Theory and Practice of Rotor Dynamics
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Module – 6
Transverse Vibrations
Lecture - 27
Transverse Vibrations: Transfer Matrix Method

Today, we will extend the transverse vibration by using the transfer matrix method, now we will consider an intermediate support when in the continuous rotor, if there are intermediate support, which is very common in practical rotors. Then how transfer matrix method can how we can able to handle such situation also gyroscopic effect in rotor system when we are dealing with the transverse vibration. It is very common especially when we are dealing with the high speed rotor cases, so that also we will we will cover in the present lecture.

How this particular gyroscopic effect influence the especially the point matrix in the transfer matrix method, also we will through some examples we will try to see how these methods we can able to apply in in some simple cases. With the main idea of that those examples will be to illustrate the method for simple case, but once we have understood the method completely. Then, the extension of that for large number of rotors or large number of disc is straight forward, so let us see what the things we will be covering.

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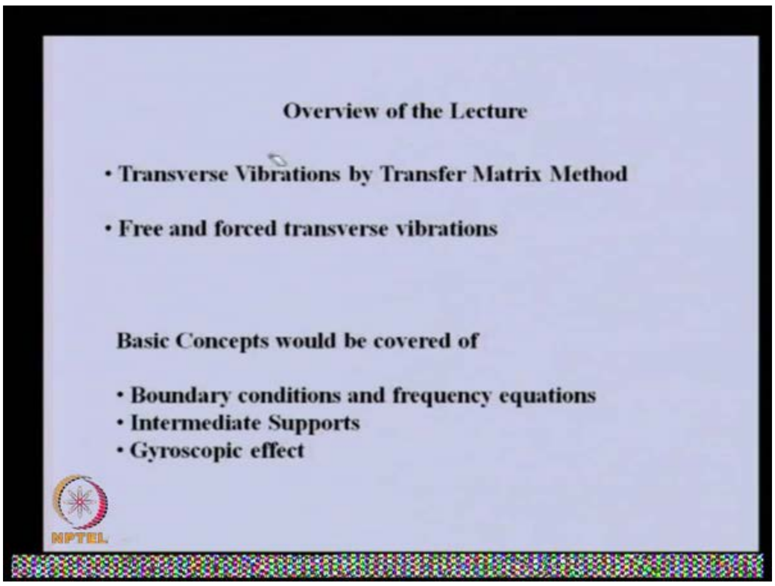

So, again we are dealing with the transverse vibration at even method free and forced transverse vibration, we will be covering and basic concepts would be similar to the previous one, some of the things like boundary condition frequency equation. Now, this frequency equation especially when we will be considering gyroscopic effect we will see that the natural frequency will depend up on the speed of the rotor itself.

So, as such the form of the equation whatever the frequency equations will be there that will be having more complexity also how to take up the intermediate support in the rotor system will taking one example. For this particular case and we will illustrate that there is some difficulty if we have intermediate support using the transfer matrix method. How the gyroscopic effect changes some of the terms in the point matrix we will see and through some example we will try to see these cases. So, let us take one example in which we have one intermediate support in a rotor system, so example is like this.

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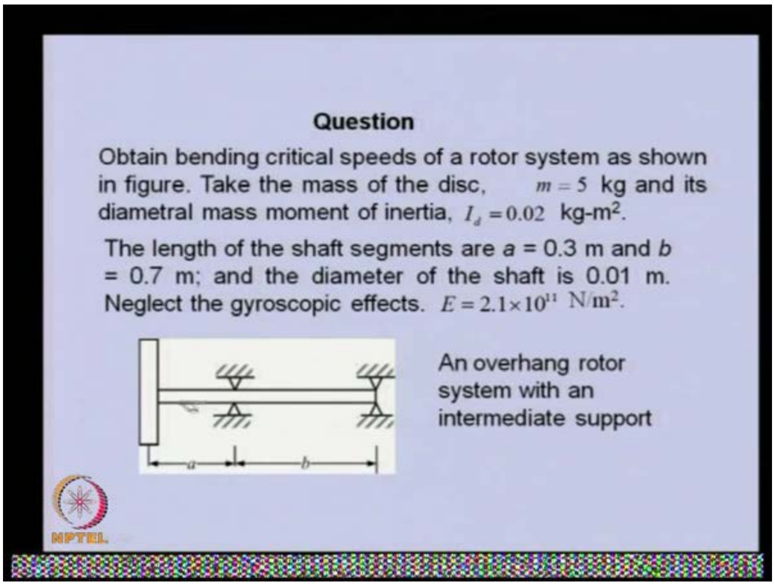

We have a shaft which is mass less, we have a disc at the overhand part of the shaft and these are the pin support or simply support and this is the intermediate support. Basically, we need to obtain the natural frequency of the critical speed of the rotor system we have a mass and diametric mass moment of inertia property of the disc various dimensions are given.

Here, k is the length of this overhand part of the shaft and b is this part of the shaft and diameter is uniform is having 0.01 meter. In this particular case we are not considering the gyroscopic effect, generally in over overhand portion we expect a lot of gyroscopic effect specially when rotor is having high speed, but for this particular case we have neglected that. So, main focus of this particular example is how we can able to handle the boundary condition at this particular intermediate support case, this support and this support will be there.

We will see that there is some difference in the conditions which create some problem in transferring the state factor of the rotor from this side up to the other side. So, because this intermediate support gives some kind of unknown reaction and that unknown reaction create a difficulty in transferring the data state vector from left of the support to the right of the support. So, we will see how this can be handled in the present example.

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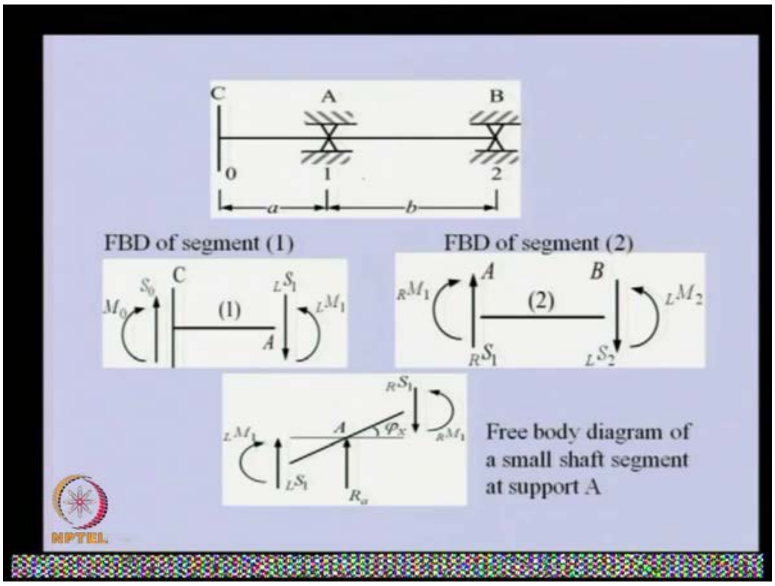

So, this is the diagram of the orang rotor you can see this is the station zero we have selected station one is at intermediate support and station two at the other support, now I have taken the free body diagram of the shaft segment. So, just left of the support up to this is the free this is the free body diagram for that particular shaft segment. So, you can able to see this is the disc this is the shaft which is just before the left side of the support in this free body diagram.

I am attaching the sheer force and binding moment at this end also at this end sheer force and binding moment corresponding to mode 1 which is basically left of the support 1. So, we are attaching l to represent that this is from the left of the support, now this is the shaft segment which is just right side of the support up to this point. So, this is the shaft segment free body diagram for the second shaft here this is the sheer force binding moment.

Sheer force and binding moment we are applying here to distinguish the sheer force and binding moment toward the right side, and the left side we have represented this as right of the support this two, and in this basically this is simply supported end condition. So, these are the sheer force and binding moment corresponding to this end, now you can able to see that the here and here because these are toward the left and right of the support. Now, we have taken the free body diagram of the shaft segment which is at the support, so this is the shaft small segment this is the support this side we cut the shaft and we drew the free body diagram of the shaft segment 1.

Similarly, we cut the shaft here and we have drawn this so obviously whatever the sheer force and binding moment is acting here will be acting here but its direction will be opposite. So, it is the left of one support sheer force is also left of one support but direction is like this downward this is upward. Similarly, this moment is counter clockwise direction this moment will be clockwise direction, similarly we will in this end of the shaft segment this sheer force and binding moment will be same only thing their directions will be opposite. We have removed the support also from this, so we expect a reaction force from the support, so I am representing that reaction force onto the shaft plus r a.

So, this shaft segment is acted by the sheer force and binding moment toward the left side and also toward the right side, and here there is a reaction unknown reaction from the support. Now, you can able to see that this reaction force which is unknown will create problem in transferring the state vector this side to this side because one unknown reaction is there now let us see how we can able to solve this problem.

