Theory and Practice of Rotor Dynamics
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Module - 6
Transverse Vibrations
Lecture - 28
Continuous System Approach

Previous lectures we studied transverse vibration using influence coefficient method and transfer matrix method. In these two methods, the shaft which we considered was having only the elasticity property. But in some cases the shaft mass is not negligible and we need to consider the shaft stiffness and mass property simultaneously as distributed property along the length of shaft. For such case, we need to model the shaft using a continuous system approach.

In continuous system approach, as we have seen in the torsional vibration case also, each and every particle of the beam is independent of each other and a continuous shaft is having infinite degree of freedom. When we model this kind of system, the governing differential equation generally becomes partial differential equation. So, when we are considering only the one dimensional model of the shaft, so generally we will be having the differentiation with respect to time as well as special quadrant, that is along the length of the shaft for 1 degree of freedom system.

For the present case, we will be considering the simple one dimensional problem, not 1 degree of freedom, but one dimensional. In this, only the property of the mass and elasticity will be changing along the length of the shaft. In this particular case, we expect that the calculation of the natural frequency and the mode shape will be more accurate as compared to when we consider the shaft as mass less.

Generally, this is the case when shaft is relatively thicker. In this particular case, we will be considering the Euler-Bernoulli beam model which we generally study in the strength of material ((Refer Time: 02:57)), not only will be obtaining the natural frequency at mode shape or Eigen function, but also we will see how the force response can be obtained using these Eigen function property along with the orthogonality property of the Eigen functions.

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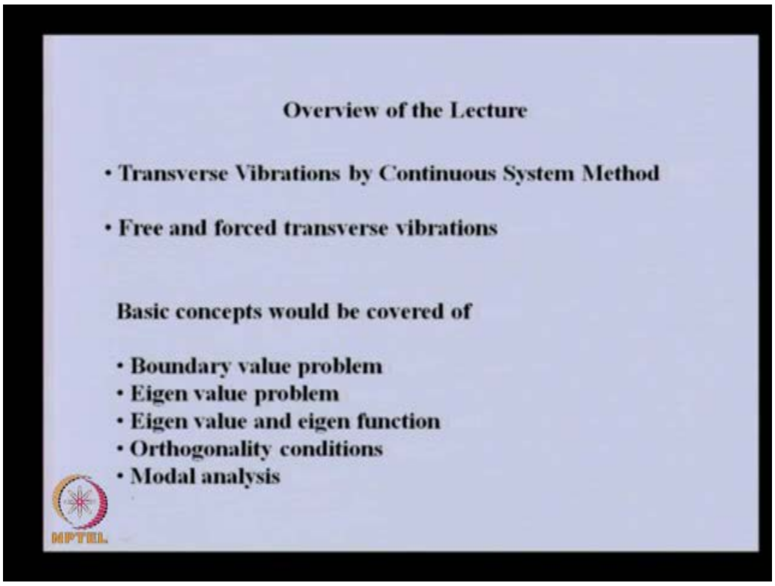

So, let us see what are the things, we will be covering in this lecture. So, basically we will be dealing with the continuous system approach for transverse vibration, free and forced vibration analysis. We will be doing for simple cases. So, various concepts like boundary value problem, Eigen value problem, Eigen value, Eigen function, orthogonality condition and modal analysis, will be covered very briefly.

We will not go in much detail on this, because our main focus would to be to use this model continuous system model for finite element formulation, which is having more versatile application. So, with this as introduction, let us see how the continuous system model for transverse vibration we can be able to do it?

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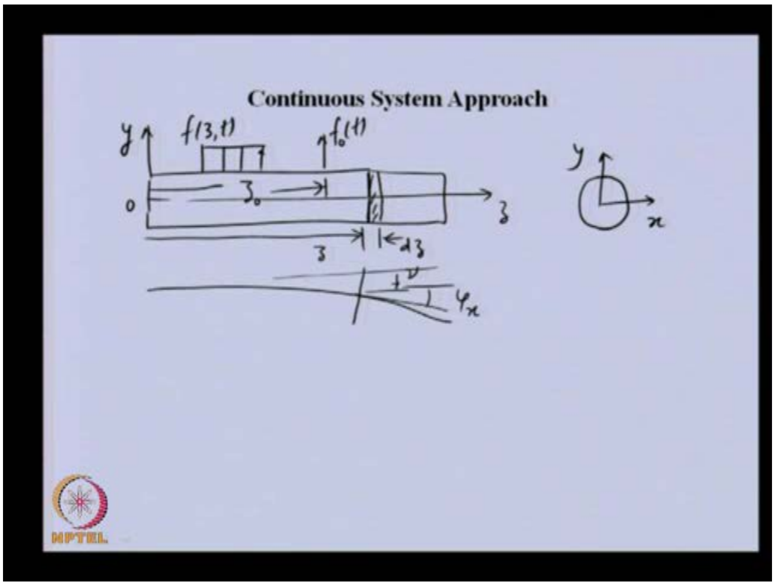

So, if we have shaft, this is the axis of the shaft and let us say we are considering 1 of the plane. In this, we can have some kind of distributed load or distributed force on to this particular beam which is function of cos z. So, this is the force per unit length or we can have concentrated force at some specified location, let us say z naught. Now, if we want to analyze a particular plane, let us say a plane which is perpendicular to the axis of the this shaft, if you see this particular shaft from side may be any cross section we can take and for this case we have considered, let us say circular cross section.

So, we have coordinate x and y here. So, during bending of this beam, generally we consider this particular plane. In Euler-Bernoulli beam hypothesis, we assume that this particular plane remains plain after bending and this is the elastic line of the shaft. This always it will be perpendicular to the elastic line of the shaft, so that if it is bending, this is the elastic line of the shaft. So, this particular plane will be perpendicular to this elastic line. Also, it will remain plane after bending. Also, this particular hypothesis will be using it. In this particular case, Euler Bernoulli beam when we are applying forces in let us say y direction, there will not be any displacement in the x direction.

So, as such there is no elastic coupling between the force and the displacement in 2 directions. So, if we are applying force in y direction, displacement will take place in that direction only. In this particular case, the displacement as we have seen earlier will be a linear displacement and the slope of the beam. So, we will be having 2 coordinates which will be required to define the displacement of this particular shaft segment. Now, let us try to obtain the free body diagram of this small segment. This is at z location. We are taking a small strip of thickness d z and we are taking the free body diagram of that. We are applying various shear force and bending moment in this particular system.

