Theory and Practice of Rotor Dynamics
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Module - 6
Transverse Vibrations
Lecture - 24
Influence Coefficient Method

Till now we have considered torsional vibration in great detail. Now, we will be extending the method, which we developing the torsional vibration for the transverse vibration also. Because transverse vibration is very important in rotating machinery, which occurs frequently, because of the inherent fault, which is there in the system such as unbalance or misalignment etcetera. In the previous lectures, we considered some of the transverse vibration, but for simple cases like we considered single mass rotor supported on may be rigid bearing or flexible bearing.

Even we considered more complexity in the rotor in the form of gyroscopic effect, but all such analysis were very simple for single mass rotor system. Now, based on the experience of the torsional vibration in the previous lectures, we will now analyze the transverse vibration also for multi mass rotor system, so that we can able to apply these methods for a real system.

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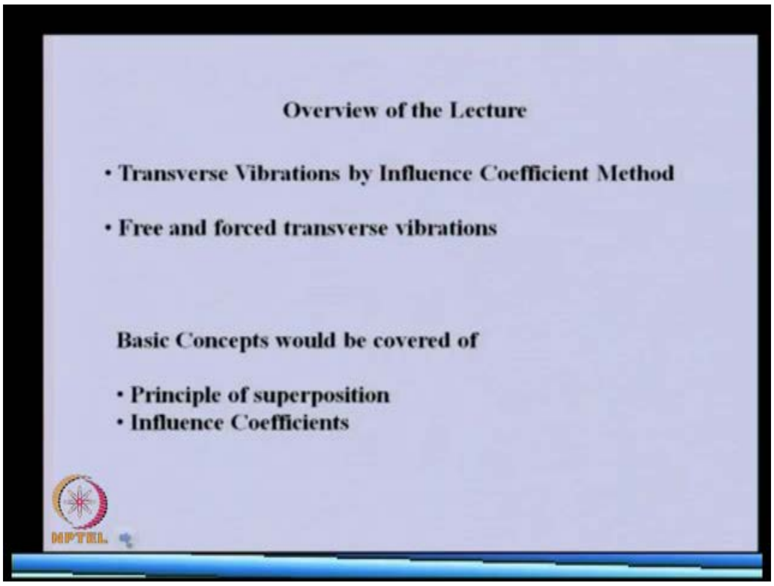

Now, let us see, what are the things we will be covering? So, apart from the methods, which we covered in the previous cases for torsional vibration one important method, the influence coefficient method which will be covering for transverse vibration and both free and force vibration can be analyzed using this particular method. Basic concepts, which we will be requiring for this is principle of superposition, basically will be dealing with the linear system.

So, this principle of superposition can be applied there, influence coefficients we will see the concept of this in more detail. So, that even for larger system we can able to calculate the influence coefficient. Now, let us see the influence coefficient method. How it can be applied to the rotating machinery? So, initially we will take very simple concepts of this influence coefficient for static case. Then we will expand that for the dynamic case.

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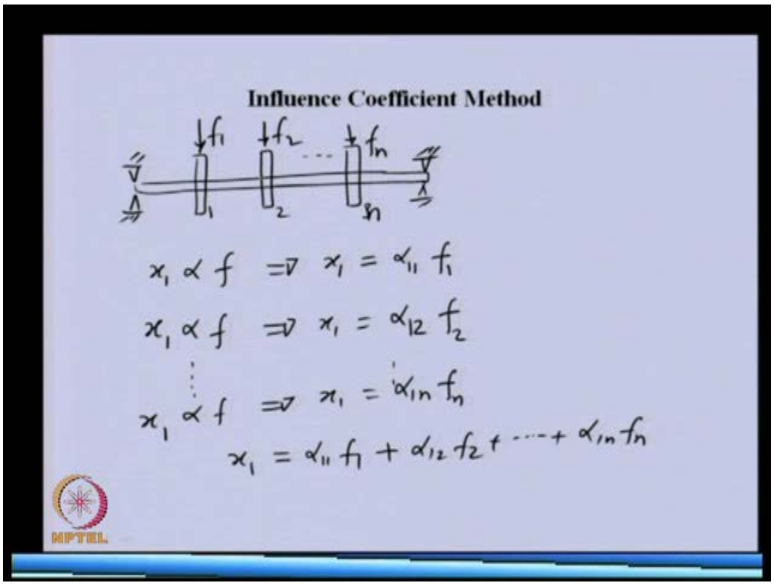

So, if we have a flexible shaft, let us say mass less shaft is there and in this number of disks are mounted, we can have n number of disks, one here only three disks. Let us say some support conditions are there or in this particular case. Let us say, this is simply supported, but any other boundary condition or end condition can be there. Now, if a static force if we are applying, let us say these are disk one, two, three, if we are applying a static force to disk one, because of this pose the displacement in disk one location, let us say that is x one will be proportional to the force applied.

So, if we increase the force in the same proportion, this particular displacement will increase. We are assuming that force we are applying gives the displacement, which is small and the force and displacement relations are linear and this proportional. If we remove it will get a proportionality constant, so x 1 is let us say equal to some constant alpha 1 1 into f. So, this particular force we applied at location one. There itself we are obtaining the displacement. Now, in second case if we remove this pose here from this, we can apply the force the same force in the second disk. Now, this force not only gives the displacement at location two, but also at disk one and disk three because the whole shaft will get deflected. We will be having displacement at one also, for this case again if you want to see the displacement at the same first disk location, because of this pose that will be proportional to the force.

If we increase the force the displacement, we expect will be linearly varying with that. This proportionality if we remove it, we will get another constant and that constant I am representing as alpha 1 2 into f, so this particular alpha 1 2 which is a constant is representing that the first subscript one is giving as the displacement location and second subscript is representing the force location. So, you can see we are applying the force at second and we are seeing the displacement due to that at disk one.

So, I have represented like this and if we have similarly, n number of disks, let us say n number of disks are there and this is the empty disk. If we are applying the force now at n th disk, so we expect the displacement due to this force which is at the end to this at disk one. Displacement will be again proportional to the force and we will be having a different proportionality constant alpha 1 n into f. Now, we can able to see that there is a another constant because the location of the force is different now. Displacement is different now. If we have a system in which, let us say we are applying f 1 force here, f 2 force here and f n force here.

So, we have n number of forces we have applied n forces in all of the disks. Now, if you want displacement at disk one, so that we can expect that there will be summation of this only thing we will be replacing this f in different forces. So, that is displacement at x will be summation of displacement due to force f 1 that will be given by this one. Then displacement due to f 2 at the same x location, so that will be f 1 to f 2 and so on up to n th force. So, we got force displacement at disk one due to various forces at various disk location from one to n. Now, the same concept we can able to extend for obtaining the displacement at disk two.

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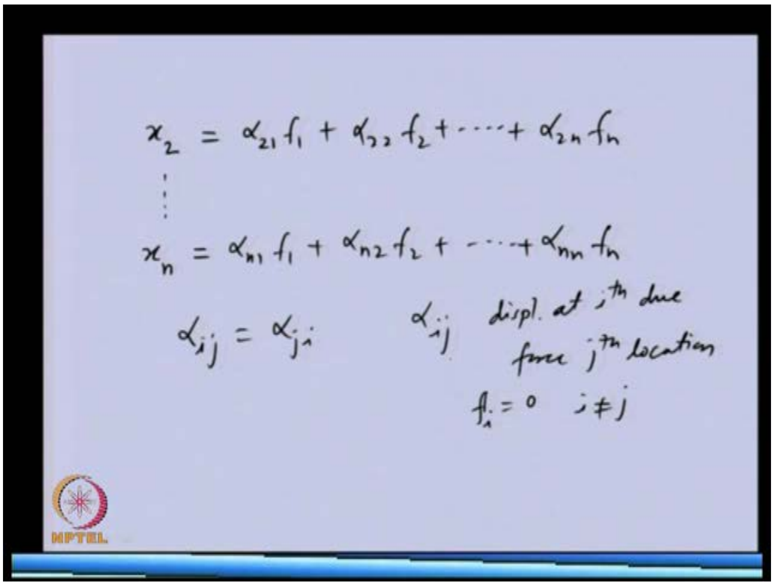

So, that we can able to write as if you want the displacement at disk two. So, this will be alpha 2 1 f 1 plus alpha 2 2 f 2 and so on. Alpha 2 n f n these constants are now different they are not same as the previous ones. Now, subscripts have changed, so if here you can able to see in these all constants. We have first subscript as 2, which is representing that we are interested in obtaining the displacement at disk location two and the second subscript is representing what are the forces at various disk locations like here 1 2. So, force 2 n so force n.

