Theory and Practice of Rotor Dynamics

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Module - 6
Transverse Vibrations
Lecture - 31
Finite Element Method

Previous few lectures, we have been dealing with the finite element method for transverse vibration of the rotor system. Today's lecture also we will continue with the finite element method. In this particular lecture, we will be incorporating two important aspects in rotating machinery; one is the gyroscopic couple. We will see that because of this, even for very small rotor system, the size of the matrices becomes large. We need to solve the Eigen value problem of force response using some kind of software like mat lab.

Apart from this, we will see how we can able to incorporate the bearing stiffness and damping into the finite element formulation of the shaft system. So, basically we will see that if rotor is mounted; if flexible rotor is mounted on fluid film bearing, how we can able to handle with the finite element method. We will see that generally, the especially the fluid film bearings, they are speed dependent. Because of that, we may find that the property of this stiffness and damping will change with the spin speed of the shaft. So, that also can be incorporated if the bearing property is speed dependent.

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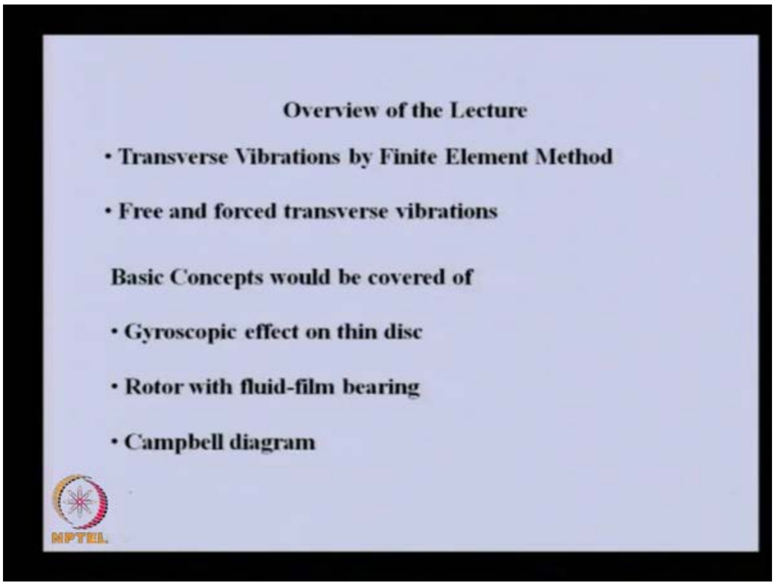

So, let us see what are the things, we will be covering. So, we will take; continue with the transverse vibration with finite element method. Both free and forced vibration can be done with the procedure described here especially; we will be incorporating the gyroscopic effect of thin disc. We will not be considering the shaft gyroscopic effect. Only the gyroscopic effect of the thin disc will be considered. But, whatever the property of the shaft we considered earlier, the distributed stiffness and mass that will incorporate. But, we will not incorporate the gyroscopic effect of the shaft as it is.

But, only of the thin disc we will be considering. Then, rotors flexible rotors mounted on fluid film bearing; how we can able to analyze such system we will try to see. In both cases as I mentioned and we know that gyroscopic couple changes with speed and fluid film bearing property also changes with speed. So, we can able to draw campbell diagram. That is nothing but variation of the whirl frequency with the speed. From there, we can able to obtain the critical speed of the system.

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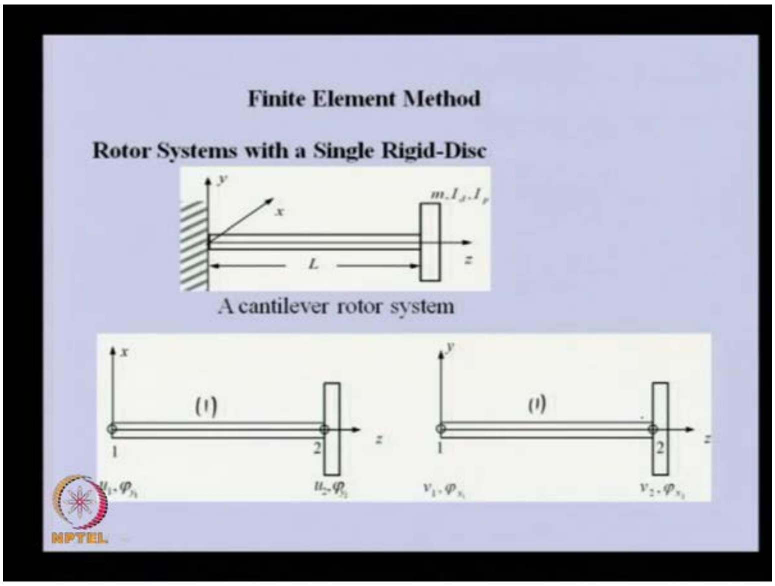

So, let us consider a simple rotor system. In this particular rotor system, this is a cantilever shaft with a thin disc at the free end. We will consider the gyroscopic couple of this thin disc. But, the gyroscopic of couple of this particular shaft, we will not consider. As I mentioned earlier, we develop the finite element in the previous lecture for Euler Bernoulli beam element. That particular model does not take into account the gyroscopic couple of the shaft itself.

Now, for that we need to have a attenuation could be modeled that we have not considered here. So, we will consider the gyroscopic couple of this. Now, because of this, we obviously the motion in two planes will be coupled. So, we need to write the element in both the planes. So, let us say for simplicity we are considering for illustration purpose only single element we are considering. This is the element 1 in z x plane. This is same element in other plane, that is y z plane because mass will be there at node 2. This is a node 2. At each node, there is 2 degree of freedom.

So, in this particular plane, which is horizontal plane, this will be a displacement and angular displacement. Here also, at node 2, the similar linear and angular displacement. In vertical plane that is y z plane, we will be having these two displacements, which will be defined as compared with the horizontal plane. So, v 1 phi x and here phi y is there. So, basically in this, we have 8 degree of freedom of the system because 2 at each whirl in at 1 node position. But, if you consider in both plane at 1 node, we have 4 degree of freedom. For 2 node, we have 8 degree of freedom.

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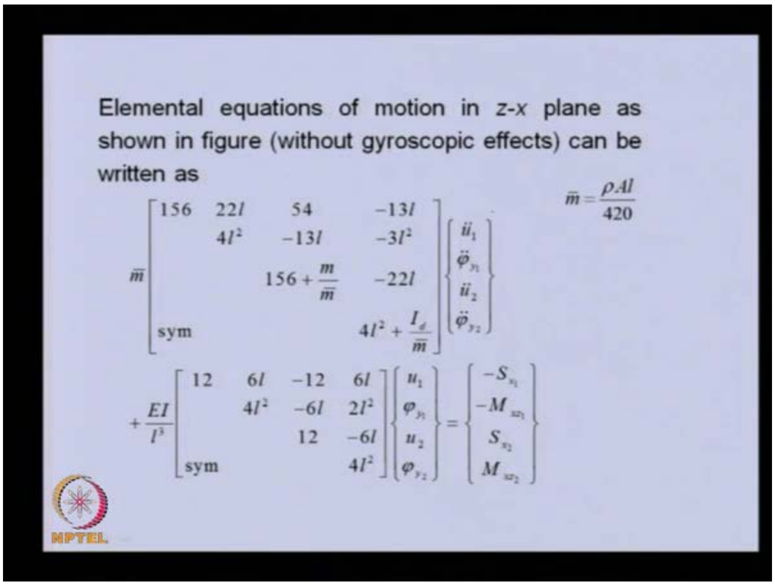

Now, we can able to write the elemental equation. First we are considering one of the plane motions. We are not considering the gyroscopic couple of the disc. Later on, we will at this particular gyroscopic couple effect. So, let us write the elemental equation in z x plane which is horizontal plane. This is the mass matrix. You can able to see that is a disc, which is there at node 2. We will contribute not only in the mass but also in the i d. This m is been divided m bar because this is common.

So, if we multiply this; basically this will be m and i d where m bar is this one, which is coming from the shaft mass matrix. This is the stiffness matrix of the shaft. Similarly, we can able to see the staking of the vectors u 2 phi y 2 u 1 phi y 1 u 2 phi y 2. So, this is the staking in one of the plane. Now, in the y z plane similar equations we can able to write.

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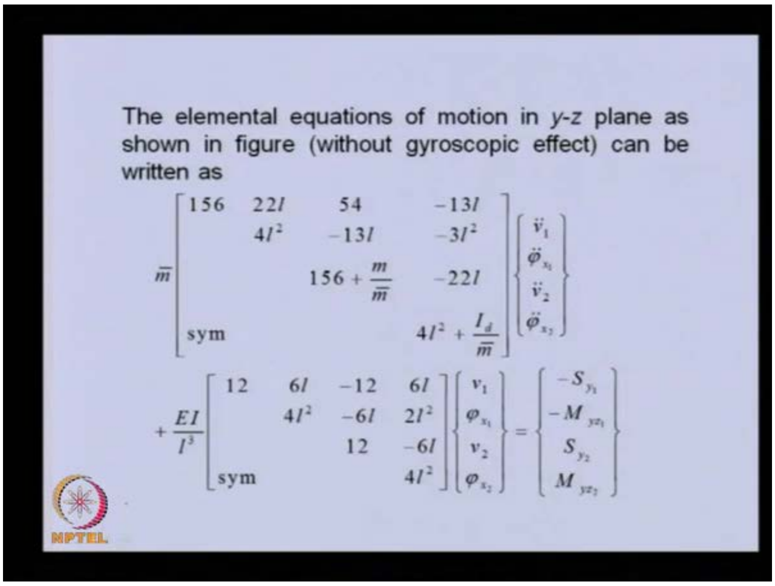

That is also without gyroscopic effect. In this also, a mass matrix is exactly same. Only thing, these variables got changed. Now, here it is v 1 phi x 1 v 2 phi x 2. The mass and diametric mass moment of inertia of the disc will appear here also because; they will be there in the same element in this vertical plane motion also. This is the reaction of the sheer force and binding moment. This two plane motion we can able to couple.

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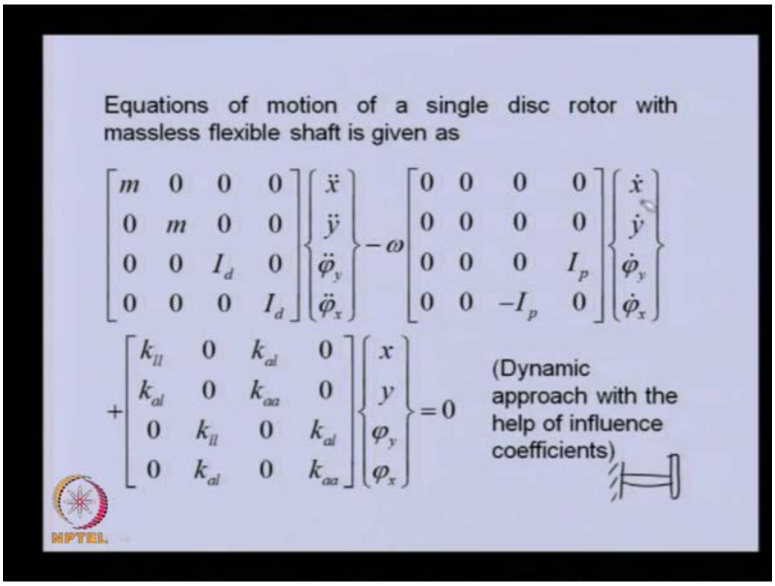

So, before coupling, let us see the gyroscopic effect and how we can able to add it. So, earlier, when we dealt with that is the gyroscopic effect of mass less shaft and the thin disc, we use dynamic approach; even we use quasi static approach. After that we discuss the dynamic approach with influence coefficient method. So, in that particular case, we develop the elemental, we develop the equation of motion of a simple single mass rotor system like this in which the mass of the shaft may neglected. So, this was the equation of motion which we derived earlier. In this, this is a mass matrix. This is the stiffness matrix, which in turn we can able to get in terms of influence coefficient matrix.