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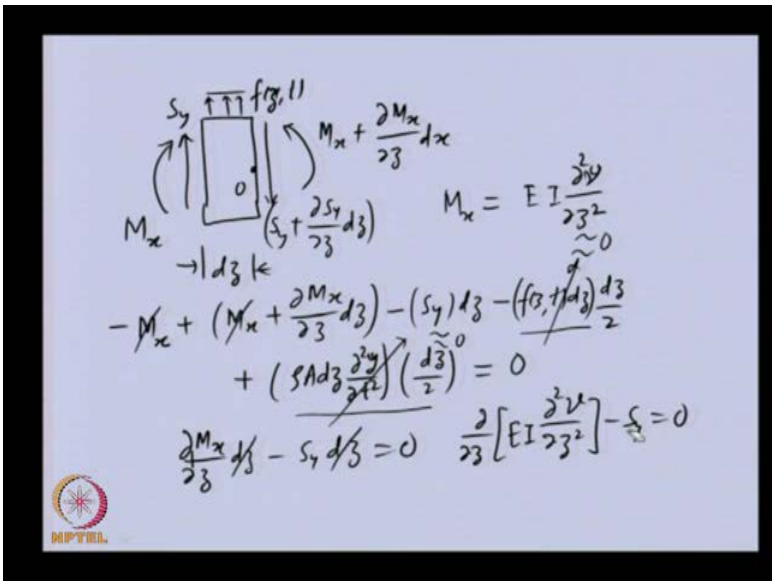

So, let us take the free body diagram of this. So, this is the shaft segment. We have sheer force in this direction, positive y direction. Sheer force on this plane will be changing because we have distributed property of the mass and stiffness. Also, we can have some kind of load, external force here. So, this sheer force is changing and this will be given as this. Similarly, the bending moment we are taking positive conventions. So, bending about x axis other plane, it will be changing again to this value.

So, 2 planes have different bending moment and sheer force which we have represented like this. Now, apart from this, this particular distributed force I have applied. This is per unit length of the shaft segment. So, total force will be this force into the d z. So, if we take, let us say, moment balance, that is if we are taking moment about this point O in counter clockwise direction as positive, this moment balance m x is negative. This moment will be positive.

Then, moment because of the S y will be again negative. We will be having distributed force and because of this we will be having, let us say, z distributed with respect to z d z. So, this is the force which is acting at the middle, let us say and the moment of this, if it is uniformly distributed can be approximated like this. Apart from this, we will be having inertia force, that will be mass per unit length into segment of the thickness of the shaft segment. So, this will be the mass of the shaft segment into this linear acceleration. So, this is the inertia and it will act at the middle.

So, moment will be this one and because this is acting opposite to the y direction, that is downward direction. So, it will produce a counter clockwise positive moment and this is equal to 0. In this particular case you can be able to see that some of the terms like this term and this term we are carrying z d z square term because d z is small quantity and square of that will be further small. So, we are approximating this equal to 0. We are neglecting these terms. So, basically now you can be able to see that this will also get cancelled.

So, we will get a relationship between S y and d z minus S y d z is equal to 0. So, even this will get cancelled. Now, we know that bending moment is given as E I del square v, v is let us say linear displacement, so this by d z square. So, this expression we can be able to write it as del z E I can be function of z. So, I am keeping I inside at present minus S y is equal to 0. Now, what I am doing is that I am differentiating this whole expression by z once more and because of that I will get del z square E I del square v by del z square minus del S y by del z is equal to 0.

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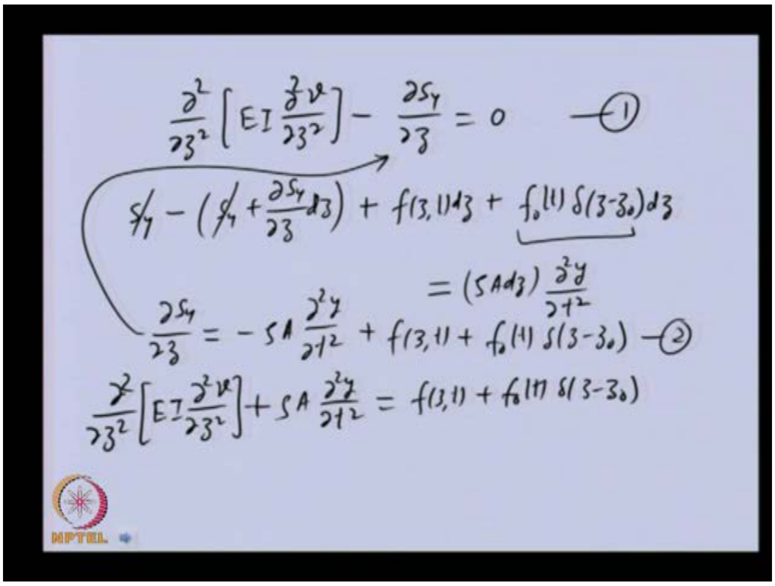

So, this is one of the equation. This we obtained by moment balance. Now, we can be able to do the force balance. For force balance, we have sheer force, external force and inertia force. So, these we can be able to create keeping the sign. So, force balance S y is in positive direction and the change in that is in negative direction. So, this and then we have external force which is distributed. If you have concentrated force, that also we can be able to incorporate in this.

So, f naught M we are using a direct delta function. So, basically this will give us a concentrated force as distributed force over a very small length and if we multiply this by d z, we get the total force. So, this is basically distributed force per unit length over very small length and into d z will give us the total force. This is the distributed force over finite length, but this is for very small length. Whenever z is equal to z naught, then only it will be having 1 unit value. Otherwise, it will be zero 0. Apart from this, this will be equal to M. That is rho a d z.

That is mass of the shaft segment and acceleration. In previous expression when we derive the moment, so these were the moment equations. We equated this equal to 0, because in the Euler-Bernoulli beam we do not consider the rotary inertia. So, that is why we have not taken the rotary inertia here. With rotary inertia, if we are considering that particular beam is called Rayleigh’s beam that is the higher order beam. So, we are not considering that here.

We are equating it to 0, but linear acceleration we are considering and mass into linear acceleration we are considering in this. Now, we can be able to simplify this. This will get cancelled. So, this will give us d Sy by d z is equal to minus rho A del square y by del t square plus external force distributed force. The concentrated force, z is equal to z naught. So, this particular expression we got is second equation. Now, you can be able to see in the first equation we have this particular term. So, we can be able to substitute this here. So, if you substitute this there, we will get an equation of this form. This term is as it is. Now, we are substituting this term.

So, we will be having rho A del square y by d t square and other terms I am keeping in the right hand side. So, this is the equation of motion of the Euler-Bernoulli beam in which we can be able to see this is the elastic force, this is the inertia force, these are external forces distributed and concentrated force. Now, we will try to solve this equation for free vibration. That means we equate the external force equal to 0 and we will solve this particular equation. In torsional vibration case, we had the special derivative that is with respect to z s 2, but here the fourth order derivative is there with respect to z. So, here the summation of this is more difficult as compared to the torsional case.

