**Theory and Practice of Rotor Dynamics**
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**Module - 7**
**Instability in Rotor Systems: Bearings**
**Lecture - 1**
**Overall Lectures 32**

That is the last lecture we have been doing the analysis of rotor bearing system in great detailed for torsional vibration and transverse vibration. Mainly those analysis we are concerned with the finding natural frequency mode shape and unbalance force response or forces due to the other sources, how to get the response for force response for that? From today we will start another topic on instability on the rotors system, but before going in to the actual subject of instability I would like to introduce the fluid film bearing concept and the rolling element bearing concept. Specially, the fluid film bearing their main source of instability in the rotor system, like force response always will be there in the system due to unbalance. On the same lines this instability is the main cause of the instability due to the fluid film bearings. So, in some detail we will try to see, how the rotor dynamic coefficients of these bearings can be obtained? And how it will be affecting then stability? That will be lending in the subsequent chapter.

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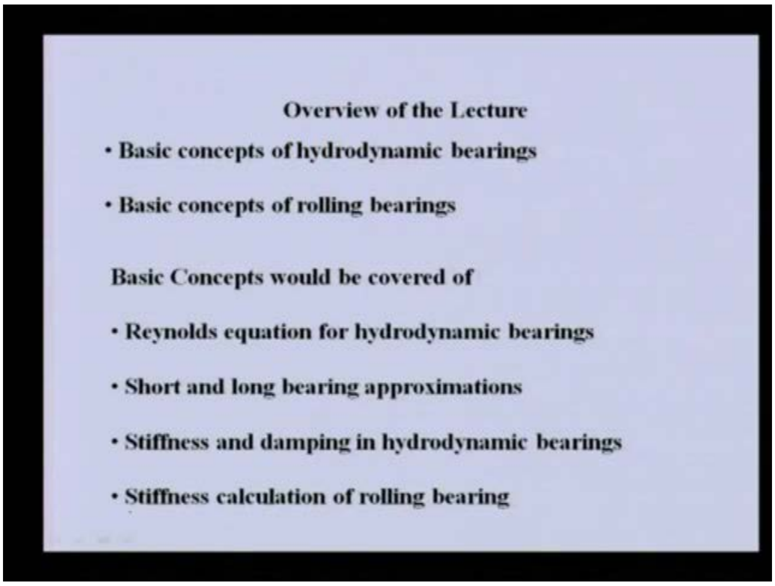

So, basically in this lecture we will concentrate on the bearing, hydrodynamic bearing and rolling bearings and how we can able to get the rotor dynamics coefficients from this? Like for bearing will introduce the Reynolds equation for hydrodynamic bearing and for short bearing approximation are long bearing approximation. How we can able to solve this particular partial differential equation? In close form will see and also we give brief idea about if the bearing is finite. How the Reynolds equation can be solve using finite difference method? Brief of the rolling element bearing stiffness is generally rolling bearings are very highly in rolling bearing are highly non-linear in nature in stiffness, but we will try to obtain the stiffness linearized stiffness is in simple procedure.

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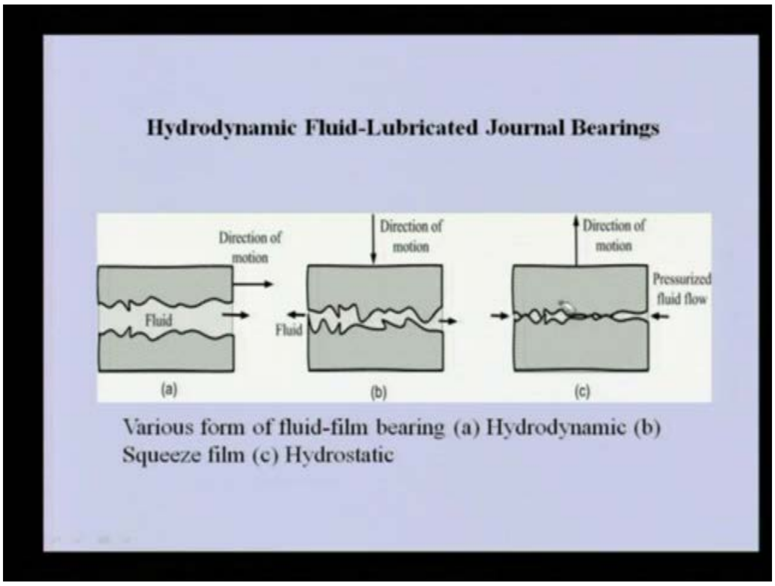


So, cutting with the hydro dynamic film lubrication in particular bearing, there are three category of bearings we have in the industry. Is a very abstract definition of the three kind of bearing like, hydro dynamic bearing; in which once surface is stationery another is moving tangential to it. In between this there is a fluid and generally we will be having some kind of converging area and because of this motion the pressure will develop in the converging area due to.. Due to that this particular body will be lifted up from the there will not be metal to metal contact between the moving body and the stationery body.

Another kind of is this squeeze film. So, in this squeeze film lubrication generally the force is in this direction or the motion is in this direction and whatever the lubricant is there in between the two body will these getting squeezed out during the motion. This generally gives some kind damping in to the system, but not the stiffness as search apart from that we have hydrostatic lubrication. In this generally the separation of the two bodies take place because of the pressurized fluid from all sides. So, because of this pressure this body will get lifted and will be having no metal to metal contact between this two body, between which the motions we expect. So, this is the three basic concept by which we separate the two bodies, so that there is no metal to metal contact them.

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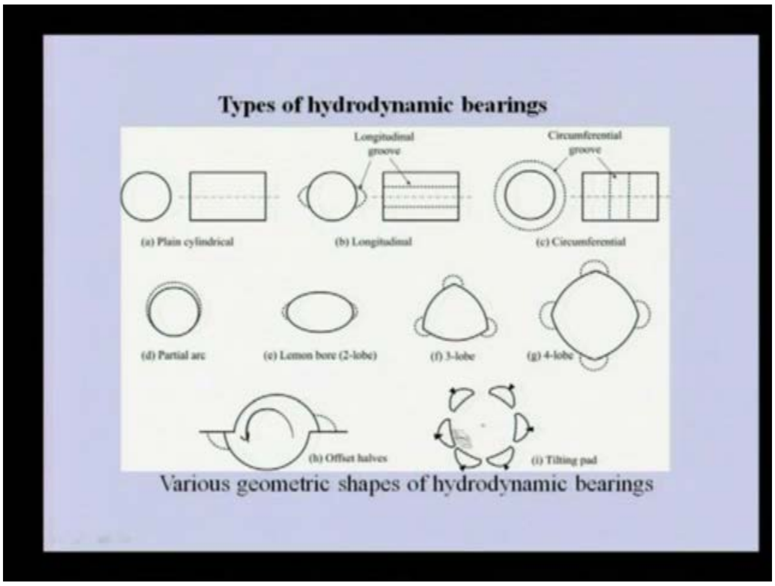


Focusing on the hydro dynamic bearing, hydro dynamic bearing the most simple hydro dynamic bearing is the cylindrical plane cylindrical bearing, in which the bearing cavity is circular and if you see from side it will be cylindrical in shape. So, shaft goes inside this and there is a small clearance between the shaft of the journal and the bearing. So, main important thing in this is this particular journal bearing the shape is circular. As search the lubrication comes and goes from the sides or some time we provides some kind of group on to the this bearing. Another kind of bearing is if the longitudinal bearing the shape is circular, but generally we provide the group at this. So, you can able to see the whole actual length of the bearing there will be groove and the lubricant will go form that side.

There is another kind of bearing in which the shape is again circular, but there is a circumstantial groove which is there at mid around midpoint of the length of the bearing. So, all over the circumference they will be groove which will give lubricant inside the bearing. So, that there is a enough lubrication between the journal and the bearing. Apart from that we have partial arc this bearing in which generally you can able to see this basically if a circular bearing, we have cut some portion of this and some portion of this and the remaining arc we have join together. So, this is that partial are and the lubrication groove are here on the top surface of the circumference not at the bottom, because generally the pressure develops at the bottom. So, if you provide lubrication grooves there that chances are there that lubricant will go out from the bottom grooves rather than coming inside the bearing.

Then we are lemon bore bearing in which there are two nodes, we can able to see there are two arc. Basically this elliptical shape and grooves are there at the and these two places from that the lubricant go inside the this bearing. Then there are three lobe bearing, so you can able to see three lobe, three bearing arcs are there and the lubrication is provided through this a portion of that grooves. Extension of this is four lobe, so four arcs are there and at each corner there is a tangent for the providing the lubrication to the a, then there is offset halves. So, you can able to see this particular two arcs are there, but they are they will offset by some amount and there is a grooves here, so that the lubrication go can go in this.

In this particular case you can able to see that the rotation can take place of the journal in one direction, but opposite is not visible otherwise this will abstract on the motion. Why we provide different shapes? Is a question revive we are going form cylindrical to various shapes. So, generally in a stability point of view the plane cylindrical bearings, general bearings are most sensitive to instability. To avoid that generally we provide different kind of shapes, in most stable bearings are the tilting pad bearings. So, you can able to see that these are the pads which can tilt about it is pivoting point and general rotates here. So, depending up on the requirement this pads can take different orientation and this is the most stable, highest is instability of the rotor system will be there with the tilting pad journal bearing.

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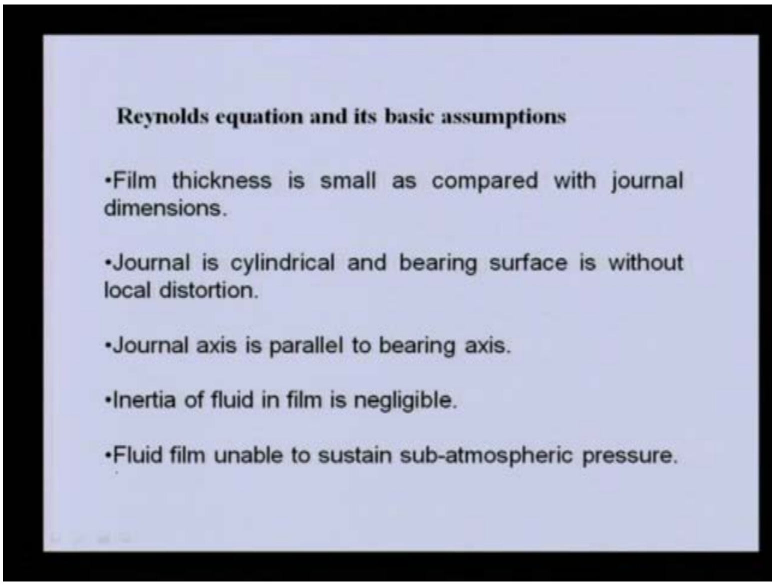


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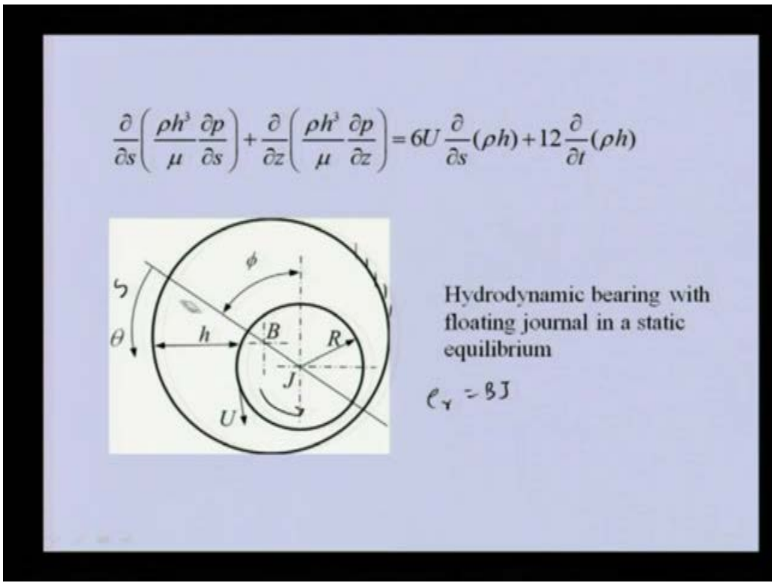


So, now will be looking to the Reynolds equation before looking in to at the junctions of this, let us see how the journal and the bearing occupy the position at some operating position operating speed. So, this is the journal this is basically circular shape. So, this is the bearing which is fix let us say and this is the journal and this clearance is generally will be very less of the order of microns, but I have just exaggerated here. So, that I can show various parameter on this, so this gape will be between the journal and the bearing lubricant will be there and this gape will be of order of microns. You can able to see this B is the journal the bearings center, which is the journal center between this two distance is a eccentricity. The angle if we join this line bearing center and the journal center line with vertical that angle is called attitude angle.

So, basically the eccentricity radial eccentricity e r, which is nothing but B J and this altitude angle defines the position of the journal. In this particular case we can able to see the journal is a rotating and this direction and basically it is pumping the fluid which is here because the fluid will get trapped here and it will pump in this narrow region. Because of this continuously converging shape will see that the pressure will developed and that will allow the this two the bearing. The journal to separate and they will be very high pressure in this region and that will basically sustain the weight of the journal or any other force, which is their all to the journal.

At any position the film thickness is given by this, so this h is the film thickness use the velocity of the journal at and capital r is the radius of the journal. This particular line, which is joining the bearing axis in the shaft text is or this will be the reference position when we want to measure this circumferential and then or may be circumferential length S. If we want to measure what is the distance in circumferential direction this will be the reference line for us reference position for this.

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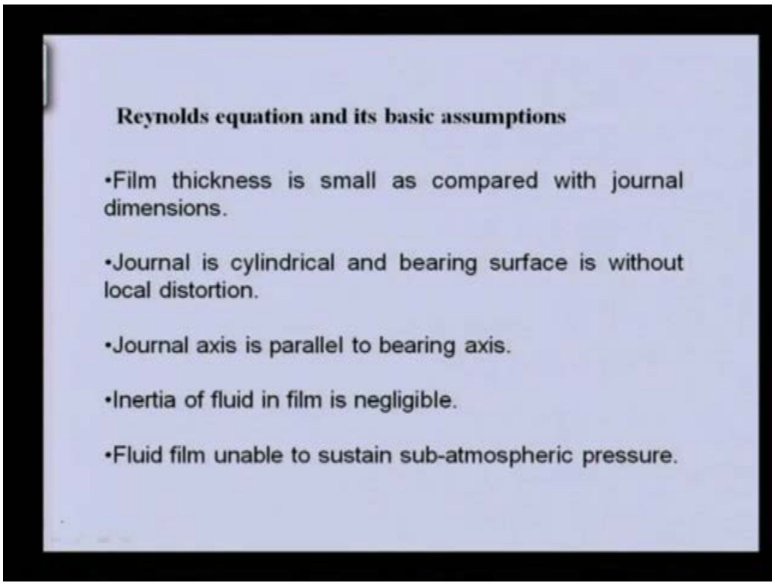


So, basically now we can look in to the Reynolds equation will not derive, but we will just show the Reynolds equation and what are the terms in it has. So, basic assumptions of this particular while deriving the reveal equations are; film thickness is small as compares to the journal dimensions. Journal is cylindrical and bearing surfaces without local distortion, journal axis is parallel to bearing axis, inertia of fluid in film is negligible. So, is another important assumption which we make inertia of the fluid we neglect, fluid film unable to sustain sub-atmospheric pressure. So, where ever negative pressure is there this will not be a cavitations may take place so we are not considering cavitations aspect here.

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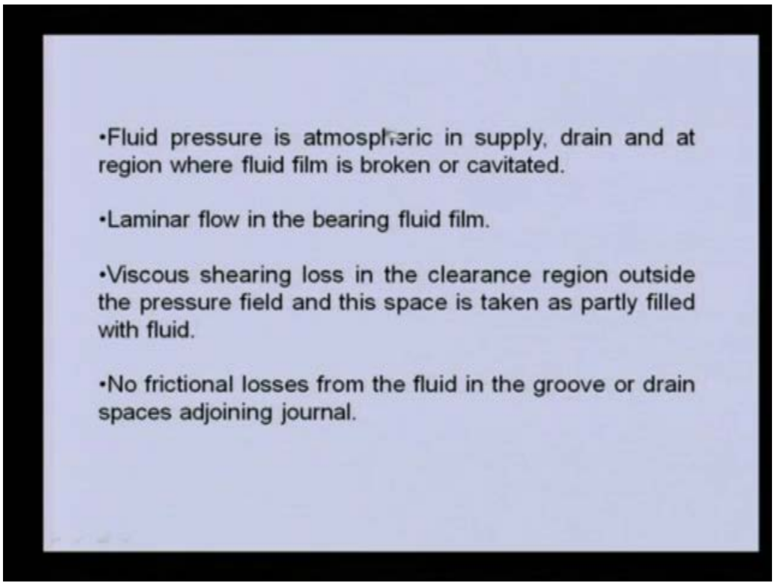


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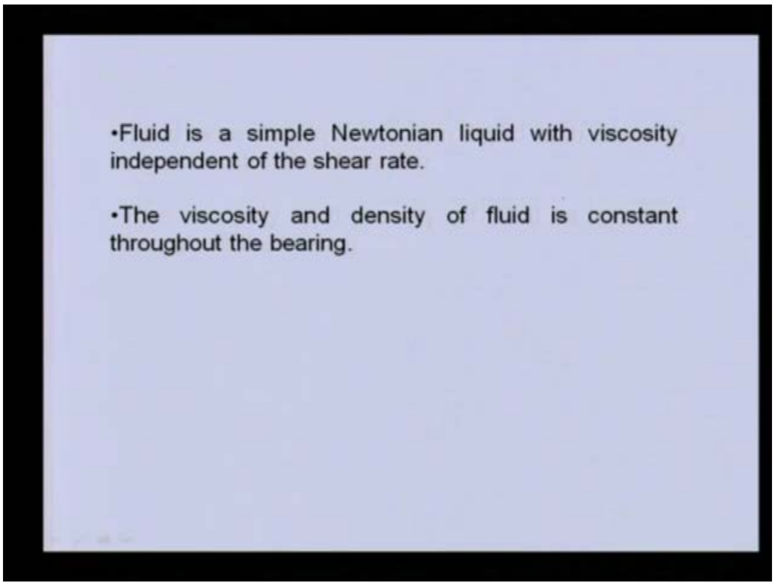

Fluid pressure is atmospheric in supply, drain and at region where the fluid film is broken or cavitated. So, where ever cavitated, cavitaion possibilities are there we are assuming that is atmosphere pressure is there. Laminar flow in the bearing fluid film, viscous shearing loss in the clearance region outside the pressure field and this space is taken as a partly filled with fluid. No frictional loss from the fluid in the groove or drain spaces the adjoining journal. So, these are the some of basic assumptions. Fluid is simple Newtonian fluid with viscosity independent of the shear rate. The viscosity and the density of the fluid is constant throughout the bearing.