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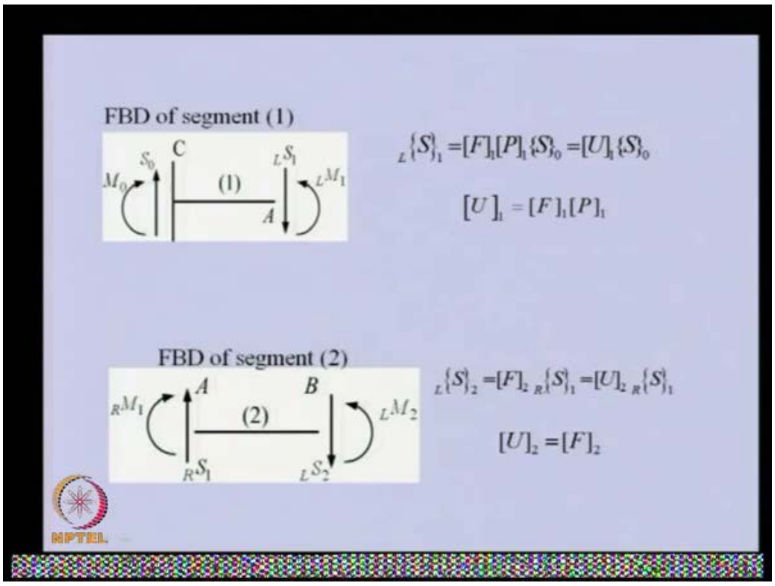

So, in this particular case, first I have taken the segment 1 and I have written the state vector which is left of support 1 and the state vector at zero station. So, you can able to see this is at state vector zero station, so we have point matrix first for the disc and then the field matrix of point matrix into field matrix. So, this is the transfer matrix for transferring the state vector from zero station to left of disc one support 1.

Similarly, this is the shaft segment 2 in this the support is this end, so you can able to see that now we are writing the state vector relating with the right of 2, sorry right of one state vector that is right of 1 support or to the left of 2. In between, we have only shaft segment there is no mass there is no disc, so only field matrix will be multiplied here to relate the state vector right of support one to left of to station, so in between these two we have the support.

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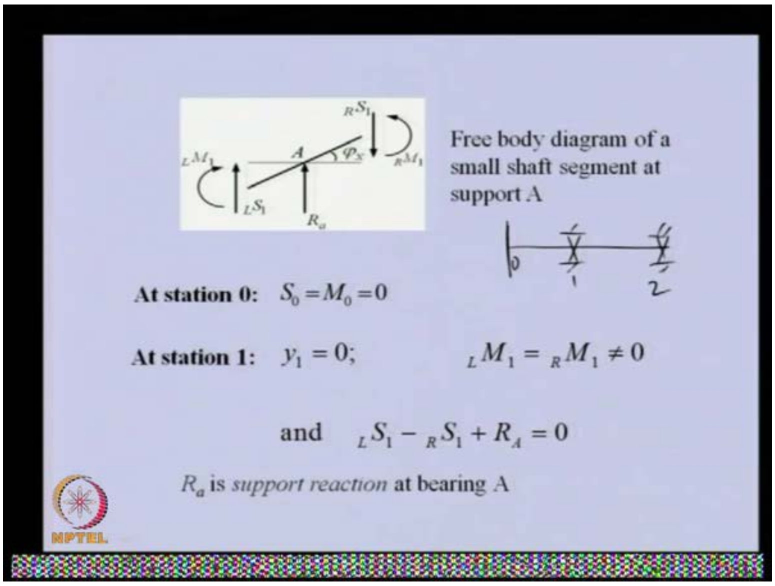

Now, let us see the free body diagram of the support the free body diagram of the support is this one which we drawn earlier. Now, you can able to see that the boundary condition apart from the support what are the boundary conditions we have in the overall system. So, at station 0, this is a free end we expect that the sheer force and binding moment will be 0 because we have a rotor system well like this. Here, one intermediate support is there, here one support is there this is 0, station 1, 2, so at zero station we have a free free end condition.

So, the sheer force and binding moment will be 0 at station 1 which is at support the displacement at station 1 y 1 will be 0. In this particular case, the binding moment which are there in the right of support 1 and left of support 1, they will be equal because from they will be equal and you can able to see that they will not be 0.

Also from free body diagram of this, if we take the moment balance or force balance you will see that there are three forces, one is left of support 1 sheer force which is let us say positive y direction right of support 1 which is downward directions negative. The reaction force at R a at station a or the support a, so that is the force balance at support at station 1.

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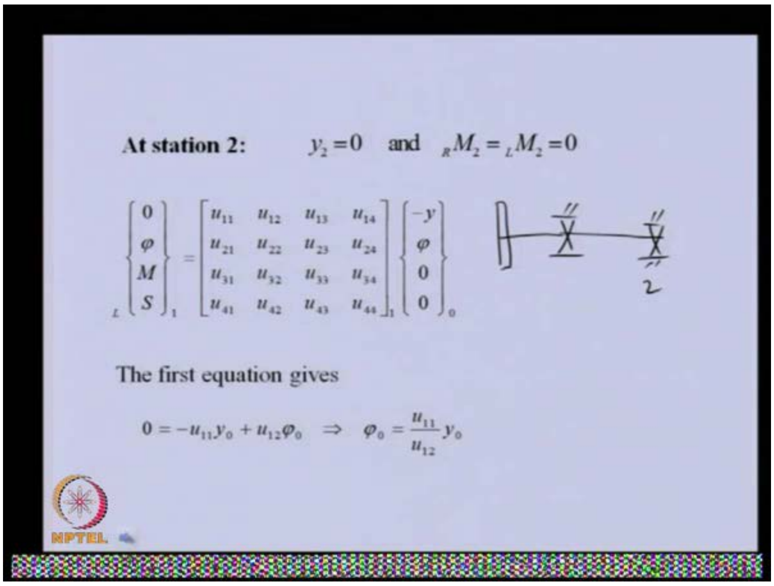

Now, at station 2, what is the boundary condition which is a simply support station 2 we have displacement y 2 is equal to 0, also the binding moment is 0, because simply supported end condition which is for station 2. Here, we have we have simply supported case and intermediate support is different as compared to this simply supported case here displacement is 0 as well as moment is 0. Here, displacement is 0, moments are not 0 as we have seen in the previous expression, so moments will not be 0 now.

So, we have all the we have we have look into all the boundary conditions at station 0, 1 and 2, now let us see let us see this is the equation corresponding to the first equation which we wrote this equation. This is relating the state vector as 0 and left of station the support 1, so 0 and left of support one in this particular case at support 1, we have seen that the displacement is 0 y 1 was 0. So, we have kept this as 0 and at station zero because this is free end binding moment and sheer force both are 0 and if we expand let us say the first equation of this. So, we will get 0 is equal to minus of u 1 1 into y naught because that is belonging to the station 0, q 1 2 into phi x naught because this is belonging to the station 0 is equal to 0. These two terms will get multiplied by this and they will not contribute to the first equation.

So, we have taken first equation because in the left side it is having 0 quantity which is known quantity in other equations, we have unknown quantity. So, that is why we have taken the first equation and with this equation we can able to relate the angular displacement at 0 station. The linear displacement at 0 station using this relation, so you can able to see, now we have related this two angular displacements at this station 0.

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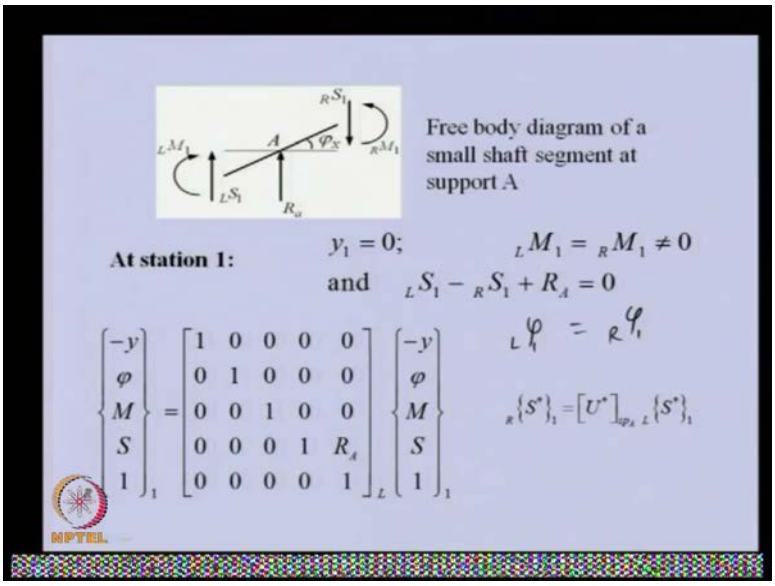

Now, we are again coming back to the conditions at the support at support what are the conditions we have we have y 1 is equal to 0, that is the displacement at station 1 is zero moments are not 0 at left and right side and sheer force. The force balance is containing the sheer force also the unknown reaction force, now I want to put all these equations in a matrix form.