So, we will derive the governing equation for Euler-Bernoulli beam for transverse vibration. Now we will solve a free vibration problem for various special boundary conditions, because whatever the partial differential equation we got, once we are specifying the boundary condition, then only the solution of this differential equation will be unique. That is why in this particular problem the differential equation associated with the boundary condition is called boundary value problem. This will be basically converted into an Eigen value problem and that can be solved for the natural frequency of the system.

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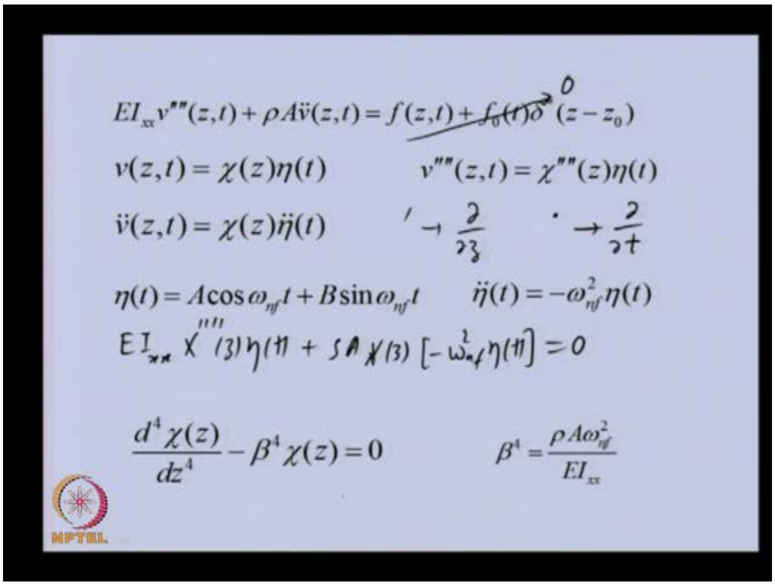

Now, let us see this is the differential equation which we derived here. I am using a notation of prime. So, 1 prime is representing that is derivative with respect to z. So, there are 4 primes. So, it is representing that the derivative with respect to z is 4 times and the dot is representing the derivative with respect to time. So, it is having twice the derivative with respect to time. So, basically we have prime as derivative, partial derivative with respect to z and dot is representing derivative with respect to time. Now, if we assume the solution of this, we will be using the separation of variable method as we did for the torsional case.

So, in this case we will take a function which is function of z alone and a harmonic function. In this particular case, now for free vibration this quantity will be 0 and the motion we expect as a harmonic. So, this particular function we are assuming as a harmonic function. So, if we take the derivative of this assumed solution with respect to z 4 times, so obviously the first function will be derivative. This will not be derivative because this is function of time. Similarly, if we take the derivative of this with respect to time this harmonic function will get derivative twice now.

Now, the harmonic function will be having this form in which this is the natural frequency of the system, and a and b are the constant to be determined from the initial condition of the problem. If we take the derivative of this twice, we can be able to see that we will be having this relation which is relation for the simple harmonic motion. Now if you substitute these assumed solution in the equation of motion for the free vibration, you can see that we will get E I x x and this is 4 times derivative eta t.

Then, this one is rho A and this when we are taking the derivative with respect to time, we will be having minus omega n f square eta t is equal to 0. Because we are considering free vibration, so there is no external force. In this, now we can be able to see that this equation because eta t is common, so this equation will give us equation of this form. Now, this equation because in this particular case only the variable is z, so this equation which was earlier partial differential equation will be now ordinary differential equation of fourth order.

This beta constant we have defined like this, which is coming from here. Now, we need to solve this particular differential equation using the boundary condition of the problem. So, till now we have not talked about what kind of boundary condition we need to be applied, but now for solving this we need to consider the boundary condition.

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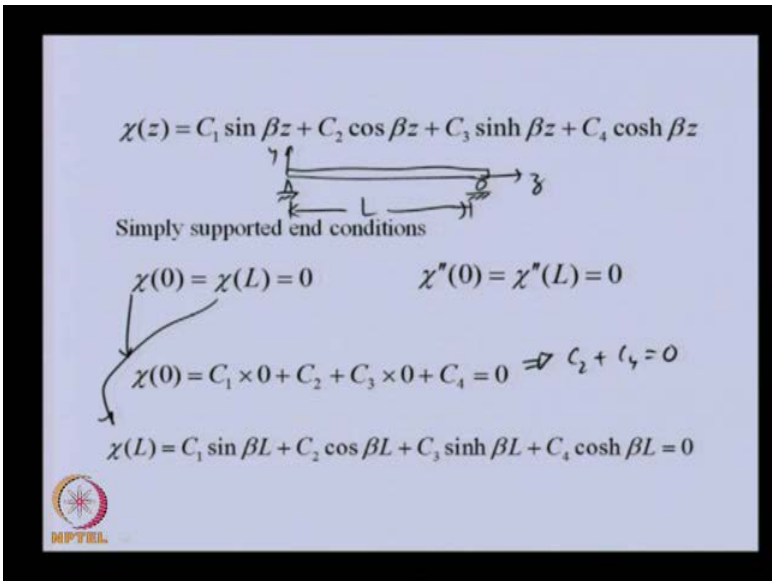

The general solution of that is this. We will be having 4 constants. So, not only sin, cosine, but also their hyperbolic functions will be appearing. In torsional case, we had only the harmonic function not the hyperbolic ones. So, now we are considering a beam of simply supported end condition like this. So, in this there is no loading, but we can able to disturb the beam. We can able to analyze the free vibration of that. So, for simply supported beam we can be able to see if we have this is z axis and this is y.

So, here we have z is equal to 0. The total length of the beam is, let us say capital l or the span of the beam is L. So, here we have boundary condition that the displacement will be 0 at both the ends, that is at x is equal to 0 and at x is equal to l. So, this is the displacement and 0 at both the end. Apart from this, we will be having moment 0 at these 2 ends. So, basically this double derivative is representing the moment if we multiply this by a constant E I. So, basically E I is constant. So, there is no meaning here if we have written it. So we have removed that.

So, double derivative of this at x equal to z equal to 0 and z is equal to n R 0, because this is simply supported condition which is representing moment R 0 at these 2 ends. Now, once we have these boundary conditions, we can be able to have 4 boundary conditions. In this equation, we have 4 constants. So, we can be able to apply the boundary condition in this problem. We can be able to solve for C 1, C 2, C 3 and C 4.

So, if we apply first boundary condition this 1 here. If we put z is equal to 0 here, this will be 0, this cos term will be 1.