That will be the influence of the influence coefficient matrix. This was the gyroscopic matrix. Now, we will be using this particular gyroscopic matrix in the finite element formulation. So, we are taking this separately. But, we will see that the staking of the displacement vectors are different here; two linear displacement and then angular displacement. But, infinite element formulation we have one linear angular; then, linear angular for node 1 and 2.

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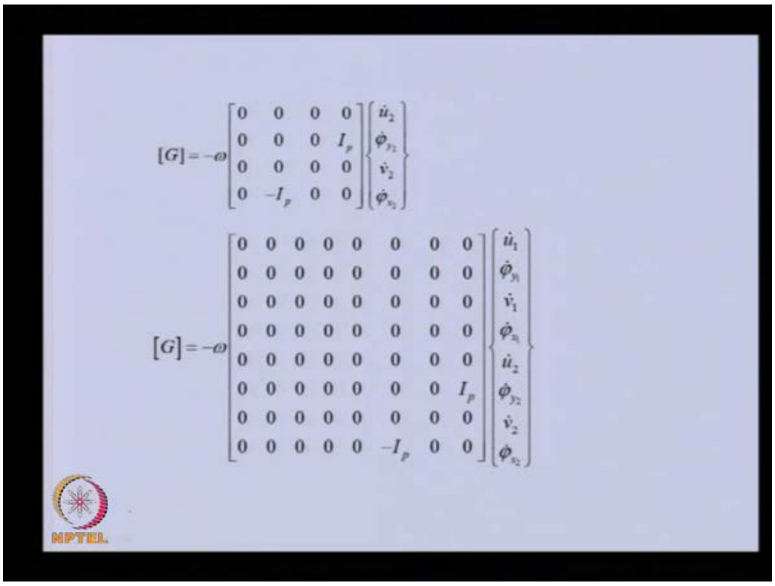

So, let us see that particular gyroscopic matrix in our finite element elemental, this free variable staking. So, in place of x, we are using u. In place of y, we are using v. So, you can able to see that the previous equation of the gyroscopic matrix, we will be taking this form. So, the change of this will take place, because of the change of the order of these variables. Now, this equation we can able to expand. So, what I am doing now? I am writing this for a node 1 and node 2 both.

So, we can able to see same. These are for node 1 and this is for node 2. So, basically this particular 4 by 4 matrix has come here. All the other terms are 0. So, this particular matrix has directly come here because these variables are corresponding to this variable. So, just we expanded this, so that we can able to couple this gyroscopic matrix with our shaft elemental equations. So, with this now we are we are coupling both plane motion.

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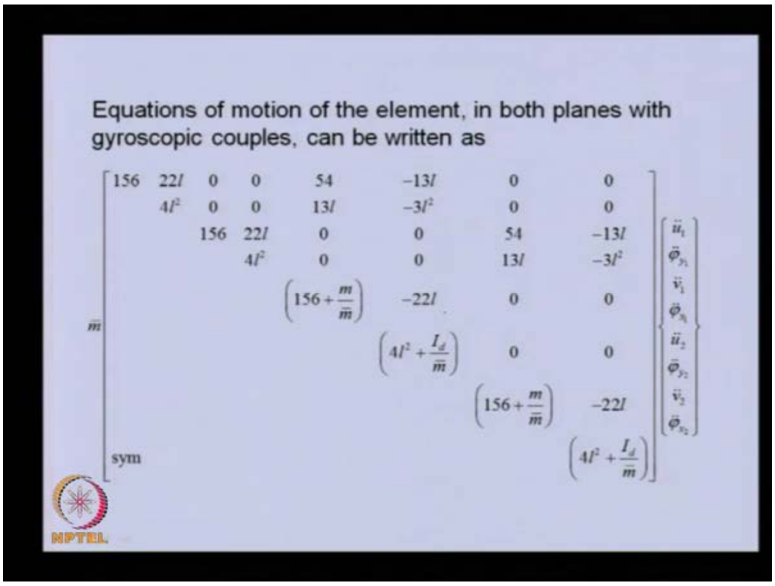

So, this is the mass matrix for that. So, you can able to see earlier, we develop for horizontal plane and vertical plane. But, now we have clubbed them. So, in this first 4 is corresponding to the node 1. So, first two is for horizontal displacement. These are for the vertical displacement at node 1; similarly here for node 2, horizontal displacement, vertical displacement. So, accordingly on the positions of these mass matrices, which we developed earlier, we will change. Now, the similar thing will be there.

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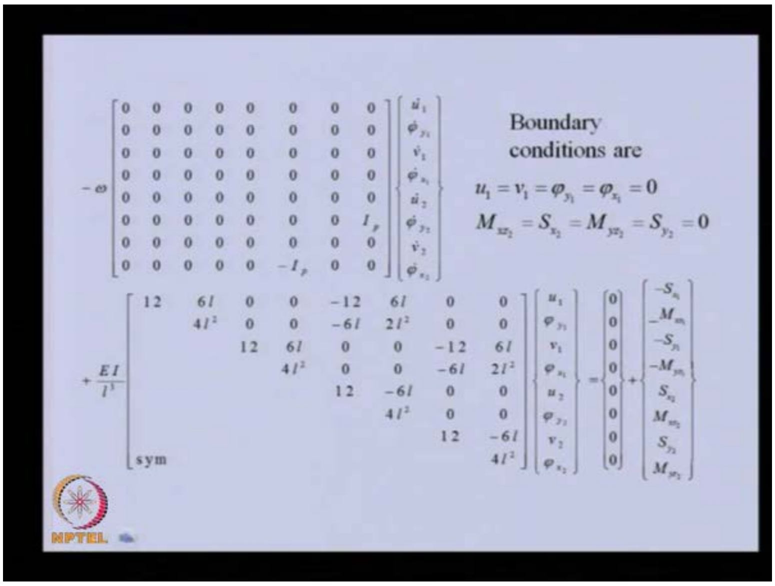

Here, this is the gyroscopic matrix, which we taken from the dynamic approach because now, order of staking is same as the finite element formulation. So, if this directly, we can plug in this is the stiffness matrix. So, the two plane motion elemental equation; we have now assembled it in this form. So, depending upon the order of this free variable, the relative positions of the mass stiffness matrix terms will change. There is no external force. These are the reaction that is shear force and binding moment. Now, let us see the boundary conditions of the problem.

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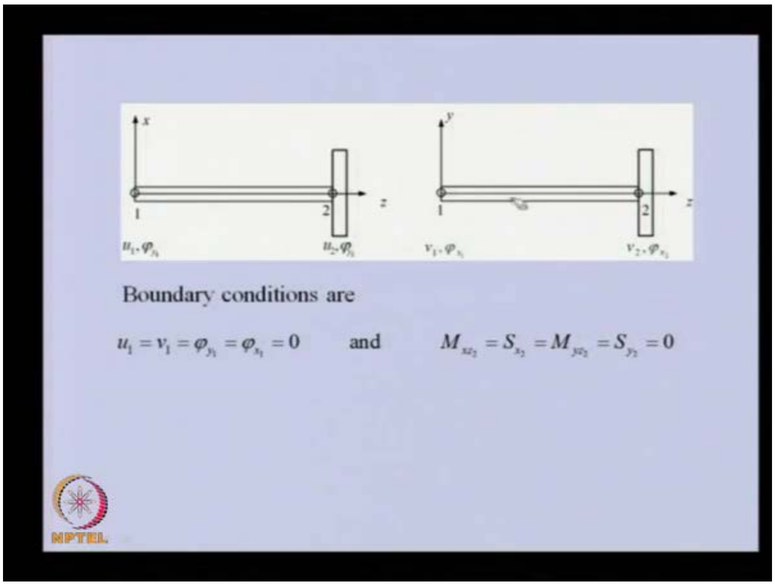

So, we have two elementary elements, one element and this node one is fixed. So, all the displacements linear and angular displacements will be 0 there. This is free end. So, we will be having shear force and binding moment at node 2 is 0.

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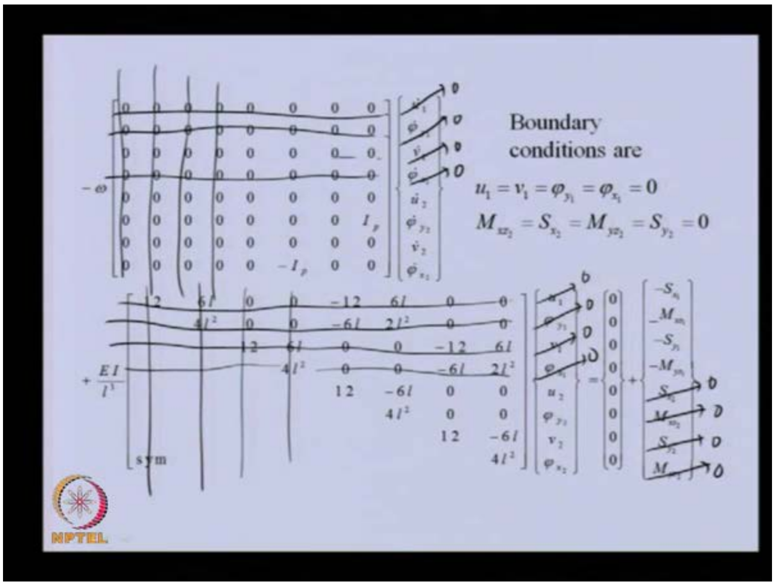

So, these boundary conditions we can able to apply here. So, we have all these shear force and binding moment at node 2, 0 and displacements at node 1 are 0. So, these displacements are 0. Similarly, we can able to equate the velocity terms at node 1 is equal to 0. Even for mass matrix, we will be having these as 0. Now, we can able to see that we need to take that because is in the right hand side.

Only the last four are having 0 values; these are unknowns. So, we need to eliminate them from the final reduced form of the equation. So, basically we need to eliminate the first four columns and first four rows of each of this. So, all these will get cancelled. First row first four equations, we will eliminate from everywhere, from here also. So basically, all these we need to eliminate.

