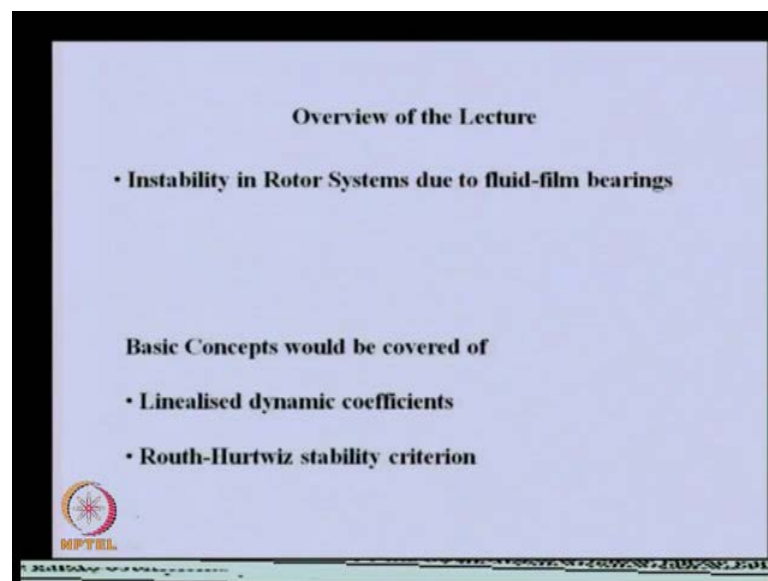


Theory and Practice of Rotor Dynamics
Prof. Rajiv Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 7
Instability in Rotor Systems
Lecture - 33
Fluid-Film Bearings

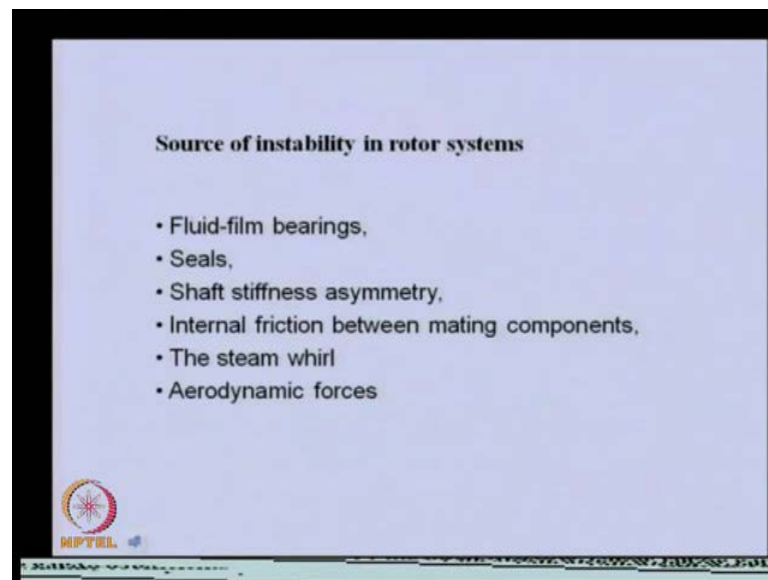
In previous lecture we studied how we can able to obtain fluid film bearing linearized stiffness and damping coefficient. These coefficients play important role in stability of the rotor system. So, today we will see how we can able to find out if we know these coefficients, dynamic coefficients of the bearing. How we can able to find out the instability of the system; that means through some criteria we can able to pinpoint that the system is stable or not.

(Refer Slide Time: 00:59)



So, in this particular case let us see, what are the things we will be covering? So, basically we will be covering the instability in rotor bearing system due to fluid film bearing. How we can able to obtain whether the system is stable or not? Apart from this we will introduce basic concepts, this we already linearized dynamic coefficients we introduced. So, we will be using this for the, for finding the stability of the rotor system. This particular criteria Routh-Hurtwiz criteria stability we will be using it for this purpose before going to the stability aspect of the fluid film bearing.

(Refer Slide Time: 01:43)

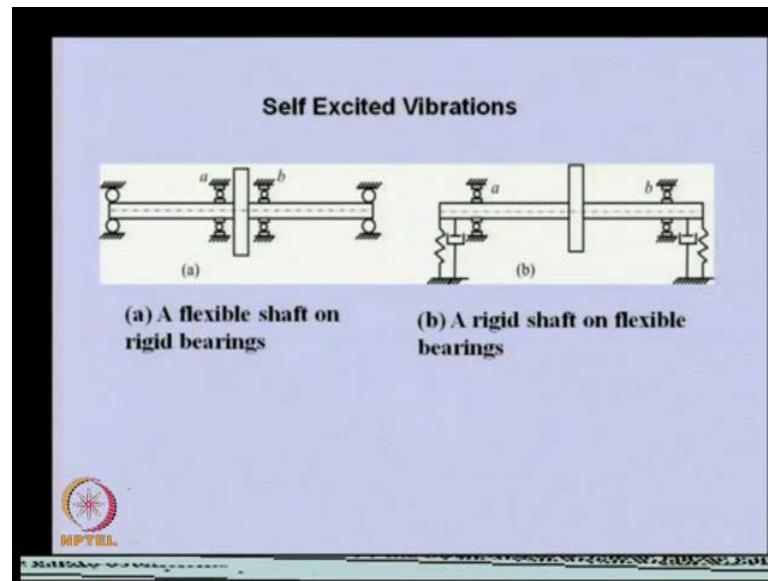


Let us see in rotor bearing system what are those various source of instability we can have. So, most common is the fluid film bearing. We know that in the forced vibration the most common force is the unbalanced force. And in instability the most common source is the fluid film bearings. Apart from that seals are there. They also give instability especially at high speed rotor applications.

Shaft stiffness asymmetry, this also gives instability in the rotor system. Apart from this internal friction or ((Refer Time: 02:29)) damping or you're like even the damping between two mating components in a rotor can give instability. This instability with simple model of rotor we will try to investigate in detail in subsequent lectures. Another important instability is the steam whirl. This particular instability is mainly occurs in high pressure turbines.

Generally, this steam whirl limits the how much power we can able to generate from turbine because this particular phenomena starts when the turbines are running very high pressure. Apart from that aero dynamical forces which comes on to the blades gives instability. So, these instability one by one we will be studying. In today's lecture we will fluid film bearing instability.

(Refer Slide Time: 03:36)



So, before going to the instability let us understand, what is self excited vibration? We know that the excitation can come from external source and that is we call it as unbalance or any other kind of a force, forced vibration system or self excited vibration is something which, in which the force comes from the motion of the system itself. So, to understand this let us take a rotor system like this. So, this is a simple rotor, Jeffcott rotor and in this there is a disc at the center which is having some unbalance.

If we rotate this particular rotor then it will start whirling, if let us say these bearings are not there then it will start whirling because of the unbalance. Now, if we want to stop the motion of this particular disc by providing this kind of two bearings very close to the disc, if we provide this. Obviously, the motion of this particular disc will be arrested. This will not be having any vibration because now this particular rolling bearing is not allowing to have whirling of this particular disc, but because the unbalance is rotating because the shaft is rotating the force will still be there in the system.