Similarly, this will be 0 and cos will be 0. So, we got this term. So, basically this equation is C 2 plus C 4 is equal to 0 if we simplify this. Similarly, if we put x is z equal to l in this expression, the second boundary condition because this will be sin beta l, all terms will be there. So, this is equal to 0. Because the displacement z is equal to l is also 0, so this is second equation. Now, we will apply this boundary condition for moment.

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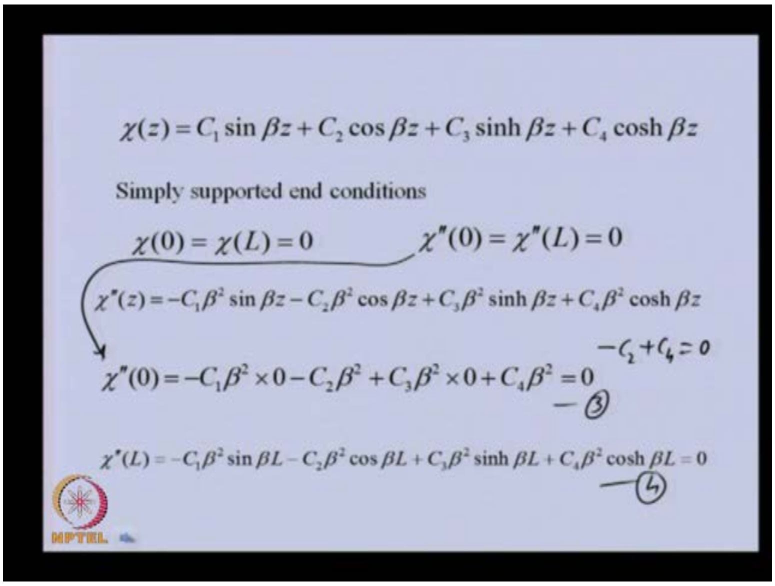

So, this is the same equation for moment. Because we need double derivative of this function, so we are taking double derivative of this with respect to z. So we will get this if we differentiate with respect to z twice. Now, we are substituting this particular boundary condition here. So, we are keeping z is equal to 0. So, first term will be 0. The third term will be 0. So, basically if you simplify this equation now, we will get minus C 2 plus C 4 is equal to 0. The last boundary condition, this one if we substitute we will get this equation. So, basically we have four equations, that is first equation is this one, second is this one, third is this one and fourth is this one. So, these four equation contain four unknowns.

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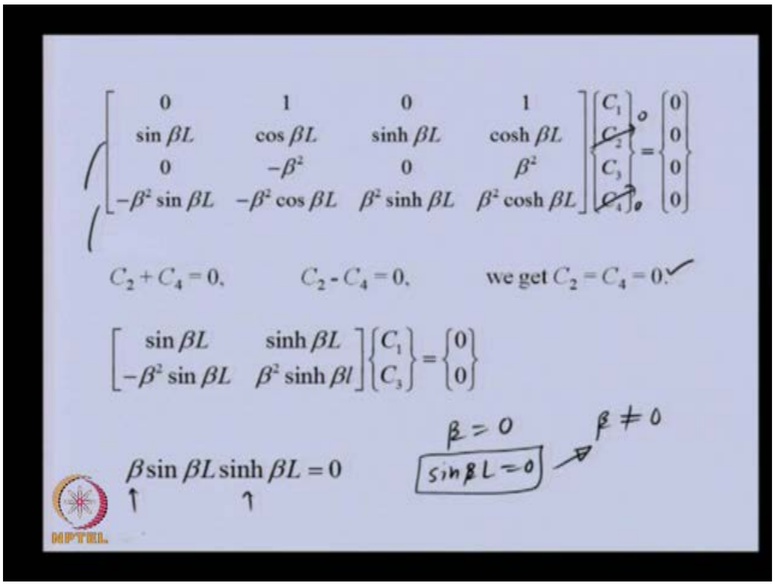

So, we can able to put them in this particular form. So, all the unknowns I am keeping here. So, basically I have rearranged those 4 equation in a matrix form, where unknowns are here. All the known quantities are here and we have seen that from the first equation and third equation we got this. From these 2 we can be able to see that these 2 conditions are only possible when both C 2 and C 4 are 0. So, if both C 2 and C 4 are 0, if we put this here, this is also equal to 0. So, basically we will see that from first, from second and third equation, and second and fourth equation we will get this, because other equations will be 0. They will not contribute anything.

So, from second and fourth equation after putting this equal to 0, that means C 2 is equal to 0 and C 4 is equal to 0, this will reduce to this 2 by 2 matrix, where C 1 and C 2 are known. Now, if we take the determinant of this equal to 0, you will get an equation like this. This particular equation we will see that all terms, beta sin beta L and sin h beta L is 1. The solution common, that is beta is equal to 0, but beta cannot be 0. Otherwise we will be having trivial solution. For non trivial solution, beta we need that should be non 0 and for non 0 beta, we can be able to see that this will not satisfy this.

The first and the third will never be 0, if beta is not 0. So, that means we need to have sin beta L equal to 0, for the condition that we do not have beta is equal to 0. So, basically we are getting a frequency equation from this. This is the frequency equation and the sin function is having infinite solution even when beta is not 0.

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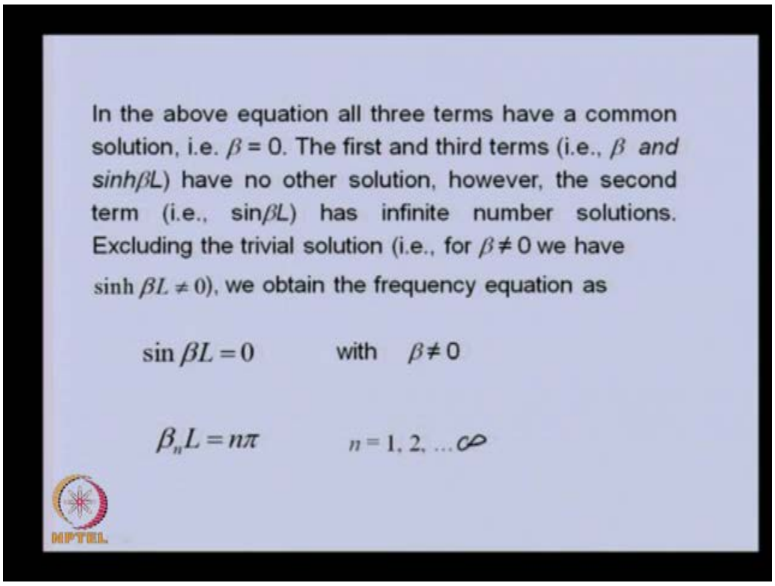

So, this is the solution. So, this is the infinite number of solutions which is possible for which sin beta value is 0. So, if you substitute this for all, we can be able to get these values. This is equal to 0 value.