So, these are the basic assumption deriving this Reynolds equation, this is basically partial differential equation. We can able to see, these the pressure inside the fluid and S is the circumferential direction and z is the axial direction. So, axial direction is the z axis direction. So, pressure variation contain be there in the axial direction or the circumferential direction. rho is density of the fluid, h is the film thickness, mu is the viscosity, here velocity journal velocity is U, also… So, all the terms this is time derivative. So, if there is some dynamic force in this so this term will be there, we are considering the study state condition then this term will be we can neglect.

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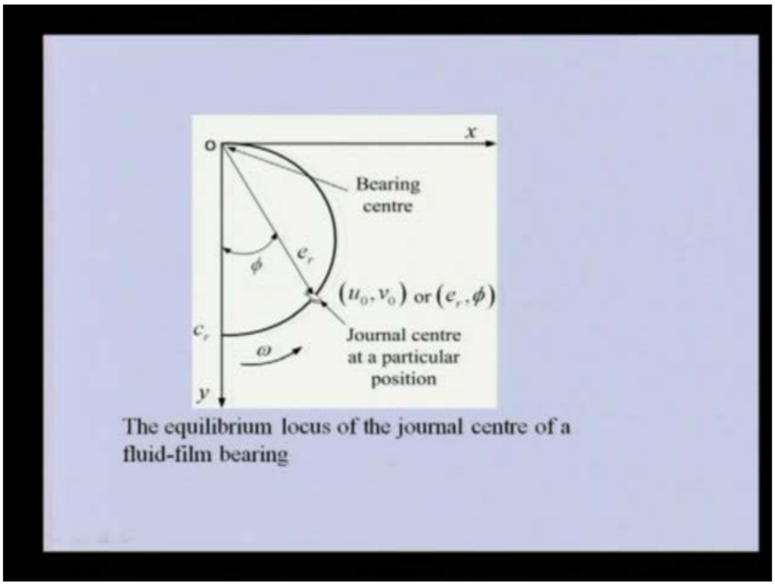

The equilibrium locus of the journal centre of a fluid-film bearing

Then once we have this Reynolds equation, let us see the concept of the linearized coefficient. Basically, this Reynolds equation gives the pressure from which we can fluid film pressure, from where we can able to get the fluid forces by multiplying this pressure with the area of the surface of the bearing. So, this particular plot is basically plot of the center of the shaft with speed how it changes? So, at 0 is speed, this is the bearing center, this is the shaft center at 0 speed, but when the speed is increasing the shaft occupies some incline position and this is this is the point where at a particular speed journal occupies at some incline position.

In the previous figure we have seen that that position we are describing by the radial eccentricity and the altitude angle. So, this is the equilibrium position of the shaft for study state force. So, if there is a dynamic force then shaft will be oscillating about this point. The linearized stiffness are define the disturbance of the journal from this equilibrium position when we are disturbing by small amount. Then what ever the change in the fluid pressure will define the this linearized coefficients.

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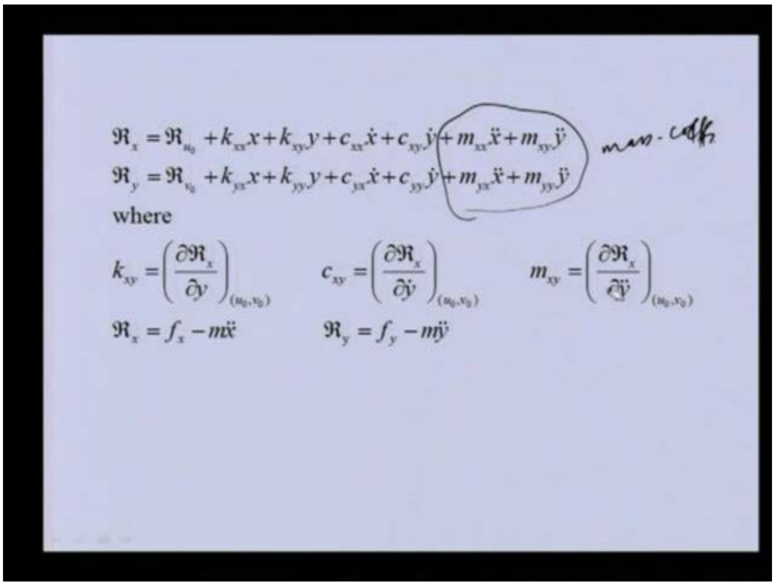


So, basically earlier we have shown the previous lectures this kind of linearized coefficient. So, this is the fluid pressure and these are the static forces in the vertical and horizontal and vertical direction. These are the terms basically for Reynolds equation, because we are not considering the fluid inertia. So, these terms are generally present when we have very we very high velocity of the rotors. So, these are called mass coefficients, so this stiffness damping and mass coefficient. So, Reynolds equation does not take care of this particular property of fluid. So, in this particular case you can able to see how the stiffness damping and mass coefficients have been defined. So, basically changing the fluid pressure for a given displacement disturbance or velocity or acceleration disturbing this property have been defined.

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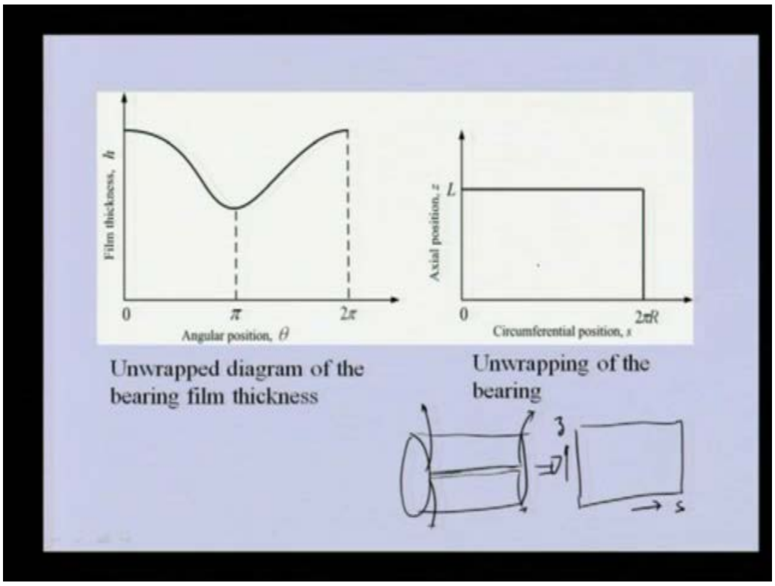


So, this is basically because in the Reynolds equation is having two variables, there is one is the circumferential direction, another is in the axial direction. So, we can able to see that if we have this cylindrical shape of the cylinder and if we cut this in one portion, if we cut form here and if we unwrap it like this. So, you can able to see that will be having we will get a rectangle R shape if we unwrap this. Let us say this is circumferential direction that is s and this is axial direction z. So, this particular inside surface of the bearing, generally we will seek the solution of the pressure at each and every point on this two dimensional space.

Because, the partial differential is differential equation is two dimensional, one is in the axial direction and other is in the circumferential direction. So, unwrap portion of the bearing surface is this in which this is the circumferential direction from 0 to 2 pie r and this the axial direction of the bearing. In the Reynolds equation defines the pressure in this region, which is function of s and z, if in the previous figure here, if we see the film thickness, film thickness is where maximum here and minimum here.

If we cut the film thickness here and if we unwrap this we will get something like this. In which this is the maximum film thickness h, which is the reference point for angle angular is displacement 0 theta is equal to 0 and then is minimum here and then it again become maximum. So, were basically this point on this points are at same point, because after 2 pie again this 2 will meet. So, this is the unwrap the film thickness along the angular position, how it changes with the angular position, this film thickness? So, now we will try to solve the Reynolds equation using final difference method in this region. So, basically we need to put various grids and will seek the solution of the pressure and various grids.

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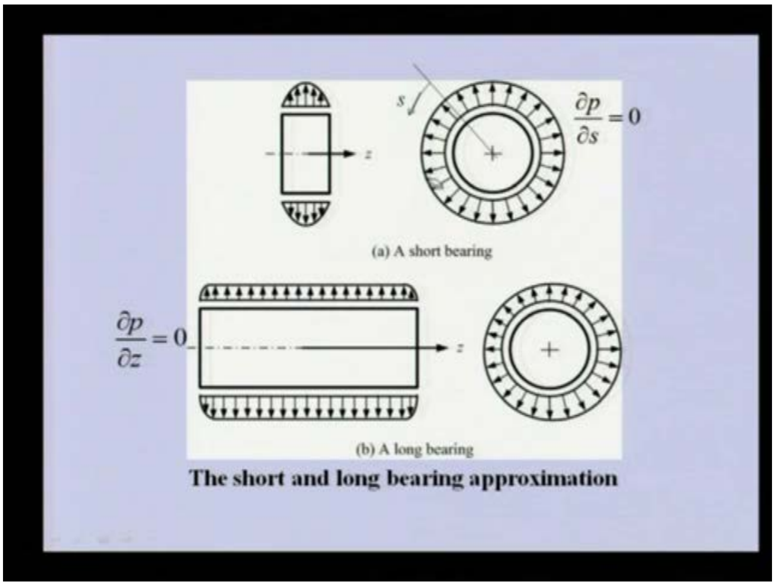


So, before that let us see if we want to solve the partial differential equation in close form. So, using two basic assumptions one is short bearing a approximation, another is long bearing approximation, we cannot simplify that Reynolds equation. For short bearing approximation, short bearing means this is the actual length of the bearing is short as compare to this one. So, if bearing length is short we will see that the pressure because, here it is atmospheric pressure, here also it is atmospheric pressure. So, at the center up to center there will be continuous variation of the pressure will be there. So, that means the variation of the pressure in the axial direction will be there in the short bearing. But if you see in the circumferential direction the pressure variation will be relatively less and this can be ignored.

So, for short bearing the circumferential direction pressure variation we ignore it. So, the Reynolds equation will be containing only derivative with respect to z. So, that will be a ordinary differential equation and that can be solved. Another approximation is the long bearing approximation and this you can able to see we get the length of the bearing is so long that the variation of the pressure will not be there in the axial direction, slight variation will be there at the end only. But if we see in the circumferential direction lot of variation will be there as compare to the axial direction.

So, in this particular case we can able to neglect the variation of the pressure with respect to z. Again in this particular case the Reynolds equation will be will be only differentiation will be respect to circumferential direction s and again it can be solved in the close form solution. I will be providing the final solution of the this stiffness coefficient and the damping coefficient for the short bearing approximation, which we can get from the pressure as discuss in the previous slide. That once we know the pressure we can able to get the fluid forces and changing the fluid forces defines these coefficients.

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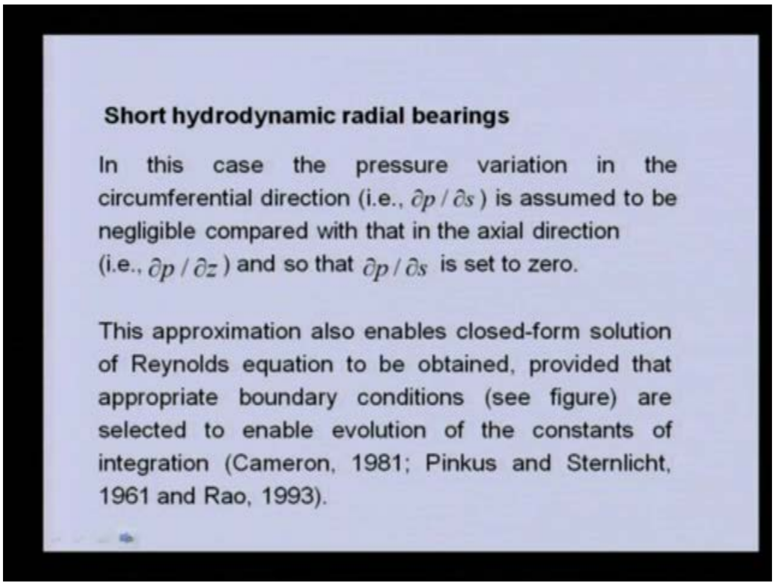

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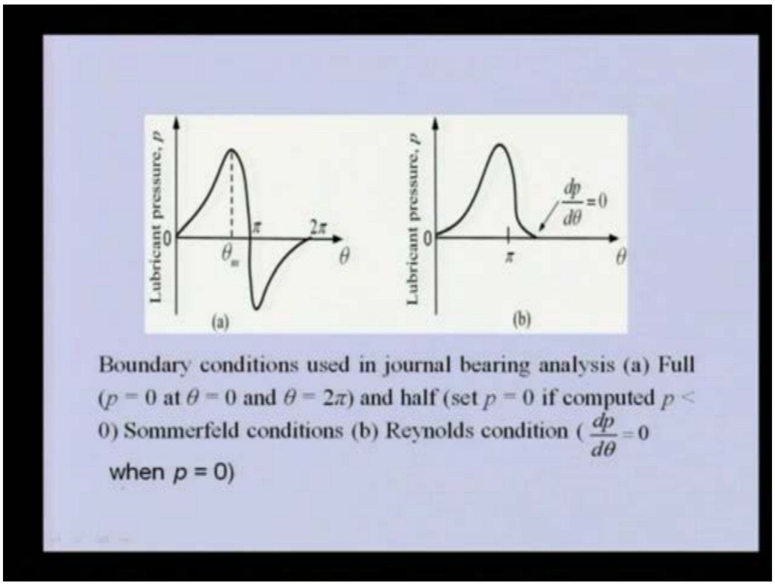


So, they can be obtained using this simplified Reynolds equation. So, we have already seen the sort bearing approximation how… so these are… So, apart from the equation obviously when we need to solve the partial differential equation boundary condition need to be satisfied. So, the previous slides are basically description of these boundary conditions. So, I am explaining the boundary condition in the figure itself rather than going in to the text. So, we can able to see the first boundary condition, which is full sommerfeld condition, full sommerfeld condition is this is the lubrication pressure and this the circumferential direction.

So, at 0 that means where the maximum thickness of the film is there pressure is 0 this is the assumption. When we are going up to the other end of this again it will become 0, but in between this around pie again there is a pressure 0. So, this is this full sommerfeld conditions, but this negative pressure will give cavitations. So, this boundary condition is not feasible ones. So, sometimes are most of an we go for the half sommerfeld condition rather than the full one in which we take the boundary condition up to this only, we do not go up to this. So, where ever the pressures are negative we take them as 0.

So, only half sommerfeld conditions is considered, another condition is or alternative is the Reynolds condition, which is more practical. In this particular case at theta is equals to 0, where the maximum thickness of the film is there pressure is 0, but here after phi wherever the pressure gradient is 0, we take p 0. So, you can able to see where ever the pressure gradient is 0, p 0. So, this particular whirl condition we need to use in the solution of the Reynolds equation, either is in sort bearing approximation or long bearing approximation or even for this finite bearing approximation, which I will describing subsequently.