So, this displacement we can able to extend for all the disk, let us say for n th disk. We will be having this relation and so on f alpha n n into f n. You can able to see these are all different in general. We will see that in this particular case the influence coefficients will be having this property cross couple terms will be same. We can able to obtain this by usual deflection relations of the binding of beam, which we have studied in the strength of material. So, let us see the definition of the influence coefficient alpha i j, so this particular quantity is the displacement at i th location due to force at j th location and location.

When we are keeping forces at all other location that is let us say, i is equal this equal to 0. When i is not equal to 0, so only force j is acting. So, what is the displacement at i th location? It is given by this and all other forces, we need to equate it to 0, so this is the definition of the influence coefficient. Now, these equations which we have developed better, now because they are becoming larger in size we can able to put them in a matrix form like this.

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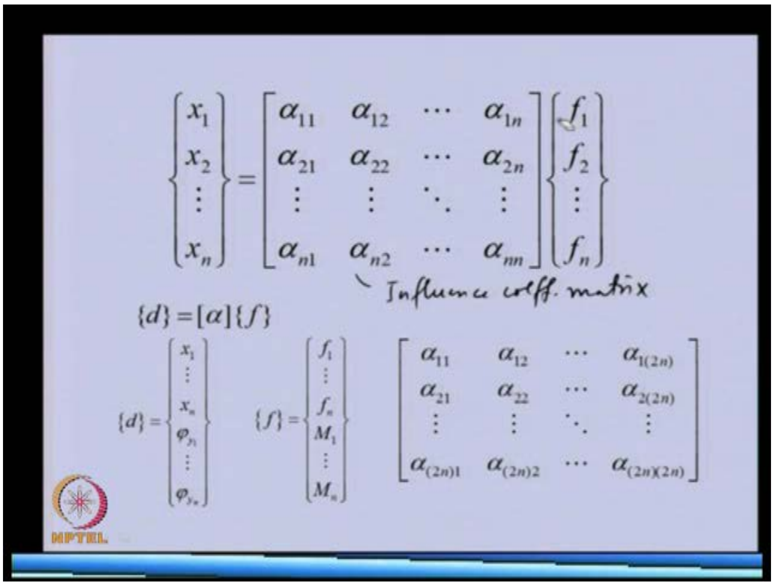

So, you can able to see that various the first equation which we derive is this one. If we expand this x 1 will x 1 will be equal to alpha 1 1 f 1 alpha 1 2 into f 2 etcetera. Alpha 1 n into f n, so this is the first equation, we derived second equation n n th equation. So, this we have kept in a matrix form and these are the various displacement vector. This is various force vector and this matrix which in a compact form we are writing alpha matrix it is the influence coefficient matrix influence coefficient matrix. This particular key in this particular keys, what we assume that the force is giving only the linear displacement, but in transverse vibration.

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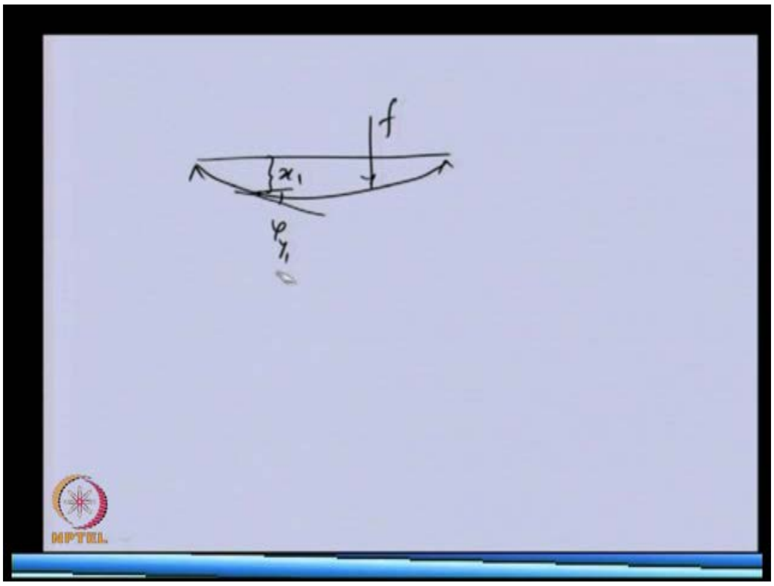

We know that if we are applying a force in a simply supported beam, if we applying some force not only it gives the linear displacement. Let us say at this location this is in the x 1, but also it gives the slope because original heat slope was 0. But now the slope has changed some angle. So, the force normally gives the linear displacement, but also the angular displacement simultaneously.

Similarly, if we apply only moment that will not only give the linear displacement, but also it will give the rotational displacement or angular displacement. This is a transverse displacement, this is a different as compared to the torsional one in this particular plane only. We are talking about on the slope of the elastic lane of the shaft, so basically when we are applying force both displacements and angular displacements are taking.

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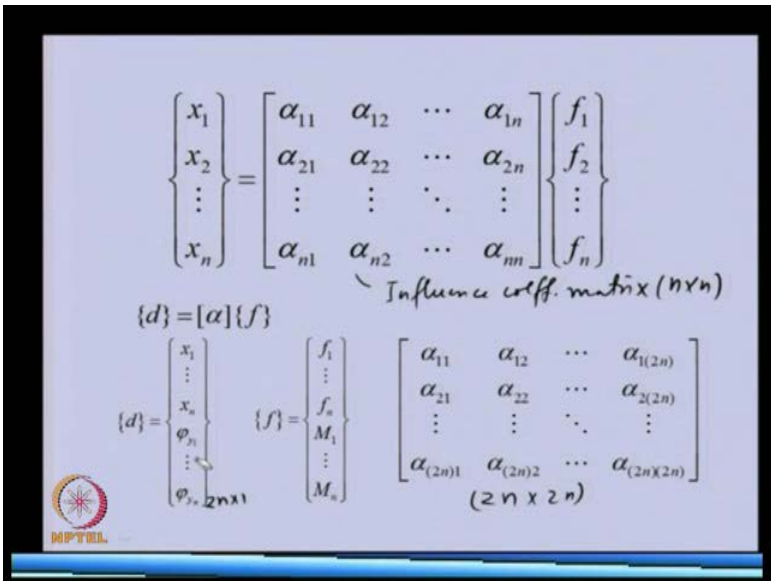

So, the previous analysis we can able to extend like this. So, instead of the displacement only the linear, we can add the angular displacements also in the force vector instead of only the forces. We can add the moment and now because the size of these vectors are 2 n into 1. So, influence coefficient matrix which was originally here, it was n into n. Now, it will become 2 n into 2 n. Now, additionally more influenced coefficients will come corresponding to a moment tan angular displacements. Now, you can able to see if we want to expand the first equation, the linear displacement is caused by not only the forces which was originally was there.

So, the first n first n this influence coefficients are related with the force and is this is a linear displacement is caused by the movement also. So, remaining n equation or n terms of the influence coefficients will multiplied by this movement to give the linear displacement. So, you can able to see now the linear displacement is caused by forces as well as movement. Similarly, if we take the angular displacement, it will cause by the not only the by the movement, but also by the force. Basically, they are coupled, so this is more general form of the this influence coefficient in which we considered both linear and angular displacement and movement and the forces.

Now, whatever the analysis we considered previously that was for static case the forces which we applied was static force. Now, we will expand this particular case for the dynamic case, so for the dynamic case the forces will be time dependent like unbalanced force. Now, we will see how we can able to apply this influence coefficient method for the time dependent forces.

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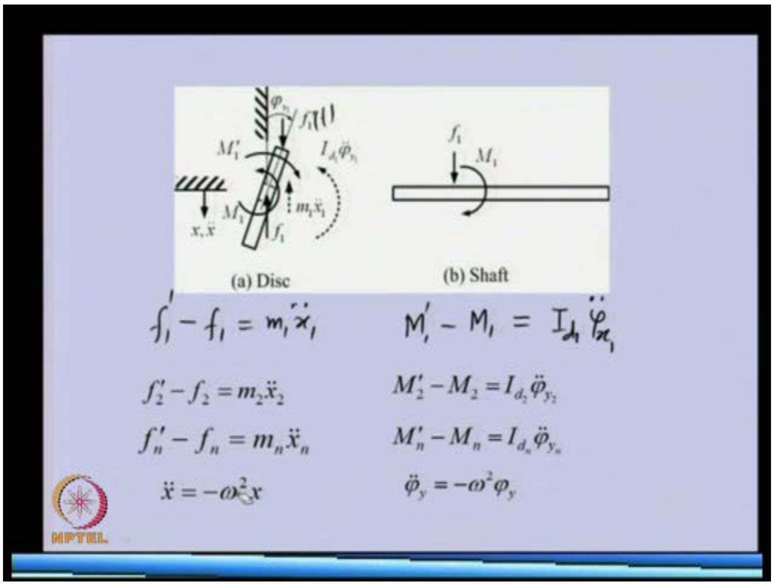

So, this is basically a free body diagram of a disk when this several forces or movements are acting, but let me first introduce the alpha 1 sorry f 1 prime f 1 prime. This is the external force, which is acting on the disk this is the external force, this is time dependent. So, basically now all the forces are time dependent. Similarly, we have movement which is external movement time dependent. This may come if disk is not perfectly perpendicular to the shaft, if it is slightly tilted.