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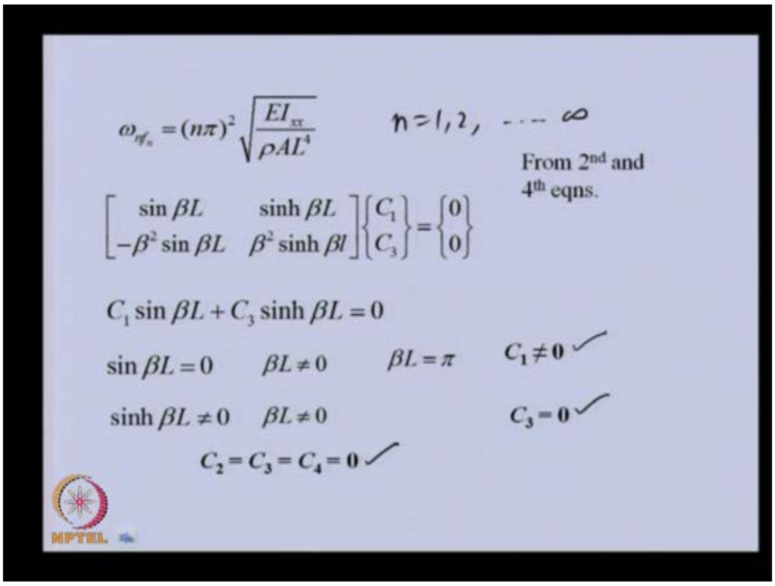

So, you can be able to see that this is the frequency equation because we related the beta with the natural frequency earlier. So, beta square we related with natural frequency earlier. So, that expression we can be able to see here. So, from that we can be able to relate the natural frequency. We can be able to get the natural frequency. So, this is the natural frequency where n is varying from 1, 2 up to infinity. So, that many number of solutions are possible. Now, let us see the same equation which we used earlier, this equation more carefully.

If we take first equation of this C l, sin beta L and C 3 sin h beta L is equal to 0. Now, first term for we are considering when beta l is not 0. That means this is a non trivial solution. When beta l is not zero, if you see the first term, this can be 0 for this condition. But C 1 cannot be 0. It may not be 0 and in this particular case this term will be 0 only when beta L is 0. But for nontrivial condition that beta naught is 0, this cannot be 0. So, C 3 has to be zero. So, basically from this we are getting that C 3 has to be 0 and C 3 cannot be 0. So, that means in the whole assumed solution only C 1 is non 0, but C 2, C 3 and C 4, we have seen that all are 0.

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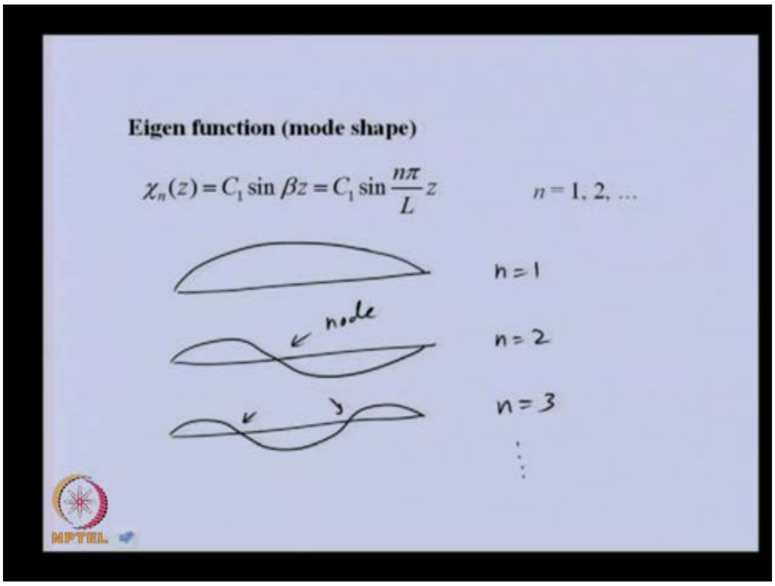

So, this will give us the Eigen function. So, out of 4 terms only single term is left out. This is the C 1 and beta which we have already obtained in terms previously. So, that we can be able to substitute here. So, this is the Eigen function, where n can be 1, 2, 3 and 4.This Eigen function is nothing but the mode shape. We can be able to draw the mode shape. How the beam will vibrate in simply supported case? Let us say for mode 1, that is n is equal to 1.

So, for n is equal to 1, this Eigen function is giving as a half sin. So, mode shape will be something like this. For n is equal to 2, we will be having full sin. This will be at the middle. So, this is your node we expect. For n is equal to 3, we will be having this. Similarly, if we increase various modes this number of modes will be increasing. You can see here it is 1, but here it is 2. So, we expect for any further increase, the mode shape will be having intermediate nodes in this particular mode shapes. So, this is a typical mode shape for simply supported beam which we obtained from the free vibration analysis.

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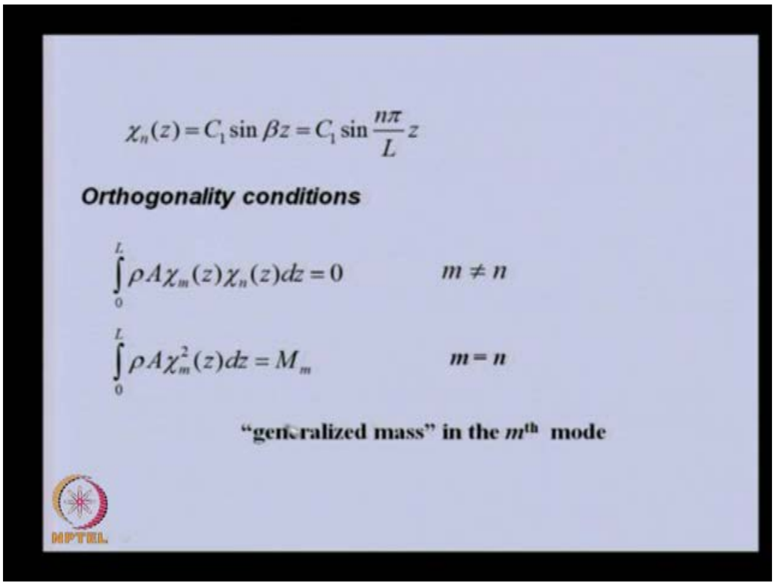

Now, let us see the orthogonality condition. So, orthogonality condition as we have seen in the torsional vibration case also, because Eigen functions are independent of each other. So, if we integrate this 2 different mode Eigen function by multiplying by this quantity and if we integrate over the whole length of the beam, this will be 0 if these 2 modes are not same. If they are same then we will get some constant term, which I am representing as M subscript m. We call this as generalised mass, in that particular m th mode. So, small m is representing that particular m th mode. So, this is generalised mass.