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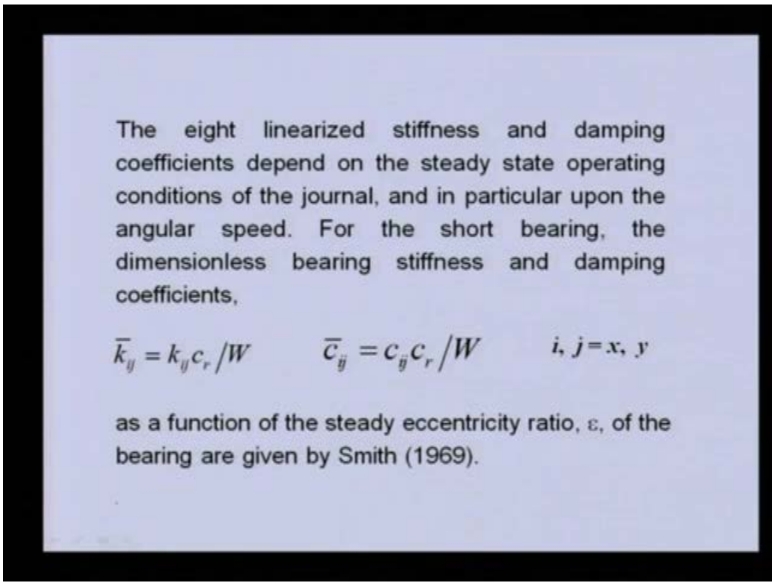


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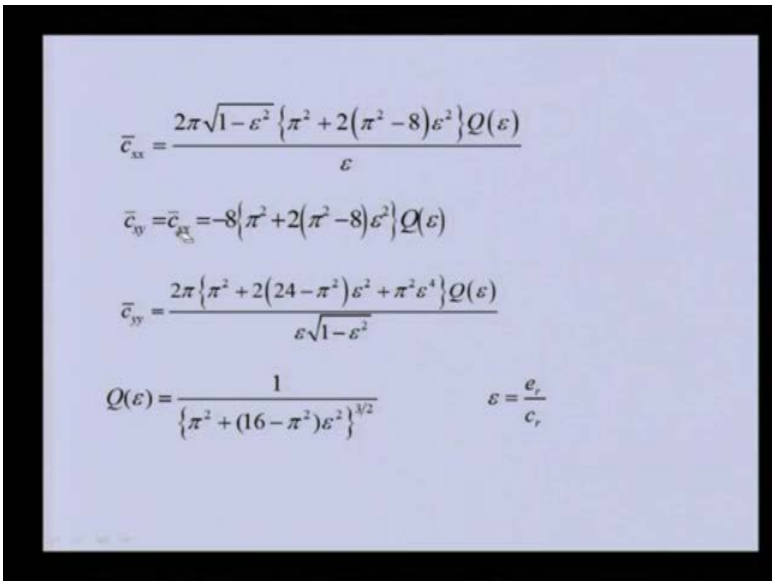

So, for short bearing approximation I am giving the linearized coefficient directly, without solving the equation as such. Here this is the clearance radial clearance, this is the weight of the journal, this is the stiffness coefficient. So, basically this is a non dimensional stiffness coefficients are defined this is damping, non dimensional damping, again this has been defined like this. In this particular case this particular expressions we have taken from Smith's book.

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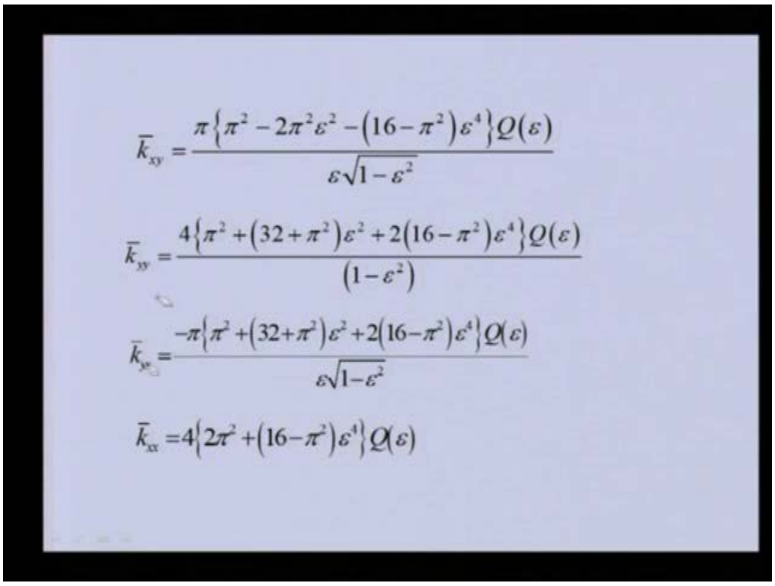


So, you can able to see that we have explicit form of these stiffness coefficients. So, you can able to see this epsilon, I will be defining what is the epsilon? Function of epsilon? I will be defining. So, basically the if we know the epsilon these stiffness coefficients can be calculated. Similarly, the damping coefficients they can be calculated, the one point is at that the this stiffness coefficient the cross couple stiffness coefficients are equal, but not the this stiffness coefficient, only the damping coefficients are equal. Generally the instability comes in to the rotor because of this cross couple stiffness this and this one.

So, they are not same and mainly the instability come because of this coefficients. So, here I have defined the Q capital this is function of epsilon and epsilon is eccentricity divided by the reveal clearance. So, you can able to see if we know the epsilon we can able to calculate these coefficients, that means if we know they are a eccentricity of the rotor we can able to calculate these coefficients.

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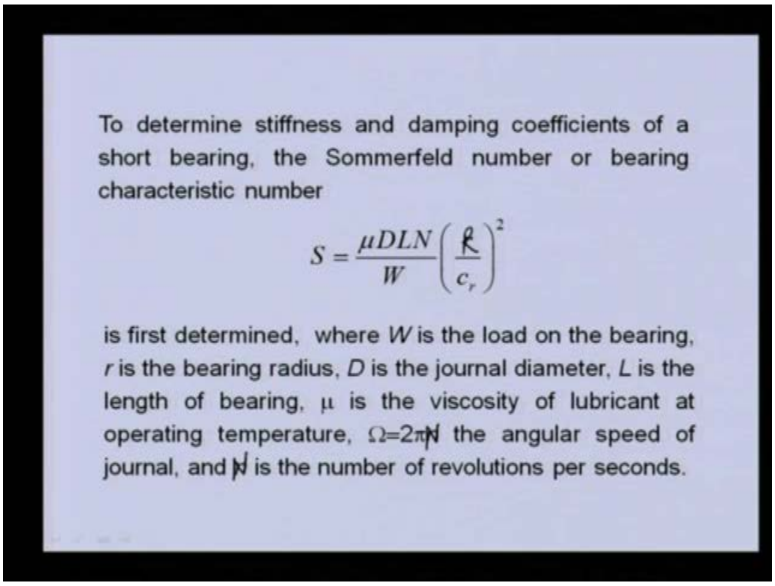


Generally, we obtain these bearings property in terms of the sommerfeld number, which is defined like this. Where D is the diameter of the bearing, where is the length of the bearing and is the revolution per minute that is s, this is the RPS and this revolution per second. This is weight of the journal, this is radius of the journal, this is the radial clearance between the journal and the bearing. So, this is basically revolution per second N is the revolution per second, this is the capital N.

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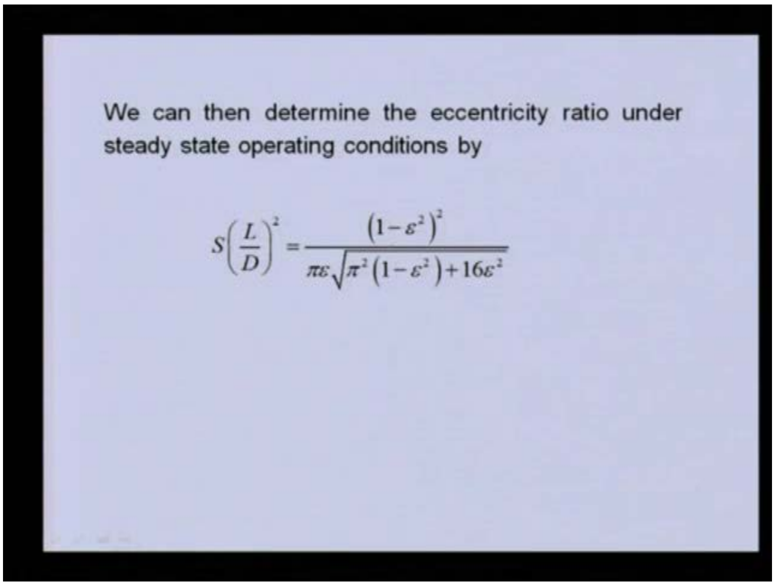

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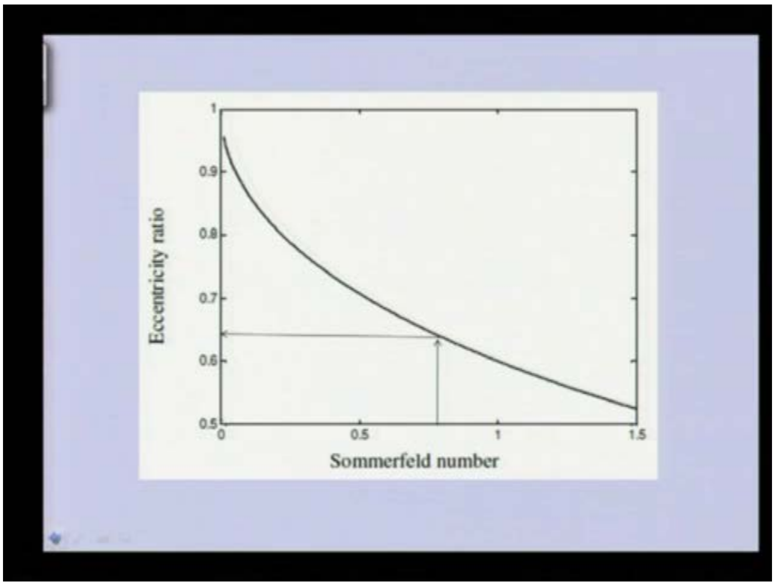


Now, this sommerfeld number we can able to express in terms of the, this epsilon which is nothing but the non dimensional eccentricity. Now, you can able to see that this sommerfeld number is depend up on the bearing operating conditions, as well as the dimensions of the bearing and the property of the bearing lubricant. Now, for a particular bearing at a particular speed we know the we will be knowing the sommerfeld number of that. From this equation we need to find out what would be the epsilon value for that particular sommerfeld number.

So, basically if we plot these two sommerfeld number with epsilon we will get a curve like this. So, sommerfeld number and eccentricity ratio epsilon, so this is the curve, so for particular bearing we can able to calculate the sommerfeld number and this plot we can able to interpolate what will able be the epsilon value for that particular bearing. Once we know the epsilon we can put in the stiffness and damping coefficient expressions to get the property of that. So, this the procedure generally we follow for calculating the stiffness and damping coefficient of the bearing.

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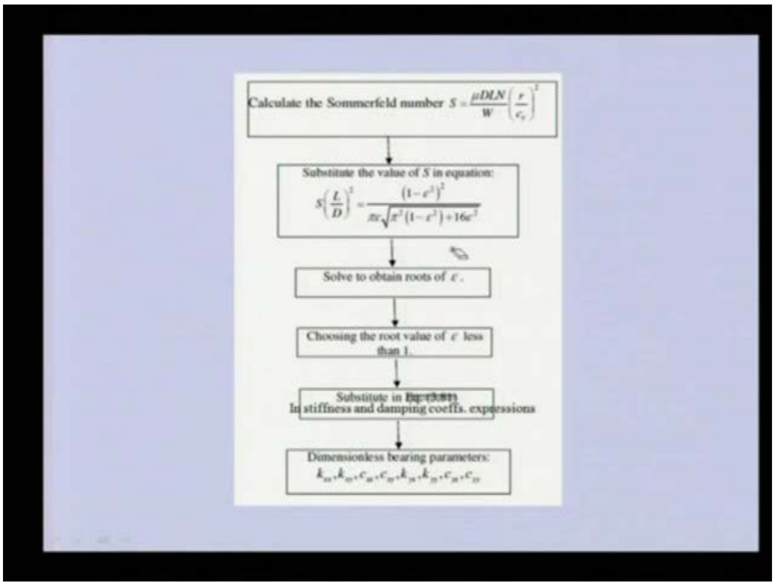


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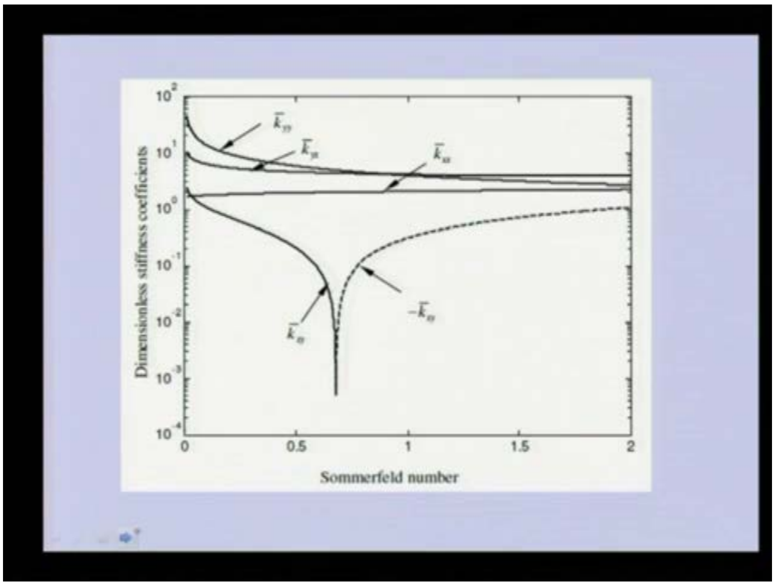


Basically, this is the same procedure which I described, how we can able to get the bearing property using sommerfeld number and the expressions which are given previously. These are the plot of the sommerfeld, the stiffness variation and the sommerfeld number. So, for various sommerfeld number we have summarize this non dimensional or dimensional less stiffness coefficient. So, you can able to see the variation of various k x x, k y y, k y x, this particular k x y it becomes negative. So, it has been shall in this region is dotted line, so this is the negative because this is the lobe log lock, semi lock plot. So, then there we cannot able to plot the negative value. So, we have taken absolute value of that and I have located by dotted line is these are the negatives value of the stiffness.

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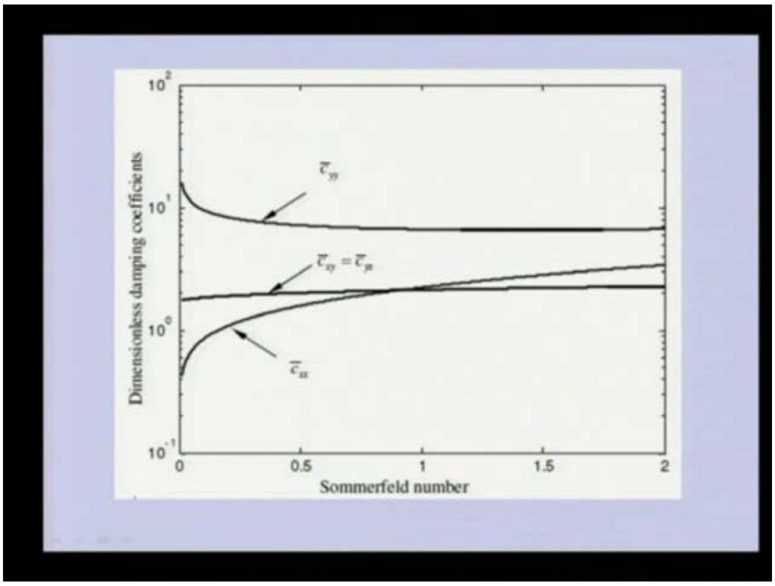


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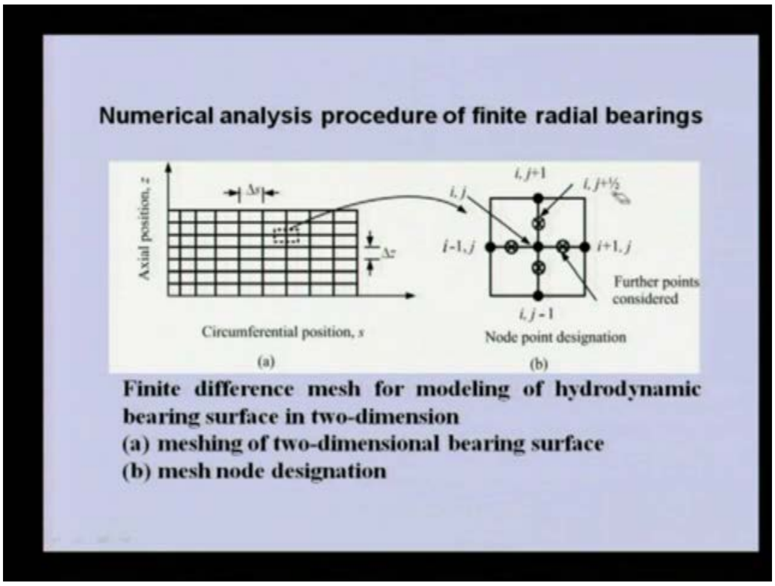


Similarly, the damping c x x, c x y and c y y and these two are same cross coupled stiffness damping are same, but not the stiffness as we are seen in the previous plot. Now, we will see how we can able to solve the partial differential equation, this Reynolds equation using finite difference method. So, brief outline of the method will be explained and for this particular case as we as explained previously this is that, once we cut the bearing and un wrap it this will be the bearing linear surface. So, in finite difference method we need to make a grid and we will be seeking the solution at how this nodes.