So, this kind of external movement may come on to the shaft on to the disk, because of the tilt of the disk. So, f M 1 prime and f 1 prime on the external forces. Now, because this is the free body diagram of the disk was here. We have removed the disk from the shaft and we have drawn the free body diagram. So, obviously that of the disk will be having reaction force from the shaft f 1, which is will which will be there at the force at the on the shaft also and also the movement will also reacting.

So, basically this is the reaction movement and this is the f M 1 prime external torque. So, this is the external torque external force and reaction force and reaction movement from the shaft. So, this is the reaction movements, which is coming onto the shaft from the disk. Now, let us take the positive direction of the displacement in downward direction. So, we will be having inertia of the disk in the upward direction, which will be opposite to the motion direction. Similarly, we will be having a rotary inertia term, which will be opposite to the tilting of the disk. So, tilting is in this particular case is clockwise, so this inertia will be acting in the quanto clockwise direction.

So, I think I have defined all the forces these are the external forces and these are the internal reaction movements. Now, if we write the force balance, so this is the external torque which is acting in the positive direction of x reaction external torque. Then reaction torque which is opposite to the motion direction motion direction is downward reaction torque is force is in the upward direction. These two forces are acting on this disk should be equal to M 1 into acceleration linear acceleration. So, this is the force balance of this disk. Now, let us take the movement balance this is the external movement reaction movement for shaft, these are two movement acting should be equal to the rotary inertia.

So, this is the movement balance or this is the force balance of this free body diagram of the disk. Now, we can able to obtain free body diagram of the disk two three and all other disks. So, you can able to see corresponding to the second disk, we have external force reaction force should be equal to inertia. Similarly, external movement reaction movement should be equal to inertia of the second disk. So, this is i d 1 here i d 2 and we can able to extend this for n th disk like this, where m n is the mass of the n th disk. Then i d n will be diameteral mass movement of inertia of the n th disk. These are all diameteral mass movement of inertia and this is the angular displacement hmm and time derivative. So, angular acceleration of the n th disk.

Now, for harmonic motion because either due to the unbalance or if we are analyzing the free vibration the harmonic motion can be represented like this, where omega is the frequency for forced vibration. It will be the like for unbalance, it will be the spin of the shaft or if we are analyzing the free vibration. So, omega will be the natural frequency of the system, so this equations are valid this we can able to substitute in this terms. Basically, now these equations we can able to put in a matrix form like this.

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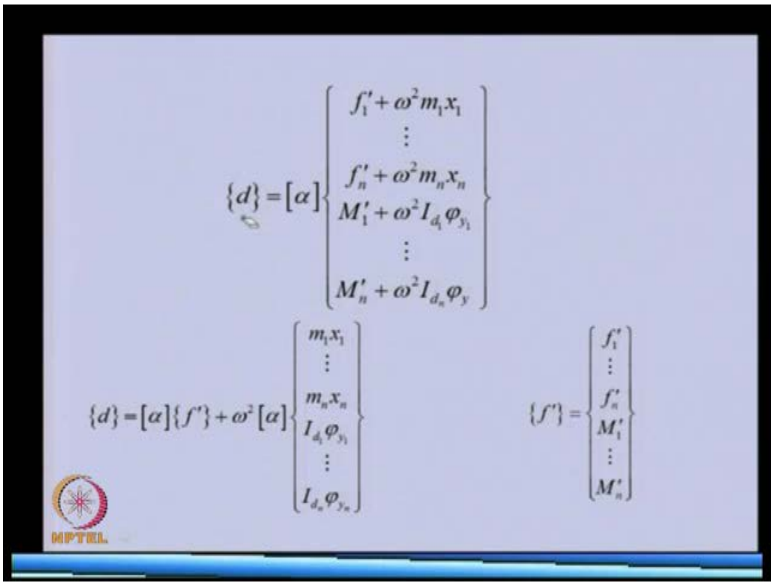

So, you can able to see we have this is the displacement x 1 x 2 all angular displacements. These are the influence coefficient which we obtained earlier this was the reaction force and movement from the, on to the shaft. So, this equation we derived earlier for static case like this.

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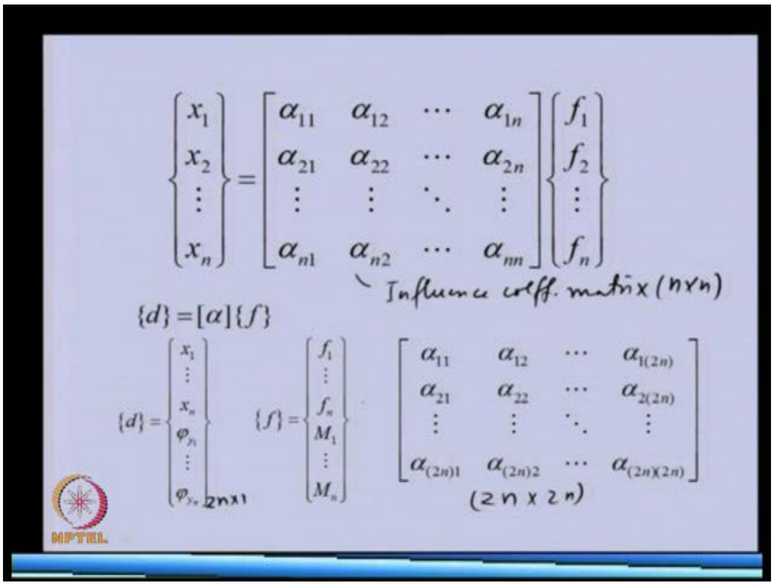

So, d is equal to alpha f f is the reaction force onto the disk onto the shaft. So, from these equations, we can able to solve for f 1 and f 1 f 2 f n M 1 M 2 M n. We can able to substitute in that vector.

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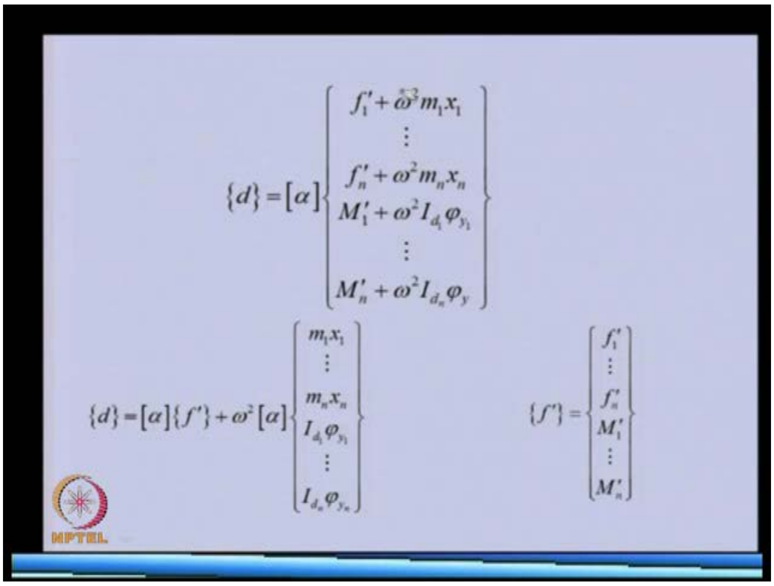

We have done this here even we have considered the harmonic motion. So, omega square terms are also coming, so you can able to see now. Earlier it was static force, now because of dynamic force we have these terms got changed. Now, this equation we can able to split in two parts this particular vector one is only the force and movement. So, that is this 1 plus omega square is common and this vector, so M 1 x 1 all these vector are basically…

I have split this in two parts and then I expanded. So, because alpha matrix was common, so it will be multiplied to both the terms. Now, here alpha prime is this force is a movement which is external torque and external force and movement which is coming from the for example, unbalance. Now, the second term this term, so what I am doing? I am multiplying these masses inside the influence coefficient and only keeping the displacements here.