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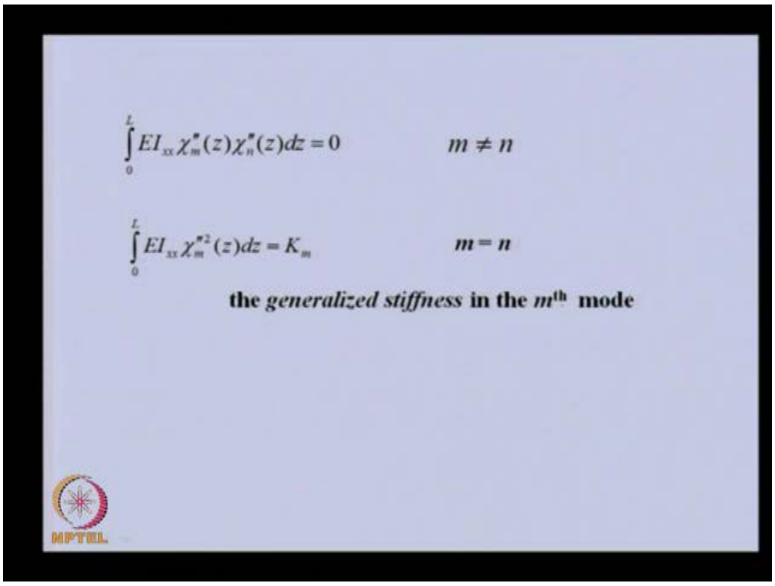

Similarly, when we integrate the double derivative of this Eigen functions, if two different modes are there, then the integration is 0. If modes are same, then we are getting a constant term and this constant term K m, we called as generalised stiffness in that particular mode m th mode.

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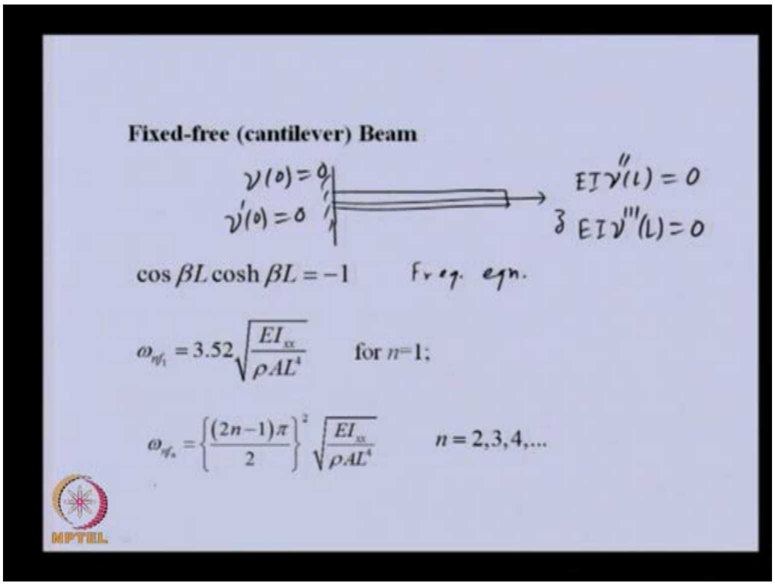

Now, let us see some other boundary condition, how the frequency, the natural frequency and Eigen function take the shape? So, this is a fixed free beam, that is cantilever beam. In this particular case, we have boundary condition that this is a fixed end, so we will be having this equal to 0. Displacement is 0. Even, we will be having prime of that. This means slope at this point will be 0. Because fixed end beam will be having displacement and the slope is 0 here. Similarly, here we will be having shear force equal to 0. So, we will be having bending moment and shear force. So, shear force, first let us say bending moment. So, double derivative at x equal to 0.

The shear force is represented by triple derivative with respect to z. This will be 0. So, these are the boundary condition. So, we have 4 boundary conditions. As, we had for the simply supported case, we can be able to apply to the chosen solution to get the C 1, C 2, C 3 and C 4 constants. Finally, on the same lines basically we will get this kind of function. This kind of equation is a transcendal equation. Earlier, we had for simply supported case, sin beta is equal to 0.

Basically, this equation is the frequency equation and roots of this will give us the natural frequency of the system. Let, beta is related to the natural frequency which we already seen. So, this is the natural frequency for cantilever beam case. This is for first mode and this for higher modes an Eigen function is in this particular case more complicated, because some of the more terms of this constant C are non 0.

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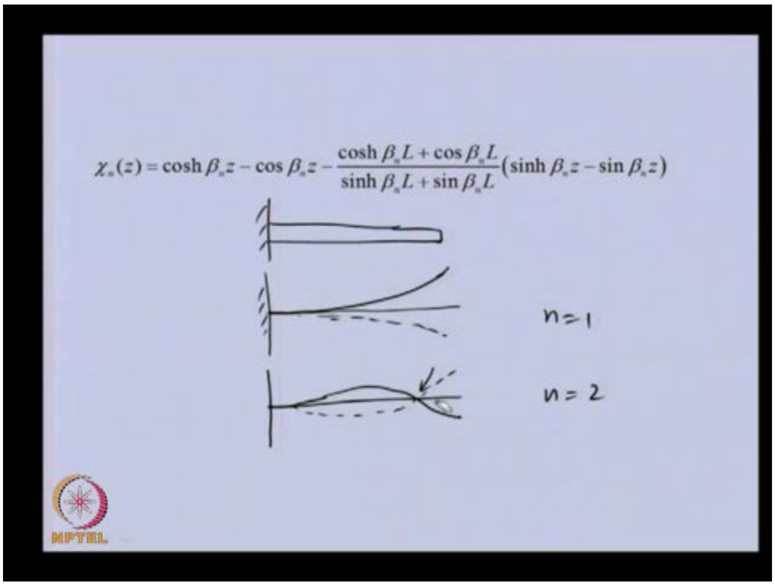

So, this is the Eigen function. If we plot this, we know that the mode shape for this particular case. Let us say for n is equal to 1, if this is the centre line we will be having mass, because at this end we have slope and linear displacement 0. So, while drawing the mode shape we need to take care of that. So, first mode will be something like this where you can be able to see displacement and slope. If we draw the tangent here that will be along this. That will be 0.

In the second mode also that particular condition we need to satisfy, but we will find another node here. So, this is 0, but we will be having 1 node which will be appearing here. So, basically during oscillation they will be having oscillation in other extreme in this direction or this will be having vibration like this third mode. Then we will be having another node here, so that can be extended. So, this is for n is equal to 2. For higher modes we can able to draw the mode shape on same lines. Let us see another boundary condition, that is when both end of the beam is fixed.