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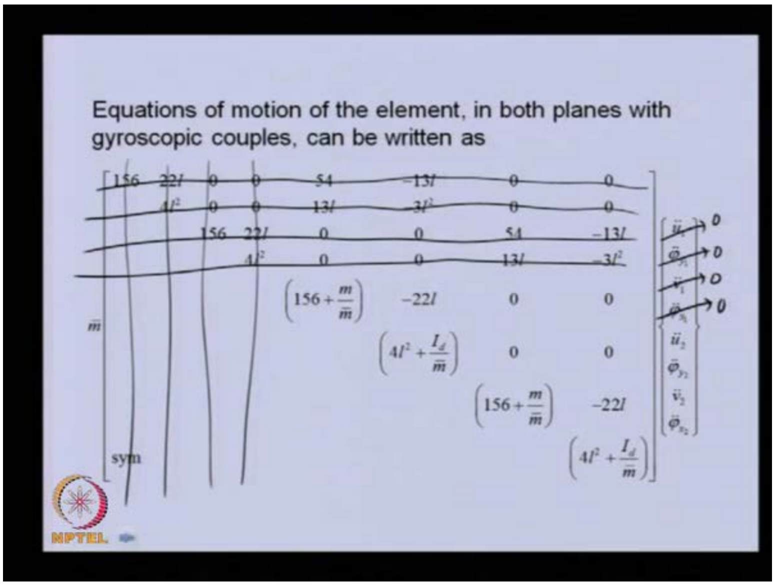

Similarly, for mass matrix, this mass matrix and stiffness matrix are symmetric. Only the gyroscopic matrix is skew symmetric. So, first four rows and column of these matrices are eliminated. The remaining one, we will write it separately.

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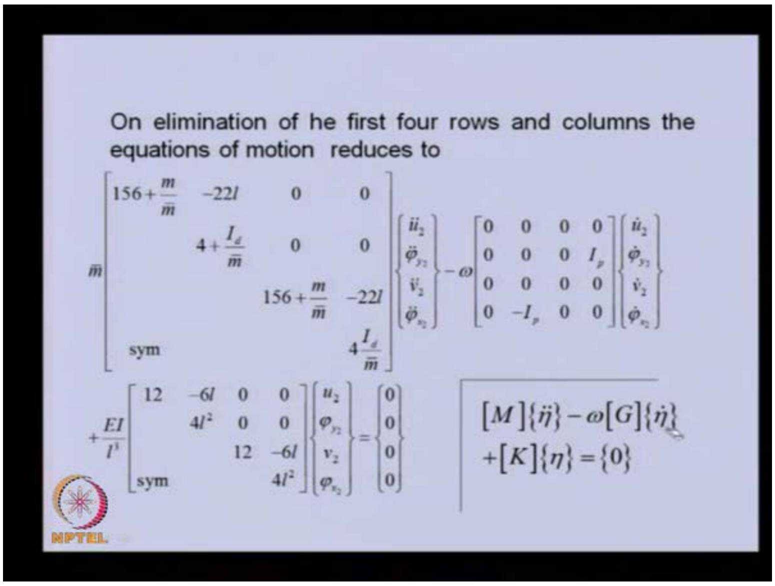

So, this is the reduced form of the equation of motion after boundary conditions. We can able to see corresponding to node 2 only. We have the field variable corresponding to 1 have been eliminated. So, this is the reduced form of the mass matrix, gyroscopic matrix. Still it is skew symmetric.

This is also asymmetric. This is symmetric. So basically, the form of this matrix is M eta double dot. That means this vector minus omega gyroscopic matrix eta dot. So, single derivative with respect to time and this is stiffness into eta is equal to 0. So, you can able to see this particular equation is having this form, in which now because of gyroscopic couple, we have additional term which is coming here.

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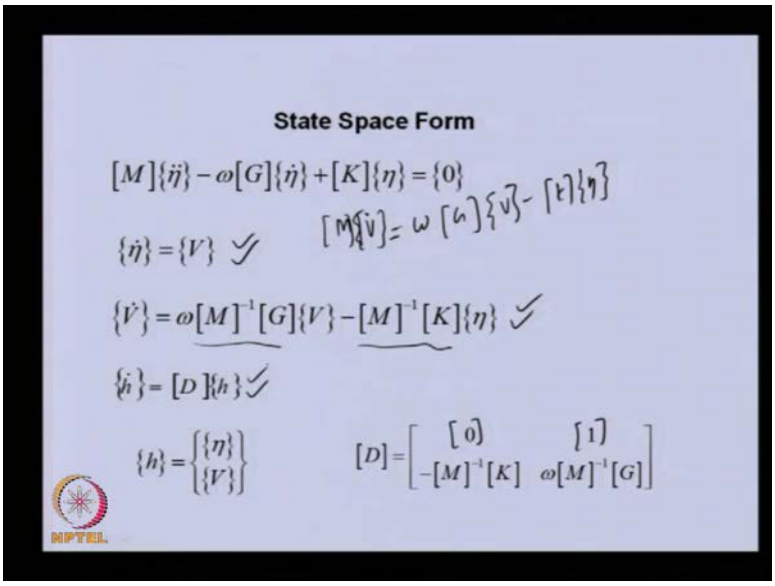

This particular when in the equation of motion, we have the field variable, first derivative term. Then the Eigen value problem, which is there earlier will no longer be valid. We need to convert this particular equation of motion in a standard eigen value problem that is the state space form. So, let us see how these particular equations of motion in which there are certain terms are also there. How we can able to convert into a standard Eigen value problem.

So, this is the state space form we will be converting. Basically, at present our equation of motion is in that is second derivative with respect to time. In the state space form, we need to convert this equation into the single derivative terms. So obviously, the number of equations will get doubled. But, derivative with respect to time will reduce by 1. So, let us see how we can able to do it. So, let us define the eta dot as V vector. If we substitute this in the equation of motion, we can able to see this will be M V dot. So, we will be having MV vector V dot because acceleration can be written as V dot minus omega G.

This can be written as V because this is a velocity plus K eta is equal to 0. Now, I can able to express this in terms of V. So, that means I need to put all these in the right hand side. So, that means this will be equal to this; will become negative. This will not be there. Then, I need to do the inverse of M throughout. So, I will get this expression; so omega inverse of M into G into V minus M inverse into K and eta. Now, this equation and this equation is required equation of motion. Now, you can able to see this equation and this equation. Now, they are single derivative terms.

But, number of equations will get doubled because earlier whatever the size of the eta was there. Now, we have this is also eta dot. So, the number of equation got doubled here. These two equations may can able to club like this. So basically, this particular equation is we have clubbed this. So, you can able to see h we are defined as eta vector and V vector. So, you can able to see that and once we are stacking this two, one over another; this will be h dot. That means velocity of these two. D will be this form. So basically, you can able to see this is in one vector.

This is identity vector. This is the term which is corresponding to this. Sorry, this is corresponding to this one. This gyroscopic term is corresponding to this one because this is velocity term and this is eta term. So, that is in the second place. So, it is coming this side and this one. So basically, if we expand this equation, we will get these two equations back. So, this is the standard Eigen value problem now we have.

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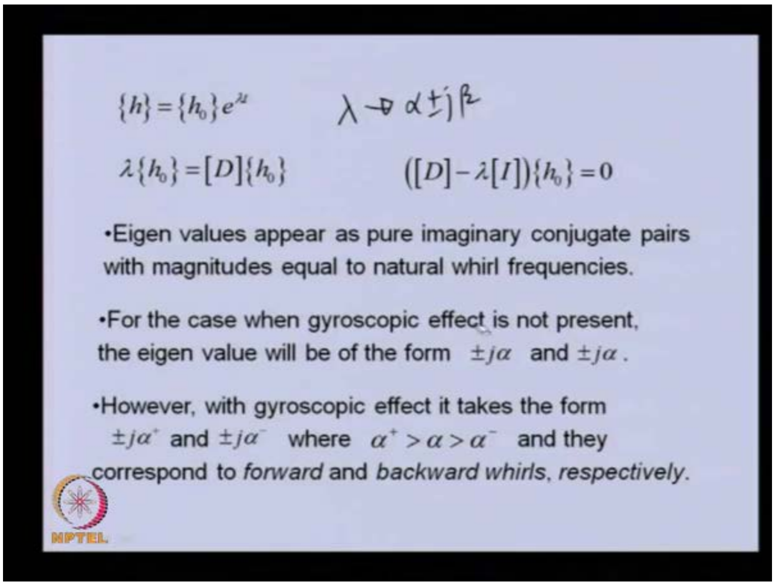

The solution of this; we can able to assume is some amplitude and e lambda t where lambda is an eigen value. So, if we substitute this because only single derivative is there in the previous equation, we will get lambda h naught is equal to D h naught, which we can able to pick. We can bring in one side take and we can take the amplitude as common. So, basically, this is the higher standard eigen value problem, where if we obtain the eigen value of D matrix; we will get the eigen values, m number of Eigen values.

Basically in this particular case, because we do not have a damping in the system so basically, this particular Eigen value should have real part plus imaginary part. The real part is corresponding to damping. Imaginary part is corresponding to natural frequency. Because there is no damping in the system, so we expect this real part will be 0 because this is corresponding to damping. Even we have velocity dot term in the equation of motion but that is due to the gyroscopic couple which is not a damping.

So, this we will find that eigen value will be pure imaginary. The magnitude of that particular term will be the natural frequency of the system. So and obviously, we will get any complex conjugate form. But, they will be pure imaginary Eigen values. In case when we do not have the gyroscopic couple, so we will find that we will get two pair of same complex Eigen values. Their magnitudes will be same if we are not considering the gyroscopic couple. But, if we consider the gyroscopic couple, what will happen among these two.

One of the magnitudes of the Eigen value that is the imaginary part will be reduced value. Another will be having some higher value. So, one will be alpha plus that is let us say, higher value alpha minus super script is lesser value. So, if this is the condition, so that will be alpha will be in between. But, alpha plus will be more than this and this will be less than this. So basically, what is happening when we are considering the gyroscopic effect? This natural frequency, which is corresponding to two different planes because once we are not considering the gyroscopic couple, these two plane motions are uncoupled.

So, we expecting two planes the natural frequency will be same. But, once we have the gyroscopic couple because now the two motions are coupled, so we will not be having the concept of the natural frequency in horizontal plane and vertical plane. But, we will be getting a system natural frequency which will be basically splitting with respect to this. So, we will be getting, having the splitting phenomena of the natural frequency because of the gyroscopic effect. One will be less than this. Another will be more than that. The lesser one will coordinate backward whirl. The more one will coordinate as the forward whirl.

Now, through a simple example, let us see this particular illustration. How we can able to obtain the Eigen values and from there how we can able to get the natural frequency of the system? Even we will see the eigen vectors, how what is the form of that and how we can able to extend the eigen mode shapes from that.