So, apart from this you can able to see that the slope of the slope of this particular shaft will be same in both ends that means we will be having additional condition that phi left of 1 is equal to phi right of 1. Similarly, you can able to see the first this matrix which I have combined all this equation what they represent, so first equation is minus y which is right of 1 is equal to minus y left of 1. Basically, both are 0, so they will be representing 0 equal to 0 terms, similarly the second equation will is this one in which the left and the right side of the slopes are same.

So, if we expand the second row second equation we will get this equation, third equation is the moment equation this is the moment equation both are same. So, this two moments you can able to see if you expand the third equation you will get moment in the right of one will be equal to moment in the left of one side, because this one is the when we are coming to the sheer force equation this one. So, we have right of one is equal to left of 1 and the reaction of this particular support that is also there, now you can able to see that we have both are positive.

So, right of one is equal to this plus this, so if you expand the fourth equation you will get the exactly same equation as this one. So, you can able to see that we have included the reaction force inside this particular matrix and this particular matrix I am representing as u star for support s p is for support. So, this particular transfer matrix is transferring the state vector which is in the left of support at station 1 to the right of support at station 1. So, as earlier we had point matrix which was relating the state vector at other end of the disc, similarly this particular matrix will relate the state vector at other end of the support.

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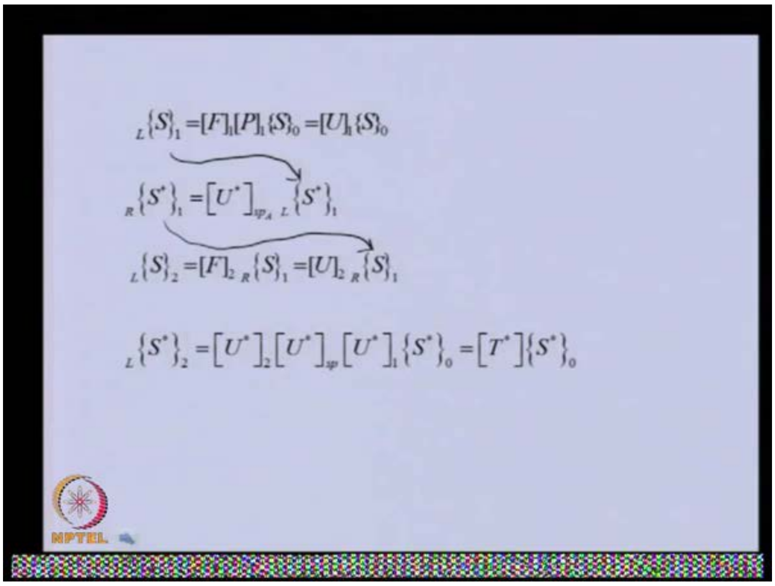

So, left to the right, now coming to the transfer matrices which we developed earlier, so first we develop for segment one so this is the relation for that in which station 0 and 1 we related in previous using support conditions. We developed the state vector between left of 1 to the right of 1 of the support and from segment 2 shaft, earlier we related right of one state vector to the left of two state vector.

Now, you can able to see we can able to combine them or we can able to combine them we can able to substitute this here also we can able to substitute this here. So, if you substitute you will see that we are relating the state vector of left of two to the zero station and in this not only the transfer matrix for shaft segment 1 and 2 is there, but also for the support is also there. So, if we multiply these matrices we will get a transfer matrix overall transfer matrix, now the form of the overall transfer matrix is very important here.

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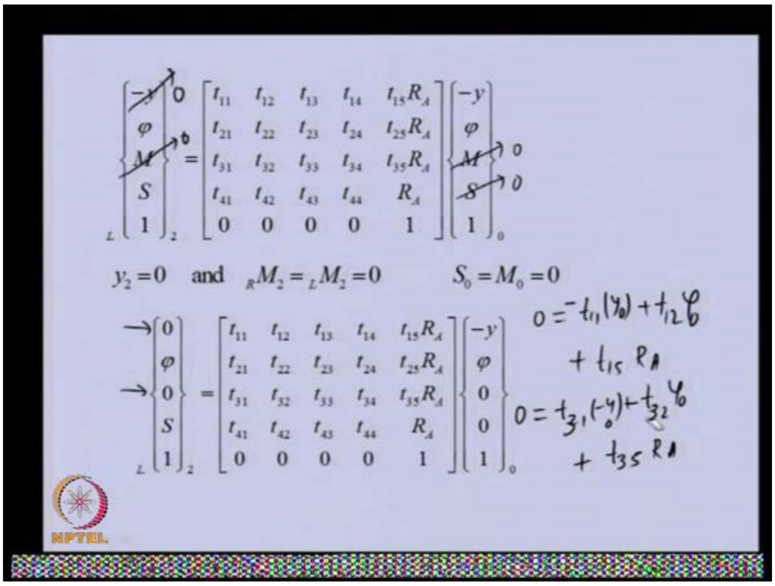

So, if we multiply these matrices for illustration, so if we multiply we will see that the support reaction R a which is unknown will be coming in the last column like this other quantity will be a known quantity only the unknown terms. I have highlighted here because there is one main important thing in this on this will be containing the mass stiffness terms of the shaft the disc. The shaft also the natural frequency which is also unknown, but the reaction which is unknown will be in this particular column.

Now, we can able to apply because now this is the overall transfer matrix, now we can able to apply the overall boundary conditions of the problem. So, at node two we have this particular displacement equal to 0 because this is a simply supported condition at 0 station. This station we have free end condition so binding moment and sheer force are 0, so you can able to see this particular equation will take this form in which these two are 0 and this is also 0.

Now, you can able to see in the left side, now we have apart from this this is at station two which is simply supported the moment is also 0. So, you can able to see this moment is also 0 so in the left hand side we have two terms which are 0, so we will be taking out the first row and second row equations from this. If we write that we will you can able to see if I write 0 the first equation of this will be t 1 1 minus y. This will get multiplied with that then t 1 2 phi at 0 station, these are for 0 station 0 y 0 minus I am taking out and other two terms are 0 0 multiplied by that is t 1 phi into R a and that is multiplied by 1.

So, this is R a, now you can able to see that is the first equation, similarly the third equation will give us another equation t 2 1 minus y naught and plus t, this is 3 1 and 3 2 phi naught. Then, t 3 5 R a, so this is the third equation. So, these two equations we can able to put in a matrix form, now how we can able to put?

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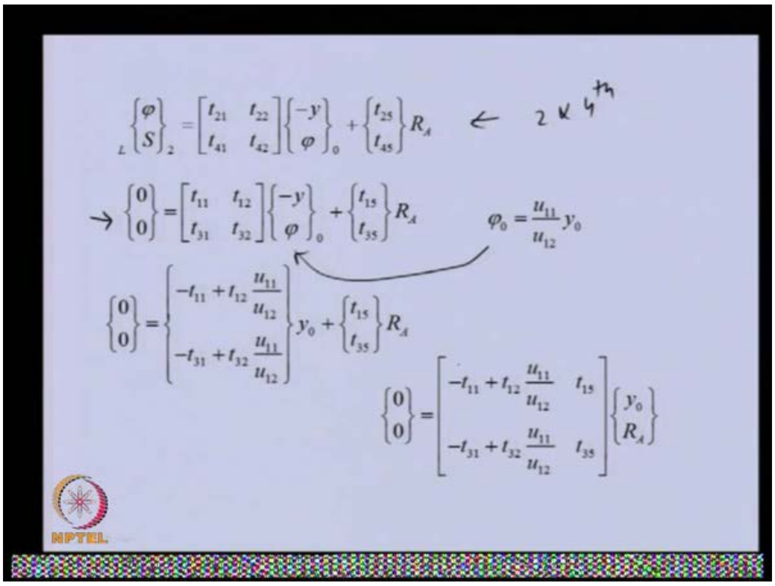

We can able to put in the matrix form like this is the particular equation, so this is the equation of the first and third. Basically, this is the equation for the second and fourth equation of the previous equation, but we are at present we are interested in this in which we have this two equations. Basically, these two equation from first and third which we expanded the second and fourth will give this equation. Now, we are focusing on this equation in which we have 0 terms in the left hand side, earlier if we you remember we had the state vector at 0 station related by this expression. We related earlier let us see back that relation, so this is the relation which we developed earlier, so this equation we will be using here.

So, I am using phi naught, so this is the phi naught, so this equation I am substituting this one. So, you can able to see that once I put the phi not here and I am multiplying with this matrix, I will get this because now this phi not will come here, and then y naught will be common and whatever the remaining terms I have multiplied. So, that multiplication will give me this and y naught is common and this is this term, now these two equation, now containing only two alone.

