Theory and Practice of Rotor Dynamics

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Module – 6
Transverse Vibrations
Lecture - 29
Finite Element Method

In the previous class, we studied continuous system for transverse vibration, we derive the equation of motion for a beam with distributed stiffness and mass property we consider the Euler Bernoulli beam model. Today, we will be extending the solution of that particular beam model using the finite element method, we already introduce the finite element method when we will do we were discussing the torsional vibration. On the similar line, we will be describing the finite element method for transverse vibration, and in this particular case we will see that for transverse vibration, one element will be having more degree of freedom as compare to the torsional vibration case.

So, the size of the matrices for the element will be slightly bigger, earlier it was 2 by 2 matrix, but now will see that it will get the doubled and even we will see that when thus other effects like the gyroscopic effect. When bearings are there in which the plane of the motion in 2 directions are coupled then the size of the mass matrix and stiffness matrix will be even bigger, so with this let us see what are the things will be covering in this particular lecture.

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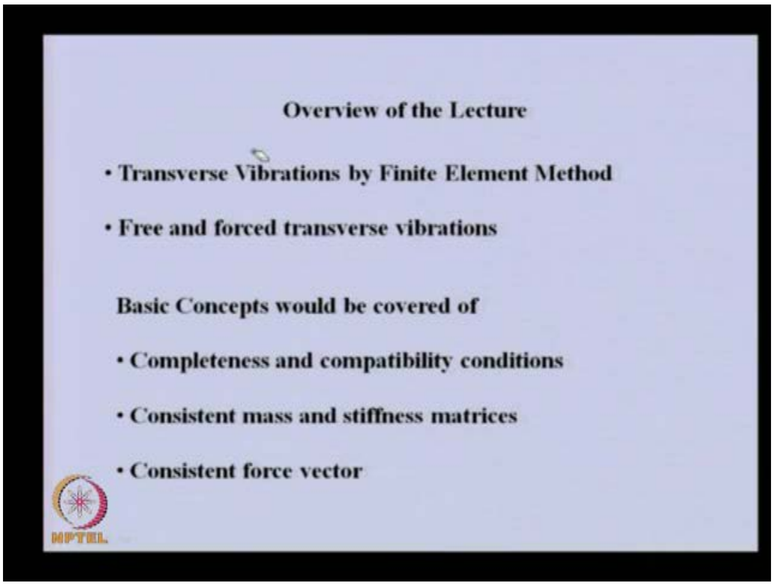

So, basically will be covering the transverse vibration using finite element method both free and forced vibration will be considering. In this, similar to the torsional vibration case here various concept of finite element method like completeness and compatibility conditions consistent mass and stiffness matrices will be deriving, even will be deriving the consistent force vector.

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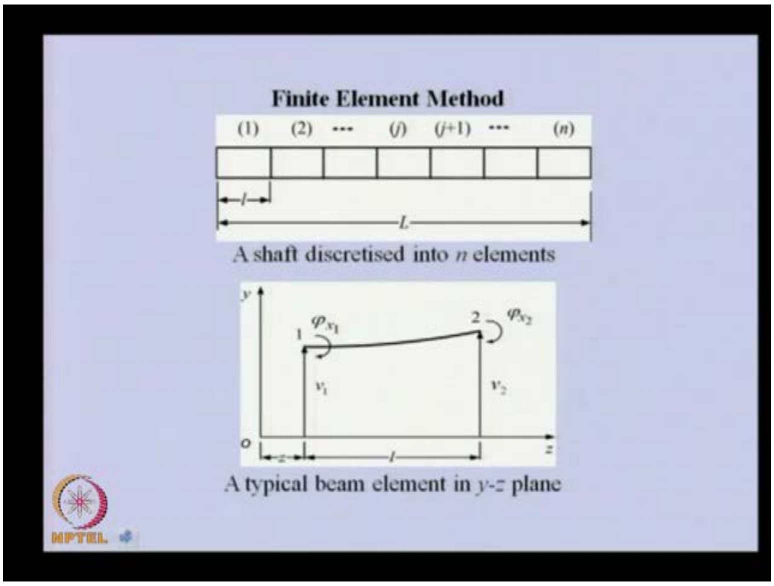

So, let us take one beam so this is a beam and we want to analysis this particular beam for torsional transverse vibration is infinite element. So, we have divided this particular beam in to various number of elements and this particular beam element is one of the representative element and this we have defined node one and node two. We can able to see this particular beam element we have shown in one of the plane that is z z y plane. As we discuss in the previous lecture in a Euler Bernoulli beam the single plane motion is that one plane motion is uncouple as compare to with respect to the orthogonal plane motion.

So, we can able to analyse the one plane motion separately and the other plane motion orthogonal to this will be identical to that that means z or x plane motion will be we can able to analysis all the similar lines. In this particular case element properties are same in this 2 planes, we can able to see that to define the position of this particular node we have the linear displacement also the slope we will be defining.

So, the positive convention of the linear and angular displacements we have consider here. Similarly, at this node we have 2 coordinates one is linear displacement or translation displacement and is the rotational displacement or the slope of the beam at this location this the basically one of the element of L. So, we can able to see this is the small element of size L and total length of the beam is L.

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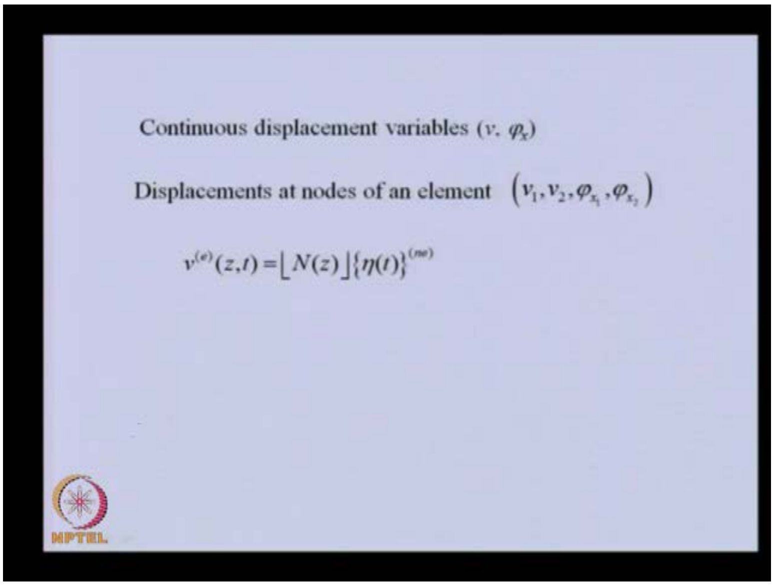

Now, in when we consider the continuous system the continuous displacement variables we had was the linear displacement or translation displacement and the slope now for this particular element we have 2 nodes. We have four variables there is this are the field variable for one element the 2 is the translation and towards slope. Now, infinite element method will be seeking solution of these displacements and if you want to known displacement other than the node position in this position. Then, we will be using interpolation function shape function to interpolate the values of the displacement in between so that means the displacements at any other location z.

In the local coordinate can be expressed as the shape function and the nodal coordinates and in this particular case, this nodal coordinates will this and shear functions will be accordingly will be choosing subsequently. As for the torsional vibration case the shear function will be choosing based on the completeness and comparability condition of the problem and that will be will be choosing at the stage, when will be developing the week formulation of the problem.

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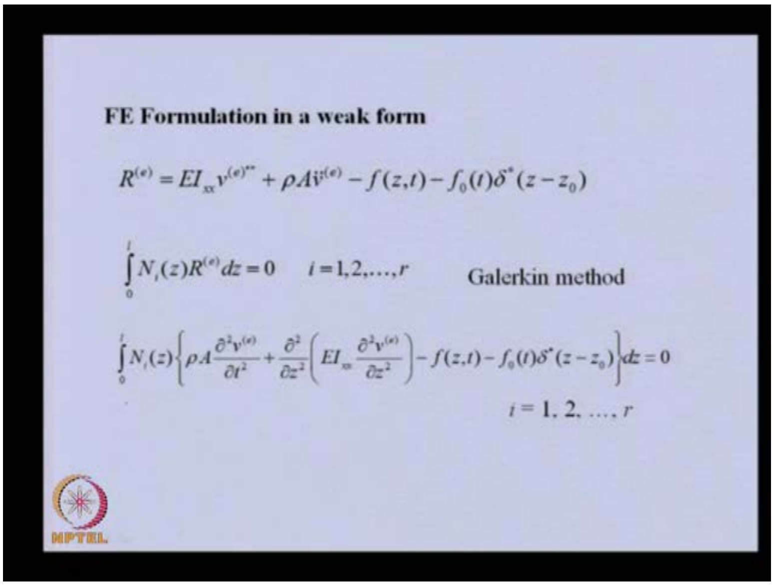

So, formulation of the weak form what are the assume solution we have the approximate solution of this form in which this is the approximate solution. If this solution we substitute in the equation of motion of the continuous system which we derived in the previous class because the solution is approximate in nature.

So, we can able to see this equation will not satisfy completely this will not be completely equal to 0, but some residual for that element will left out this govern equation which we derived earlier was valid for each and every point out of the beam.

So, obviously it will be valid for a particular element also that is why we have substituted the solution of the element on this.

