

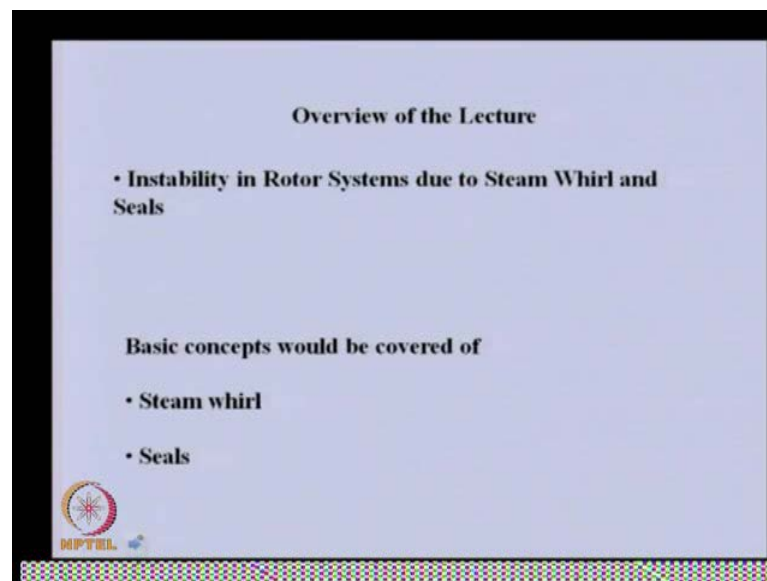
Theory and Practice of Rotor Dynamics
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Module - 7
Instability in rotor systems
Lecture - 4
Steam Whirl and Seals Introduction to the Course

Last few lectures, we have been seen the instability in rotor system. Today, we will take up another kind of instability, which is called steam whirl. And will see how also the seals can give the instability in to the, so basically this steam whirls will find that will be present, when the turbine is having very high pressure. So, is mainly of course, in the high pressure turbines and basically it limits the, how much we can able to generate the power, in a particular turbine. So, the steam whirl generally limits the how much maximum power, we can able to generate in a turbine.

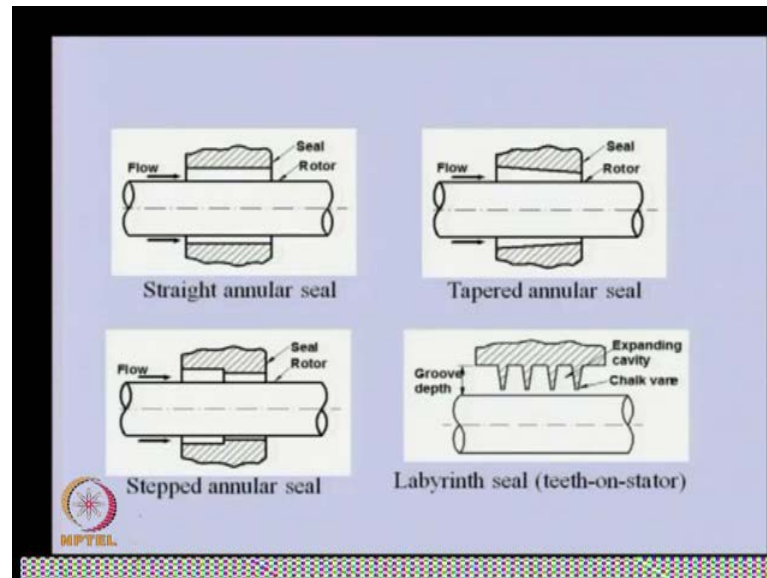
So, with a very simple model, we will try to study this steam whirl and also how the seals can able to, introduce the instability into the rotor system. Before that we introduce this seals, various types of seals we used in the rotor, between the rotating part and stationary part. And then, we will with simple model, try to see how this can stabilize the rotor system.

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So, basically in this we will be dealing with the instability rotor, due to steam whirl and due to the seals and basic concepts will introduce of these two terminology. So, let us see the dynamic seal.

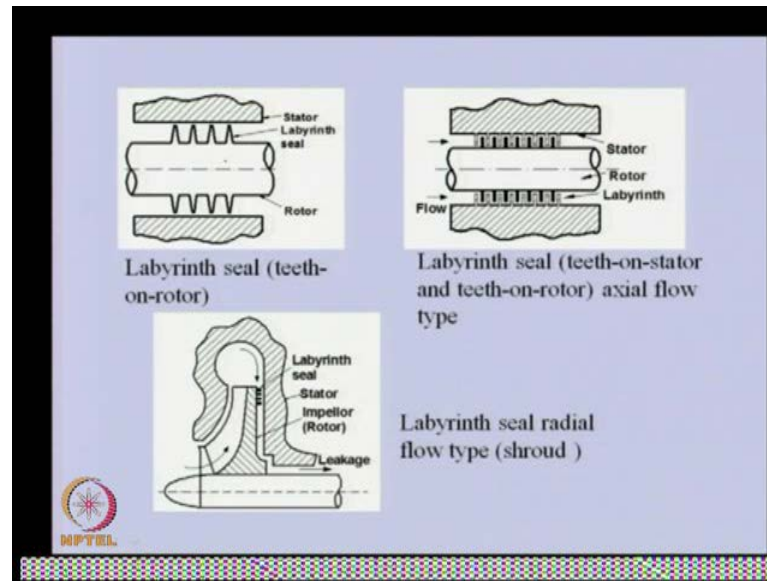
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So, in rotors system, we will find various kind of seals like let us say, rotor which is rotating about its own axis. This is the either, this is stator and basically this is seal which is provided generally to stop or to minimize the flow of the working fluid or the lubricant from one end of the shaft to another one. So, here simple surgical shape is there of the seal. Generally, the seals are having larger clearance as compared to bearings.

So, in this hydro dynamic action is not present. And this is another kind of seals in which, the path is stepped, so that the flow of the lubricant from one end to another end is less. There are some other kind of seals like the stepped seals, so instead of the single straight cylindrical or the upper, there is stepped annular seals are there. Apart from this, there are different variations of the seals like this is the labyrinth seal and the in this particular case, the teeth is on the stator. The so, this is the stator and teeth is on this stator. And basically they will prevent the flow of the lubricant and because of this abstraction, the flow of the lubricant will be less.

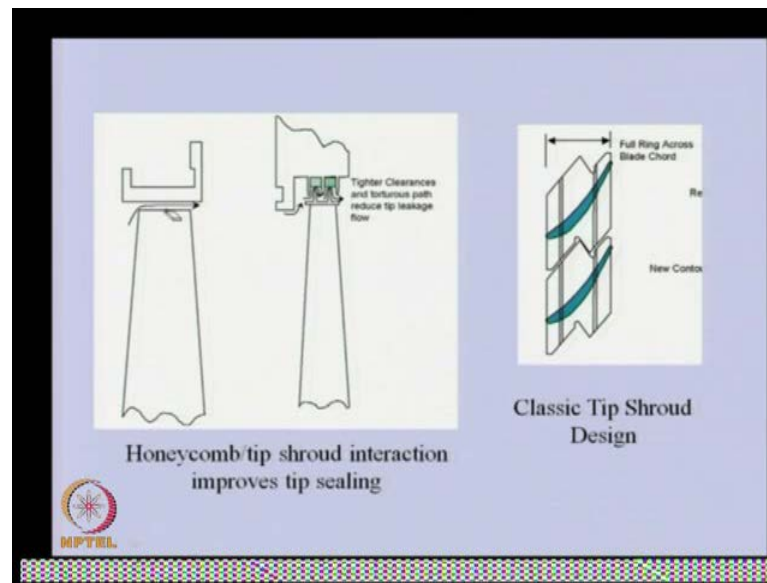
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And similarly, we can have the labyrinth seal on the rotor, instead of stator. And basically, they are all over the circumference and we will see, we see that the, this prevent the leakage of the fluid, because of the various abstraction. Sometimes, this labyrinth seals are provided in not only in the stator, but also in the rotor. So that the path of the working fluid or lubricant is zigzag, while it when it is passing through this.

So, with this prevention of working fluid is more and better sealing can be done, dynamic sealing can be done with such arrangements, apart from this, in the turbine blades, sometimes like this in the pillar, in which fluid can come from this side. So, to prevent the leakage of the fluid from this side, some kinds of seals are provided, so that the leakage is minimized. In this particular case, fluid is coming like this and we want to prevent the leakage through this, because this is rotor this is stator. So, there has to be some clearance between them. And we do not want the lubricant to flow in this direction.

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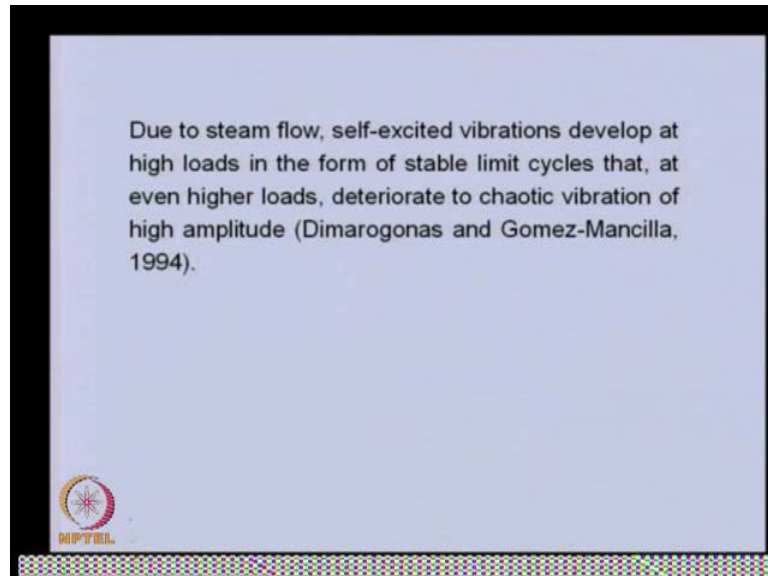
So, this prevents the leakage of the fluid. This is another thing, you can see this is rotor tip and the fluid we pass through, the gap between the rotor tip and the stator. And to prevent this, sometime some kinds of shots are provided, at the tip of the rotor. So, you can able to see this shots and similar to the labyrinth seal and it prevents the leakage of the fluid, from the tip clearance. And this is generally required, so that the leakage, the steam or working fluid should not take place at the tip, because that will be loss, because it has pass through the blades.

And this is the face see from the top this cross section is like this. So, you can able to see there are weans also at the top. So, classical tip shots designed, which gives better stability of the system. So, basically the scenes and this shots, they provides some kind of instability and we will try to study, in the present lecture this kind of instability. So, with this small introduction to the seals, especially seals, dynamic seals and this are out. And now, we try to see how we can able to understand the steam whirl, in a rotor system.