One is y naught, another is R a, these two equations we have obtained by using this, so basically we have eliminated the phi naught from this and this equation we can able to write it like this. So, you can able to see we have rearrange basically this is a two equation and containing two unknowns and these are homogeneous equation. So, I have substituted I have rearranged this in this form in which two unknowns I am writing. Now, in this form now you can able to see this is a homogenous equation non trivial solution of this that means when do not want the r a and y naught to 0 we need to have determinant of this to be 0, so that will give us the frequency equation.

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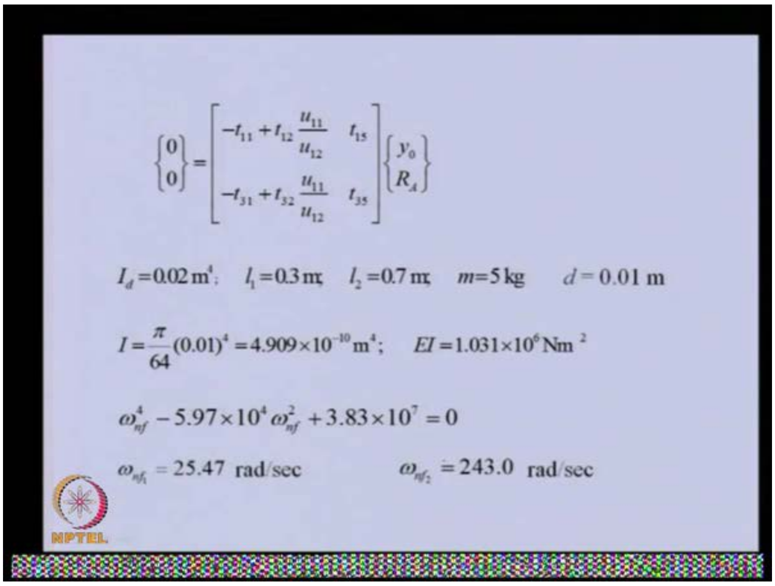

So, this is the frequency equation determinant of this and for various values of the parameter if we substitute in this because t is an in which they will be containing only unknown as omega. So, we will get a polynomial in this particular case only single disc is there and we are considering two degree of freedom of that disc that is the linear and angular displacement. So, we expect two natural frequency for that obviously in this frequency equation will be quadratic in nature and we can able to get the two natural frequency from this.

This same problem we solve using transfer matrix method earlier using in force coefficient method and the values are very close to those values. So, through one example we have seen that how we can able to develop the transfer matrix for the support because support contains one unknown reaction. So, we based on the free body diagram and the boundary condition and the support condition, we develop the transfer matrix and then from that we have obtained the frequency equation.

This particular method we can able to apply for multiple supports also and now we will take up another complexity in the rotor system that is a gyroscopic couple. As we know gyroscopic couple generally occurs onto the disc having large polar moment of inertia. We expect that the change in the transfer matrices will be at on the point matrix only because shafts when we are considering mass less as we have modeled till now. They will not be having any gyroscopic couple only the discs will be having gyroscopic couple, now let us see how the gyroscopic couple will affect the analysis.

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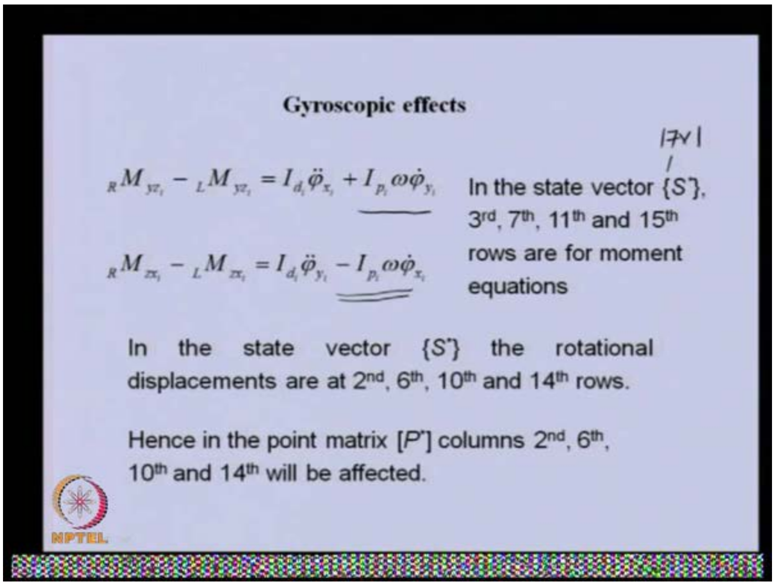

So, in the previous lectures when we have dealt with the gyroscopic effect we obtain the gyroscopic effect expressions. Now, we will be using them directly here only thing is in the disc case when there is no gyroscopic couple. Earlier when we have developing the point matrix, earlier in the previous lectures we had these kind of moment equation only this term was not there.

So, we had moment balance of the disc from free body diagram on the disc and these are the binding moment and these where the external binding acting was equal to the inertia rotor inertia of the disc. Now, because of gyroscopic couple apart from this there will be additional term of the gyroscopic couple which will be coming in this you can able to see that this particular angular displacement will be a of the other two. So, when we are considering the y z plane the angular displacement in that plane of the disc is phi x but the gyroscopic couple will be because of the angular velocity of the angular precession in the other plane that is z x plane.

So, that is why this angular velocity or angular precession is phi y instead of phi x. Now, this is one of the equation which will be effective apart from this the binding moment in z x plane also we need to write here and in this particular case one additional gyroscopic term this will appear apart from the rotary inertia. In the development of the point matrix we consider only single plane, but now because of gyroscopic couple what is happening both plane are involved.

So, we need to write these equations for both plane, so that is why we are written for y z plane and for z x plane now you can able to see the state vector. If you want to see the state vector in this particular case in the previous lecture I explained how we should step the state vector. If we are considering damping or the gyroscopic couple and I mention that the state vector will be having 17 into 1 size.

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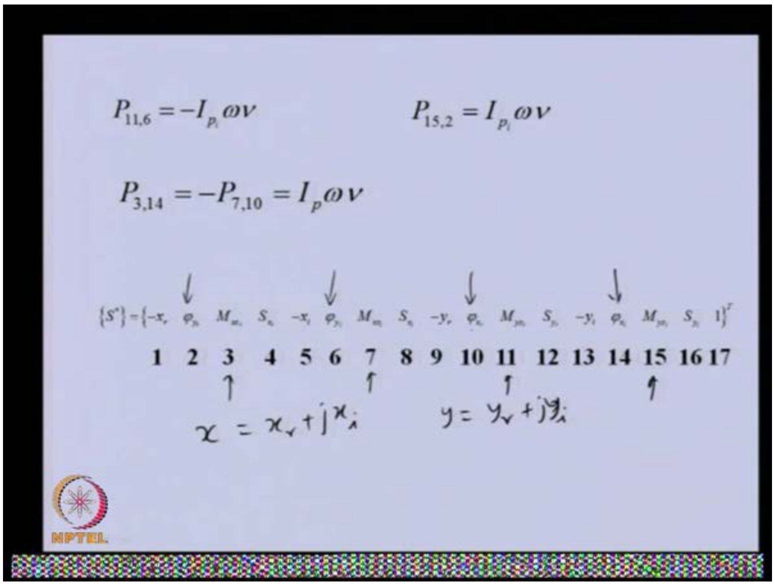

So, let us see how the state vector will be representing in this particular case, so this is the expanded form of the state vector. So, basically transverse of this will give a a column vector, so in this particular case we can able to see that the x displacement, but real part of that I must taking first. The angular displacement about y axis binding moment in the z x plane sheer force in y direction in x direction. So, all these are corresponding to the real part of the state vector this next 4 from fifth to eighth is the imaginary part of the same state vector in x phi y and m y z z x and s x direction.

So, first eight vector are belonging to one plane that is z x plane first four is real and next is the imaginary part of that. Similarly, the 9 to 16 this another eight state vector will be representing corresponding to z x plane z y plane. First 4 is for real part and next 4 is for the imaginary part that means each displacement which is there we will be representing as imaginary and real. Similarly, y another displacements and also sheer force binding moment everyone will be having two parts.

Now, we are considering two plane, so there phase may be different, so to take care of that we are writing them in a form a vector or a complex quantity so you can able to see this is the staking of the state vector. Now, coming back to the previous equations you can able to see that in this state vector the third 7, 11 and 15 terms belonging to the moment equation. That means again go back to the previous one third seventh eleventh and fifteenth these are belonging to the moment equation. So, that means the changes will be there in this particular rows in the point matrix because gyroscopic couple terms are giving additional terms in these rows third seventh eleventh and fifteenth rows.

Now, when we are talking about the angular displacement, because this gyroscopic couple is containing the angular displacement. So, again when we look into that state vector you will see that the second sixth tenth and fourteenth are belonging to angular displacements second sixth tenth and fourteenth.

