Theory and Practice of Rotor Dynamics
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Module - 6
Transverse Vibrations
Lecture - 26
Transverse Vibrations: Transfer Matrix Method

Today, we will study transverse vibration using transfer matrix method. This particular method earlier we studied for torsional vibration case. We have seen the advantage of this method that this can be applied to multi rotor system. We have seen this particular method once we develop the point matrix and field matrix for the disc and the shaft. We can able to use this for larger system also in torsional vibration case. The size of the matrixes was small that was 2 by 2 matrix, but will we see that for transverse vibration. These matrixes point matrix and the field matrix will be of larger size, especially when will be considering both plain motion of the rotor.

In transfer direction, especially when the gyroscopic couple, when the system then because this two plain motions are coupled. So, we will be having the size of the matrixes 16 by 16 on that order, but with the single plain motion we will be having the size of the point matrix and the field matrix of the order of 4 by 4.

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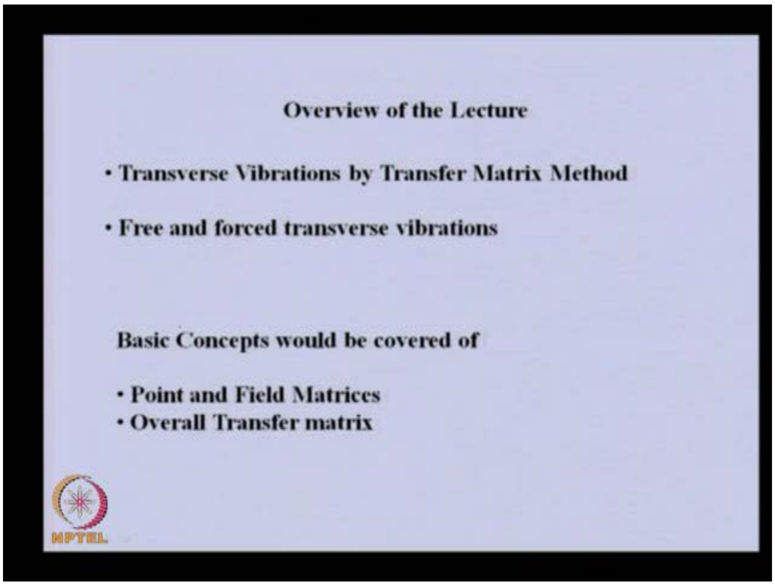

So, let us see what are the things, we are covering in this particular lecture. So, transverse vibration, by transfer matrix method free and force method both we can able to analyze and this similar to the torsional vibration is will be having point and field matrix matrixes development. For transverse vibration and similar to torsional vibration case, overall transfer matrix concept and then we can able to apply the bonding condition of the problem to either to solve pre vibration and post vibration ratio.

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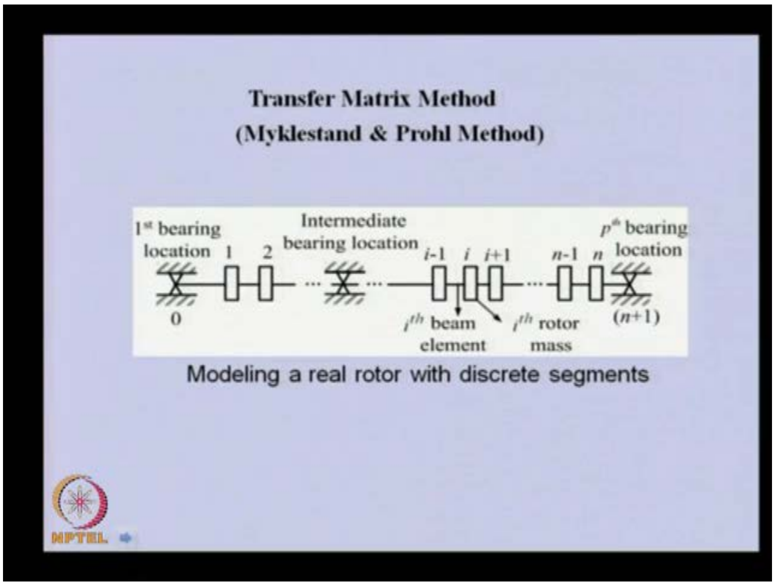

So, let us take a multi mass rotor system in which we can have multiple support also and in this we have leveled various station from 0, 1 up to n plus 1 and all the important disc bearing disc. We need to assign some station number and now for developing the point matrix or field matrix we need to take out some representative. This beam segment or rotor segment, then from the free body diagram of that we will try to derive the field matrix and point matrix for this segments. For support we will have separate segment because support will give some unknown reactions.

So, to tackle that we see trough some of the example in substituent lecture, how the support can be taken care if there is inter median support are there. In this transfer matrix is also called Myldestand Prohl method because these two developed independently or this method. So, that is why this method is also called Myldestand & Prohl method.

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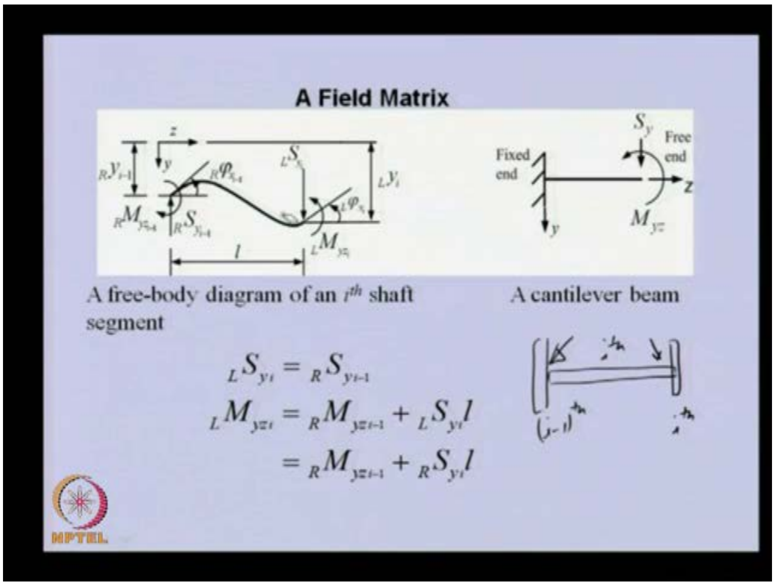

Now, if we want to develop the field matrix for this case, it is the basically a particular shaft segment ith shaft segment. In this v axis z axis along the shaft axis, y is in the transverse direction. We have taken the free body diagram in one of the plain and a similar relation will be there for the other plane because this shaft is symmetric. So, for y z plane one end of the shaft is having linear displacement y, this is ith shaft. So, we will be having i minus one disc in the hand side of this shaft and toward the right side will be having i th disc and to represent the various field variable or state variable.

We will be representing the right or left with respected to the particular disc. Here, we have i plus i minus 1 disc. So, field variable which is here is y i minus 1, but right of that particular disc. So, backs of the disc R is representing that this field variable is right side of the i minus 1 disc. Apart from this, we see there will be slope of the shaft during binding. So, that slope we are representing pi i minus 1 backs of script R. So, these are the two field variable linear displacement and angular displacement for representing the displacement of shaft at i minus nth disc.