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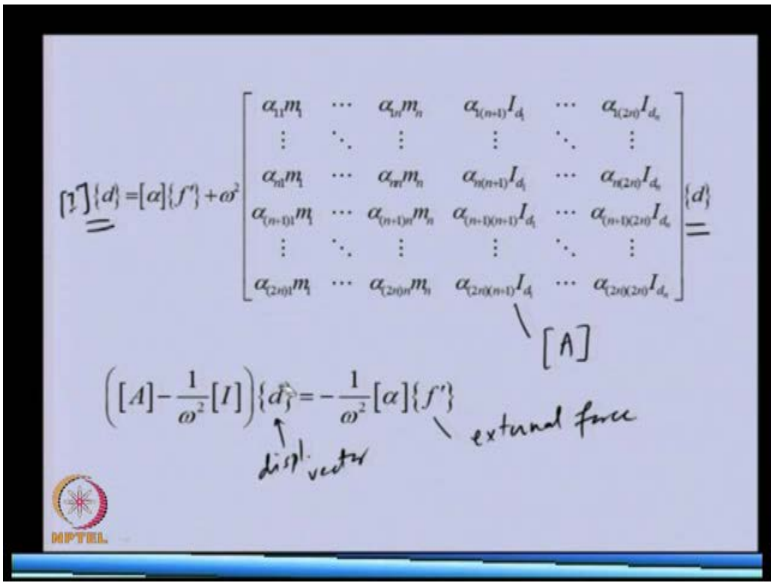

So that particular term is so first two terms are first term is same as previous one only thing here only the displacement. I am keeping here at mass terms, I have included in the influence coefficient matrix. So, you can able to see those mass terms and diametral mass movement of inertia terms are at the matrix. This you can able to see now here we have d displacement vector here displacement vector. They can be combined, so what I am doing? I am I am putting this particular d vector in the other side. So, this can be multiplied by additive matrix, so this is the that one so you can able to see a is this matrix d is common m I have multiplied by…

I have divided throughout the expression by 1 by omega square. So, this 1 by omega square term is coming here also here. This is in the other side, so minus 1 by omega square alpha into f. This is the external force term external force and movement term. This is the displacement vector and alpha is the influence coefficient matrix and a is this matrix, which contain not only the influence coefficient terms. But also the masses now this particular equation is the required form of the equation. You can able to see, if you want the force vibration to be solved? If we know the, what is the force, external force in the system, we can able to get the displacement by inverting this matrix.

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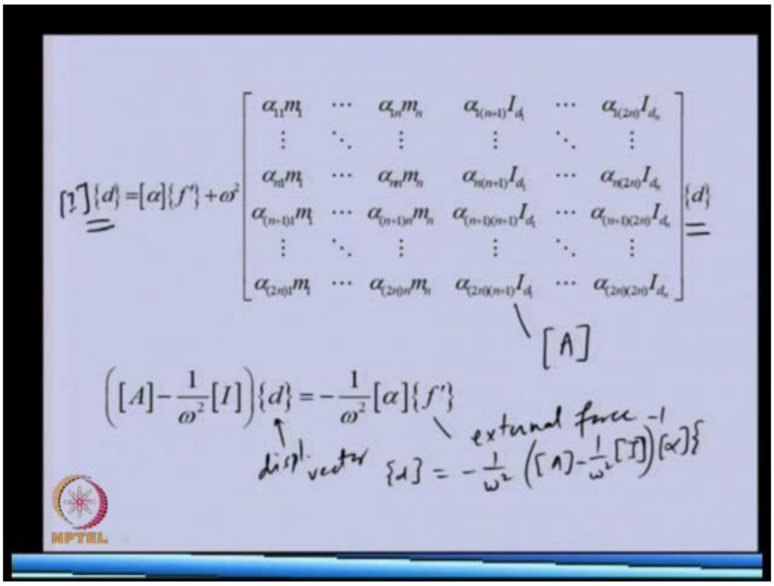

So, we can able to see that d we can able to get displacement as r into f prime where r is the inverse of this. Basically, I have inverted this matrix to get the d, so here d is 1 minus omega square inverse of this whole term omega square will be the spin split for this case inverse of this into alpha into f prime. So, this particular equation is this one which is I have written in more compact form and this a matrix is given by this. So, this form various angular speed, if we know the force values we can able to get the displacement. This is the force vibration case, now if you want to analyze the free vibration, so obviously we need to keep the external force equal to 0.

So, this whole term will be 0, so we will get this equal to 0 that is this one, so this is for the free vibration. So, if we want the natural frequency of the system or the critical speed of the system. We can able to use this expression where a is given by this expression i is the identity matrix, this is the natural frequency. Now, omega will be natural frequency for free vibration and because this is a homogeneous equation. We need have determinant of this equal to 0 that will give us the characteristic equation or as we have done earlier, the Eigen value of the a matrix will give us the natural frequency and the mod shapes will be given by the Eigen vector of a.

Now, we have seen the basic concept of the influence coefficient method with this method not only we can able to analyze the free vibration also the forced vibration. Now, through some simple example we will see how this method can be applied to obtain the natural frequency of the rotor system or critical speed of the rotor system so for this particular case .

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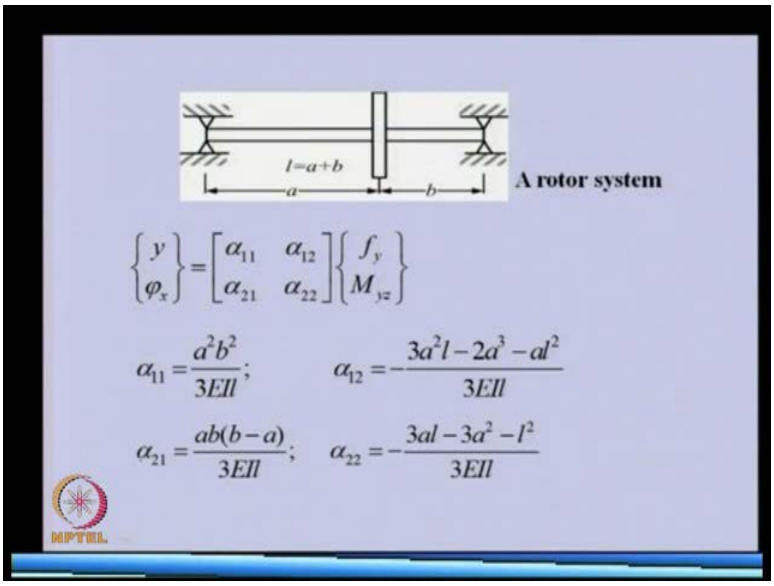
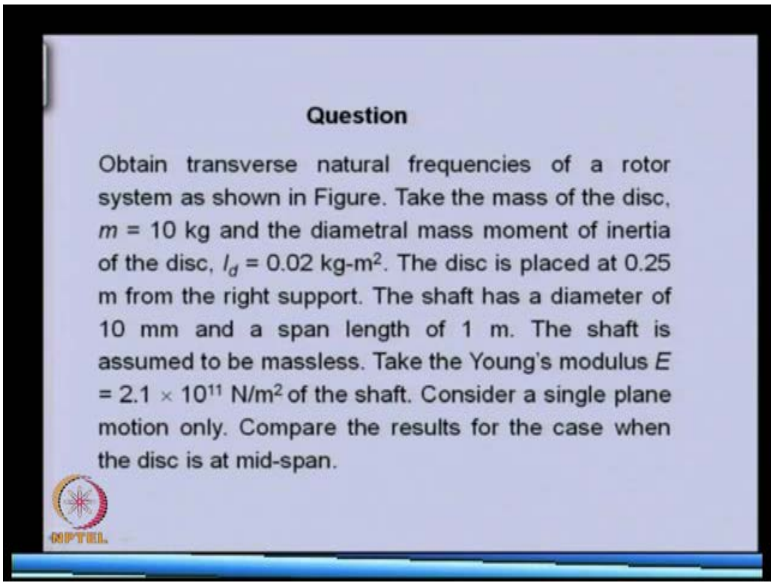


Very simple rotor like this we have we have considered this is simply supported rotor system mass of the shaft. We are not considering only the inertia of the shaft is there and we are considering both mass and diameteral mass movement of inertia of the disk. (Refer Slide Time: 28:25)



Now, various property of this are given here mass is 10 k g i d is 0.02 k g meta square disk location is also given that from right support it is 0.25 meter and total length of the shaft is 1 meter diameter of the shaft is 10 millimeter Young's modulus is given. So, and this particular case we are considering only the single plane motion because as such this influence coefficient. They do not couple the a vertical motion and the horizontal motion, so single plane motion can be considered.