So, we got some residual and using Galerkin method by multiplying the some weight and in this particular case this weight will be the shear function of the problem itself. So, if you multiply this shear function with the residual and if you integrate this residual over domain equal to 0. As we know this shear functions will be equal to the number of coordinates the degree of freedom of the elements, so in this particular case we have substituting this residual and this expression. So, this is expression for that, so we can able to see this is this inertia term and this one is this term can in this particular case we have taken out if we have properties varying.

Then, we need to write the equation like this that we have discuss earlier while deriving the equation of motion of the continuous system. So, if it is constant let it will come out and will be having derivative of v with respect to z 4 times which we are represented as prime here. Explicitly, we have express in terms of the partial derivative with respect to z and this force terms will be as it is. This z is distributed force is a concentrated force acting at z equal to z not location, so we have substitute the residual here and we are equating into 0. These expressions are we have order number of such equations are there, now we will because in this particular equation the highest order differentiation is 4.

So, for this particular case if we want chose this shear function here in form of polynomial. So, we have to choose fourth degree because the fourth order differentiation is there, so unless we choose 4 degree, it will it will vanish. So, what we do in this particular case will we can the requirement of this particular shape function degree by during integration by parts of this particular term in which we have z derivatives. So, let us see so here are expanded those terms, so this is inertia term.

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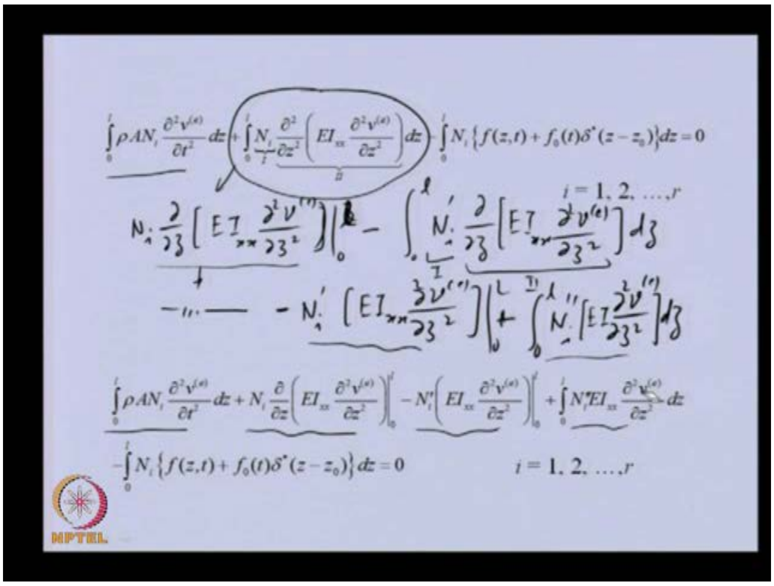

This is the term which we want to do integration dot part and other terms will be as it is because they do not have higher derivative with respect to z. Now, let us see how we can able to differentiate this with respect to this, so I am differentiating, I am taking this term only, so if we do the integration by part of this particular term this particular term.

So, this will be first part integration of the second part this is integration of the second part limit 0 to z or L 0 to L this we are doing at for the element length minus 0 to L differentiation of this particular first term. So, differentiation with respect to z, I am writing as prime this will be integration of the second term that means same as this term d z. So, we have integrated we have done integration by part of this, once will be doing one more time so that we can able to we can able reduce the requirement of this particular differentiation.

So, if you do one more time so this term will be as it is, now this will considering this as first part and this as second part. If we do the integration by part we will get this as differentiation of the first part and integration of the second part. So, that will be 0 to L and then this will be positive 0 to L differentiation of first part, differentiation of this and integration of this. So, what we have done here we have we have done integration by part of this in 2 step first was this one and then second of this. So, if you see finally, you will get this so this expression will be as it is there is no change this first term is this term and this particular is this and this integration is this one and this force terms are unchanged there as it is equal to 0.

So, you can able to see that how you have done the integration by part wise to get the weak form the formulation finite element formulation. In this particular case, now if you see the highest order of differentiation is 3 over within the integral and outside the integral and within the integral highest derivative is 2. So, the completeness compatibility conditions. Now, we will be defining first is in whole expression so whatever the highest derivatives with respect to z, there up to that we require the completeness, that means we need to have the polynomial which should satisfy that we can able to see.

Now, with cubic polynomial we can able satisfies this, this will not vanish and for comparability condition we need to look into the within the integral what is the highest differentiation with respect to z so that is 2. So, as per rule we should have the comparability one order less that means up to first order, so that means we need to have comparability at the common mode of 2 elements of the variable itself and its first derivative that means slope.

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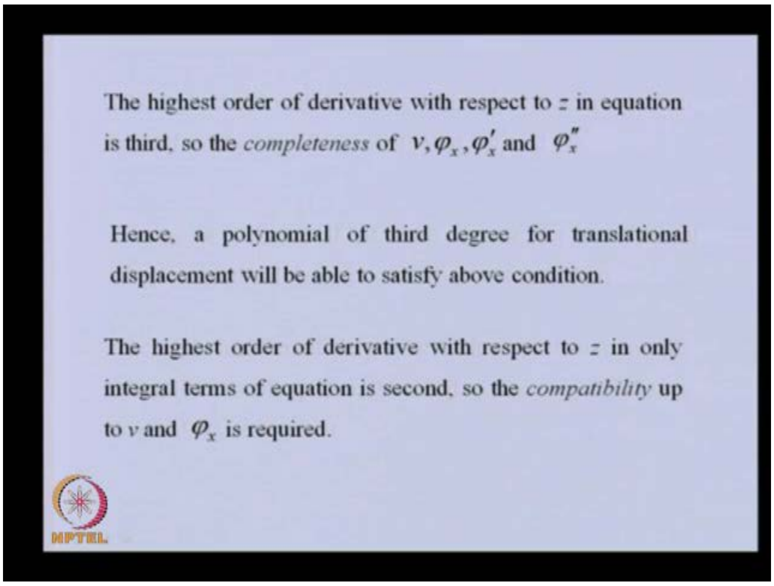

So, let us see this 2 requirements, so highest order of derivative with respect to z in the whole expression is third, so completeness of these are required. So, up to the third differentiation with respect to v was this slope is itself is first differentiation of the displacement linear displacement. So, a polynomial of third degree is sufficient 2 z for the shear function or delta affiliation function and for comparability the highest derivative within the integral is 2. So, up to 1 or less that means the linear displacement and the angular displacement we require the comparability condition.

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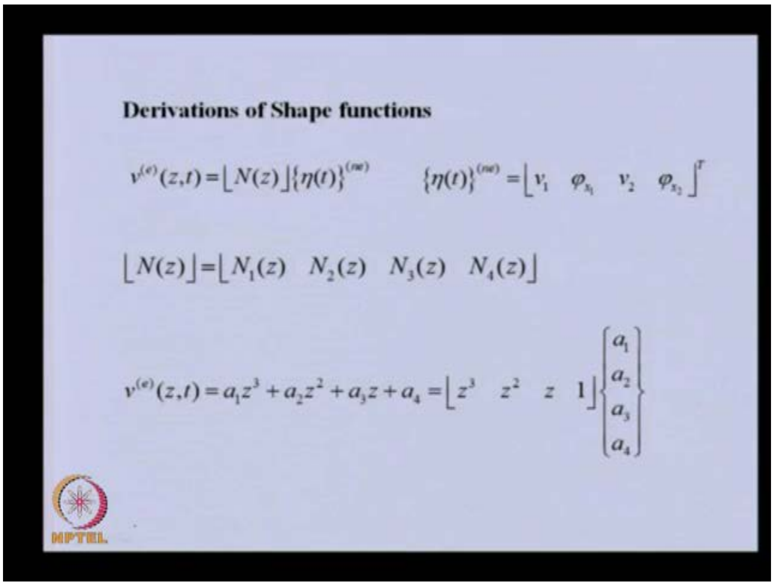

So, with this information, now we can able decide about the shape function, so as we have chosen the shape function as approximate and we had are the nodal displacements the element as this four because four degree of freedom element is there for 2 nodes. So, we expect there will be four shape functions and this shape functions as we are seen in the previous condition that cubic polynomial is sufficient to represent the shape function for the this particular problem. So, this expression we can able to write in a row vector and column vector like this so we can able to see this a one a 2 a 3 a four are unknown this, we need to find out from the boundary condition of the element of the element, and let us see how we can able to obtain the this constants

So, we have obtain the weak form of the formulation and based on that we have known chosen what should be the polynomial degree and this polynomial degree polynomial is having 4 constants. We are obtaining this using the boundary condition of the element and let see how we can able to obtain these constants.