Apart from this we will be having the force, we are taking the positive convention of strength material of representing the sheer force. So, sheer force that is S y is the y direction and i minus 1 disc at right of that. So, this is the representation of sheer force and this is the binding movement which is again in y z plane. It is acting at i minus 1 th disc right of that. So, this is the representation of the binding movement at this location.

So, you can able to see there are four state components; one is linear displacement, angular displacement, sheer force and binding movement.

Similarly, in the right side of the disc that means towards the right side of the i minus 1 th disc because disc is acting here, we are removed that. So, this is the left part of the that particular i nth disc variable, like displacement. Linear displacement will be having y i left of i so l is representing that particular displacement is representing the left of I th disc, and this particular is the angular displacement this slope. So, basically if we want to see the i th shaft, it is the i th shaft here, we have i th dischere we have i minus 1 th disc.

So, we can able to see that this particular end of the shaft which is here is representing the right of i minus 1 th disc and other side, this side of the shaft which is here is representing left of i th disc. So, we always representing left or right with respected to particular disc, not respected to shaft, but particular disc. Now, here apart from this we have sheer force, that is S y i left of i th disc then the similar of binding disc. So, here also we have four state variable and now we need to relate this state variable which are there in the right of i th, i minus 1 th disc and left of i th disc.

So, if we see the free body diagram of the disc force balance, only two forces are acting in this particular shaft, one is sheer force here and here. So, they can be equated sheer force of left of i th disc is equal to sheer force right of i minus 1 disc and binding movement for binding movement, maybe we can take this movement about. Let us say here left of the this shaft we will be having this movement equal to this movement plus this sheer force and momentum. Momentum is nothing but the length of the shaft because this sheer force is acting here itself, were we are taking the movement. So, this will not contribute to this movement. So, this is the movement balance and using this first relationship we can able to see because both the shape was the same where we can able to replace the left with the right.

So, were we all need a change, so we can able to see that how we represented the field variable here all are in the left side and here all are in the right side of the i th disc. Now, we want to relate the disc plain, now we want to relate the displacement, linear displacement of this end the disc. This end of the shaft will be I am taking about the fluid displacement of the disc will be relative displacement of this end with respect to left end and the displacement of this end itself.

So, you can able to see there are two component of displacement, one is displacement of this displacement to left hand displacement of this itself. So, to take care of this we can able to take two cases, first will consider let us say this end of the shaft is fixed like this. So, one end is fixed now because of whatever the sheer force binding movement, which is acting. What is the displacement of this end we need to calculate and once we calculate this then we need to add to that displacement the displacement of itself here.

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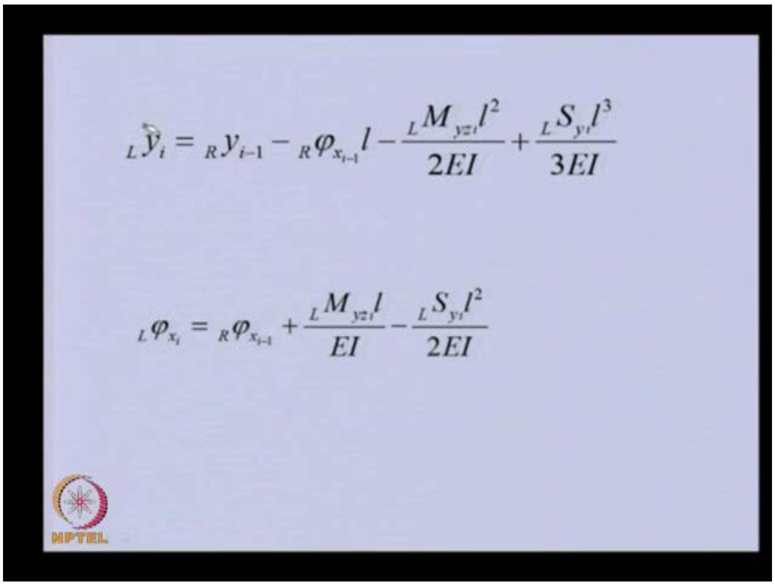

Let us see in the substituent life displacement of the left of the i th disc is given by this is this two term are coming from the cantilever when we are considering the shaft as fixed. So, from the strength of material book we can able to get the relation that when we are sheer force acting. So, what is the linear displacement when we are having this binding movement what will be the displacement of the free end of the shaft because we are linear system. So, they can be combined, so this is while the shaft is flexible and support left hand support recite that means cantilever case.

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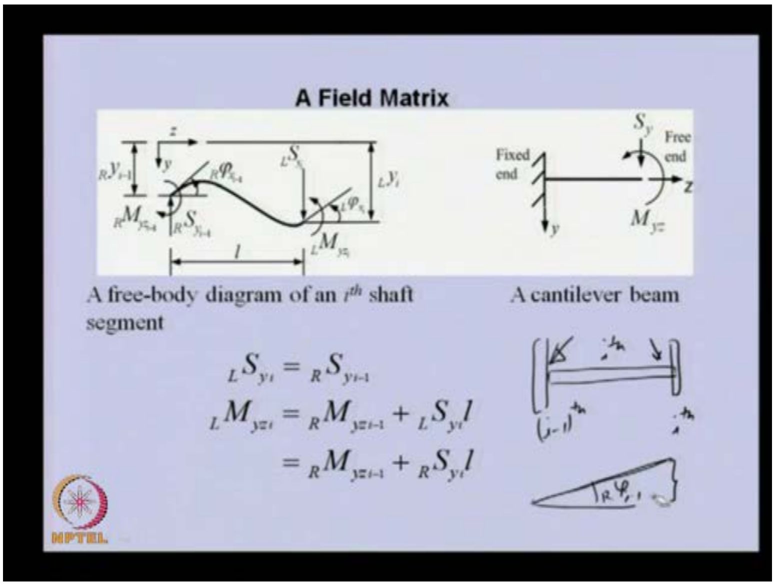

Now, in the second case will consider the shaft as recite and then will give the these two displacement. Now, this y and pi displacement to the this end of the shaft which is now in the second case we are considering as recite. We can able to see that whatever the displacement of the ith minus 1 right of i minus because disc is there linear displacement, same will be there. Put this value here and for the slope for slope again we need to refer the figure for slope.

So, whatever the slope of this end we are giving, so in this particular case we can able to see that if the shaft is recite this second case, this is the slope of the shaft which is right of i minus 1. So, because of this angular displacement which we are giving to this end, what is the linear displacement of this? This will be slope into length of the shaft.

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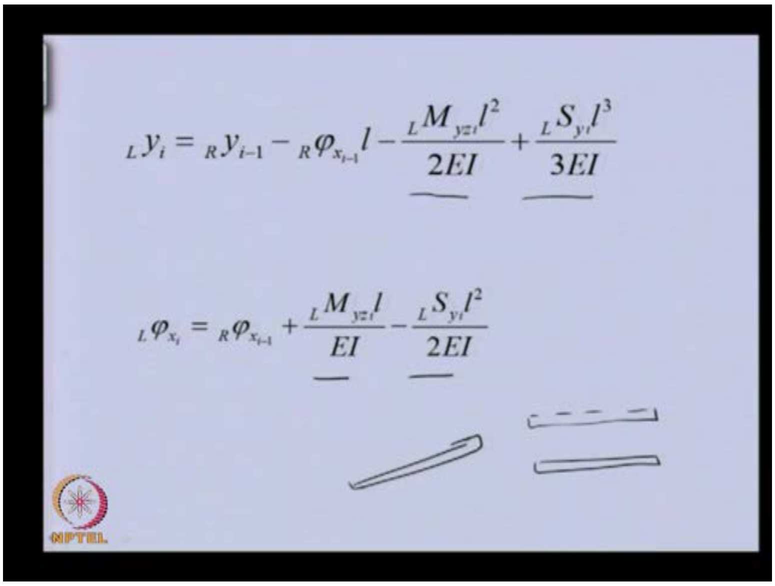

So, this is the same thing we have done here and because it is going towards the negative y direction. So, this negative sign is there, we can able to see that the total displacement of the left of ith disc. That means this disc is summation of two component, one is linear displacement of the right side of the i minus disc, angular displacement of i minus right of i minus 1 disc binding movement and sheer force. So, four components are there.