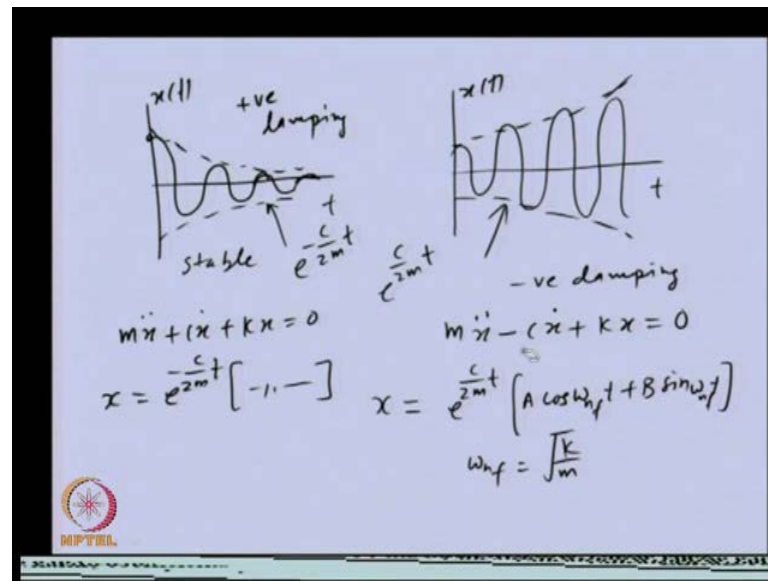
The unbalance force will be there, that will not diminish because of stopping the motion. In other case the same rotor which is now let us say perfectly balance rotor is there, it is mounted on fluid film bearing. Let us say at present initially these two whirling bearings are not there. So, only it is mounted on two fluid film bearing and rotor is balanced perfectly. Now, if let us say the property of the bearings are such that the system is under unstable region. That means at that particular speed it is having large oscillations.

Now, and the, there is even because there is no unbalance, no external force the system is getting excited because of self excited vibration from the parameter of the bearing itself because this is instable in the unstable zone. Now, if we try to arrest the motion of the, this particular shaft at the ends by providing a rolling element bearing and fixing that particular bearing with some kind of stator here. So, we are preventing the motion of the shaft ends by providing these two bearings.

Then we will see that when we are stopping the motion of the shaft ends, the whatever the self excited force was generating earlier that will also diminish, that will or dies down. So, that will not be pleasant. So, you can able to see that this self excited vibration due to the fluid film bearing will take place only when the motion of the shaft ends are there. If we arrest the motion of the shaft, the self excited vibration will automatically it will die, die down. So, now it is very clear, what is the difference between the self excited vibration and the external exaltation? So, in this particular case we have seen that the self exaltation vibration will, in fact the motion will generate this particular kind of vibration. Now, let us see how we can able to analyze this particular case.

We have seen that how the self excited vibration generates and we have seen that when the motion take place then only this particular exaltation can be there in the system. And let us try to analyze this particular system in terms of the positive damping and negative damping in the system. So, when there is a positive damping in the system we know that if the system is perfectly balanced, if we disturb the system by some small amount in a form of either giving a small displacement or small tap, the system will come to its steady state position after some time.

(Refer Slide Time: 08:55)



So, that means if we have let us say some disturbance of the system at t time is equal to 0, we expect if damping is positive the system response will gradually dies down than this. So, this is a stable system. Now, if we have a negative damping. In negative damping what happens? Generally, in positive damping at every cycle some energy of the system is dissipated as in the form of energy and the system energy is getting reduced. So, we have amplitude which is gradually decreasing with time.

In the negative damping case, this is a positive damping. In negative damping case we expect this opposite of the positive damping that means at every cycle we expect that there will be increase in the energy of the system. So, whatever the disturbance if we give after sometime we will find that the amplitude will grow like this and for linear system we expect this amplitude will become infinity after sometime. So, this is, this in the equation of motion for this we can able to write it as for negative damping case like this. And since free vibration response of such is given as e raise to c by $2m$ t . This is the exponential part and then the harmonic part is this one where ω_n if it is nothing but $\sqrt{k/m}$ ((Refer Time: 11:11)) frequency.

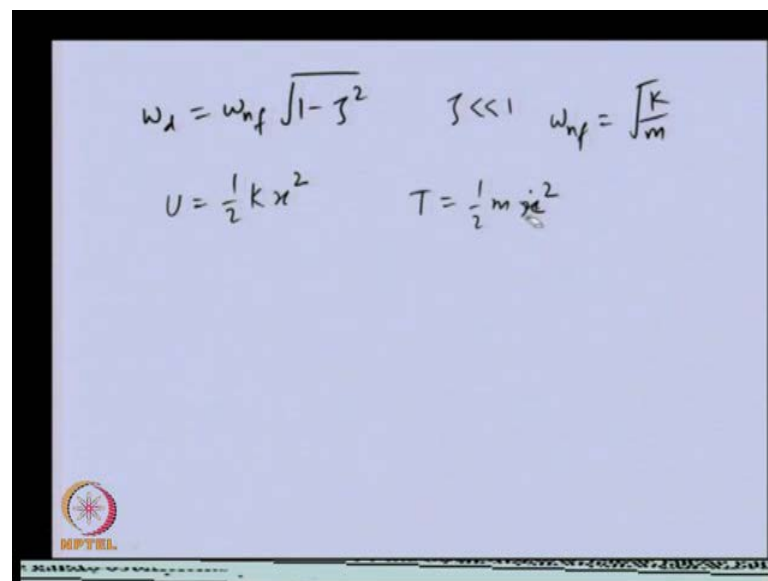
So, you can able to see if this when this is negative the exponential term is exponentially it will be increasing. So, in this particular curve this will be the, that particular plot in which, this is the exponential function and for the stable system we have a equation of motion is like this. And in this case x will be minus c by $2m$ t and this term is exactly

same as this one. So, you can able to see here is minus. So, we have exponentially decaying curve for this case.

So, this is the exponential, but decaying because of the negative term in the here. So, we can able to see that how the negative damping gives instability into the system. And this particular damping we know that if motion stops there is no damping force. When there is no motion obviously, the damping force will not be present because velocity will be 0. So, this particular concept of negative damping is similar, is identical to the previous concept in which we described the self excited vibration that if motion stops there is no self excited vibration.

So, in this particular case also you can able to see with this concept also the same is true, when there is no motion the self excited vibration will not be there. So, by two different concept we have seen the self excited vibration concept. Now, let us see at what particular frequency this self excited vibration will take place because in this particular case the, we assume that there is no external force in the system. Whatever the vibration is taking place in the self excited condition is due to the internal forces. So, we expect the system will be having the frequency same as the whirl frequency of the system or damp natural frequency of the system.

(Refer Slide Time: 14:11)



The slide contains the following handwritten equations:

$$\omega_d = \omega_{nf} \sqrt{1 - \zeta^2} \quad \zeta \ll 1 \quad \omega_{nf} = \sqrt{\frac{k}{m}}$$
$$U = \frac{1}{2} k x^2 \quad T = \frac{1}{2} m \dot{x}^2$$

In the bottom left corner, there is a logo for NPTEL (National Programme on Technology Enhanced Learning) featuring a stylized sun or flower icon and the text "NPTEL".