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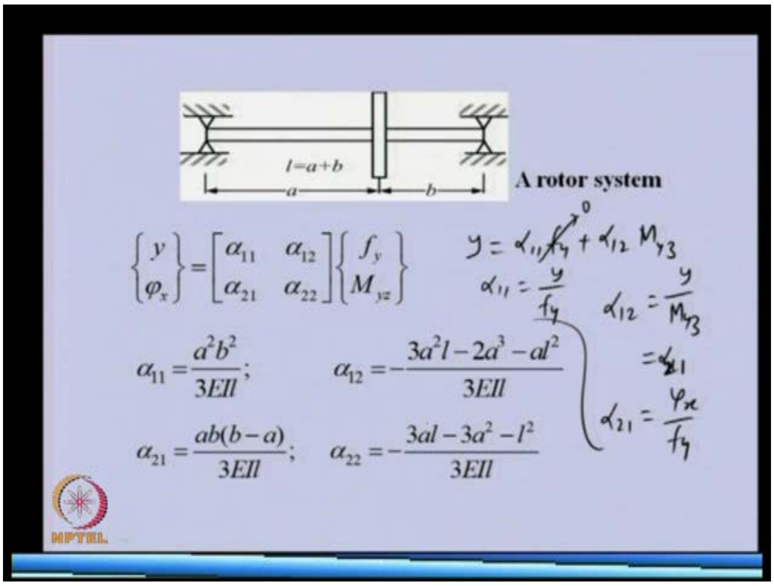

Now, for this particular case the displacement linear and angular are related with the force and movement with such influence coefficients this influence coefficient, we can able to get from any strength of material book. So, you can able to see that definition of the influence coefficient 1 1 is nothing but we can able to expand this. So, alpha 1 1 is nothing but displacement linear displacement of the disk for a force f y keeping the movement equal to 0. So, basically alpha 1 1 is nothing but linear displacement divided by the force at the same location.

Similarly, if we want to obtain the alpha 1 2, so that will be the linear displacement due to only movement when it is present in n th system. So, we need to keep the force 0 in the second case. This is 0 only movement is applied, so whatever the linear displacement take place at the same location will be given by that displacement divided by the movement, will give us the influence coefficient 1 alpha 1 2. I already mentioned alpha 2 1 will be same as this one and definition of alpha 2 1 is different. So, that is nothing but the angular displacement due to force at the disk location when are keeping the movement 0.

So, alpha 2 1 is angular displacement divided to the force, so definition is different, but the values will be same for both the case alpha 1 2 and alpha 2 1. From strength of book material book, we can able to get such influence coefficient relations or we can able to derive also using this basic concept. Just we need to obtain the linear displacement and angular displacement corresponding to other force or movement. That will give us the influence coefficient, so for this particular case in which we have a total with offset disk where a and b are given like this. Alpha 1, alpha 1 2, alpha 2 1 all are expressed in terms of the location of the force geometry of the shaft and property of the shaft. So, once we know on this property we can able to calculate the influence coefficients.

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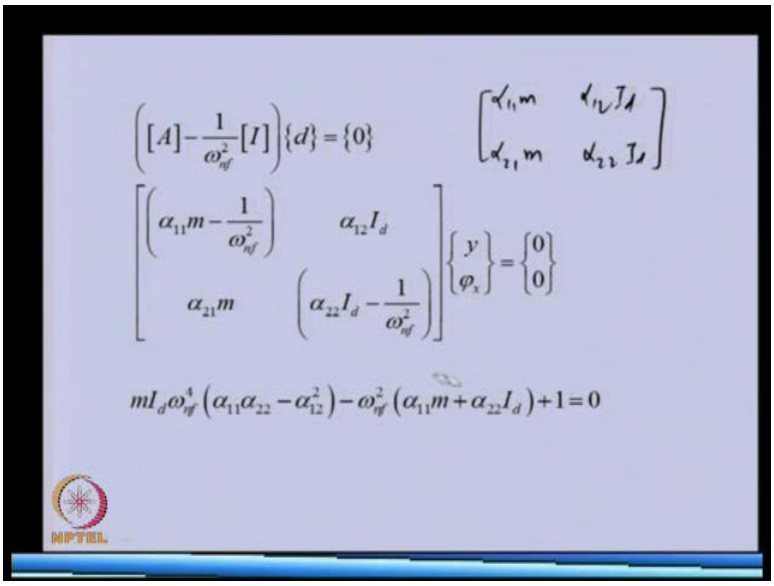

For free vibration we had the, this kind of relation earlier which we derived and for because now we have only 2 2 terms in the a matrix. So, this A matrix will be having the form of alpha 1 1 m. This we had earlier alpha 1 2 i d alpha 2 1 m alpha 2 2 i d. So, if you substitute this here, then club this two we will get this matrix. Now, because this is homogenous equation determinant of this will give us the characteristic equation, you can able to see, this is a quadratic m, this will frequency square. So, we expect two roots out of this.

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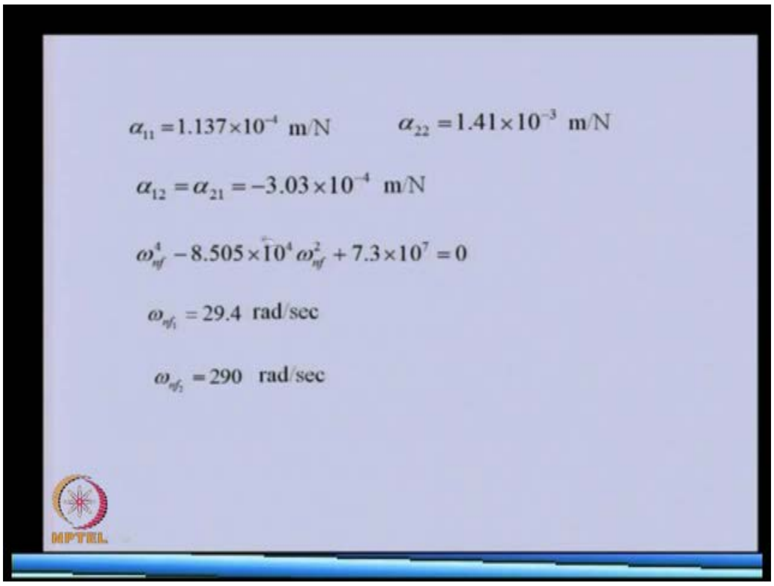

For various values of the data given in the problem, these are the influence coefficient values. So, if you substitute that in the previous polynomial, we will get the polynomial of this form and we can able to get the roots the solution of this 2. So, this is the first natural frequency and this is the second natural frequency of the system. So, we can able to see that once we know the influence coefficient is quite straight forward to obtain the natural frequency of the system. Now, the same problem if we have disk at the center, because in the previous case, we had the disk at the at slightly offset position disk at the center. What will happen the tilting of the disk will not take place?

If we are applying the force at the middle because slope is 0, so that means this coupling terms will be 0. Because if we are applying force only linear displacement will take place and if we are applying movement only angular displacement will take place at the middle. So, this coupling terms will be 0 and influence coefficients for these values will be alpha 1 1 and alpha 2 2 will be these. Now, you can able to see that the natural frequency will be given by this because now there is no coupling terms. So, this equation will get simplified these terms will be 0. So, this equal to 0 and this equal to 0 will be giving us two natural frequency.

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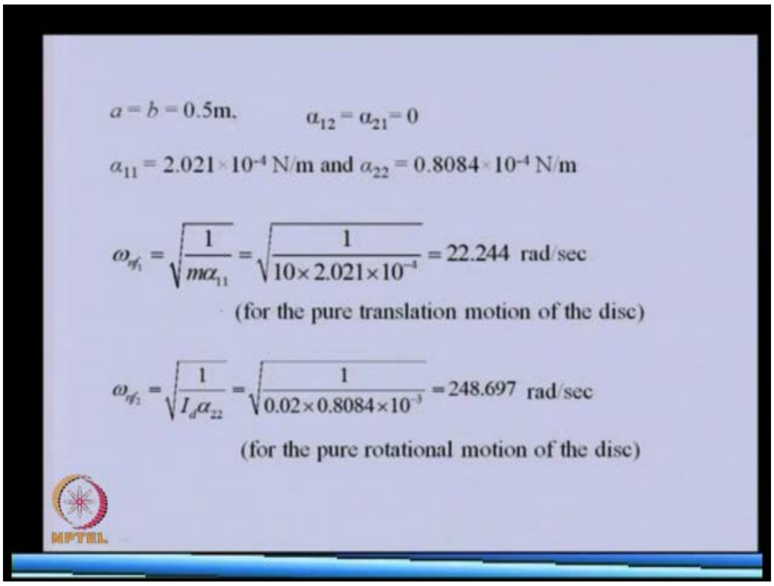

These are the two natural frequency. Now, for when disk is at the middle we are getting 22.244 radius per second as one natural frequency and 248.697 is second natural frequency. These are different as compared to the previous one in which coupling was there, so 0. So, even though if we are changing the disk location we expect change in the natural frequency, this particular example which we considered was very simple, but it was good in clearing the concept of the influence coefficient.

