Theory and Practice of Rotor Dynamics
Prof. Dr. Rajiv Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 7
Instability in Rotor Systems
Lecture - 36
Subcritical Speed Whirl

In previous lectures, we have seen various kinds of instability in rotor system. Today, we will take up another instability system especially this occurs for a case of heavy rotors in which the gravity effect is more. This is called secondary critical speed or the sub category speed. As the name implies, we have resonance conditions at half the critical speed of the rotor system and we will see that through that in detail with analytical expression, how we can able to analyze this particular phenomenon? Basically this particular phenomena, we will see that it is a non linear in nature, and we will try to get the analytical expressions by linearizing them. Then by improving the linearized solution, we will try to get this particular solution.

Apart from this, we will take up a case study for fluid film varying mounted rotor, in this particular case, we will be taking a relatively bigger system, and will try to see how the Campbell diagram can help in finding the stability of the rotor system? In the previous lectures, we have seen that with very simple rotor model, either the two degree of rotor model or 3 degree of rotor model. We try to analyze various phenomena by introducing this kind of instability causes like material damping or unequal mass moment of shaft, this kind of parameter. We introduced and try to get some analytical relations, so that we can able to interpret the results better way.

With that as background and we will try to see for bigger system, how we can able to get the instability directly by looking at to the agon value of this system in which the more general case, when we are considering the damping and other effect. We expect the real part and the imaginary part of the agon value. We will see that by plotting them in a Campbell diagram, how we can able to find the instability into the system?

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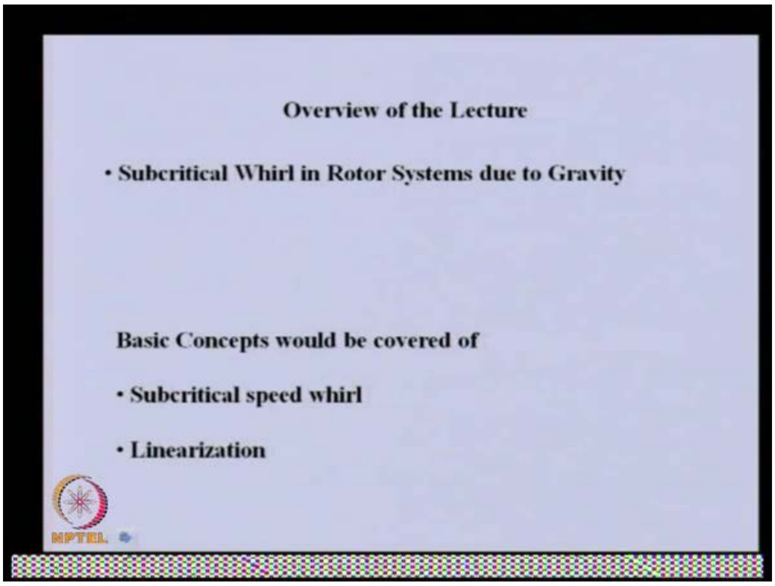

So, in this module basically prime focus will be on the subcritical whirl in rotor system due to gravity. Apart from that as I told, I will take up one case study, because this lecture will be concluding the instability of the system. So, for bigger system how we can able to apply the more powerful the agon value problem solution technique by which, we can able to get the Campbell diagram for finding the stability of the system. In this a new terminology like subcritical speed of whirl, and as I mentioned this equation of motion for such phenomena will be non linear and through some linearization method will try to solve this equation.

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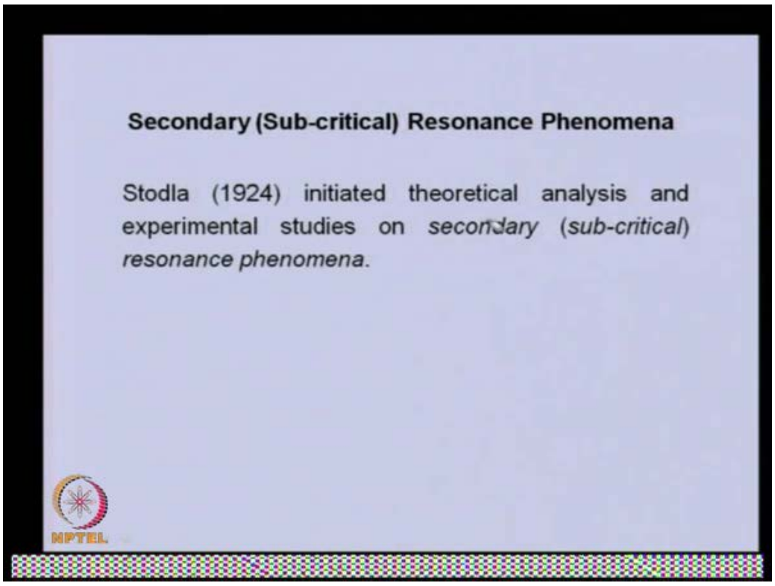

So, regarding the secondary or subcritical resonance phenomena, Stodla in 1924 initiated theoretical analysis and experimental studies on this particular phenomena.

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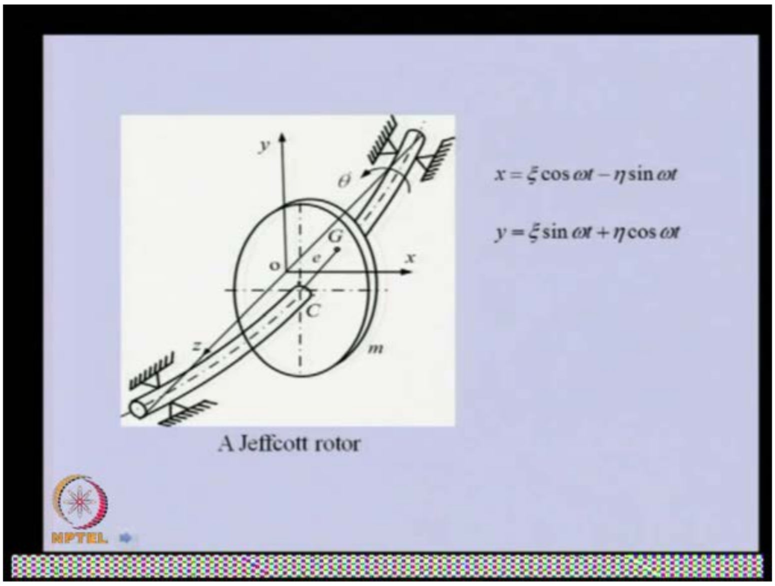

He basically considered a Jeffcott rotor.

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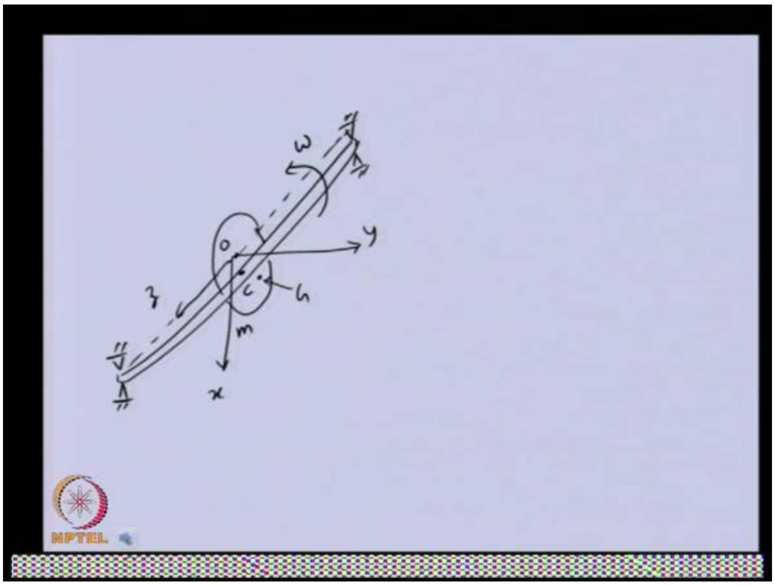

So, I will explain this by simple Jeffcott rotor, so in this we have one Jeffcott rotor like this where this is the ((Refer Time 04:33)) disc? This is mounted on some rigid bearing, may be rolling little bit bearing, so flexibility of the bearing we are not considering. Apart from this we, so here we can able to take, this is the un-deformed, so this is the bearing axis and or axis system origin is here. Let us take this as x system, let us see horizontal system is x and this is y and accordingly we have z direction this one.

In this rotor is rotating with let us say nominal speed of omega, it is the center of the disc ((Refer Time 05:44)) of the disc could be somewhere here, let us say this I have ((Refer Time 05:50)) the location of the center of the gravity, this is the mass of the disc. Now, let us try to draw the free body diagram of this particular disc separately so that we can able to see various forces which are coming on to this.