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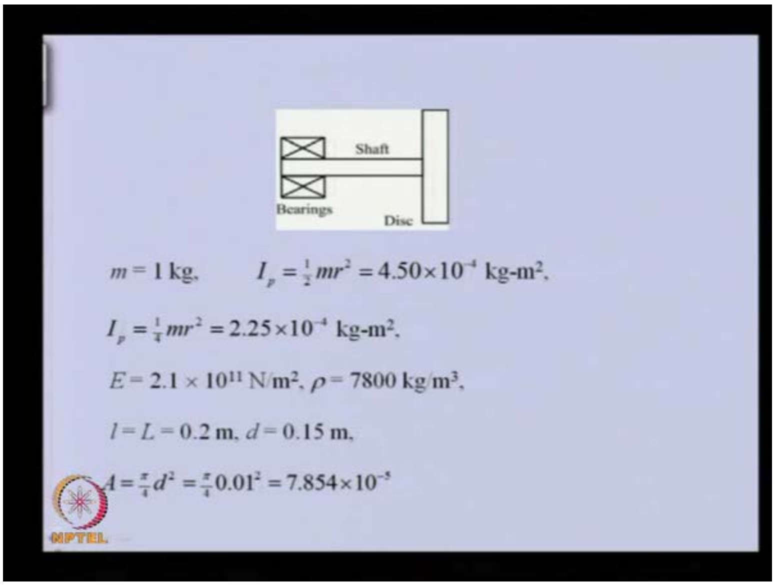

So, basically we have, we are taking the same cantilever shaft and with a disc. Various properties are given here. So, even we have given all the property a mass, stiffness. In this, basically we need to obtain the Campbell diagram and obtain the critical speeds. If these are the property of the shaft and initially we are taking the single element for illustration purpose. But, we will be showing the results for more elements. So, these are the property.

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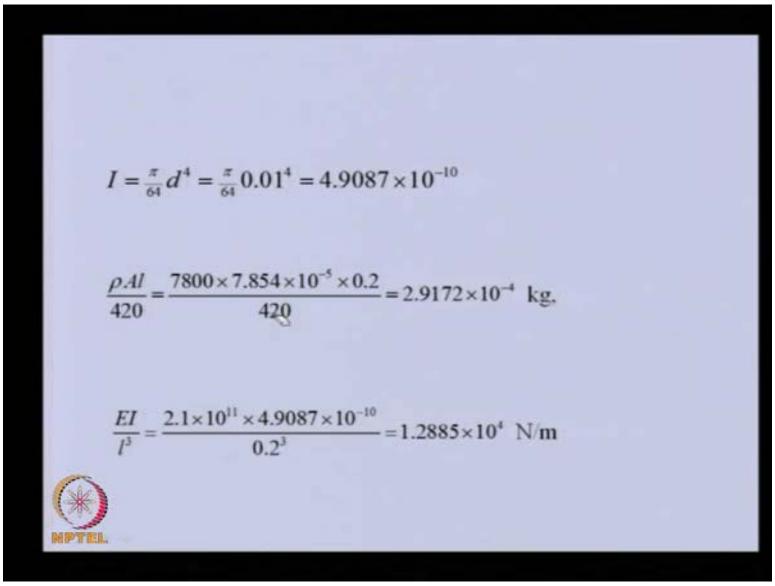

So, these we can able to calculate based on the data given.

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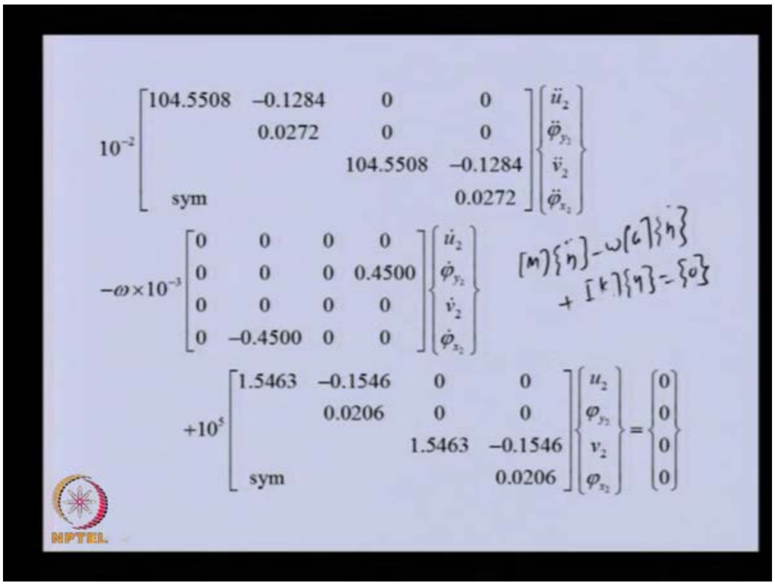

Now, the elemental equation; so this is the elemental equation. Basically, this is the directly, we are, we have taken the reduced form of the equation of motion. Because in the previous discussion, we have already shown that how we can able to apply the boundary condition. How we can able to get the reduced form of the equation of motion? So, this is directly, we are using that particular expression, in which we already applied the boundary condition.

So, corresponding to node 2 only, we have the field variable 1 corresponding to 1 already eliminated because of the fixed end boundary condition. So, this is the reduced form of the equation of motion, which is of the form of M eta double dot minus omega G eta dot plus K matrix into eta is equal to 0. So, this is the final form of the matrix. Now, if we want the state space form of this, we can able to write like this where D is this matrix.

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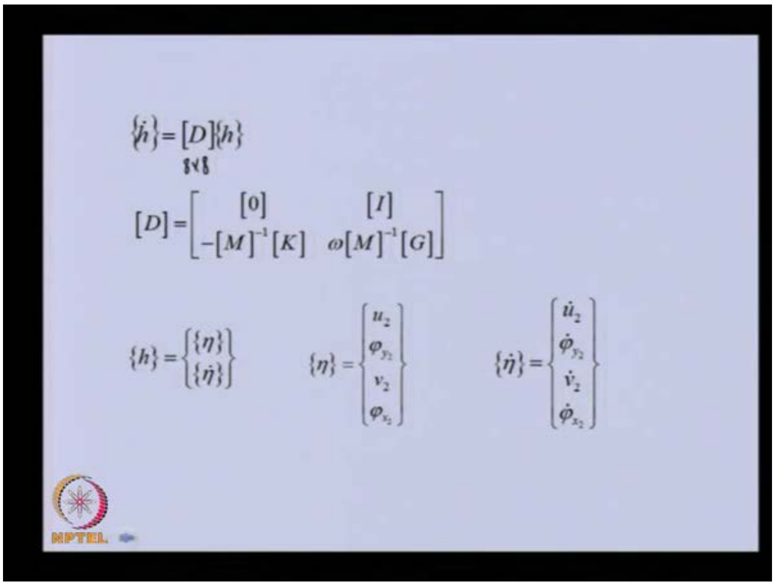

So, we need to multiply all these matrices to get the D matrix. Now, you can able to see that D matrix will be double the size of the K matrix. K matrix is 4 by 4. So, D matrix will be 8 into 8 and whereas the state vector variables are given like this corresponding to node 2, only they are appearing.

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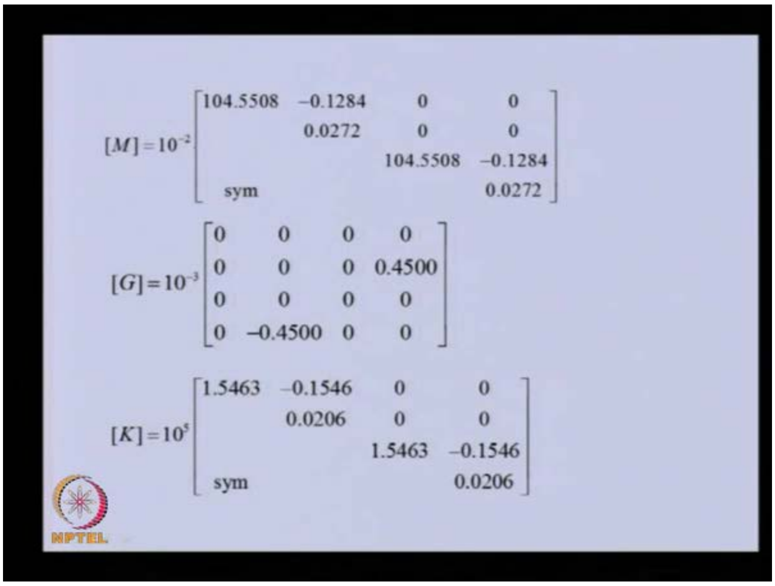

Now, this mass matrix gyroscopic matrix K matrix in the reduced form; we earlier wrote it. So, this is the same one. So basically, we will be multiplying and will be because the size will be bigger. So, I am not showing this. But, these matrices we can able to substitute. We can generate the D matrix; from for which we can able to obtain the eigen value. If we are operating at 0 speed that means gyroscopic couple will not be there.

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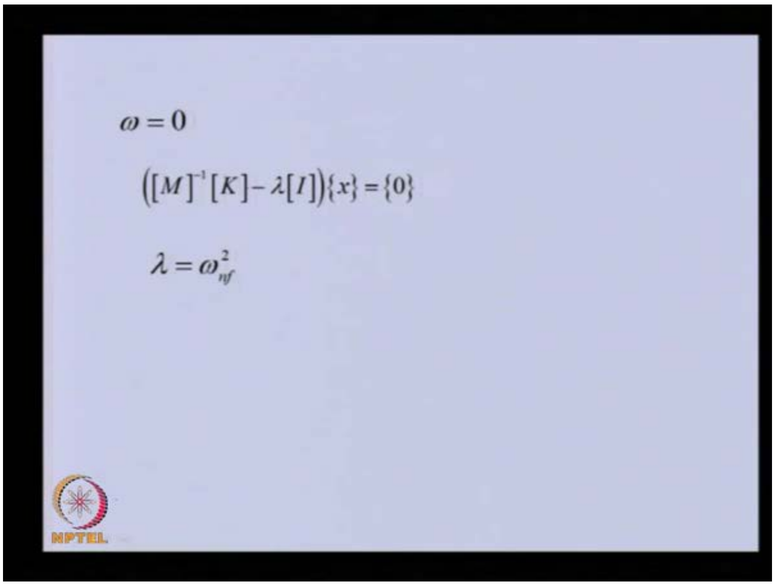

G matrix will be 0. In that case, the earlier standard eigen value problem itself will be valid because gyroscopic term is not there. That eta dot term is not there. So, whatever the eigen value problem was there earlier; that is valid. In that case, this eigen value will be square of the natural frequency. But, in the state space form, that is equal to the natural frequency.

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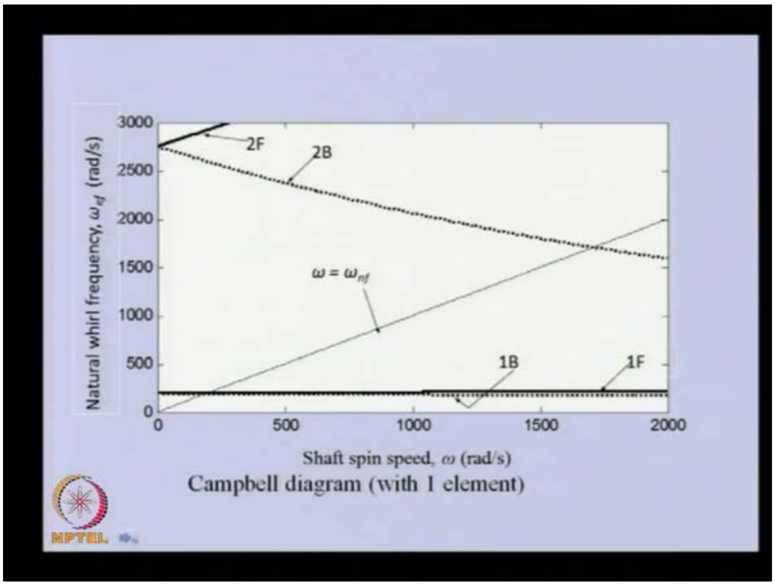

So now, I am giving directly the natural frequencies. So, this is the variation of the whirl frequencies we will be showing in the tabular form how this has been extracted. So, you can able to see this for a single element. So, we are getting here. We can able to see as we are increasing, the speed slight splitting of the natural frequency is taking place. So, one is corresponding to the forward whirl the lower one is corresponding to the backward whirl. This is more predominant splitting in the second this one.