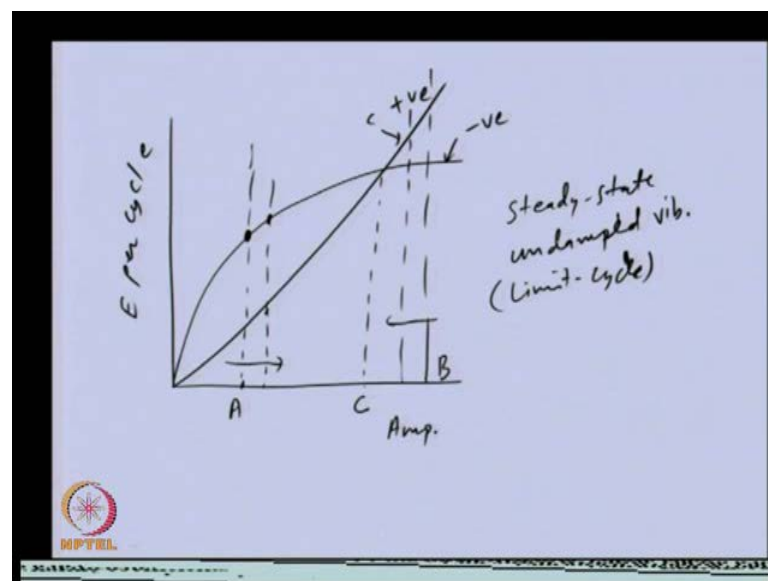
And we can able to see that the damp natural frequency we have related with the undamped one earlier like this. So, we have this is the damping ratio. So, if damping is

there either positive or negative because this is square term the damp natural frequency will be always decreasing. So, we can able to see that the whirl frequency during self excited vibration will be the damp natural frequency and in this particular case zeta is very small generally. So, we expect that whatever the, that whirl damping is there, their damped natural frequency is there that will be very close to the undamped natural frequency of the system which we defined as $\sqrt{K/m}$.

So, in this particular case let us see in the terms of energy when we have positive damping in the system we expect that the recycle the energy of the system decreases. If we disturb the system, recycle the energy of the system will decrease and it will after sometime it will come to this steady state condition. In the negative damping we expect the recycle the energy of the system will increase. So, gradually the amplitude will grow in this particular case. So, energy is also growing.

So, amplitude will also grow because energy is something like square of the amplitude like in the case of we have strain energy. So, strain energy is square of the displacement or kinetic energy that is square of the velocity. So, basically because this for simple harmonic motion we can able to write again in the form of displacement. So, energies are nothing but something like square of the amplitude of the vibration. So, we expect that the energy if you want to plot with respect to the amplitude that will be a parabolic in nature.

(Refer Slide Time: 16:39)



So, let us see how we can able to plot let us say amplitude is there and this is the energy per cycle. This is energy per cycle. For negative damping we will be having parabola like this, for positive damping, because in this system this negative damping and positive damping exist simultaneously and independent of each other. So, this is corresponding to negative damping and this is corresponding to positive damping. If your amplitude is at A let us say. So, we can able to see that if amplitude is relatively small and at here the energy here is more than the energy which is dissipated.

So, this is the energy input in one cycle and this is the energy dissipated in one cycle. So, we have surplus energy from here to here and because of this we expect that the amplitude will grow. So, amplitude will come somewhere here. So, here also the energy input per cycle is from here to here and energy dissipated is this much still surplus of energy is there. So, again we expect the energy, the amplitude will grow toward this side. Now, let us say this particular plot I am drawing again.

So, let us say we are somewhere here. So, in this particular case we can able to see that this is the energy which is getting inside the system, but this is the energy from here to here which is getting into the system, out of the system. So, the dissipation of the energy is in this particular case is more because positive damping, dissipate the energy. So, from here to here the energy is getting dissipated, that is more. So, this much energy is getting dissipated in this particular cycle.

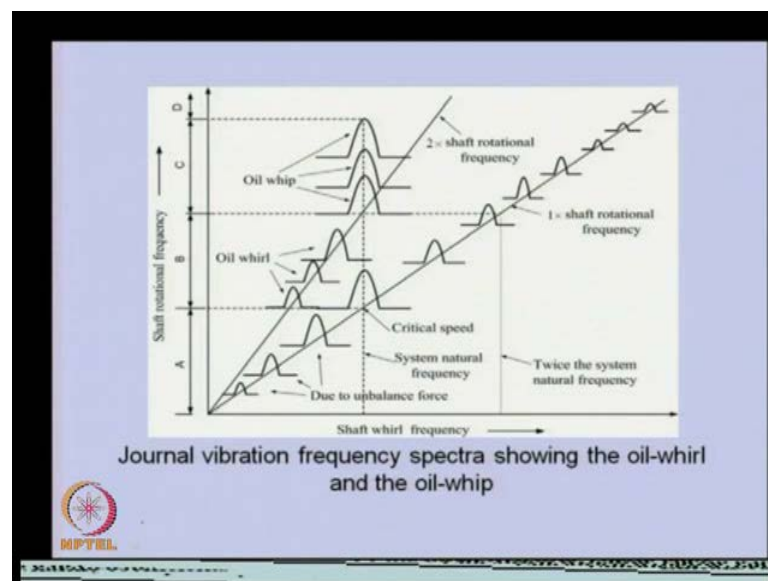
So, we expect that the amplitude which is A, B here will go toward the, this side. So, here amplitude is less, but still the energy of dissipation is more than the energy of input to the system here. So, again it will go toward this side. So, if the tendency of the system either from this side is going toward this or if we are here is going towards this and this is the position where these two energies are same. The intersection of the, these two curves.

So, here whatever the energy is getting into the system is equal to which is dissipated from the system. So, net damping in this particular amplitude will be effectively 0 because the net energy is 0, into the system is 0. So, at this particular case the system will be having a constant amplitude, constant amplitude. So, this is something like steady state vibration will take place. The amplitude will be having some value and sometime in non-linear vibration it is called limit cycle.

So, you can able to see there is no unbalance in the system, there is no force in the system, but still this particular system is having some finite amplitude and that is the system is having vibrations at that particular amplitude. In this particular case what, whatever we have studied because we for negative damping we set the energy is going into the system and we are saying there is no unbalance force or there is no external force.

But there has to be some source of energy from where the system can extract the energy. So, the energy can be in the form of pressure of the water or steam or may be wind pressure or any such source of energy should be there from where the system can extract the energy. There may not be any excitation force frequency, but only the energy should be there in the form, so that the system can take energy from there. So, that it can have a self excited vibration. Now, we will ah study two very important phenomena in the fluid film bearing which take place because of instability. One is the oil whirl and another is oil whip. They are somewhat related. So, let us see how they take place.

(Refer Slide Time: 22:35)



So, in a Campbell diagram I am showing this particular phenomena. So, the horizontal x is the shaft whirl frequency and the vertical axis is the shaft in the spin speed or the rotational speed at when we are starting the rotor at low speed in the zone A. In this particular case we can able to see this is one x line that means in, along this line we have

the frequency of speed and the whirls are same. Basically, this is synchronized whirl condition that is $1 \times$ of the rotational frequency, shaft rotational frequency.

So, if there is unbalance in the system we expect those amplitude will be present in this particular region. This line is twice of the shaft rotational speed. Generally, if rotor is mounted on fluid film bearing we do not expect any amplitude along this line. The vibration frequency will not be having the frequency equal to twice the rotational frequency in this region. So, only unbalance amplitude will be present and when we reach here, this is the first, this is the natural frequency of the system.