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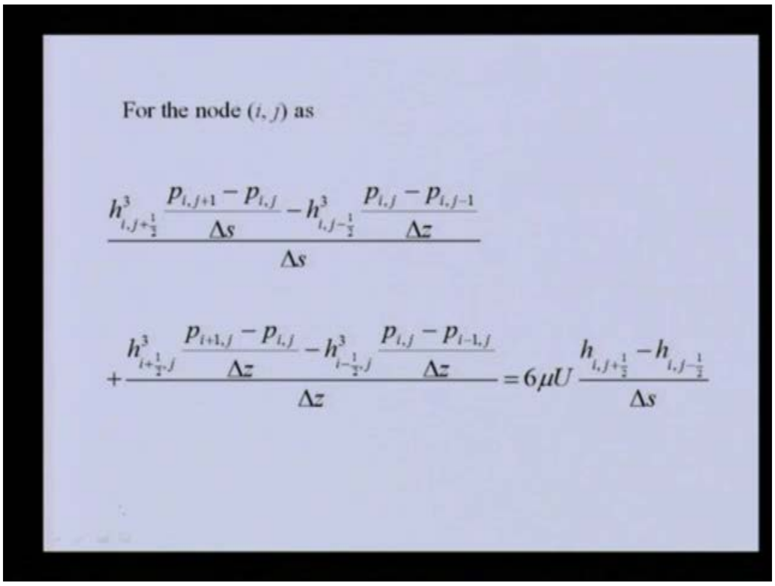


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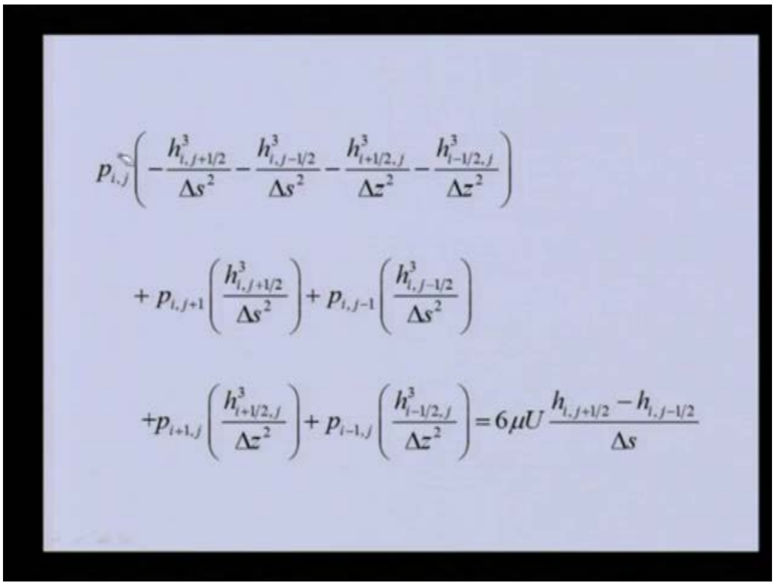


So, basically you can able to see if we take one particular node this is that particular i j node and neighboring nodes are like this one is i plus j, i minus i comma j minus 1, this is i plus 1 j and this is the i minus 1 comma j. So, will be predicting the node displace node pressure here with the help of this neighboring nodes. If we require further points we can able to considered in between and for that particular case we node description will be like this.

Now, the Reynolds equation we can able to write is in the partial differential equation like this. So, each and every term because this was second derivative, so we can able to express the derivative with respect to S of the pressure like this, this is the film thickness. Similarly, the other variation with respect to Z, which also second derivative, so that will take this form, right hand side we had remove the time independent term we are considering the study state condition. So, that is the variation with S was there of the second derivative, first derivative. So, only this term will be there. So, this equation are now can we can able to arrange such that we can able to club the terms containing like a pressure variation at i j in one place, i comma j plus 1 at another place like that. So, all five node positions pressure terms we have collected and we express like this.

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$$p_{i,j} = \frac{a_6}{a_1} + \frac{a_5}{a_1} p_{i+1,j} + \frac{a_4}{a_1} p_{i-1,j} + \frac{a_3}{a_1} p_{i,j+1} + \frac{a_2}{a_1} p_{i,j-1}$$

$$a_1 = -\frac{h^3_{i,j+1/2}}{\Delta s^2} - \frac{h^3_{i,j-1/2}}{\Delta s^2} - \frac{h^3_{i+1/2,j}}{\Delta z^2} - \frac{h^3_{i-1/2,j}}{\Delta z^2};$$

$$a_2 = \frac{h^3_{i,j-1/2}}{\Delta s^2}; \qquad a_3 = \frac{h^3_{i,j+1/2}}{\Delta s^2};$$

$$a_4 = \frac{h^3_{i-1/2,j}}{\Delta z^2}; \qquad a_5 = \frac{h^3_{i+1/2,j}}{\Delta z^2};$$

$$a_6 = 6\mu U \frac{h_{i,j+1/2} - h_{i,j-1/2}}{\Delta s}$$

This can be written now in a more regression way. So, the pressure at the i j we are predicting with the help of neighboring four of nodes. This various constants are known quantity either a film thickness or the circumferential distance or radial distance for axial distance, so with this equation is very important. So, you can able to see that we are predicting the pressure at the central node by four neighboring pressures and we can able to write i j for the whole domain, so starting from one corner to another corner of the whole grid. So, basically will be getting several equations like this they will be simultaneous equations and we need to solve them one by one.

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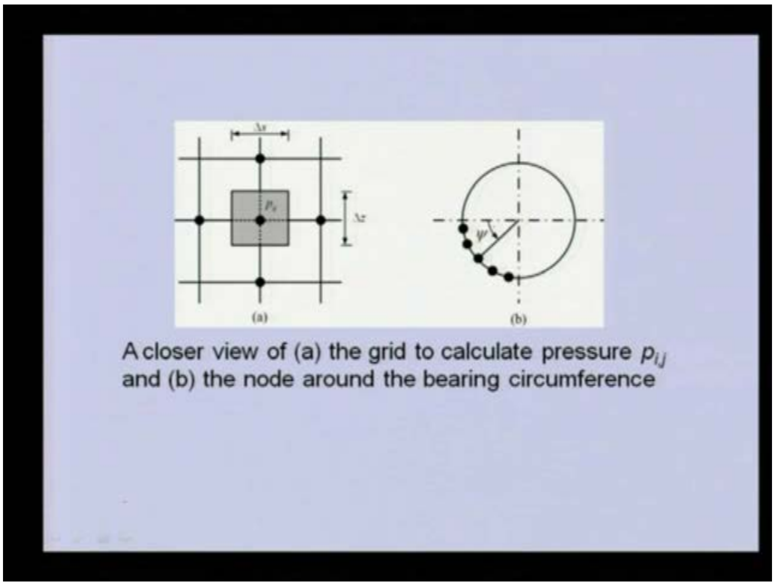


So, we are predict at this pressure with the help of the neighboring pressures. So, basically these recreation equations we need to solve iteratively that will see the procedure. But let us see this particular point, which is there in the unwrap position. If you want to see that particular point in the bearing this is the point the position of that angular position we had representing by xi. So, this particular pressure if we multiply the area this particular area.

So, we will get the force how much it is exerting onto this bearing, because this is the pressure at this point if we take half of this also it multiply by this area. So, that will give how much force, which it is giving in the radial direction this particular fluid. So, this the way we will be obtaining form pressure the force value and then we can able to take the component of these forces, which are in different direction in the horizontal direction vertical direction and then we can able to see they are force balance.

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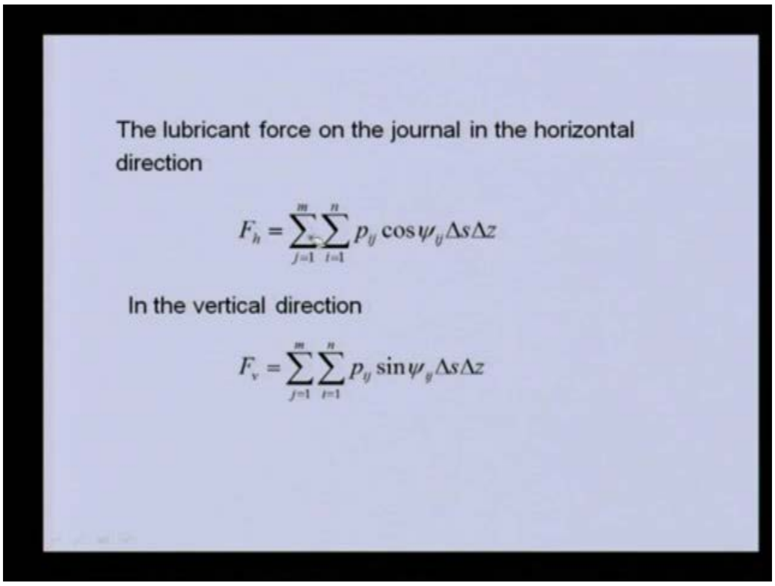


So, you can able to see this is the force balance the horizontal direction. So, component of that pressure in to the area and this is the force and this is the component in the horizontal directions. Similarly, if we take sin of that xi than will get the horizontal direction, vertical direction force. Basically, in this the journal weight which acts in the downward direction, but there is as such known there is a horizontal force. So, basically we need to find out the, this pressures iteratively using the previous recursion relation. Such that this force is equal to the weight of the journal, but this should be 0, because there is no net force in the horizontal direction. So, this is the basically outline in which this I already explained.

So, basically this particular case we need to obtain the pressure at each and every node and we need to and we are obtaining from neighboring nodes. Once we have obtained at that we can at switch over to a next node. In this particular case one important thing is if we are predicting the pressure negative at any point of time then because Reynolds equation does not take care of the negative pressure. So, we need to put that pressure equal to 0 and proceed. So, we will see that iteratively we will be solving the pressure at each and every node from one end to another and once we have done once again we can come back to the original position.

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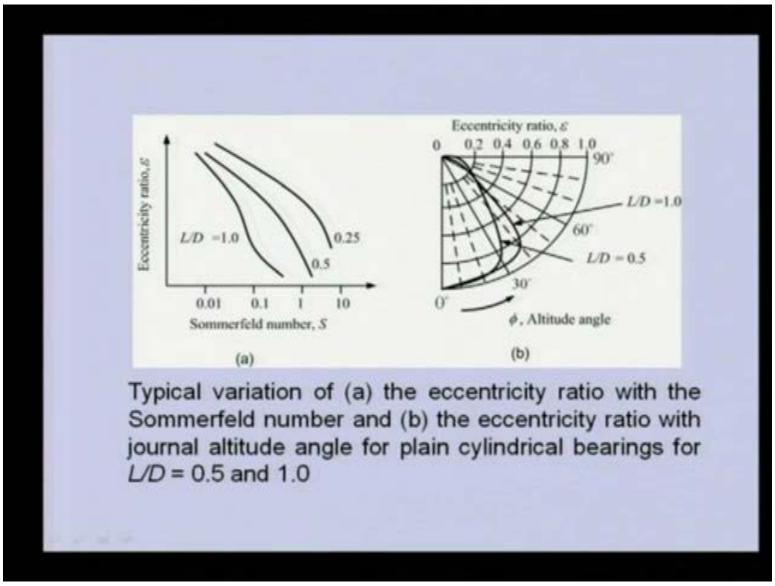


So, we will be keep repeating this till we will get no variation in the pressure that means in the subsequent iteration the variation in the pressure is negligibly small up to the desired decimal point. Once we get the this particular pressure variation that we have seen that we need to check that the horizontal component is 0 or not or the whole bearing. The vertical component of the pressure is equal to the weight of the bearing or not. If that is the case then we are use the solution part and from there then we can able to, we can able to get basically position of the eccentricity and the altitude angle.

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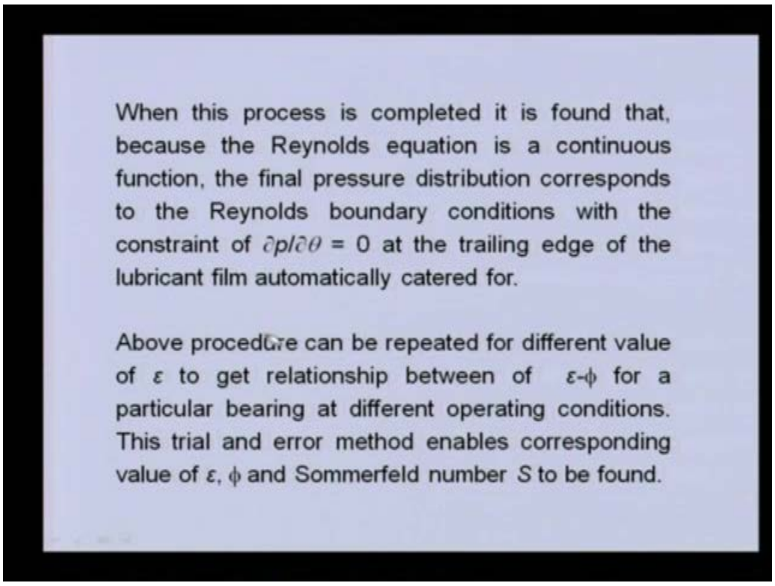

So, for that let us see; so, basically we need to follow this procedure and for various values of the epsilon. That means we need to once we have chosen, once we have got the convergence. Then basically we will be getting the equilibrium position of the journal that means eccentricity and the amplitude angle. We can able to find out various combinations of this that means the amplitude angle and the eccentricity. So, in this particular plot we can able to see that the sommerfeld number and this eccentricity ratio variation have been provided for different L by D ratio. So, for finite bearing you can able to combine this kind of non dimensional parameter, because for a particular bearing we can able to calculate the sommerfeld number from there we can able to predict the eccentricity ratio.

So, this basically gives the equilibrium position of the bearing. So, you can able to see here even if we want to plot the variation of the path for different L by D ratio. So, this contain both the eccentricity as well as the this altitude angle. So, this is having more information regarding the bearing equilibrium position. So, this was this was again I am repeating, this was very brief description of, how we can able to solve the Reynolds equation using finite difference method or using shaft bearing approximation? How we can able to get the coefficients? Now, I will introduce very briefly the rolling element bearing and how we can able to get the linearized stiffness coefficient from this.

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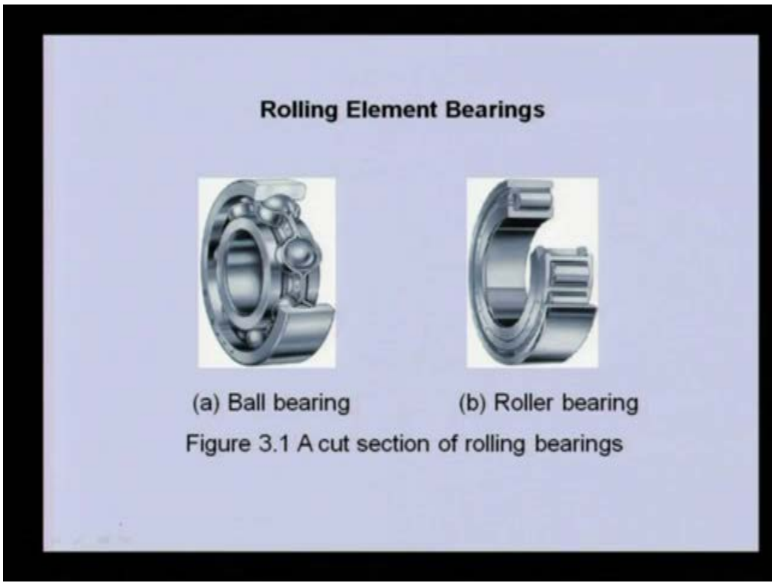

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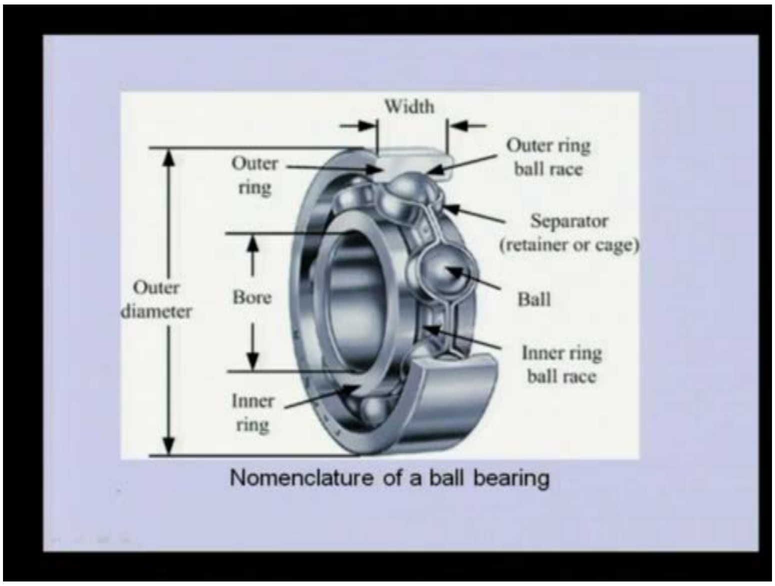


So, in this particular case you can able to see this is a ball bearing, this is a roller bearing and this is the shape of the rolling element is cylindrical, here it is in the spherical shape. The close view of the bearing is that this is a rolling element ball and there is a inner race ring, outer race ring, there is a groove on which this particular rolling element role. So, you can able to see this groove and this rolling element bearings are totally a case kind of thing that is called separator or retainer. So, this separate this ball with each other. So, that they should not collide and other dimensions like bore where the shaft will go over the outer diameter. So, the housing dimensions we need to make up the size, so that the bearing can go in the housing, this is the width of the bearing, it is a basic nomenclature of the ball bearing.