How we can able to get the characteristic equation to get the natural frequency was quite clear. Now, we will take up a more complex example in which now instead of our 2 1 disk. We have two disk in the shaft system and this is a cantilever rotor system. In this particular case for simplicity, we are considering only the force and the linear displacement. So, we are ignoring the movement and that is a angular displacement. So, let us see how we can able to analyze this system?

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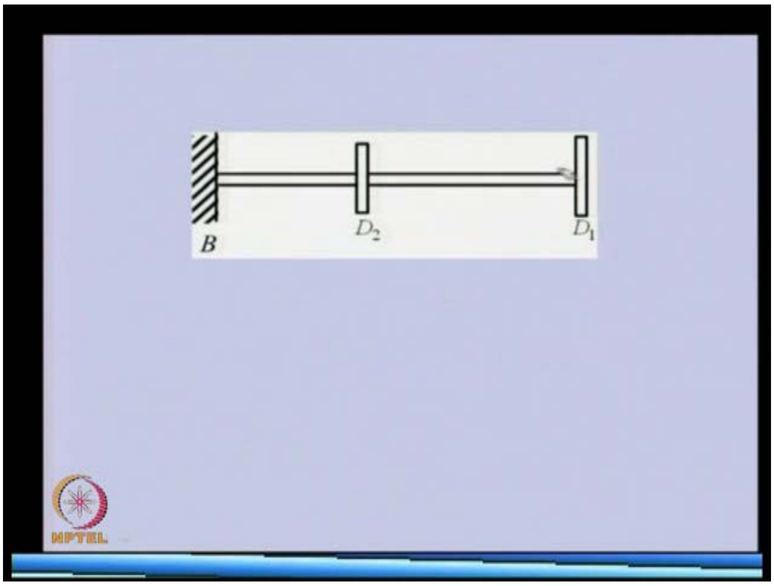

So, we have a rotor system like this in which we have two disks, mass of the shaft we are not considering. We are considering that the disk is having mainly the up and down tilting motion linear motion. Tilting motion we are ignoring it.

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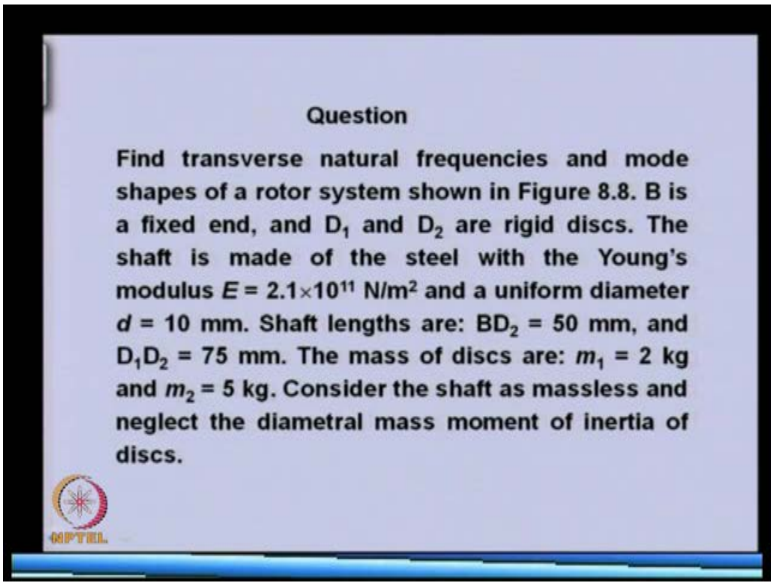

Various property of the shaft and disk locations I have given here. Like diameter of the shaft is given, disk various geometries are given, mass of the disk first is 2 k g and second is 5 k g and diametral mass of movement of inertia of the disk we are ignoring it.

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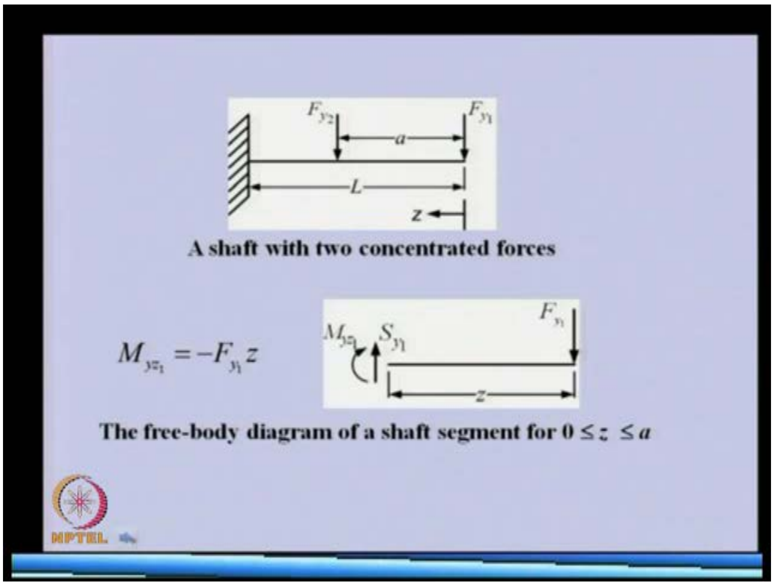

So, for this particular case we are first deriving the influence coefficient just for illustration purpose. So, basically we have two forces f y 1 here, f y 2 here and this is the distance between forces. This is the l, now we can able to obtain the z axises from here. So, we are measuring from this end. So, by bending movement concept, we can able to obtain the influence coefficient. So, first we are obtaining the strain energy, so for that we are obtaining the bending movement. So, for first segment from 0 to A, this is the free body diagram. This is the bending movement in shear force, so if we take the balance this is the bending movement for the first shaft segment.

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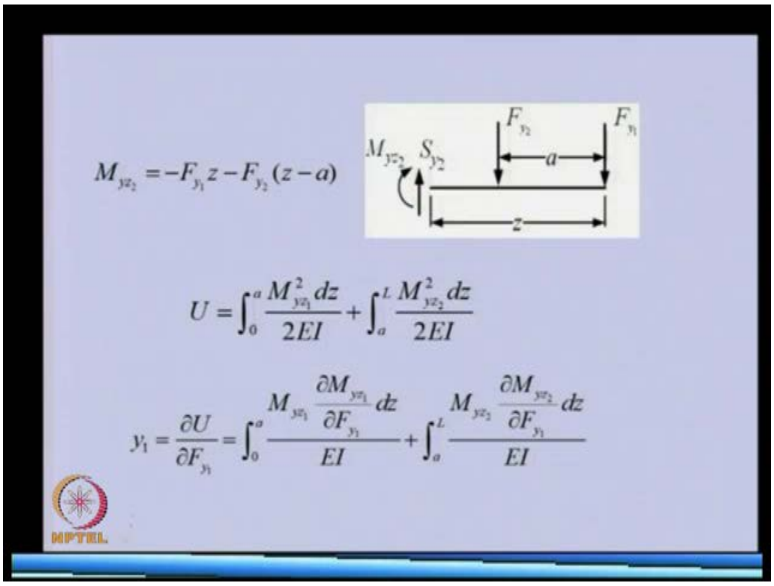

Similarly, we can able to get for the second. So, in this particular case z is varying from 0 to l and from sorry from A to z is varying from A to l. So, for this case the bending movement will be given by this expression. This is standard procedure in the strength material and now we can able to get the strain energy of the system by integrating from 0 to A and from A to l. Once we have strain energy, we can able to get the displacement let us say if we want the displacement at the force one location, we need to differentiate with respect to that force using caustic linear theorem. We will get the displacement at one location, so we can put the expression of the this bending movement and we can able to get the displacement at location one due to force one.

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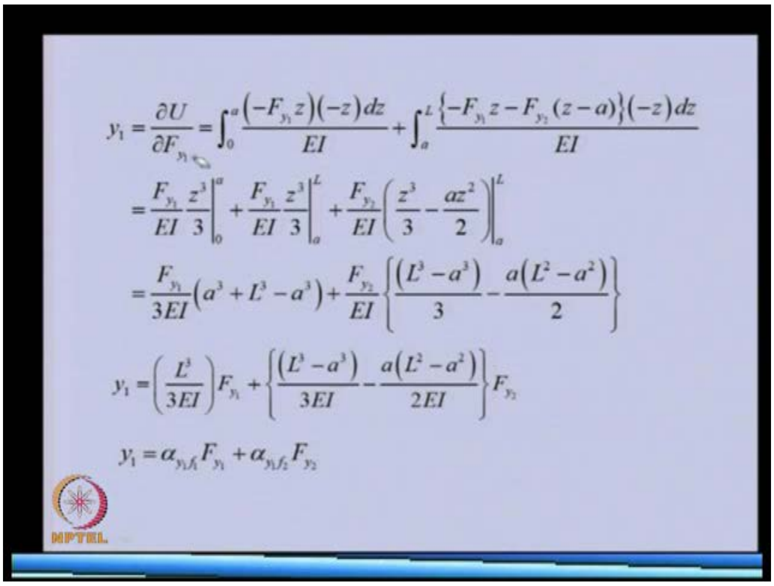

So, if we simplify this is a simple strength of material steps. So, finally we will get displacement, we can see because there are two forces. The strain energy is due to both the forces. So, the displacement at one location will be not only due to force one, due to force two also. This expression we can to write it as y 1 is equal to alpha y 1 f 1 that is we can say alpha 1 1, this is alpha 1 2, so this is the alpha 1 1 and alpha 1 2 for this present problem.