So, when the speed is equal to the natural frequency of the system this is the resonance condition. This is the critical speed. So, we expect very large amplitude of vibration here, which is equal to the natural frequency of the system, and because rotor is rotating at the same speed. So, we have at $1 \times$ rotor shear frequency this is amplitude, but if we see in the $2 \times$ of the shaft rotational frequency, there also there is a small amplitude of vibration.

If we increase the speed further this whirl frequency, this is a, at half the, at the, this particular speed they are having these amplitudes. As we see in the $1 \times$ line the amplitude we expect will be decreasing gradually. If you increase the speed we, because the critical speed is here; if we increase further speed we expect the amplitude corresponding to $1 \times$ will be decreasing gradually. But in the, in this particular case we can able to see these amplitudes are growing and their frequency is half the speed of the rotor.

So, rotor is at the critical speed, but the frequency of the whirl is at the half of the whirl frequency. And as we are going that is always corresponding to the half of the whirl frequency it is taking place, if we increase the speed up to the twice the natural frequency. So, this particular point is twice the natural frequency. Natural frequency is this, twice of this here. So, the rotor is rotating now at twice the rotor speed and in this particular speed we do not expect much vibration in the $1 \times$ component, but here we have very large amplitude of vibration.

Let me define this amplitude. These are called oil whirl. These amplitudes are relatively smaller. Oil whirls phenomena are not so dangerous as compared to this particular condition in which we are rotating the rotor at twice the speed of the critical speed and because of that the, at this position the whirling frequency is equal to the natural

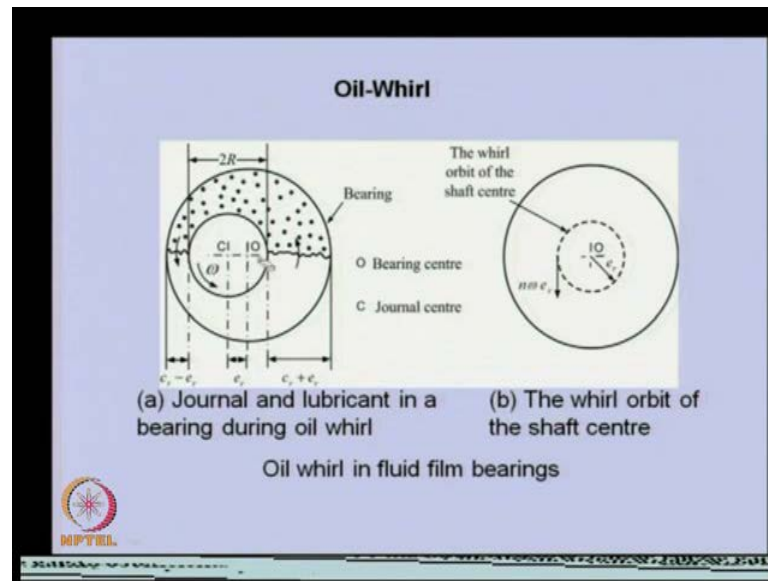
frequency of the system. You can able to see this whirling frequency is equal to natural frequency of the system and this is more dangerous.

And even if we increase the speed of the rotor further, the oil whip, the frequency that locked to the natural frequency of the system here it will not follow this particular line, but it will lock to the value of the natural frequency. And this is the oil whip which is far more dangerous condition in which catastrophic failure of the rotor bearing system can take place. Now, these two phenomena's are very clear. Basically, oil whip is a very special condition of the oil whirl itself.

At oil whirl basically we are rotating the rotor around the twice of the critical speed and because of that the frequency of whirl is equal to the natural frequency of the system itself and this is more dangerous, that itself more dangerous instability conditions. This particular phenomena of oil whirl and oil whip is basically non-linear in nature. And we need to, we can able to analyze them only through the non-linear analysis through the Reynold equation.

In the subsequent slides I will be showing how we can able to get these amplitudes with the help of the pressure variations in the fluid film bearing. So, some glimpse of the procedure I will be showing so that if we need to analyze this system how the Reynolds equation can help us in finding out the, this particular phenomena. Now, through very simple linear analysis we will try to understand the oil whirl condition and this, in this particular case we expect that the system should be very light and it should be rotating at relatively slow speed.

(Refer Slide Time: 29:09)



And in this particular case let us say, so we are considering this particular rotor bearing system. As I told the speed should be relatively low and the weight of the rotor should be smaller. So, in this particular case we expect the rotor will be having very low eccentricity. So, let us consider this a rotor which is inside the bearing. In between this there is a fluid and the fluid is having motion because of the motion of the shaft. We are not considering the variation of the pressure due to the hydro dynamic action.

Only, we are considering, we are assuming that the motion of the fluid is taking place because of the motion rotation of the shaft. So, basically the shaft is whirling in this position. So, this is the center of the shaft which is whirling like this. So, in this particular configuration the, because now we can able to see this is the eccentricity at the shaft e_r and the spin of the shaft is ω . So, that tangential velocity of the rotor in this particular position that means at the center of the shaft, the tangential velocity of that will be $m \omega e_r$.

So, in this particular configuration that velocity will be toward the downward direction. This particular circle that is a, the whirl orbit of the shaft center, we are analyzing here. So, you can able to see during the there are two motions of the rotor. One is spinning about its own axis, and then it is whirling in this direction and because this is spinning. So, the velocity of the fluid here will be same as the rotor velocity. Here it will be 0

because there is, this particular bearing is stationary and in between we can assume that the variation of the velocity of the fluid is linear.

Similarly, here also, here velocity will be equal to the velocity of the rotor and here it will be 0. And in between we can assume this is linearly varying because in this particular case rotor is rotating anti clockwise direction. So, we expect that the fluid will come from this portion in the downward direction and it will go up toward the upward direction. Now, because this O is the bearing center, C is the center of the shaft, it is slightly offset and clearance is C_r , total clearance.

So, that means when the C and O are at the same place we have clearance in both sides they are equal. That is equal to C_r , but now we have displaced the rotor toward this side by e_r . So, we expect that this length will be C_r plus e_r and this length will be C_r minus e_r . Now, we are trying to calculate, what is the volume of the fluid which is going here and here? And how much the volume of the space which this particular shaft will be creating here because of the motion in the downward direction and we will be equating these two volumes, three volumes.