Similarly, the angular displacement at the left of ith disc, that means again we have to refer here. So, in this also will be first considering this as cantilever beam and whatever the angular displacement, because the sheer force binding is there that will come. So, this also we can get it from length of material book by reflection of beam radiation, these are available. So, these are the angular displacement due to sheer force and binding movement. When we are giving the linear displacement of this, there is no change in the slope because in this particular case shaft.

If we are giving a linear displacement like this, there will not be any change in the slope. This is in the second case where we are shaft has recite, but when we are given a change in the slope like this up to the recite shaft, we will be having same amount of slope at this end also. What are the slope? We are changing at this end, left hand right hand also having same displacement. So, this is that particular component. Now, we can able to see that we have related not only the sheer force binding movement, this translation displacement, rotational displacement.

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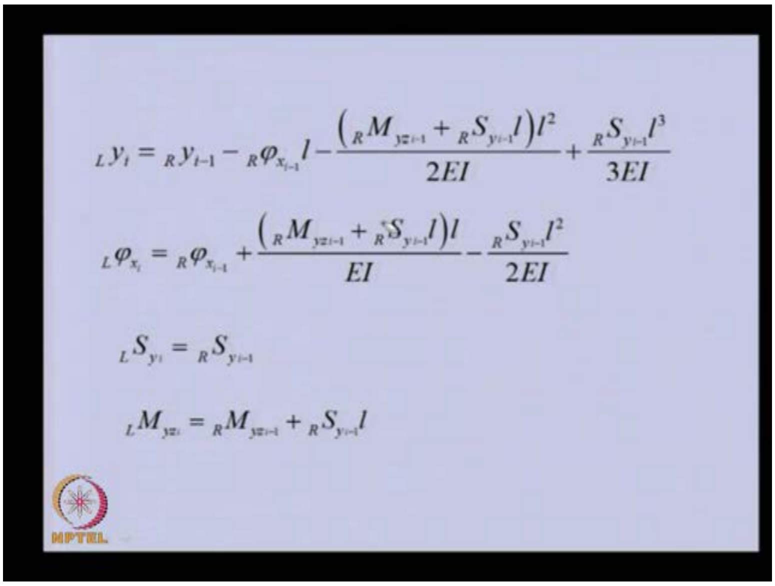

This relation we can able to summarize like this. This is a first equation linear equation here, if you go back here can see this particular movement. Now, I am replacing with that movement because we have already these relations. So, I am just substituting this expression here. So, that all the terms this side is toward the right of the ith disc term. Similarly, in the angular displacement cases are replace this term and this are the two relation, which we obtain for sheer force and binding movement. There is no change in this, now we can able to rearrange this equation and we can able to put them in matrix form.

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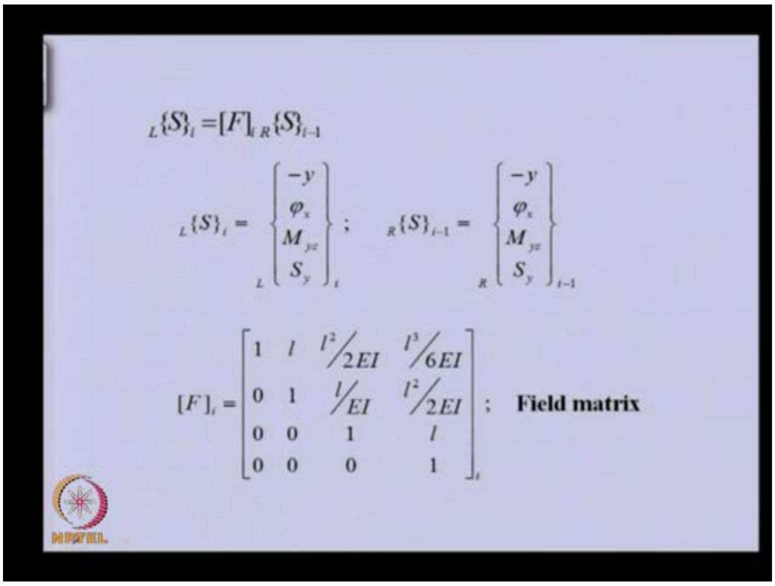

So, basically we are stacking the state factor like this, first linear displacement minus this minus, we are taking that in matrix, all the terms are positive, then angular displacement binding movement sheer force. So, this particular state factor which we have written for left of ith disc this for right of i minus 1 disc and this is the field matrix which is for i th shaft. Now, if you go back to this four equation, basically you can able to see these are the vector which we arranged. I think if we put here then this will be of similar form, this is left of i, left of here all are left of i in the side of we have right of i minus 1 component. Similar four components are there.

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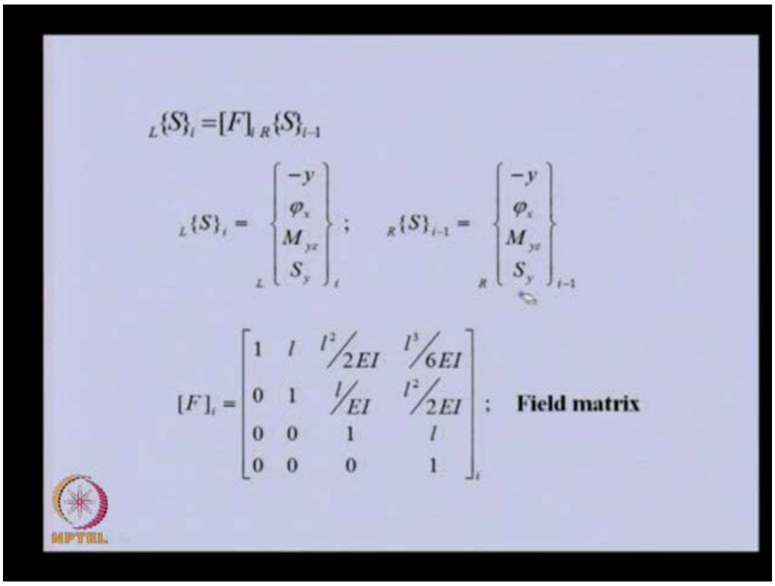

That means this components which are right of i minus and whatever the coefficient are there in the right side we are keeping in the field matrix. So, this is the field matrix, you can able to see sheer force relation, if we want to see sheer force is at the end and here because these field variable are getting multiplied by 0. Here, will be having the sheer force in the left of i is equal to the sheer force of toward the right of i minus 1. Similarly, for the binding movement this is for angular displacement for the linear equation. So, basically if we substitute this here and multiply we will get all the four equation which we have derived this. So, you can able to see that fluid variable and field matrix earlier in the torsional vibration the size of that was 2 by 2, but now it is 4 by 4 matrix.