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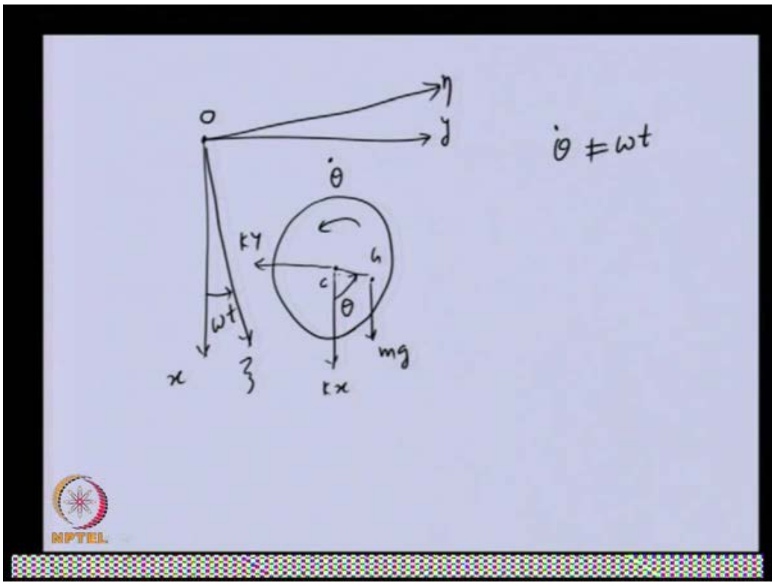

So, this is the axis system which is the stationary rotating axis system, let us say this is the disc, this is the bearing center line of the point and this is the center of rotation of the disc. Because we have removed disc from the shaft we expect elastic force will come, so this is because this distance is k and this is y. So, this will be having k y and k x axis which will be having at the center of rotation c. Let us see center of gravity is here g and m g is acting down, because now our reference axis is at the bearing axis not at the static equilibrium position.

As in the previous case of run up or run down equation of motion in that also equation of motion was non-linear and because of that we considered the bearing axis as the reference for x and y axis. So, here also we will be considering the gravity effect in this and let us say a reference for the angle measurement is x axis, so this is the theta, so this is the direction of the rotation of the disc, which is theta dot angular velocity.

So, in this case also we do not have the angular velocity theta dot is not equal to nominal speed into this, but basically it varies, so we will see how we can able to simplify this particular in the subsequent analysis. So, now various forces are we already applied in the disc once we take the free body diagram of disc, again apart from this if we want to analyze we need to take rotating coordinate system that is more convenient, as we already seen in some of the case it is convenient, this is rotating with omega t. So, now we want to obtain first the equation of motion of this rotor in x y and theta direction and then we will try to analyze the rotor system.

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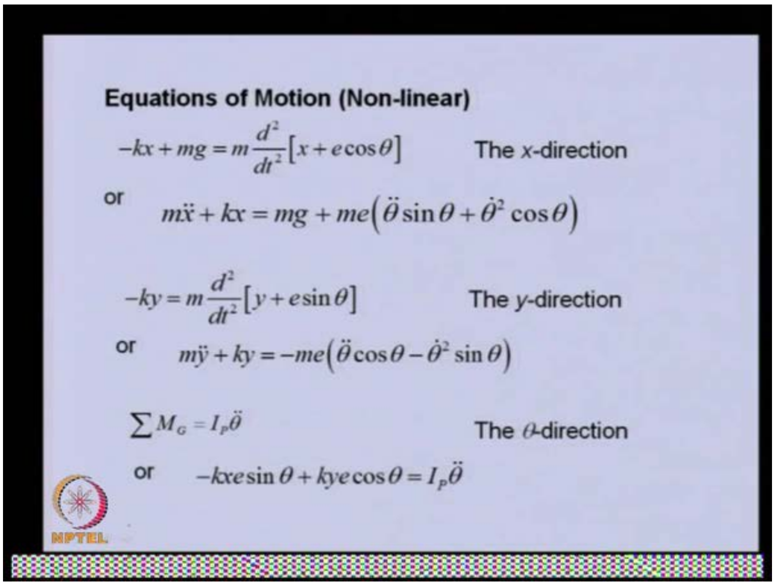

So, in equation of motion in x direction this is the elastic force which is acting here, apart from that we have gravity force. In this particular case the choice of the axis is basically in this equation of motion is in the vertical direction is positive and that is why this is negative, this is positive. So, basically in this particular case for derivation of equation of motion x will be vertical y horizontal itself, so like this. So, because of that it is negative otherwise there is no problem with sign.

So, this is the equation of motion you can able to see, this is the weight and in this particular case as you have seen in the previous lectures also this is the position of center of gravity of the rotor and in this particular case this is the linear acceleration. So, if we differentiate this; this will be having a differentiation which will come here, but differentiation of this will give us a two components like this, in this e is constant it is not the variable, but this will give two components like this once we differentiate twice.

Similarly, in the y direction, we have elastic force and this is the mass into acceleration term. If we simplify this, basically we will get this, here also we will get two terms. The third equation which is moment equation we are taking about center of gravity, so moment of center of gravity will give moment, because there is two elastic force, this two elastic force should be equal to the polar moment of inertia and to the angular acceleration. So, basically these are the three equation of motion and the third equation we can able to simplify like this.

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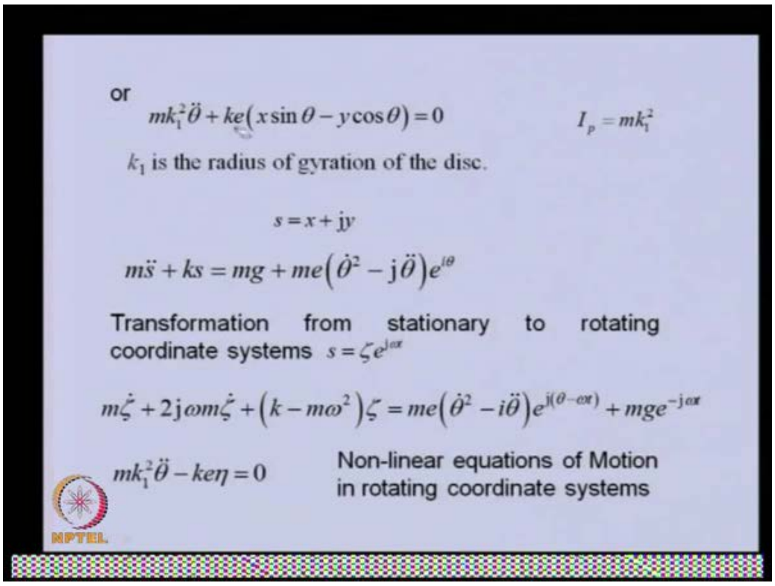

So, i p we can able to write it as mass into radius of gyration of square, so k 1 is the gyration of the disc, so we can able to write in that particular form, k is the stiffness of the shaft, so this two are different. Now, to analyze this particular system, first we are defining a complex displacement s equal to x plus j y, so first equation, this equation and this equation can be combined like this. So, basically we are multiplying the second equation and adding to first equation, so these two term we are getting and then this is from this, so we can able to see various expressions we will be getting like this, from this we will get this. So, basically we can able to get the expression like this.

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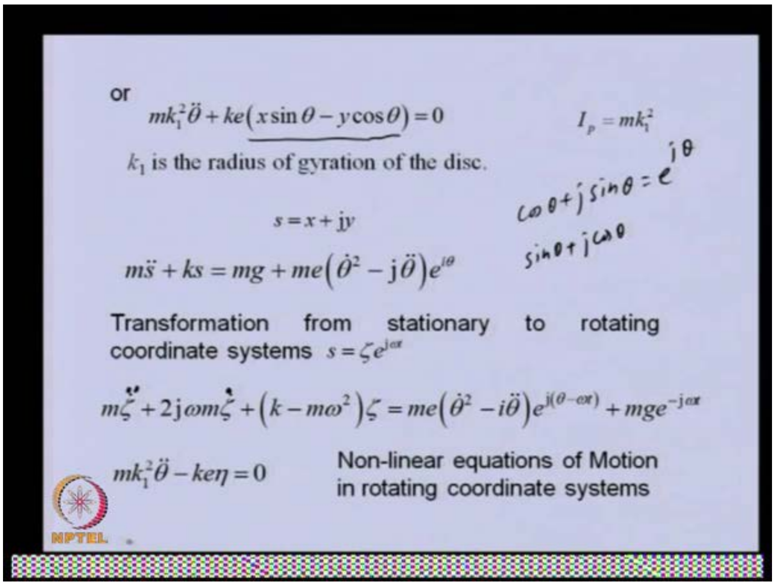

So, basically we are using cos theta plus j sin theta is equal to e j theta, this is thing we are using and wherever there is the problem in the something like this sin theta j plus cos theta. These kinds of terms are there, then we are converting this into this form and by taking some common to j and because of that these terms we are getting. So, basically this is the equation of motion in complex domain, but stationary coordinate system we want to covert this into rotating coordinate system. For that we have already developed in the previous lectures the transformation, so this is the transformation in which zeta is the complex displacement in rotating coordinate system and this is in the stationary coordinate system.

This two coordinate system we already seen that they are rotating with respect to each other with omega. If we use this here we have already done earlier, this several time, this double dot. This kind of transformation, so if we take derivative of the first second and substitute in this equation of motion. Basically we will get equation like this, where some terms even we will get with xi dot also, there is no damping term. But because of the differentiation which we do, because of that we are getting these terms and these equations will also get transformed.

Now, we can able to see this gravity force which is in stationary coordinate system is a constant force in rotating coordinate system. It will appear as rotating in the opposite direction. Because if a person is sitting in the rotating coordinate system and if he is observing the static force which is there in x direction it will appear to him as it is rotating in the opposite direction of the axis on which it is sitting. Similarly, here there will be difference in the frequency because of the observer will be sitting in the rotating coordinate system, so that much difference will come, so this will come directly in the transformation.

In the third equation the moment equation which we had here so basically we will see that this particular term within the bracket is nothing but nu. So, the nu and xi are basically two axis which we have chosen and which is rotating coordinate system. So, this is in the previous lecture we have already seen this relation, so that will be nu, other terms are same. So, basically this equation and this equation is the non linear equation. Now, we can able to see because some of the terms are having square. So, this is the non-linear equation of motion and this is the rotating coordinate system.