(Refer Slide Time: 32:46)

$$Q_{in} = \frac{1}{2} (R\omega + 0)(c_r + e_r)$$

$$Q_{out} = \frac{1}{2} (R\omega + 0)(c_r - e_r)$$

$$Q_{vol} = (n\omega e_r)(2R)$$

$$Q_{in} - Q_{out} = Q_{vol}$$

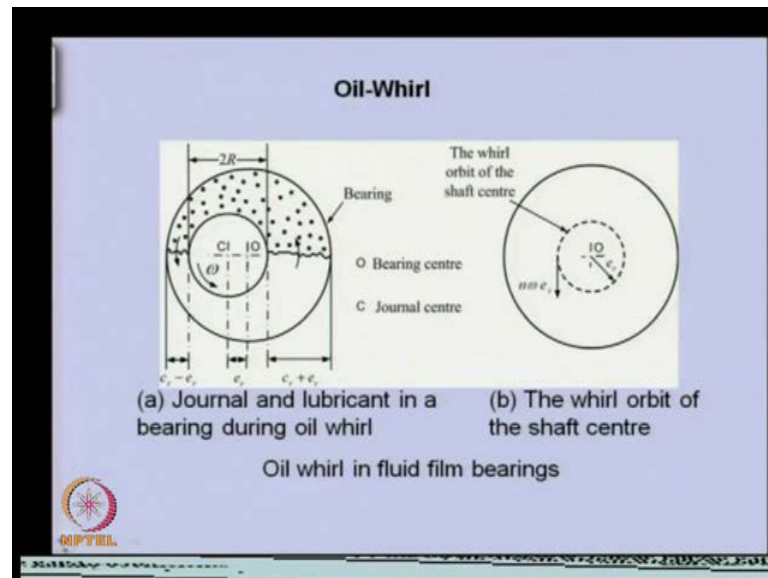
$$0.5R\omega(c_r + e_r) - 0.5R\omega(c_r - e_r) = 2Rn\omega e_r$$

$$n = 0.5$$

So, let us say we are considering what is the volume of the fluid, which is going inside the upper half of this particular bearing. So, this is Q_{in} . So, Q_{in} we can able to obtain as this is the average velocity because $R\omega$ is the velocity of the fluid here. So, first is the average velocity, half on this and this is the, this into let us say depth of the bearing is

1. So, this distance into 1 will give us the average area, area through which this fluid is flowing. So, this will be the input volume of the fluid which is going from this side. From this side we will be having same velocity, but this distance is now c_r into e_r . So, this is the area through which this volume is passing. So, fluid is passing. So, this will be the fluid which is going out.

(Refer Slide Time: 33:53)



Now, because of the motion of the shaft which is going down toward the downward direction if we see; so here this is the whirling motion. So, basically whirling the frequency is let us say ν , and if ν is the whirl frequency of the rotor, and if we multiply by the e_r that will be the velocity of the journal which will be going toward the downward direction. Now, we are taking ν as some factor of speed of the rotor. This factor we do not know n . So, that is why here the downward motion of the rotor we have taken as if you substitute this $n \omega e_r$.

(Refer Slide Time: 34:40)

$$Q_{in} = \frac{1}{2}(R\omega + 0)(c_r + e_r)$$

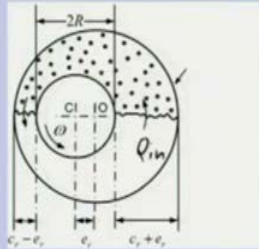
$$Q_{out} = \frac{1}{2}(R\omega + 0)(c_r - e_r)$$

$$Q_{vol} = (n\omega e_r)(2R)$$

$$Q_{in} - Q_{out} = Q_{vol}$$

$$0.5R\omega(c_r + e_r) - 0.5R\omega(c_r - e_r) = 2Rn\omega e_r$$

$$n = 0.5 \qquad \nu = 0.5\omega \qquad \underline{0.46 - 0.48}$$



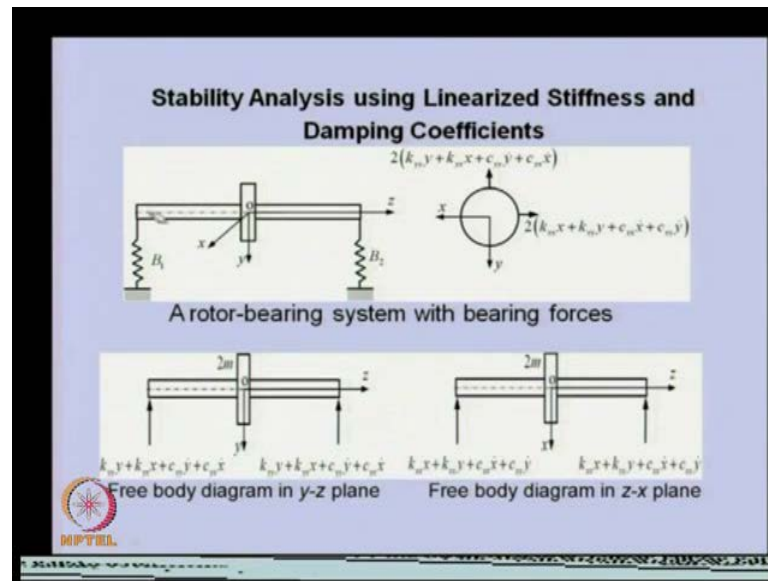


And this is the quantity. It is the $n\omega e_r$ is the velocity of the journal and projected area is $2R$ into the depth is unity. So, that is the volume it is getting created in the upper half. Now, the difference in the volume in and out should be equal to this particular volume which the journal is creating. If we equate this three volume and solid we will see that n is equal to 0.5 that means the whirl frequency is equal to 0.5 of the speed of the rotor.

This we have seen in the previous Campbell diagram that the whirl frequency due to the, this point whirl take place at the approximately half of the spin speed of the rotor. In practice people have found that this particular factor n is upon, it varies from 0.46 to 0.48. And the difference is due to the because in theoretical calculation we assume the motion of the fluid only due to the motion of the shaft, but there will be pressure gradient that we have not considered and because of that this particular difference of the theoretical and the experimental is there.

So, if there is a pressure gradient the fluid flow will take place because of that also and if we incorporate that also in the analysis then we can expect, we can get the, this particular factor that is 0.46 to this, in this particular range. Now, we will study how we can able to find out the stability of the system of a rotor bearing system using the linearized stiffness and damping coefficient. So, we will be studying basically Routh-Hurwitz criteria of stability and in this particular case we are taking very simple rotor model.

(Refer Slide Time: 36:47)



So, let us say there is a rotor. The mass of the rotor is let us say $2m$ is mounted on two identical fluid film bearing. Their properties are identical and this is basically a model using all 8 linearized stiffness and damping coefficient as we discuss in the previous class. Now, if we draw the free body diagram of this rotor we will see that the fluid film forces in the x and y direction will be given by this. So, this is basically fluid film forces in the x direction and in this we will be having contribution from stiffness as well as damping terms and because cross coupled damping is and stiffness is also there.

So, those terms will also be there. Similarly, in the y direction we will be having this. Now, the same thing is drawn here. So, this is a free body diagram of the rotor in this particular plane, one of the plane. Let us say $y-z$ plane. So, in this $y-z$ plane we will be having bearing force in the, from this bearing and this bearing; they are having similar coefficients. So, they look like and similarly, in the, this is the free body diagram in the $z-x$ plane and this is basically we can able to see because of the cross coupled stiffness and damping both x and y are pairing in this.