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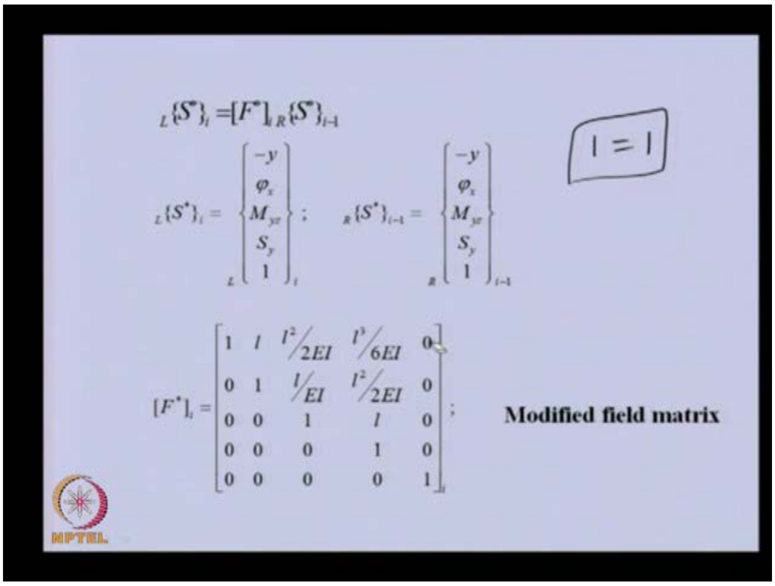

Now, if we have external force then as we did in the torsional vibration case, we need to modify field matrix by identity at the lower position. So, basically we are applying 1 equal to 1, we can able to see this field matrix is added all the 0 except the diagonal term and here also 0. So, again if we substitute this here and these state factor here, if we expand the last equation. We will get this because this is identity such it will not effect the equations, but this will help us in of putting the unbalance force which will be there at the disc location, when we will be driving the point matrix. That particular force will come in the last column, so that will see once we derive the field matrix point matrix.

Now, as I told so we derive the field matrix for one plain motion of the shaft, but generally the rotor have both the plain motion that is in vertical plain and horizontal plain. So, we expect that if we want to analyze the both plain motion simultaneously, then we need to modify this particular matrixes to cooperate both plain motion, also as I pointed out when we have gyroscopic couple in the system or damping in the system.

Then we will be having face difference of the displacement of factor and the velocity component factor by 90 degree and because of that we will see that we will be getting the complex quantity within the field matrix and point matrix. And to take the to make the as we seen in the torsional vibration case also with damping. As we have seen in the torsional vibration case, with damping and the point matrix and field matrix became complex, but if we want to work with the real matrixes only. We need to separate this matrixes real part and imaginary part separately. Then we can able to recombine these equations so that matrixes are in the always in the real form.

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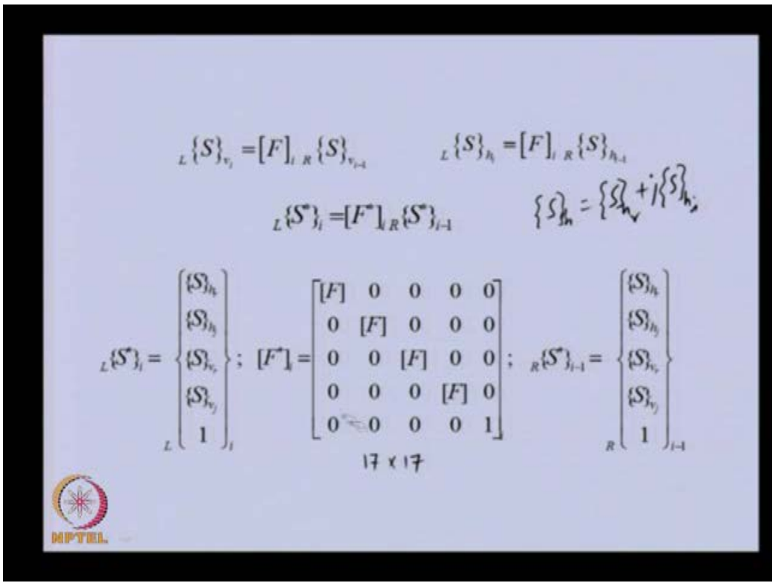

So, for that case when we are considering both the plain motion and when we want to consider the damping over gyroscopic aspect also then we need to. So, here we can able to see the state factor which is in the horizontal plain. This two are in horizontal plain, these two are in the vertical plain, in horizontal plain itself we have taken the real component, the imaginary component.

That means a state factor which is there in the horizontal plain; we will be having two component because of the face real component and imaginary component. Now, what we have done it, when we are substituting this in such field matrixes, we are separating the real part and imaginary part separately and recombining them. So, you can able to see that if we expand these matrixes after substituting here, this will be equal to F into this part.

So, horizontal real part multiplied this matrix and horizontal real part and left of this particular disc i n d right of i minus 1 disc. So, basically here we can able to see these field matrixes which we obtain earlier having four size of them, each of them and we have added the identity also at the end. So, basically this modified matrix is having 17 into 17 size and if we want to consider damping or the gyroscopic effect or any other aspect, then this is the most general form of the field matrixes.

Only thing some extra term will be added up in this considering damping, but as such this are ready for addition of any other complexity and other in the form of damping gyroscopic couple or any other complexity which we want analyze. So, this is the most expanded form of the field matrix which will be having when will be considering the gyroscopic effect. Then we will see that especially the point matrix will get effected and some of the terms which are here 0, we have some matrixes.

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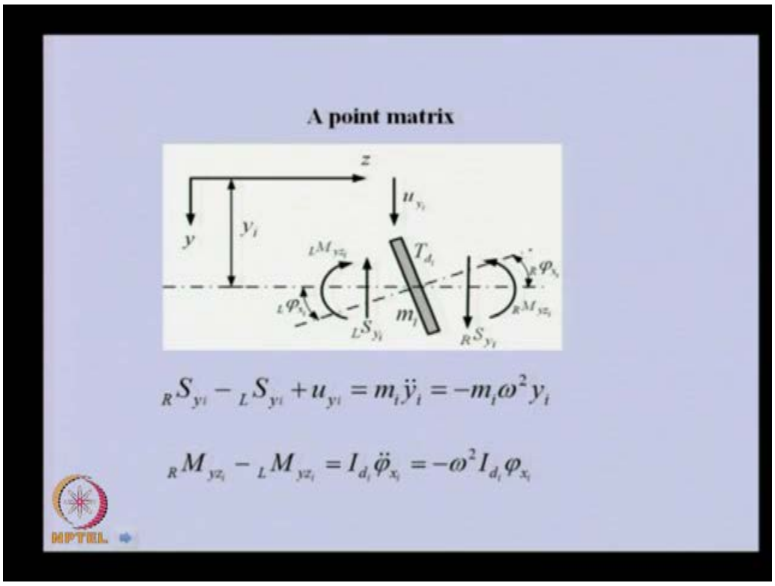

So, once we are done with the field matrixes, now let us come to the point matrix for that I have taken the free body diagram of the ith disc. Now, will be having linear displacement, there is translation displacement which will be ith of the disc because this is a thin disc. So, we will be having displacement in the left and right of the discs are same. Similarly, the slope you can able to see the left of ith disc slope, that is the right of that and this is the thin disc. The slope will also be the same apart, from that we have sheer force at both end and binding movement and apart from that if there is some eccentricity, the rotor.

