Theory and Practice of Rotor Dynamics
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Module - 7
Instability in Rotor Systems
Lecture - 34
Internal Damping and Asymmetrical Shaft

In previous lecture, we have seen how the fluid film bearing imparts instability in the rotor bearing system. Today, we will take up two another kind of damping; one is internal damping, and another is asymmetric shaft. Because of these two cases, how the instability comes into the rotor system we will try to study. In this internal damping, we will see how it comes into the system. Initially and then we will with simple mathematical model, we will try to analyze this instability in which we will find that there will be a threshold speed above which instability can occur. And for asymmetrical shaft case, we will see there will be a band of instability zone where the rotor can be unstable and below and above these bands; we will find that there will be stable system.

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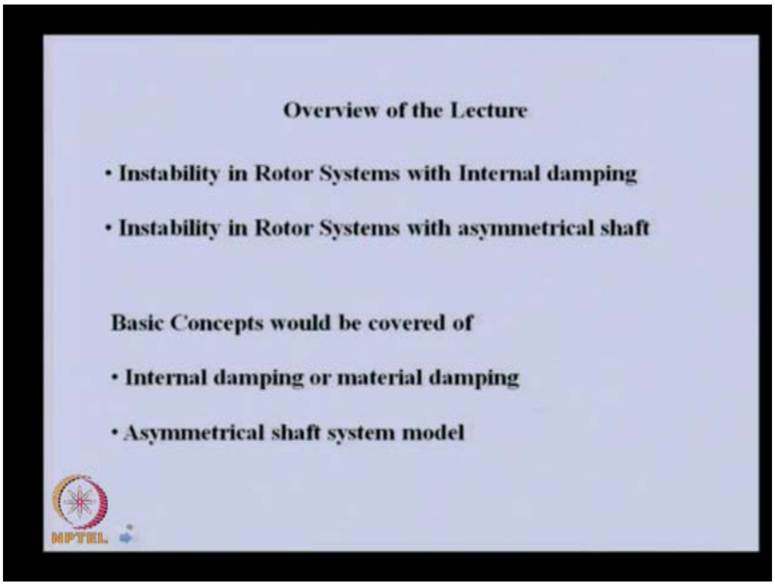

So, before going into detail, just let us see what we will be covering here; so instability in rotor system with internal damping and with asymmetrical shaft. We will see that internal damping or material damping or sometime we call it as hysteretic damping, how it comes into the system. This asymmetrical shaft's stiffness rotor model, we will see how we can able to analyze with this, the instability in the rotor system.

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So, coming to the internal damping, which comes from various sources like elastic hysteretic of shaft material. Due to fractural vibration of the shaft, the intermolecular interaction takes place within the shaft material. That gives some kind of heat generation within the shaft material. So, basically this kinetic energy converts into the heat in the form of hysteresis. Even in the shaft during the fractural vibrations; the shaft fiber shear, it takes place during whirling, because of tension and compression of the fiber.

Apart from this frictional forces between two mating parts with in interference fit can have this kind of internal damping. So, in this particular case all such damping sources can impart a common damping which we call as internal damping. This internal damping is having slightly different characteristic as compared to the viscous damping.

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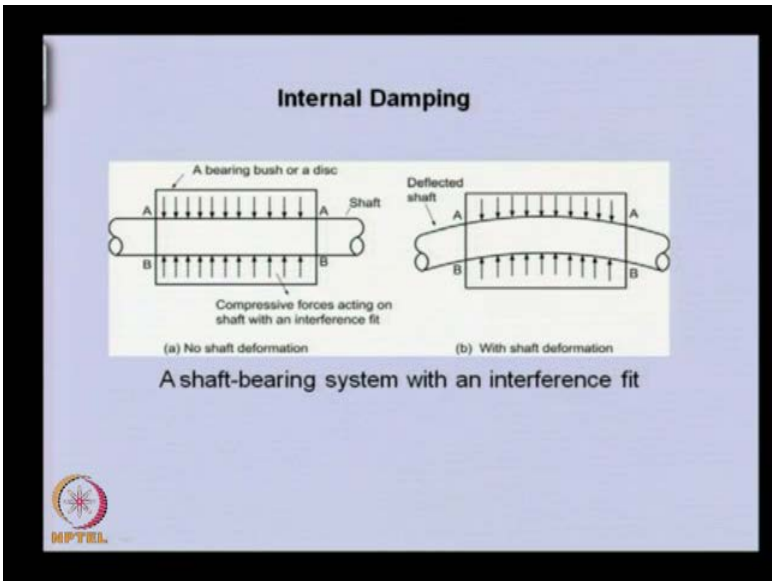

So, like now, let us understand the internal damping when we have one shaft and there is a hard disk. This is ring fit all this. So, during vibration or whirling, this shaft bends because of the compression here, already onto the shaft because of interference. During the binding, we will see that the upper fiber of the shaft will elongate and the lower one will get compressed. These forces will resist that particular motion; extension of the shaft or contraction of the shaft and this case internal damping between two mating parts having some kind of interference fit.

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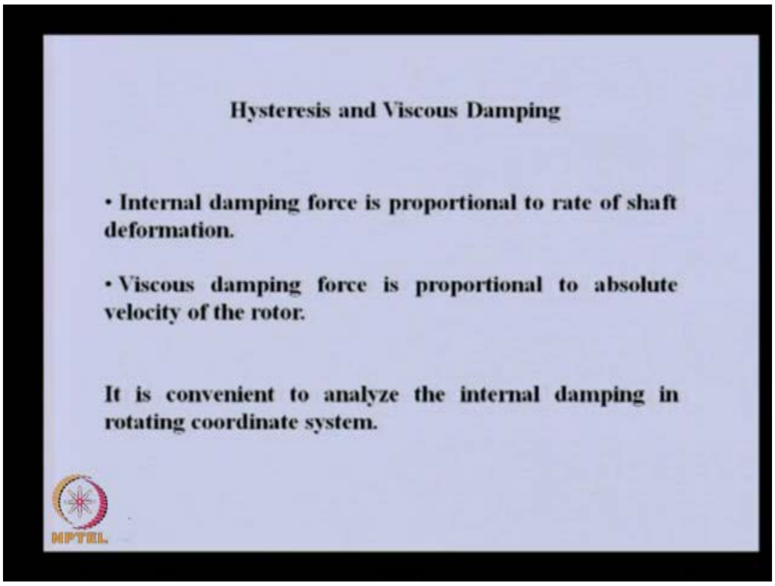

Now, coming to the difference between the hysteretic damping and viscous damping. Internal damping force, which is nothing but hysteretic damping is proportional to the rate of shaft deformation. But, the viscous damping is proportional to the absolute velocity of the rotor. So, one is rate of shaft deformation. Another damping force is proportional to the absolute velocity of the rotor because of this internal damping direction changes along the shaft rotation.

It is convenient to analyze the internal damping in rotating coordinate system. Now, we will analyze the internal damping with a very simple mathematical model of rotor system. In this particular case, we will derive the equation of motion in rotating coordinate system. Once we have obtained the equation of motion in rotating coordinate system, we will introduce the hysteretic damping or internal damping at that stage.

So, initially the equation of motion; we will be deriving only with the internal that is only with the viscous damping. Subsequently, we will be introducing the hysteretic damping. So, before going to the equation of motion, let us see the rotating coordinate system and its transformation.

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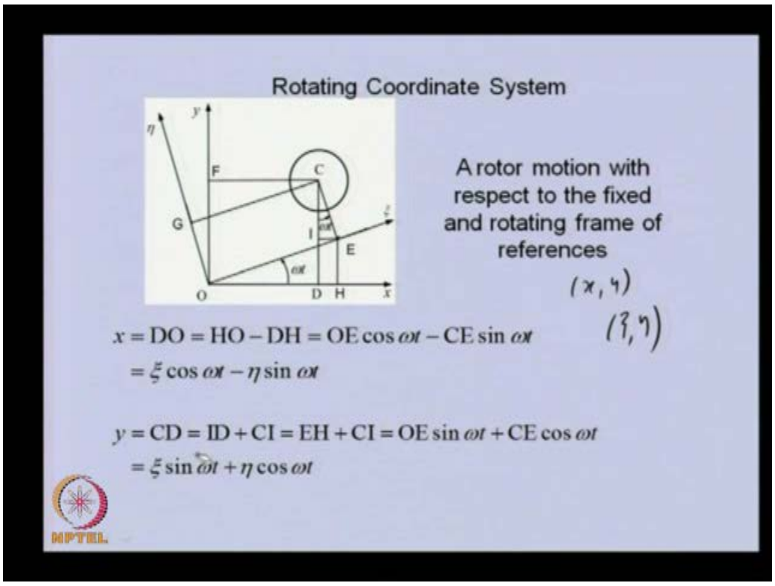

So, x y is stationary coordinate system. Psi and eta is rotating coordinate system. We want to transform the equation of motion in x y, x's coordinate system to the rotating coordinate system. Before that, we can able to relate these two coordinate system. So, with elementary geometry, we can able to relate that. Here, you can able to see. This is the centre of the shaft. Here, OD is the x distance. OF is the y distance. Similarly, OH is the xi. OE is the xi distance. OG is the eta distance.

We need to relate these two coordinates. That means, we have x and y. The geometrical centre of the shaft is the coordinate of the geometrical centre of the shaft in x and y coordinate. Another is xi and eta. We need to relate this two. So, we can able to see with simple geometry. We can able to write x is equal to xi omega cos omega t minus nu sin omega t. This omega t is the angle at particular time t of the rotating coordinate system. At t, time is equal to 0. We can assume that both coordinates are in the same position. But, as time passes, this rotating coordinate rotates with the spin speed of the shaft. Similarly, y can be related as xi sin omega t plus eta cos omega t. These are standard trigonometry relations.