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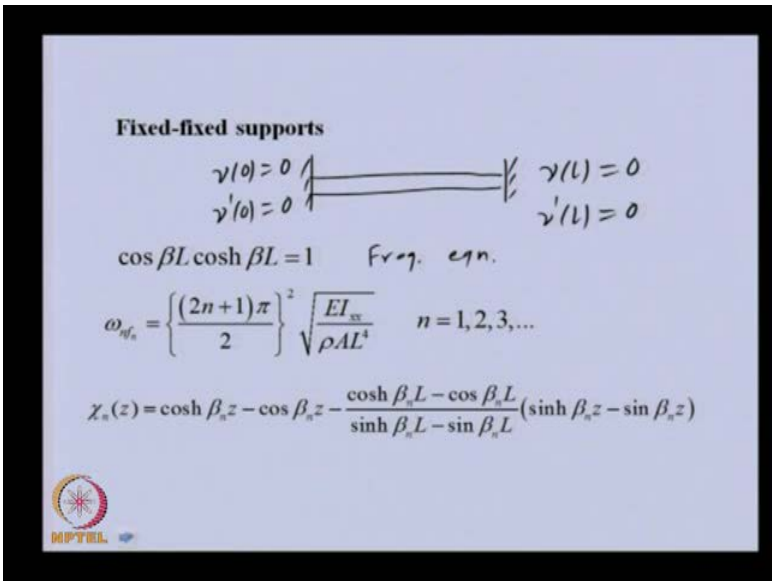

So, in this particular case we have displacement and slope 0 at this end. Here also, the same condition is there at z is equal to n. So, now we have again 4 boundary conditions. We can be able to obtain the 4 constants from the chosen function that is C 1, C 2, C 3 and C 4. We will get a frequency equation like this. This is the frequency equation. This is transcendental equation and solution of this will give us the natural frequency. This is the Eigen function. Once we are applying this to C 4, Eigen function will be in the explicit form.

Now, we can be able to plot this and we can be able to get the mode shape out of this. So, I am not drawing that particular shape. So, we have seen that how we can able to obtain the governing equation. If we have different boundary condition, how we can able to get the frequency equation, like for simply supported case. We have given more details as to how we can be able to get the frequency equation, generally in the frequency equation, in this particular cases, the transcendental equation. In previous case, we found that this particular frequency equation for discrete system is a polynomial form, but here now it is a transcendental equation which is more difficult to solve.

Apart from that, we have seen that the natural frequency are infinite in number and corresponding the Eigen function or the mode shape also we can be able to obtain. Once we plot them, we can be able to see that each and every point on the beam, that shape is defined here. As we know that Eigen functions are not unique, their shape is unique, but not the size. Now, we will see that these natural frequency are mode shaped. How we can be able to use it for the forced vibration?

We have already seen the orthogonality condition, that these particular modes are orthogonal to each other. So, this particular property we will be using it for the forced vibration analysis of such system. So, we will just outline the method how it can be applied for the force vibrations.

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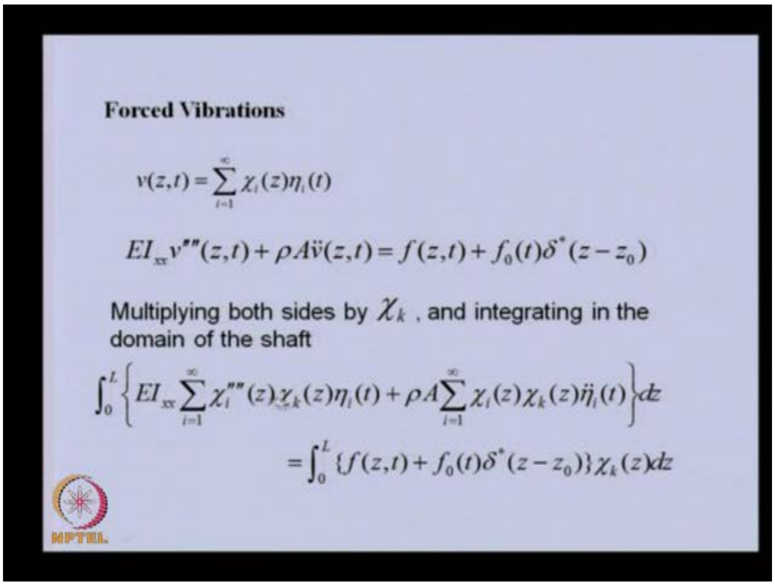

So, now we are trying to solve the forced vibration. So, this is the governing equation which we derived earlier in which, now we are considering the force. This is a distributed force and this is the concentrated force. Summation of this now, we are assuming in a form of the Eigen function. Any harmonic function, in this particular case this is basically Eigen function which we have obtained from the free vibration analysis. We know that this Eigen functions are infinite in number. So, we are assuming here, whatever the force response which we will be getting will be basically the summation of various modes of the vibration multiplied by some constant, because in this we will be having initial condition constants.

So, basically here what we are assuming that any force response can be represented by summation of various modes, that is Eigen function multiplied by some initial condition constant. This is similar to the Fourier series. If you think of a Fourier series in which we have we to represent the periodic function as a summation of sin and cosine terms and some constant term and their higher harmonics, so we represent a whole periodic function by summation of various terms. Depending up on the accuracy required there we truncate the series up to certain value, certain terms.

So, similarly here you can expect that when we want the summation approximate solution, we need to terminate this particular series up to certain value depending up on the initial condition. So, let us substitute this and assume solution in the equation of motion. Then we are multiplying that particular equation both sides by an Eigen function which is, let us say x k, which is other than the x i as such. So, this x k we have multiplied throughout. Then we even integrated over the domain. So, that means first we are substituting the assumed solution, multiplying by another Eigen function x k and integrating over the domain.

So, we can be able to see this particular term is belonging to this one where we have substituted this. So, once you substitute this here, the fourth derivative with respect to z will come here. This is not a function of z. So, this will be separate. We have multiplied by x k also and integrated over domain, that means d z. Similarly, this is the second term. This term in this derivative will go into this function. This is directly multiplied by x k and integrated over domain.

Now, you can be able to see that earlier orthogonality conditions we had that if this two are not same, integration of this will be 0. So, that means in this fourth series when i is equal to k, then only the terms will be non zero. Otherwise, they will be 0. So, out of all infinite terms, only one term will be non zero.