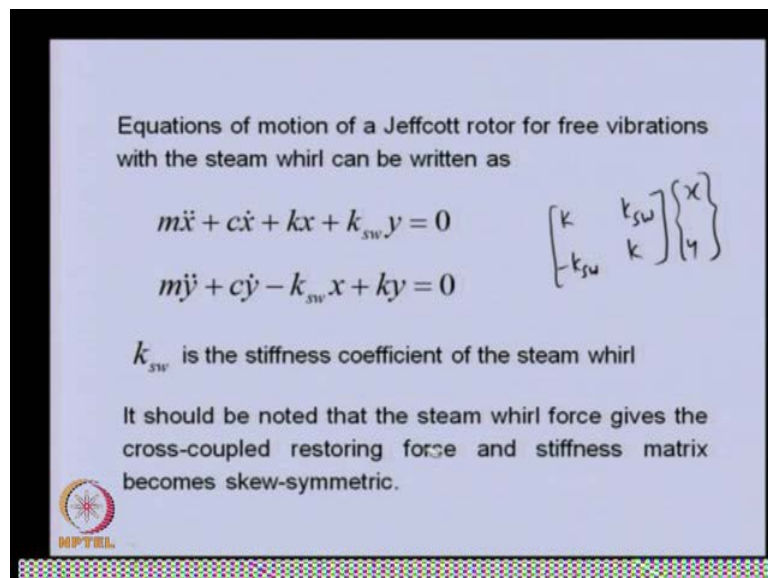
So, basically the steam exited whirls of high pressure turbine rotors is caused by blade tips out and shaft labyrinth seal forces and by steam flow forces on the blades. And the seals whirl forces is de-stabilizing, for some condition and stabilizing for other condition. The problem of steam whirl is one of the technological limit, that now beat the development of power generating turbo machinery substantially below 1 giga watts. So, due to steam flow, self-excited vibration developed at high loads, in the form of stable

limit cycles that, at even higher loads deteriorate to chaotic vibrations of the high amplitude.

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So, let us see the equation of motion, of a Jeffcott rotor and in which the steam whirl is taking place. So, for that, these kind of equation of motion is valid. So, you can able to see the first 3 term is belonging to the rotor, but this is due to the steam whirl. So, basically steam whirl stiffness is coupling the y direction equation of motion with the x

direction equation of motion. And if you see the y direction equation of motion, here the stiffness is negative.

So, that means you can able to see because of the steam whirl, we are getting the cross-couple, cross-coupling. Also the stiffness matrix become the symmetric, it becomes skew symmetric. So basically, if we put the stiffness matrix in the form of matrix, we will find, this is skew symmetric stiffness matrix like this. And it should be noted, that the steam whirl gives the cross-coupled restoring force and stiffness matrix becomes skew symmetric.

(Refer Slide time: 11:05)

$$s = x + jy$$

$$m \ddot{s} + c \dot{s} + k s + (j^4 k_{sw} y + j^3 k_{sw} x) = 0$$

$$j^3 k_{sw} [jy + x] = j^3 k_{sw} s$$

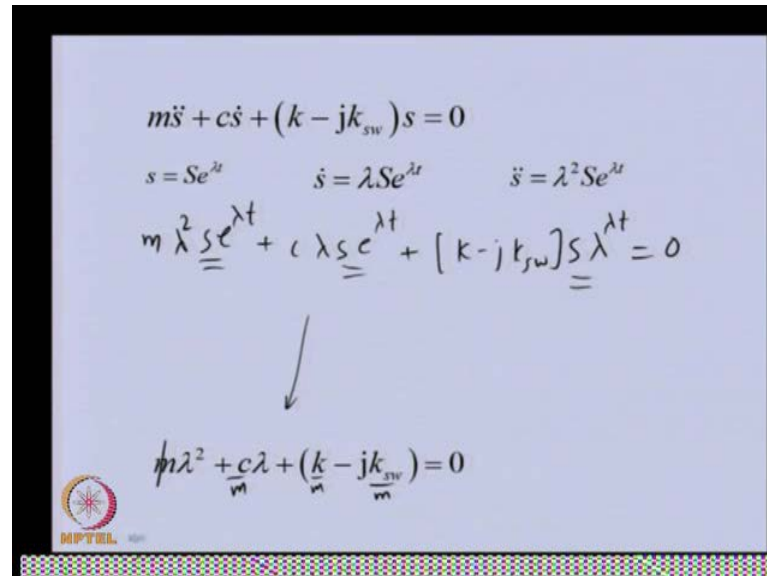
$$m \ddot{s} + c \dot{s} + (k - j k_{sw}) s = 0$$

And now, we will we will try to solve this equation, by in the form of complex displacement, which we have to find as x plus j y. So, with this we will multiply the second equation by j and add it to the first equation. So you can able to see that the first term will be simple. ms double dot plus c s dot. So, this is the first 2 terms, will give this. And then, this K x and K this K x and K y, will give this. Then, K s w that is for steam whirl, x plus minus K s w j y is equal to 0.

So, if we multiply this by, this by j and this by, so basically we will get equation of this form. This is x and now this 2 quantity, we can able to see this. s w is common, so basically we can able write this also as plus and then this will be cubic and here we have plus 1 so we can able to write this. So, if we remove the cubic term, then we get j y plus x and which is nothing but, s. So, this will give us this quantity, this will be minus j k s w

s. So basically, we can able to get the equation of motion, in the complex domain in the stationary coordinate system like this.

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$$m\ddot{s} + c\dot{s} + (k - jk_{sw})s = 0$$

$$s = Se^{\lambda t} \quad \dot{s} = \lambda Se^{\lambda t} \quad \ddot{s} = \lambda^2 Se^{\lambda t}$$

$$m\lambda^2 \underline{Se^{\lambda t}} + c\lambda \underline{Se^{\lambda t}} + [k - jk_{sw}] \underline{Se^{\lambda t}} = 0$$

$$\downarrow$$

$$m\lambda^2 + \frac{c}{m}\lambda + \left(\frac{k}{m} - j\frac{k_{sw}}{m}\right) = 0$$


So, this is the equation of motion which we derived. Now, we are assuming the solution of this as amplitude and some frequency component or eigen value component and if we differentiate this, we will get this quantity. If we substitute this in the equation of motion, we will get $s \lambda^2 t + c \lambda s \lambda t$ and then this will be as it is. And now you can able to see as λt is common and that will go out and we will left with this equation. So, basically this is frequency equation, this is frequency equation. Now, we are simplifying this equation, basically if you see this, previous equation is in terms of m c n k , so if you divide full expression by m , so we will get this as c by m and k by m and this by m . And now, we can able to simplify by defining natural frequency square as k by m and this ω_{sw}^2 steam whirl, this is parameter we are defining by this. And this the dumping ratio and critical dumping is defined like this. So, if we use this non dimensional parameters, we can able to write the previous equation in this form.

(Refer Slide time: 12:15)

$$\lambda^2 + 2\zeta\omega_{nf}\lambda + (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$

$$\omega_{nf}^2 = \frac{k}{m} \quad \omega_{sw}^2 = \frac{k_{sw}}{m} \quad \zeta = \frac{c}{c_c} \quad c_c = 2\sqrt{km}$$

$$\lambda_{1,2} = -\zeta\omega_{nf} \pm \sqrt{\zeta^2\omega_{nf}^2 - (\omega_{nf}^2 - j\omega_{sw}^2)}$$

$$\lambda_i = \alpha_i \pm j\beta_i \quad i = 1, 2$$



Now, this is quadratic equation, in terms of lambda, so we can able to solve for lambda. So, we will get 2 roots form this. So simplified form of that, so this is quadratic equation that we can able to get in the closed form solution. So, this is the quadratic equation solution and we have 2 roots of this lambda and the form of this is like this, in which we have alpha i as a real part and beta i as imaginary part, where i can be 1 to 2. And this alpha i, will see that is a real part and some quantity of the real part may come from the terms within the square bracket also. And the remaining part will be the imaginary part, which will come from this square bracket, some from the some of the parts.

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It should be noted that some real part may come from the term within the square bracket of equation as it will be clear in the following case at *the boundary of stability*.

The system is unstable when the real part of λ_i is positive and it is stable when the real part of λ_i is negative. Hence, at the boundary of stability the real part of λ_i must be zero.

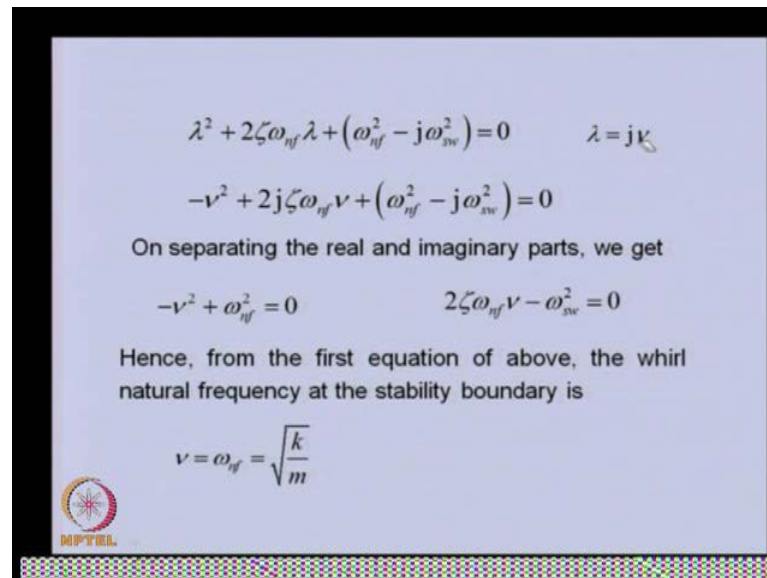
Hence, let us defining the eigen value at the stability boundary as $\lambda = j\nu$



So, now we should note that some of the real parts may come from the comes within the square bracket of the equation, as it will be clear in the following case, at the boundary of stability. So, here we are defining the boundary of stability, basically in the previous Eigen solution, Eigen solution the system is unstable when the real part of lambda is positive. And it is stable, when real part is negative and has the boundary of stability will be when this real part of lambda is 0. So, in one case when it is positive, we have unstable regions. If it is negative, we have stable regions, so in between this when this lambda is 0, we will be having that will be the boundary of instability.

Hence, it is defining the Eigen value at the instability, at the stability boundary as pure imaginary. Let us say, the real part is purely 0, so this is the ha we are assuming that this lambda as j nu, where nu is the same frequency and is no real part of this. So this is the condition of boundary of stability, if we so this was the equation frequency equation if we substitute in this lambda is equal to j nu that is boundary of instability condition we will get this equation, so we can able see this becomes minus nu square and other places we can replace lambda by j nu.

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$$\lambda^2 + 2\zeta\omega_{nf}\lambda + (\omega_{nf}^2 - j\omega_{sw}^2) = 0 \quad \lambda = j\nu$$

$$-\nu^2 + 2j\zeta\omega_{nf}\nu + (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$

On separating the real and imaginary parts, we get

$$-\nu^2 + \omega_{nf}^2 = 0 \quad 2\zeta\omega_{nf}\nu - \omega_{sw}^2 = 0$$

Hence, from the first equation of above, the whirl natural frequency at the stability boundary is

$$\nu = \omega_{nf} = \sqrt{\frac{k}{m}}$$

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So, we get this expression. So, in this we can able to separate the real part and imaginary part. So, if we separate the real part in to sites, we will get this and for imaginary part we will get this. From this, we an able to see that nu is equal to omega n f, which is defined


as k by m . So, this is the basically the frequency will be at the boundary of instability, this frequency will be equal to the natural frequency of the system.

And it is obvious also because when the real part is 0, that means is there is no dumping in the system. So, frequency of the oscillation will be un-damped natural frequency. And from the second equation, this second equation of the imaginary part we will get this condition which we can able to write like this, if we express this in terms of k and m and this will be k s w y m and this will be k by m .