So, these terms will be coming in the second sixth and then fourteen columns, so there in the row, so basically in point matrix they will be coming in the second sixth tenth and fourteenth rows. So, the position of these terms will be third row seventh row eleventh row and fifteenth row in the point matrix their column will be corresponding to the angular displacement that will be second sixth tenth and fourteenth. Now, let us see one of the equation we will expand and see how they will change the point matrix.

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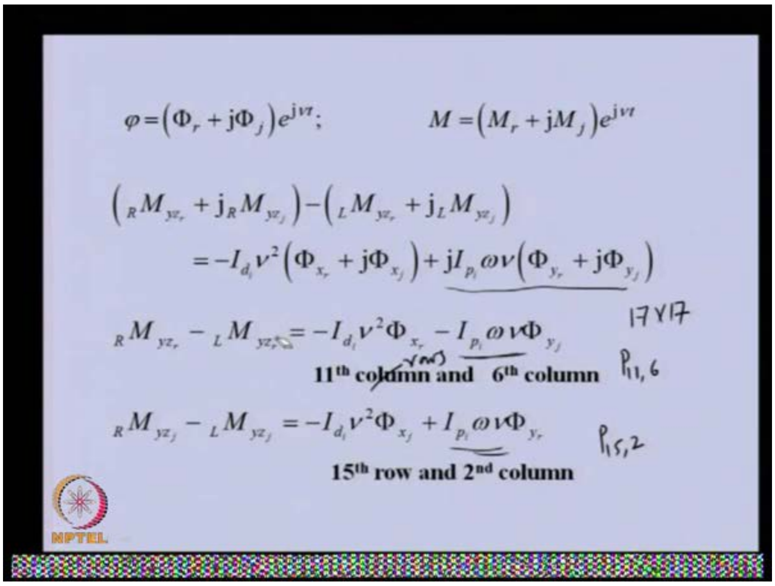

So, I am writing the angular displacement in the real and imaginary part and similarly moment also I am writing the real and imaginary part. So, basically this is a quantity will give them the amplitude and phase information and this is the frequency component of the moment similarly, this will give the phase and amplitude part of the angular displacement and this is the frequency. So, if we substitute this in the first moment equation this moment equation so this is belonging to the right of m y z. So, the first moment equation moment will give this minus the second moment and is equal to this is the rotor inertia.

So, because here second derivative is there so we need to take second derivative of this so we will get this nu omega square second derivative we can derivate it because they are not time dependent. The binding this gyroscopic couple will give one term because this is only derivative with respect to one time t, so that means we evaluate differentiate this once will get j nu j nu, so that is why this complex term is coming here.

So, this is belonging to the gyroscopic a couple in that y z plane, now if we take the real part of this and imaginary part of this, if we equate us will get this as a real part. So, you can able to see first term this term the real part of this and because this j is here, so this will be real now. So, that is why this is coming here so this term will come here similarly, this is the imaginary part of this equation now let us focus on this equation.

So, this equation is basically we have the real part of the binding moment in y z plane so let us see what is the where it will it will be there, so this is basically row, so y z real. So, if you see in the state vector this is y z and real this is the y z imaginary, but that expression is y z real, so this is the eleventh row. So, we have a eleventh row in the point matrix and this this is the additional term basically in the gyroscopic couple, so this is phi y imaginary j is imaginary.

So, let us see what is the position of that phi y imaginary is here sixth, so we will be having sixth column and eleventh row this particular term will come in the point matrix because point matrix is as I mention will be seventeen into seventeen. This particular term will come in eleventh row and sixth column, similarly if we solve this equation this is the y z in the imaginary we have seen that this was coming in the fifteenth row. This angle phi y real phi y real phi y real is here that is the second position, so that will be in the second column, so this term which is coming from gyroscopic couple will be effecting the fifteenth row and second column in the point matrix.

So, this is the re written again here so eleventh column six eleventh row and six column this term will be coming and fifteenth row and second column will be this. So, this two are coming from this two equations which in turn is coming from the first equation, so because of this we got two terms in the point matrix additional term. Similarly, if we take this equation if we expand it we will get additional two terms and there, you can able to check that at the third row and fourteenth column and seventh row and tenth column this term will be coming. So, in total we will be having 4 entries which will be new because of gyroscopic couple.

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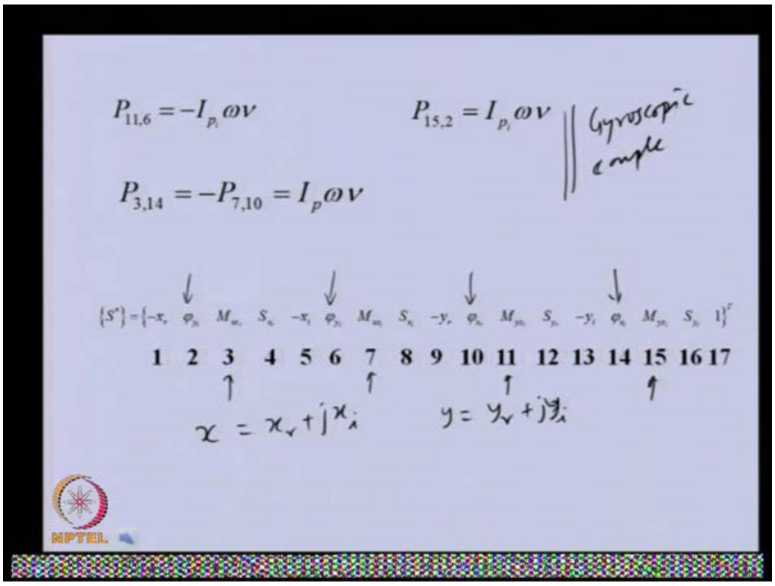

These are only coming because of gyroscopic couple other terms will remain same as the previous one and these four entries will be new in the or point matrix.

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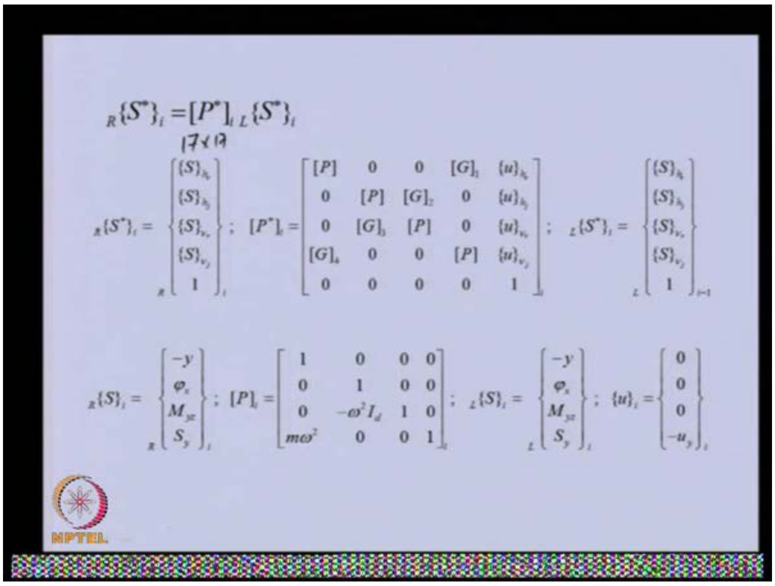

So, let us see what will be this transfer matrix over one disc, so this is the overall transfer matrix sorry this is the point matrix modified point matrix for a for a disc i th disc earlier we had this form of the p in which we had this as 17 into 17 size. We had p matrix at the diagonal phase earlier, this is the p matrix which we had already obtain earlier without gyroscopic couple.

If we refer back to previous lectures, we had these diagonal terms all over the half diagonal terms was 0 earlier except these are because of the unbalance. So, here also we have unbalance component in the horizontal direction real part imaginary part vertical direction real part and imaginary part. Gyroscopic couple terms are coming in this G matrices G 1, G 2, G 3, G 4 where this G 1, G 2, G 4 will be having this form.

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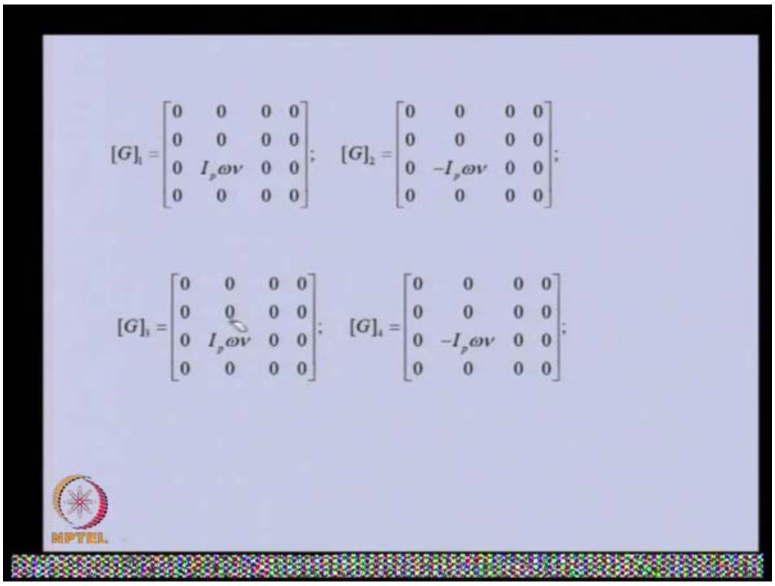

So, G 1 G 2 G 3 G 4, so you can able to see that if you want to check one of them like this p eleven comma six eleventh row sixth column.