So, we can able to see. This is the forward whirl. So, initially where 0 split is there; there is no splitting. But, as we are increasing the speed the splitting is more. So, this is forward whirl. This is a backward whirl. This particular line is the line. The spin speed is equal to the natural frequency. So, wherever this line will intersect the curves, they will be the critical speed. So, here we will be having critical speed corresponding to the forward whirl and backward whirl. Similarly, here we will be having corresponding to the backward whirl.

But, we found that for second, there is no intersection. So, the second natural frequency is not feasible. This phenomenon we already seen while discussing the gyroscopic couple; that this gyroscopic couple certain form of critical speed is not feasible for the discrete system.

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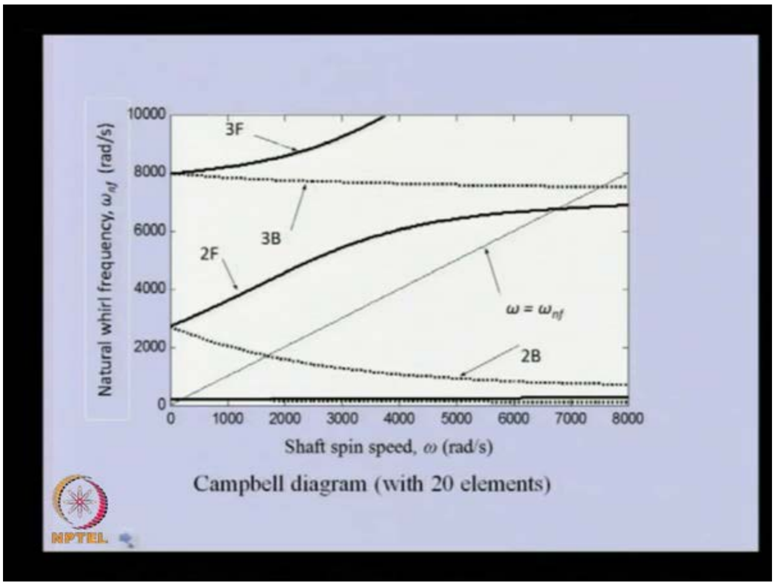

But, the same thing if we obtain for more number of elements because we can able to divide the shaft into more or number of element; we can able to get the natural frequencies more accurately. So, this lower whirl is corresponding to the first forward and backward whirl. This is second; this is corresponding to the second whirl. In this particular case, as we are increasing the number of elements. Now, because shaft modes are also coming into the picture; so basically now the shaft is having consistent mass matrix. So, these even for the second mode; now, we are getting the intersections. But, we can able to see that the third mode.

Now, it is more. This t, this curve is more straight. So and likely, this will intersect that for particular mode. So, this is we expect the more accurate because of more number of elements we have chosen. We can able to see these are corresponding to the 0 speed. So, there these all this curve meet. But, as we increase the speed, the splitting is more and more. So basically, this is a Campbell diagram.

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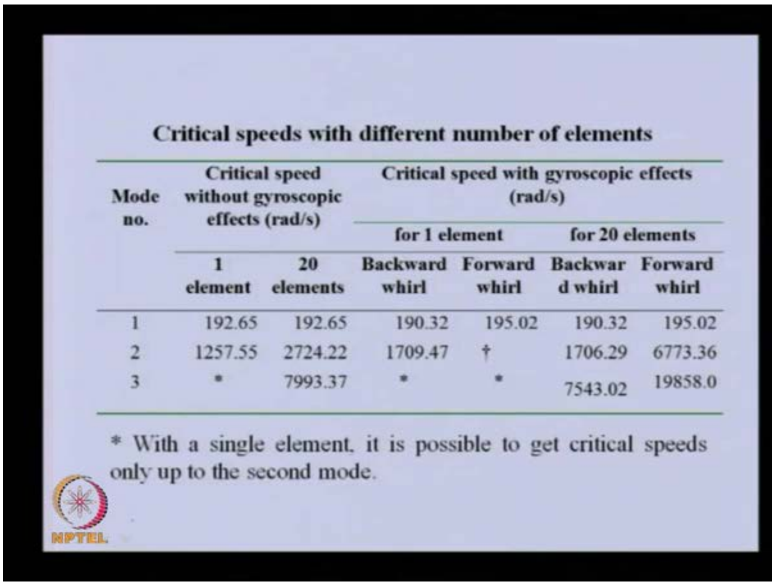

Critical speeds with different number of elements

Mode no.	Critical speed without gyroscopic effects (rad/s)		Critical speed with gyroscopic effects (rad/s)			
			for 1 element		for 20 elements	
	1 element	20 elements	Backward whirl	Forward whirl	Backwar d whirl	Forward whirl
1	192.65	192.65	190.32	195.02	190.32	195.02
2	1257.55	2724.22	1709.47	†	1706.29	6773.36
3	*	7993.37	*	*	7543.02	19858.0

* With a single element, it is possible to get critical speeds only up to the second mode.

So, now let us try to summarize this. So, critical speed this is without gyroscopic couple that means when the speed is 0. For element 1, we want this. For element 20, we want quite close value. But, for second there is a difference. But, we expect this will be more accurate. For 1 element, we could able to get even third mode. But, if you take more number of elements, we can get the third mode critical speed also. This is with the gyroscopic effect for element 1 and element 20, number of element 20 y. So, in this splitting was there.

But, we can able to see small splitting is there and always it this will enclose the value of this 1 because you can see one is less than another is more than this. When we took one element, there was no intersection of the second forward as we have seen in the graph. Only the backward second was present and higher we are not getting because only one element was there. For more number of elements, we are getting intersections. You can able to see they are increasing these particular values, so because splitting will take place around this. Similarly, for this is the splitting; so one will be lower than this and another will be higher than this.

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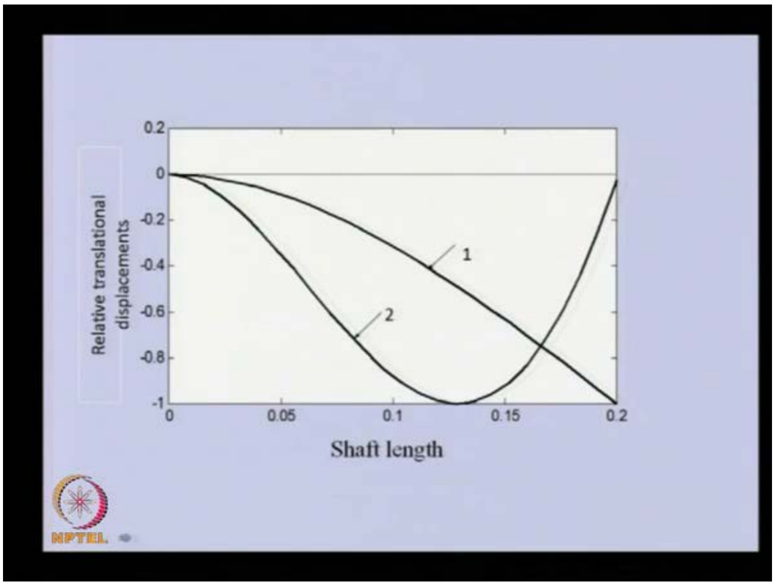

This is the mode shape. So, this is the amplitude of their mode shape, because eigen vector also we will be getting as complex. But, only real part or imaginary part will be there. If you take the amplitude and plot it with respect to the shaft length, this will be for first mode. This is for the second mode and higher modes also we can able to plot it.

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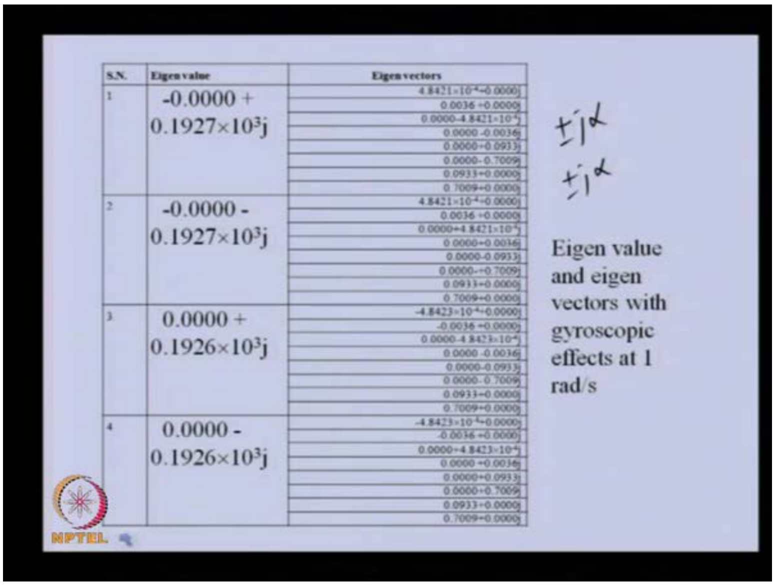

Let us see that how a eigen value generally we will get it. So, this is for one element. So, we can able to see that for one element. Then, we are considering the very lower speed that is the omega nearly 0, so you can able to see the eigen values. Four sets of eigen values are same. So, these are complex conjugate. But, another set is there having same value. So as we discussed, we will get two sets of similar eigen values. They will be same magnitude. This is because the coupling between two planes is not present. But, so this is for high speed.

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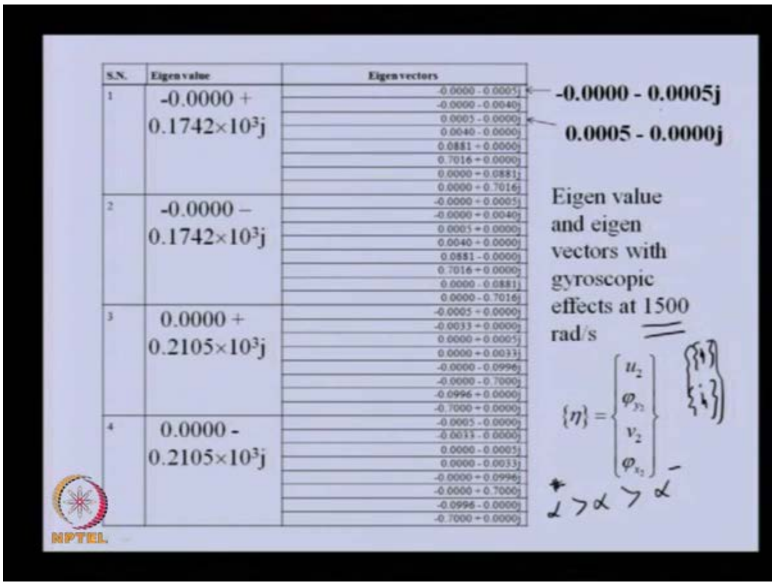

So, for high speed, we expect that now this eigen value. So, this is one complex conjugate this is another. Now, they have different values. We can able to see one is less as compared to the previous one and another is a more. So basically, the splitting of the natural frequency is taking place. So, we can able to see that it was 1927. Now, one of the pair has smaller value another is a higher value. So, as we discuss earlier, so we will get one is the less which will be higher than the without gyroscopic couple. This will be higher and this will be lesser. So, one is higher.