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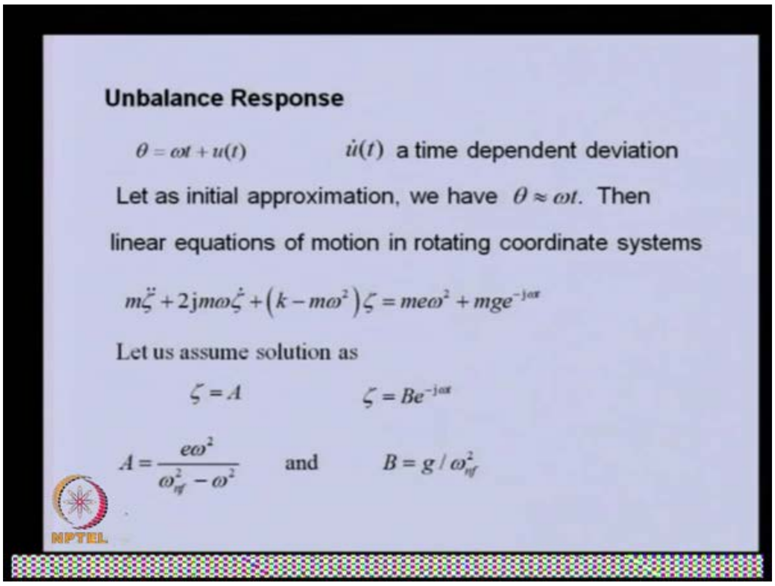

Now, first we want to obtain the unbalanced response of that particular equation of motion and theta which is angular displacement of the disc, which is the theta dot is not equal to the constant angular velocity. So, what we are assuming here that there is equal to omega t plus some small quantity, which is time dependent and this u dot is time dependent deviations of the angular velocity. So, because this is the nominal speed, whatever the variation is taking place of that speed is there in this. So, basically u is nothing but the torsional angular displacement, so as the initial approximation to solve the previous equation. We are assuming that this torsional displacement is negligible and we are saying theta is equal to omega t and with this condition we are substituting in the previous equation of motion here, it is equal to omega t.

So, basically this equation will become linear equation, right hand side will take a simple form like this. So, this in the rotating coordinating system, complex displacement equation, right hand side is now a known quantity theta is now vanished, because we have chosen this approximation. So, now this is the linear equation now this we can able to solve it, we have now in rotating coordinate system, we have one constant force, another is time dependent force. So, we can able to choose solution for this and this separately because this is the linear system, we can able to obtain the solution of these two independently. Then we can able to ((Refer Time 18:40)) these two solutions.

So, first solution is corresponding to this constant term let us see, zeta is equal to a constant term we need to obtain. So, if we substitute this in this equation of motion by not considering this one, so we will see that this term will be 0 because this is constant, this will be 0. So, this will give A is equal to this divided by this quantity, so e omega square by this quantity, so this will be the A corresponding to this. This is the amplitude of the solution corresponding to this constant term this. Similarly, for the solution of this we can able to choose some constant and this frequency, so e minus j omega t, if you derivate this twice substitutes in this equation by neglecting this one.

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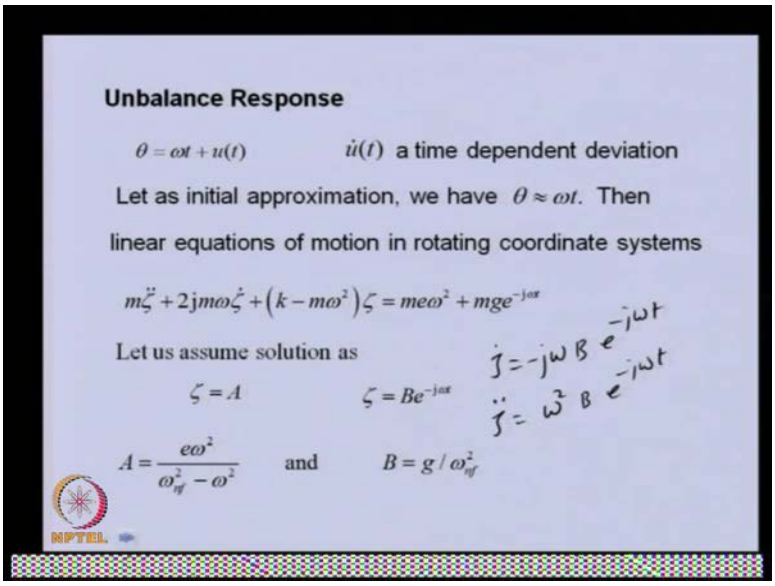

So, basically we need to take the derivative of this, so that will give minus j omega B, e minus j omega t double derivative that will give, that is omega square B, e j omega t and these three we can able to substitute here. Then we can solve for B, because B is quantity which will be obtaining. So, we will see that B will get simplified to this simple expression of B is equal to g by this quantity.

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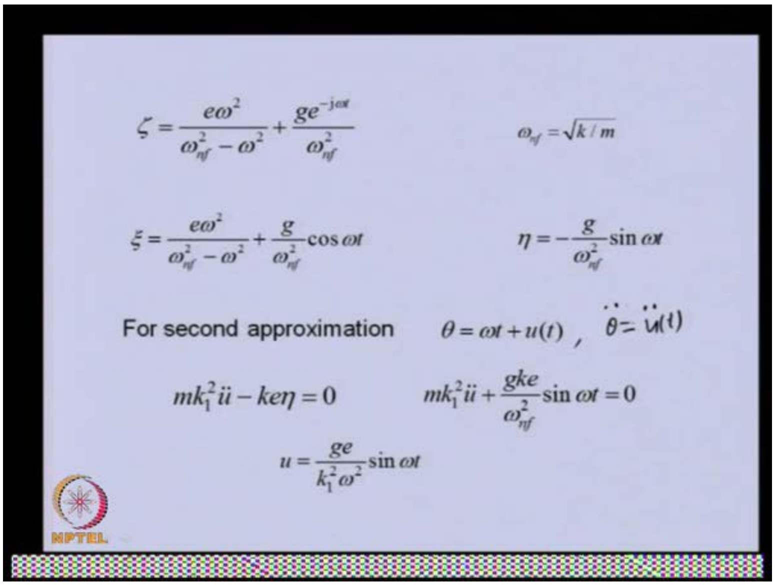

So, now we can able to write the solution for this, the simplified equation, linearized equation like this. So, this is the constant term and this is the b term, so this is the solution of previous equation. This is the omega n f finite un-damped natural frequency, we can able to take the real part and the imaginary part of this separately, because in the third equation this equation we have eta. So, we want to substitute this assumed solution here and whatever the solution we obtain we want to substitute here and then solve for theta. So, basically we can able to substitute in real part and imaginary part, because the imaginary part is having this is only real part, so that constant term will not come in this.

So, this eta now you want to use in the third equation to get the theta variation. So, now we can able to see if we substitute, so now as a second approximation, so that means now we want the updated solution. For that again we are introducing theta is equal to omega t plus u t and this we are substituting in the third equation and that requires theta double dot. So, theta double dot you can see this will vanish, this will give us u double dot t and this we are substituting and this in the moment equation, this moment equation. So, basically with this we are trying to get what is the variation u t, so we have substituted eta from here.

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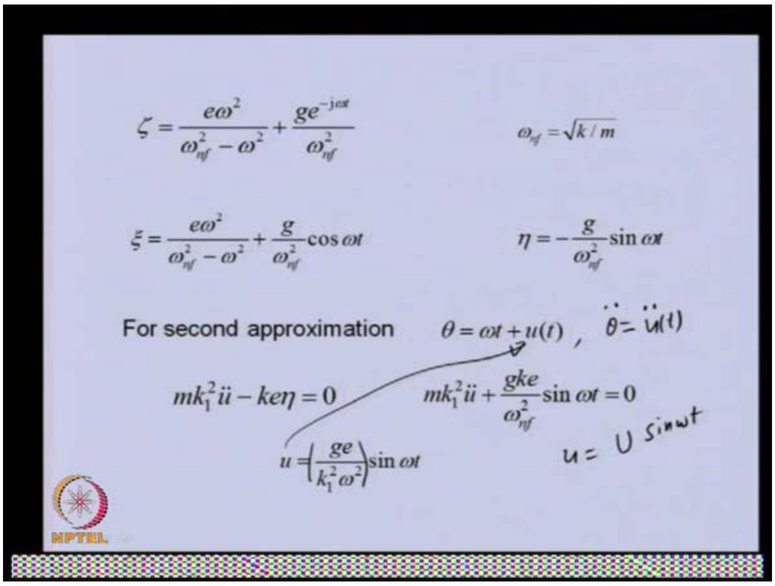

So, now this we can able to solve for u and the solution will be something like u is equal to some form constant sin omega t and if we differentiate these twice and substitute in this equation and we can able to solve for capital U. Basically this will be the capital U within the bracket term, so this is the solution of this equation of motion. Now, with the third equation, we can able to see that we could able to get what are the variations of the theta. So, in the first approximation, we assume this is equal to 0, we got the value of zai and eta. That solution which was approximate we used in the third equation to get the variation u and now we have variation of u.