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and from the second equation, we have the following condition

$$\omega_{sw}^2 = 2\zeta\omega_{nf}^2 \qquad k_{sw} = 2\zeta k$$

$$\begin{aligned} \lambda_{1,2} &= -\zeta\omega_{nf} \pm \sqrt{\zeta^2\omega_{nf}^2 - (\omega_{nf}^2 - j\omega_{sw}^2)} \\ &= -\zeta\omega_{nf} \pm \sqrt{\zeta^2\omega_{nf}^2 - (\omega_{nf}^2 - 2j\zeta\omega_{nf}^2)} \\ &= -\zeta\omega_{nf} \pm \sqrt{\zeta^2\omega_{nf}^2 - \omega_{nf}^2 + 2j\zeta\omega_{nf}^2} \\ &= -\zeta\omega_{nf} \pm \sqrt{(\zeta\omega_{nf} + j\omega_{nf})^2} = -\zeta\omega_{nf} \pm (\zeta\omega_{nf} + j\omega_{nf}) \end{aligned}$$



Now, the roots which we obtained earlier, so in that this was the roots, in which we had real part and imaginary part, but now we are substituting the condition of boundary of stability in this, so that so here we are substituting the condition of boundary of instability. That means this condition, so if we substitute this here we will get this and if we simplify this, finally we will get this expression. So, you can able to see, when we have positive quantity here, then this 2 parts are canceling each other and the real part is 0. So, that was the condition of a instability, but when we are taking negative, then the real part is imaginary that is giving a stable solution.

(Refer Slide time: 18:12)

It should be noted that the real part is zero for the positive sign and for negative sign the real part is negative, which is the case at the boundary of the stability.

The stability condition can be obtained as follows by the Routh-Hurwitz stability criteria. For the polynomial with complex coefficients of the following form

$$(a_0 + jb_0)\lambda^2 + (a_1 + jb_1)\lambda + (a_2 + jb_2) = 0$$

$$\lambda^2 + 2\zeta\omega_{nf}\lambda + (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$


So, one of the root gives the boundary of instability and another is giving the stable solution of this. Now, we should be noted that the real part is 0 for the positive sign and negative sign, the real part is negative which is the case at the boundary of stability, that we are yet to discuss. The stability condition can be obtained as follows by Routh-Hurwitz stability criteria.

(Refer Slide Time: 19:51)


$$-\begin{vmatrix} a_0 & a_1 \\ b_0 & b_1 \end{vmatrix} > 0 \qquad \begin{vmatrix} a_0 & a_1 & a_2 & 0 \\ b_0 & b_1 & b_2 & 0 \\ 0 & a_0 & a_1 & a_2 \\ 0 & b_0 & b_1 & b_2 \end{vmatrix} > 0$$

It should be noted that for the present case $\lambda = j\lambda_0$

Hence, the frequency equation becomes

$$\lambda^2 + 2\zeta\omega_{nf}\lambda + (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$

$$-\lambda_0^2 + 2j\zeta\omega_{nf}\lambda_0 + (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$

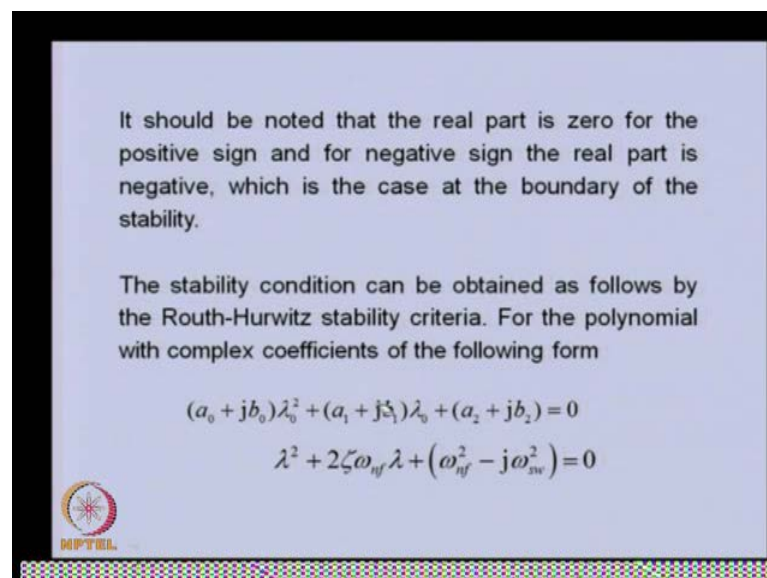
$$\lambda_0^2 - 2j\zeta\omega_{nf}\lambda_0 - (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$


This particular criteria which we described earlier was for real polynomial, but now the present polynomial which is of this form, is having some complex coefficient. So, more

general form of the complex coefficient for a polynomial, quadratic polynomial it could be like this. Here these are coefficients, if we compare this 2, we can able to get various coefficients like a naught will be one and there is no imaginary part here so b naught is 0.

Like this, we can compare this 2 polynomial and we can able to get a naught, b naught, a 1, b 1, a 2, b 2. And the condition of stability is the first condition is this one, determinant of this should be 0 here minus is there. Similarly, determinant of this quantity should be greater than 0, so for stability of the system. In this particular case, this previously we used it, when we previously we used in previous lectures.

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It should be noted that the real part is zero for the positive sign and for negative sign the real part is negative, which is the case at the boundary of the stability.

The stability condition can be obtained as follows by the Routh-Hurwitz stability criteria. For the polynomial with complex coefficients of the following form

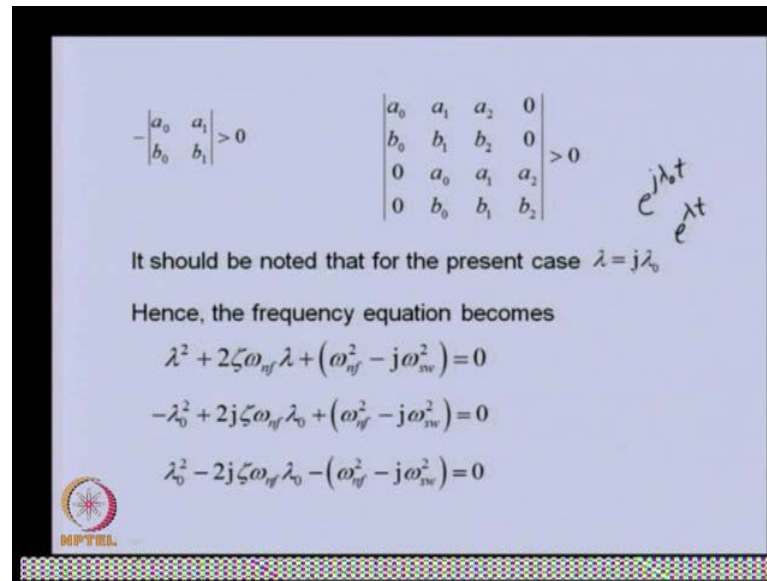
$$(a_0 + jb_0)\lambda^2 + (a_1 + jb_1)\lambda + (a_2 + jb_2) = 0$$

$$\lambda^2 + 2\zeta\omega_{nf}\lambda + (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$

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But the difference of this polynomial and this is you can able to see, there is there is a lambda naught and there is lambda.

(Refer Slide Time: 19:51)



$$-\begin{vmatrix} a_0 & a_1 \\ b_0 & b_1 \end{vmatrix} > 0$$

$$\begin{vmatrix} a_0 & a_1 & a_2 & 0 \\ b_0 & b_1 & b_2 & 0 \\ 0 & a_0 & a_1 & a_2 \\ 0 & b_0 & b_1 & b_2 \end{vmatrix} > 0$$

$e^{j\lambda_0 t}$
 $e^{\lambda t}$

It should be noted that for the present case $\lambda = j\lambda_0$

Hence, the frequency equation becomes

$$\lambda^2 + 2\zeta\omega_{nf}\lambda + (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$

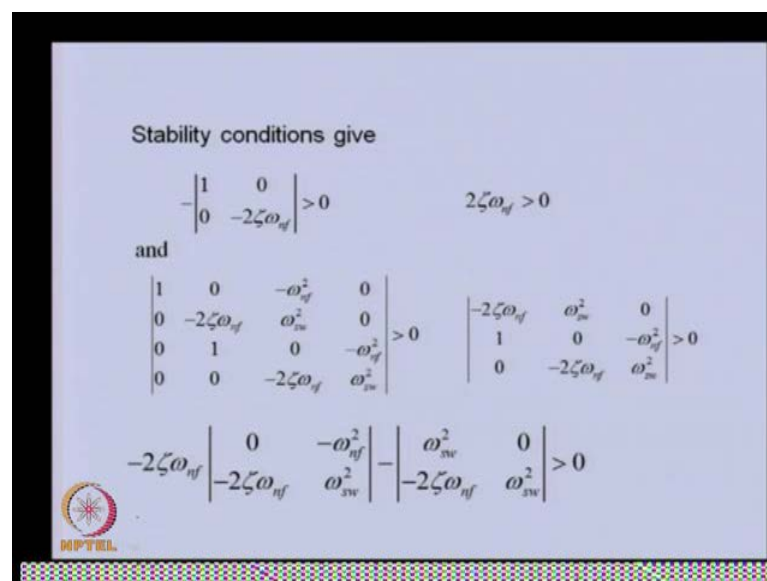
$$-\lambda_0^2 + 2j\zeta\omega_{nf}\lambda_0 + (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$

$$\lambda_0^2 - 2j\zeta\omega_{nf}\lambda_0 - (\omega_{nf}^2 - j\omega_{sw}^2) = 0$$

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So basically, lambda and lambda naught is defined like this, because earlier the solution was something like this and now we are chosen the solution like this. So, we have the difference of j here, so that means if we substitute lambda is equal to j lambda naught and then this 2 polynomial will be equivalent. So if you substitute that we will get this expression, and basically now we have converted the present polynomial in the form of solution of this form. And now, we can able to compare this 2 polynomial. This polynomial and general form of this to get the coefficients a 0 b 0 etc.

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Stability conditions give

$$-\begin{vmatrix} 1 & 0 \\ 0 & -2\zeta\omega_{nf} \end{vmatrix} > 0$$

$$2\zeta\omega_{nf} > 0$$

and

$$\begin{vmatrix} 1 & 0 & -\omega_{nf}^2 & 0 \\ 0 & -2\zeta\omega_{nf} & \omega_{sw}^2 & 0 \\ 0 & 1 & 0 & -\omega_{nf}^2 \\ 0 & 0 & -2\zeta\omega_{nf} & \omega_{sw}^2 \end{vmatrix} > 0$$

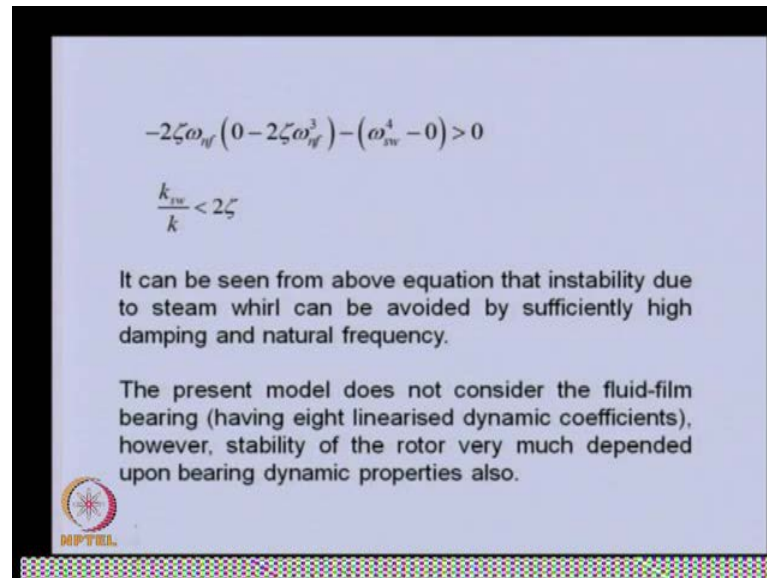
$$\begin{vmatrix} -2\zeta\omega_{nf} & \omega_{sw}^2 & 0 \\ 1 & 0 & -\omega_{nf}^2 \\ 0 & -2\zeta\omega_{nf} & \omega_{sw}^2 \end{vmatrix} > 0$$

$$-2\zeta\omega_{nf} \begin{vmatrix} 0 & -\omega_{nf}^2 \\ -2\zeta\omega_{nf} & \omega_{sw}^2 \end{vmatrix} - \begin{vmatrix} \omega_{sw}^2 & 0 \\ -2\zeta\omega_{nf} & \omega_{sw}^2 \end{vmatrix} > 0$$

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So, from first condition this determinant if we are putting 0, we are getting this condition from second one, after simplification we will get this condition, that k_{sw} by k should be less than 2ζ which is damping ratio.