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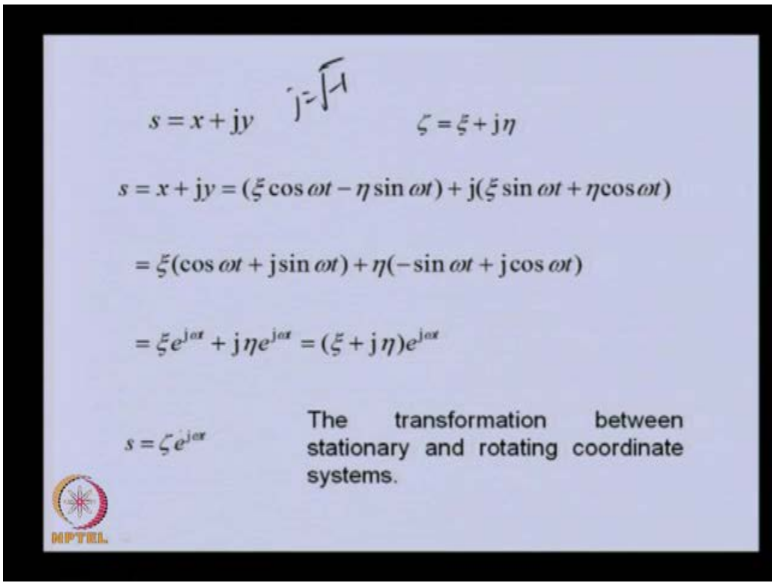

Now, we are defining a complex displacement s which is defined as x plus j y. j is the complex quantity that is root of in the rotating coordinate system. Similarly, we are defining zeta is equal to xi plus j eta s. xi are the complex coordinates in stationary coordinate and rotating coordinate system. Now, this two in place of x and y; from previous expression, we can able to write this in terms of zeta xi and eta. Similarly, y we can able to write in terms of this two.

This we can able to rearrange by taking common of xi term and eta term. We can able to see within this bracket is e j omega t. This if we take common, this can be converted into j e j omega t. If we are, we can able to see this when this e j omega t is common. So, that can be taken out. So, xi plus j eta and xi plus j eta is nothing but zeta here. So, that can be substituted. So, this is the transformation basically in complex domain on stationary coordinate system to rotating coordinate system. This will be using to transform the equation of motion of the rotor system.

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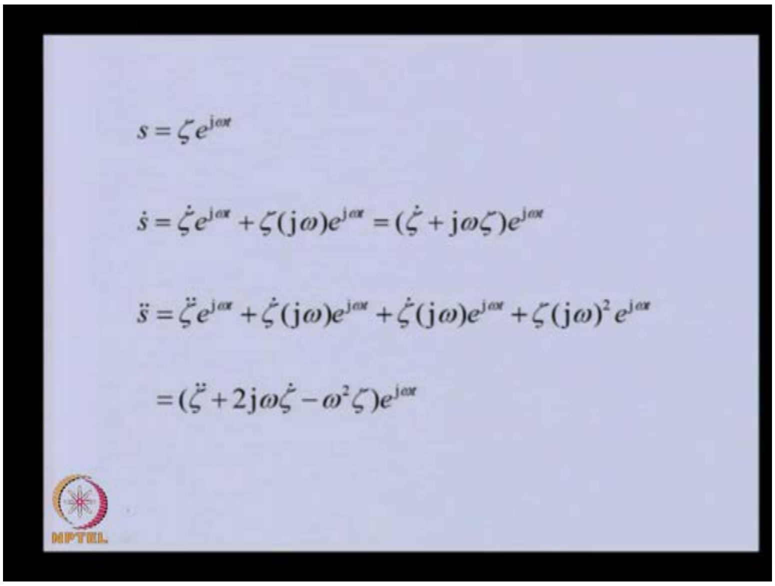

Because we will be having derivatives of this complex displacement in the equation of motion, so we can able to take the derivative with respect to time. Zeta and this, both are time dependent. So, once we differentiate, we will get two components; two parts. So, first is derivative and then derivative of the second term. This can be clubbed like this. Similarly, if we take another derivative of this, we will be having derivative of this. Then we will be getting basically four terms. So, these are the four terms after differentiation of this, which can be simplified as this because two terms are common. So, they will add up to give this. So, we have complex displacement in stationary coordinate system and its derivative we have already obtained.

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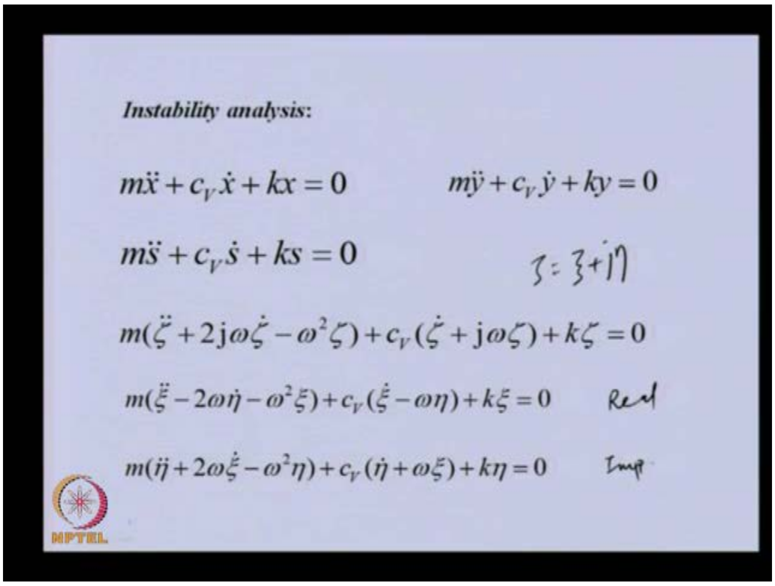

Now, let us see the equation of motion of a simple rotor system in which we have mass of the rotor, viscous damping which may come from bearing or any other source. k is the stiffness of the shaft. So, this is the equation of motion in the horizontal direction. Similarly, this is the equation of motion in the vertical direction. In this case, we have not considered any cross coupling of the stiffness or damping. Only simple direct stiffness and damping we have considered. CV is the viscous damping.

At present, we have naught introduced the hysteretic damping. As I told earlier once, we will transform this equation of motion in the rotating coordinate system. Then we will introduce the viscous damping, because it will be convenient to introduce at that stage. Now, we are multiplying the second equation by j, adding it to one first equation. So, we can able to get the equation of motion in a complex wobbling in stationary coordinate system. So, m S double dot plus CV S dot plus k x is equal to 0.

Now, we can able to substitute the transformation, which we developed in the previous slides. So, S double dot we obtained earlier. So, this is the S double dot. S dot two terms was there. So, this is the S in terms of zeta. So, basically this equation of motion after substituting the transformation. It has come into the rotating coordinate system. Now here, it is in the terms of zeta that is in the rotating coordinate system complex coordinate. Because zeta is defined as xi plus j eta, so we can able to split this equation in real part and imaginary part with the help of this expression.

So, in the direction of xi, we will get this equation. We can see that j term is there. So, that will give basically after multiplying this eta term here otherwise, xi terms will be there at any other place. Wherever j is there, we are getting the eta term and plus and minus sign. You need to take care of that. This is as it is only real part, we have considered. So, this is the real part. This is the imaginary part of this complex equation. So, this equation is in the direction of xi. This is the equation of motion of the rotor in the eta direction.

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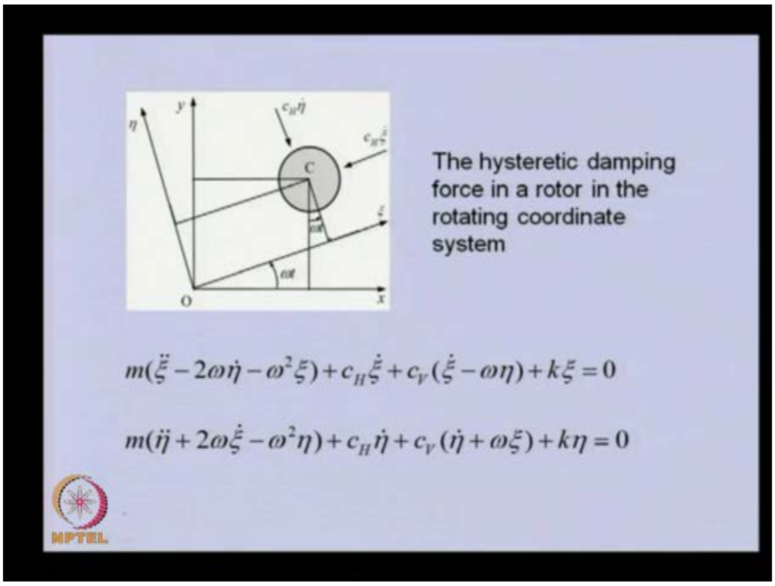

Now, coming to the hysteretic damping; here, this is the xi direction. This is the eta direction. We have earlier noted that the hysteretic damping or internal damping; it acts proportional to the rate of deformation of the shaft. Because this coordinate system xi and eta is attach and is rotating with spin speed, so the rate of deformation of the shaft will be xi dot and eta dot.

If you multiply with hysteretic damping coefficient; this will give the damping force, hysteretic internal damping force in the direction of xi and in the direction of eta. Now, you can able to see that because we have already resolved the equation of motion in the xi direction and eta direction. If we want to include the hysteretic damping, just you need to include these two terms in the previous equation of motion.

So, this equation of motion is exactly same. Only thing, now we have added this hysteretic damping in this model. So, this equation is exactly same. Only thing, now additional term of the hysteretic damping CH we added. CV is already there. In this now, this equation is ready for doing the stability analysis of the system. So, again we will combine these two equations. That means, the second equation will multiply by j and I add it to one first equation.