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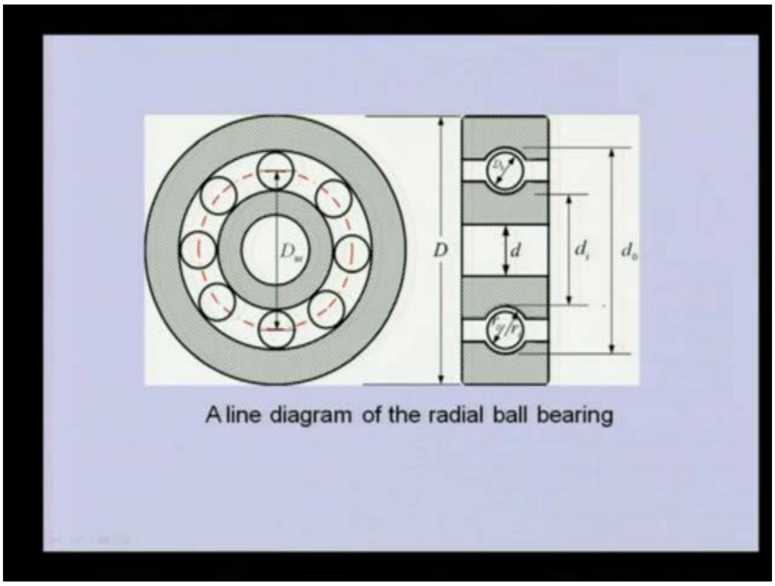


More close view of the this particular ball bearing you can able to see, if we join this circle this ball centers, this is a circle imaginary circle, which we called it as a pitch circle. Diameter of that is pitch diameter and this is the ball and these are the grooves and this ball the roll over inside this groove and this grooves are having different radius as compared to the ball. Generally, this radius is more, so that the ball can freely role on this grooves. Apart from that some of more dimensions are there this is inner groove diameter outer groove diameter.

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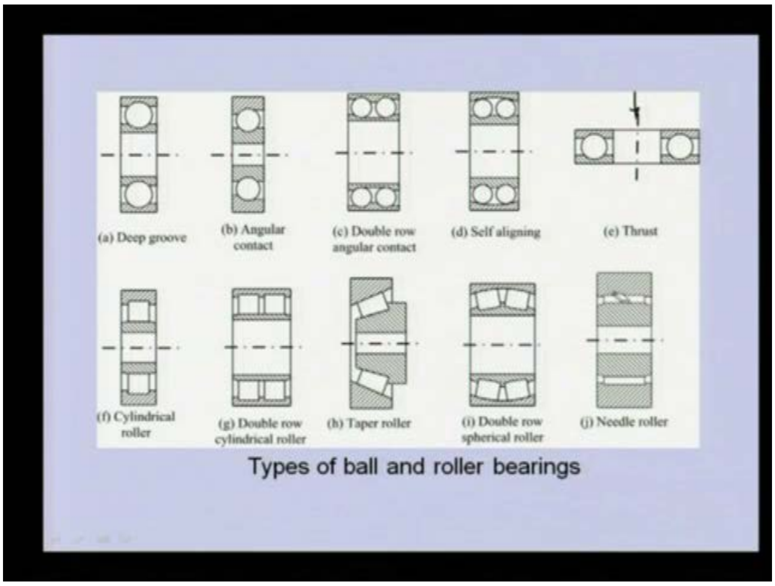

Now, let us see various kind of now ball and roller bearing types. So, that means this is a deep groove ball bearing, this is angular contact ball bearing. In this particular case the angle of contact of the ball bearing is large, we will see, what is the contact angle in the subsequent slide? Double row angular contact ball bearing, so two rows are there, this is self aligning bearing. So, in this particular case the inner ring, inner ring can tilt with respect to the outer ring by large amount. Then this is a thrust bearing generally the load comes in the vertical direction, so that they take the load here.

So, load comes like this in this particular case, this is cylindrical bearing, cylindrical roller bearing. So, shape of the roller is as cylindrical, double roll cylindrical bearing, tapper roller bearing. So, this is first stem of cone, double row spherical bearing. So, the shape of the this contacting surface are spherical in shape, similar to the cylindrical bearing, but curvature are more at the on the contacting surface. This is the middle bearing in which the length is relatively long as compared to diameter of the bearing.

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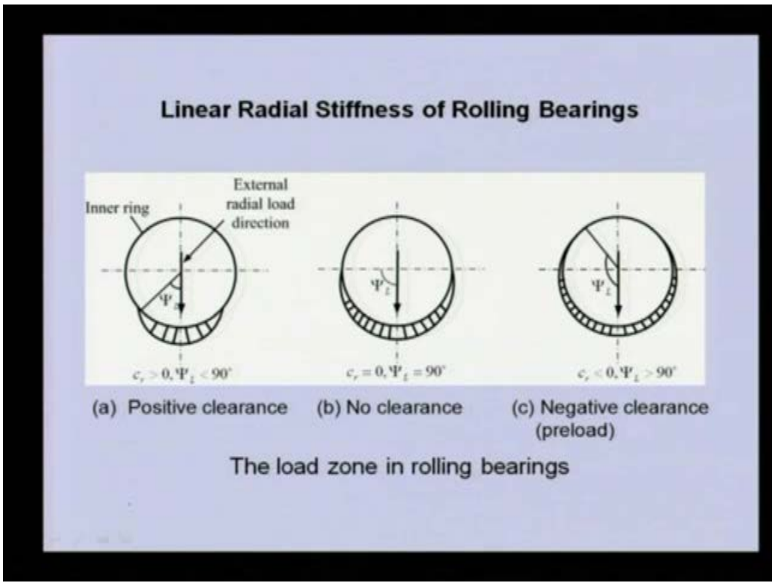


So, this is the basically load zone in rolling bearings. So, this is the ring let us say and when we apply the load into the inner ring or outer ring, generally not the all the rolling elements take the load. Partly some of the rolling elements take load and how much is the load zone is defined by the clearance. So, if clearance is positive then we will see that the node zone, which is defined half of this total angle will be less than 90 degree.

So, only the rolling elements, which are within this area, will be loaded. If there is no clearance then this load region is 90 degree that means total 180 degree, here whatever the rolling elements are there they will take part in the load, they will be free they will not take any load. Here also the rolling element, which are here they will not take any load. But if we have neglected clearance or the preload if ball are preloaded then we will see that the contact zone is more than 90 degree and most of the rolling element take the load. So, generally if we are providing the preload we will find that we have more number of rolling element taking part in the load sharing.

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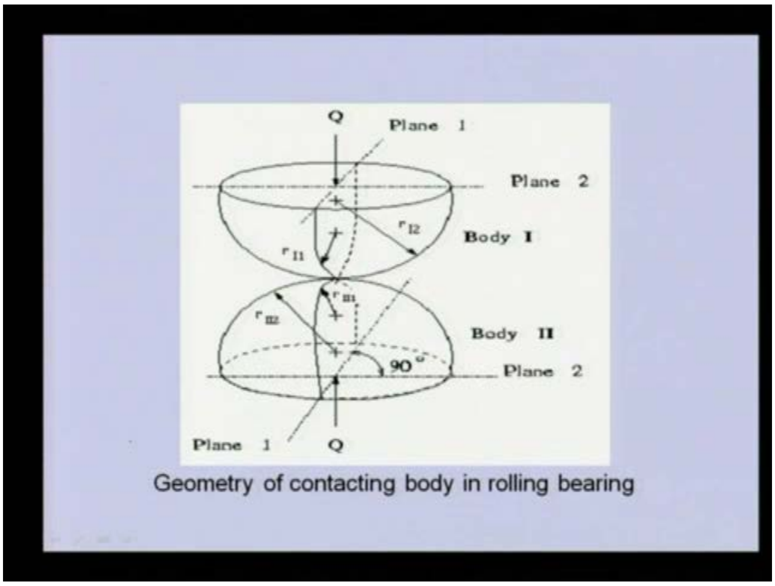


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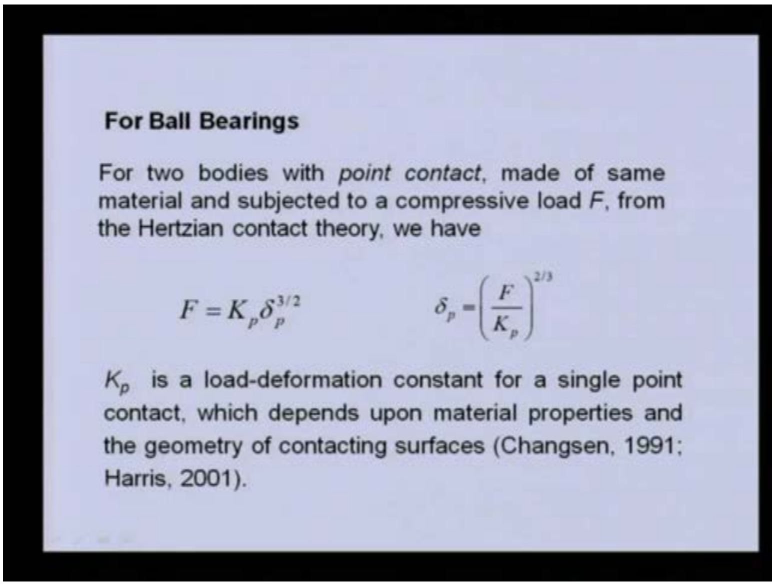

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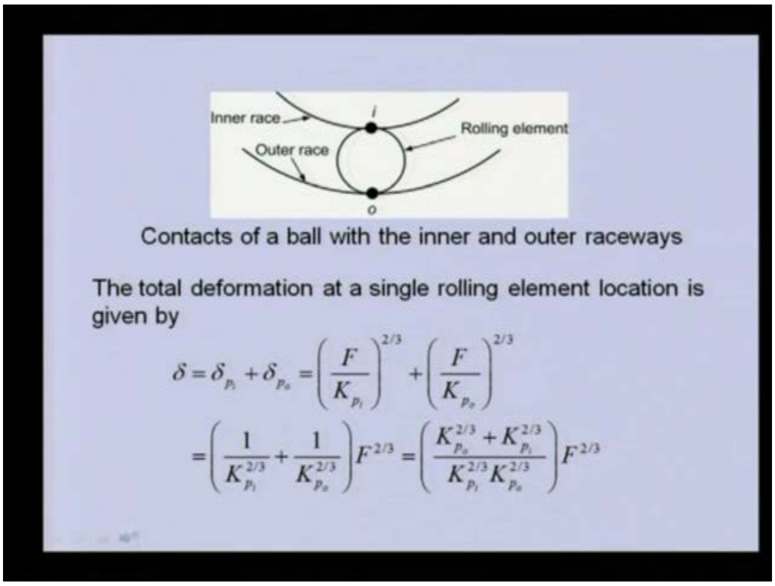


So, generally using the Hertzian contact theory, we define the load versus deformation relation between two contacting bodies and for point contact that means two spherical contacts we have this relation. So, you can able to see this is the load, this is the deformation at the contact point or point load, this is the exponent 3 by 2 for ball bearing that is point contact. This is the load deformation constant, which depends upon the geometry of the bearing and material property of the contacting bodies this can be written like this.

Now, this is for single point contact at one of the contact, if we have in the ball bearing we have two point contact; one at the inner ring and at the outer ring. So, these two deformations the total deformation can be defined as summation of this two and the previous expression can be substituted for inner contact and outer contact. So, it can be simplified like this because load is same, so because load transmitting the same, so we will get this expression.

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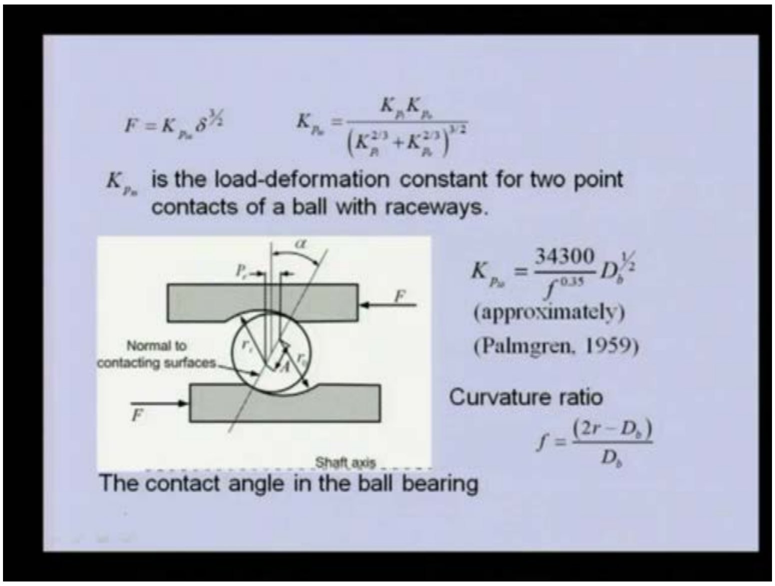


So, basically this is some kind of equivalent the load deformation constant for the two point contact, which is defined like this for inner contact and outer contact this. Now, this particular approximately has given by a palindrome like this where D b is the diameter of the ball. f is given as the curvature ratio, which is given like this and r is the groove radius, in this figure you can able to see the groove radius clearly. So, it is the radius of the groove, which is groove radius this is the deep groove ball bearing if we apply axial node this inner ring and outer ring they shaft relative to each other.

Now, you can able to see the points of contact of the ball with the races are here. So, this is the line where the load will be acting in this direction, because point of contact is here and the angle of this with respect to the vertical is called contact angle. This distance is the end play P is the end play, so this happens because of the clearance in the bearing. So, this contact angle is important because it defines how much axial load it can take.

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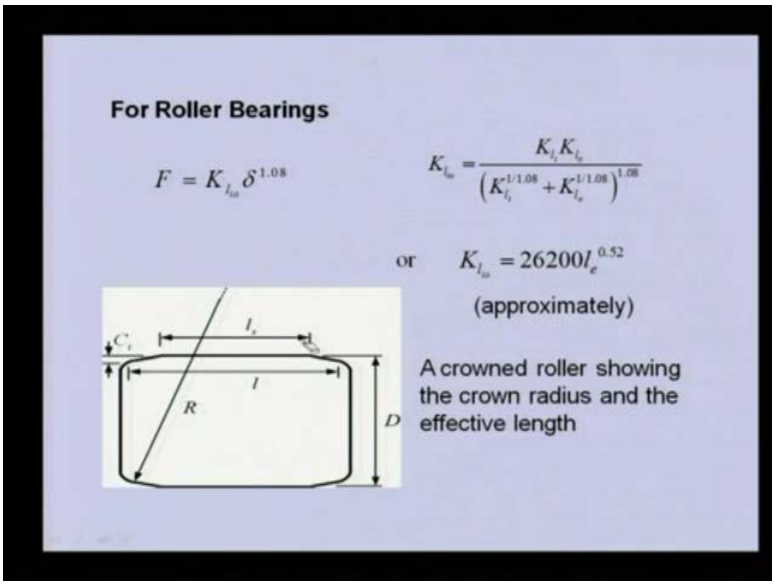


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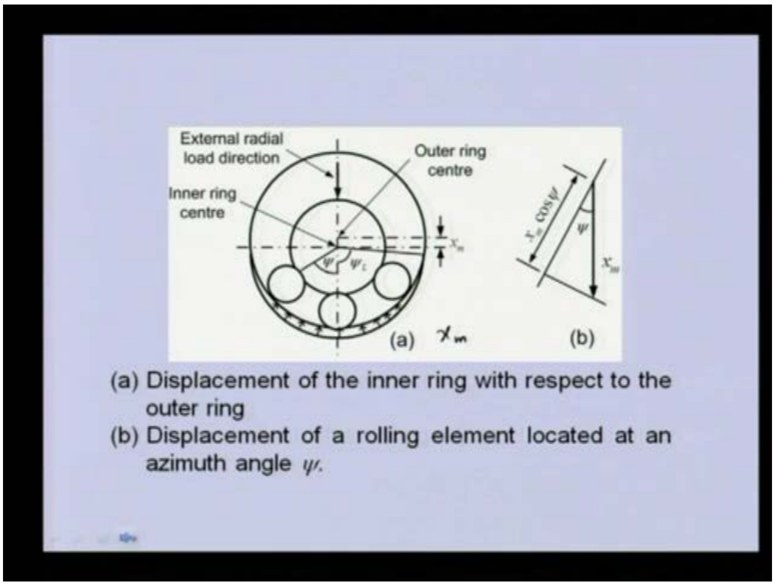


Similarly, for roller bearing similar expressions only the exponents are different and for two point contact this will be defined like this. Approximately this is given as this only the effective length of the roller is incorporated in this. This is the roller and you can able to see at the ends we have chamfering. So, effective contact will be in this region and that is we need to consider in the calculation of this. So, this is the deep groove bond bearing, we are applying a external load here, let us say to the inner ring also bond are getting compressed.