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$$-2\zeta\omega_{nf}(0 - 2\zeta\omega_{nf}^3) - (\omega_{sw}^4 - 0) > 0$$

$$\frac{k_{sw}}{k} < 2\zeta$$

It can be seen from above equation that instability due to steam whirl can be avoided by sufficiently high damping and natural frequency.

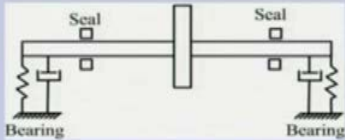
The present model does not consider the fluid-film bearing (having eight linearised dynamic coefficients), however, stability of the rotor very much depended upon bearing dynamic properties also.



So, it can be seen from above equation that instability due to steam whirl, can be avoided by sufficiently high damping, if damping is high then this condition is more, we can be able to satisfy and if k is the natural frequency is more that means, this is more then we can be able to better stability in the system. The present model, we considered only the this steam whirl stiffness, but in actual case apart from this will be having the damping and stiffness from the bearings. And that also we need to consider, while analyzing the steam whirl and for that case, we will be having the conditions slightly more complicated. But this is the basic premise regarding steam whirl and this particular case we will see in the subsequent, in the subsequent thing in which we will be considering the seals as the system, in the system along with the bearing.

(Refer Slide Time: 22:16)


Instability due to Seals



A Jeffcott rotor with seals

$$\begin{bmatrix} m + m_{sd} & 0 \\ 0 & m + m_{sd} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} c + c_{sd} & c_{sc} \\ -c_{sc} & c + c_{sd} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} k + k_{sd} & k_{sc} \\ -k_{sc} & k + k_{sd} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

sd and sc subscripts represent, respectively, the seal direct and cross-coupled terms.



So in the last, we have seen that how we can able to analyze a steam whirl and how to we can able to get the condition of the steam whirl and how we can able to improve the stability of the rotor system with the steam whirl. Now, we will take up a case of seal along with the bearing in a rotor system and in this particular case, we will try to get the condition, but we will see that the expressions will be more complicated to analyze in a as a general. But for every specific cases, we can able to analyze that particular system, so that means we will take this kind of rotor system. We are considering only the translation motion and these are the bearings and they are symmetric and seals are also symmetric. And for equation of motion for this system, we can have like this.


So in this, basically this is the mass coefficient of the seal, so seal direct coefficients of mass. These are the damping coefficients of the seals, direct one and these are the cross-coupled one. Cross-coupled you can able to see this queue symmetric is there. Similarly, this s d, this is shield direct coefficients of the time stiffness and this is the cross-coupled seal coefficients. Now, other parameters like m c and k they are coming from rotor and the bearing. Bearing in this particular case, we have taken simple direct stiffness, direct damping and direct stiffness. We are not considered cross-coupled stiffness and damping, only this couplings are there in the seals, damping and the seals stiffness because we want to analyze this particular seal parameters in more specific case. These parameters of the seals will be there, generally the when the speeds of the speed of the rotor is very high, otherwise these parameters can be ignored.

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$$s = x + jy$$

$$(m + m_{sd})\ddot{s} + (c + c_{sd})\dot{s} + c_{sc}(\dot{y} - j\dot{x}) + (k + k_{sd})s + k_{sc}(y - jx) = 0$$

$$(m + m_{sd})\ddot{s} + (c + c_{sd} - jc_{sc})\dot{s} + (k + k_{sd} - jk_{sc})s = 0$$

$$s = Se^{j\omega t} \quad \dot{s} = j\omega Se^{j\omega t} \quad \ddot{s} = -\omega^2 Se^{j\omega t}$$


So those 2 equations, which we had in the matrix form, we can able to write in a complex displacement s by defining s is equal to x plus j y . So, if we multiply the first, the second equation by j and add it to first one, we will get equation of motion like this. So, we already seen how to take care of some terms which are having y plus j x kind of term, so we will get complex coefficients for such cases. So this is the equation of motion, for that case now we, this we can able to simplify and all the terms we can able to write interns of s . So this is the equation of motion for that. x


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$$(m + m_{sd})v^2 - \{c_{sc} + j(c + c_{sd})\}v + \{-(k + k_{sd}) + j\omega c_{sc}\} = 0$$

From stability conditions, we have

$$-\begin{vmatrix} m + m_{sd} & -c_{sc} \\ 0 & -(c + c_{sd}) \end{vmatrix} > 0 \Rightarrow (m + m_{sd})(c + c_{sd}) > 0$$

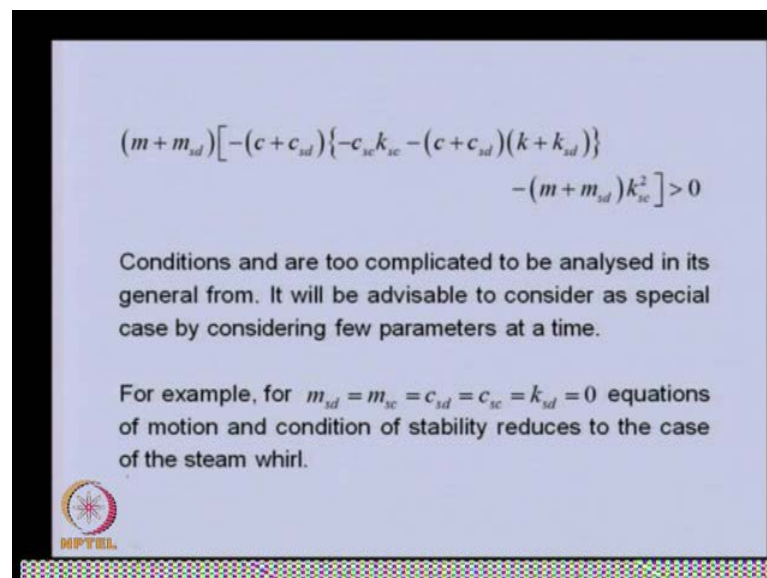
and

$$\begin{vmatrix} m + m_{sd} & -c_{sc} & -(k + k_{sd}) & 0 \\ 0 & -(c + c_{sd}) & k_{sc} & 0 \\ 0 & m + m_{sd} & -c_{sc} & -(k + k_{sd}) \\ 0 & 0 & -(c + c_{sd}) & k_{sc} \end{vmatrix} > 0$$


This particular criteria which we described earlier was for real polynomial, but now the present polynomial which is of this form, is having some complex coefficient. So, more general form of the complex coefficient for a polynomial, quadratic polynomial it could be like this. Here these are coefficients, if we compare this 2, we can able to get various coefficients like a naught will be one and there is no imaginary part here so b naught is 0.

And this frequency equation is, so this is similar to quadratic polynomial with coefficients as complex, so the previous stability conditions in terms of a naught b naught, that we can able to apply here, Routh-Hurwitz criteria. So this is the first condition, so this gives this condition. Second condition was 4 by 4 matrix. So, if we if we simplify this, we will get a equation like this.

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$$(m + m_{sd}) \left[-(c + c_{sd}) \{ -c_{sc} k_{sc} - (c + c_{sd})(k + k_{sd}) \} - (m + m_{sd}) k_{sc}^2 \right] > 0$$

Conditions are too complicated to be analysed in its general form. It will be advisable to consider as special case by considering few parameters at a time.

For example, for $m_{sd} = m_{sc} = c_{sd} = c_{sc} = k_{sd} = 0$ equations of motion and condition of stability reduces to the case of the steam whirl.

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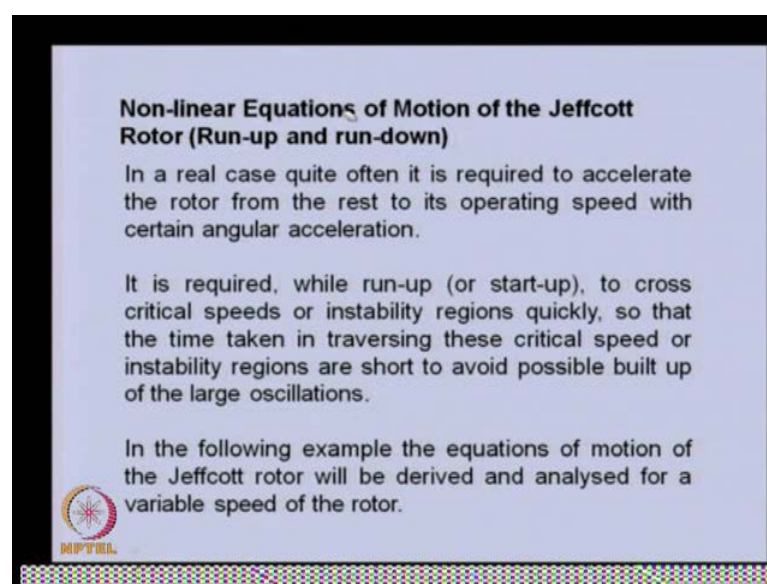
So in this, you can able to see so many parameters are there, so it is not easy to interpret which parameter is will gone and what. But this is the general expression and if you have the values of this, we can able to see the stability of the system clearly. So, for a special case like if we substitute the mass coefficients, of the seals and also of the damping coefficients and this is equal to 0, then equation of motion and condition of stability reduces to the case of the steam whirl. So similar to steam whirl, case will be there if we take the this particular example, this coefficients are 0. So you can able to see that, this particular conditions are more general and very special cases we can able to analyze, by putting these parameter one by one or in combination is equal to 0.

Now, we will take up another case, in which generally we need to accelerate the rotor at certain angular acceleration. Basically not to avoid the critical speed or to avoid the instability zones, we need accelerate the rotor at a particular rate, so that we do not give much time, to the rotor to be there in the at the at the near the critical speed or within the instability zone.

And if we cross this zones quickly, then the possibility that the rotor get high amplitude is will be very less. In this particular case, we will be considering two case; one is will be having constant torque. So, the supply is unlimited. Supply is having limited power, so the constant power it can apply. The second case is constant acceleration, especially the angular acceleration in that depending upon requirement, the whatever the drive is there, it can change its torque and it can give required torque.

So in this particular case, we have unlimited power source. So these kind of practical difficulty, we may have in the real system. So, let us start to analyze this two cases and in this basically the equation the of motions, will find that in especially when we are considering that we are giving a constant torque, the equation of motion will be ordinary in nature. In this particular case, we will solving this using some kind of direct numerical integration in time domain and we will get the response of the system with time by these numerical techniques.

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Non-linear Equations of Motion of the Jeffcott Rotor (Run-up and run-down)

In a real case quite often it is required to accelerate the rotor from the rest to its operating speed with certain angular acceleration.

It is required, while run-up (or start-up), to cross critical speeds or instability regions quickly, so that the time taken in traversing these critical speed or instability regions are short to avoid possible built up of the large oscillations.