Now, this solution we can able to substitute in the theta expression and again we can go to the equation. This equation in which the theta variation are there, because we need the more accurate solution of this, first solution we obtained by considering the angular velocity as constant. But now again once we obtain the variation of the angular displacement, we want to use this expression here. So, this theta now we will be substituting in the right hand side of that equation.

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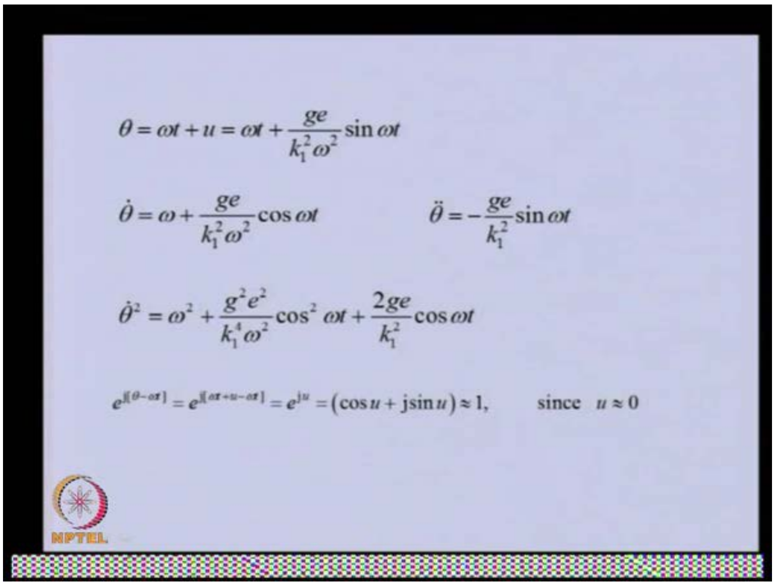

So, basically theta is now defined like this, so if you take derivatives of this double derivative, because that contains some square terms also and some other terms, so they can be simplified. In this we can able to see this because u is very small variations some kind of approximations will be using it wherever we will be getting some kind of non-linear terms, we will try to substitute drop those non-linear terms.

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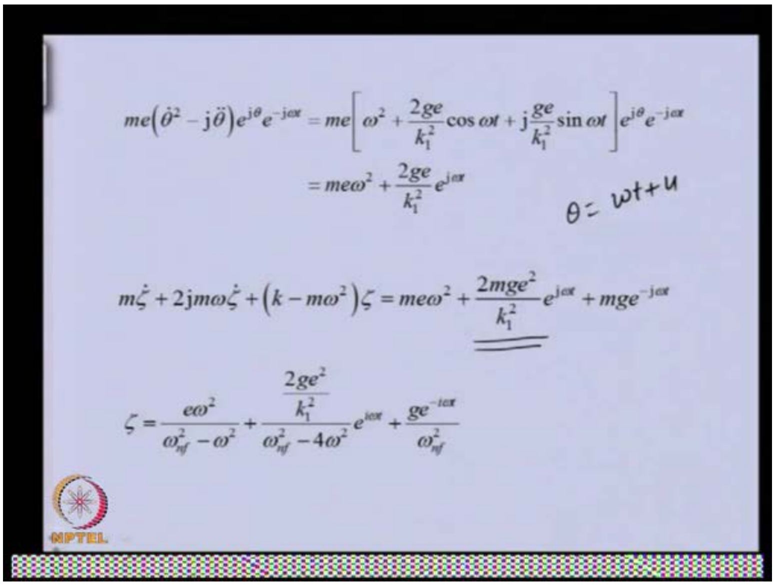

So, basically if we substitute the theta is equal to u omega t plus u in the equation of motion, we will see that in right hand side this term basically will give this, which will give this simple expression. So, basically equation of motion will now be having this form, so this additional term which will be getting this and this was already there in the previous expression, when we linearized the equation of motion. So, if you go back to this equation of motion to which we have linearized it by putting theta is equal to omega t, we had this two terms.

Now, with this variation we are getting a additional third term and this term we can able to see it contains the gravity terms also, apart from the other parameter, but this is the main parameter. Now, we want to solve this particular equation, for solving this again because now with this equation is linear again, we can able to solve for this, this and this for separately and we can add it. We already solved for this, in this previous expression, now we will be solving for this. We will see that in the solution of this for all three, it will be like this.

So, basically this is the additional term which we are getting, which was not there in the previous solution. So, in the previous solution two terms were there like this, only two terms were there. Now, we are writing a third term, because of the variation in the theta. So, now if you look into these equations, so better we can transform this in the stationary coordinate system, because now this solution is in the rotating coordinate system.

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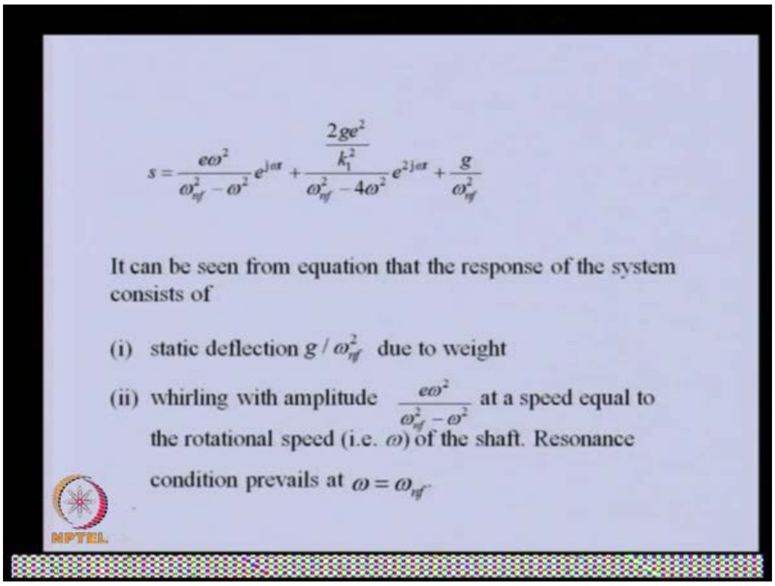

So, if we transform this in the stationary coordinate system, we will see that basically here, whatever the transformation we had earlier. This transformation now it will be opposite.

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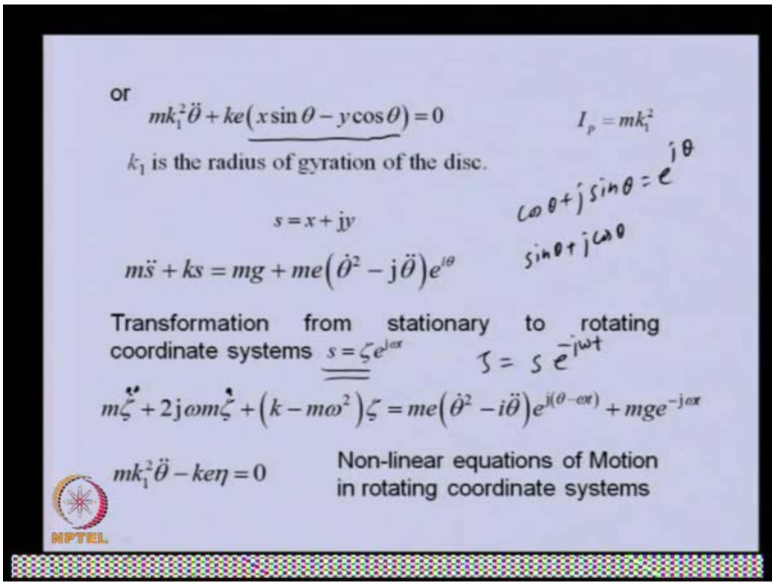

So, to transform back into the original system, we need to use minus of omega t, so this transformation we need to use for transfer this in the…

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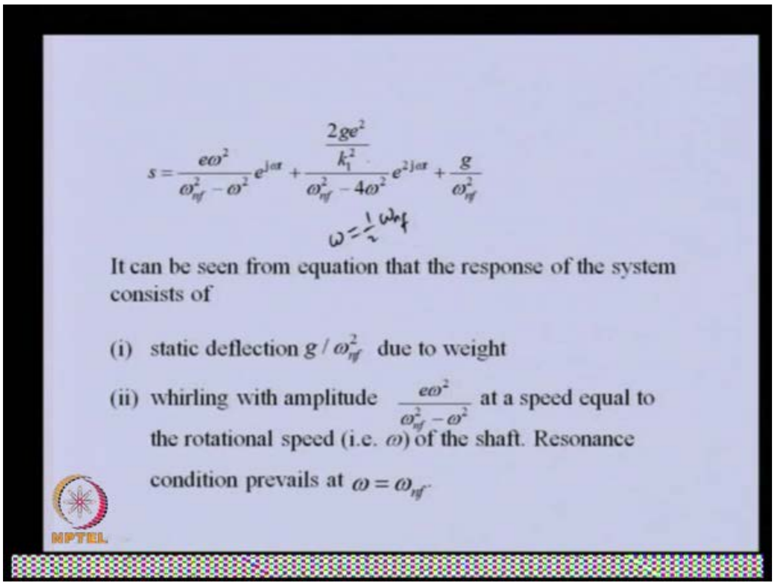

So, basically that will get multiplied by the j omega t, so basically this is the complex displacement in stationary coordinate system. So, if you see this equation, this term, this is the deformation of the shaft due to gravity, because this is no time dependent thing, so this is static force. So, whatever the gravity force is there because of that decimal deformation will take place. On the first term this is the amplitude, the frequency of this particular response will be equal to the spin speed of the shaft, frequency is equal to spin speed of the shaft.