Let us say u i, u y i is the unbalance force which is the toward the y direction, positive y direction. Now, we can able to obtain the relations as we obtain for field matrix. So, first the force balance, so we can able to see the shape force plus 1 which is acting towards the y direction. This is acting in the negative direction, unbalance force is acting in the positive direction should be equal to inertia m i into y i double dot is stand derivative and for harmonic motion which will be there for force vibration. We can able to write this like this where omega is spin speed of the shaft.

If we are dealing with the free vibration, unbalance will not be there and this frequency will be the natural frequency of the term. Similarly, the movement equation we can able see this is the write, this movement which is in the positive direction. This is negative direction or positive to previous one is equal to rotor inertia i d, diameter mass inertia into a angular acceleration about x axis. So, for previous simple harmonic movement we can write like this.

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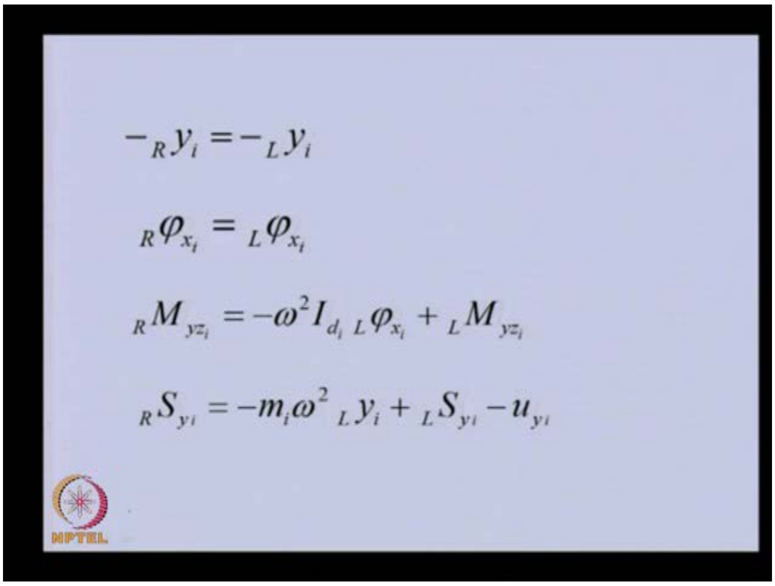

So, these are the sheer force and binding movement and force balance as I pointed out the displacement translation displacement at both ends are same slope is also same. This are the previous movement equation force, the force equation I have written such that the terms related with the write of disc is this side. All the terms which are left of i terms are this side. So, if we are all the terms are ready l that subsequent, now this we can able to combine similar to the field matrix.

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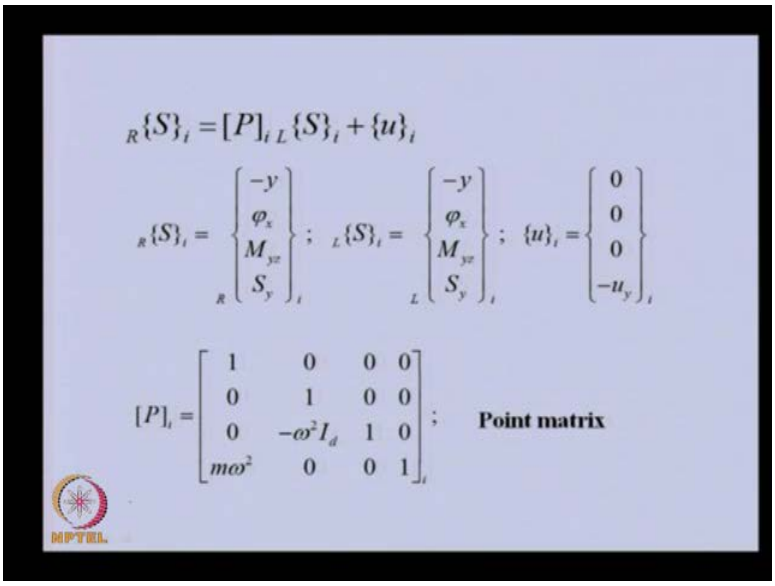

So, you can able to see that this is the point matrix, this one which is coming from various coefficient of the these state factor components. We have additional factor which is unbalance factor which is coming from the unbalance force, because we have stack the force sheer force at the last. So, it will so basically this equation if we expand this equation, the fourth equation we will get this equation and this is the last term which is corresponding to the external force. Now, to accommodate this external force inside the point matrix, we need to modify the point matrix by adding identity as we did earlier.

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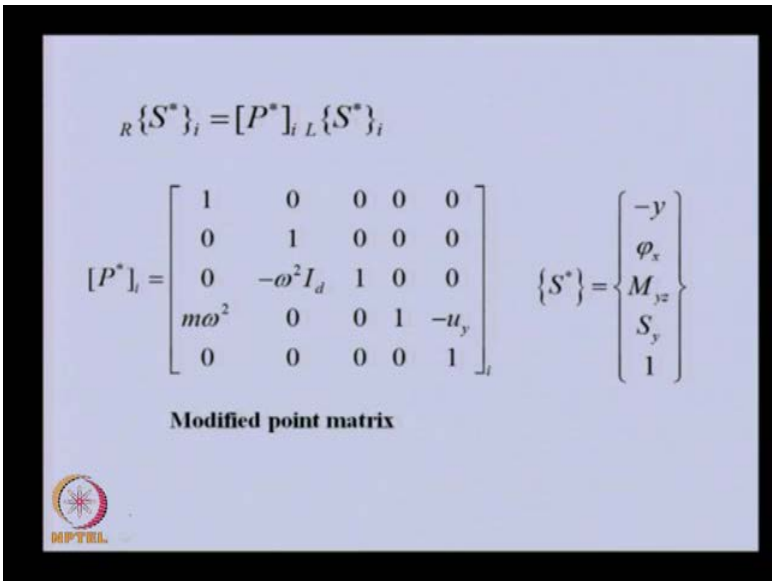

So, if we add identity, now you can able to see that we can able to include this external unbalance force inside the point matrix, which will be calling as modified point matrix. So, basically fourth equation if we substitute this equation point matrix here, state matrix here if we expand the fourth equation, we will get the same equation as the previous one. This one, but now the unbalance they are inside the matrix. Now, on the seam line as we did for the feel matrix, if we want to analyze both plane state motion. Also if you want to analyze damping or gyroscopic effect, we need to have the both plane state factor here, there real part and imaginary part components.