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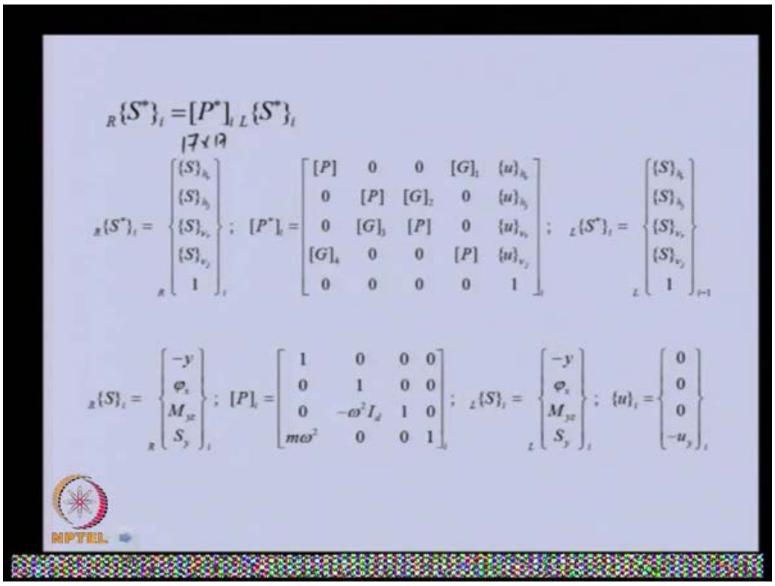

So, in this row is this is four row because this is 4 by 4 p is 4 by 4 and then this is another 4, so this is 8, then this is another this will be 12 and this will be 16, but this entries at eleventh row. So, that means it should come in this particular row, so 4, 4, 8 and here third in this third row will be the entry now the column wise column wise is sixth. So, column wise is here we have four columns because this 4 by 4 matrix and then here 4 so sixth will be second column in this particular vector. So, you can able to see here it will be there and this is belonging to that particular new entry, so you can able to see this G 3 which is there.

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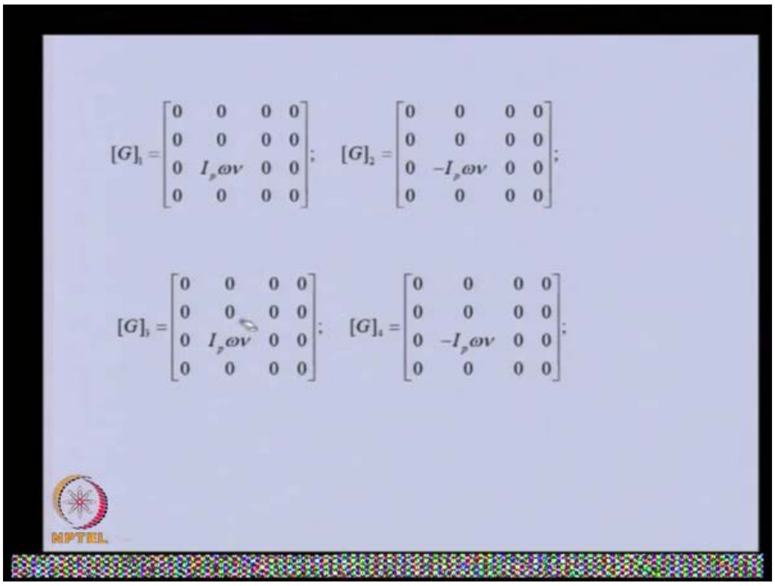

This particular G 3 will be belonging to this particular vector, similarly other four entries are there you have this this and this. So, they will be coming in this G 1 G 2 G 3 only single terms will be coming this whole 4 by 4 matrix most of the all terms are 0 expect one terms which are belonging to the gyroscopic couple terms. Now, you can able to see that the overall the point matrix is of this form.

Now, we can able to take up some example to illustrate how this we can able to use it and we can able to calculate the whirl frequency and in this particular case we can see the G is this one. Now, omega is the spin speed of the shaft nu is the whirl frequency that is the natural frequency of the shaft and now both whirl frequency and the natural frequencies are appearing that means natural frequency will depend up on. Now, on the spin speed of the shaft that we have already seen in the gyroscopic effect when we discussed that particular topic in detail.

So, we have seen the derivation of the gyroscopic effect how the point matrix get effected with that. Now, we will through one example very simple example we will try to see how we can able to obtain the Campbell diagram that is nothing but the variation of the whirl frequency with speed using this transfer matrix method, so for this I have taken very example a cantilever rotor.

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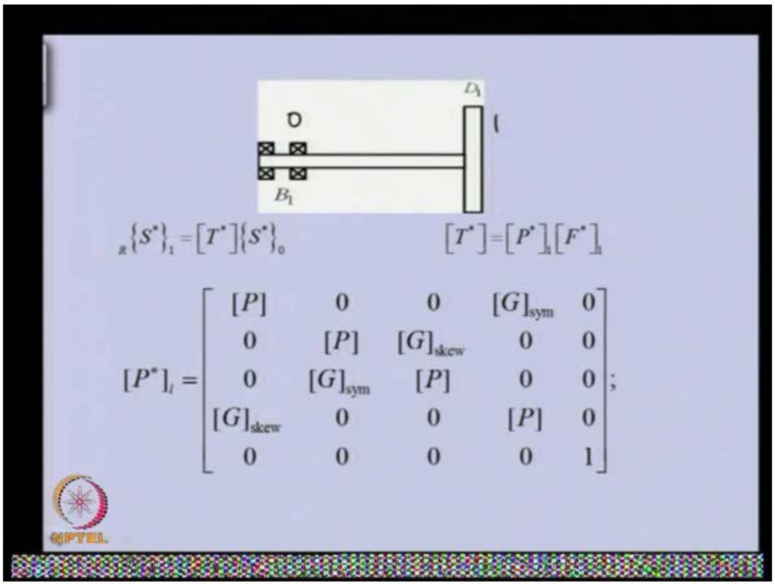

Basically, it is a mass less shaft the disc is having both the mass and diametric mass of moment of inertia even the polar mass moment of inertia and we are considering the gyroscopic couple in this case. Now, because we have only two stations 0 and 1, we can able to write the state vector illusion like this where t is the overall transfer matrix which is multiplication of the field matrix and the field matrix.

The point matrix field matrix will not change as we have explain in the previous lectures it will remain same only the point matrix as we have discuss today will change some of the components. Now, this point matrix is given like this where G symmetric and G skew matrices we will be defining.

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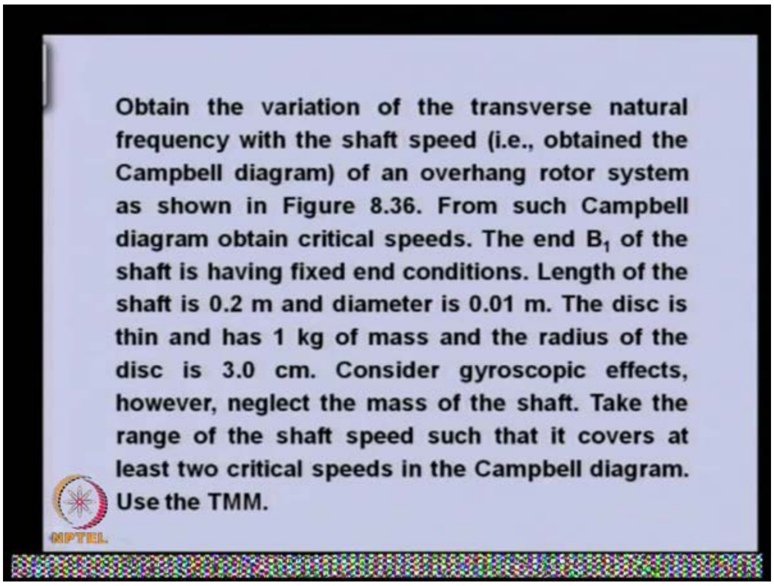

So, first let us see this particular problem various dimensions, so you can able to see shaft length diameter is given mass of the disc is given its radius is given this is the disc, so from there we can able to get the or there inertia property. So, we are considering gyroscopic couple and obtaining the Campbell diagram also, that we can able to calculate two critical speed of the rotor system.