In this we are getting a condition that when the frequency of the shaft is equal to un-damped natural frequency of the shaft, we have resonance condition. So, this is standard due to the unbalance, because this e is there. So, this response is purely due to unbalance in which we know that there is the resonance whenever the speed is equal to the un-damped frequency of the system. The whirl frequency for the unbalance we know is equal to the spin speed, so this is also we know from the previous analysis.

Now, coming to this term in this if we say this frequency component, we have frequency of the whirl corresponding to this is twice the spin speed, so the frequency is twice the spin speed. As I mentioned earlier we are this is due to the gravity, mainly this particular phenomena is coming there is no gravity of it, then this whole term will be vanished. The in-combination with that there is a because of the eccentric application of the gravity. So, if e is also 0, then it will also it not be 0. So, this will this particular phenomenon is the combination of the gravity effect and when gravity is acting eccentrically to the rotor system which is always present, e is always present as we know in the practical rotors.

So, the whirl frequency is twice the spins speed then if we see this denominator here, so from here we can able to see that there will be resonance, when the speed is half the natural frequency. So, when omega is equal to half the omega n f then the resonance will take place. So, the resonance is taking place when the speed is half the un-damped natural frequency, which is quite different as compared to the this particular case, in this particular case when the speed is equal to natural frequency then we have the resonance. But here when the speed is half the natural frequency then the resonance is taking place, but the frequency of whirl is twice the speed of the shaft.

So, this we need to observe carefully that the resonance is at the half the critical speed that is why it is called secondary or subcritical speed. Because when the speed is below the critical speed then also the resonance is taking place, but the whirling frequency of that is twice the natural frequency. That means, at resonance condition when we are operating at this the resonance will take place and the frequency of this whirl will be twice of this that means equal to the natural frequency.

So, this is the very interesting phenomena of the subcritical speed in which the resonance is taking place at half the critical speed or the natural frequency, but the whirl frequency is at the natural frequency. So, this is we have explained here so this is static deflection, this is unbalance, this is eccentric application of the gravity, because of that this second resonance phenomena is taking place.

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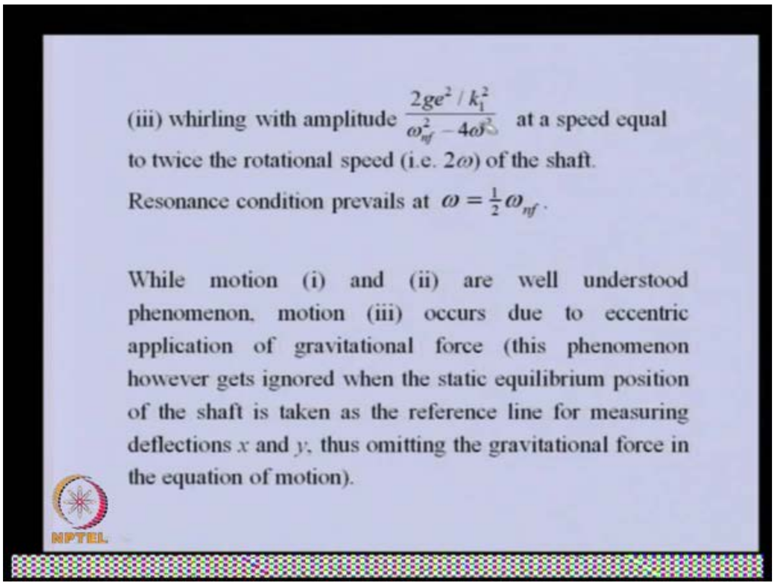

So, this I have explained in detail how this particular phenomena is taking place and what is the amplitude of that particular phenomenon. Commonly we know that gravity and the unbalanced response is well known, but the second resonance phenomena is not well known in literature, but it has been explained quite early in 1924 by Stodla. Now, we will take up case study in which we will be taking a slightly bigger system .Will try to demonstrate that through finite element method, which is more versatile method for modeling any rotor system, either it is to bearing or to multiple bearing support system.

Finite element method we have already seen that very conveniently we can able to model a rotor system, even in that we have seen that the bearing can be added quite conveniently or gyroscopic effect we can able to analyze. So, in this particular case we will try to analyze the rotor system and will try to see how they can, this particular method can be mounted on fluid film bearing with all the eight linearized coefficient of this stiffness and damping. With that will try to obtain the whirl frequency of this particular rotor system.

In this not only we will be considering the bearing at eight linearized, but will in this second case study we will be considering the speed dependency of this particular bearing property. Because in the previous lectures, we already seen that especially in fluid film bearing, they can have some kind off, when the speed changes there static ((Refer time: 34:00)) changes. Because of that the property of these bearing stiffness and damping changes with speed. So, using short bearing approximation, we will try to incorporate that particular speed dependence in the rotor system and will obtain the Campbell diagram. In the Campbell diagram apart from the frequency, we will put the deplement also, that will give the idea about the stability of the system, but various speed ranges. So, there is another way of finding the first stability of the system especially in the complex system.

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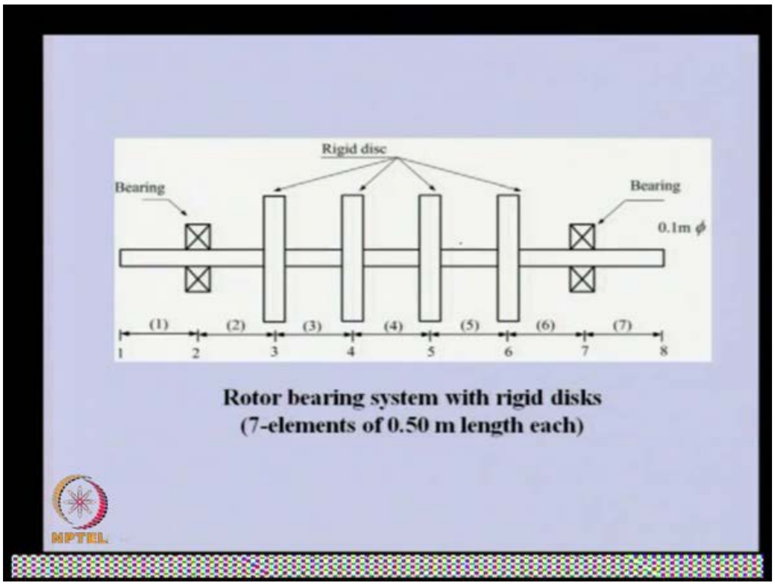

So, in this particular case we are taking basically a rotor system like this, so there is a huge rotor with four discs are there, they are also heavy discs. These bearing fluid film bearings and we will be modeling this particular rotor system using finite element method, various property of the rotor and bearings are given here.

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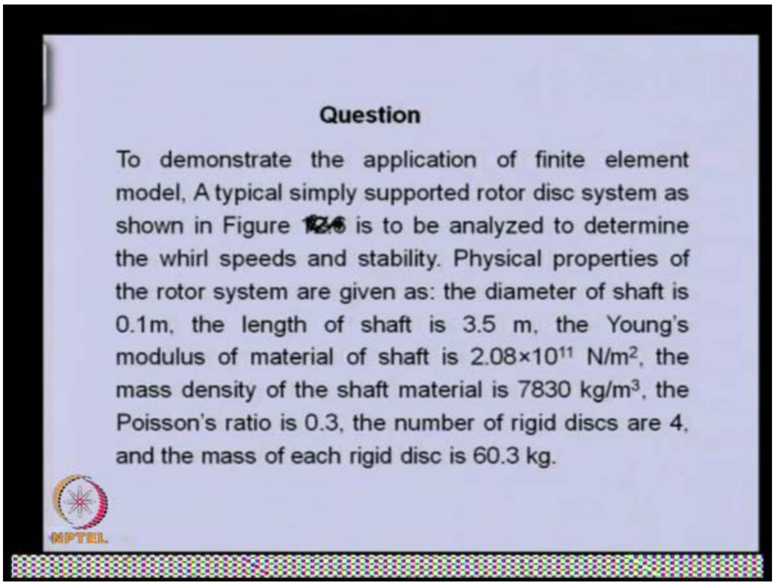

So, basically here we want to demonstrate the application of the finite element method, for typically simply supported rotor disc system, but this boundary conditions is not limitation for finite element methods as we know. Various physical properties of the rotor system is like shaft diameter is 0.1, meter length of the shaft is 3.5 meter, module properties are given here. Young's modulus and density of this ratio, number of disc are 4 as we have seen in the figure and they are heavy discs 60.3 kgs each of them, both bearings are linearized.