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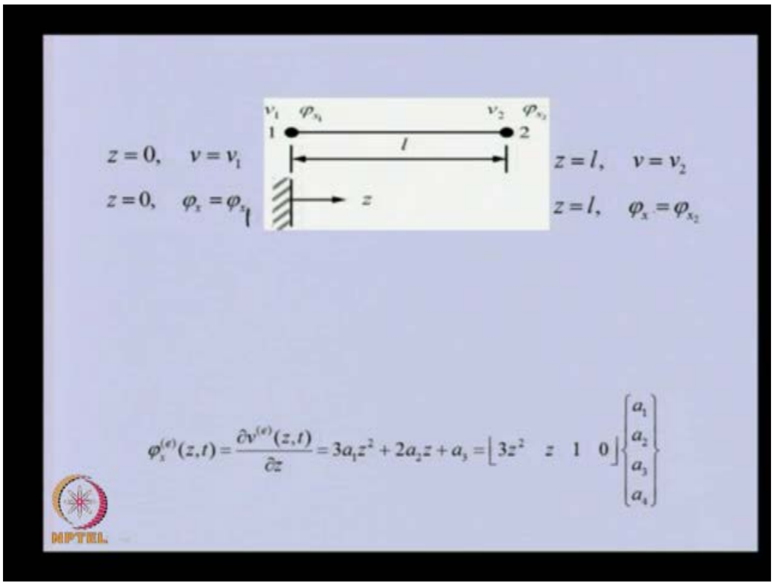

So, this a particular element which we have chosen it is having 2 nodes at 2 ends at each node. We have 2 displacements one is v translation displacement another is the phi, there is phi which is slope. So, you can able to see there are 4 variable attach to this particular element and the boundary conditions for this considering this as local coordinate system z. So, origin is here and this hand is that is equal to L, so we can able to see at z equal to 0 the translational displacement v 1 slope is equal to phi x 1 this should be phi x 1. Similarly, here at z equal to L we have displacement v 2 translational 1 and the slope is phi x 2, now we can able to.

So, in this particular case because we need the slope also, this the derivative of the shape function, which we choose the polynomial form. So, if we take the derivative we will get this expression because one term one degree will get reduce at this x l. We can able to write again in the similar form as the previous one only thing is we have, now a 4, 0, but to take care of that we are keeping this as 0. So, basically here in this particular expression only 3 constants are there 4 is not contributing, now will be satisfying this boundary conditions.

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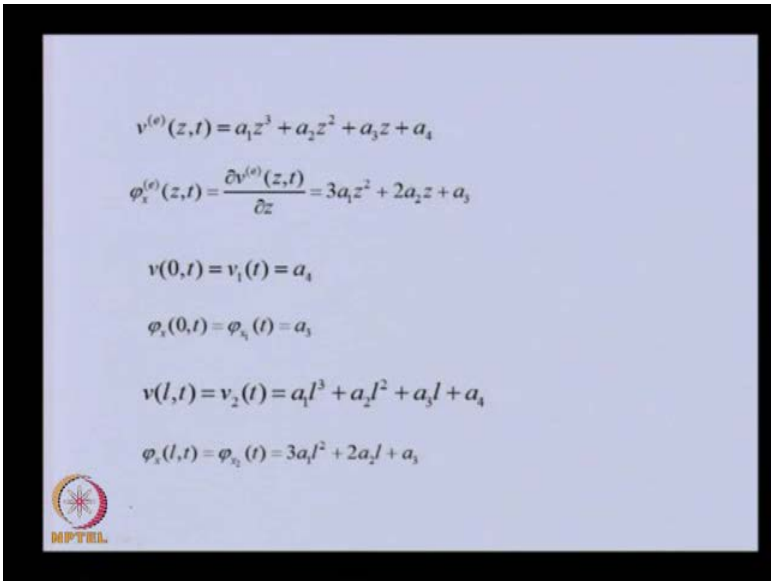

So at z is equal to 0 slope that displacement is v 1 0, so if you satisfies here z is equal to 0 we will see that a four will remain other terms will vanish. If you put z equal to 0 in the slope term, this 2 term will vanish only a 3 will be left out, similar if you put z equal to L in this displacement term will get this. In the slope term, we will get this so basically 4 equations and we have 4 unknowns a 1, a 2, a 3, a 4, so and they can be solved because they are simultaneous linear equations.

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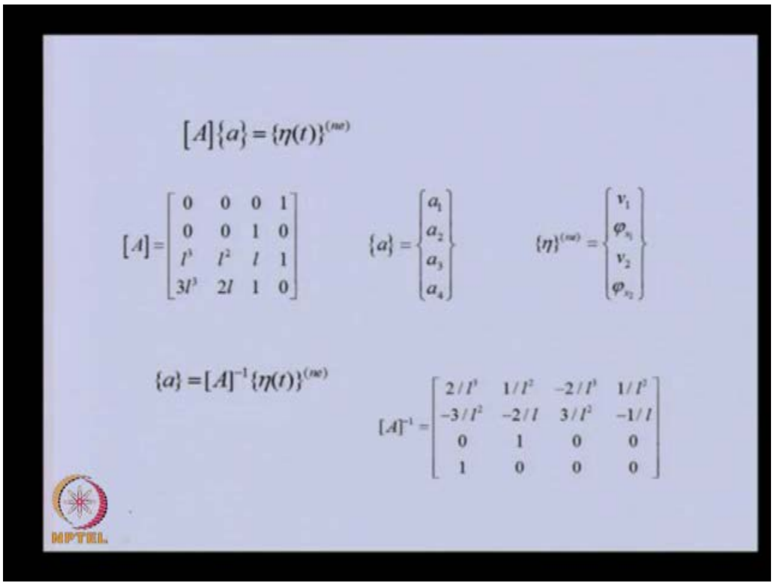

These expressions have written this particular form, so we can able to see that have expressed.

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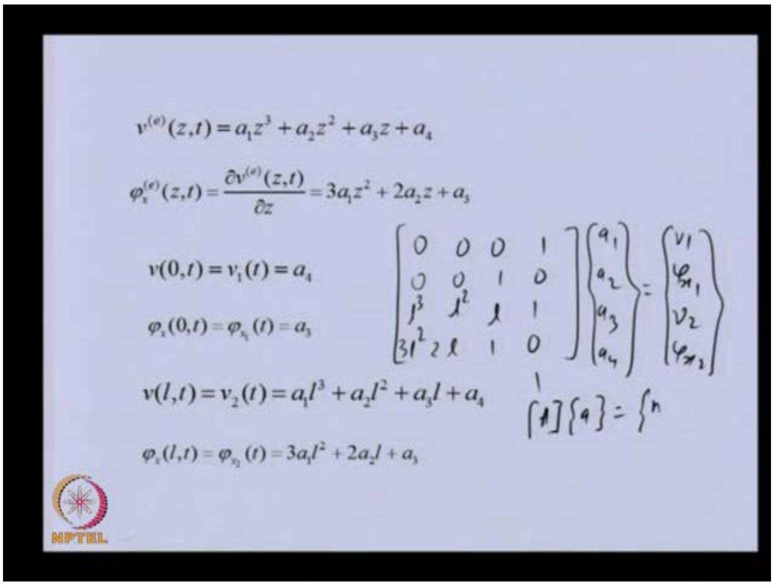

These four equation in a matrix form something like this, so we have a 1, a 2, a 3, a 4 is equal to a 1, phi x 1, p 2, phi x 2, so the first equation if you want to substitute here will see this will be 0, 0, 0, 1. Second equation will be 0, 0, 1, 0, this expression will be L cube L square L 1 and this expression will be 3 L square 2 L 1, 0.

So, basically I have put this in a four systematic way in a matrix and this I am calling this a, this is small a in the next slide and this is the eta that nodal displacement, now this constants are unknown so we can able to get this by inventing a matrix. So, that we have done here we invent a matrix to get a multiplied by the nodal displacements and if we if we invert the matrix the form of the a will be like this we have obtain.

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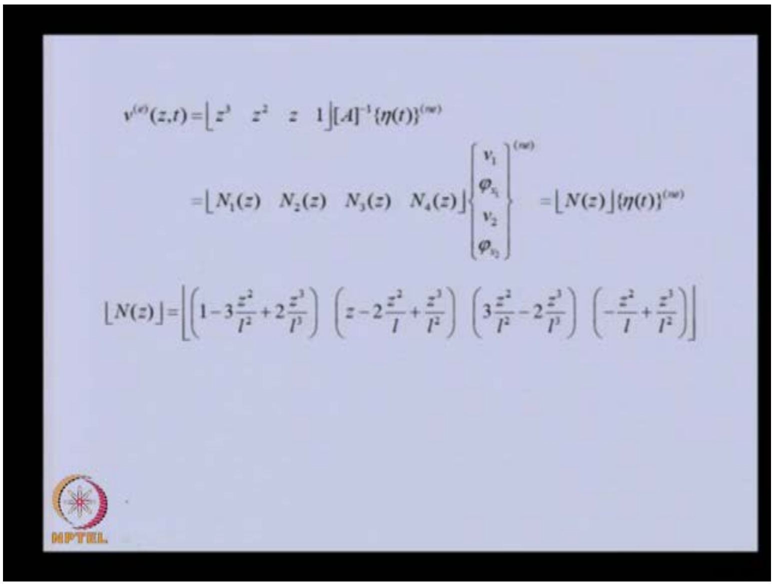

Now, a constants of the polynomial, now we earlier expressed the particular displacement shape function displacement as go back to this we express like this.

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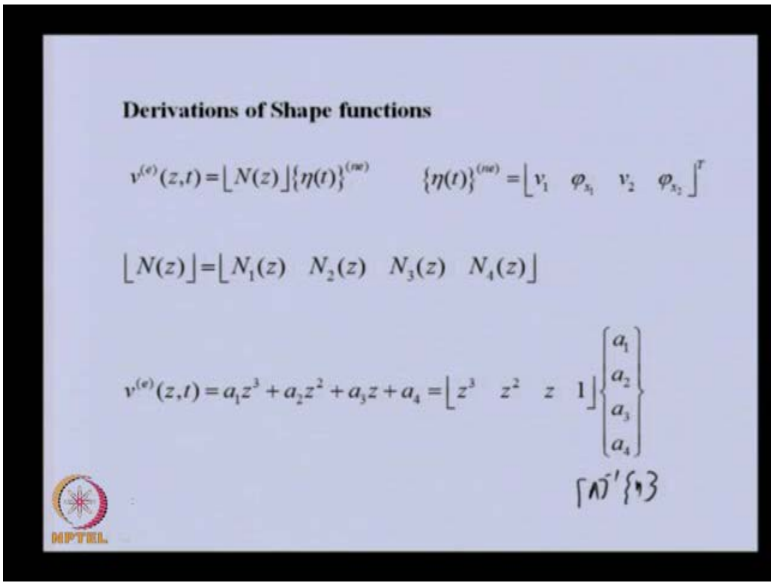

So, basically we had a here which is now we obtain as A inverse into eta, now we are substituting this in place of a this quantity in this expression.