(Refer Slide Time: 38:32)

$$\begin{aligned}
 k_{xx}x + k_{xy}y + c_{xx}\dot{x} + c_{xy}\dot{y} &= -m\ddot{x} \\
 k_{yx}x + k_{yy}y + c_{yx}\dot{x} + c_{yy}\dot{y} &= -m\ddot{y} \\
 x &= X_0 e^{\lambda t} & \dot{x} &= X_0 \lambda e^{\lambda t} & \ddot{x} &= X_0 \lambda^2 e^{\lambda t} \\
 y &= Y_0 e^{\lambda t} & \dot{y} &= Y_0 \lambda e^{\lambda t} & \ddot{y} &= Y_0 \lambda^2 e^{\lambda t} \\
 k_{xx}X_0 + k_{xy}Y_0 + c_{xx}X_0\lambda + c_{xy}Y_0\lambda &= -m(X_0\lambda^2) \\
 k_{yx}X_0 + k_{yy}Y_0 + c_{yx}X_0\lambda + c_{yy}Y_0\lambda &= -m(Y_0\lambda^2)
 \end{aligned}$$


So we can able to now write the equation of motion using this because we have taken the mass of the rotor as 2 m. So, basically we will be getting very simple equation like this. So, external force is equal to inertia by that we can able to obtain this equation of motion. Now, because there is no unbalance in the system, so let us take the solution of the response in x and y direction as amplitude and some frequency lambda t. We can take the derivatives of these 2 and we can substitute in the equation of motion. So, basically now we will be getting this equation and all the time dependent terms will vanish. In this the lambda will be there. Now, you can able to see basically this is a homogeneous equation.

(Refer Slide Time: 39:31)

$$\begin{aligned}
 X_0(-m\lambda^2 - k_{xx} - c_{xx}\lambda) &= Y_0(k_{xy} + c_{xy}\lambda) \\
 X_0(k_{yx} + c_{yx}\lambda) &= Y_0(-m\lambda^2 - k_{yy} - c_{yy}\lambda) \\
 \begin{bmatrix} (m\lambda^2 + c_{xx}\lambda + k_{xx}) & (c_{xy}\lambda + k_{xy}) \\ (c_{yx}\lambda + k_{yx}) & (m\lambda^2 + c_{yy}\lambda + k_{yy}) \end{bmatrix} \begin{Bmatrix} X_0 \\ Y_0 \end{Bmatrix} &= \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \\
 (m^2)\lambda^4 + m(c_{xx} + c_{yy})\lambda^3 + (mk_{xx} + mk_{yy} + c_{xx}c_{yy} - c_{xy}c_{yx})\lambda^2 & \\
 + (k_{yy}c_{xx} + c_{yy}k_{xx} - k_{xy}c_{yx} - c_{xy}k_{yx})\lambda + (k_{xx}k_{yy} - k_{xy}k_{yx}) &= 0
 \end{aligned}$$

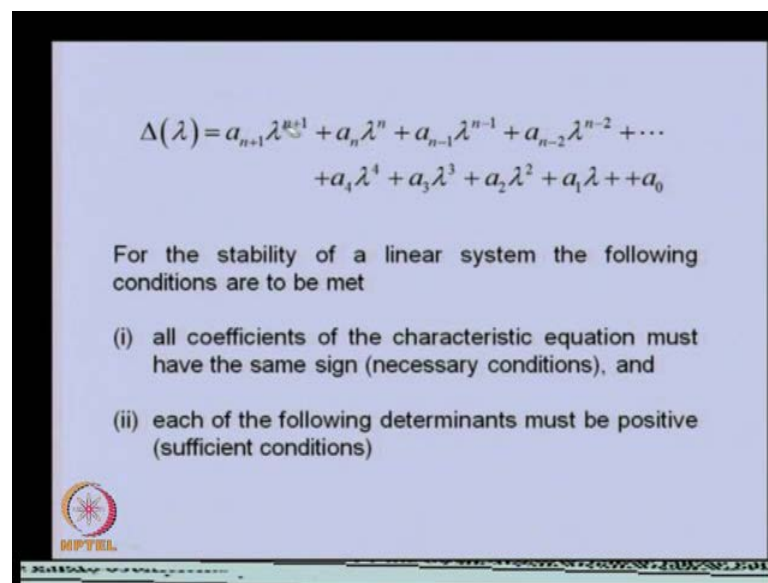
In general the roots of λ will have both the real and imaginary parts.



Now, we can able to write them in more compact form like this. So, all the X term I have collected in one place and Y in other place. So, these two equations we will be getting. Basically, they are homogeneous equation. So, we can able to put them in a matrix form like this and because this is homogeneous for non trivial solution, determinant of this should be 0. So, if we take the determinant of this matrix 0 we will get a polynomial, fourth degree polynomial in terms of lambda and a polynomial will be of having this form.

So, each and every term of the lambda will be having some coefficients. And these coefficients basically will help us in finding out the stability of the system. So, generally because in general this lambda the roots of this will be complex. It will be having both real and imaginary parts and so we got a fourth degree polynomial and that will help us in finding the instability of the system. Now, I am generalizing this particular polynomial and we will try to find out the condition for which we can get the system will be stable.

(Refer Slide Time: 40:57)



The slide displays the general form of a characteristic polynomial $\Delta(\lambda)$ and lists conditions for system stability.

$$\Delta(\lambda) = a_{n+1}\lambda^{n+1} + a_n\lambda^n + a_{n-1}\lambda^{n-1} + a_{n-2}\lambda^{n-2} + \dots + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$$

For the stability of a linear system the following conditions are to be met

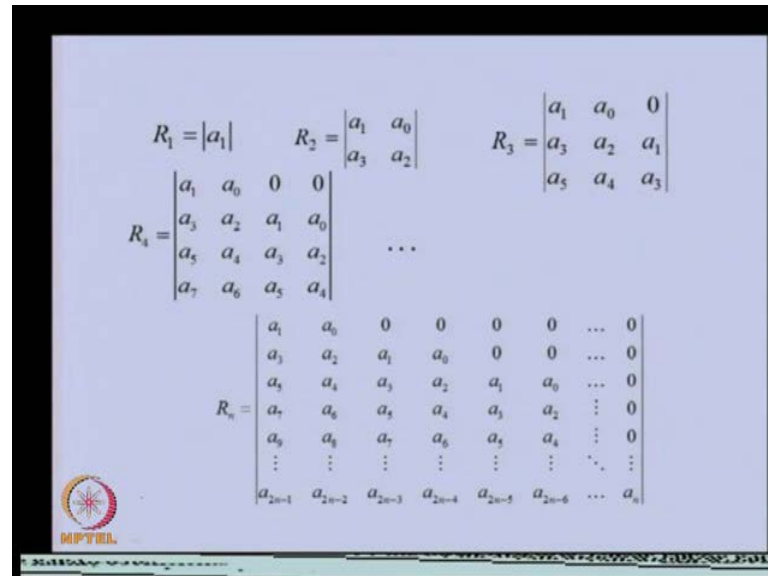
- (i) all coefficients of the characteristic equation must have the same sign (necessary conditions), and
- (ii) each of the following determinants must be positive (sufficient conditions)

The NPTEL logo is visible in the bottom left corner of the slide.

So, this is the general form of that particular polynomial which we obtained. In this, this is 10 plus 1 degree polynomial. So, for the previous case basically we will be having n is equal to 3 for stability of the, this particular linear system we will be having this, these conditions. So, all the coefficients of the characteristic equation must have the same sign. This is the necessary condition that means all these a's should have same sign then the

system will be stable. For sufficient condition each of the following determinants must be positive.