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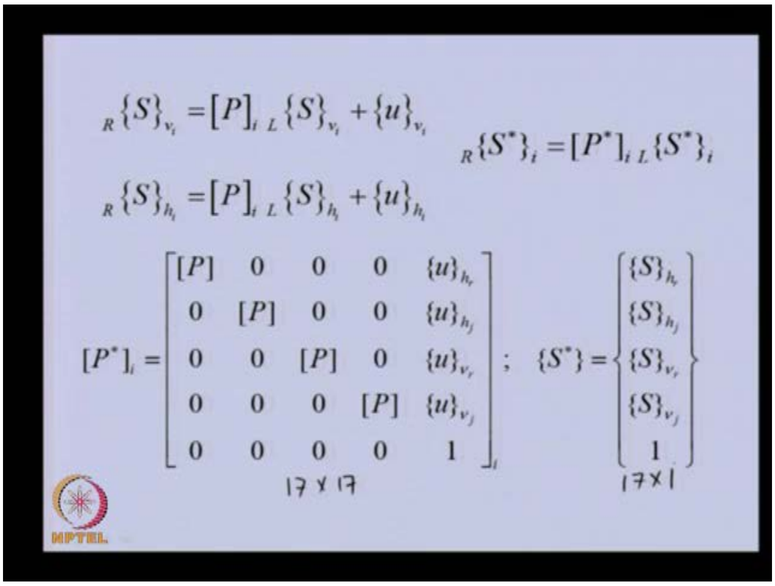

So, again I am expanding that thing here, so we can able to see point matrixes are here. So, here again the size of the modified point matrix is 17 into 17. Here, we have the component of the unbalance force in vertical direction or horizontal direction, real part and imaginary part, real part and imaginary part of the vertical direction. So, here additional force, external force factors are coming and so these are unbalance force. So, you can able to see that the state factor for such case is this is in horizontal plain. This two, this is in vertical plain the real part and the imaginary part. The size of the imaginary factor is again 17 into 1, this will be same for the field matrix. Also we have derived the field matrix and find the point matrix, basically for most of the case we are analyzing single plain motion.

So, point matrix and field matrix which will be considering d 4 by 4 size or there is unbalance, than is a 5 by 5 size, but that 17 into 17 size of the field matrix and point matrix, just illustrate it in case. We are considering the damping or gyroscopic couple, then we need to consider both plain motion and because there is a face component because of the velocity of t term we need to take care of real part and imaginary part of the state factor.

So, we need to expand this into 17 into 17, but most of the cases which will be illustrating here through example. We will see that the point matrix and field matrix will be 4 by 4 size unless otherwise unbalance force is there. In that case we will consider 5 by 5 matrix. Now, the procedure for obtaining the overall transfer matrix and the application of the bonding condition is similar to the torsional vibration case, but here because the size of the matrixes are bigger. We will see some more complexity in the frequency equation we were getting earlier.

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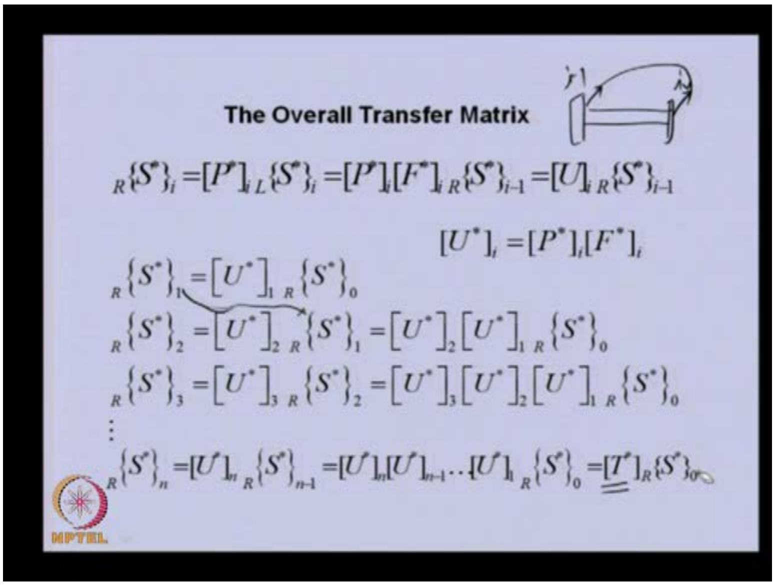

So, let us see how we can able to get the overall transfer matrix. So, earlier we derived the point matrix, so that was relating the state factor the either disc. The state factor in of the left of ith disc, also we related the state factor left off ith disc to the right of i minus 1 disc using field matrix. So, that if you substitute we can able to see that we can now able to relate the state factor right of i minus 1 disc, two state factor right of ith disc and this P into F call it as transfer matrix. And this is basically transferring the state factor from.

If we see from figure this is ith disc this is I minus n disc state factor here, which is i A minus on disc and this state factor here which is right of i nt minus 1 disc. And the state factor right of ith disc we are directly getting. So, information from here directly we are getting to the right of i nth disc. Now, this i disc can able to take 1, so if we are taking i as 1. So, we can able to see the state factor right of one, we can able to relate the u 1 and this is the state factor at the zero th station. Now, if we want i is equal to 2, the right of 2 is equal to u 2 and right of 1 state factor.

Now, we can able to substitute, the substitute this here and if we substitute this here. This u 2 into u 1 state factor at zero th station, single we can able to write for 3. The multiplication of u 3, u 2, u 1 and state factor at zero th station. We can able to extent this up to nth station. So, in that particular case we will be having multiplication of all this transfer matrixes upto 1 and multiplication of these all these are we are representing as time, which is overall transfer matrix. Now, once we have got transfer overall matrix, we can able to apply the bonding condition because we have nth station in the extreme right side of the rotor system and zero station at the extreme left side of the rotor system.

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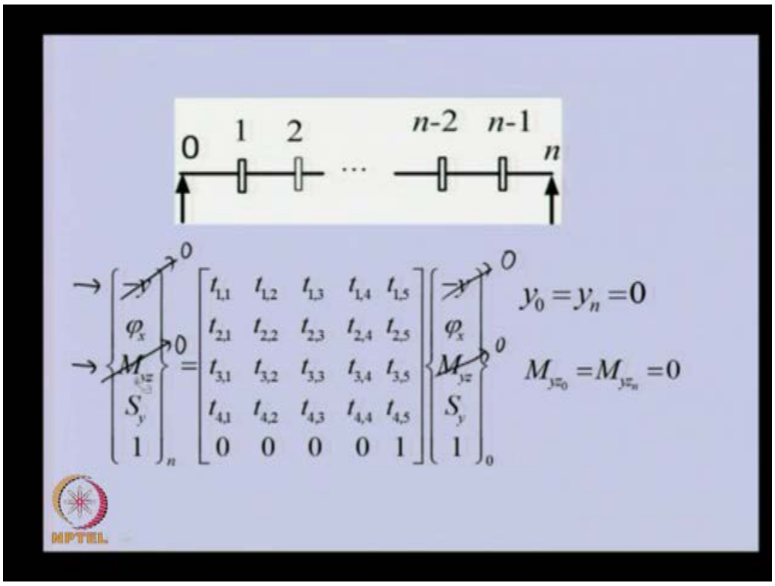

So, like this, so this is the nth station zero station. In the previous overall transverse matrix t relates the state factor here to the state factor here. Now, that particular last equation expanded here. So, this is state factor at nth station, these is the state factor of 0 station and because if there is unbalance in the system, the form of the overall transfer matrix will be less. So, last row will be 0 0 1, these are correspondence for unbalance force, other terms are multiplication of various point matrix and field matrix and basically all these will be the function of omega. The which is spin speed of the shaft for post vibration and for free vibration this will be 0 and omega. We can able to take as natural frequency of the system.