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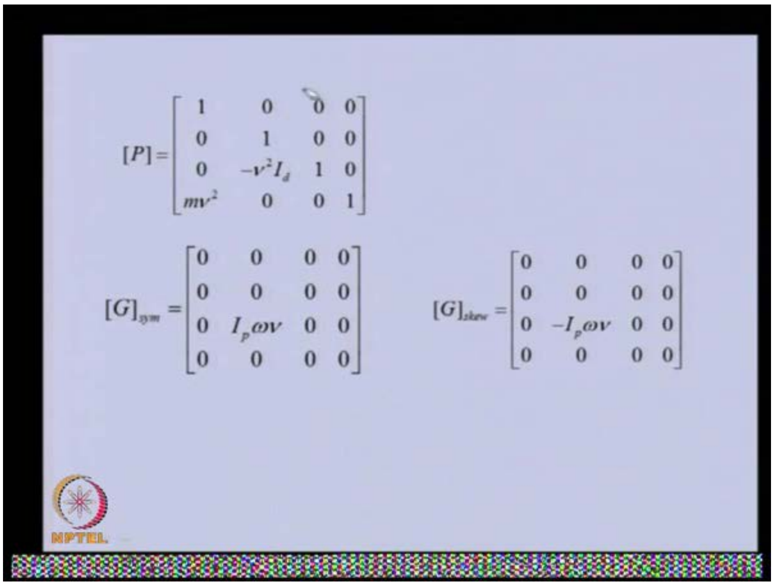

So, this matrices p matrix is this which is exactly same as the without gyroscopic case this gyroscopic matrix additional terms which are represented as symmetric and skew symmetric is given here. So, we can able to see what will be the form of the point matrix modified point matrix for this case we are not considering any external force here.

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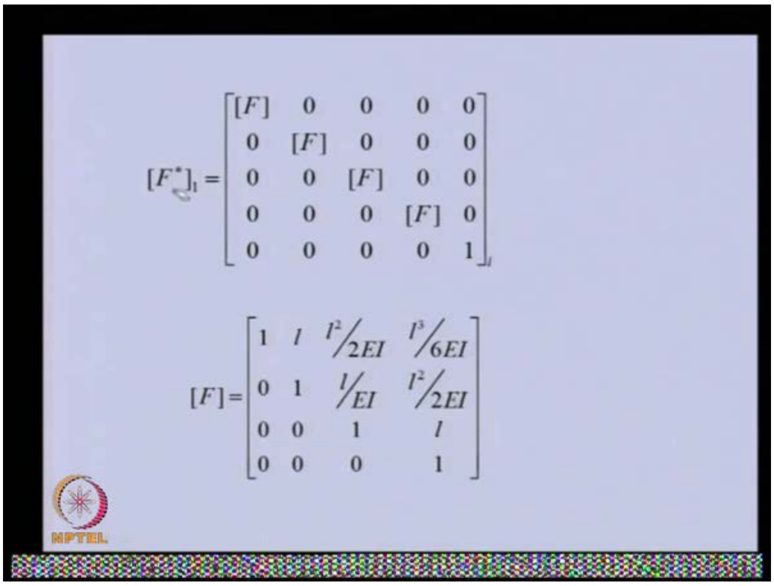

So, field matrix will be of this form only diagonal terms will be there will not be any of diagonal terms and one of the f will be same as we obtain earlier, for the without gyroscopic case the state vector staking is in this form. So, horizontal real horizontal it is varying vertical real and vertical imaginary components and one of the state vector is represented like this.

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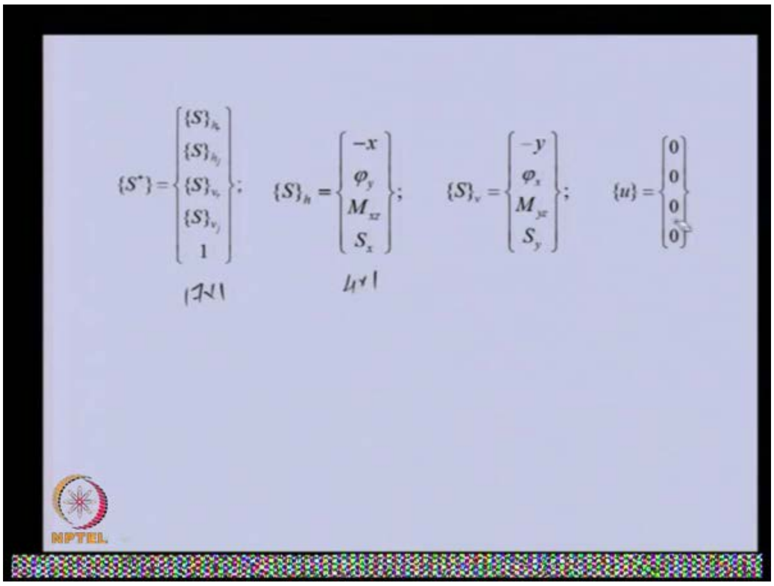

So, one is having four by 4 into 1 size, so total will be the 17 into 1 size and this is 4 into 1 size, there is no unbalance, so this vector will be 0.

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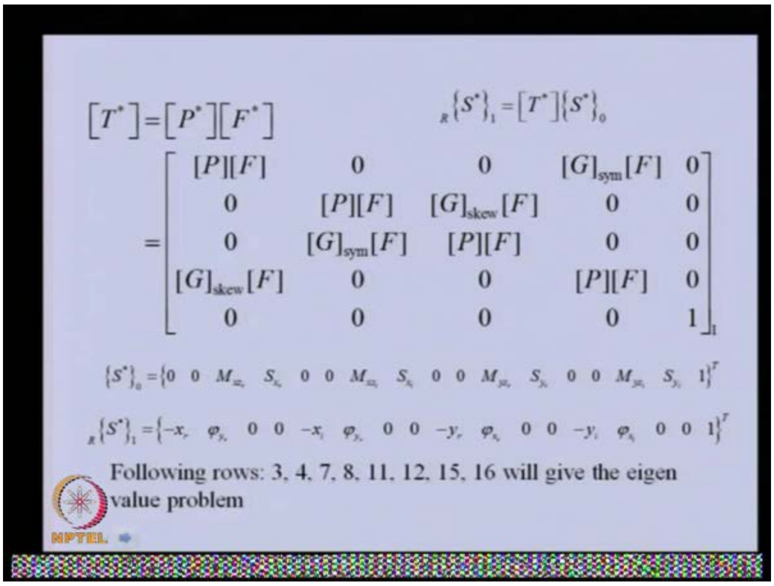

Now, the multiplication of this two to get the overall transfer matrix will give this form of the matrix. The state vectors at zero station and station one are given like this here we applied the boundary condition also because at 0 station will be having fixed we have fixed. So, displacement linear and that will be 0 both real part and imaginary part of that, similarly here we have for even in the vertical direction and horizontal. This is for the horizontal direction this is the vertical direction, so vertical direction real part vertical direction imaginary part these are the linear displacement and angular displacements.

So, this is coming because of the fixed end condition and that node one we have free end, so binding moment on sheer force components will be 0 for horizontal real part horizontal imaginary part, horizontal the vertical real part and vertical imaginary part. So, these are the application of the boundary condition on to the state vector, now you can able to see in in this basically we have this form of the transformation, so in this particular vector whatever the terms are zero we need to take out those equations.

So, in this we have third equation fourth equation seventh eighth eleventh twelfth fifteenth sixteenth which will be having 0 entry here. So, we take out those equations only and we will stack them in a and we will we will be staking them, so basically we will be getting total 8 into 8 matrix 8 into 8 matrix. So, this will be the homogenous equation because this big equations we cannot able to write it, so numerically we need to solve this, but basically we will be getting the homogenous equation as 8 to by 8 matrix. That can be solved for various values of omega, so you can able to see that we are solving.

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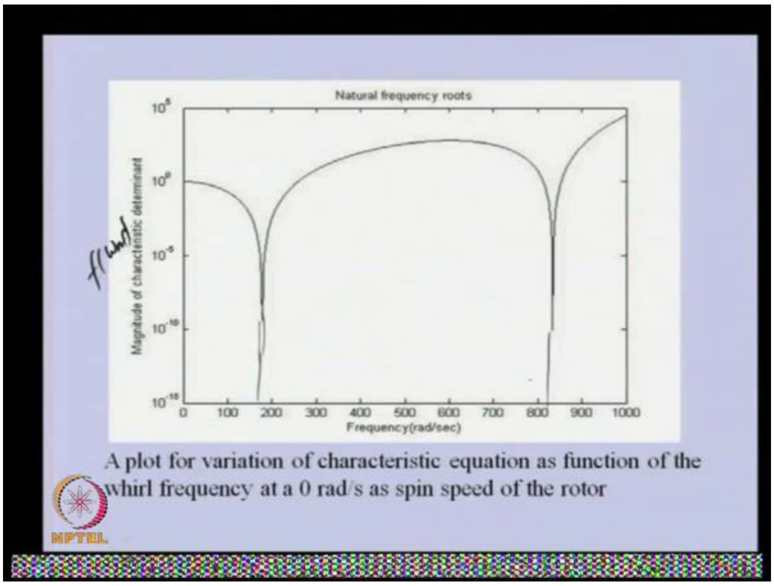

That particular 8 by 8 matrix determinant for one of the speed, so here we have taken 0 speed and when we are solving it we are getting 2 terms. You can able to see basically they are approaching to 0 because of the numerical difficulty they are not approaching here but these functions are becoming 0. So, that means this is one of the natural frequency of the system this is one of the natural frequency of the system. This is the determinant of that homogenous equation basically the frequency equation this is for the frequency equation. So, these are the two routes because speed is 0, so there is no gyroscopic couple, so we are getting only two critical speeds or two natural frequency.