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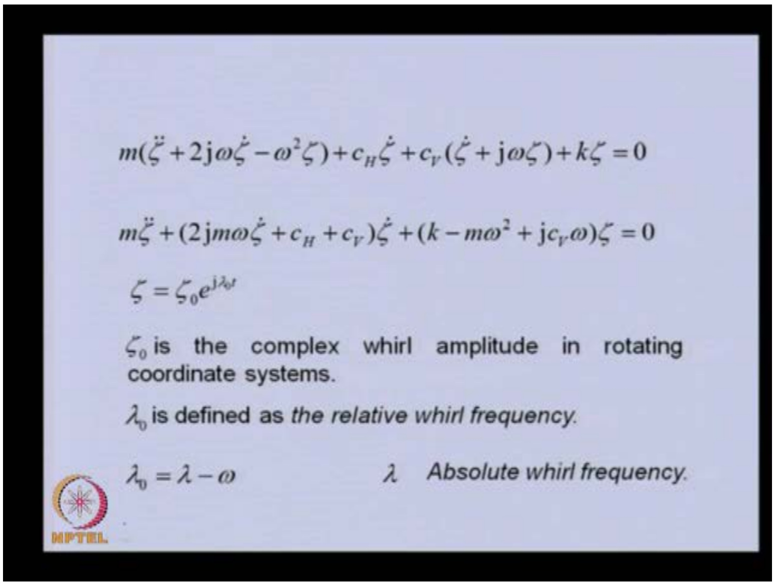

So again, we can get the equation of motion in the rotating complex coordinate system that is zeta. All other terms are similar. Only the additional term you can able to see is coming due to the hysteretic damping. This was already there. Now, we can able to because we have some terms of zeta dot and zeta. So, we can able to rearrange this equation so that we have terms of zeta double dot in one place, zeta dot in another place and zeta in other place.

So, now we can able to assume the solution of this in this form in which zeta naught is complex whirl amplitude in rotating coordinate system. j lambda naught t lambda naught is the relative whirl frequency or Eigen value of the system. This is relative because we assuming the solution in the rotating coordinate system. The absolute whirl frequency is will be and the relative frequency will be defined like this. That means, because the differences at which this particular rotating system is rotating; so relative whirl frequency or the Eigen value is defined as absolute whirl frequency minus the speed. So, with this you can able to get with this relation. We can able to get the absolute whirl frequency also. So, the assumed solution, we will be substituting in the equation of motion in the rotating coordinate system itself.

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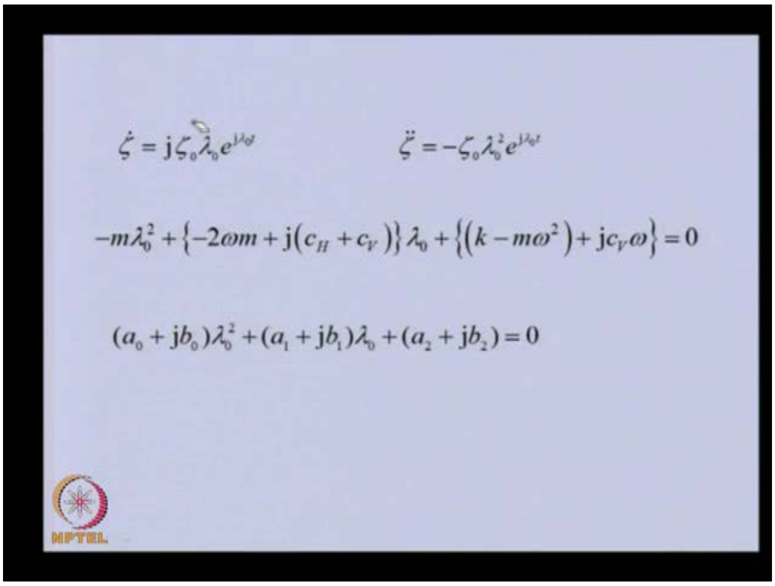

For that, we need derivatives of the zeta and zeta double dot. So, we can able to substitute this in equation of motion and with that we will get a polynomial in the lambda naught square. So, you can able to see this is a polynomial lambda naught square. This square with a quadratic polynomial and the form of this polynomial, because this is slightly different as compared to the previous case of the damping, a fluid film bearing damping case. We can able to see some complex terms are also there. So, basically more general form of this quadratic equation could be like this in which the real part and imaginary part of the epsilons where this is lambda square term is there. Similarly, for lambda naught term and for the constant term, so this is the more general form of this. So, stability criteria for this equation are provided in the next slide.

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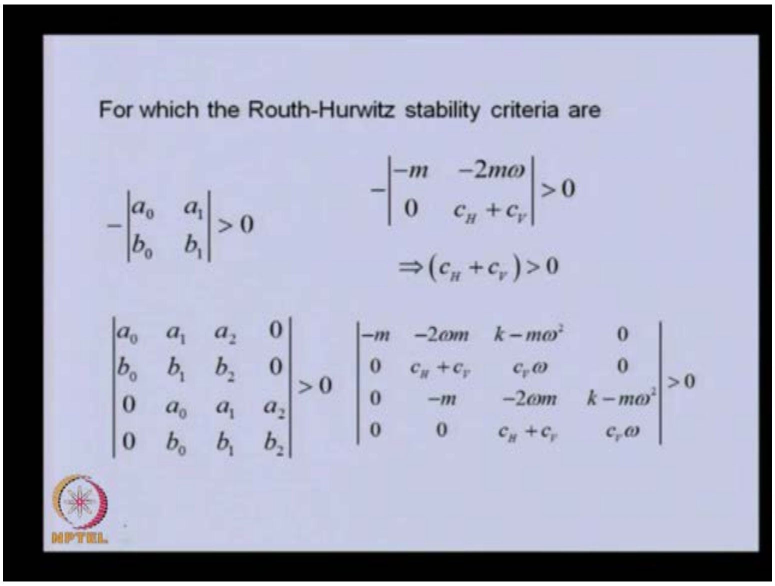

So, this is the stability criteria; Routh-Hurwitz stability criteria for a polynomial with complex coefficients. So, this is the determinant should be minus of this should be greater than 0. So, if we compare these two equation, you can able to see that a naught is minus m and b naught is 0. So, like that we can able to compare these two equation and we can able to get these a and b coefficients. If you substitute this here from first determinant, we will get C H plus V is equal should be greater than 0. This is the one condition, second criteria is this one.

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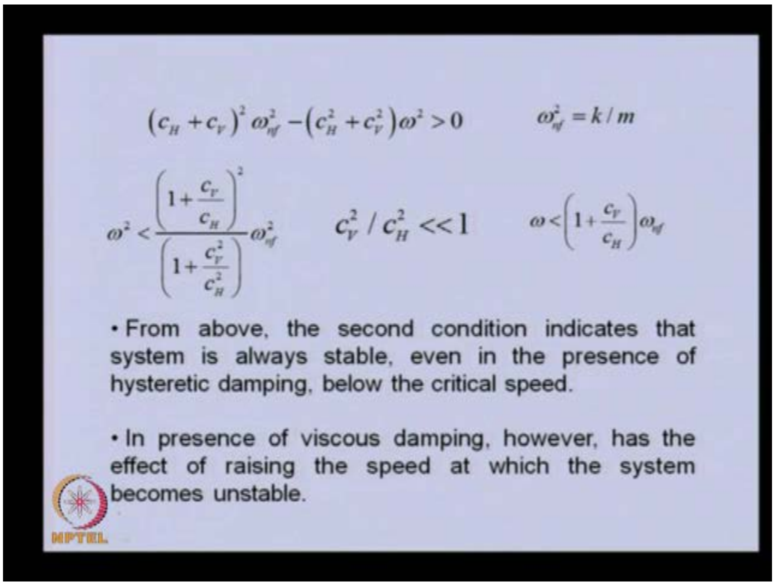

So, if we substitute various coefficients and if we simplify this, we will get expression like this in which we have defined the omega n f square as k by m. In turn, this can be written as omega square less than this quantity. Because this ratio is generally small, so this can be neglected. So, if we neglect this, we will get. Basically, if we take the square root the speed when it is less than this quantity, we will be having system stable. That means when there is no damping in the system.

So, when the speed is less than the natural frequency, undamped natural frequency of the system, we will be having stability. But, if there is a viscous damping in the system and this then the stability because this factor will be positive; so the total factor will be more than 1. So, we see that with the damping the stability the speed below which the system is stable increases. Especially if we increase the viscous damping in the system, we can able to increase the stability of the system by some amount.

So, from this, we can able to see that the system is always stable even in the presence of hysteretic damping below the critical speed. So, that we have already seen that if below the critical speed there is no instability and the presence of viscous damping. This one has the effect of raising the speed at which the system become unstable. So, if we can provide more viscous damping in the system, by in the form of the damper or the in the bearing or we can have this instability speed, we can able to raise up to certain level.

Now, through a simple example, we will see that how we can able to analyze by a numerical integration, this kind of instability. So basically, we will be integrating this kind of equation of motion for various speeds. We will try to see the response how it changes especially if you giving some small disturbance, how the response changes. We will be solving this for without any external excitation.