So, this is the load zone, because of clearance the load zone is less than 90 degree. Now, one particular bond which is at the xi angle this will, in this particular case this particular rolling element, which is let us say just below the direction of the load is having, let us say x m displacement. So, you can able to see the inner ring is getting displaced in the direction of the load by x m. So, this particular rolling element, which is at the xi angle will be having displacement x m into cos xi. Similarly, this xi can be this bearing or this ball or any other angle.

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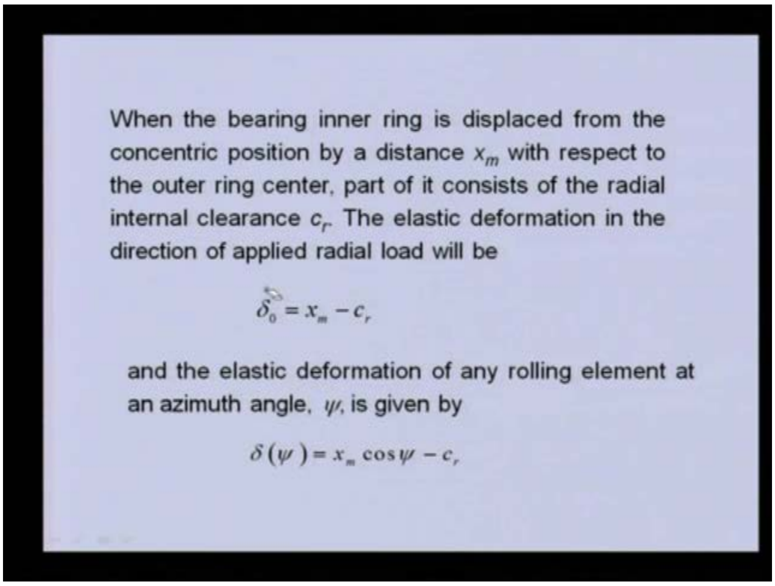


So, basically you can able to see that the actual deformation of the bond will be displacement of the ring with respect to the let us say, inner ring with respect to the outer ring minus the clearance, because the clearance we cannot define that is a deformation. So, displacement of the inner ring with respect to outer ring center minus clearance will be the deformation of the deformation. As we have seen in the previous slide the deformation at an angle xi of the bond will be x m cos xi minus clearance, because in that direction also radial clearance is c r. So, this is the deformation at any angle of the ball and if we keep that deformation equal to 0 that means deformation will be 0, only when the ball is outside the this inner zone here.

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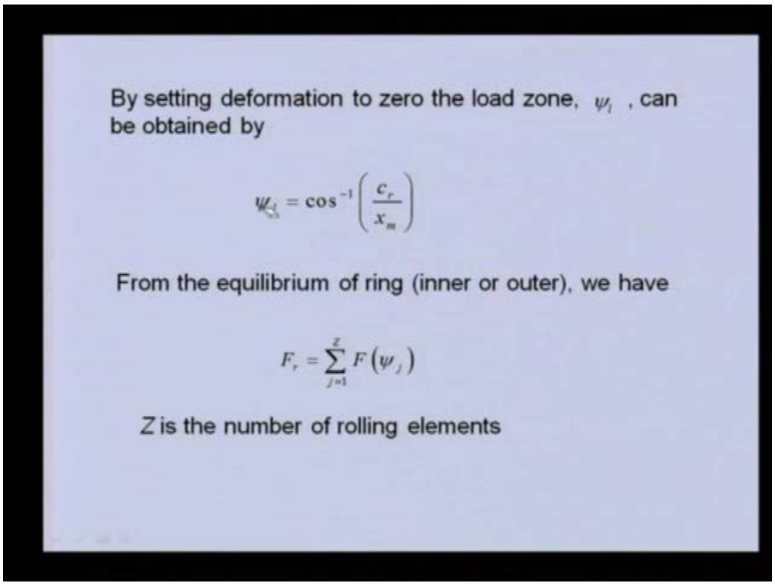


So, basically putting this equal to 0, we are trying to find out what is the angle xi for which there is no deformation. So, this will give us a load zone relation and this load zone relation we can able to see for 0 clearance we will be having pi by 2, pi by 2 load zone. Now, we can able to take the equilibrium of a ring that means all such forces because of the ball at various angular position xi. We can able to sum up for all the rollers if this rollers are outside the contact zone these terms will be 0. So, that should be equal to the external applied load where z is the number of rolls.

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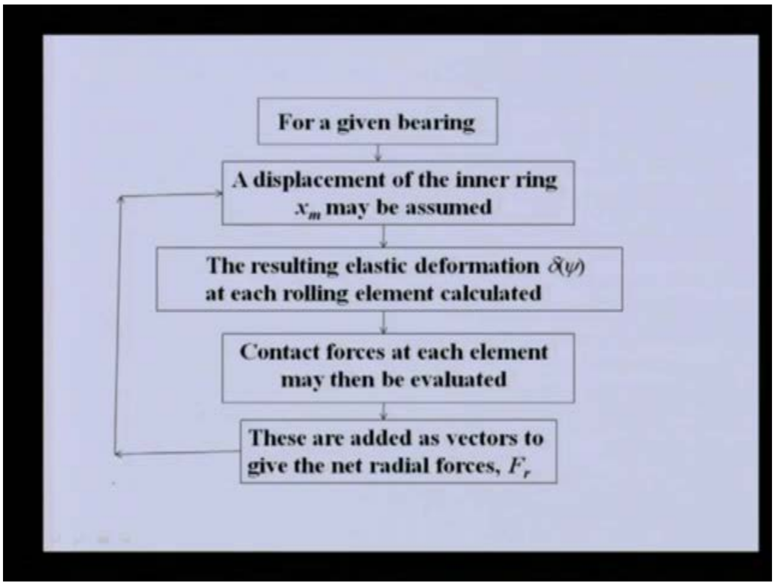

So, basically for a given bearing we can able to chose this particular displacement from that we can able to get various displacements at various ball locations. From there we can able to get the contact forces, because once we know the this deformation we can able to its additional contact and relations to get the contact forces. These contact forces then we can added up we can add the component of that in the radial direction and check whether that is equal to the external load or not, if not then again we need to give a we need to the another displacement.

Basically, here totally we are try to find out what should be the displacement for that particular given radial load. If this displacement is not equal to the actual one this will not satisfy then we need to repeat this iteration till we get the close value of the x m for which this the bearing is having that much displacement. In rolling element bearing, for a given force, obtaining the displacement of the inner ring with respect to outer ring is iterative procedure. If we if we are we can able to obtain for a particular load, how much the deformation? And if we vary the load and again, what is the change in the change in the deformation? Then basically what we are obtaining we are obtaining a variation of the load with displacement and from this we can able to get the stiffness.

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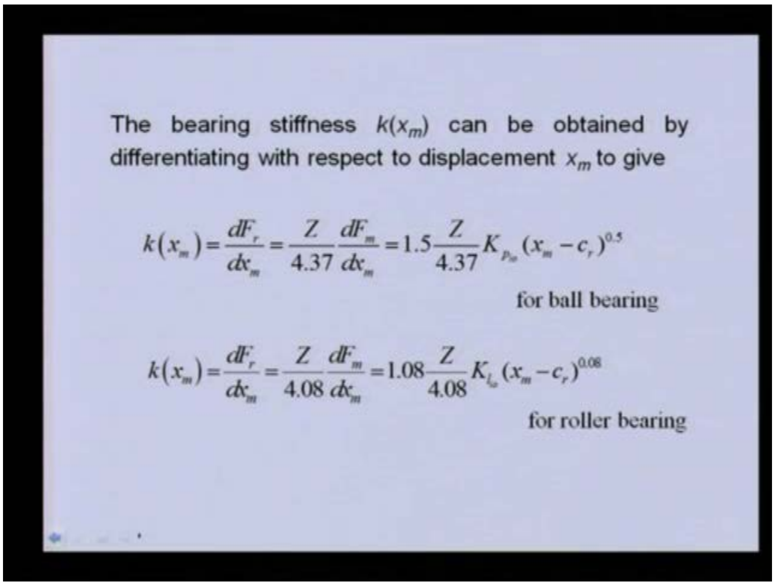


So, for this I am giving the expression of the stiffness of the rolling element bearing, the first one is from ball bearing. So, this particular stiffness expression is this one. So, you can able to see this is a number of rolling element, this is the load displacement constants. This is the deformation or displacement of the inner ring with respect to outer ring, this is the clearance, this is a factor which we have used for ball bearing. Basically, it gives a relationship between the actual force, which is we are applying like if we are applying to the bearing F r force. So, what is the force F m that the ball which is just below the in the direction of the radial load is taking this particular factor defines that. So, let us see that particular expression.

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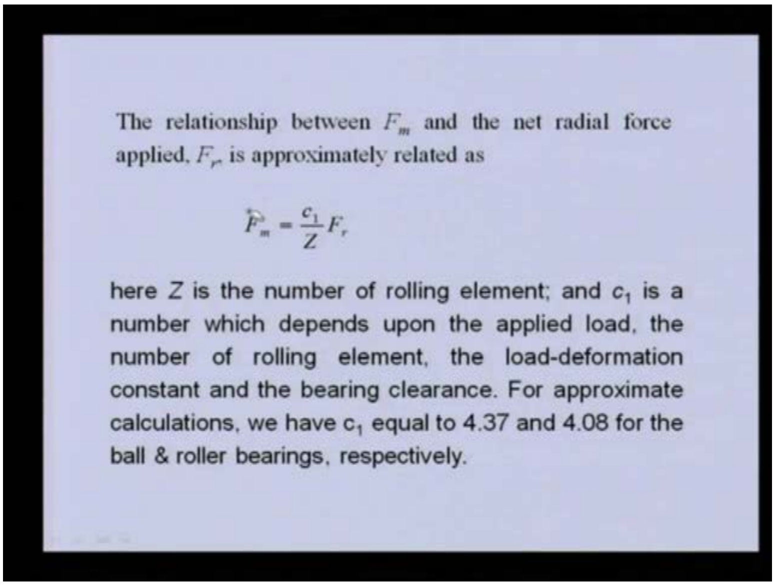


So, this is the force which a particular ball, which is just below the radial direction of the load is there. c 1 is the constant, which is given as 4.37 for ball bearing and 4.08 for the roller bearing, z is the number of roller. So, this expression have we have used in this calculation of the stiffness and this is coming from the exponent of the, at zonal contact for ball bearing and roller bearing. So, these two expressions can be used to obtain the linearized coefficient of the stiffness. So, these two expressions can be used to obtain the stiffness coefficient for the ball bearing and the roller bearing directly. For more detailed calculation of these load deflection calculation.

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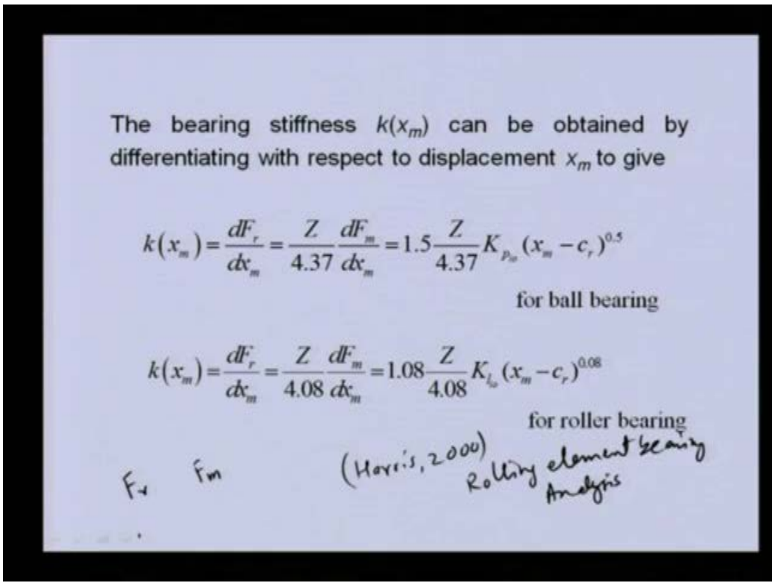


People can refer the Harris that is 2000 book on rolling bearing analysis rolling element bearing analysis book, so because a detailed is not possible to cover in this particular, in single lecture. So, I am referring this particular book, which itself is 1000 page, for more detail analysis of the rolling element bearing. In today's lecture we cover very brief idea about the fluid film bearing and the rolling element bearing.

It is not possible to cover the concept of these two bearing in a single lecture. But just to have idea of; what kind of bearings fluid film bearings are there? What type of rolling element bearings are there? Specially, how we can approach the calculation of the rotor dynamics coefficient? That was important that I try to outline in this, but for more detail obviously there are dedicated books on ((Refer Time: 55:37)) that can be referred for calculation more detailed calculation of the this kind of bearings.

For bearing designed to carry a vertical loads only, for example; the gravity load. The relation between the eccentricity ratio and journal attitude angle phi may be determined by investigating different value of phi for a given a value of epsilon until the value of F h that is a vertical horizontal force is found to be 0. So, if gravity is the only load on to the rotor. So, F h if it becomes 0, then we can have the combination of the eccentricity ratio and phi for the rotor for that particular operating condition. For example, once we choose an arbitrary value of epsilon and phi then corresponding film thickness can be obtained.

Since, phi and epsilon and phi determines the position of the shaft with respect to the bearing bore.

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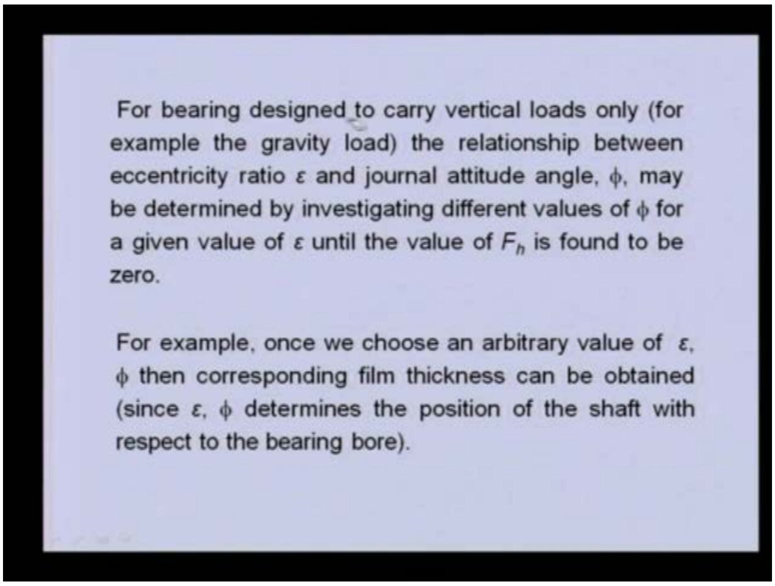


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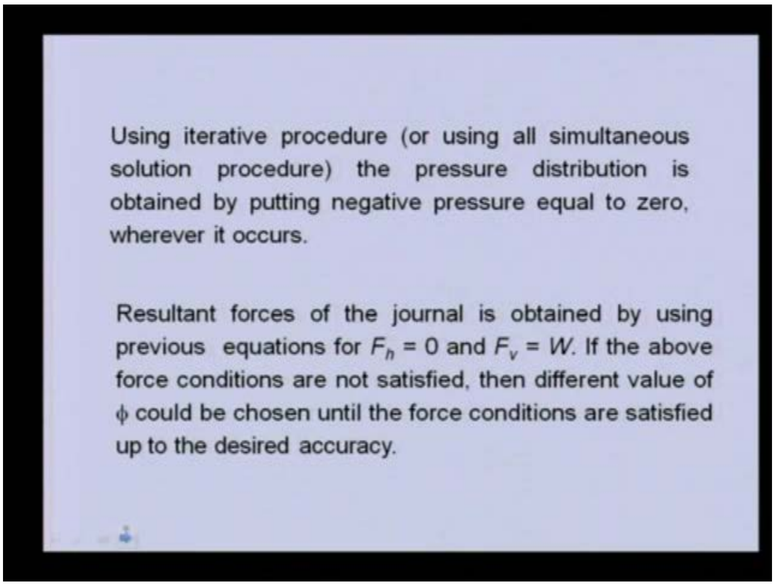


Using iterative procedure or using all simultaneous solution procedure by solving the finite difference equation simultaneously the pressure distribution is obtained by putting negative pressure equal to 0. So, whenever we are obtaining negative pressure we are keeping that equal to 0. Because, resultant forces of the journal is obtained by using previous equations for horizontal force equal to 0 and vertical force is equal to weight of the journal. If the above force conditions are not satisfied then different value of phi could be chosen until the force conditions are satisfied up to the desired accuracy. So, basically here we are trying to find out for a particular epsilon value what should be the phi value to satisfy these two conditions.