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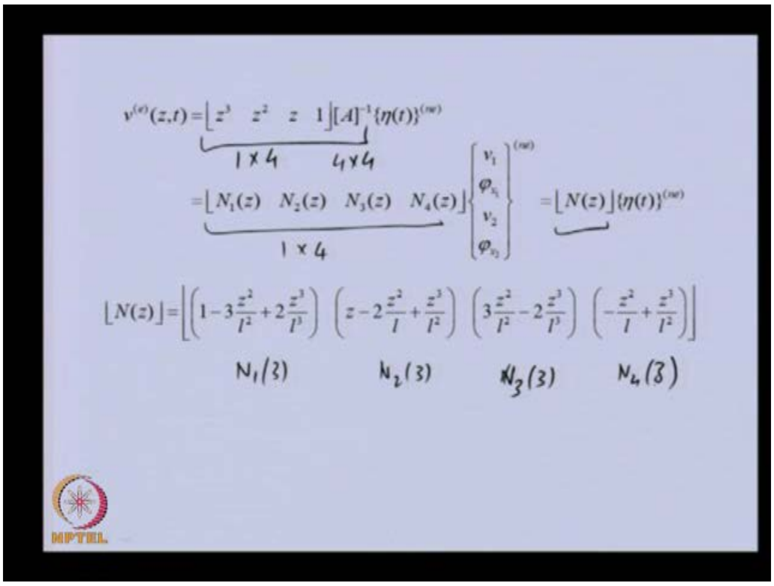

So, there is the z terms and inversely into nodal displacements and if we multiply this 2 we are expressing because the size, let us see the size of these. So, we have 1 into 4 of the first rho vector and this is a matrix is 4 by 4, so finally, we should get this as 1 into 4. So, only one rho is their four columns are there and the corresponding terms will be we have define as the shape function N 1, N 2, N 3, N 4. If we multiply this, basically this shape function which is written like this will get this form. So, if we multiply inverse into this you will get this so here we have this as N 1 shape function this is N 2, this is N 3, this is N 4 which is function of z.

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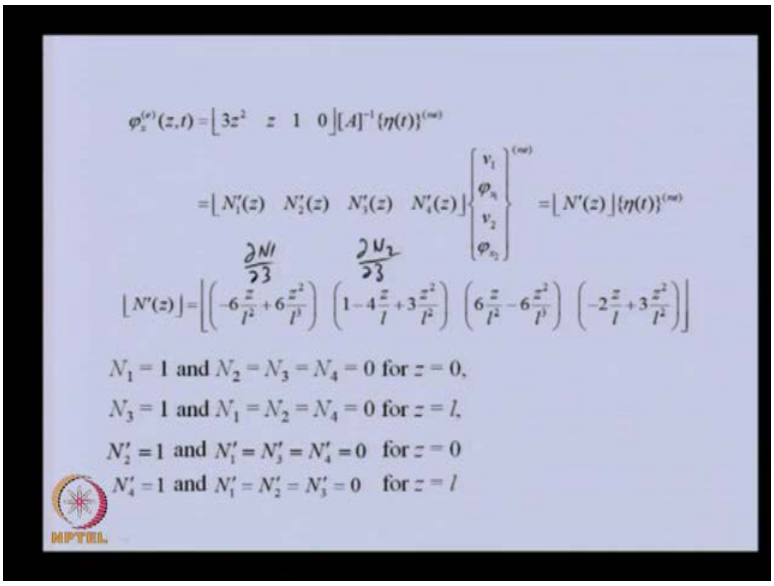

For slope also, we wrote on the similar lines, so here in place of a we have substitute the value of a if we multiply capital a matrix. This we will get this the differentiation of the that is shape functions so these are so these are nothing but differentiation of the N 1 with respect to z and N 2 with respect to z etcetera. Now, the property of these shape functions, so we are seen in the torsional vibration case here the N 1 will be 1 if we put z equal to 0.

So, in the previous here if you put z equal 0 this will be 1 or this will be 0 so N 2 N 3 N four will be 0 similar for z is equal to L N 3 will be 1 z is equal to L N 2, N 1, N 2 will be 0, only this will be 1 this will be 0 similar for z equal 0 in the slope. So, if we are keeping a N equal to 0 we will see that and 2 prime will be 1 others will be 0 so here and to prime this is 0 1 others are 0. Similarly, for z equal to L, we will be having and four prime is 1 and all other 3 terms will be 0.

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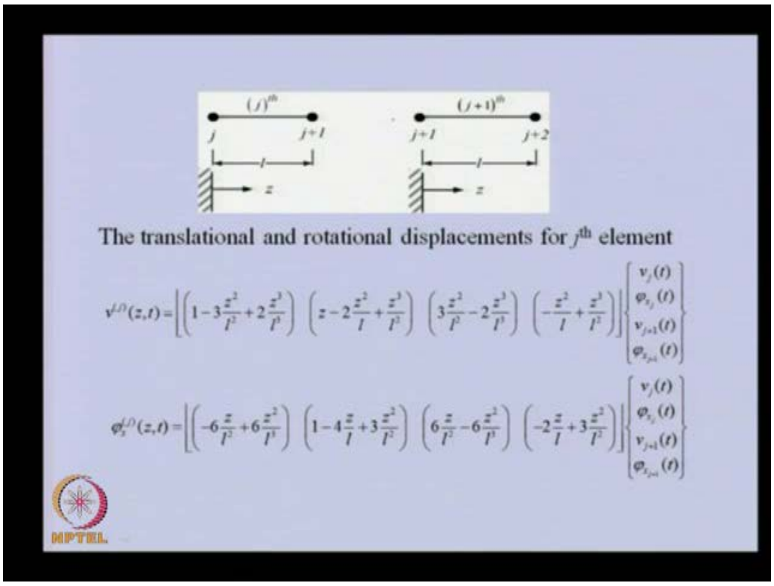

Now, let us see the 2 elements, basically we are trying to see the comparability condition at the common node. If we predict the displacement from this element and this element whether we have the similar production of the translational displacement as well as slope because we have seen earlier. We require for this particular shape function comparability up to the first derivative. So, this is the shape function which is predicting the displacements at node position also at the intermediate position, so will be using this 2 expressions to predicate the displacements at the common node, but 2 element 2 separate elements.

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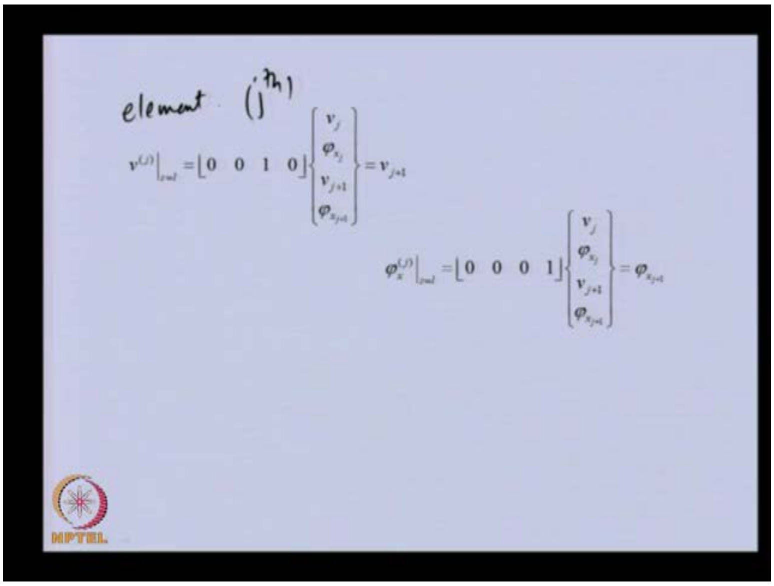

So, for z is equal to L for first element 1 that is a z is equal to L, so we are getting if you substitute in the previous expression this expression z equal to L we will get only this as 1 other are 0. So, we will get v is z plus 1 and slope we will get phi x z plus 1, so this is for element in this particular case j th element this is j th element, now for j plus 1 element, this is the expression, so this was a z plus 1 element.