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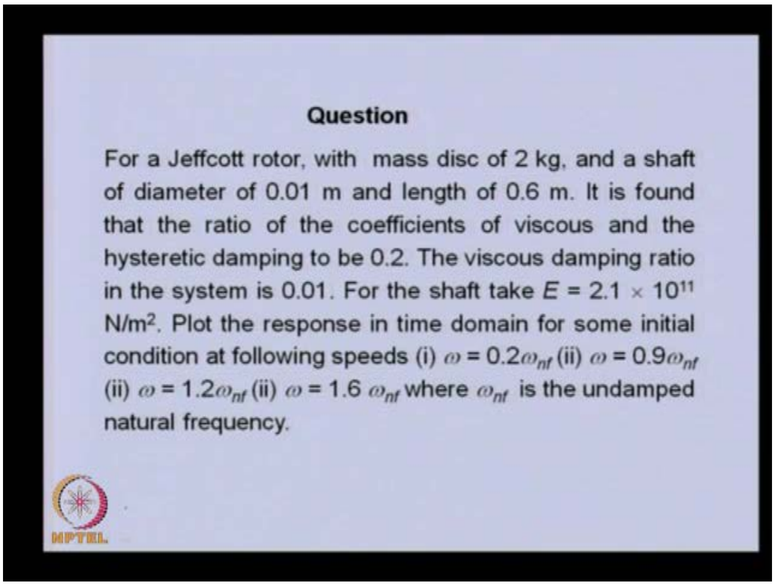

So, this is the problem in which we are considering a Jeffcott rotor of mass is given. Diameter of the shaft is given. Length is given. The viscous damping and hysteretic damping ratio let us say we are assuming as just 0.2 and viscous damping ratio of the system that zeta is 0.01. That is the damping ratio for the shaft material property like E is given here. Now, we want to obtain the response in time domain or the orbit plot for some initial condition at various speeds. This is the undamped natural frequency. So, at various speeds, we will be obtaining these plots and we will see how it behaves.

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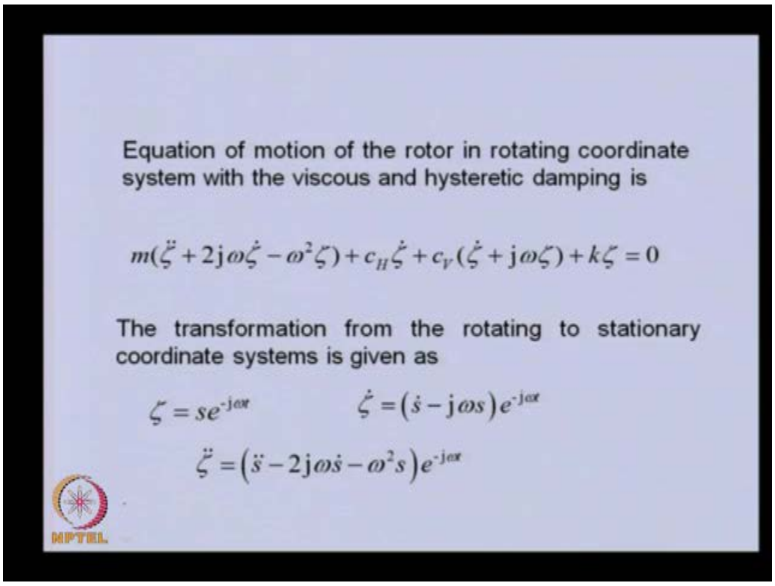

So, coming to the equation of motion in the rotating coordinate system; this was the equation of motion with a hysteretic damping. We choose where here the transformation from the rotating to the stationary coordinate system will be just opposite. So, if we take the derivative of this, we will get this. The second derivative will give this. If you substitute this in equation of motion, we will get back the equation of motion in the stationary coordinate system like this.

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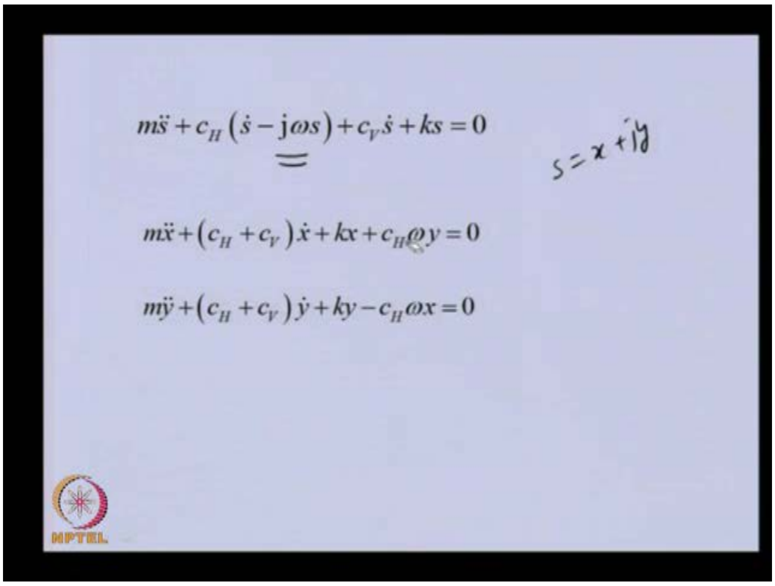

So, you can able to see. This is the additional term which is coming because of the hysteretic damping. Other terms were there in the equation of motion. Now, we can able to take the real part of this and imaginary part of this to get these two equations because we define the s is equal to x plus j y. So, this is the equation of motion in the x direction and this is in the y direction. Now, you can able to see there is hysteretic damping terms coming in these places. Not only it is coming here but here also it is coming. These two equations are now coupled because in x direction, y term is also coming because of internal damping. Also in this, we have x term in the y direction. So, to solve this equation by direct integration method, obviously we need to solve these two equations simultaneously.

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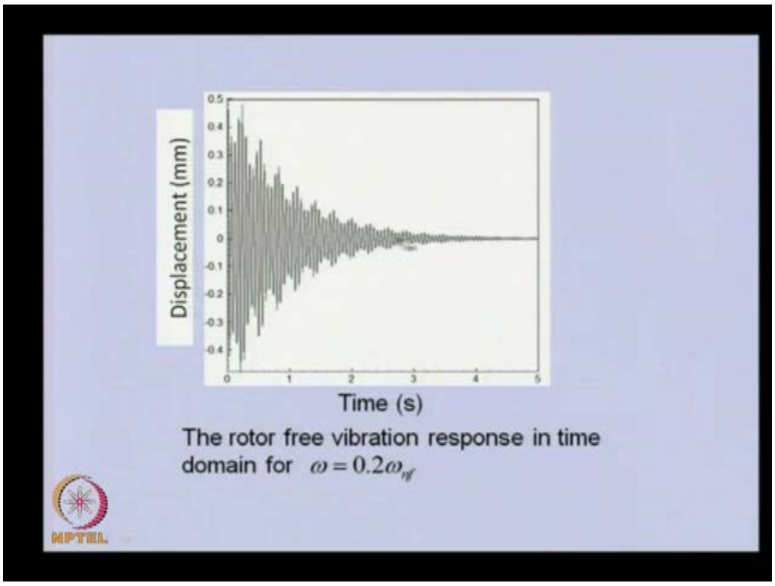

We can use any numerical integration method. This is the displacement with time when we are considering the speed is equal to 0.2 of the natural frequency of the system. Because there is no excitation in the system, so we have given some initial condition, so that once we have disturbed the system, we will see that it stabilizes after some time.

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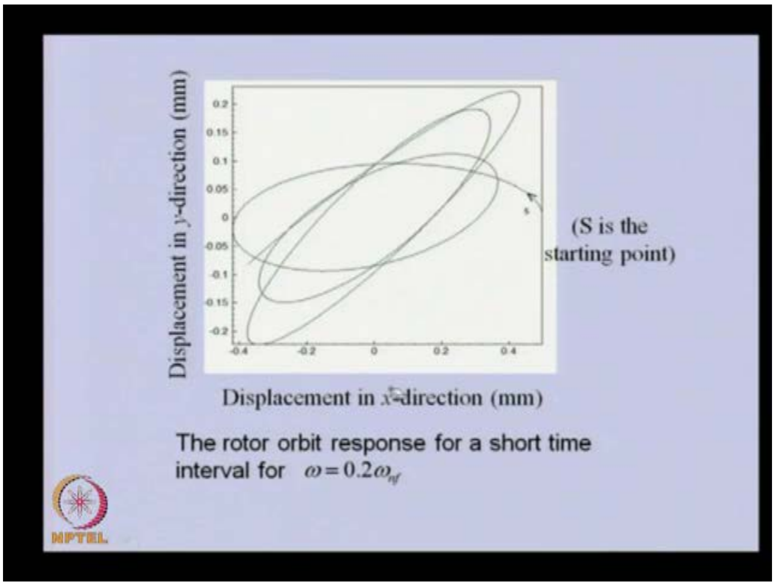

This is the orbit plot that means, x direction and y direction displacement plot with respect to time. So, we have given some initial disturbance. So, this is the initial condition. From here we have leave we are leaving the rotor. So, it is going like this. It is trying to stabilize. So, this is for very short duration of time.

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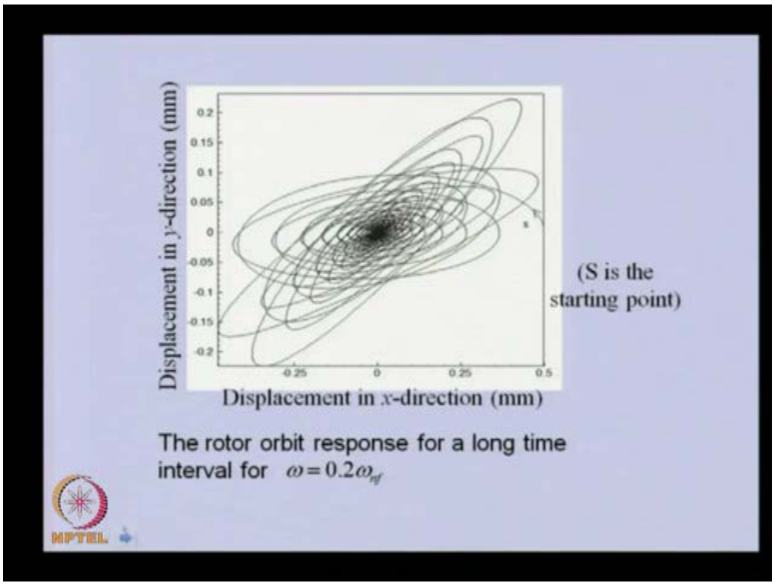

But, if we take more time, we can see that we started from here and gradually it is converging toward this point. So, because here clarity is not there, so initially we showed this for very short duration how to it goes. But, it after sometime it goes to here as we have seen in the very first slide also. After some time it goes to the stable solution. So, this is for one of the speed.