This is higher and lower one is this one. If we see the eigen vector, so you will get because the reduced form of the eigen vector had this form. The state space form of the eigen value had 8 by 8 size that means we had staking of this eigen vector is eta and eta dot. So, the eight values of the eigen vector these are very small. Some of the values are shown here. That is more important. So basically, corresponding to one eigen value; we will get 8 number of eigen vector for one element and staking will be first four will be this. Next four, will be velocity of these.

So now, we can see that here, first is the horizontal displacement. The third is the vertical displacement. First is horizontal displacement. Third is vertical displacement. Now, you can able to see because they have 90 degree phase difference. So, they are like this. So basically, if we multiply minus j with this, we will get this. This is because they are corresponding to two different planes; one is horizontal plane vertical plane. They have 90 degree phase. So, we expect them to be 90 degree phase. Then, Eigen vector also. Another observation is we are getting the purely either imaginary quantity or real quantity because there is no damping in the system.

So, that is why we have this particular typical behavior of the eigen vector. But, when we will consider the damping especially like, fluid film bearing; we will see that both quantity will be complex. Then, plotting of the mode shape will be more difficult in that case. Now, we will take up another case, in which we will be mounting a flexible rotor on fluid film bearing. Through one example directly, we are explaining how the fluid film bearing elemental equation can be clubbed with the finite element equation of the shaft system.

As we have done in the previous case, this gyroscopic coupled matrix. We developed separately and we clubbed with the finite element original shaft elements. So likewise only, we will be developing the equation of motion for the bearing separately. Then, we will club with the elemental equation of the shaft.

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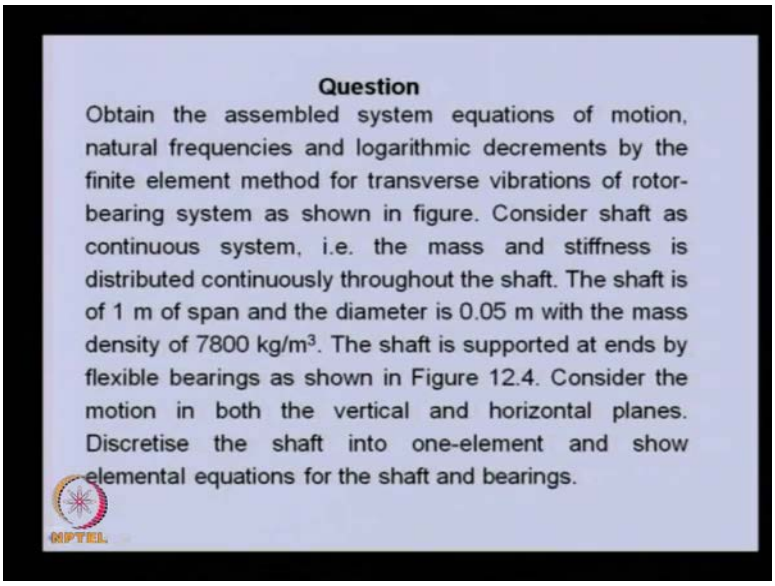

So, for this we are considering a simply supported shaft like this.

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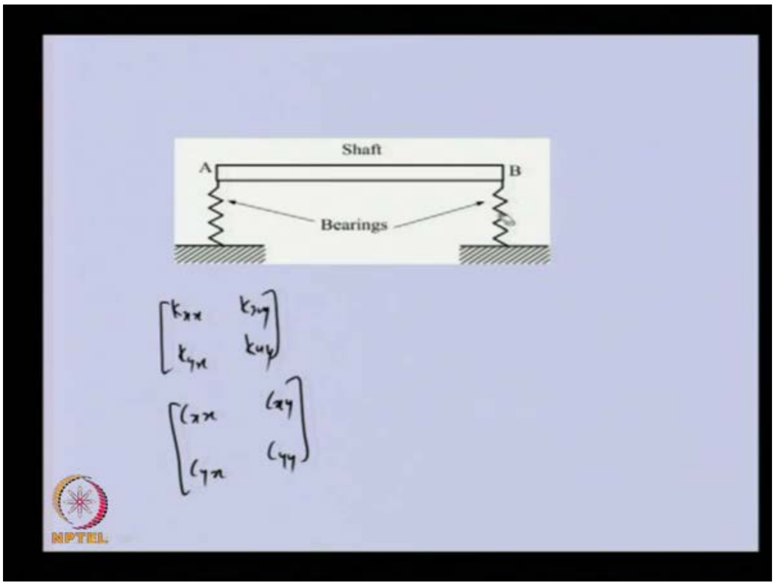

This is flexible shaft. In this particular case, we will be considering the Euler-Bernoulli beam model. So basically, we will be treating this as finite. We will be discretizing. This will be one element for illustration. But, we can able to increase the number of elements using. These bearing are basically fluid film bearing. They each of them are described by four stiffness and four damping torques. So, total eight properties are there for each of the bearing. So, we have coupling of the motion in both the plane because of this particular fluid film bearing.

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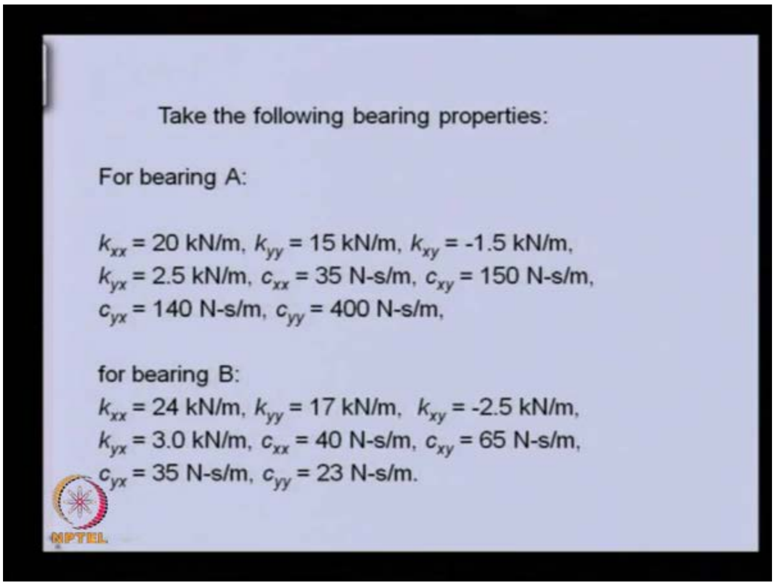

The properties of these bearings are given here. So, let us say left hand side bearing is A. So, this is the property and bearing B this is the property. In the subsequent class, we will explain how we can able to get this property for a typical fluid film bearing especially, if we have some kind of approximation like shaft bearing approximation. So, this is the various other property of the shaft and metallic property of the shaft. So, for illustration we will show only one element. But, that can be extended for high level high elements. Now, let us see how we can able to model this.

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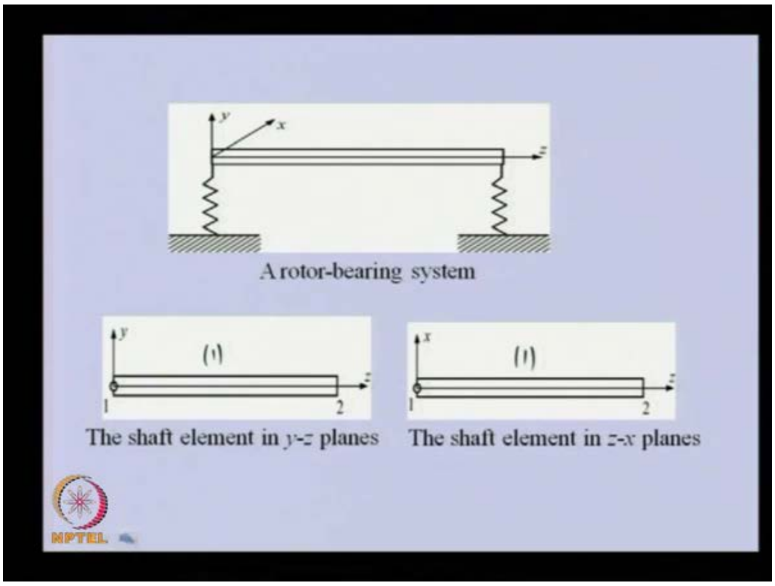

So, we have taken one element. So, this is the shaft element in one of the plane because now, both plane motions are coupled. So, only one element we have considered. So, similar to the gyroscopic couple case, we need to write the elemental equation for both of them. Then, we can combine them in the two planes.

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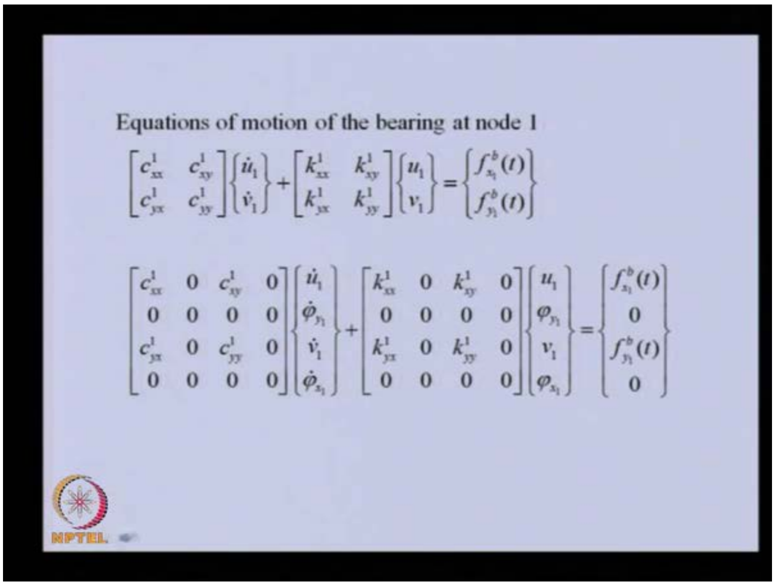

So now, let us take the bearing model. So earlier, we know that the bearing fluid film forces in horizontal and vertical direction. For one of the bearing, we can able to express like this. These are the property of the stiffness or damping and stiffness of the bearing.

Now, this we can able to expand like this because this is for node 1. Bearings, we have defined in terms of the linear displacements only. So, we are not considering their stiffness and damping because of the tilting of the disc tilting of the shaft. Only the linear motion we were we generally consider.