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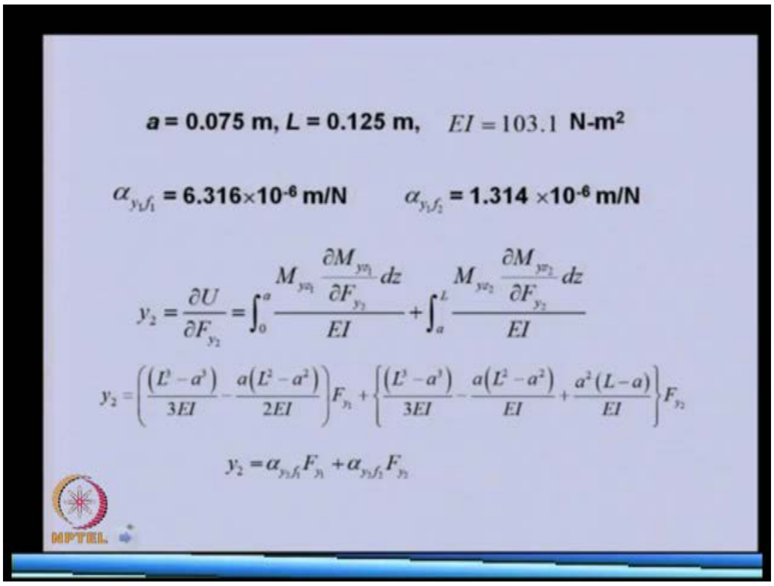

Similarly, so if we put the values of various property of the shaft, we can able to get the this influence coefficient alpha 1 1 and alpha 1 2 from the previous expression. Similarly, if we want the displacement at y 2, because of the force 2, we can able to once we differentiate this we can able to get the expression for y 2. The term within this bracket will be alpha 2 1 and this will be alpha 2 2.

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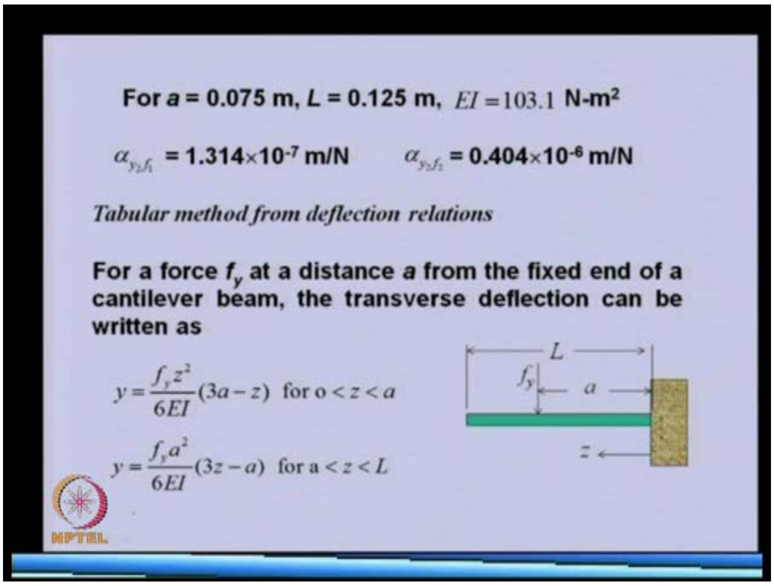

If you substitute the values again, we can get the these influence coefficients also, more systematic way of obtaining this influence coefficient by tabular method. Now, I am explaining, because this can be extended for multi rotor system also. So, for we have formula for such cantilever system in which let us say z is measured from here, a is the position of the force total length of the shaft is l. So, the displacement at force displacement at any location can be ah given by this expression. This particular expression is valid for 0 to a.

So, between this range, this particular displacement relation is valid. Second expression is valid from a to l, because there is no force here, so we expect a linear variation of the displacement here. So, you can able to see this is a linear variation of the displacement at that location. Now, these two equation, which is generally available in the table in the strength of material book can be used to obtain influence coefficient, which we obtained earlier.

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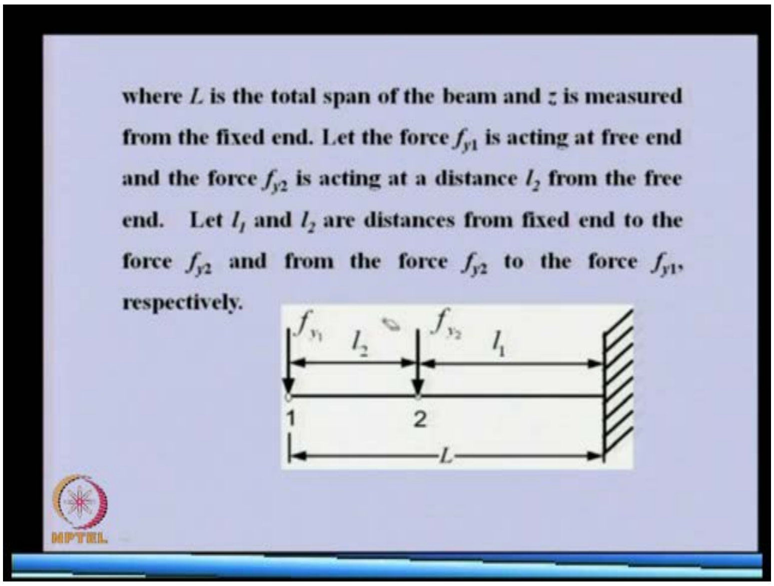

So let us see, how it can be done? So, two forces are acting, the first force distance from the fixed end is l 1. This is between two forces, the distance is let us say l 2 total length is l.

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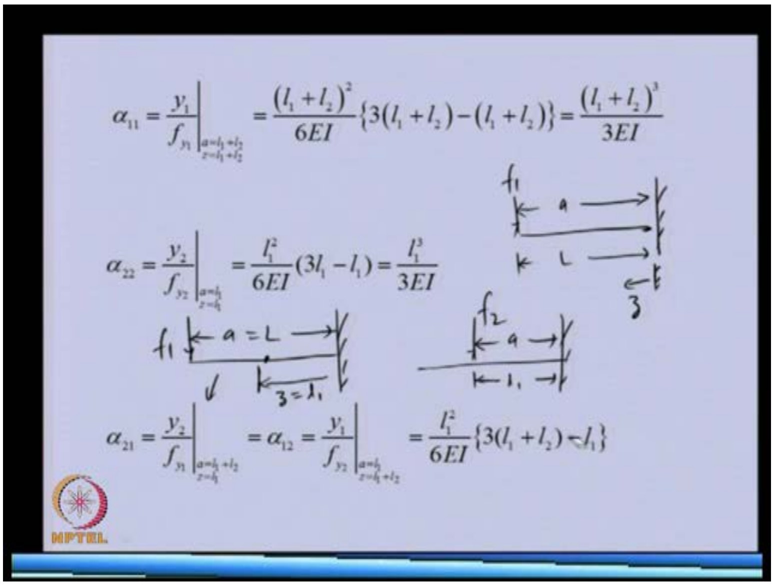

Now, we if you want alpha 1 1, for alpha 1 1 that means we have force acting here. This is the z location measurement of the z. Now, you can able to see that if we go back here a is the this distance from fixed end to the force. So, for this case the distance a is this one and which is also equal to total length total length l 1 plus l 2 and we want the displacement at the same position where the force is acting. So, z is defining the position where we want a displacement a is the position of the force.

So, both are same here l total l, so if we in the previous expression of y here, if we put z is equal to l a is equal to l we will get the displacement at free end. So, this is the expression for that and that will give us if you divide by f f 1, that particular displacement we will get the influence coefficient f alpha 1 1. So, similarly if we want the displacement alpha 2 2 in this particular case, we will be having alpha f 2 is here. So, you can able to see that basically a distance was this which is l 1 and z is also at l 1, because we want the displacement at the same position, where the force is there.