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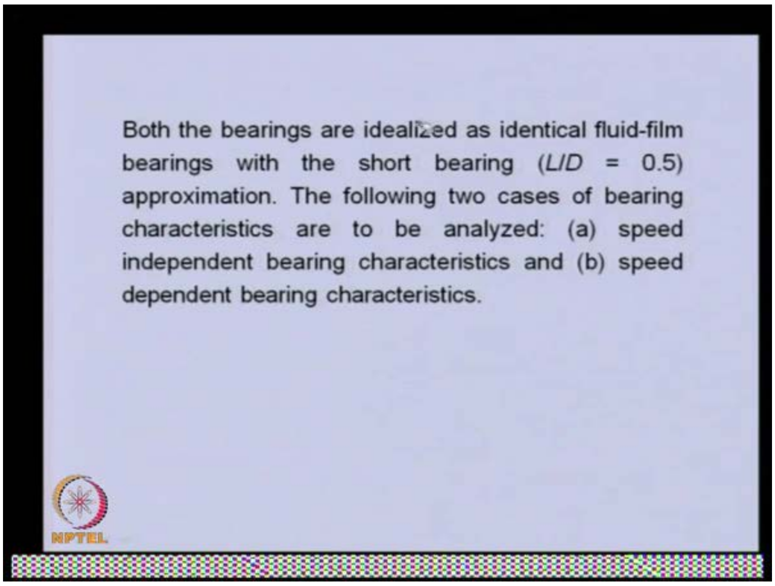

They are idealized as identical fluid film bearing and they are l by deviation is 0.5, which we can able to consider as short bearing. We need to consider two cases, one is speed independent bearing characteristics. In this we are considering we are choosing one bearing parameter, we will keep constant that during the change of the speed. In this particular case another is speed dependent bearing characteristics in which we will be varying the speed of the property of the bearing with the speed.

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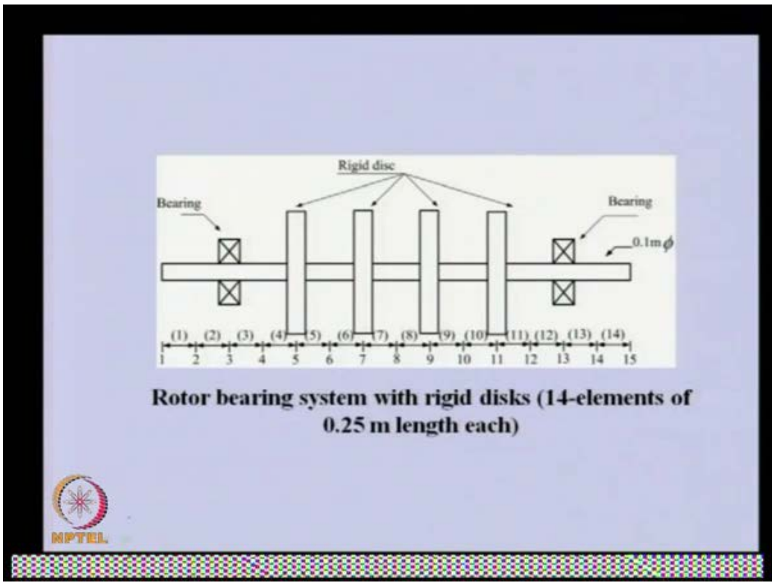

So, in this particular case just to show the convergence we have taken two cases, one is we have idealized this as 7 elements. In this second case we have considered that as 14 elements. So, we have divided this various shaft segment into two so that the number of elements becomes 14.

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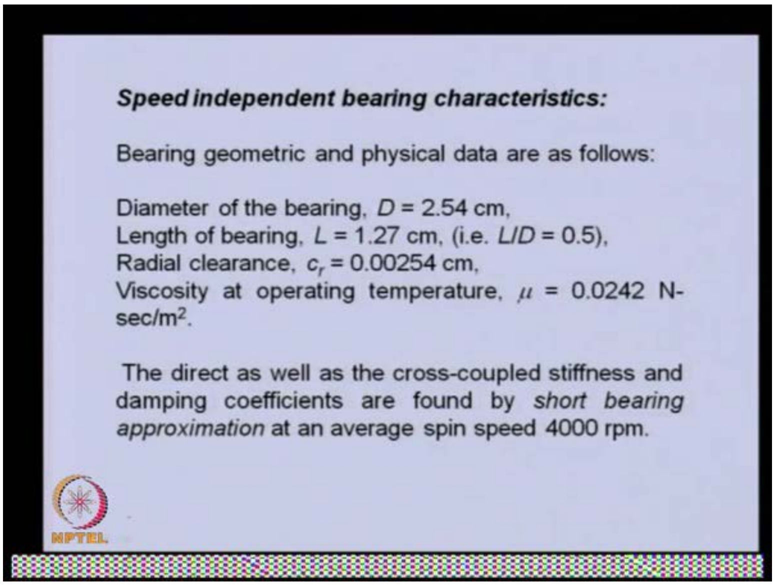

So, for speed independent bearing characteristics various parameter of bearing like d l where algorithm is this radial clearance, viscosity at operating temperature we have provided. So, direct as well as the cross coupled stiffness and damping coefficients, we can able to find for any particular speed. So, what we have done at 4000 rpm, we have obtained the bearing property and we are assuming that remains constant for all other speed, so something like average speed we have chosen.

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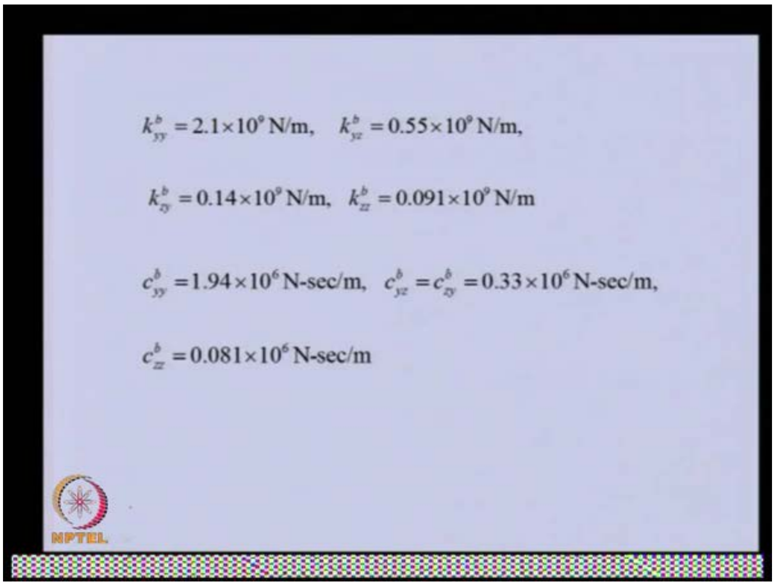

So, these are the by short bearing approximation which we described earlier, for mean speed of 4000 rpm and various bearing property. We can able to bearing parameter, we can able to get the stiffness coefficients and damping coefficients of bearings like this.

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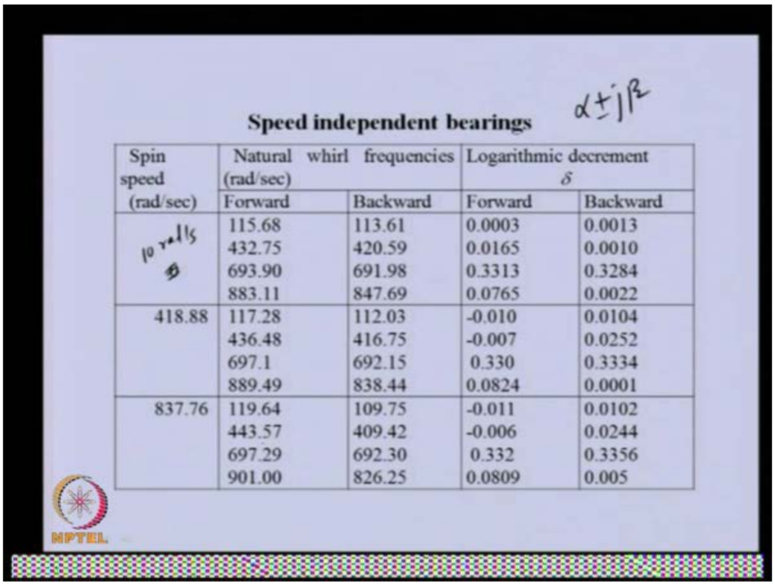

Speed independent bearings $\alpha \pm j\beta$

Spin speed (rad/sec)	Natural whirl frequencies (rad/sec)		Logarithmic decrement δ	
	Forward	Backward	Forward	Backward
10 rad/s	115.68	113.61	0.0003	0.0013
	432.75	420.59	0.0165	0.0010
	693.90	691.98	0.3313	0.3284
	883.11	847.69	0.0765	0.0022
418.88	117.28	112.03	-0.010	0.0104
	436.48	416.75	-0.007	0.0252
	697.1	692.15	0.330	0.3334
	889.49	838.44	0.0824	0.0001
837.76	119.64	109.75	-0.011	0.0102
	443.57	409.42	-0.006	0.0244
	697.29	692.30	0.332	0.3356
	901.00	826.25	0.0809	0.005

Now, in this particular case because I am not describing the detail of the rpm formulation that we already discussed in detail earlier, so directly I am giving the whirl frequency, because so here we can able to see 1 is the 0 speed. So, at 0 speed we have, basically this is nearly 0 speed, this is not 0 speed, so that is why you can able to see that there is slight splitting of the frequencies are there. So, this is very low speed, basically this is not 0 at very low speed at obtain that is radius 10 per second is there. We found some splitting of the frequency at this so forward and backward whirl and these are the logarithmic decrement.

So, basically in the agon value as we described earlier, this beta gives the whirl frequency and alpha is the damping parameter and with alpha we define the logarithmic decrement. Whenever this logarithmic decrement is basically positive or negative, we have instability. So, you can able to see that in this particular case, this is low we have stable, so in this particular case the logarithmic decrement as been defined when they are negative the instability is there. So, you can able to see that in this particular case as we are increasing the speed the splitting is more in various. So, these are the first four whirl frequencies as for fun, because they are changing in speed because of the gyroscopic effect in the disc.