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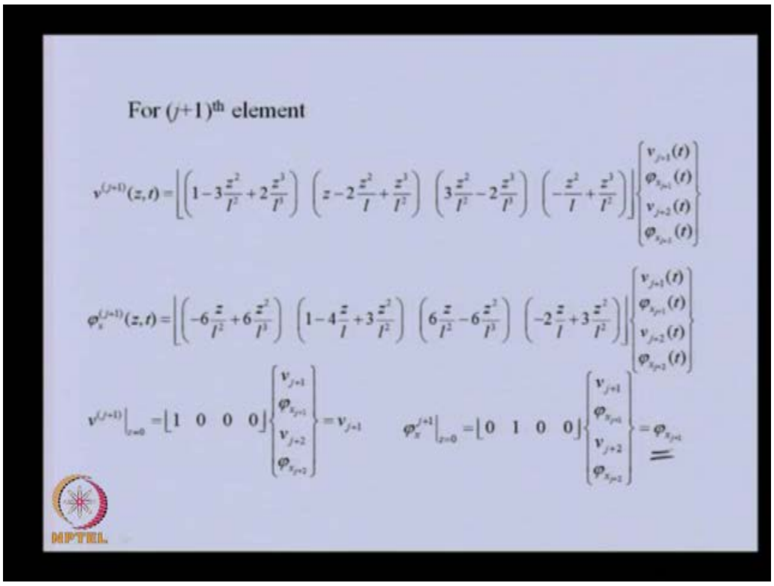

This is the linear displacement and angular displacement and in this particular case, if we put z equal to 0 because for z plus 1 element this common node is at z equal to 0. So, if you put z equal to 0 when this expression we will get this as here only this will be 1 remaining terms will be 0. So, this will give v z plus 1 which is same as the previous displacement from the element j and if you put in this z is equal to 0 we will get this is as one other terms as 0. From here, we are getting phi x z plus 1 which is same as the previous predictions this, so that means from two different elements at the common node.

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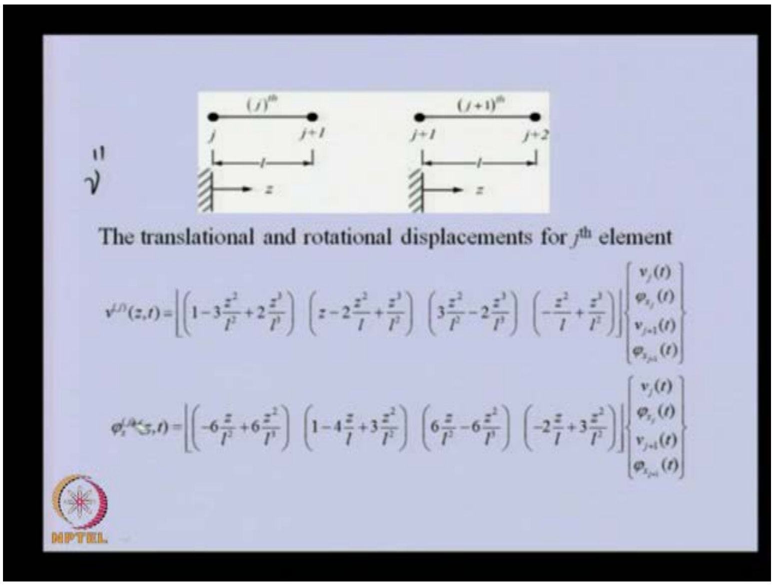

These shape functions are giving same translational displacement as well as slope that mean there comparability up to that. If you want to check for higher order, then will see that this may not be this shape functions may not be comparability for second derivative. So, if we take derivative of this once more and predict the value at common nodes from this two different elements will see that they will not match.

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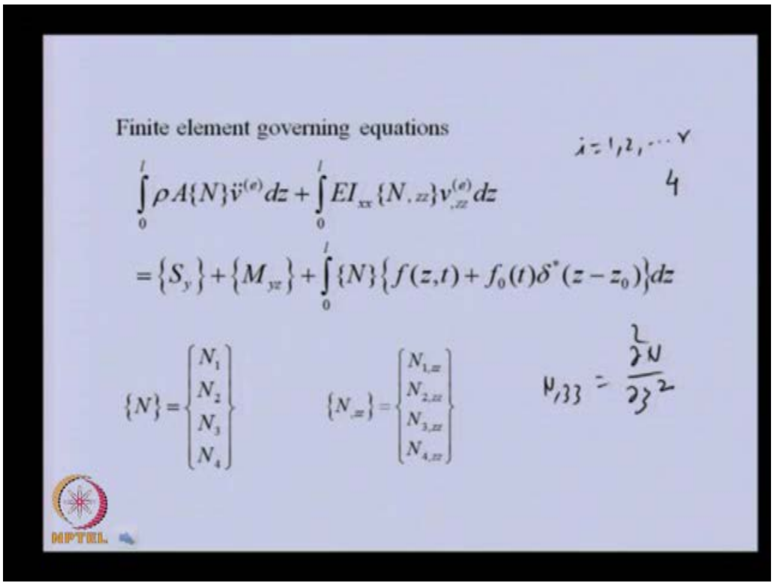

So, with this now we are choosing the shape function of third degree even we have obtain the explicit form of the shape function. We will check comparability condition of the of this shape functions that they satisfy not only the translational displacement, but also the slope comparability between 2 L elements at the common node.

So, now this approximate solution which we obtain in the explicit form, now will substituting in our weak formulation and then we will try to get the elemental equation. So, we if we see if we substitute, so this particular equation is basically what we have done here earlier we had weak formulation equation. That was i is equal to up to r, so we had r number of equations those r number equation in the weak form. Now, we have put in a matrix form, so that means in that particular equation if we look back that particular equation we will see that in this expression and is containing the subscript i which is varying from 1 to r. So, only N r there, so these r number equation we can put in a matrix form a.

So, basically this N is a vector which is because now we know the r is equal to four of the present problem because degree of freedom of element is 4. So, will we have 4 shape functions, so here it was double derivative, so comma double z is representing that double derivative of N with respect to z. So, this is share force binding moment this terms I be explaining explicitly this was the force term, now let us see.

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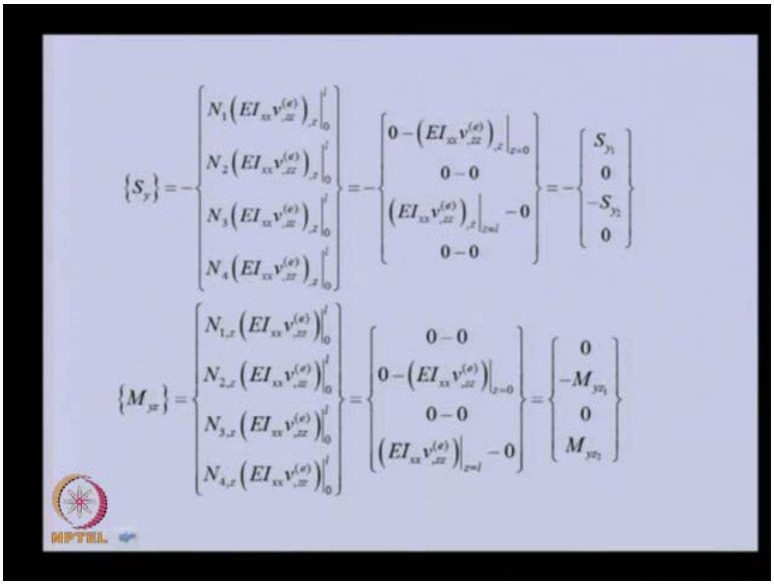

What is shear force expression? So, we had after integration by part this kind of terms, so I have represented in the first term after integration by part like this here also because N was having substitute 1 to r. So, they are in a vector form, now similarly this one is the binding moment is from the second term we are getting it.

Now, let us see what we are getting from this so if we substitute z is equal to L because suppose N 1 will be 0 there, so that expression is 0 for z is equal to 0 N 1 is 0, so we are getting this one and this I am expressing at z is equal to 0. This particular term is basically shear force because here you can able to see this a prime z that itself is representing at whom the term within the bracket is differentiated with respect to z. So, basically this is a shear force term having third degree in the translational displacement.

So, I am expressing that negative of that term as shear force y one here will see that for z equal to 0 and z equal to L and 2 will be 0. So, this will not contribute anything, similarly here and 3 will be 1 for z equal to L for z equal to 0, this will be 0. So, this will contribute only one term in that I am expressing as minus of as y 2, so I have define this s y 2 as minus of these. So, these are the shear force at node 1 and 2, similarly from binding moment we will see that first and third term will not contribute anything. This will contribute and they will belong to binding moment at node 1 and node 2 of that particular element.

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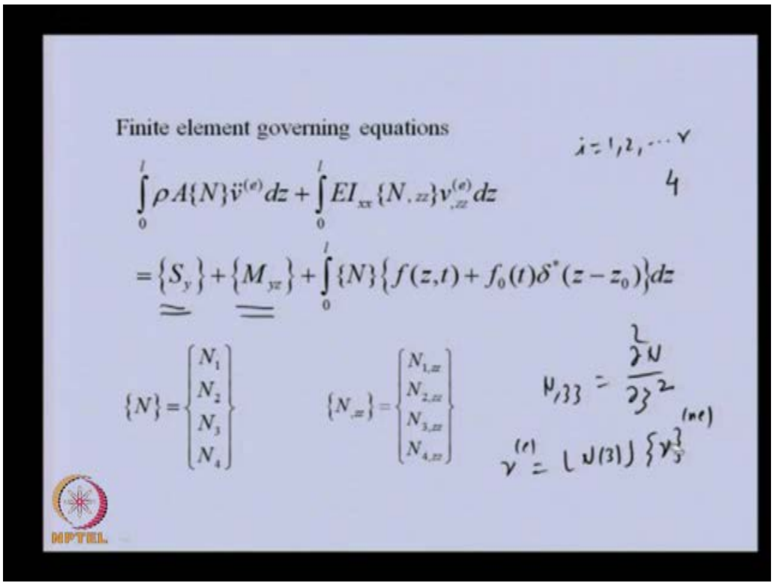

So, coming back to the previous equation in this particular expression, we have we have seen explicit form for these two. Now, because this is the displacement we are chosen in a form of shape function like this and from the nodal displacement. So, this for the element this for the nodal displacements, so this approximate solution we are substituting here.