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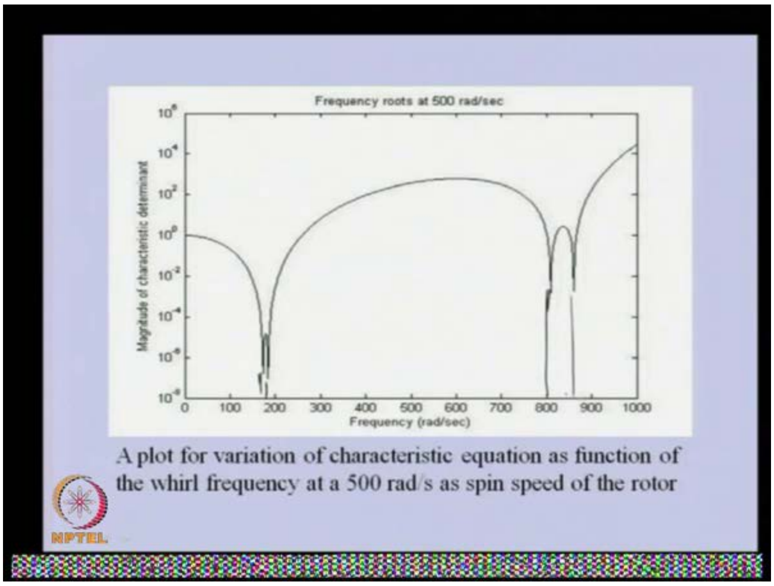

If we take some other speed and solve the same determinant, then we will get you can able to see that there is splitting of the frequency is taking place here also. So, now we are getting four routes, so basically they are all intersecting to this zero line to some value so we will be getting four natural frequency.

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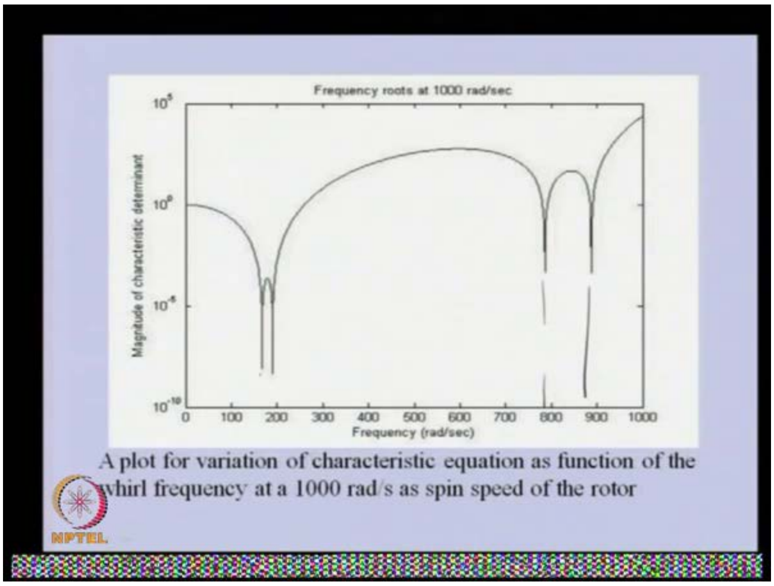

Similarly, if we. increase the speed to 1,000 we will see that separation of the speeds will be more and more separation will be more as compared to the previous one even if you increase to 1,500 you can be able to see separation is more.

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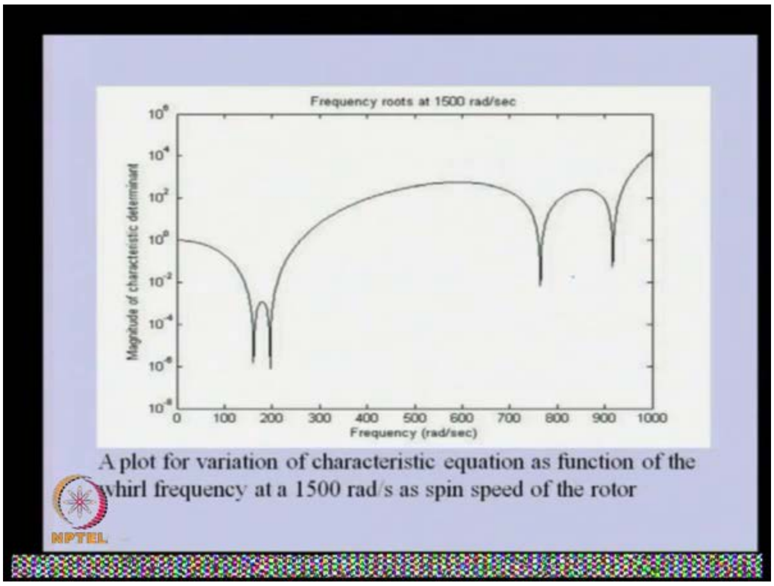

So, basically if we vary the speed and pick up all these frequencies and plot them with respect to speed we will get the Campbell diagram.

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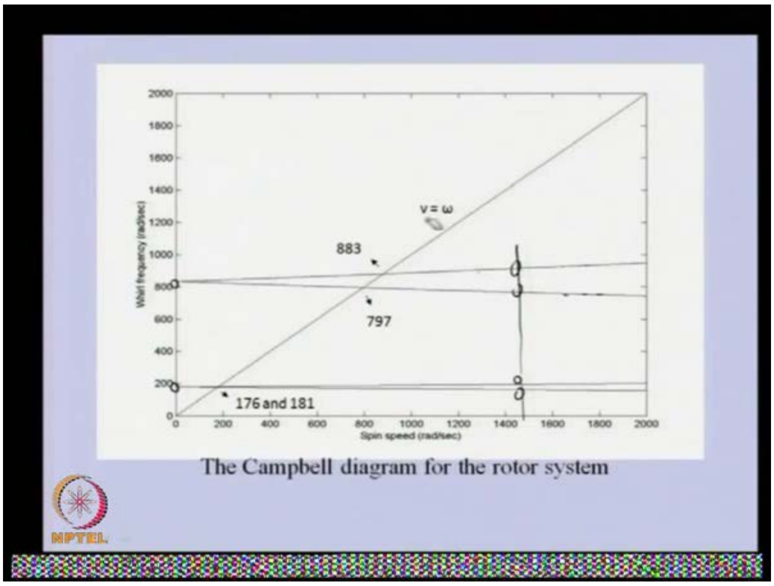

So, basically you can able to see this is the spin speed and these are the four natural frequency which we are getting. So, you can able to see that at one particular speed let us say 1,500 here we had four intercepts 1, 2, 3, 4, so those four values are belonging to 2 here and 2 here. So, similarly we are varying the speed in this range and we are obtaining various points along this lines along this lines and at 0 speed we have seen that they are meeting at the same point, so this is most splitting at 0 speed.

So, you can able to check the zero speed what was the value, so it was around below 200, so around and this is above 800, so approximately I am checking. So, this was below 200 the first route and this was above 800 second route. As we increase the speed this splitting is more and more, now if we draw a 45 degree line here which will belong to the whirl frequency equal to spin speed, we will see that they will intercept these lines at 4 places. They are basically four critical speed of the rotor system, so one two three four intercepts will be there and all these values on the critical speed of the rotor system.

So, this is the Campbell diagram which is nothing but the variation of the whirl frequency with spin speed which is occurring because of gyroscopic couple and if we draw the this particular line which is the synchronous whirl line in which the whirl frequency is equal to speed we will get the critical speeds of the rotor system ah.

Today we have seen the transfer matrix method two main aspects one was the how to handle the support intermediate support on the rotor system. We develop the transfer matrix for that support itself, and we have seen how we can able to get the frequency equation for such cases then we extended the method for more complex phenomena that is gyroscopic couple phenomena.

We have seen that only four terms in the point matrix the modified point matrix are getting affected. Only difficulty is with that that the whirl frequency becomes the speed dependent and because of that the solution of the whirl frequency for each of the speed we need to obtain. Basically that we can able to represent using Campbell diagram and Campbell diagram, then we can able to use it to obtain the critical speed. Now, in the subsequent lecture there we will be considering the finite element method which is more powerful and more popular. Now a days in that particular case, we will see that how the boundary conditions support reaction, there they can be easily handled.