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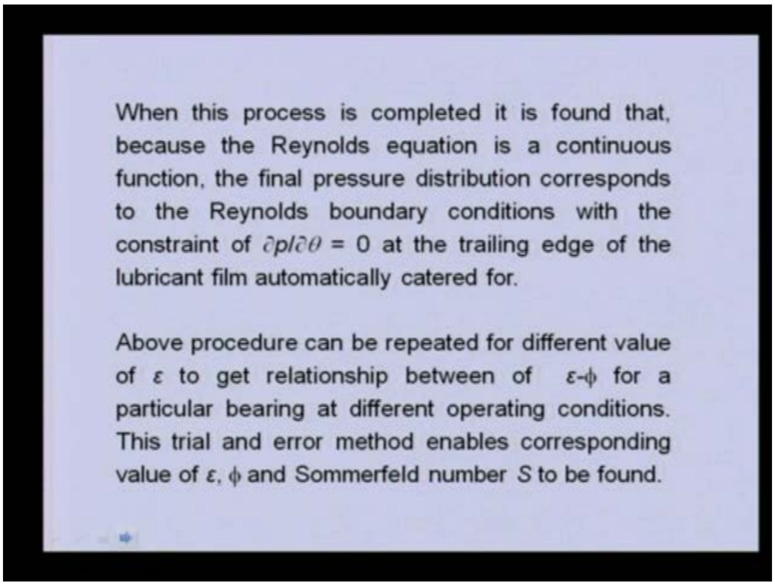


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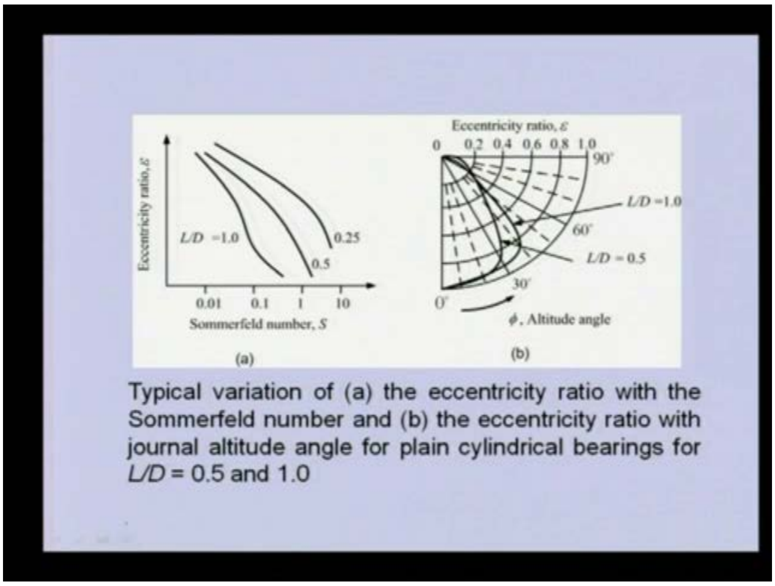

When this process is completed it is found that because the Reynolds equation is a continuous function. The final pressure distribution correspond to Reynolds boundary condition with the constraint of this gradient is equal to 0, which we described earlier at trailing edge of the lubricant film automatically, which is catered for. Above procedure can be repeated for different value of epsilon to get relationship between epsilon and phi for a particular bearing at different operating conditions. This trial and error method enables corresponding value of epsilon and phi and sommerfeld number S to be found.

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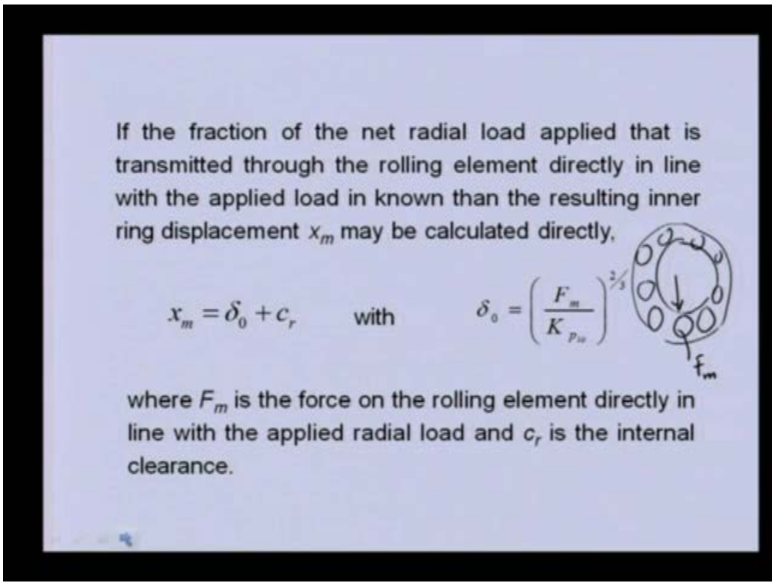


So, with this we can able to plot the eccentricity ratio with sommerfeld number, which we defined earlier. So, basically it depends up on various operating parameter of the bearing including speed for various geometry we can able to plot these relations. Even we can able to plot the epsilon and the altitude angle for various geometrical conditions of the bearing like for L by D ratio 0.5, this is the curve and the second curve is for L by D, L by Dis equal to 1. If the fraction of net radial load applied that is transmitted through the rolling element directly in line with the applied load is known than the resulting inner ring displacement may be calculated directly.

So, we have that is the bearing we having, then let us say this is the rolling element which is here. So, if we are applying a radial load on to the inner ring and this particular roller or ball which is just below this particular radial load. So, if you can find out what is the load shared by this particular rolling element as because, this load changes with the angular position of the ball. So, if let us say the load shared by this particular rolling element is F m. So, if we can obtain what is the load shared by this particular bearing load element, then it is easy to obtain the load distribution also the stiffness of the bearing.

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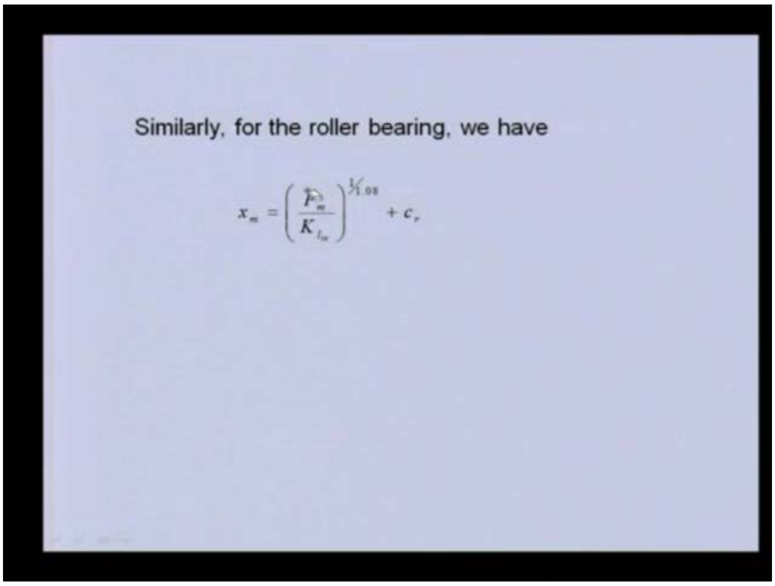


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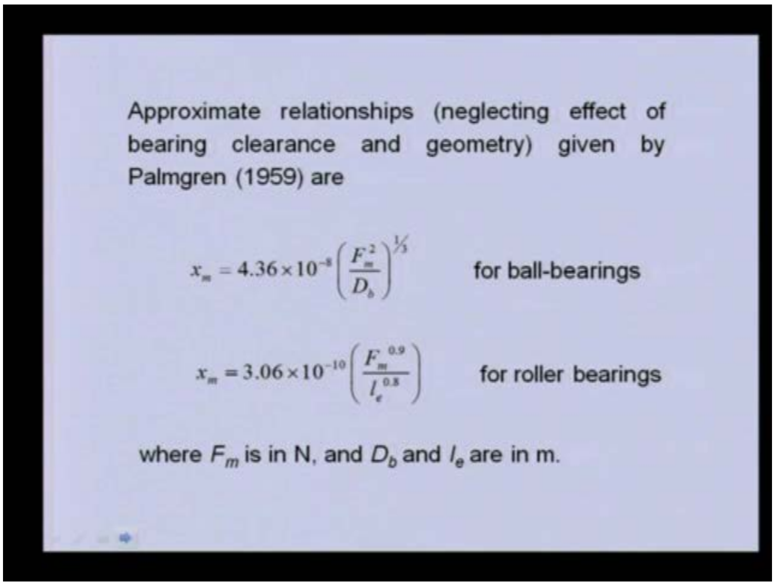


So, this is the total displacement of the bearing or the displacement of this particular ball also they will be same. So, this is the deformation plus the radial clearance and this deformation is of the ball is due to the load shared by that particular rolling element. So, we can able to get using the axial relation in which the exponent will be 2 by 3 and in this particular case this C r is the clearance radial clearance of the bearing.

Similarly, for the roller bearing we will be having a relationship between the deformation of the maximum loaded roller often like this in which the exponent will change it is radial clearance of that particular roller bearing. The same relations is given by Palmgren these are the approximate relationship in which neglected the bearing clearance and geometry. So, the similar relation is provided based on the experiment. So, this can also we used, where D is the ball diameter, l is the effective length of the roller. So, this is for ball bearing and this is for roller bearing.

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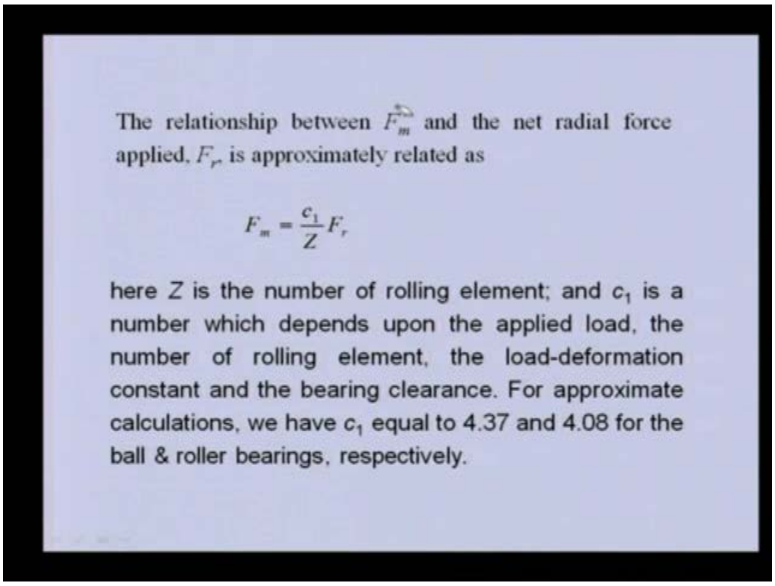


The relationship between F m the maximum load in which a particular rolling element is carrying. The net radial force F r, which we are applying to the bearing is approximately related as like this. This is also approximate formula in which the c 1 constant and Z is the number of rolling element. c 1 constant is basically it depends upon the number of rolling element, load deformation constant and bearing clearance. Approximately it can be taken as 4.37 for ball bearing and 4.08 for roller bearing. So, this relation gives directly what will be the a particular roller or ball, which is just below the radial direction share that particular load. So, once we have obtained this particular load from previous relations we can able to obtain the differentiation also.

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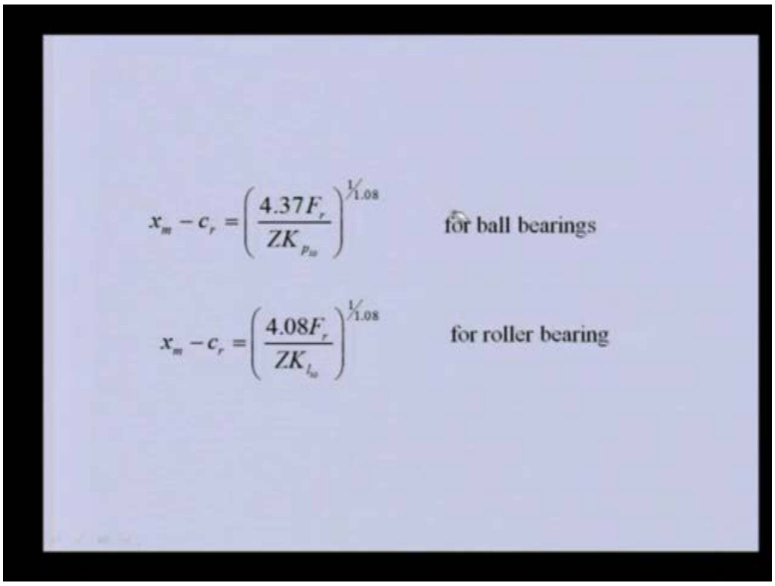


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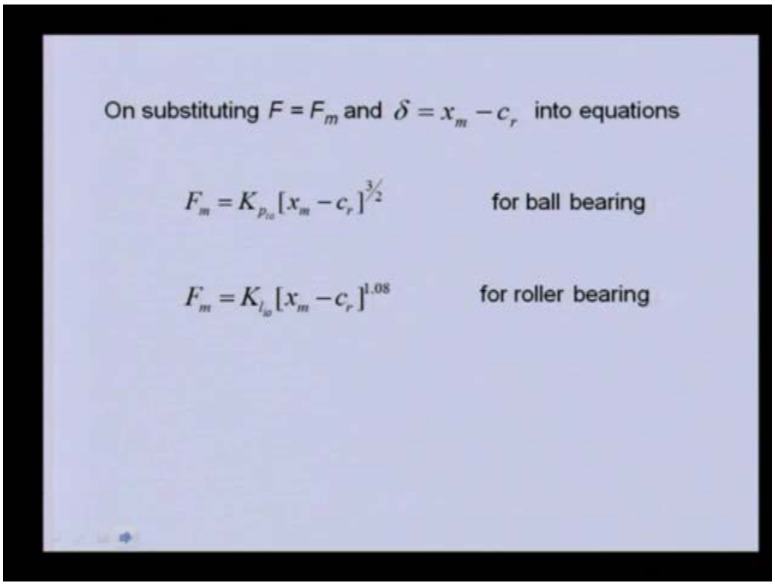


So, for ball bearing; so, in place of F m we have substituted this 4.37, F r by Z in the previous relation for ball bearing, similarly for the roller bearing. Now, for particular load F m the deformation of that particular which is just below the radial direction of the external load is given by this. They are related by Hurwitz relations like this for ball bearing and roller bearing. So, when this is for one particular ball or roller, so once we have this relation.

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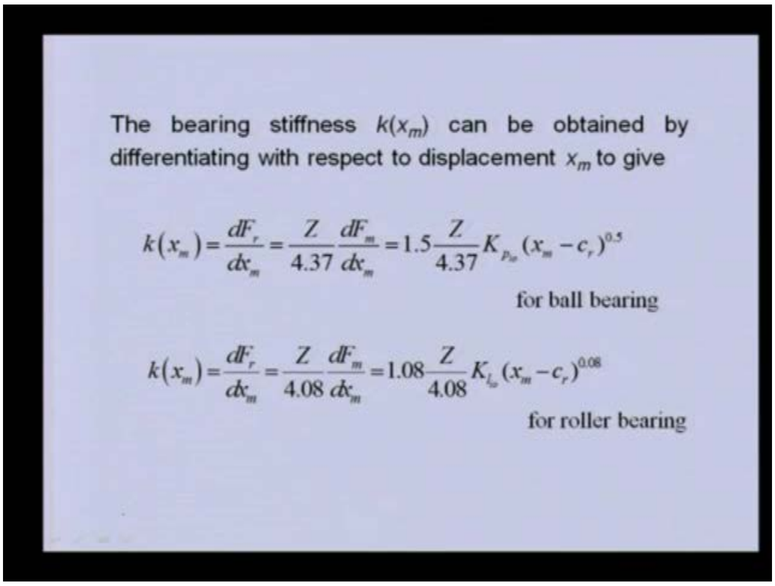


Now, we are defining the stiffness bearing stiffness, which is non-linear in nature. So, that bearing stiffness is defined as the change in the radial load applied and the deformation of the that particular roller or ball, which is getting maximum deformation. Even that is equal to the deformation of the of the rolling element that is a inner raceway or outer raceway also. So, basically displacement is the relative displacement between the inner raceway and outer raceway.