(Refer Slide Time: 41:36)



Slide showing Routh-Hurwitz determinants:

$$R_1 = |a_1|$$

$$R_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}$$

$$R_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix}$$

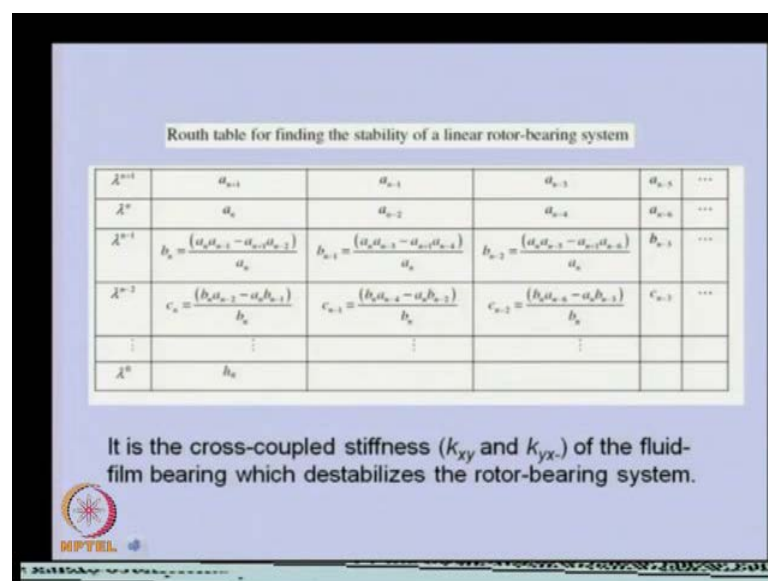
$$R_4 = \begin{vmatrix} a_1 & a_0 & 0 & 0 \\ a_3 & a_2 & a_1 & a_0 \\ a_5 & a_4 & a_3 & a_2 \\ a_7 & a_6 & a_5 & a_4 \end{vmatrix}$$

$$\dots$$

$$R_n = \begin{vmatrix} a_1 & a_0 & 0 & 0 & 0 & 0 & \dots & 0 \\ a_3 & a_2 & a_1 & a_0 & 0 & 0 & \dots & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & \dots & 0 \\ a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & \dots & 0 \\ a_9 & a_8 & a_7 & a_6 & a_5 & a_4 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{2n-1} & a_{2n-2} & a_{2n-3} & a_{2n-4} & a_{2n-5} & a_{2n-6} & \dots & a_0 \end{vmatrix}$$

That means if we R_1, R_2, R_3, R_4, R_n these are the determinants if they are more positive then we will be having the system is stable. So, you can able to see these are the coefficients of the polynomial. And if we take the determinant, this determinant should be positive to have stability of the system. This is called Routh-Hurwitz criteria.

(Refer Slide Time: 42:07)



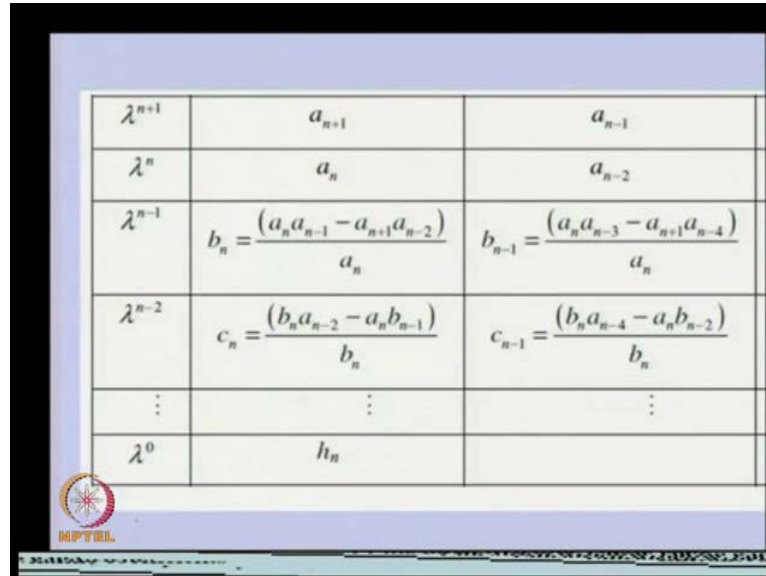
Routh table for finding the stability of a linear rotor-bearing system

λ^{2n-1}	a_{n-1}	a_{n-1}	a_{n-1}	a_{n-1}	$\dots$
λ^n	a_n	a_{n-2}	a_{n-4}	a_{n-6}	$\dots$
λ^{n-1}	$b_n = \frac{(a_n a_{n-1} - a_{n-1} a_{n-1})}{a_n}$	$b_{n-1} = \frac{(a_n a_{n-2} - a_{n-1} a_{n-2})}{a_n}$	$b_{n-2} = \frac{(a_n a_{n-3} - a_{n-1} a_{n-3})}{a_n}$	b_{n-3}	$\dots$
λ^{n-2}	$c_n = \frac{(b_n a_{n-1} - a_n b_{n-1})}{b_n}$	$c_{n-1} = \frac{(b_n a_{n-2} - a_n b_{n-2})}{b_n}$	$c_{n-2} = \frac{(b_n a_{n-3} - a_n b_{n-3})}{b_n}$	c_{n-3}	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
λ^0	b_n				

It is the cross-coupled stiffness (k_{xy} and k_{yx}) of the fluid-film bearing which destabilizes the rotor-bearing system.

The same criteria in a tabular form is given like this. This is for the linear system only, in this particular case because this is very small.

(Refer Slide Time: 42:21)



λ^{n+1}	a_{n+1}	a_{n-1}
λ^n	a_n	a_{n-2}
λ^{n-1}	$b_n = \frac{(a_n a_{n-1} - a_{n+1} a_{n-2})}{a_n}$	$b_{n-1} = \frac{(a_n a_{n-3} - a_{n+1} a_{n-4})}{a_n}$
λ^{n-2}	$c_n = \frac{(b_n a_{n-2} - a_n b_{n-1})}{b_n}$	$c_{n-1} = \frac{(b_n a_{n-4} - a_n b_{n-2})}{b_n}$
$\vdots$	$\vdots$	$\vdots$
λ^0	h_n	

So, I am enlarging only two columns of the previous one. Other two columns I will be showing subsequently. So, first three columns I have shown here. So, these are the λ^{n+1} , λ^n like this up to λ^0 . This is a_{n+1} , this is the coefficient of the polynomial, a_n , a_{n-1} , a_{n-2} . Basically, in this we have, we have recalculated various coefficients like b_n , c_n and the condition of the Routh's table is that the first column coefficient after calculation should not have any change in the sign.

If there is change in the sign then the system will be unstable. So, like b_n . b_n we have calculated based on the previous coefficients. So, $a_n a_{n-1} - a_{n+1} a_{n-2}$ divided by a_n . So, this quantity is defined as b_n . Similarly, $b_n a_{n-2} - a_n b_{n-1}$ divided by b_n . So, these have been defined and if we calculate this, this coefficient should not have change in the sign.

(Refer Slide Time: 43:43)

a_{n-1}	a_{n-3}	a_{n-5}	...
a_{n-2}	a_{n-4}	a_{n-6}	...
$b_{n-1} = \frac{(a_n a_{n-3} - a_{n+1} a_{n-4})}{a_n}$	$b_{n-2} = \frac{(a_n a_{n-5} - a_{n+1} a_{n-6})}{a_n}$	b_{n-3}	...
$c_{n-1} = \frac{(b_n a_{n-4} - a_n b_{n-2})}{b_n}$	$c_{n-2} = \frac{(b_n a_{n-6} - a_n b_{n-3})}{b_n}$	c_{n-3}	...
$\vdots$	$\vdots$		

So, now I will be showing the third and fourth column. So, like continuation of the previous one was a_{n-1} , a_{n-3} , a_{n-5} , a_{n-7} like this and here b_{n-1} , b_{n-2} , b_{n-3} , b_{n-4} like this. So, all these coefficients we will define based on the previous coefficients that can be calculated. But only we need to see in the first column, this particular column there should not be any change in the sign. Now, through very simple example, let us see this particular Routh Hurwitz criteria.