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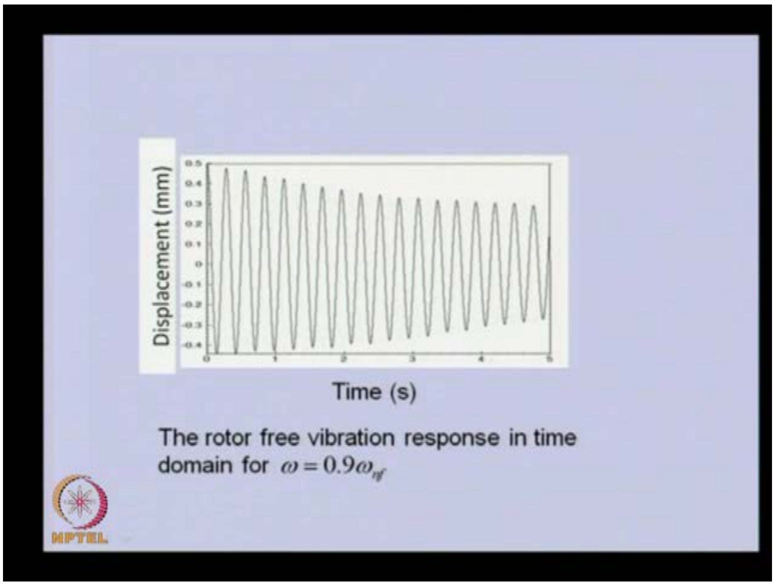

Now, when we are close to the natural frequency; this is for that given disturbance. This kind of here again it is going toward the lesser amplitude. So, it looks the system is stable.

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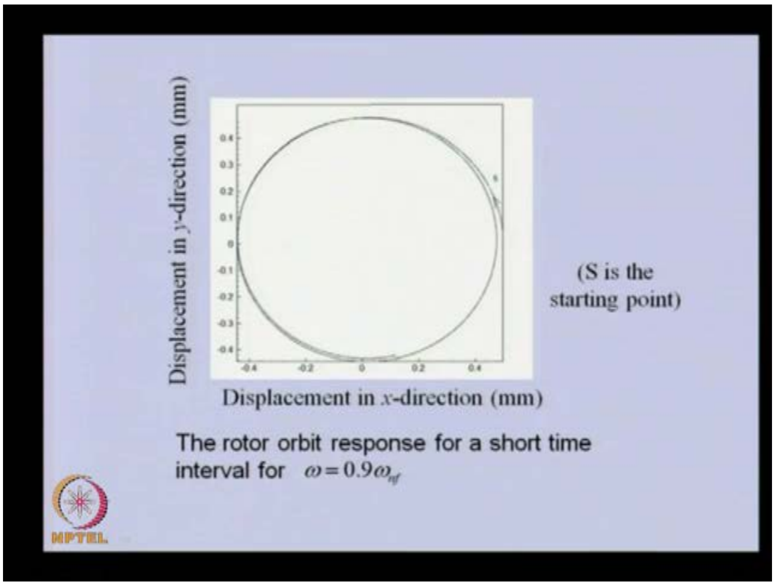

This is the orbit plot. So, this is for very short duration.

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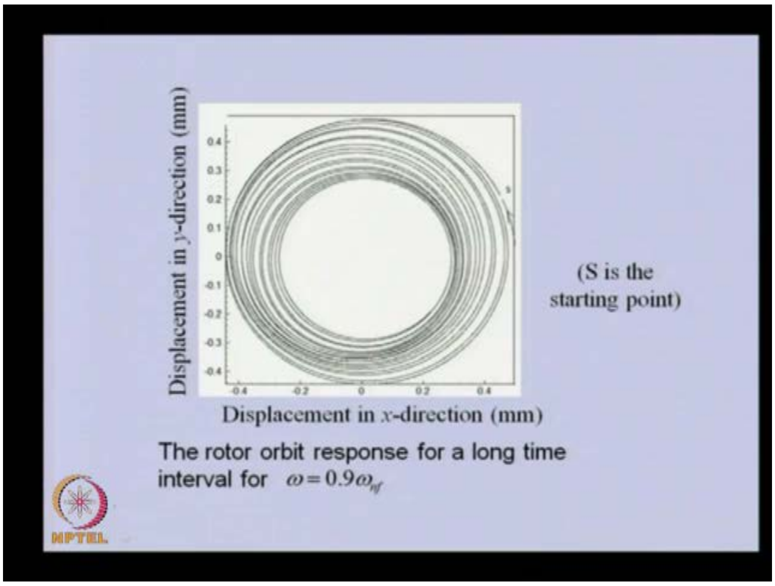

But, if you take long time then gradually you will see that it is going toward the stable zone. Similarly, you have different speed you are tried like 1.6 times the natural frequency. So, in this case also it stabilizes.

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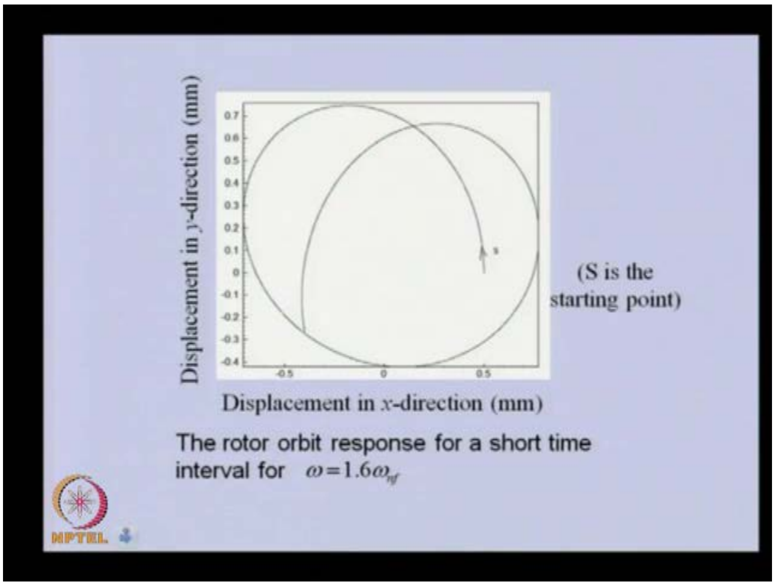

So, it is for short duration. This is the initial condition. Then it comes like this but for long duration.

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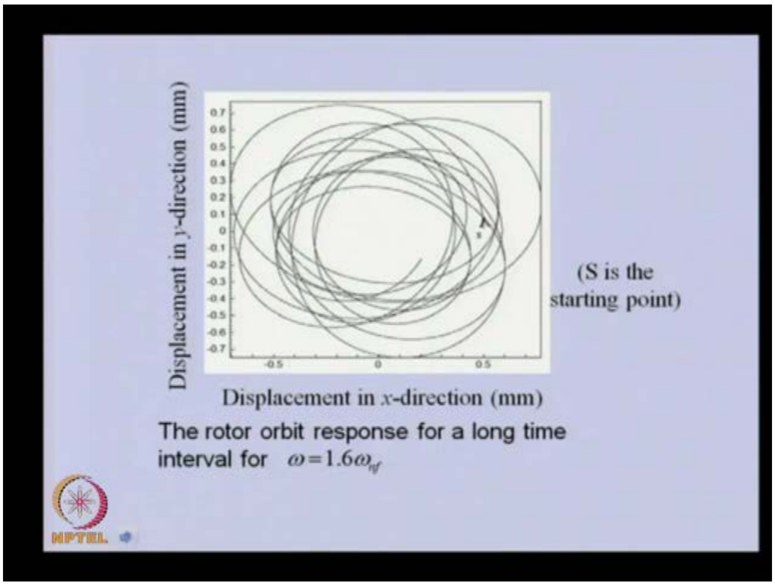

So, gradually it is converging to steady state solution. Then if we take omega is equal to 1.6 then we found that the system is unstable.

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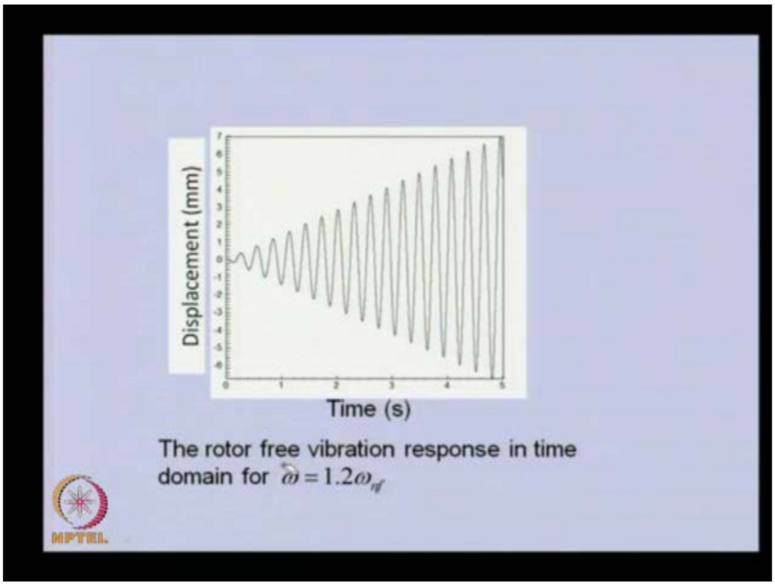

For a given initial disturbance, the amplitude grows with time.