Now, bonding condition, for bonding condition here replacement at 0 1 0 station. Similarly, in nth station we have b displacement at 0 because it is simple support ends the movement. The binding movement will be 0, this two ends. So, these are the bonding condition of the system, so you can able to apply this. This is 0 here, this is u, then binding movement is 0 at both the end. Now, we can able to see that we can able to take out the first row and third row, because in the left hand side which is a vector we have 0 value in this vector. So, I am taking out the first row and third row and in second case I will be taking the reaming rows.

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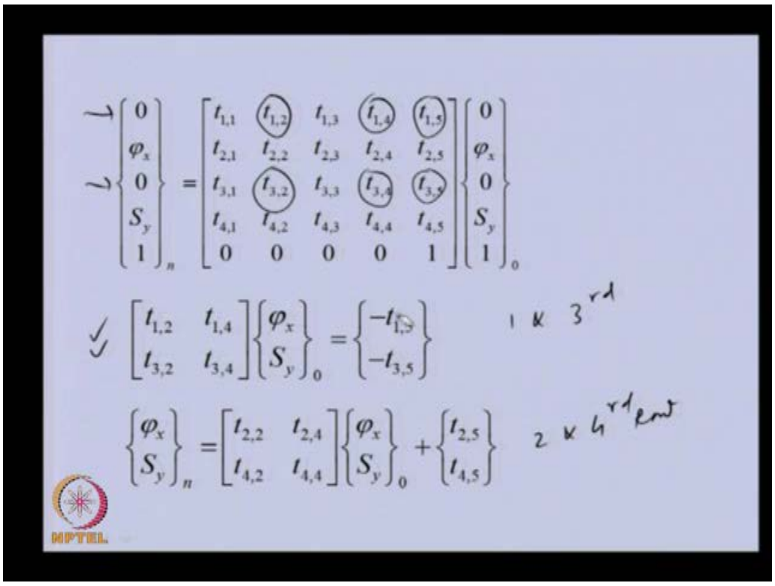

So, basically this is the application to the bonding condition and this is the first row and third row, first and third row if we take out from the above equation. So, you can able to see that first row is t 1 1 is multiplied by 0. So, this will not contribute, t 1 2 is multiplied by this angle. So, this is coming here, so similarly this will not contribute. So, we will be having will be having these four terms in this corresponding to the first row and third row. So, they are here and they will also come which are coming this side, this is the external force component.

So, basically we have taken out first and third row, recombine them. Similarly, second and third row if I take out, I will get this equation. This is from second and fourth row, so here also the unbalance terms are coming. Now, if we dealing with free vibration, so from this equation these are as I told they are coming from the unbalance component here. So, this terms will be 0, here also this terms will be 0, but if you see comparing this two equations, this equation is homogeneous equation. We are equating this 0 to free value vibration.

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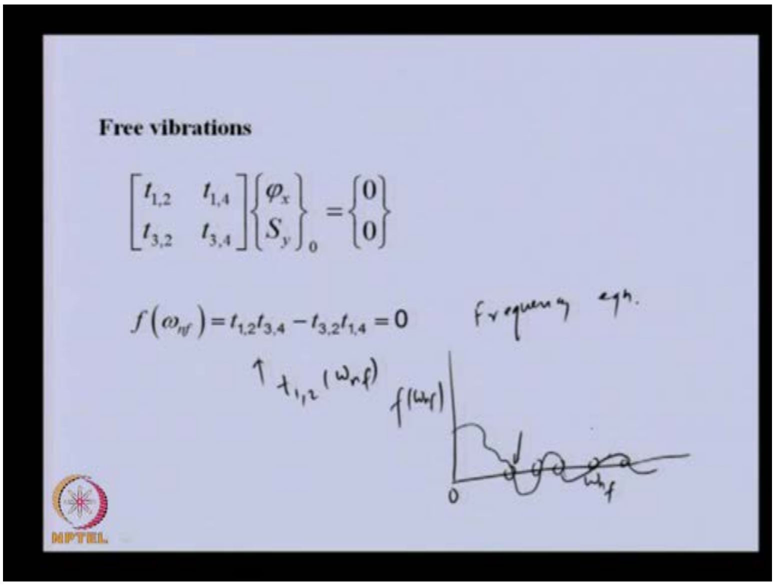

That is the equation we brought here homogeneous equation and for non-linear solution of this determine matrix of this determine is should 0. So, we will equate this equal to 0. So, basically this is the frequency equation, this t 1 2, t 1 4 all these are they are basically function of natural frequency terms on t terms are function of natural frequency terms. So, basically we expect here a polynomial for transverse vibration case. This generally polynomial will be higher degree because we are dealing with larger mass rotor system, obtaining this polynomial will not be easy.

So, generally solve this frequency equation, numerical analysis technique. We explain that in torsinal vibration case also, kind of rot searching technique. We used to obtain this particular frequency equation, basically if we are expressing this function of omega lf. So, if we are plotting this function with respect to omega n various value, let us reform 0 to some value. So, we expect this function will vary and it will intersect the line of 0 to various locations, and all this intersection will give us the rots which will be the natural frequency of the system. And depending up on the degree of the system that many intersect we expect from this function.

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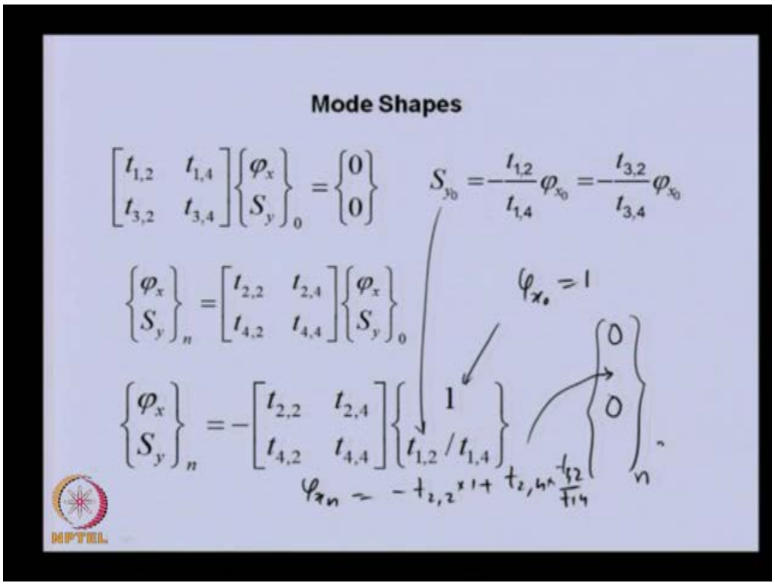

For mode shape, once we have obtain the natural frequency we can come back to this equation. And now if we expand this equation, let us say the first equation. Let us say first equation will give this term. Similarly, second equation will give this, so we can able to use any of them here. So, if we have, if we have so here we have related this factor zero station there is there is the sheer force with the angular displacement. This is the equation second set of equation which we obtain this one. So, here we are equating this as 0 on balance force and using this equation here now let us assume this angular displacement as unity.

So, we will be obtaining the mode shape with respect to this reference. So, I have substituted that has 1 here and this sheer force which is at 0 station. Then will be given either this or this, because this now one this we are substituting here. Now, can able to see that the state factor which is angular displacement at nth disc and the sheer factor nth disc is known. If we see the bonding condition at nth disc, we have linear displacement zero because the simply supported case and angular displacement we are taking we can able to get from here.