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Speed independent bearings

Spin speed (rad/sec)	Natural whirl frequencies (rad/sec)		Logarithmic decrement δ	
	Forward	Backward	Forward	Backward
1047.2	120.85	108.59	-0.011	0.0099
	447.37	405.47	-0.006	0.0239
	697.39	692.36	0.3367	0.3337
	907.43	819.20	0.0802	0.00723
1256.64	122.08	107.43	-0.0121	0.0096
	451.21	401.43	-0.0056	0.02340
	697.49	692.42	0.335	0.337
	917.02	811.80	0.0089	0.0797
1675.52	127.56	105.13	-0.0128	0.0090
	458.94	393.25	-0.0045	0.0224
	697.69	692.51	0.337	0.339
	927.29	796.32	0.0792	0.0114
2097.4	127.07	102.85	-0.0136	0.0084
	466.63	387.98	-0.0034	0.0216
	697.89	692.58	0.341	0.3422
	940.40	780.27	0.079	0.01261

So, these are the various speeds so basically you can able to at various speed we obtain the first four whirl frequencies and their logarithmic decrement and these can be plotted in Campbell diagram.

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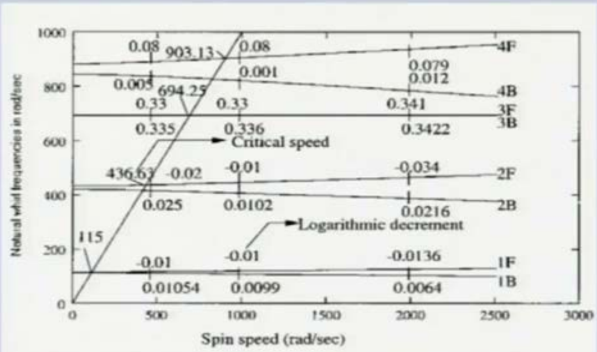

Natural whirl frequency map and stability of rotor–bearing system supported on speed independent bearings

So, this is the speed and this is the natural whirl frequency, so you can able to see more spin. Because of this, we are considering the gyroscopic effect because of that this splitting of the frequency is taking place as the speed is increasing. These are the upper one is the forward whirl and the lower one is the backward whirl, so wherever the negative logarithmic decrement are there unstable reasons of and this particular case, this line is the speed is equal to whirl frequency line and these are the critical speeds. So, the forward critical speeds have been shown here, so in the third natural frequency this splitting is not much, this is the fourth, these are the forward critical speeds and the lower one is the backward critical speeds.

So, you can able to see the logarithmic decrement is defined both for the forward and backward, so as such the backward whirls are stable here, they are unstable, so here also they are stable, so here they are unstable. But if you are going to the higher one, this third mode is stable, because they this logarithmic decrement is positive, here also it is stable. So, in this particular case at low speeds, the first forward and backward whirls at both lower and higher both they are unstable. But at the higher modes are stable, lower modes are unstable, but the higher modes are stable.

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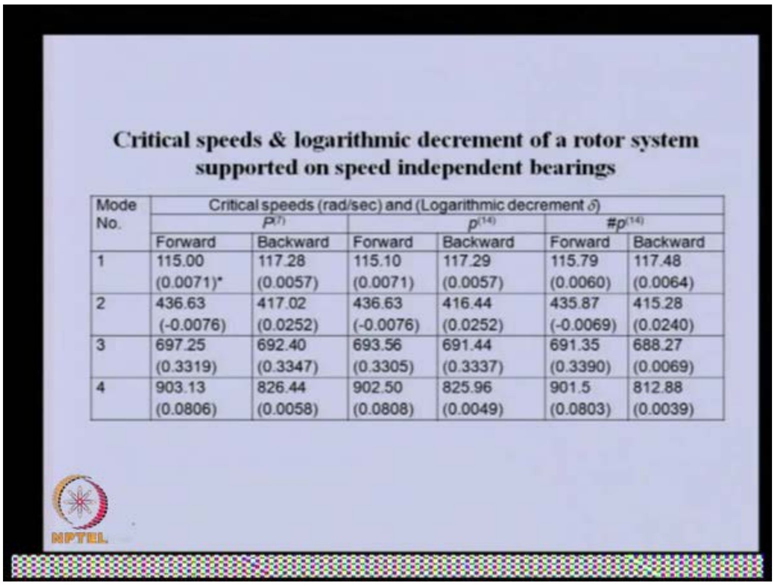
Critical speeds & logarithmic decrement of a rotor system supported on speed independent bearings

Mode No.	Critical speeds (rad/sec) and (Logarithmic decrement δ)					
	$P^{(7)}$		$p^{(14)}$		$\#p^{(14)}$	
	Forward	Backward	Forward	Backward	Forward	Backward
1	115.00 (0.0071)*	117.28 (0.0057)	115.10 (0.0071)	117.29 (0.0057)	115.79 (0.0060)	117.48 (0.0064)
2	436.63 (-0.0076)	417.02 (0.0252)	436.63 (-0.0076)	416.44 (0.0252)	435.87 (-0.0069)	415.28 (0.0240)
3	697.25 (0.3319)	692.40 (0.3347)	693.56 (0.3305)	691.44 (0.3337)	691.35 (0.3390)	688.27 (0.0069)
4	903.13 (0.0806)	826.44 (0.0058)	902.50 (0.0808)	825.96 (0.0049)	901.5 (0.0803)	812.88 (0.0039)

So, this is one of the Campbell diagram, critical speed and the logarithmic decrement of a rotor system supported on speed independent bearings. So, basically previous tables were for whirl frequency, now this is for the critical speed. Now, for the intersection of the 45 degree line, so here that somebody we are giving. So, basically and in this we are given for 7 element and 14 element. What are the difference in forward critical speed and backward critical speed? This is using some kind of condensation scheme first statically length scheme how the resonance effects?

So, basically this order or the rotation degree of freedom we eliminated and we saw the effect of the, not much effect we observed, especially at the low frequency, lower modes this particular condensation scheme, but higher frequency. Because of static condensation, we expect more error for high speed frequencies. So, in this we can able to see these are the critical speed forward backward and these are the logarithmic decrement corresponding to that. Second mode this is forward critical speed, backward critical speed, this is with the more number of elements, so we expect these will be more accurate as compared to these ones, but up to second modes we are getting close values. So, this is the summary of the Campbell diagram critical speeds and their logarithmic decrements.

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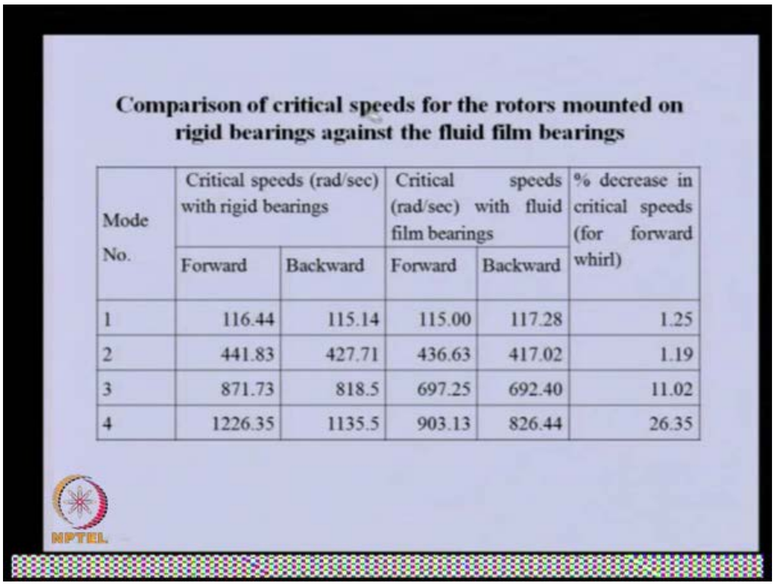

Comparison of critical speeds for the rotors mounted on rigid bearings against the fluid film bearings

Mode No.	Critical speeds (rad/sec) with rigid bearings		Critical speeds (rad/sec) with fluid film bearings		% decrease in critical speeds (for forward whirl)
	Forward	Backward	Forward	Backward	
1	116.44	115.14	115.00	117.28	1.25
2	441.83	427.71	436.63	417.02	1.19
3	871.73	818.5	697.25	692.40	11.02
4	1226.35	1135.5	903.13	826.44	26.35

This is the comparison of the critical speeds just for you want to compare same rotor mounted on the some kind of rigid bearings, simply supported bearings. What will be the difference that we wanted to see? So, we can able to see that critical speed with rigid bearing and fluid film bearing, so obviously we expect with fluid flow bearing, there will be decreasing in the natural frequency or the critical speed. So, we could able to observe that especially at higher modes the decrease is more, the critical speed the decreases more, when we are comparing with rigid bearing and fluid film bearing.

So, you can able to see that if we consider the fluid film bearing as rigid, how much error we can able to incorporate in the calculation, especially for the higher modes? So, this is very interesting comparison between the rigid bearing and the fluid film bearing model. So, obviously this module is more accurate and if we consider the fluid film bearing as simply support them, we get enormous results and especially in higher modes, we get error for may be for 26 percent. So, that is a typical result, but it is not valid for all cases.