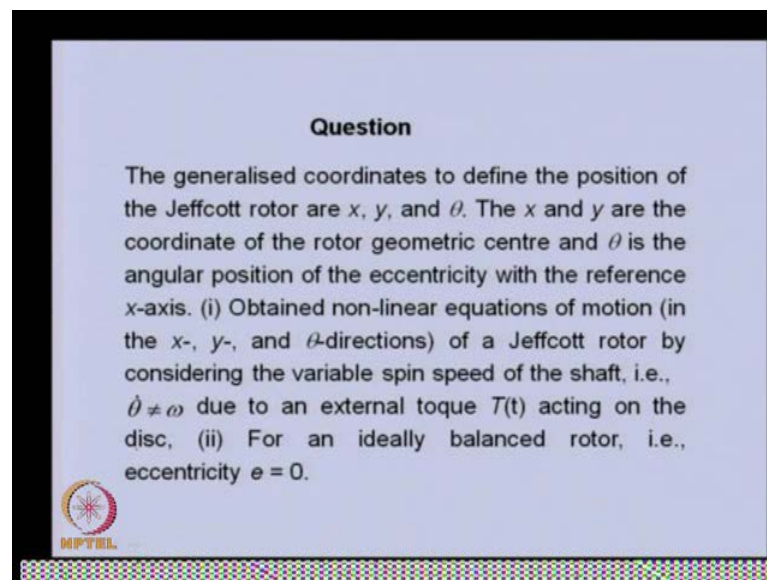
In the following example the equations of motion of the Jeffcott rotor will be derived and analysed for a variable speed of the rotor.



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So basically in this particular case, we will be having ordinary equation of motion, Jeffcott rotor for run up or run down of the rotor as we have already mentioned. So in real case, it is quite often required to accelerate the rotor from the rest to its operating speed, with certain angular acceleration. It is required while run-up or start-up to cross critical speed or instability regions quickly, so that the time taken in traversing these critical speed or instability regions are short, to avoid possible built up of the large oscillations. In the following example, the equations of motion of the Jeffcott rotor will be derived and analyzed for a variable speed of the rotor. So in this particular case, angular acceleration is there, so the speed of the rotor will be variable.

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So, this is problem in which will be, through this example we will be basically explaining this particular run-up case. So, we have in this 3 generalized coordinates: x , y , z ; x , y and θ of the rotor. x and y are the transfer displacement which of the geometrical center of rotor and θ is the angular position of the eccentricity with reference to one of the axis, which is x -axis. So basically, θ is the torsional displacement and x and y are the transfer displacement. And so, in this first we obtain the equation of motion in this x and y and θ direction and then especially in this, we will not be considering the speed s constant, but it is variable and this will be variable because there is external torque, in the rotor system. And in the second case, we will be analyzing for ideally balanced rotor.

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obtain the time require to reach the angular velocity $\dot{\theta} = \omega_{\max}$ from rest. Assume a constant toque T_e and

(iii) When eccentricity, e , is present, obtain equations of motion for a known constant angular acceleration with zero initial conditions, i.e.,

$$\ddot{\theta} = \alpha, \dot{\theta} = \alpha t, \theta = \frac{1}{2} \alpha t^2$$

A constant toque : Limited power source

A known angular acceleration : Unlimited power source




This what will be the basically, time required to reach the angular velocity. Whatever the maximum velocity from rest, how much it time it take to reach that up to that point. In this particular case, we assume the torque as constant and another case when the eccentricity is present in the system, will be obtaining the response, for known constant angular acceleration, with some 0 initial condition.

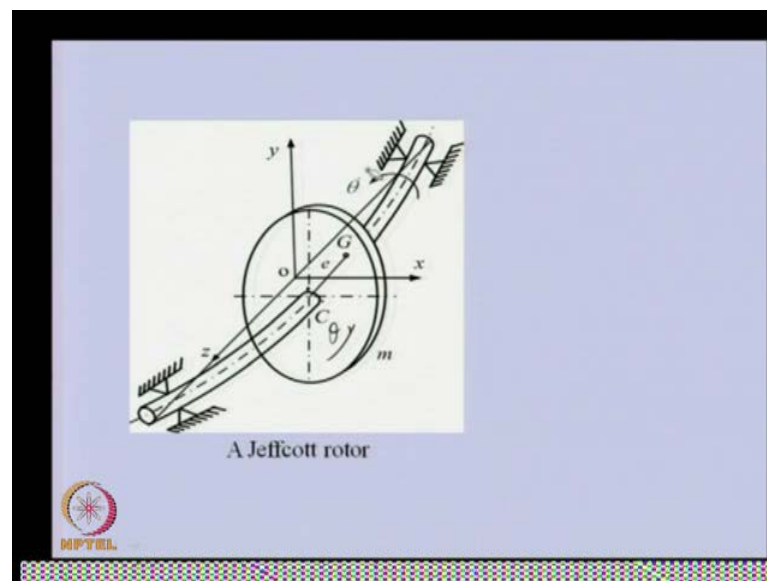
That means, will be having angular accelerations conditions like this, even the angular velocity and this is the angular displacement. So, you can able to see angular displacement is changing with time, square of the time. In this particular case, that is why I mentioned earlier the constant torque, that is a limited power source required. Another case is the known angular acceleration, in this the power source is having unlimited power. So first, let us try to derive the equation of motion.

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Solution: (i) Equations of motion of the Jeffcott rotor in the x , y , and θ directions can be written as

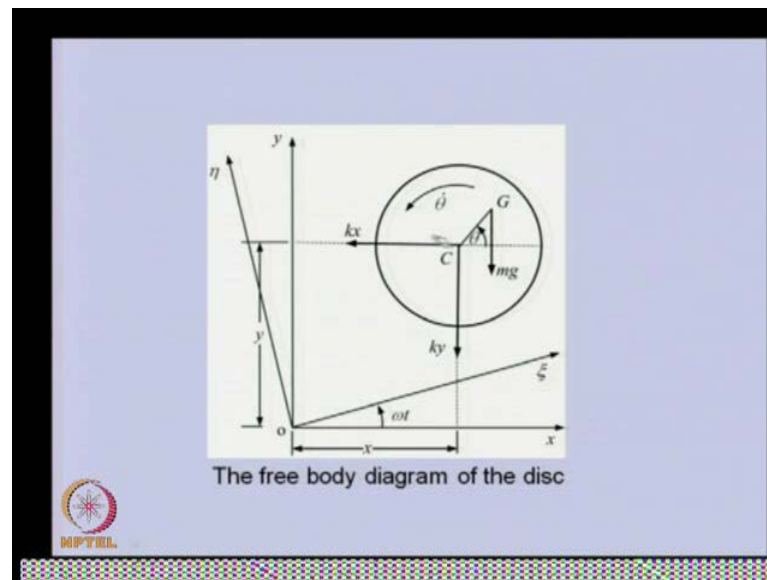
$$-kx - c\dot{x} = m \frac{d^2}{dt^2} (x + e \cos \theta)$$
$$-ky - c\dot{y} = m \frac{d^2}{dt^2} (y + e \sin \theta) + mg$$
$$T(t) - kx(e \sin \theta) - ky(e \cos \theta) - c\dot{x}(e \sin \theta) - c\dot{y}(e \cos \theta) = I_p \ddot{\theta}$$


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So basically, the rotor is like this; x , y and z is in this direction and is rotating with contact clockwise direction, which is θ direction. This is θ velocity. Now if we take the free body diagram of this, we will see that these are the elastic force from the shaft.

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This is gravity force. In this particular case, the static equilibrium is at the bearing center and there will be some static deformation. So to take care of that, the way we need to consider here, especially in the non-linear system, we need to consider this because we cannot able to shift this. The coordinate axis to static equilibrium position because of the non-linearity in the system or linear system generally we shifted. So this will act and we have this x y stationary coordinate system and this another coordinate system psi and eta is a rotating coordinate system, which we have encountered in the previous lectures also. So with this, we can able to write the equation of motion.

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Solution: (i) Equations of motion of the Jeffcott rotor in the x, y, and θ directions can be written as

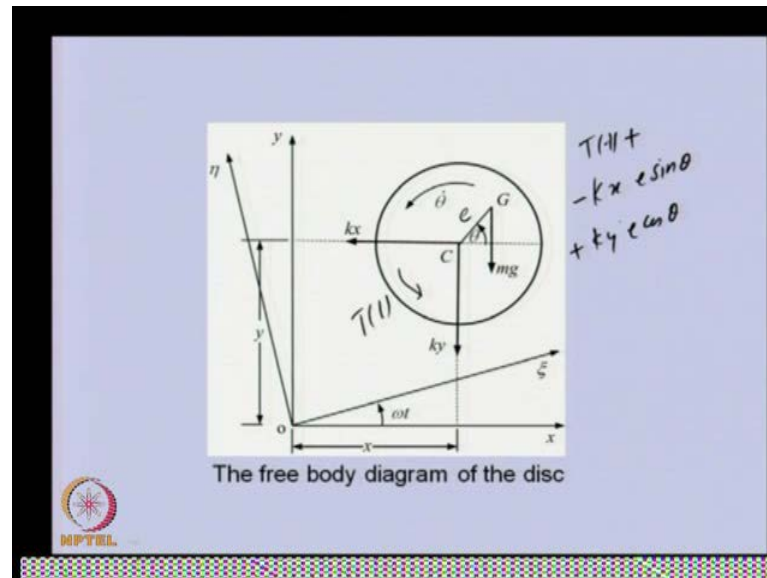
$$-kx - c\dot{x} = m \frac{d^2}{dt^2} (x + e \cos \theta)$$

$$-ky - c\dot{y} = m \frac{d^2}{dt^2} (y + e \sin \theta) + mg$$

$$T(t) - kx(e \sin \theta) - ky(e \cos \theta) - c\dot{x}(e \sin \theta) - c\dot{y}(e \cos \theta) = I_p \ddot{\theta}$$

So, you can able to see apart from the stiffness, we can able to add the dumping force also. This external force should be equal to the mass into angular acceleration and this is the angular position, this is the linear position of the center of gravity of the rotor.

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So, this is x and e , this is the eccentricity e . This is the center of gravity, so $e \cos \theta$. So, x plus $e \cos \theta$ will be the location of the center of the gravity, similarly in the y direction y plus $e \sin \theta$ plus weight also, we are considering here because positive y direction is in the up-ward direction. So basically, this should have been this side as minus and then this are the external forces equal to the inertia, mass into angular mass in to the linear acceleration. And the third one is the torque, the torque balance. So, in this we can able to see in, if we taking moment about of this.

So, we will be having $k x$ into $e \sin \theta$, $k x$ into $e \sin \theta$. That is the moment and that is acting opposite to the rotational direction, that will be negative. From this, we will be having $k y e \cos \theta$, but this is positive. And torque let us say, is acting in the positive direction. So t and this is θt . So, these are the and form damping also similar terms will come. So, you can able to see torque stiffness damping, should be equal to polar moment of inertia, mass moment of inertia into angular acceleration. So, these are the 3 equation of motion in x direction y direction and θ direction. Basically, they are coupled θ and these equations, we can able write like this. So, these are the 3 equations which we derived in x and y and θ directions.

(Refer Slide time: 36:38)


On simplification of above equations, we get

$$m\ddot{x} + c\dot{x} + kx = me(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) \quad (d)$$

$$m\ddot{y} + c\dot{y} + ky = me(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) + mg \quad (e)$$

$$I_p \ddot{\theta} + e(kx + c\dot{x}) \sin \theta + e(ky + c\dot{y}) \cos \theta = T(t) \quad (f)$$

It should be noted that, for a known torque, equations (d) to (f) have to be solved simultaneously to get the resultant angular acceleration of the rotor.