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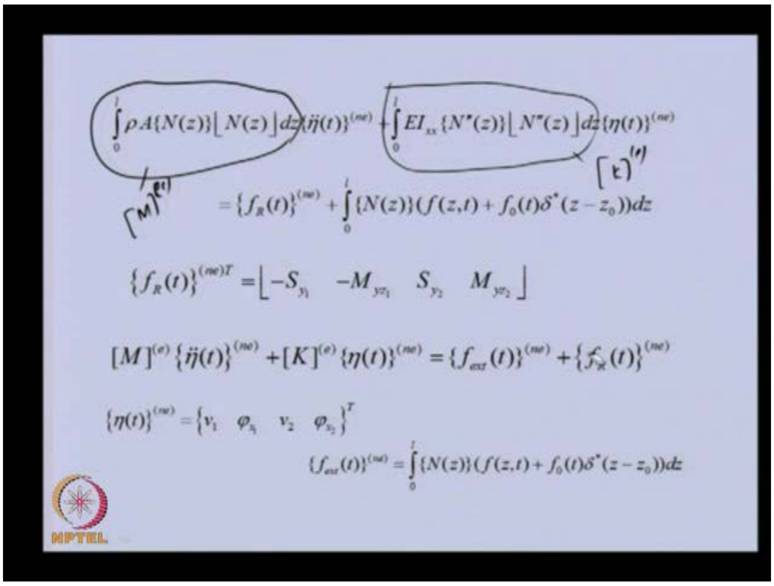

So, once we substitute we will see that multiplication of another rho vector of the shape function will come and this particular term nodal displacement will get the derivative of the v with respect to time because this is independent of z. So, it will be outside the integral, similarly from second term we will get the derivative with respect to z which was there in v we will get in the shape function.

There is eta will be outside the integral this will not get any differentiation because there no differentiation with respect to time in the second term. Similarly, this is basically we are we are club the shear force and binding moment in one this is the reaction force have force vector have represented. If you see the shear force which we got and the binding moment we have kept in a single vector and this is the force term.

Now, will be representing this term as mass matrix for the element and this term as stiffness matrix for the element this one, and this is the external force belonging to this particular vector and reaction force belonging to this. This vectors nodal displacements are this external force we are already express like that, so you can able to see that. Now, we have got the elemental equation of the finite element elemental equation, now let us see explicit form this mass matrix and the stiffness matrix.

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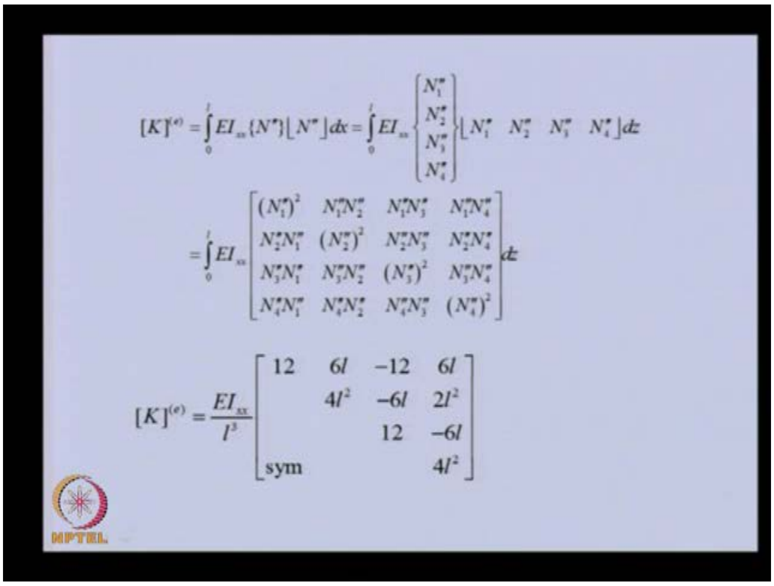

So, stiffness matrix as we have express the integral, so this is if you multiply this 2 vector if you substitute and multiply will get this and each term we need to integrate with respective z because we know the this shape function in the form of z. So, if we do the integration of each terms, we will be getting the stiffness matrix like this. This will be symmetric matrix only shown the one half site of the diagonal, so it is matrix so we can able to write the other half without any problem.

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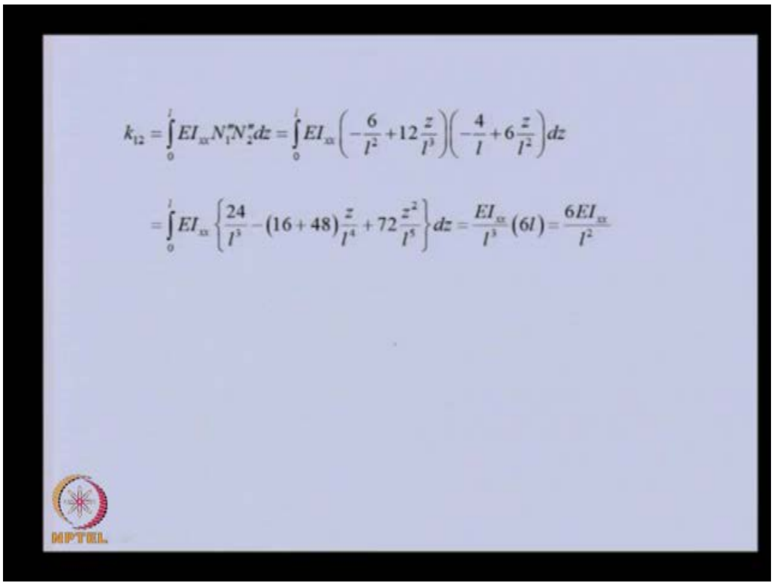

Now, let us see for a typical stiffness let us say element one k 1 2, so we need to substitute that corresponding shape function and their second derivative. If we integrate will get that particular term, similarly we can able to integrate other terms.

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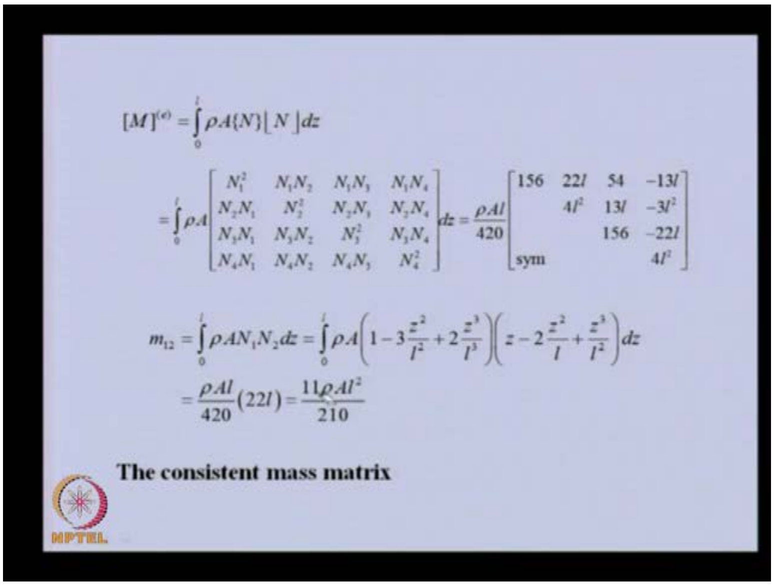

Now, coming to the mass matrix we have this two multiplication, so if we integrate this terms we will get this as mass matrix this is also symmetric matrix and is fully populated.

That is why we are we call this is consistent mass matrix and consistence stiffness matrix one particulars mass matrix. If we want to integrate we can able to, so this particular term is I am going to, so this will able to get.

So, if you multiply these two we will get that particular term from this this is the consistent mass matrix because all the element of the mass matrix is fully populated. Now, we have obtain mass matrix and stiffness matrix in explicit form we have seen that in the right hand side we have reaction torque and reaction force that contain the shear force and binding moment. There is an external force also, if we know the explicit form of the force, then we can able to calculate that particular vector, so let us try to calculate that vector for particular distribution of the force.

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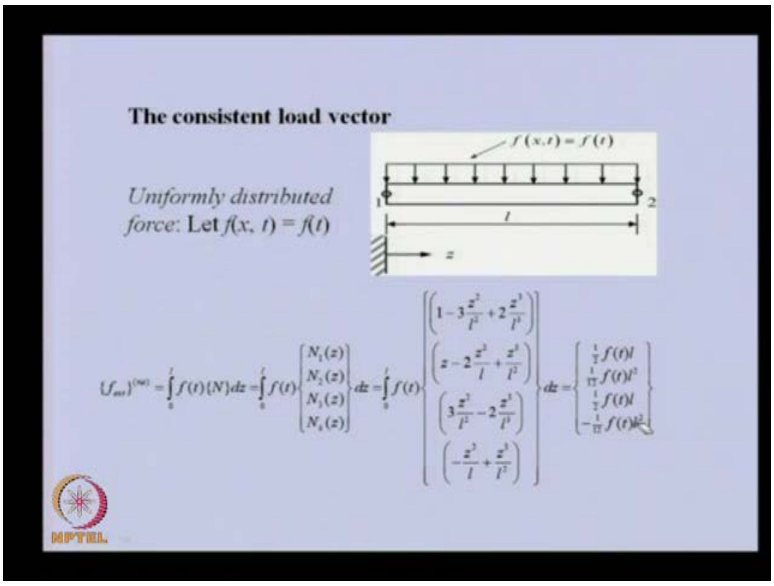

So, in the first case let us say we if you want to obtain the consistent node vector for uniformly distributed load that is so f t is a load, which is uniformly distributed over the element. So, this is the load per unit length. So if we if we substitute this in the distributed load component they is no concentrated force because that was containing 2 terms one was distributed and another was concentrated, so this only we considering the distributed one.