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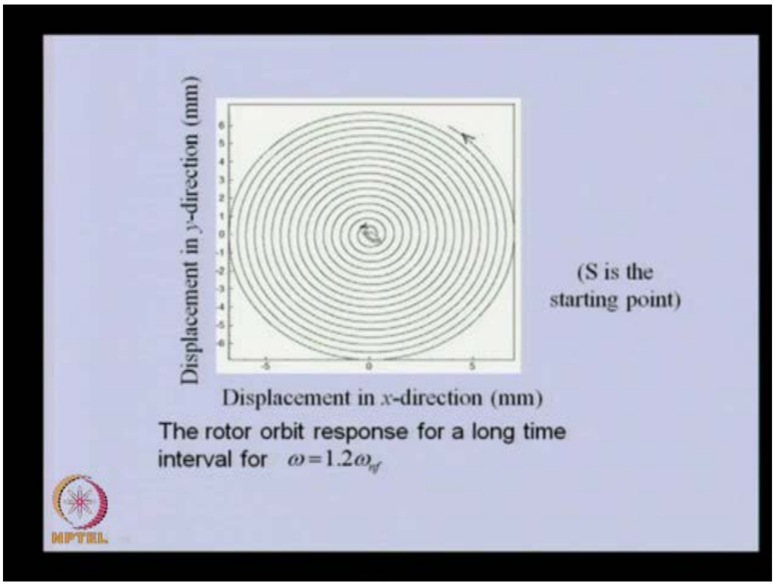

Here also, you can able to see it is gradually increasing. So, this is the orbit plot. So, this condition is giving us the instability when we are operating the rotor at 1.2 times this. So, we have seen that this particular instability is always occurs above the natural frequency of the system and not necessary at all speed above the natural frequency. But, at some of the speeds, this instability can occur that we have demonstrated here. Now, we will take another case in which the rotor is having the shaft is having asymmetry.

That means, it has shaft stiffness different in two principle direction. Generally, in rotors especially in generators, we find that we cut some kind of groves for providing the windings. Those are not symmetrically placed onto the rotor and because of that, we have two principle directions. The stiffness varies in this two principle direction and continuously. Basically, it varies when we rotate the rotor.

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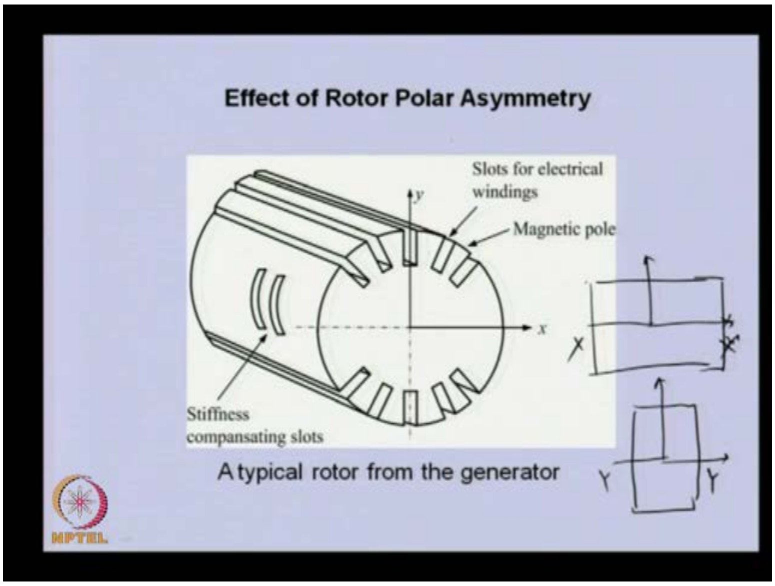

So, this is a one particular a typical rotor of the generator, in which you can see there has slots which are running axially. These are provided generally for providing windings in the generator. Here also it is, but some part of the rotor is solid. So, we can expect that in this particular case, the effective area of the shaft will be something like this because we have removed the material. So, it will be something like this.

We expect that now because of this effective area here that and here are different, so when rotor is having this orientation and this orientation. First case and second case the stiffness let us say; about one of the axis XX, this will be different as compared to this axis which is YY. So, here we expect the about this axis the stiffness will be less as compared to this one. So, we expect because of its own weight, the rotor will deflect more in this configuration than this configuration.

Generally to compensate the decrease in the stiffness, some kind of slots are provided in the solid part which is called stiffness compensating slots; so that we can able to reduce the stiffness in this direction also. But, even with that we have variation of the stiffness where the shaft rotates. So, you can able to see that it is having some different orientation; we will be having change in the stiffness. So, basically for such system, the stiffness changes with time. This particular behavior gives the instability into the rotor system.

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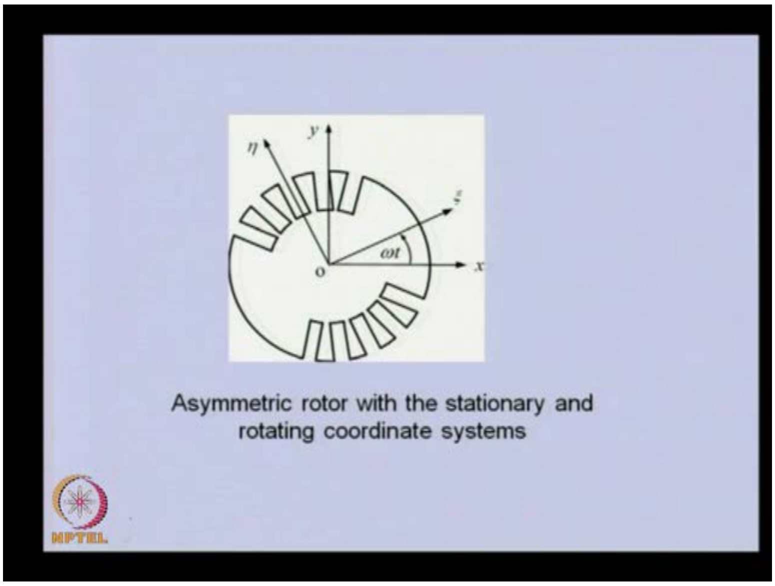

Now, let us see this. If we attach a rotating coordinate system along the two principle direction of the rotor and if we analyze the equation of motion in this rotating coordinate system then it will be convenient to analyze. So here also, we will be choosing the rotating coordinate system. We will be obtaining the equation of motion in the rotating coordinate system itself. In fact, we will be defining the stiffness of the shaft in these two directions in the rotating coordinate system itself.

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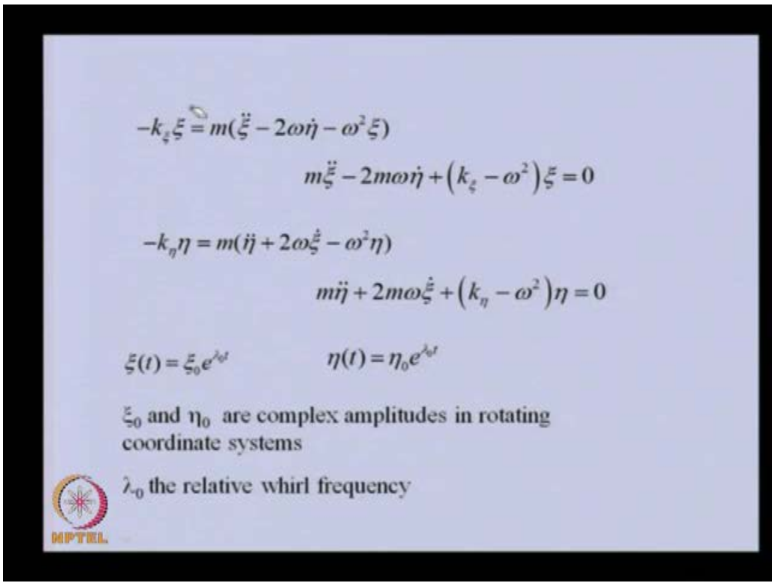

This is the equation of motion. So, basically this is m x double dot. We have seen that x and zeta, xi and eta, how they are related earlier. We have defined. So basically, this is the S double dot term. This, sorry, this is x double dot term. This one is the stiffness in the xi direction. If you multiply by the xi displacement, we will get the elastic force. So basically, this is the Newton's second law of motion, in which this is the sum of external force. In this particular case, only the stiffness force, we are considering is equal to the inertia force.

So basically, this whole term is x double dot. But, it has been transformed to the rotating coordinate system. This, we can able to write like this. This is the rotating coordinate system equation of motion. Similarly, in the eta direction, this is the force. This is the elastic, the shaft stiffness in the eta direction. Now, you can able to see. There is the stiffness in two directions, two principle directions are different; is eta xi and eta. This is basically y double dot transform into the rotating coordinate system.

So, this is the equation of motion in the rotating coordinate system in the eta direction. So, we have two equations; this and this in the rotating coordinate system. Now, we can able to assume the solution of this. This is the amplitude and the frequency part. Here, the lambda naught is again relative whirl frequency or the Eigen value of the problem. Eta naught and xi naught; they are the complex amplitude in rotating coordinate system. So, these equations which we assumed, we need to substitute in these two equations. So, for that, we need to take derivative of this with respect to single derivative and double derivative. So, those things if we do it, we will get the equation in this form.

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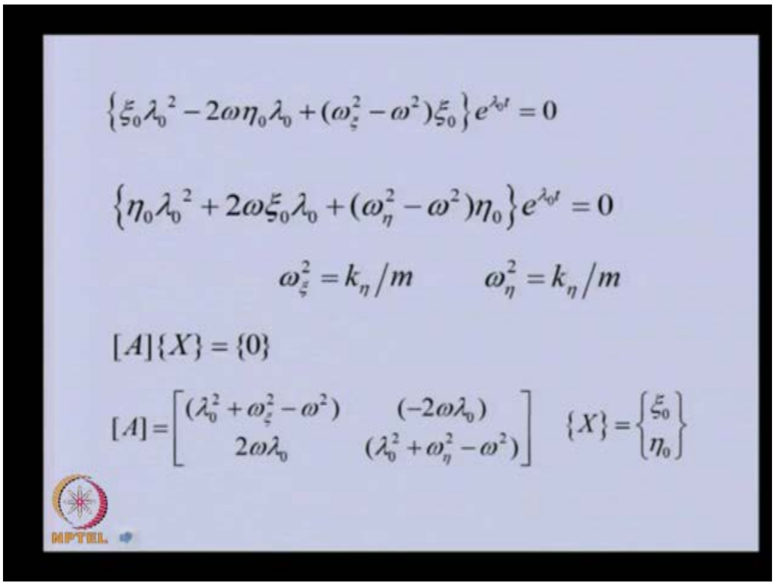

So because this will be common, so this can be taken out; similarly, in the second. So, this is in the xi direction and this is in the eta direction. So, these are the two equation of motion. Now, it has been converted into the basically frequency domain. They cannot be 0. So, this they can be eliminated. So, the remaining term, we can able to put in A in this form. So basically, we are writing this in a matrix form, in which that is xi naught and eta naught is this particular vector. All the coefficients of these equations are.