We can able to see theta of a these 2 equations. So, if you want to solve this, you need to solve simultaneously. And in this particular case, if you see there is a non-linear terms of the theta also as square of velocities and sin theta cos theta they are also non-linear term. So basically, these are already coupled equations but for special cases, we can de-couple them. So we will see that, how we can able to de-couple. So for non-torque, so if know the torque how much we are applying, the equation or 3 equations we need to solve simultaneously because if we know the torque, then we can able to un-coupled these.

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On simplification of above equations, we get

$$m\ddot{x} + c\dot{x} + kx = me(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) \quad (d)$$

$$m\ddot{y} + c\dot{y} + ky = me(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) + mg \quad (e)$$

$$I_p \ddot{\theta} + e(kx + c\dot{x}) \sin \theta + e(ky + c\dot{y}) \cos \theta = T(t) \quad (f)$$

However, for required angular acceleration, equations (d) and (e) are uncoupled and can be solved for x and y , which can be used in equation (f) to get the torque required to produce the desired angular acceleration.




In another case, if we have required angular acceleration, so instead of we know the torque. If we have required angular acceleration, these are known, then the equation d and e will be un-coupled. And we can able to solve for x and y independent of each other and once we solved x and y, we can able to come to this equation, to get what should be the torque required to give that particular kind of angular acceleration or angular velocity. So in this particular case, the second case in which the angular acceleration and velocity are specified, these 2 equations are un coupled and we can able to solve this, to get the torque, the third equation. So in this particular case, the simultaneous solution of this is not required. So, now let us take the case when the rotor is perfectly balanced.

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(ii) For perfectly balance rotor, i.e., eccentricity $e \approx 0$ and for a constant torque T_c , from equation (f), we have

$$I_p \frac{d\dot{\theta}}{dt} = T(t) = T_c \qquad \int_0^{\omega_{\max}} I_p d\dot{\theta} = \int_0^{t_{\max}} T_c dt$$


is the maximum angular velocity of the rotor to be achieved and t_{\max} is the time taken for the same. On integration the above equation gives

$$I_p (\omega_{\max} - 0) = T_c (t_{\max} - 0) \Rightarrow t_{\max} = \frac{I_p \omega_{\max}}{T_c}$$


The eccentricity is 0, that means all the terms in the equation of motion on the right hand side are 0 and we have constant torque. So, in if we have constant torque, then we can able to see the last equation, all the terms will be 0. Only the this inertia rotor inertia and the torque will be present, which is constant. So this, we can able to rearrange like this I_p into $d\theta$ and t into dt . And we can integrate this from 0 to what is the maximum angular velocity we required and what is the time from 0 to t_{\max} , what is the time required to reach that particular speed. If we if we integrate it, basically we can able to get the time required to reach ω_{\max} . So, this will be expression for that, this is valid for the constant torque and when there is no eccentricity of the rotor.

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(iii) For a known constant angular acceleration with zero initial conditions, i.e., $\ddot{\theta} = \alpha$, $\dot{\theta} = \alpha t$, $\theta = \frac{1}{2} \alpha t^2$ for the unbalanced rotor equations of motion takes the following form

$$m\ddot{x} + c\dot{x} + kx = me \left\{ \alpha \sin\left(\frac{1}{2} \alpha t^2\right) + (\alpha t)^2 \cos\left(\frac{1}{2} \alpha t^2\right) \right\}$$
$$m\ddot{y} + c\dot{y} + ky = me \left\{ \alpha \cos\left(\frac{1}{2} \alpha t^2\right) - (\alpha t)^2 \sin\left(\frac{1}{2} \alpha t^2\right) \right\}$$



The next is, now for the known constant acceleration, with these initial conditions. Let us say, these are given to us these are specified, then the unbalanced rotor equation of motion which we had written earlier, will take this form. So, now you can see in place of theta double dot, theta we wrote this and theta dot we wrote this. And now these equations are independent, we can solve this equation independent of each other.

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$$m\ddot{x} + c\dot{x} + kx = me \left\{ \alpha \sin\left(\frac{1}{2} \alpha t^2\right) + (\alpha t)^2 \cos\left(\frac{1}{2} \alpha t^2\right) \right\}$$
$$m\ddot{y} + c\dot{y} + ky = me \left\{ \alpha \cos\left(\frac{1}{2} \alpha t^2\right) - (\alpha t)^2 \sin\left(\frac{1}{2} \alpha t^2\right) \right\}$$

It should be noted that above equations are now uncoupled and linear and can be solved independent of each other.

The forcing is general in nature, hence, to get the response either of equations can be time integrated by using numerical integrations, e.g., the Newmark's method.




And now, like this, now and so basically, here now we will be obtaining this because here the forcing is general in nature because this is external force. Because of angular

acceleration, the force is not a linear. It is varying continuously with time, so we will be using some kind of time integrated integration method, numerical method like Newton new marks method we can able to use it .

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By introducing the following non-dimensional parameters

Dimensionless displacement	$\bar{x} = x / e$
Dimensionless time	$\tau = \omega_{nf} t$
Dimensionless acceleration	$\bar{a} = \frac{a}{\omega_{nf}^2}$


$$\omega_{nf} = \sqrt{k / m} \quad \zeta = c / c_c = c / (2\sqrt{km})$$


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$$m\ddot{x} = m \frac{d^2 x}{dt^2} = me \frac{d^2 (x / e)}{d(\tau / \omega_{nf})^2} = me\omega_{nf}^2 \frac{d^2 (\bar{x})}{d\tau^2}$$

$$c\dot{x} = c \frac{dx}{dt} = ce\omega_{nf} \frac{d\bar{x}}{d\tau} \quad kx = ke\bar{x}$$

$$\frac{ce\omega_{nf}}{me\omega_{nf}^2} = \frac{c}{m\omega_{nf}} = \frac{\zeta(2\sqrt{km})}{m\sqrt{k/m}} = 2\zeta$$

$$\frac{ke}{me\omega_{nf}^2} = \frac{k/m}{\omega_{nf}^2} = 1$$


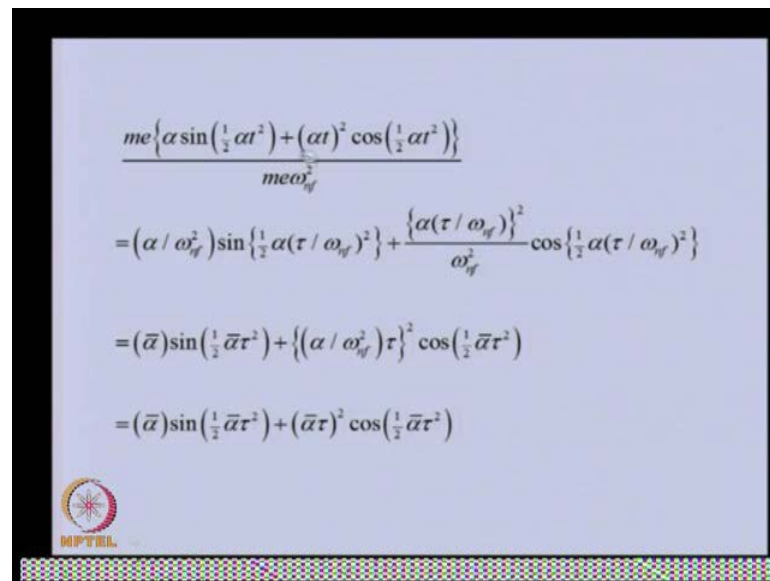
Before using it, we are using we are non dimension analyzing the previous equation like dimension less displacement, x by eccentricity. We are solving one of the equation in the x direction, y direction displacement will be similar to this one, a non dimensional time frequency into time, non dimensional acceleration we are defining like this. This is the

natural frequency and damped one. This is the damping ratio. So, these non dimensional terms, we can introduce in various terms of the equation of motion like $m \ddot{x}$.

These can be written like this and intern we can able to define non dimensional displacement and the time. So basically, $m \ddot{x}$ can be written like this. Similarly, $c \dot{x}$ we can able to write in the terms of non-dimensional terms like this. kx can be written like this. Then some other terms like this is coming into the mass term, so equation of motion we are dividing the whole equation of motion by this.

So that means, in the damping term this divided by, which is this 2 terms, again we can able to write in the non dimensional term. Similarly, in the stiffness term, this divided by this we will community. So like that, we introduce the various non-dimensional terms. Similarly, in the forcing term, we this division is there because we have divided this throughout the equation of motion.

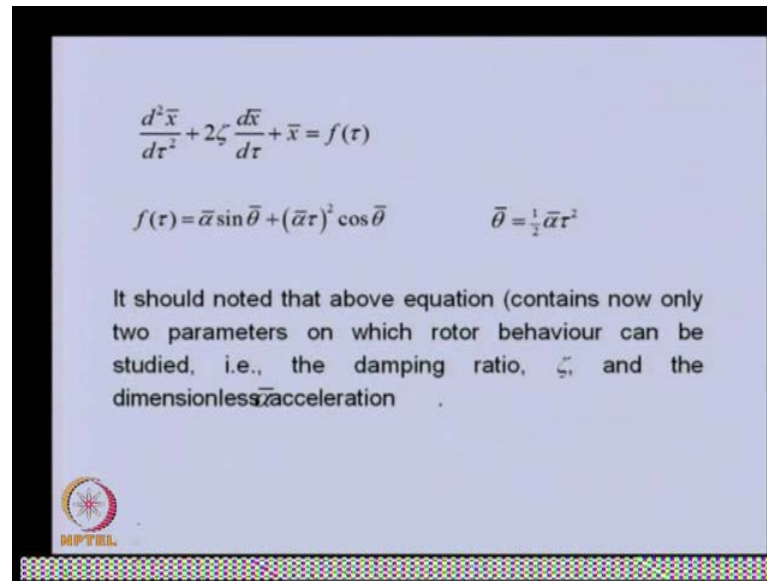
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$$\begin{aligned} & \frac{m e \left\{ \alpha \sin \left(\frac{1}{2} \alpha t^2 \right) + (\alpha t)^2 \cos \left(\frac{1}{2} \alpha t^2 \right) \right\}}{m \omega_n^2} \\ &= \left(\alpha / \omega_n^2 \right) \sin \left\{ \frac{1}{2} \alpha (\tau / \omega_n)^2 \right\} + \frac{\left\{ \alpha (\tau / \omega_n) \right\}^2}{\omega_n^2} \cos \left\{ \frac{1}{2} \alpha (\tau / \omega_n)^2 \right\} \\ &= (\bar{\alpha}) \sin \left(\frac{1}{2} \bar{\alpha} \tau^2 \right) + \left\{ \left(\alpha / \omega_n^2 \right) \tau \right\}^2 \cos \left(\frac{1}{2} \bar{\alpha} \tau^2 \right) \\ &= (\bar{\alpha}) \sin \left(\frac{1}{2} \bar{\alpha} \tau^2 \right) + (\bar{\alpha} \tau)^2 \cos \left(\frac{1}{2} \bar{\alpha} \tau^2 \right) \end{aligned}$$

So this equation, we can able to several steps, finally we will get a simple equation like this, for the forcing term.


