and phase.

$$w_R = -3.3519 \times 10^{-3} + j 7.123 \times 10^{-3} \equiv 7.893 \times 10^{-3} \text{ kg at } 295^\circ$$
$$w_L = 3.90356 \times 10^{-3} - j 2.7136 \times 10^{-3} \equiv 4.7541 \times 10^{-3} \text{ kg at } 325^\circ$$

Handwritten formulas:

$$\sqrt{R_i^2 + I_m^2}$$
$$\tan^{-1} \left(\frac{I_m}{R_i} \right)$$

The NPTEL logo is visible in the bottom left corner of the slide.

So, once we got this in some complex form again we can take the magnitude. That means real part square, imaginary part square if we take square root of that will be the magnitude. So, that means if we have real part square R square plus imaginary part and square root. So, this will be the magnitude and angle phase will be imaginary divided by real. So, this will use the symbol; so similarly for the W_L .

So, this is the correction mass in right plane and this is the correction mass in the right a left plane. So, now we can able to see we could able to obtain; what will be the correction mass in 2 planes is in the influence coefficient method. But in this we require the phase information that is very important. And because of that we could able to reduce the number of measurement drastically as compared to the cradle balancing machine procedure.

So, in today's lecture we have seen first a graphical method for cradle balancing machine; in that we have seen that how we can able to reduce the number of measurements as compared to the heat and trial method. And in that systemic particularly only we need to measure the 8 total measurement to obtain the balance correction masses in the both the plane of the rotor. And then we took up another method which is more advance method of influence coefficient method; in that because of the non availability of the sophisticated instrument for measurement of the amplitude as well as the phase; we could able to exploit that information.

And we could be able to reduce the number of measurement to only 3 runs. Basically we need to run the rotor only for 3 times and that is enough to find out what should be the correction masses in both the correction planes not only their magnitude but also the orientation of that. In the subsequent class now we will take up methods for balancing the flexible rotor which are more challenging. But because of the sophistication of the instruments which we have now these procedures are quite developed. And now people have started practicing this in industry also. So, we will see first a very basic outline of these methods in the subsequent class; specially the model analysis first we will cover. Then we will go for the influence coefficient method for flexible rotor balancing.