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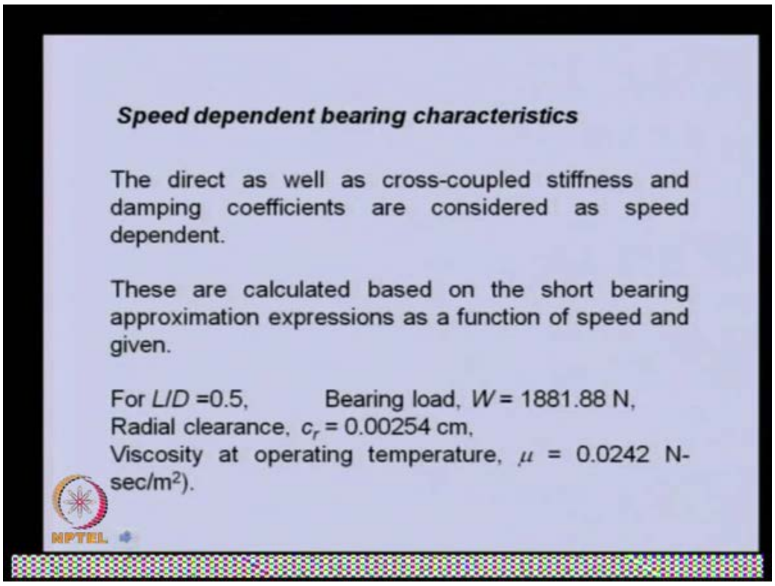

Then next we will be considering the speed dependent bearing characteristics, in this direct as well as the cross coupled stiffness and damping coefficients are considered as speed dependent. These are calculated based on the short bearing approximation expressions as function of speed, so here these are the same bearing parameter.

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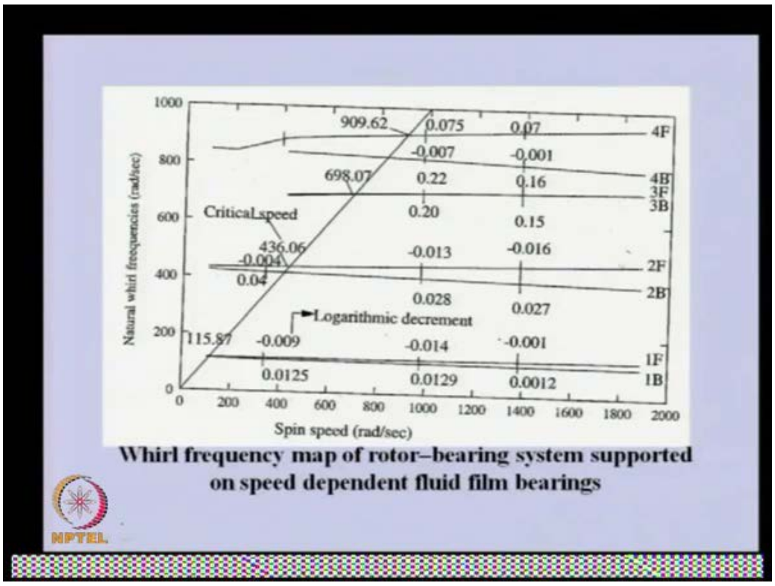

Now, we are calculating the stiffness and damping parameter that is variation of the, they are varying with the speed. So, when we discussed this particular short bearing approximation in previous lectures, so because they were in the non dimensional form. So, in this particular bearing we can able to get those parameter and we know that they vary with speed, so those values we have directly fed into our programs.

So, this is the Campbell diagram when the whirl frequency map of rotor bearing system supported on speed dependent fluid film are considered. So, here you can able to see the similar one, two, we are considered up to fourth mode. We have given the even the logarithmic decrement of this. So, you can able to see that, especially then your modes are unstable, but higher modes are unstable at very high values of the speed.

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For speed dependent fluid film bearings

Spin speed (rad/sec)	Natural whirl frequencies (rad/sec)		Logarithmic decrement δ	
	Forward	Backward	Forward	Backward
107.72	115.84	113.23	0.015402	0.012393
	432.98	423.15	0.002445	0.012912
	577.80	573.03	3.223702	2.882490
	847.36	*	0.003701	*
418.88	117.22	111.93	-0.00961	0.012579
	436.23	416.23	-0.00420	0.040687
	688.97	685.16	0.565563	0.530957
	886.48	838.27	0.097201	0.002730
837.76	119.58	109.69	-0.01337	0.012954
	443.54	409.17	-0.01158	0.029706
	701.63	700.24	0.267101	0.258887
	907.03	826.74	0.076321	-0.005129

* Natural whirl frequency becomes infinity.

So, this is the same values of the whirl frequency, in some cases we are not getting these values because what are the models we consider. Here we are not getting the natural frequency because previously we can see that in some cases. If we are in the unstable joule, we may not get these frequencies or sometimes, because of the gyroscopic effects especially the separation of this kind of frequency critical speed occurs.

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For speed dependent fluid film bearings

Spin speed (rad/sec)	Natural whirl frequencies (rad/sec)		Logarithmic decrement δ	
	Forward	Backward	Forward	Backward
1047.2	120.80	109.69	-0.01468	0.012999
	447.45	405.28	-0.01378	0.028400
	707.69	705.60	0.225162	0.209098
	911.44	820.17	0.075242	-0.00795
1256.64	122.04	109.69	-0.01589	0.013021
	451.42	401307	-0.01550	0.027825
	708.42.	708.98	0.192820	0.177773
	918.57	813.27	0.076812	-0.009921
1466.08	123.28	109.69	-0.01691	0.012911
	455.39	397.26	-0.01676	0.02741
	711.33	711.61	0.169752	0.15542
	925.42	806.11	0.078994	-0.00110
1887.96	125.79	103.96	-0.01904	0.012770
	463.33	389.11	-0.01883	0.027341
	717.10	716.83	0.142446	0.128098
	938.74	791.30	0.087177	-0.011441

So, these are the variations of the whirl frequencies, which we have plotted earlier and also the logarithmic decrement.

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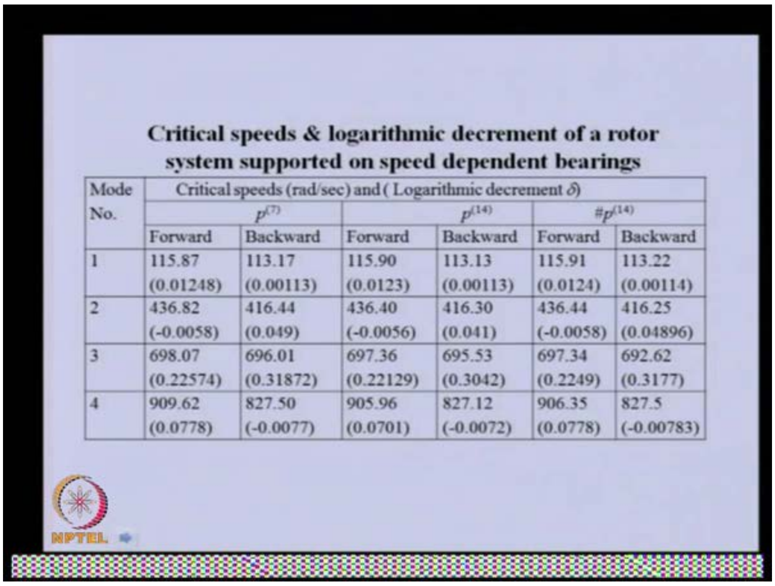

Critical speeds & logarithmic decrement of a rotor system supported on speed dependent bearings

Mode No.	Critical speeds (rad/sec) and (Logarithmic decrement δ)					
	$p^{(7)}$		$p^{(14)}$		$\#p^{(14)}$	
	Forward	Backward	Forward	Backward	Forward	Backward
1	115.87 (0.01248)	113.17 (0.00113)	115.90 (0.0123)	113.13 (0.00113)	115.91 (0.0124)	113.22 (0.00114)
2	436.82 (-0.0058)	416.44 (0.049)	436.40 (-0.0056)	416.30 (0.041)	436.44 (-0.0058)	416.25 (0.04896)
3	698.07 (0.22574)	696.01 (0.31872)	697.36 (0.22129)	695.53 (0.3042)	697.34 (0.2249)	692.62 (0.3177)
4	909.62 (0.0778)	827.50 (-0.0077)	905.96 (0.0701)	827.12 (-0.0072)	906.35 (0.0778)	827.5 (-0.00783)

This is the summary of the critical speed and logarithmic decrement for speed dependent bearings. So, for 7 elements, 14 elements and with condensation of some intermediate support and the angular displacements, so basically you can able to see some of the modes are stable or whenever negative logarithmic decrements are there, those critical speeds are unstable, like this one. Today, we initially started with the subcritical resonance phenomena or secondary resonance phenomena. We have seen that in that particular phenomenon that takes place because of the eccentric application of the gravity.

In this particular case, we saw that not only the response due to the weight of the rotor which will be like something static deflection will be there in the response. We will be having even in the response of the unbalance, because the eccentricity is always there in that system. Apart from that we got the secondary resonance amplitude, those amplitudes we saw that in that the resonance takes place half the un-damped natural frequency of the system. But the whirling frequency at which the rotor takes place at this is twice the speed of the rotor that means at the critical at the natural frequency of the system. So, that is the very interesting phenomena, which we observed that is due to the eccentric application of the gravity.

Apart from that when we tried to apply the finite element method in more general case in which we considered the gyroscopic effect, the bearing property. We considered as speed

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In the subsequent lecture now, we will go for more practical applications of various methods, like specially various kinds of faults are there in the rotor system, how they can be identified or how they can be rectified? So, we will begin with balancing of rotors not only the rigid rotor, but also the flexible rotor balancing theory we will study. With some numerical example, we will try to understand those methods in more detail.