So, this is coefficient of the eta naught. In the first equation, this is zeta naught of first equation. This is for the eta naught. Similarly, this is for the zeta naught, the sorry, xi naught in the first equation and eta naught in this equation. So, basically, these two equations, I have put in a matrix form. This is a homogenous equation. For non trivial solution of this, the determinant of this matrix should be 0.

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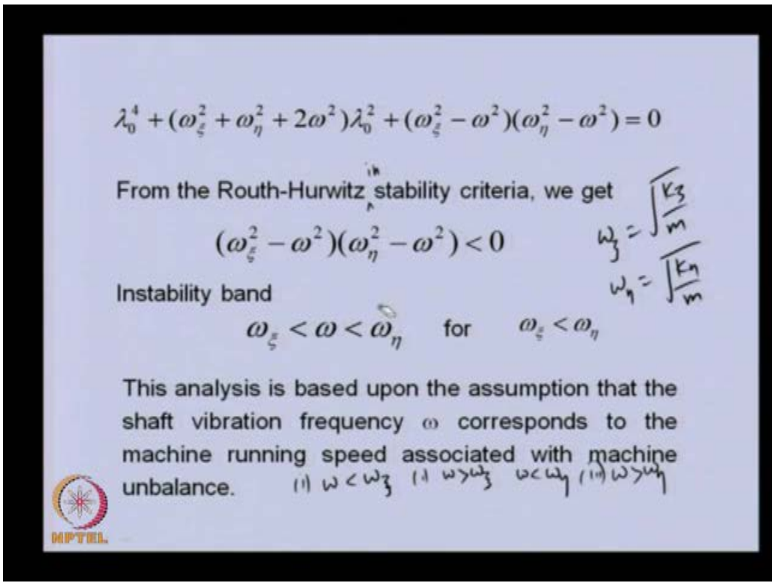

That gives us a polynomial of this form; quadratic polynomial in lambda naught square; in this particular case because this is a now polynomial with the real coefficients. So, we know Routh Hurwitz instability criteria. First criteria were that all the coefficients should have same sign. So, here we have 1. So, this is a positive. Here, these terms are all square terms. So, they will be positive. This term should be positive. Then only, we can have the stability.

So, if this is positive that means this should be basically, this is the instability. For instability, this should be 0. If it is below, if it is less than 0, that means the system this will be negative. That means system will be instable or unstable. So, this is the condition that for instability. We will be having this condition. Now, in this we have defined this lambda naught, sorry, omega xi S root of k x i by m. This terminology, it is a naught, an initial frequency.

Similarly, this is another terminology. So, we have defined like this. The stiffness is such that omega xi is less than omega eta. So, the choice of the coordinate system is such that that we have this condition let us say. So, that means this is less than this. So, in this particular case if we see, there are three conditions possible. When we have the speed, if first case is when speed is less than omega xi. So, if this is the case, we will see that this is less than this. So, this positive and this also be positive.

So, this will be this will naught be satisfied that means system will be stable. If this is satisfying, we will be getting the unstable condition. So, when both are positive, this one quantity is positive, so that is naught less than 0. So, we will be having stable solution. Second case; when omega is more than omega xi. But, omega is less than omega eta. In that case, this is more than this. So, this quantity is negative. But, this is less than this. So, it is positive. So, negative and positive becomes negative that means for this case the system will be unstable because now it is satisfying this condition. This is the condition.

So, when omega is in this range, we are finding that this is becoming negative. The third condition is when speed is greater than omega eta. Then this is positive. This is also positive. So, it is not satisfying this instability condition. So, that means only rotor will be unstable when it is operating between this two ranges. Otherwise it will be stable. So here, we have seen that basically we are getting a band of frequency ranges at which the system will be unstable, but beyond that band the system will be stable.

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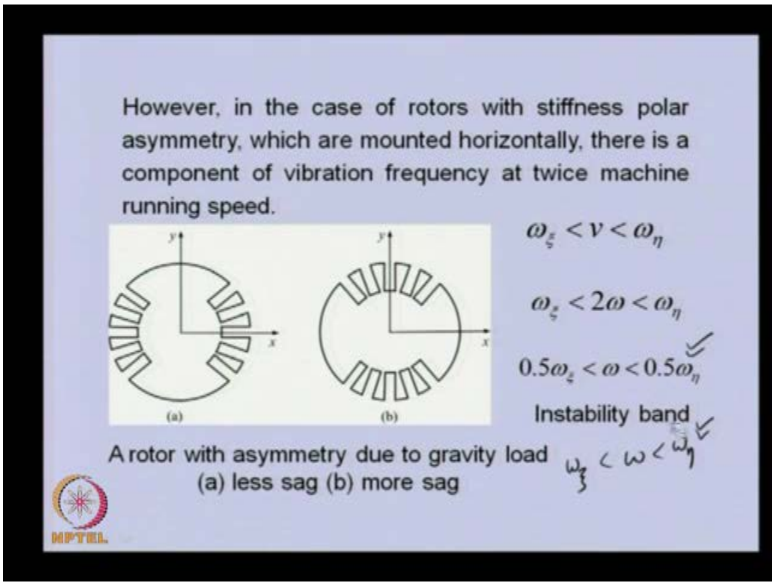

This particular analysis we did by assuming that whatever the vibrational frequency is there, that is same as the spin speed of the shaft. But, vibrational frequency can be different as compared to this spin speed of the shaft. For that particular case, the band which we obtained earlier; we took in this vibrational frequency is equal to spin speed of the shaft. But, if we generalize, so this vibrational frequency whenever is between this two bands, we will be having instability. For a case of such system in which we have asymmetrical shaft in two principle directions, we find that during one rotation, they will be twice the change at the stiffness would take place so that it is rotating.

We will find that the speed equal to twice of the speed. We will be having vibrational frequency because of this asymmetric shaft property. Because of that, this nu will be 2 omega because vibration frequency is twice for this case. Earlier case, when we took nu is equal to omega, that could be because of may be unbalance in the system; because of that, that kind of frequency of vibration can take place. But, this 2 omega will take place in this particular case when we have the symmetrical shaft.

So, from this we can able to see that we can divide throughout by 2. So, we will get this band also. When omega is between these two ranges, also we will be having instability zone. So, apart from the previous one which was this band, we have additional band in which we have unstable vibrations. That is due to asymmetric part of the shaft and because of that, the whirling frequency itself is twice the spin speed of the shaft. So basically, we are finding that this is one of the band in which a system will be unstable.

There will be another band on which a system will be unstable. We may find that these two bands may be independent of each other or sometime they may overlap depending upon the values of this omega. So, we will take up one example in which we will show this particular case; in which we have asymmetrical shaft how the instability zones can be obtained.

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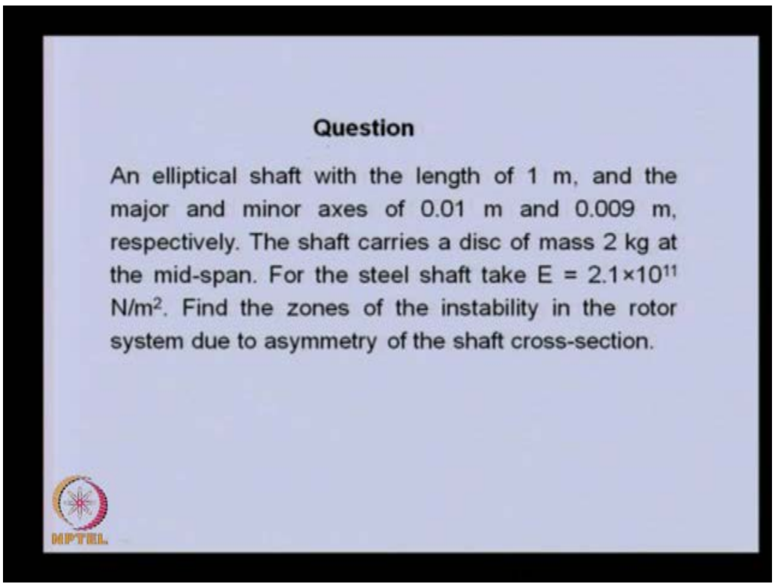

So, in this particular case, we have taking an elliptical shaft with a length of 1meter. The major and minor axes of the shafts are this. So, this may be due to the manufacturing defect. This kind of geometry we may get or sometimes may be the requirement of the system. The shaft carries a disc of this mass. The material property of the shaft is this. We need to find out the zone of instability in the rotor due to asymmetrical cross section of the shaft. Because of this asymmetrical cross section, we will be having asymmetrical stiffness of the shaft.