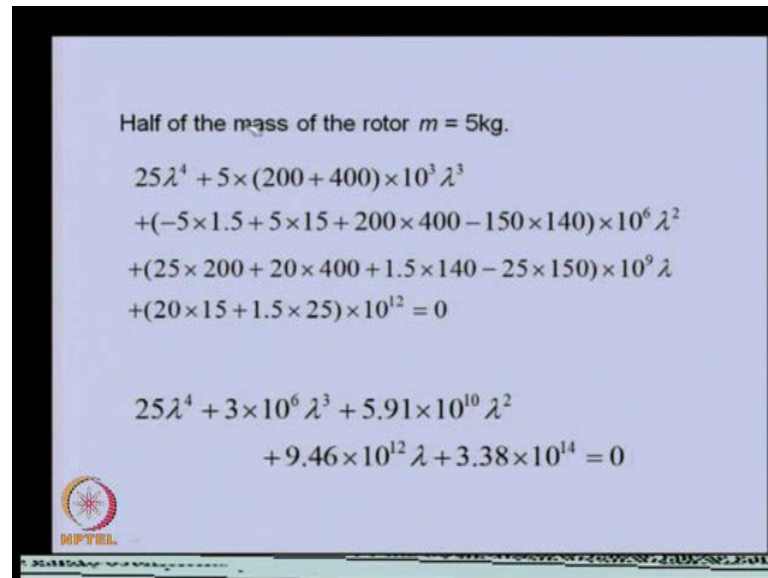
(Refer Slide Time: 44:18)

Question

For a rigid rotor of mass 10 kg is supported on two identical fluid film bearings with the following properties: $k_{xx} = 20 \text{ MN/m}$, $k_{yy} = 15 \text{ MN/m}$, $k_{xy} = -1.5 \text{ MN/m}$, $k_{yx} = 25 \text{ MN/m}$, $c_{xx} = 200 \text{ kN-s/m}$, $c_{xy} = 150 \text{ kN-s/m}$, $c_{yx} = 140 \text{ kN-s/m}$ and $c_{yy} = 400 \text{ kN-s/m}$. Find the stability of the rotor.

So, in this particular case we have taken very simple rotor model of 10 k g supported on two identical bearing. The properties of the bearings are given here. We need to find out the stability of the rotor system.

(Refer Slide Time: 44:33)



Half of the mass of the rotor $m = 5\text{kg}$.

$$25\lambda^4 + 5 \times (200 + 400) \times 10^3 \lambda^3$$

$$+ (-5 \times 1.5 + 5 \times 15 + 200 \times 400 - 150 \times 140) \times 10^6 \lambda^2$$

$$+ (25 \times 200 + 20 \times 400 + 1.5 \times 140 - 25 \times 150) \times 10^9 \lambda$$

$$+ (20 \times 15 + 1.5 \times 25) \times 10^{12} = 0$$

$$25\lambda^4 + 3 \times 10^6 \lambda^3 + 5.91 \times 10^{10} \lambda^2$$

$$+ 9.46 \times 10^{12} \lambda + 3.38 \times 10^{14} = 0$$

So, we can able to substitute all these parameter in the polynomial which we have obtained earlier, the fourth degree polynomial which we obtain earlier for the rotor systems similar to this. So, various coefficients can be obtained. So, basically I have substituted the various values of the stiffness and damping and mass of the rotor and basically we will get this polynomial having coefficients given like this. This is a fourth degree polynomial.

(Refer Slide Time: 45:05)

$$a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$$

$$R_1 = |a_1| = 9.46 \times 10^{12} \quad n+1=4 \quad n=3$$

$$R_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = \begin{vmatrix} 9.46 \times 10^{12} & 3.38 \times 10^{14} \\ 3.00 \times 10^6 & 5.91 \times 10^{10} \end{vmatrix} = 5.58 \times 10^{23}$$

$$R_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix} = \begin{vmatrix} 9.46 \times 10^{12} & 3.38 \times 10^{14} & 0.00 \\ 3.00 \times 10^6 & 5.91 \times 10^{10} & 9.46 \times 10^{12} \\ 0.00 & 25.00 & 3.00 \times 10^6 \end{vmatrix} = 1.67 \times 10^{30}$$

It can be seen that all determinants are positive, hence, the rotor-bearing system is stable



Now, we will apply the Routh-Hurwitz criteria for fourth degree. So, R_1 is this one. They are determined. First criteria, is that all the coefficients should be same sign. So, you can see they are of same sign. So, necessary condition is satisfied. Now, next is this determinant should be positive. So, you can see R_1 , R_2 , R_3 , they are positive. So, by this we can conclude that the system is stable.

(Refer Slide Time: 45:40)

$$a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$$

λ^4	$a_4 = 25$	$a_3 = 5.91 \times 10^{10}$	$a_2 = 3.38 \times 10^{14}$
λ^3	$a_1 = 3 \times 10^6$	$a_0 = 9.46 \times 10^{12}$	$a_1 = 0$
λ^2	$b_1 = \frac{(a_1 a_2 - a_4 a_0)}{a_3}$ $= 5.902 \times 10^{10}$	$b_2 = \frac{(a_0 a_2 - a_4 a_1)}{a_3} = a_0$ $= 3.38 \times 10^{14}$	$b_3 = \frac{(a_1 a_3 - a_4 a_1)}{a_3} = 0$
λ^1	$c_1 = \frac{(b_1 a_1 - a_4 b_2)}{b_1}$ $= 9.443 \times 10^{12}$	$c_2 = \frac{(b_2 a_1 - a_4 b_3)}{b_1} = 0$	$c_3 = \frac{(b_3 a_1 - a_4 b_3)}{b_1} = 0$
λ^0	$d_1 = \frac{(c_1 b_2 - b_1 c_2)}{c_1}$ $= 3.38 \times 10^{14}$	$d_2 = 0$	$d_3 = 0$

It can be observed that there is no sign change in the first column of the Routh table and hence the system is in stable condition.



The same thing with Routh table, we can calculate. So, like now in case of this coefficient I have given the numbers also and with these formulas we can calculate.

calculate these coefficients in this particular column. And we can be able to see that all are positive in this case also. So, by this we can be able to conclude that the system is stable and if there is a sign change in any of this then the system will be unstable. Today, we have seen instability of the rotor system specially for fluid film bearing.

Initially we introduce various instability sources in the rotor bearing system and we pointed out that the most common source of the instability is the fluid film bearing and for fluid film bearing specially for the linearized stiffness and damping coefficient we provided a method of Routh-Hurwitz criteria by which we can be able to find out the stability of the system. That means we using these parameter we can be able to find out if really whether the system will be stable or unstable. In subsequent lectures we will be analyzing other kind of instability one by one and in this case always we will be taking very simple rotor model. So, that we can be able to go in more insight into the problem especially the physics of the problem we can be able to understand.