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$$\frac{d^2 \bar{x}}{d\tau^2} + 2\zeta \frac{d\bar{x}}{d\tau} + \bar{x} = f(\tau)$$

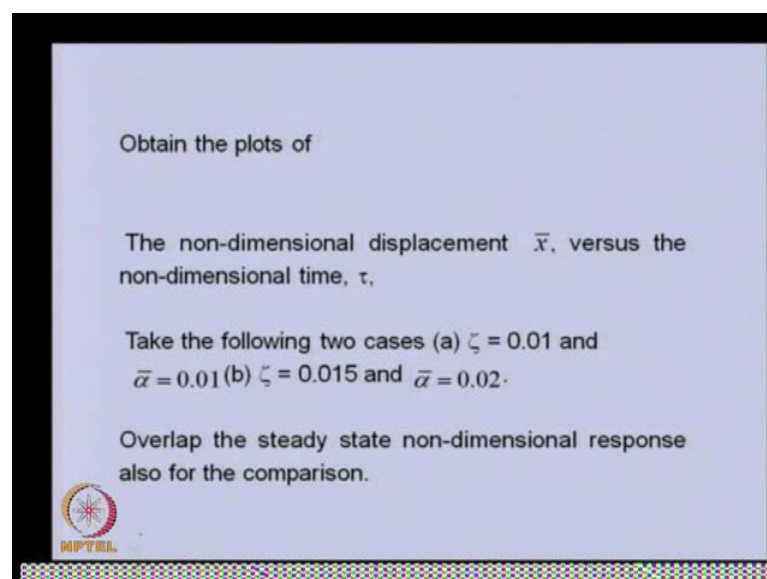
$$f(\tau) = \bar{\alpha} \sin \bar{\theta} + (\bar{\alpha}\tau)^2 \cos \bar{\theta} \quad \bar{\theta} = \frac{1}{2} \bar{\alpha} \tau^2$$

It should be noted that above equation contains now only two parameters on which rotor behaviour can be studied, i.e., the damping ratio, ζ , and the dimensionless acceleration $\bar{\alpha}$.



So basically, equation of motion in the non dimensional form, will be of this form, where it is given by this, where theta itself defined like this. So basically, now equation is ready for numerical simulation and in this equation of motion, you can able to see that there are only 2 parameter now. One is the damping ratio and the another is non dimensional angular acceleration. So, 1 is damping ratio and other is non dimensional angular acceleration. We can able to study, the effect of these parameters on the response.

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Obtain the plots of

The non-dimensional displacement \bar{x} , versus the non-dimensional time, τ ,

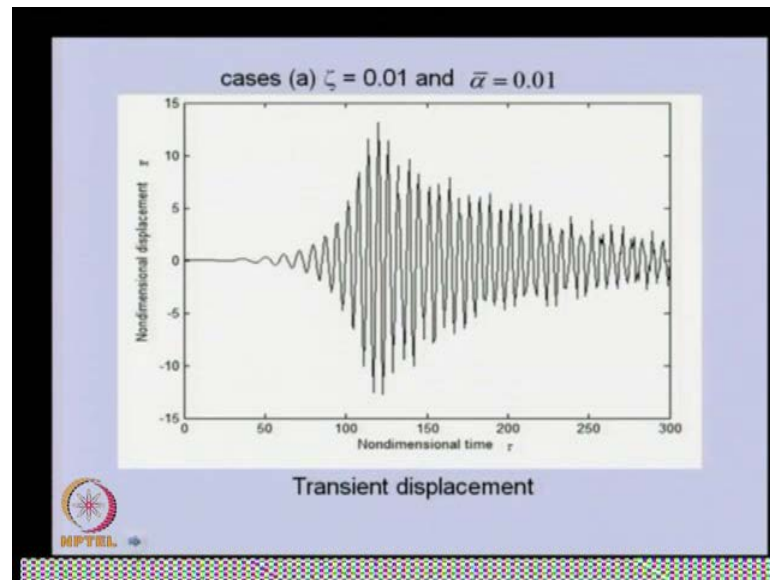
Take the following two cases (a) $\zeta = 0.01$ and $\bar{\alpha} = 0.01$ (b) $\zeta = 0.015$ and $\bar{\alpha} = 0.02$.

Overlap the steady state non-dimensional response also for the comparison.



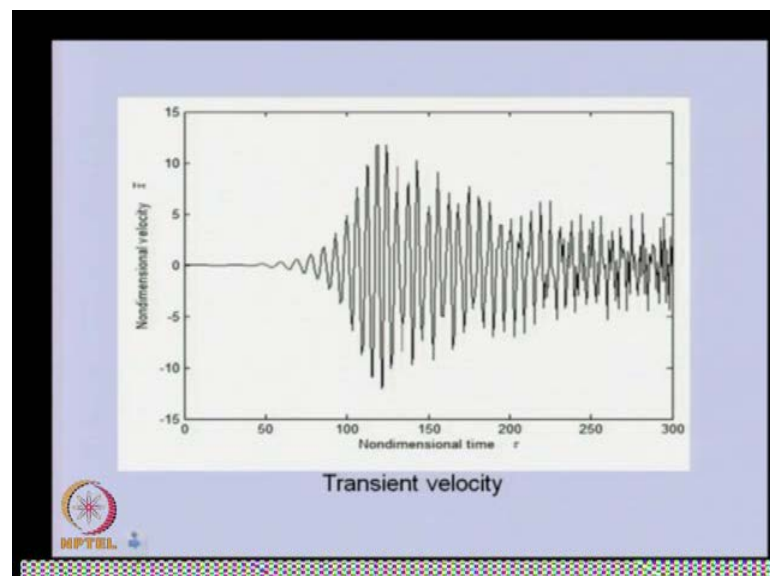
So now, we will be obtaining the plot of the response with non dimensional time. So, as we have 2 parameters to play with, so we can able to take different cases. So, these 2 cases, we have simulated in which damping ratio was in this angular acceleration and non-dimensional was chosen and we have obtained the various responses.

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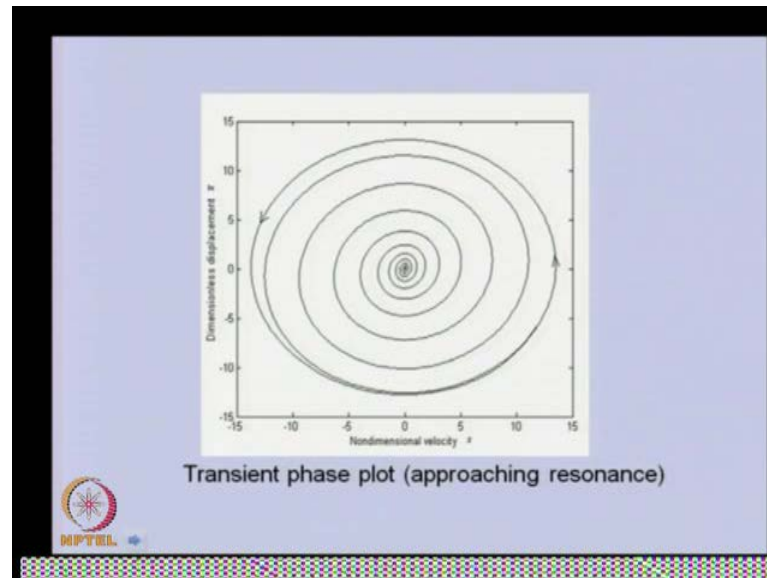
So, this is a typical non dimensional displacement x -prime with non dimensional time. So, you can able to see with time how the response changes. This is for one particular case.

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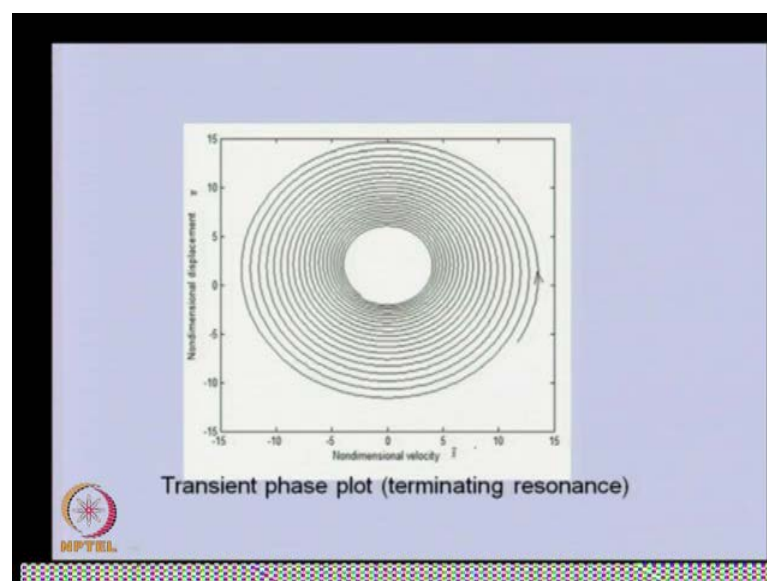
In this particular case, again this behavior is there because there is a angular acceleration of the rotor. This is this is the transient velocity, in place of displacement velocity is there.

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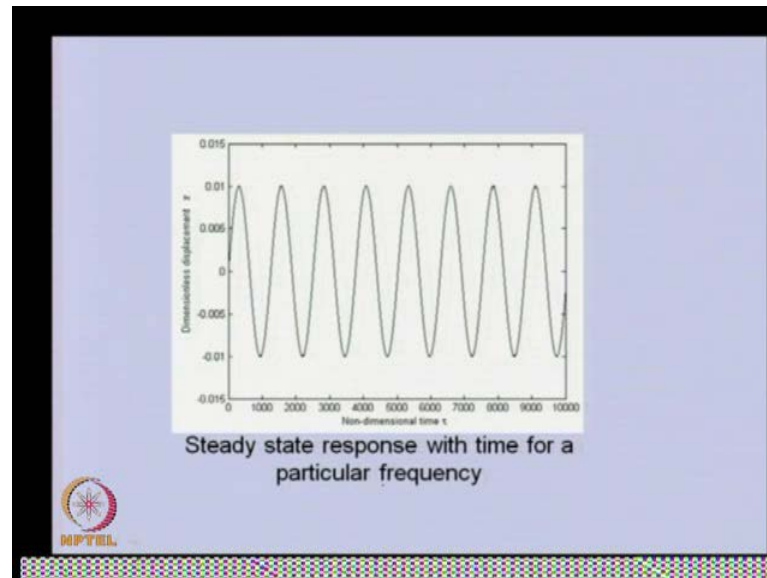
This is the basically the plot of the x and the \dot{x} , x -prime and the \dot{x} -prime. So, this is the phase plot, transient phase plot. So, when resonance is approaching, we can able to see the gradually the amplitudes are increasing.

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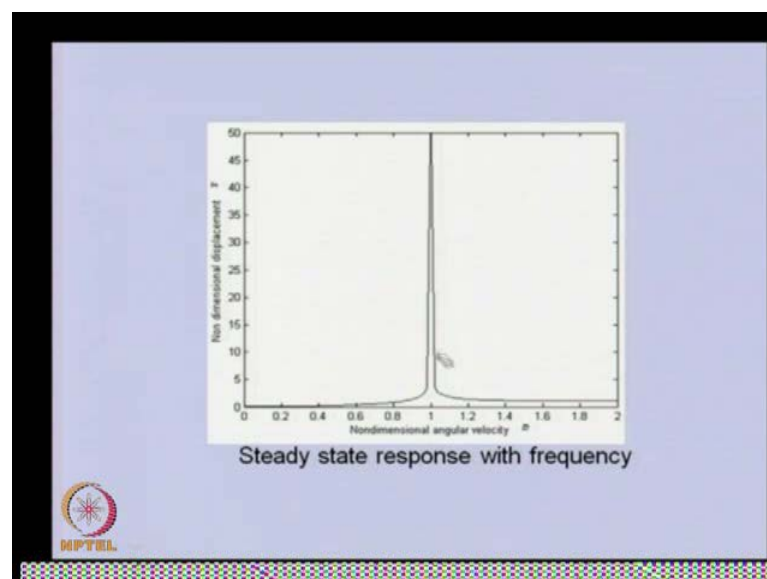
And once we cross the resonance, this is a terminating resonance, so it is coming down. This is the steady state response with time, for particular frequency. So at one particular frequency, if we are not having acceleration, this will be in the form of response.