So, that force multiplied by the shape function term was there and shape function term having four terms so because this is a constant quantities it will go out and we need to integrate this shape functions if we integrate we will get the consistent force vector for a distributed force like this so we can able to see that because of this distributed force over the element we are getting some kind of equivalent force at 2 nodes as well as equivalent moment at those 2 nodes because the first and 3 belongs to the force and second and fourth belong to the moment.

So, for uniformly distributed force we need infinite element method basically we see the solution at the nodal position. So, we need to obtain the equivalent of this distributed load at the node position, so this is the equivalent of that if you see if we some of the first and third term that will give the total load. So, f t half and this is f t half, so this if you add it that will give you get f t into L, so that is the total force acting.

Apart from that we are getting the some moments also and because there is another model in which instead of consistent load vector. We could have we use the lump force model in which obviously the straight forward will be that whatever the force as acting on this element half of that. If we apply here half of that if we apply here that is the lump force model and in that particular case this is fine, but moments will be 0, so that will be the in fact consistent load vector or lump mass load vector.

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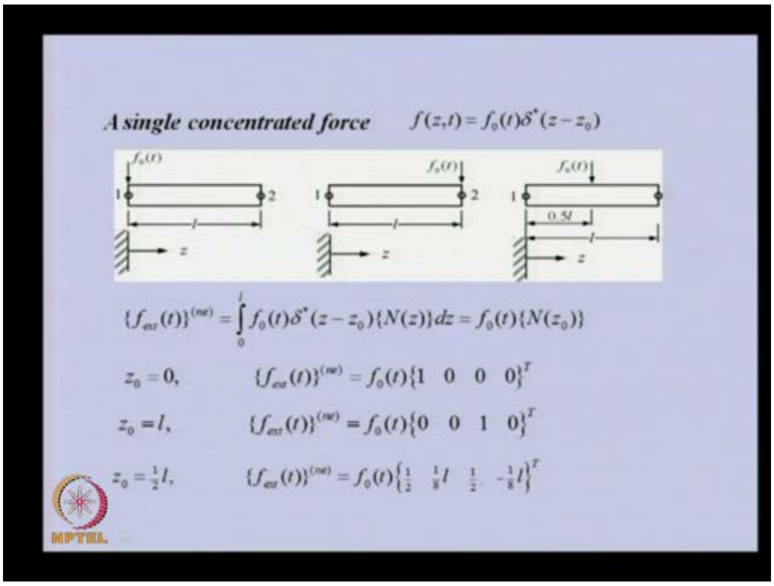

Let us see some more cases so if we have single concentrated force so if of this form at z is equal to 0, so let us say for one case it is it is at node 1. If it is at node 1, we can able to substitute here because it is acting at node 1 that means at z is equal to 0. So, we know that this direct delta function if it is there if you want to integrate this term because this is constant this will go out, but this function will be evaluated directly by putting z is equal to z naught. So, that is basically the explicit integration of the this and for first case when z not is at 0, this one we will see that on the this will be one this will be 0.

Because N 1 is 0 for rest of the case except at N 1, so you can able to see that directly we are getting the node nodal force at 1. Similarly, if node force is here we will if you substitute there we will get at corresponding node because second and third is fourth is for moment this is for the force at node 2, but if we keep this particular force at the middle. Then, we will find that all the all the terms including the moments terms will not be 0 and this is a consistent force factor for the for the concentrated load. So, half of this is coming here and here as a force, but apart from that moments are also coming so you can able to see y what is that voltage of the consistent force factor that it is more accurate and this is not a.

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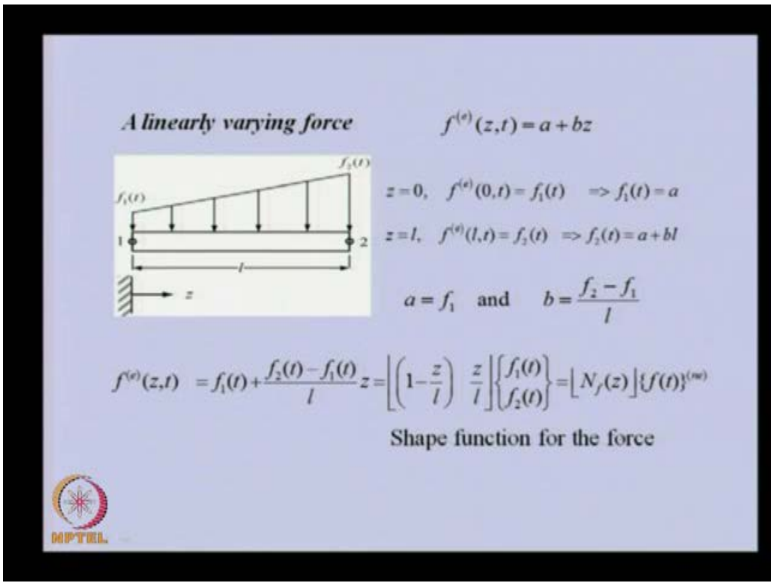

Based on the lump force analysis and other case, let us say if the force is linearly varying like this amplitude of this are given because it is linearly varying. So, let us express the force as it is a linear polynomial where a and b are constant to be found out from the end condition of the force distribution. So, at z is equal to 0 the amplitude of force is f 1, so if we substitute here we will get a is equal to this one for z is equal to L, here the amplitude is f 2.

If we substitute in this polynomial, we will get this these 2 equation will give us a and b like this. If we substitute a and b in this polynomial like this, then we are rearranging f one and f 2 terms separately. So, we will see that basically we obtain a shape function for the force these are for node 1 and 2, but this is the shape function for force.

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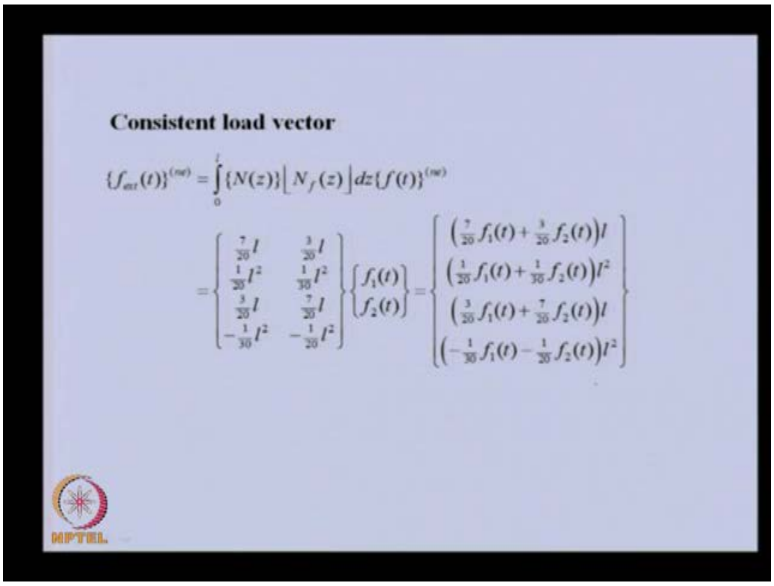

This can be used now for calculation of the consistent load vector and now we have usage of force we have this shape function into this. So, not only the shape function of the element is there, but shape function of the force is also coming, this is coming out because this is time dependent term. So, if we integrate this we will see that we will get the consistent force vector for linearly varying force like this. So, at each, this is the force these are the moment, so this and this is a consistent force vector for linearly varying force this analysis.

Now, we can able to see that we can able to extend for if this distribution is parabolic or cubic or the similar lines now we have obtained a force vector also the mass matrix and stiffness matrix. So, once we have obtained the elemental equation, now the thing is how we can able to assemble if various elements equations we have once we have written. How we can able to assemble them, and especially if apart from the element which we have considered. If some concentrated of mass like discs are is there how to incorporate in the assembly assembled equation. So, let us see the assembly of the equation for very simple 2 element case.

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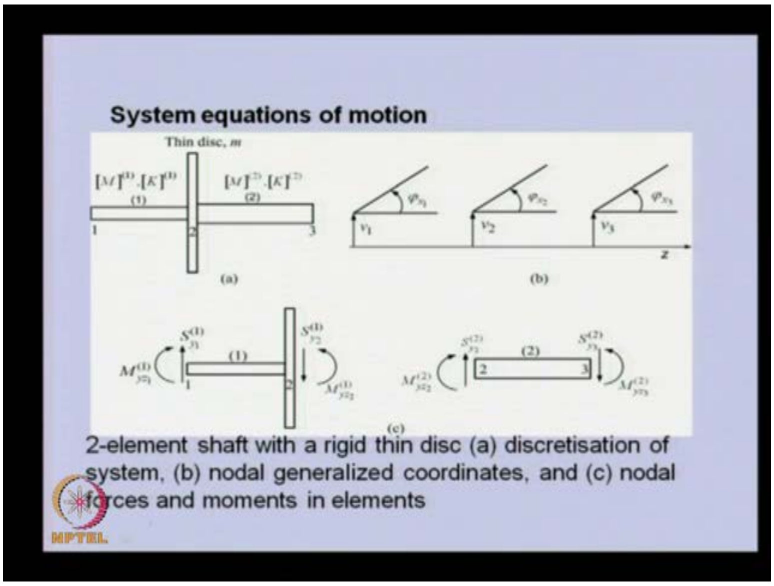

So, in this particular case we have two elements so this is one element so this is another element now the dimensions of these two elements are different and in between this there is a thin disc. Now, we can able to see that these two elements we have 3 nodes 1, 2 and 3 and at this 3 nodes we have at each node we have 2 degree of freedom. So, total 6 degree of freedom is there, so we have divided this two elements like this, so disc we are keeping with the first element the second element we are not keeping the disc.