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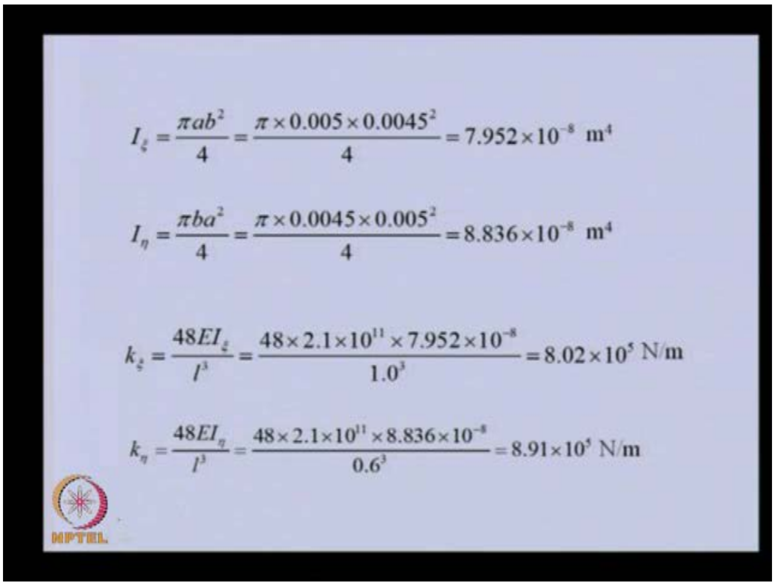

So, various geometrical property, we can able to obtain. So, this is the second moment of area for an elliptical shaft. We can able to get in two principle directions and with that we can able to get for the boundary condition of the problem. The stiffness in two directions, two principle directions, we can able to obtain. So, there is high variation because of the elliptical cross section in this two.

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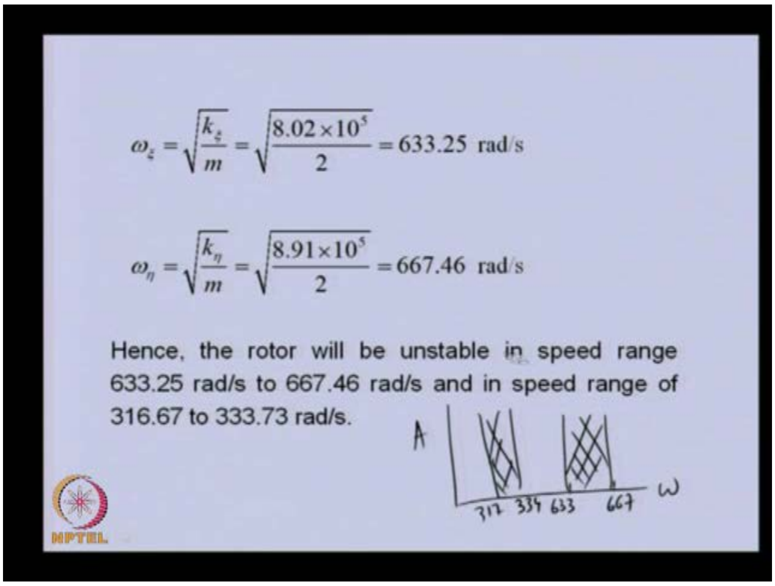

Now, this two parameter omega xi and omega eta can be obtained from as it has been defined. So, the range is this one. The omega xi is this, sin omega eta is this much. So now, as we analyze already that if the rotor is operating below this two speed range, we will be having unstable operation. So, one band of unstable operation will be from here to here. Another one because of the twice speed of the whirl frequency, due to the asymmetric nature of the shaft stiffness half of this.

That means, this will be additional zone where the system will be unstable. So, in the whole range, if we want the amplitude versus this, so we will be having two parameters this. So, this is 633 and this is 667. So, in this range system will be unstable. Similarly, we will be having another band, so 317 and 334. This will be another band, in which the system will be unstable.

So, and if we are operating in this region or in this region or above this, you expect this system will be stable. In the previous lecture, we did a instability analysis of fluid film bearing that was linearized case. If we want to consider the non-linear bearing fluid film fluid film forces then we need to use the Reynold equation to obtain that.

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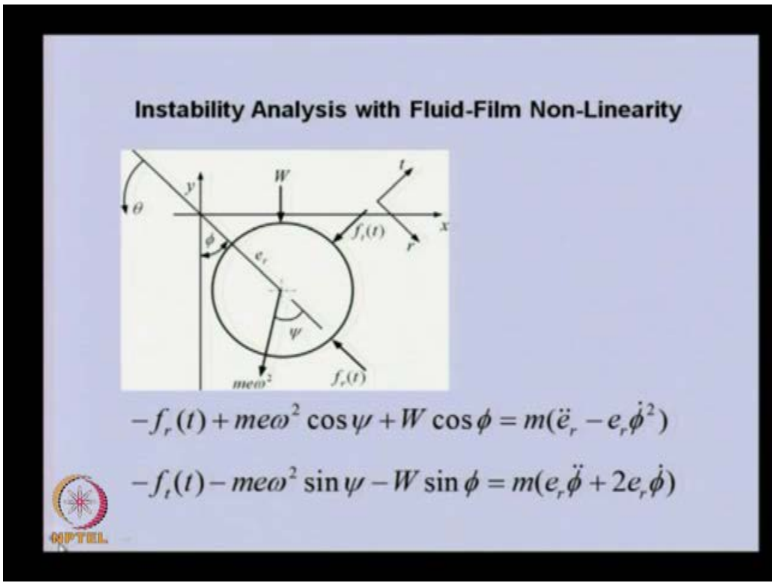

So, here briefly I am outlining how the instability analysis can be done for fluid film force. If non-linearity we want to consider so in this particular case, this is the rotor. In this, we have fluid film forces in two directions; radial direction and the tangential direction. Weight of the journal is here. This could be unbalance force which is rotating with some speed. The position of the rotor is given as eccentricity and the altitude angle.

Now, if we want to write the equation of motion of this; obviously, we can able to balance the force in the radial direction and tangential direction. So, this is the force balances the radial direction. So, this is the fluid film force in the radial direction. This is the unbalanced force. Component of this in the radial direction will be cos x i. Then weight, component of that in the radial direction because that angle is phi; cos phi is equal to the mass and the radial acceleration.

So, this is due to the radial motion and this is due to the angular motion. So, this is the radial acceleration. This is centripetal acceleration. Similarly, we can able to write the force balance in the tangential direction. So, this is the fluid film force in tangential direction. This is the unbalance force. This is the weight component. This is the mass and the acceleration in the tangential direction.

So, this is due to the rotation. This is basically; this is cartelized component of acceleration because of radial motion and the angular velocity. So, this is equation of motion. Only thing is this radial force and the tangential force from the fluid film need to obtain from the pressure of the fluid like this.

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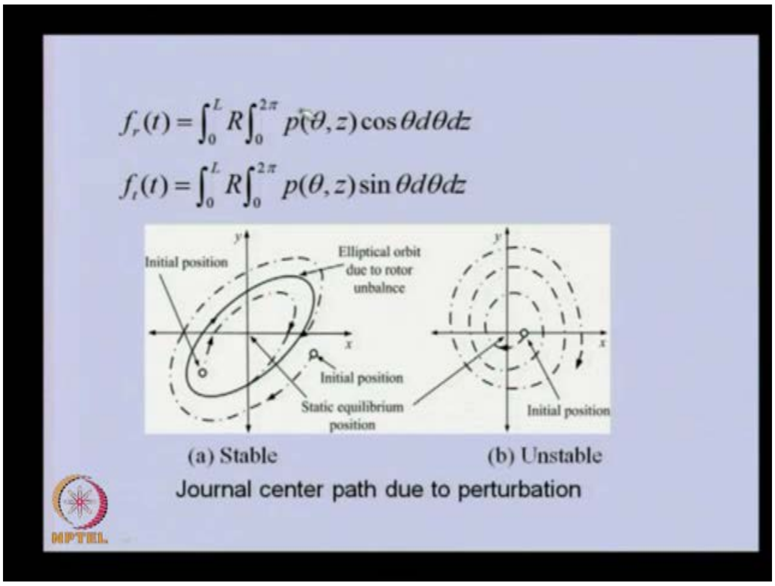

So, over the inside the surface of the bearing from 0 to 2 pi angle and from the length of the bearing, we need to integrate this pressure, which we can obtain. This pressure, we can able to obtain from the Reynold equation, so component of these pressures in the radial direction.

This cos theta we are measuring from this. So, this will give the force in the radial direction and then in the tangential direction. This equation of motion can be integrated with respect to time for e r, the eccentricity pressure, which is time dependent and from the altitude angle. We can plot let us say, eccentricity ratio. So, if for stable system, if we had an initial condition is somewhere here.

After sometime, it stabilizes to this solid line. We can call the system is stable or even if we disturbing the rotor inside of this orbit to come to the solid elliptical path. Then the system will be stable for whatever the initial condition. The system will stabilize. For unstable case, the response v r will increase like this. This kind of thing was having seen in the case of hysteretic damping also.

So, by direct integration of the response; the differential equation to get the response will give us this kind of whether the system is stable or unstable. So, this is another way by which we can able to obtain the instability of the system especially when the fluid film forces are non-linear in nature. So, in today's lecture we considered two three aspects. Initially, we started with the internal damping. We saw that how the internal damping can provide the instability in the rotor system; especially we saw the role of viscous damping when the internal damping is also there.

The internal viscous damping basically stabilizes the system. So, if we are providing more viscous damping the system, the speed of threshold stability threshold can be increased. In another, when we consider the shaft asymmetry, we found that naught a single speed. But, a band of speed at which the rotor can become unstable. That band of speed below or above those bands in the rotor will be stable. But, that kind of bands may be there at other reasons. Also as we have seen, when we considered the two, twice the speed of the rotor; if some vibrational components are there.

So, a certain band of unstable zone can also be there. Apart from that, we have seen that, if we want to consider the non-linearity of the fluid film bearing how we can can able to obtain the system stability. Basically, we need to in time integrate the equation of motion. If the response is increases continuously for a particular disturbance, we can able to conclude that the system is stable unstable.

If it is stabilizing to a particular orbit then we can able to say the system is stable. In the next class, we will take up some more kind of instability, in which we can have the rotor can go into the unbounded response. That means the system may become unstable because of other kind of. In the next class, we will see some other sources of instability. We will try to analyze; how these instability can give some kind of unbounded response to the rotor system.