So, we can put the F r from previous expression. So, we will get the stiffness term like this and F m again we can able to put from previous relations, which we have here. So, to get the ball bearing stiffness in terms of various parameter of the bearing like clearance, this is the maximum loaded roller deformation and this is the number of roller, this is the load deformation constant for point contact. Similarly, for roller bearing we can able to obtain the stiffness, in this the exponents got changed and even this load deformation constant will be different

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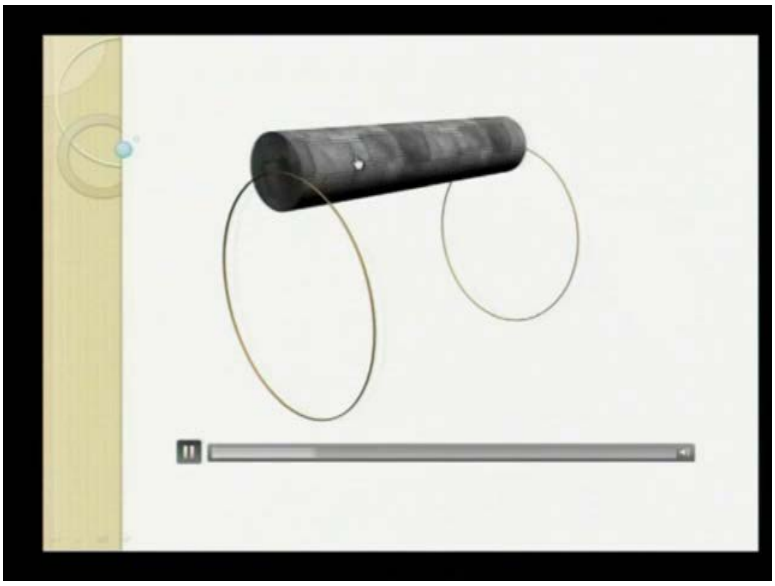

So, this is the animation for translatory forward whirl. So, we can able to see that in this particular case shaft is rotating clock wise also it is whirling in the clockwise direction. So, again we can see the shaft is rotating clockwise and also the whirling direction is clockwise. So, this is a forward translatory whirl, this is the animation for transverse or translatory backward whirl and this the shaft is rotating clockwise but, the whirling is counter clockwise direction. Again we can see once the shaft is spinning about its own axis in the clockwise direction and this whirling in the counter clockwise direction. so, this is a backward translatory whirl.

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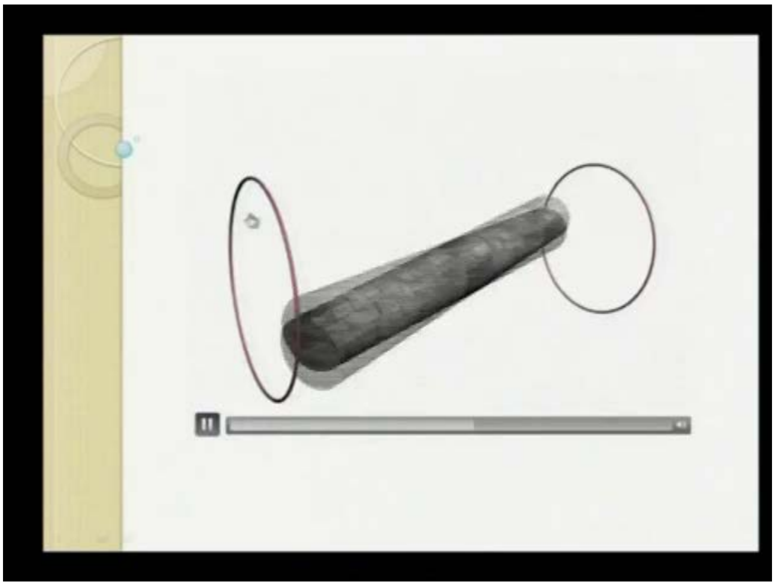

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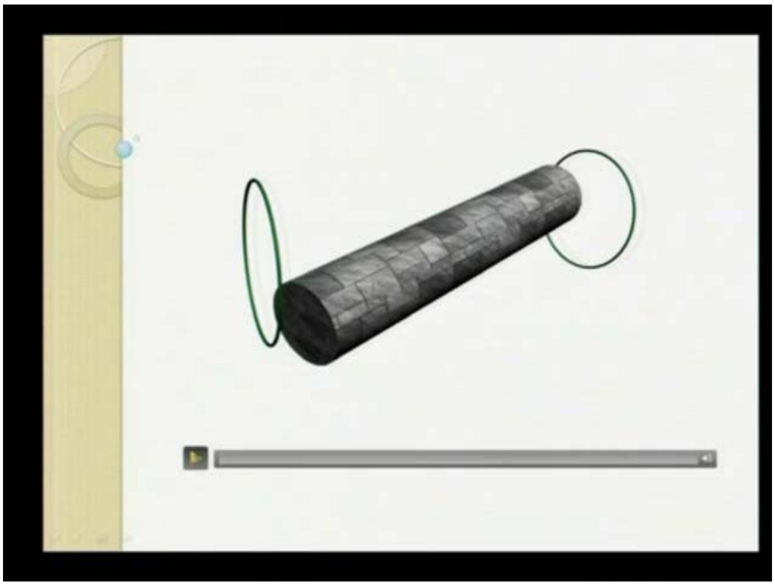

This is the animation for forward conical whirl and this shaft is rotating about its own axis in clockwise direction, but if you see end of the shaft that is also whirling in the same clockwise direction. So, that is why it is forward conical whirl again we can see this animation. So, shaft is rotating clockwise about its own axis also it is spinning whirling at both ends in this clockwise. This is the animation for backward conical whirl, in this again shaft is having same direction clockwise, but it is whirling in the counter clockwise direction. So, again you can able to see this shaft spinning and the whirling directions are different. So, whenever this is the case we will be having backward conical whirl.

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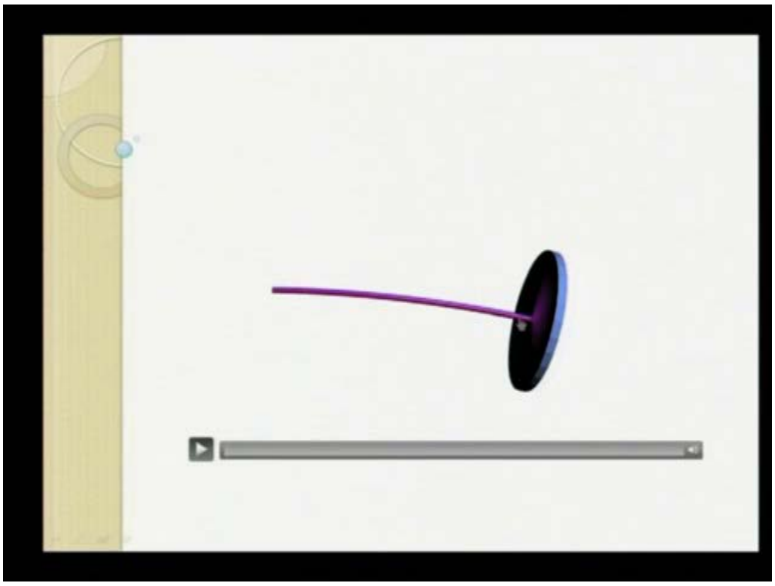

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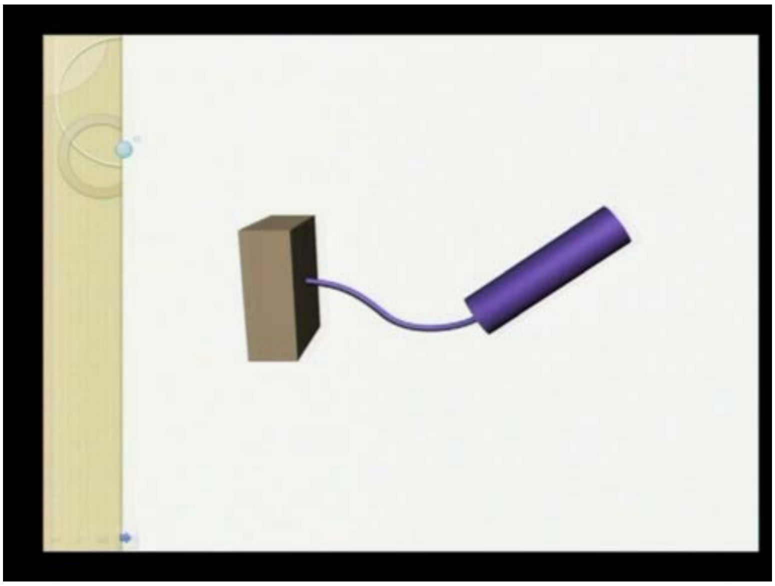

This is the animation for, this is the animation for pure whirling of the disk about its bearing axis. So, here disk is not spinning, but the pure whirling is taking place. This is the animation for the long rotor in which the whirling in the forward whirl direction is taking place. So, in fact when we are having synchronous whirl the shaft bends in particular configuration and remains in that position and the whirl take place. So, you can able to see this is a pure rotation of the shaft is taking place, because we assumed here it is having titling about its centre of gravity.

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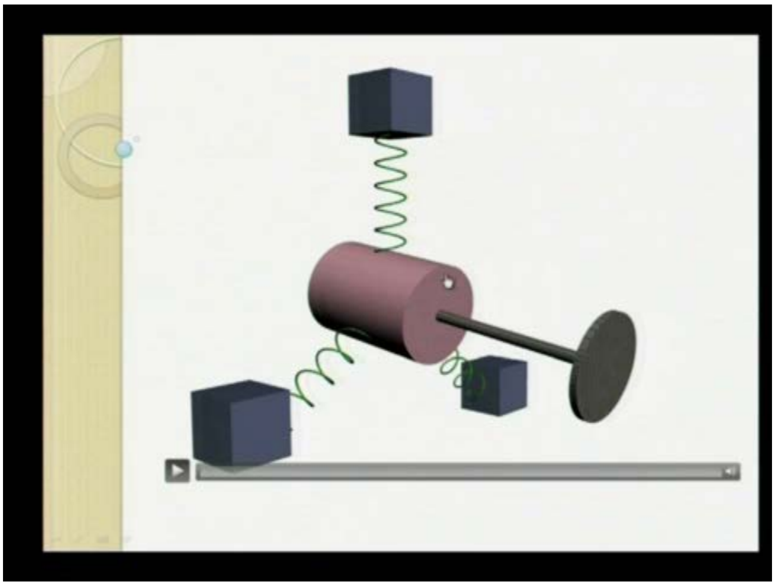

This is the animation for the motor and the shaft whirling. So, this particular motor is whirling, because this is mounted on some spin which is covering torsional stiffness and the spinning is very fast. So, here we will see the whirling and the spinning frequencies will be different. So, generally the this particular shaft will be rotating at very fast speed, but the whirling of the whole rotor system will be along with the motor will be slow. So, again you can able to see once more this animation. So, here we can able to see the whirling frequency and the spin frequency will be are different.

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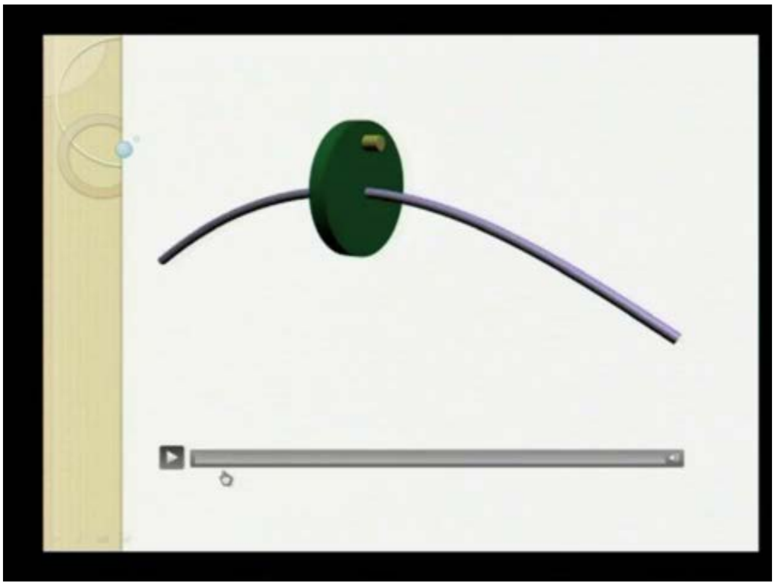

This is the animation of a Jeffcott rotor in which the disk is at the centre and the this particular unbalance, which we have shown here is away from the bearing axis that is that means we are operating below the critical speed. Because, the shaft is at the centre the titling of the shaft is not taking place in this particular case, this remains vertical during motion, but if we so… This one is another case in which we have cross the critical speed and in that particular case unbalance will come inside toward the bearing side. So, animation this is for synchronous whirl, so this unbalance remains and try to rotate about the bearing axis.

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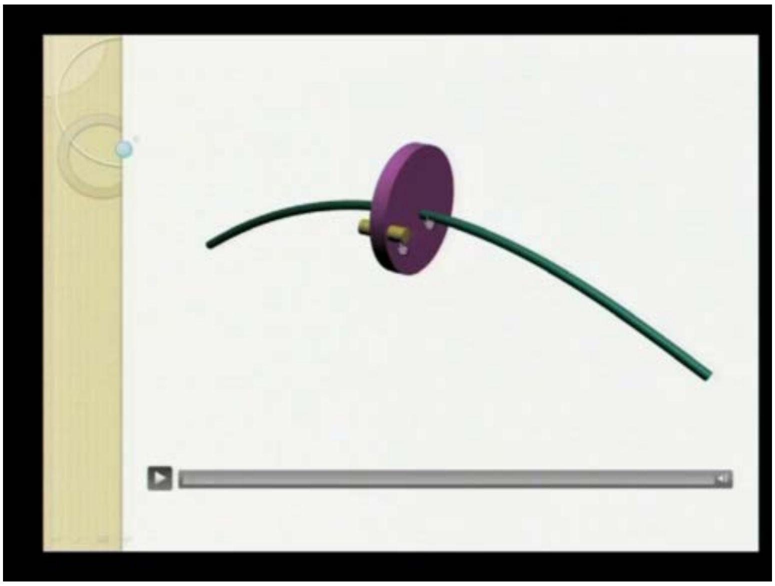

In the third case this particular case in which we are at the critical speed and at that position the unbalance is in the 90 degree with the response and basically, this will give a tangential force. So, we can able to see this unbalance is always ahead of the or toward the disk direction of motion. So, this is the critical speed condition in which the response is in this direction and the force in this. So, they have 90 degree phase difference.

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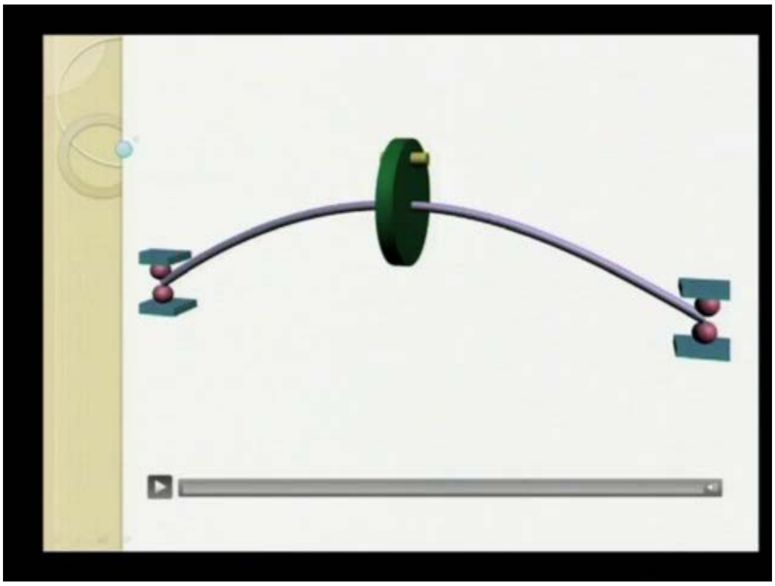

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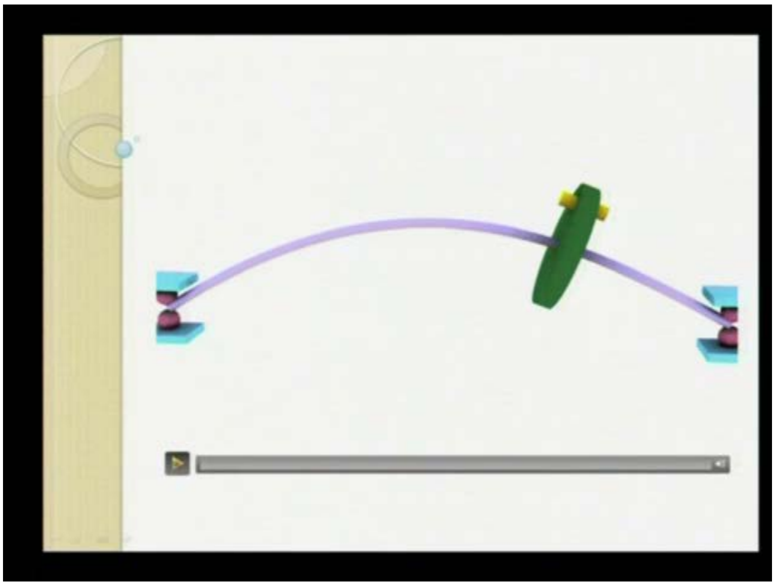

This is the Jeffcott rotor, which is mounted on bearings simply supported bearing, it is a the disk is at the centre. So, here we will try to see the motion of the disk in which there is no titling of the disk is taking place. So, titling of the disk is not taking plate. In this particular case that particular disk is offset by it is not at the centre and we will see that there will be titling motion of the disk also as it will is whirling. Because, of this we except the gyroscopic couple will be acting on this particular disk, because spinning about the shaft axis at high speed and also it is whirling. Because, of the precession of the disk about its diameter there will be gyroscopic couple. So, this animation we can again able to see when the shaft disk is offset from the centre the tilting of the disk take place along with the spinning and because of that gyroscopic couple act on to the disk.