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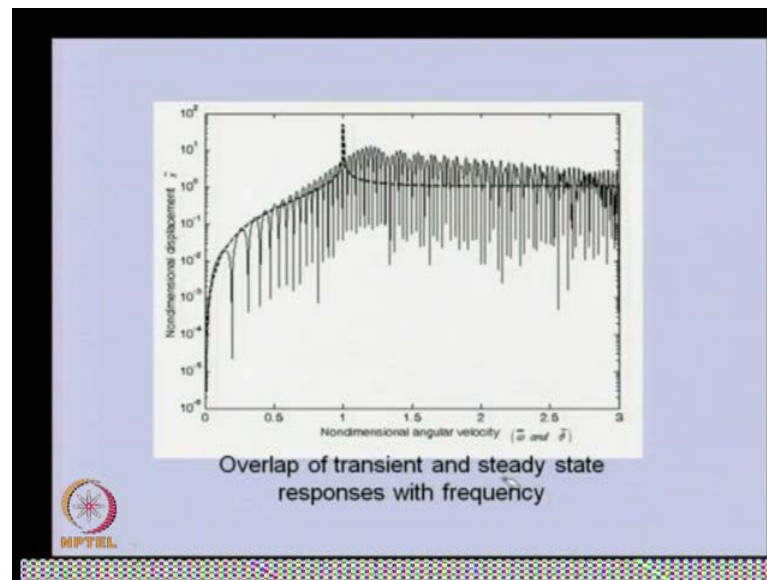
And for that particular steady state response, these will be the frequency domain data. So, the peak will be there corresponding to that particular frequency.

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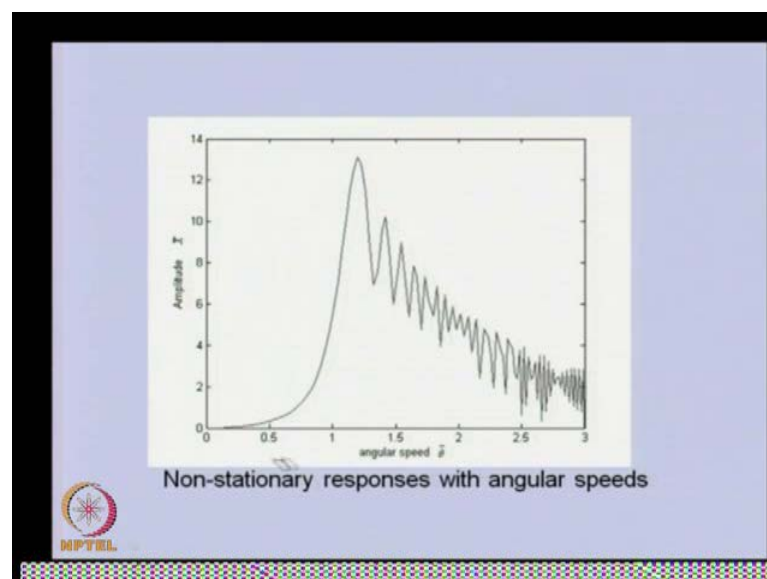
This is the overlap the transient and the steady state response with frequency.

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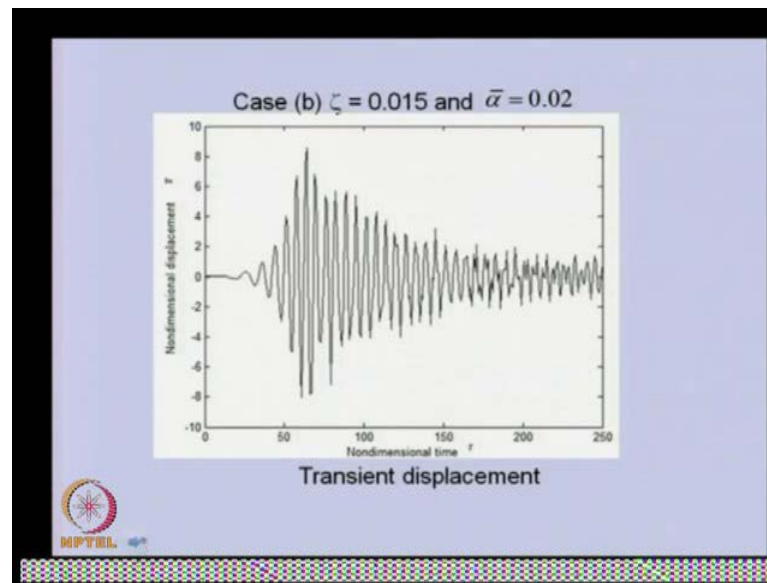
So, you can able to see this is the semi log case. So this is the case, with without transient and with acceleration, angular acceleration we have we overlapped these 2 responses in 1, for compression purpose.

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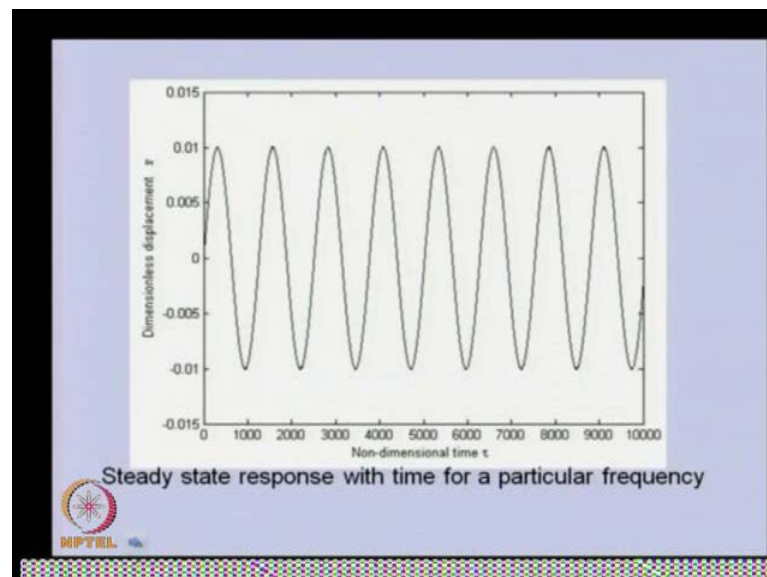
And this is the non stationary response, with angular speed. So, how the amplitude x is changing with \dot{x} x -prime is changing with angular speed.

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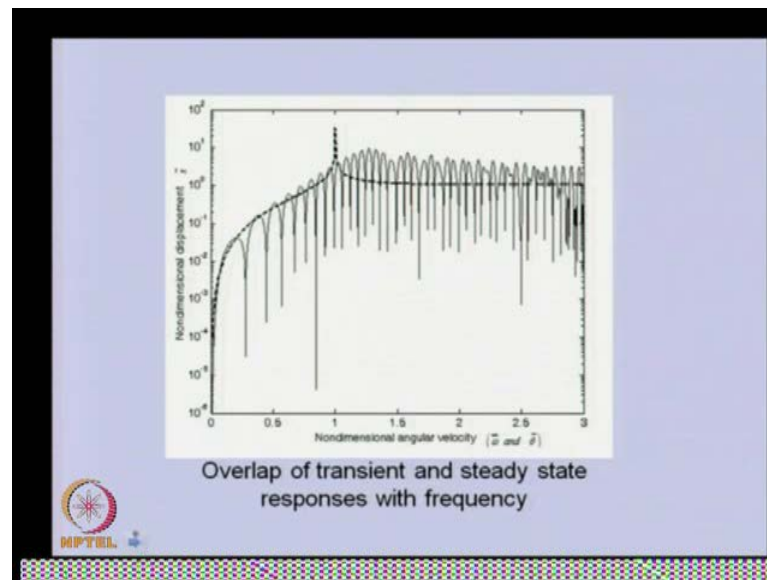
So you can able to see, these is the resonance after that there is a this kind of non-transient behavior. This is for another case, similar plots are there, for different damping and angular acceleration, so these are this for velocity. This for approaching resonance. This is for terminating resonance.

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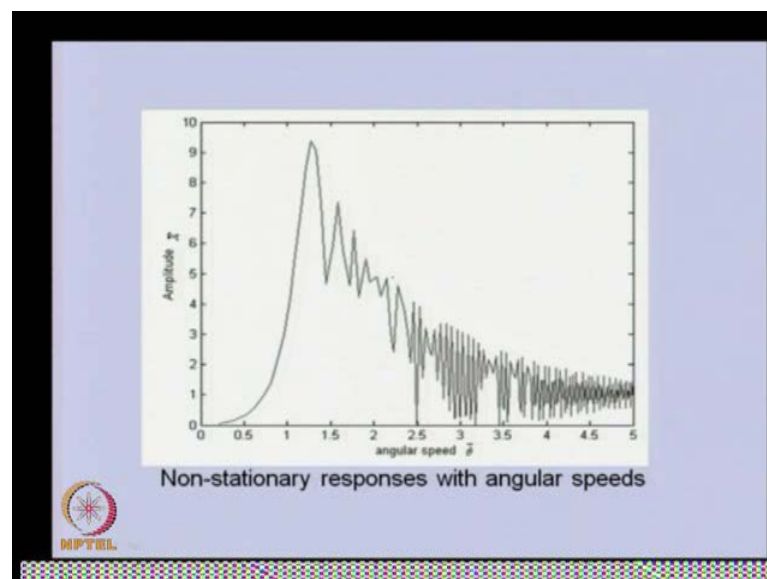
This is steady state, just for compression, we have made the steady state response also.

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So, this the steady state response, overlapped with the transient response.

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And this is with angular velocity, how the transients are coming once we are going away from the resonance condition. Today, we have seen one is the steam whirl and how the steam whirl can give instability to the system and how we can able get the condition of instability, which are the parameter which can govern the or which can able to stabilize the system, in which there is a possibility of the steam whirl.

So, we have seen that if we have more damping in the system or if more the natural frequency is more of the system, then that means if we have more rigid rotors, we can have the more stable the threshold speed. That means, we can able to increase the speed of threshold or steam whirl with these 2 parameter. Apart from this, we have seen how the how the order of the seals, we can have in a simple Jeffcott rotor and what are the conditions, we can get for a rotor with seals. Those expressions were more general in nature and that can be used for more specific cases or to understand various parameter and the seal parameter may be one by one, those parameters can be considered and we can able to analyze the stability of the system in more detail.

Apart from this, we took up the case of the run-up or run-down of the rotor which is often is there in the practice, to avoid the critical speed or to traverse the critical speed and the instability zone quickly, we need to give some kind of quick acceleration, angular acceleration to the rotor. And but because of that, we can have some kind non stationary response. So, how the rotor will behave in such cases we try to understand in the particular case.

In this, we had the equations are non-linear, so we use time integration to solve to get the responses. And that is more practical to because when we are dealing with a more complex system, bigger system then that is only possibility, that we can able to analyze the such bigger system, using direct time integration. There are methods, the analytical methods for non-linear system analyses. So, that we are not dealt with this in this particular course. So, in the subsequent lecture we will see some more cases of the instability and we will try to see how these can be analyzed, more effectively especially for large systems. We will take up some case, in which the large system with fulfill bearing flexibility, how we can able to check.