So, if we introduce the tilting also, so corresponding terms will be 0. So basically, we expanded these equations in the 4 by 4 by introducing some variable. So basically, if we multiply these four equations finally, we will get this only. So, this is the expanded form of this for node 1. So, it is for bearing 1. On the same line, for bearing 2, we can able to write. Only thing with the bearing property, the superscript is representing the. So, this is representing the bearing 2. It is not the square. If we expand this, we will get similar to the previous one. Only thing is here, subscript is coming. Here, free variables are corresponding to node 2 because this is attached to the node 2. These are the fluid film forces.

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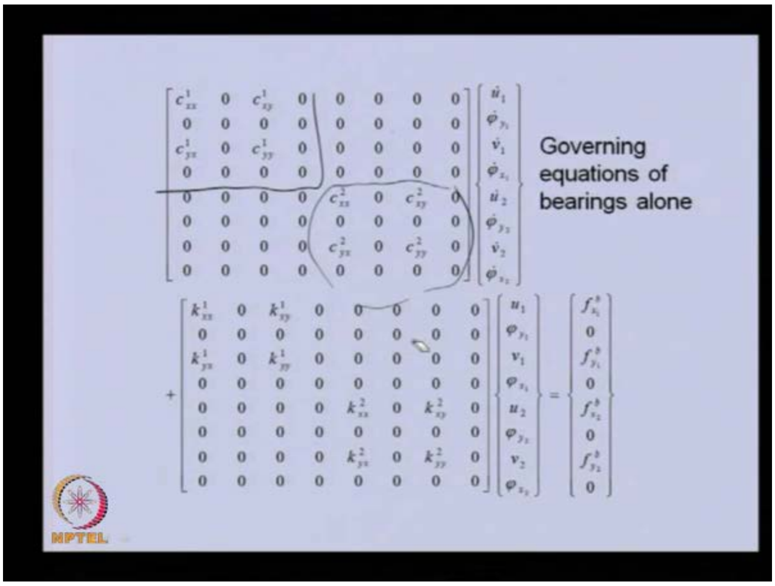

Now, governing equation for the both the bearing. So, what we have done? We have combined both bearing equation motion. Now, you can able to see the staking of the vector is for node 1 and node 2. So basically, for node 1, we have the first four this equation. For node 2, this was the equation. These two are the null matrices. Similarly, for stiffness the first four rows and columns are corresponding to bearing 1, which is multiplied by this. The last four row and columns are corresponding to the node 2. These are the bearing forces at node 1 and 2. So, this is the equation of motion for bearing alone of both the bearing.

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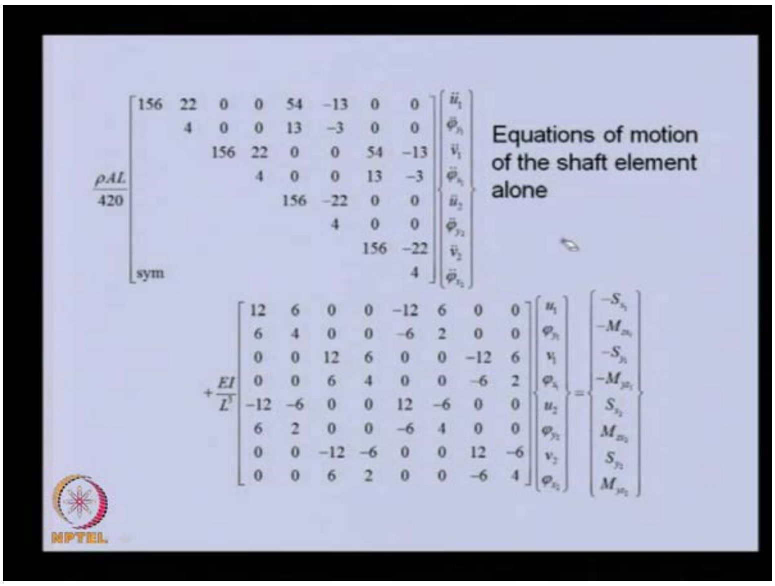

Now, we can able to; now, this is the equation of motion for the shaft alone, so you can able to see. Now, we have clubbed the two plane motion as we did for the gyroscopic case. So, this is the mass matrix. This is the stiffness matrix. These are the reaction shear force and binding moment. So, this is the equation of motion of element of the shaft.

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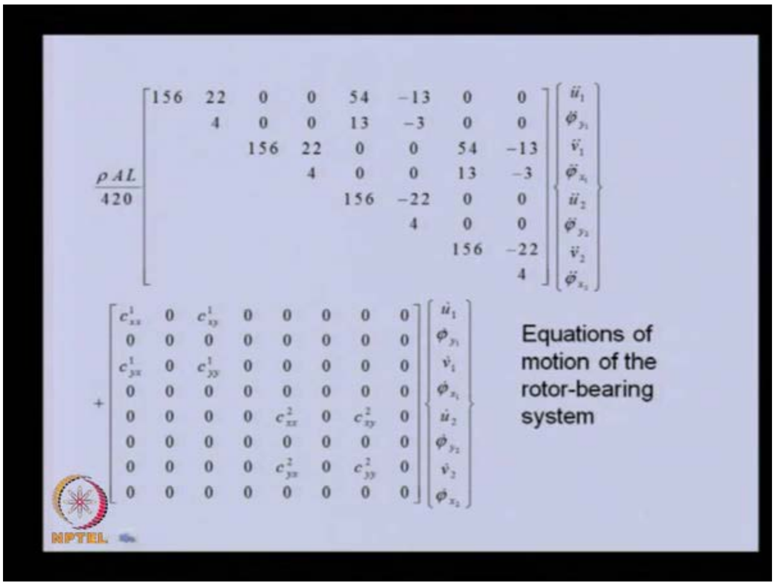

Now because both the equations are having similar staking of the free variable; now, we can combine them. So basically, this is the mass matrix as we had earlier. This is the bearings damping terms. So, you can able to see just we have added because now order of these free variables are same as this one. So, we added this. Similarly, the stiffness of the shaft as well as of the bearing, we have combined here.

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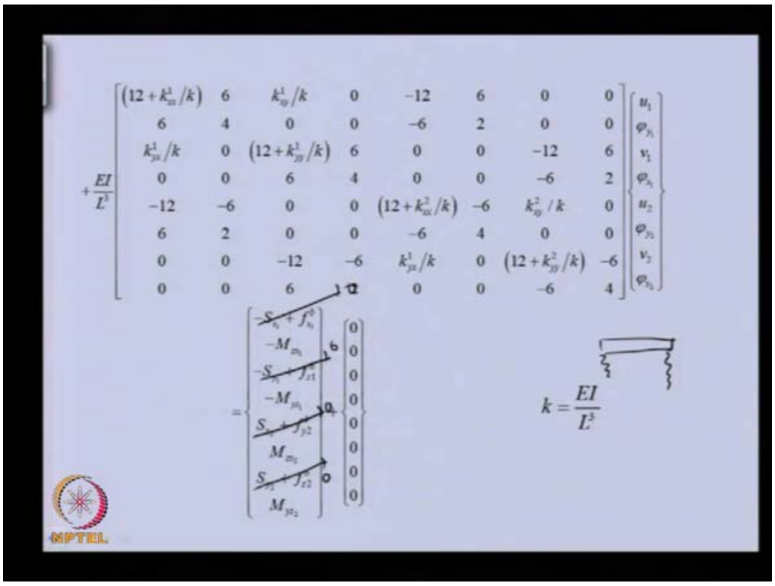

So, you can able to see contribution of the bearing to the shaft element stiffness are these. So, these are the contribution from the bearing 2 and these are from bearing 1. So basically now, we could able to get. These bearing forces will obviously cancel the shear force because once we have at the end of the shaft.

Once we are joining the bearings, there will not be any net force there; moment will be there. But, net force will not be there. So, they will cancel each other. So, these terms will get cancelled to each other. Bearing force and the shaft projection force, they will cancel each other. So basically, if you see this equation of motion is now basically of this form.

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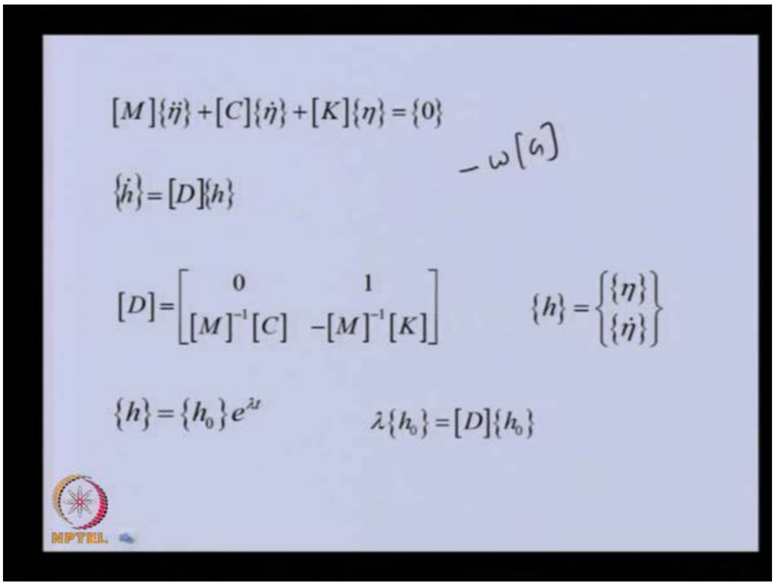

Here, because this is a free end, so moment will also be 0 here. So, you can able to see. These are also 0 because it cannot, the bearing cannot resist any moment. So, this moments are will also be 0. So, this is the form of the equation of motion. So, now as compared with the previous case earlier, it was minus omega minus omega G was there for gyroscopic couple case. Now, in place of that, C has come.

So, the straight space from formulation will be exactly same. Only thing will be, in this particular D matrix that minus omega G will be replaced by C. So, this will become positive. Omega will not be there and C. So, C is now a damping matrix. It is not the gyroscopic matrix. The eigen value now, we expect to be complex. Even eigen vector, we expect to be complex in this particular case.

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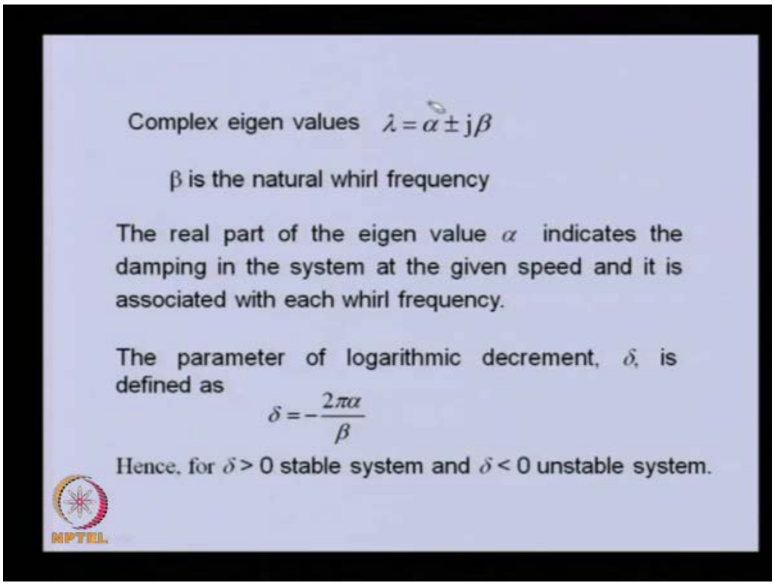

So, the complex eigen value of e be of this form; both real part and imaginary part will be there. Second will be corresponding to the natural whirl frequency; the real part will indicate the damping in the system at a given speed. It is associated with each whirls frequency; so because this damping will be corresponding to that particular natural whirl. If bearing properties are changing, the speed, so they will change with the speed both. So, for this present illustration, we have not taken the bearing property as a speed dependent.