Now, if we x n is given by the first equation here, so pi that will be given as pi x n is equal to minus t 2 comma t into 1, this one plus t 2 coma 4 into t 1 2 by t 1 2 by t 1 4, this term. So, we can able to see that we know this factor here. So, this will be substitute in second place, then the third place is for binding movement, that is 0 for simplicity supported case because nth disc.

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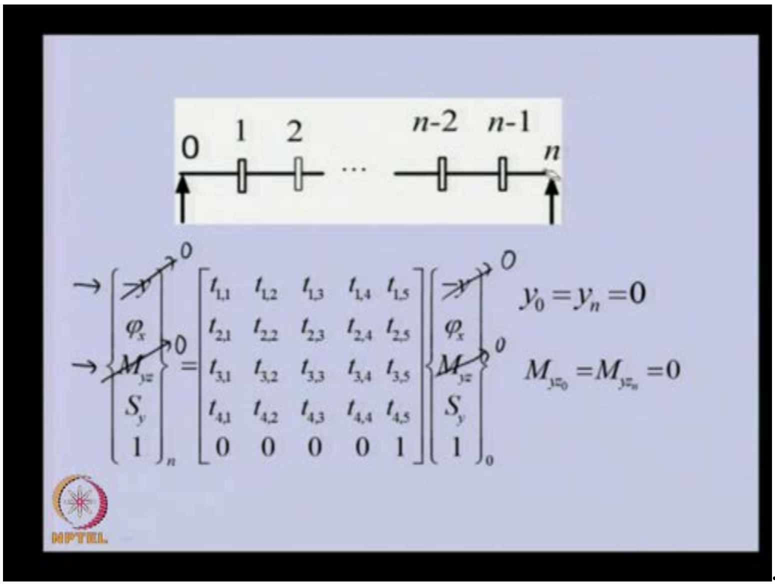

We have already seen is having these bonding conditions. Now, the sheer force, sheer force will be given by this. So, t 4 to 1 and t 4 4 into this term will give us the sheer force. So, we know the state factor at nth station completely and once we know the state factor at 1 station completely, we can able to use transfer matrix to transfer back the state factor from one station to another. So, we need to know only the state factor at one end location. So, these can be used in previous transfer matrix which we develop.

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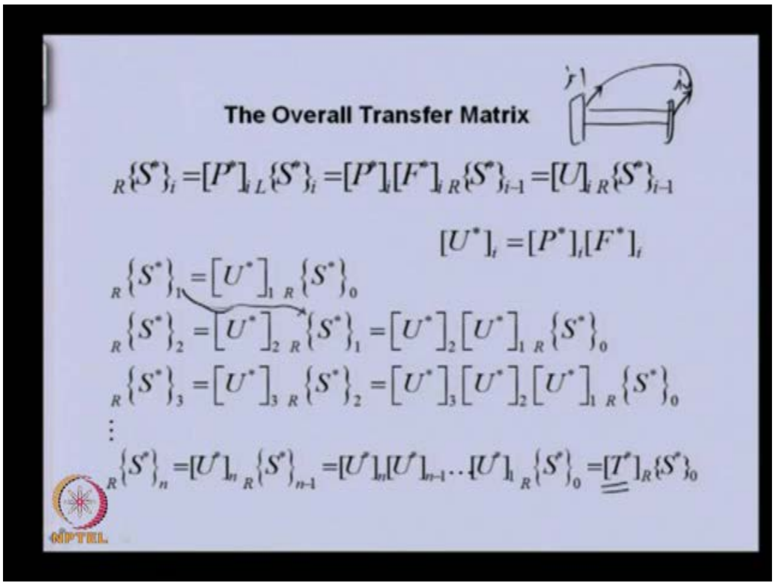

Here, to get back various state factor in between, now once we have got all the state factor, we need to pick up linear displacement separately, angular displacement separately and at various station whatever the displacements are there. If we want to plot the linear displacement, can pick up various linear displacement from various sate factor and we can plot them. Or if you want the angular displacement, we can pick them from these state factor. So, we have seen out to obtain the natural frequency and the mode shape, if we want to solve the force vibration.

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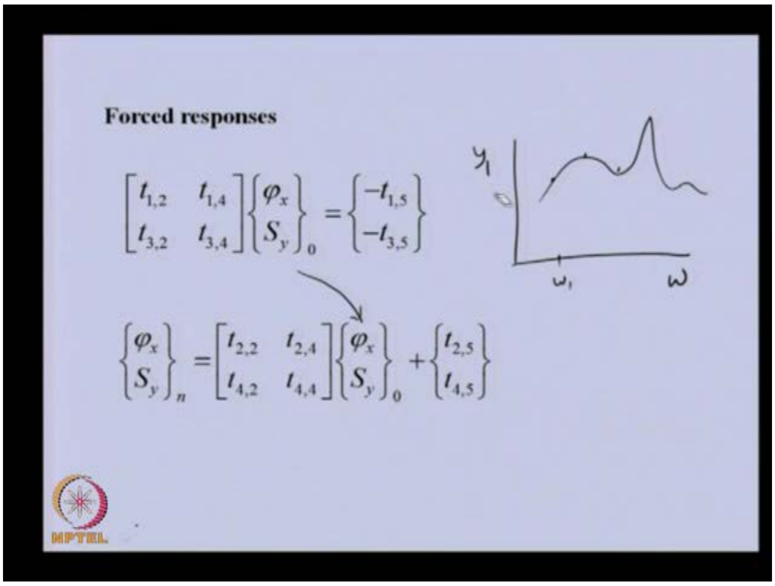

Again we have to come back to previous two equations which we obtain from first and third row. So, here now we are not keeping that unbalance force, so if we know this unbalance force, we can able obtain this factor because they will be known. Now, because pi omega will be speed and we can we can able to inverse this to get the state factor here at 0 station, and this then we can able to substitute here to get the state factor at nth station. To get the unbalance will known and all this term will be field and point matrix including this speed will be known. So, we can able to see that we can able to obtain these responses for various speech and we can able plot the response with respect to speed, how they vary.

So, this is how we can analyze the forced vibration in the system. So, basically if we are interested in one of the station that is transverse vibration linear vibration. So, this respect to speed, we need to solve this several time. Once speed at a time we will get one point, for another speed we get another like this one. We can able to get that is like this two numerical example will illustrate how we can able to get the these kind of response in substituent class now can conclude.

So, today we started with transverse vibration using transfer vibration method which we call myldestand method, also we develop the field matrix and point matrix by free body diagram of the shaft segment. We have seen that compared in torsional vibration case these are the size of point matrix and field matrix is more and especially it will be very large size of 17 into 17. When we want to consider both plain motion and other complexity like damping and gyroscopic couple through simple support bonding condition, I illustrated how the frequency equation can be obtained. And how we can able obtain the natural frequency from that.

Once we obtain the natural frequency how the mode shape can be obtained, that is nothing but the relative displacement at various station. How we can able to get then even force vibration using can be done once we know the unbalance of what is the unbalance in the system. Then we can able to analyze the force vibration, also in subsequent class in numerical example we will see the illustration of this method. Also We will be dealing with the, if there is intermediate support in the shaft how to deal with that substituent class. Then even we will consider the gyroscopic effect in that how the size of the matrixes becomes very large.