This disc we could have kept here, but only thing is we need to keep it one of the element because once we join back these two element we should have only single disc mass contribution. So, in this particular case we have kept the disc in the element 1, so you can able to see that with this reaction forces and moments shear force and that is coming now we can able to write the elemental equation for these.

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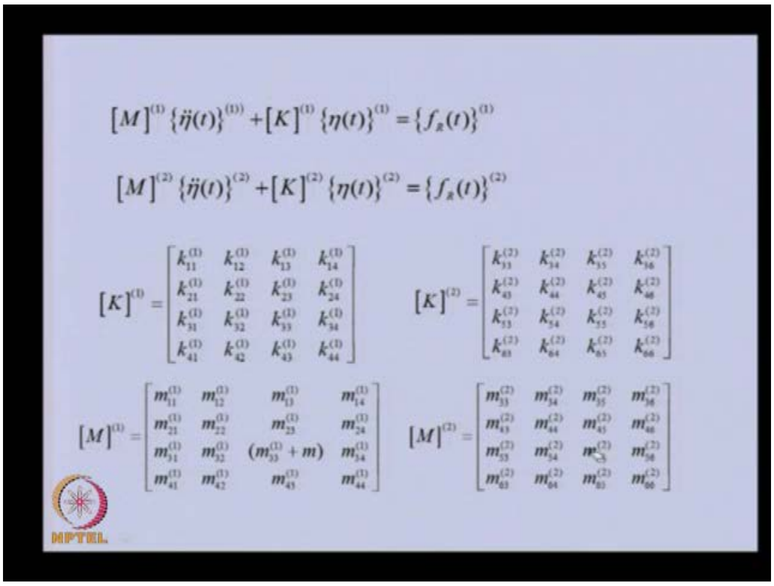

So, let us say this is the elemental equation for first element this is for second element various individual mass and the stiffness matrices are given here. So, for element one this is the stiffness matrix this is the mass matrix so you can able to see that because at node 2 we have concentrated mass that is a point mass. So, that is why and diagonal term mass of the disc is also appearing apart from the mass of the shaft.

So, other terms are belonging to mass of the shaft only this term is from the disc diametric mass moment of inertia of this disc is not considered. Otherwise it could have come here you can able to see in the element 2 because the dimensions of the element 2 are different. So, this stiffness matrix will be different as compared to element one here are the mass from the shaft is coming not form disc of the element 2.

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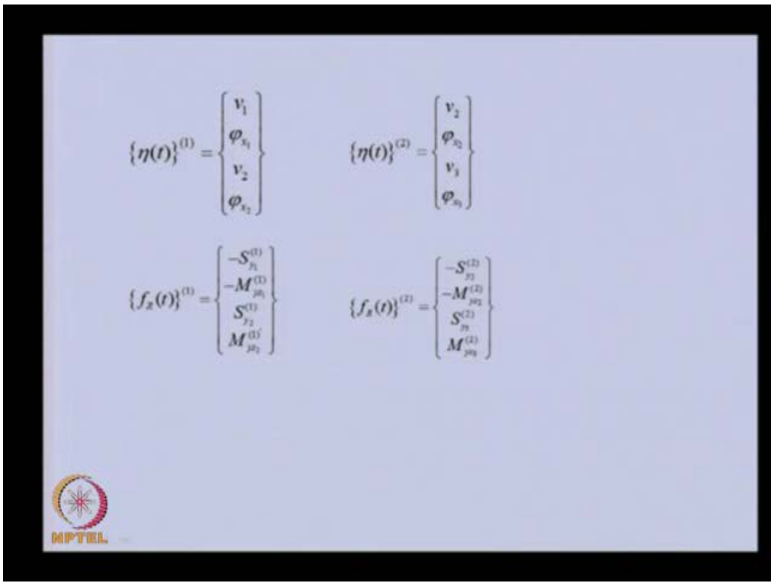

Now, these equations, now let us see the nodal displacement so this is corresponding to element 1 and this is corresponding to element 2 and this is force factor for element one and element 2.

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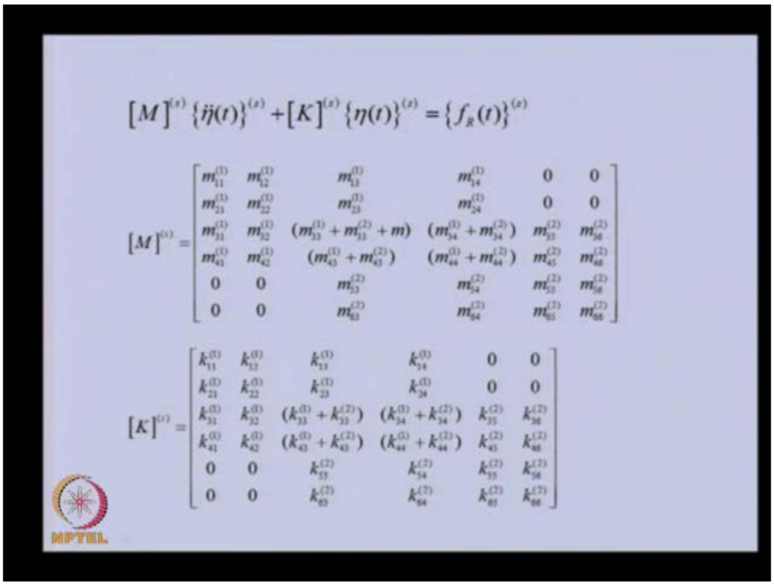

If we assemble then this is the assemble equation for that, so you can able to see that this four by four is for element 1, but because that is a common element. So, we have some of the terms related with the common elements of 2 elements are coming, so you can able to see these are for element 1 and these are for element 2 and this mass the disc is also there. Similarly, this is the assembled equation of the stiffness matrix, so we can able to see with the subscript one they belong to element 1 and this belong to element 2.

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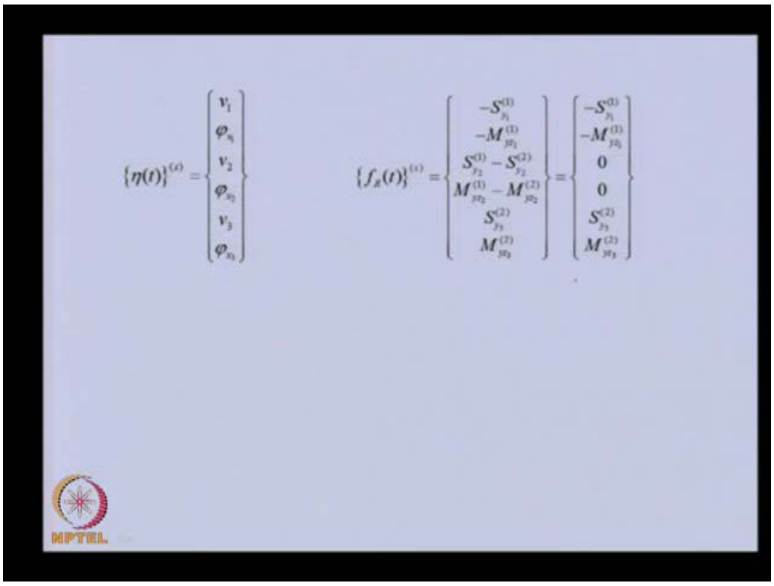

So, this is the assembled equation and the nodal displacements because there are six degree of freedom that will be may be there shear force and binding moment the reaction force. So, obviously they will cancel at the common mode because once we assemble this element they will cancel each other. Now, once we have obtain the assemble equation now we can depending upon various kind of boundary condition, we can able to solve this problem.

So, in today’s lecture we started with the development of the elemental equation we used Galerkin method for the development of the elemental equation. In the previous class we develop the Euler Bernoulli beam equation of motion and for that particular case we started the approximate solution and we obtained the weak form or that particular equation based on the weak from.

Then, we obtain the shape function in the explicit form and once we have obtained the shape function we use that for obtaining the explicit form of the mass matrix, the stiffness matrix and through some cases for the force vector the consistent force vector. We have seen that the how the force vector takes the form if it is uniformly distributed or concentrated force or linearly varying force. How this particular consistent force vector takes the form, and then with a simple 2 element case you try to explain, how the assembly of this elemental equations can be done.

Now, in the next class we will see through some example how we can able to solve the calculation of the free vibration or forced vibration, for a particular boundary condition of the problem. Till now, for when we are developing the elemental equation we did not consider the boundary condition of the problem as a whole. So, in the next class we will take up some example in which we will be obtaining the natural frequency of the system as well as the forced vibration of system using finite element method.