We have taken this property as constant. So, and from this real part, we can able to define a logarithmic decrement like this which contain both alpha and beta. So, and negative sign is there. So, that means if this is 0, more than 0, positive then, system will be stable. If it is negative then, system will be unstable. So, in the subsequent, we will be seeing this stability aspect in more detail. But, with the finite element formulation of the state space form, we can able to adjust the stability of the system directly using the logarithmic decrement.

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Natural frequencies and logarithmic decrement

S.N.	Natural whirl frequency (rad/s)	Logarithmic decrement
1	47.4	0.6275
2	51.1	0.2585
3	82.4	1.1679
4	88.5	0.3695
5	1743.4	0.0992
6	1744.0	0.0245
7	5945.5	0.0407
8	5946.0	0.0101

[D]

Now, this is the natural frequency and logarithmic decrement which we have obtained. So basically, we have solved the D matrix because we know the D matrix completely; mass stiffness, damping matrix. We have obtained the eigen value and eigen vector of that. Using the imaginary part of that, we got the natural whirl frequencies of the system and from the real part and the imaginary part.

By the definition of the logarithmic decrement, we could able to get this. Here, we can able to see all are positive that means system is stable for all the frequencies. It is not unstable, but you can able to see that gradually those are decreasing. These damping’s are very high, may be because of whatever the bearing property was taken; is because of that. So, but at higher speeds, we are having low logarithmic decrement.

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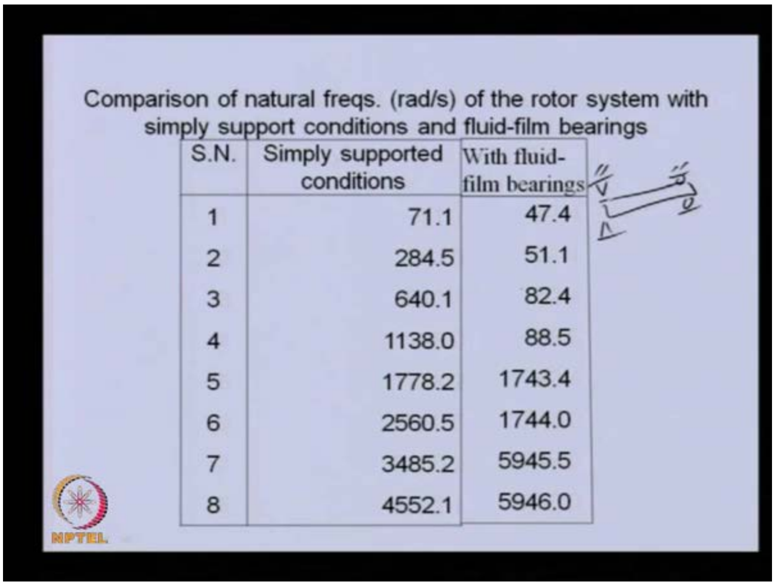

Comparison of natural freqs. (rad/s) of the rotor system with simply support conditions and fluid-film bearings

S.N.	Simply supported conditions	With fluid-film bearings
1	71.1	47.4
2	284.5	51.1
3	640.1	82.4
4	1138.0	88.5
5	1778.2	1743.4
6	2560.5	1744.0
7	3485.2	5945.5
8	4552.1	5946.0

Now, just to compare how the if in case of fluid film bearing, if we have simply supported end condition like we consider the fluid film bearing; if it is simply supported end condition. So, what will be the change in the natural frequency? So, you can able to see with fluid film bearing basically the value of the natural frequency got decreased. So, this is just comparison of two boundary conditions.

So, if we want to model the same system, which is actually mounted on fluid film bearing by a simply supported condition; we can able to see how much difference will be there in the prediction of the whirl frequencies. In the previous shaft model, using Euler-Bernoulli beam, we did not have the damping. So, let us see some simple damping model which we can able to consider.

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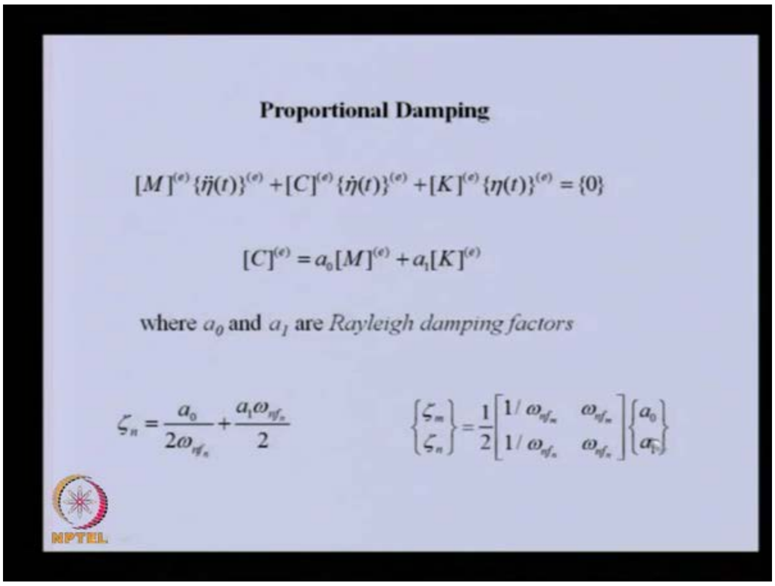

So, this is a proportional damping. So, in this particular case earlier, we had this elemental equation for mass stiffness. Now, we want to add the damping also in the system. So, this we can able to in the proportional damping. We can able to write the damping in the terms of mass and stiffness proportional to mass and the stiffness. There are some constants. Rayleigh’s damping factor, this M, M1 are defined like this. So, for a particular mode, let us say, nth mode, this is the damping ratio.

This is having a constant. This is a natural frequency. So, if we are interested in certain frequency range; so we can pick up that two natural frequency within which we want to analyze the system. So, let us say one is the m th mode, another is the nth mode. If you write this for m th mode and nth mode, we will get two equations that can be written like this. Now, we can able to see, we can able to get the a by inverting this matrix.

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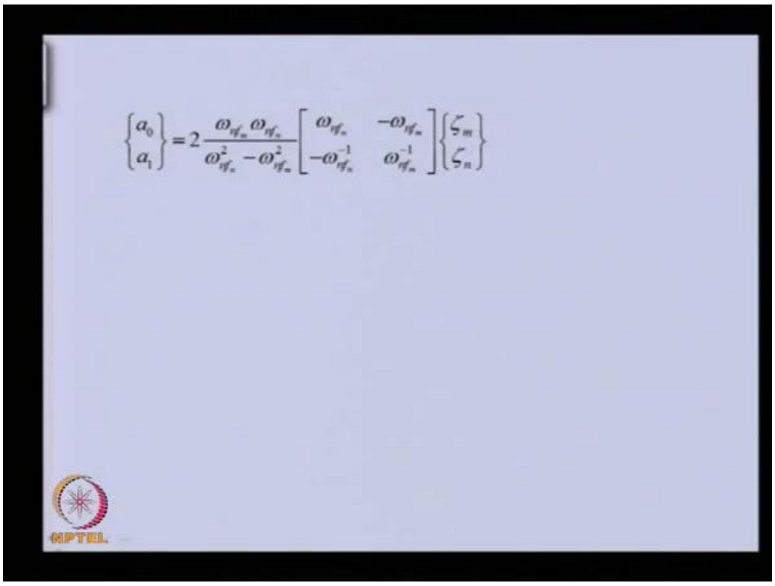

So, if we know the natural frequency of the system a, two frequencies within which we want to operate over system and damping ratio; if we can able to obtain corresponding to that mode then, we can able to obtain the a naught and a 1. So, that only damping ratio we can able to get through the experiment with relatively easy as compared to the damping matrix itself. So, once we get this, we can get the a naught and a 1.

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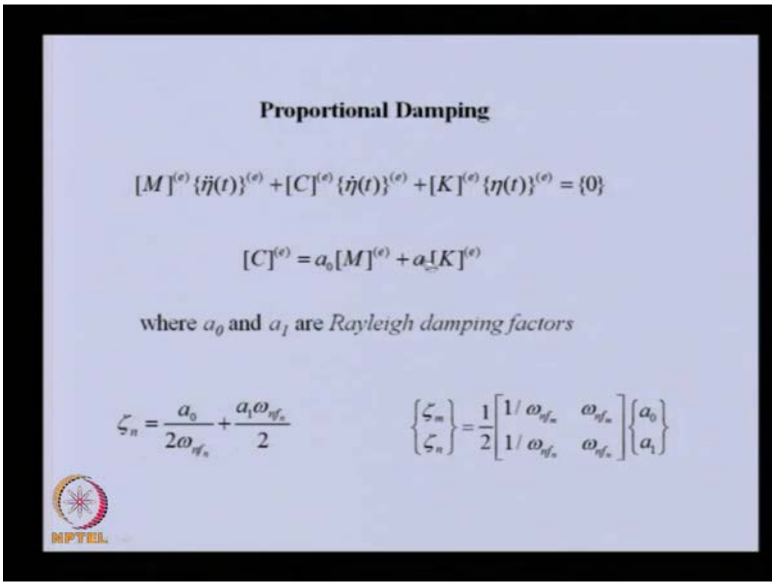

Then, our C matrix is defined. So, this generally we will use it if we want to introduce some kind of damping in the system to have more realistic analysis of a rotor system. So, this particular damping is at the shaft damping itself. But, it is proportional to mass and stiffness. Today's lecture, we first started with the gyroscopic couple of a disc; how we can able to incorporate infinite element formation. As such, we did not consider the gyroscopic couple in the shaft itself because we need to have attenuation beam model for that. Till now, we have only considered the Euler Bernoulli beam model.

So in that particular case, when we considered the gyroscopic couple of the disc, we saw that the two plane motions were coupled. The whirl frequency was depending up on the speed. For applying the critical speed of the system, we need to obtain the campbell diagram. Then we saw how we can able to incorporate the fluid film bearing model into the equation of motion of the rotor bearing system. In this particular case, we saw that coupling of the motion in the two planes.

Also, the eigen value and eigen vector were more complex specially, the eigen value we have seen that, it become complex. Not only we have the natural frequency formation, but also damping in terms of the logarithmic decrement. We can able to adjust the system stability. Then we took one simple Rayleigh damping model, which generally we can able to use it if we want to introduce some damping into the shaft model so that we can have more realistic analysis on the rotor system.

Now, with this we will be completing our analysis of the transverse vibration of the rotor bearing system. Now, in the subsequent class, we will study the instability in the rotor system. We will see that various sources of instabilities are present in the system. We will see in more detail of this in the next class.