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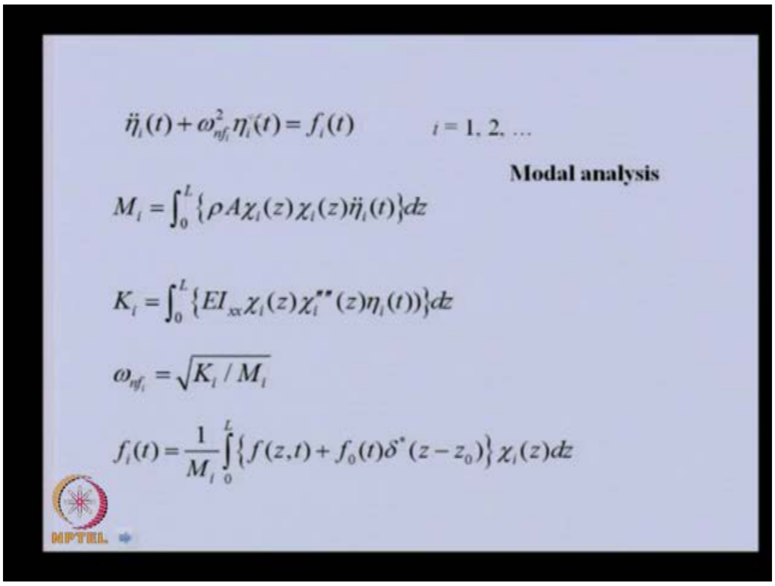

So, let us say there is a particular term i. So, from previous equation basically, here you can be able to see this will be having some value. So, we are getting this one. So, this is this term and we have already seen that this particular term gives us the generalized stiffness. This term gives the generalized mass and here we have simplified them even we have divided by generalized mass, so that this omega n f i square is represented like this. This is the integration of that particular force. Here, we have not done any change. So, I just represented f i s. This only thing is because we have divided the whole equation by generalized mass. So, this is also appearing.

So, you can be able to see generalized mass stiffness. This is the natural frequency of that particular mode. So, here this particular equation which we obtained here for one of the x k, but x k can vary from 1 to infinity. So, that means such equations will be getting infinite number of equations, but this equation is if we see carefully, this is an equation similar to the single degree of freedom system. So, basically what we are doing here is we call it as modal analysis.

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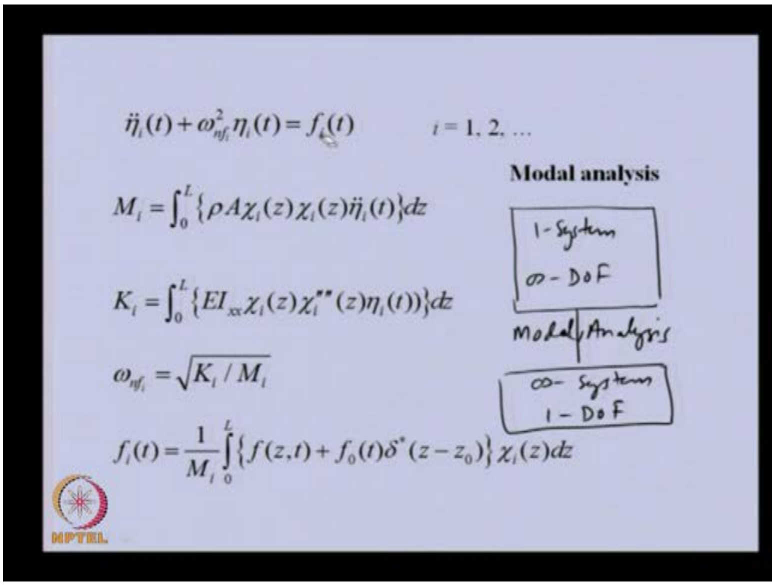

In this, we have a single system and it is having infinite degree of freedom. Using modal analysis, we have converted this single system to infinite systems and each system is having one degree of freedom. So, this is modal analysis. Now, you can be able to see this is simple to solve. Because this is single degree of freedom system, this particular force is more general in nature. If it is harmonic force, then the solution is straight forward.

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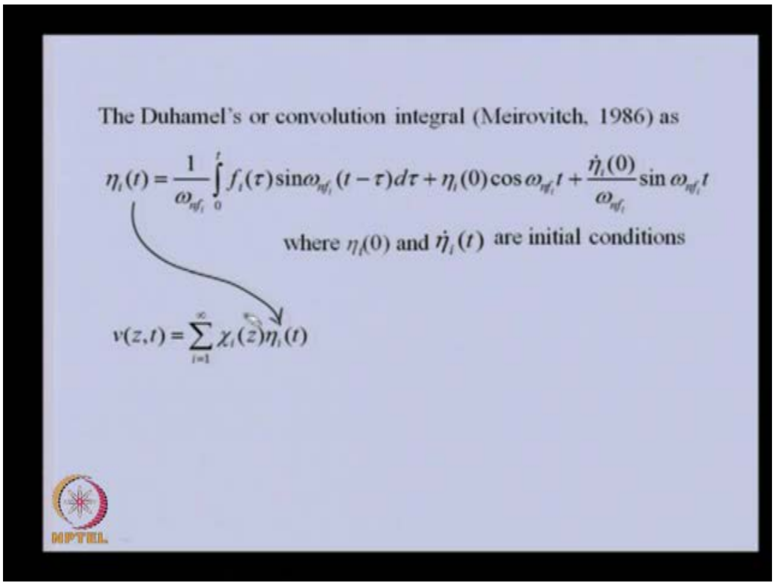

If it is more general, a periodic function, then we need to solve using the Duhamel's integral or convolution integral. This is represented by this integral and in this you can be able to see this is the force and these are the initial conditions of the problem. So, depending upon the explicit form of this function, you can be able to get this particular term. Once, we get this we can be able to substitute this in the initial assumed solution. That will be the force response because this we already know already are the Eigen function which we have obtained from the free vibration.

So, in today's lecture we have briefly outlined the continuous system approach for the transverse vibration of beam. In this particular case, we have considered the Euler-Bernoulli beam, which is more simple beam and very simple boundary conditions like simply supported case. We have explained this and how to get the Eigen function, natural frequency equation and mode shape from the governing equation. We described the idea of this, is that we can get the solution of the partial differential equation for simple boundary condition.

If we have multiple supports and some more number of disc in the shaft along with the distributed property of the shaft, we may find difficulty in the solution of such partial differential equation. So, in this particular case we will be having the finite element method which will be helpful. So, in the subsequent class we will be explaining how this continuous system approach can be approximated using the finite element method. Then we can be able to have the advantage of the continuous system. That is the distributed property of the system and even the more complex boundary conditions we can be able to handle with the finite element method. Now, after 2 to 3 lectures, we will be devoted to finite element analysis of the transverse vibration.