So, same expression which was earlier there, this will be used. Now, in place of z, we need to keep l 1 and at in place of a we need to keep l 2; that means the l 1 position. This is the force location, so at both a and z we have l 1 l 1. So, in the previous expression, if we will get the alpha 2 2 expression, now this is the third one in which we have force at 1, but displacement at 2. So, in this particular case, we have force at 1 f 1, but displacement we want at the 2. So, that means the force location is a, which is equal to total l, but z position is this one which is equal to l 1. So, you can able to see a is equal to l 1 plus l 2 total length, but displacement we want at the second position displacement at second position.

So, z should be equal to l 1, so if we substitute this in the previous expression we will get this expression. So, basically we are using this equation all the time in this particular case. So, this particular case we will get the expression this particular influence coefficient. Similarly, this can be obtained, if you are applying the force at 2, I am measuring the displacement at y a will be l 1 and z will be total length of the shaft. That will give the same expression of this. Now, you can able to see this from basic formula of this, we have obtained the influence coefficient for of the two mass system, but it can be extended for n mass system also.

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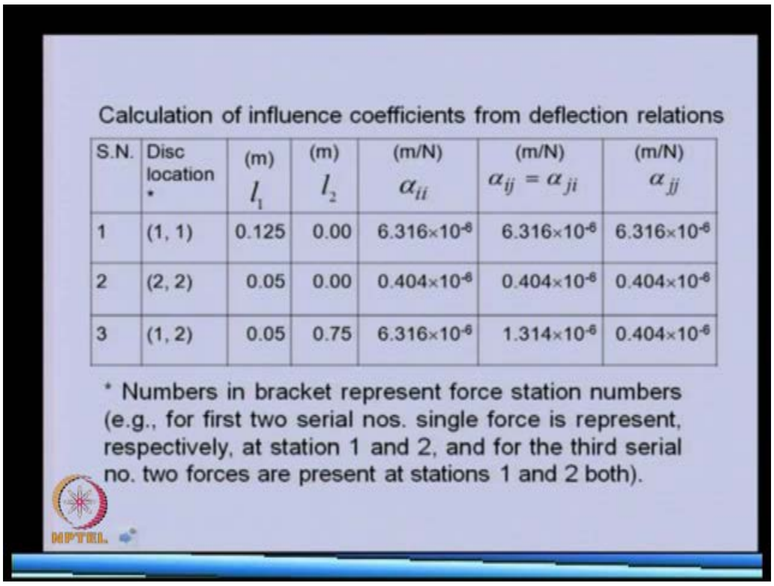

Calculation of influence coefficients from deflection relations

S.N.	Disc location *	(m) l_1	(m) l_2	(m/N) α_{ii}	(m/N) $\alpha_{ij} = \alpha_{ji}$	(m/N) α_{jj}
1	(1, 1)	0.125	0.00	6.316×10^{-6}	6.316×10^{-6}	6.316×10^{-6}
2	(2, 2)	0.05	0.00	0.404×10^{-6}	0.404×10^{-6}	0.404×10^{-6}
3	(1, 2)	0.05	0.75	6.316×10^{-6}	1.314×10^{-6}	0.404×10^{-6}

* Numbers in bracket represent force station numbers (e.g., for first two serial nos. single force is represent, respectively, at station 1 and 2, and for the third serial no. two forces are present at stations 1 and 2 both).

This can be tabulated also, so whatever you have done for alpha 1 1, we have the position of the force second force is not there. This is alpha 1 1, we can able to calculate. So, this is nothing but this more systematic way what we have explained. I have summarized here, so this is for alpha 2 2. This is for alpha 1 2, so this influence coefficients can be obtained using this.

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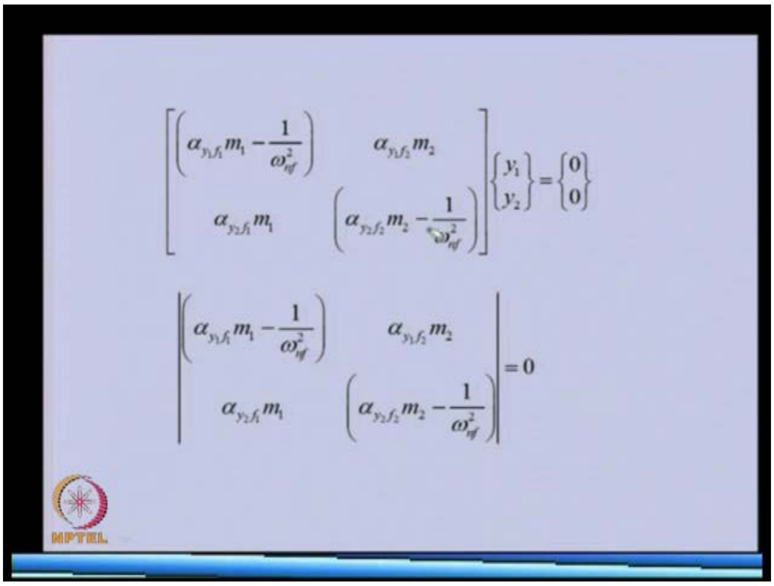

So, now once we have this influence coefficient. We can able to formulate the free the Eigen value problem in this. We have not considered the i d, so this will take this form and because this is a homogenous equation, the determinant of this will give us the there is a characteristic equation the frequency equation of this form.

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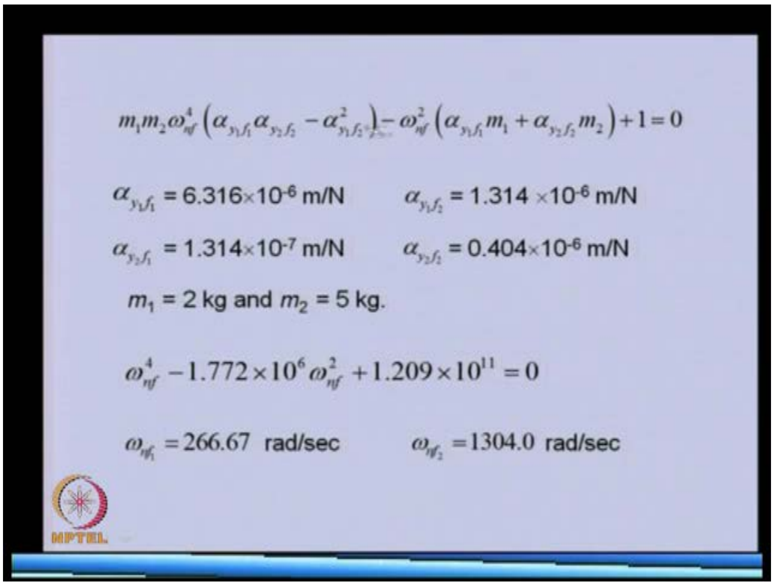

These we can able to substitute, so we will get this polynomial. From there we can able to get the two natural frequency of the system. Now, you can able to see there was two rotor system, only linear displacement we considered. So, we got two natural frequency here. If we could have considered the tilting motion also, then we expect four natural frequencies out of this system.

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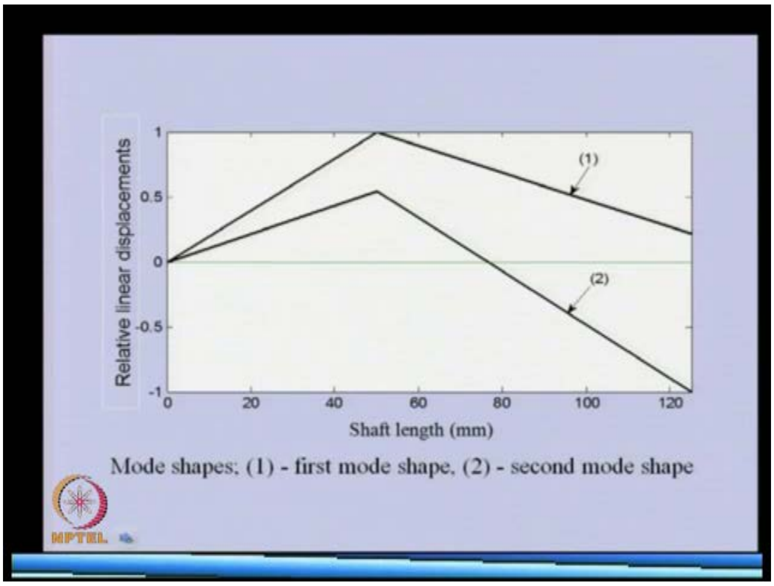

This is the relative displacement for the first mode and second mode. So, this is the, this is the free end, so this is the displacement of the two disk first disk second disk. Similarly, these two disk you can able to see the first mode both the disk are having synchronous motion, but in this they have anti synchronous motion, because this is in the negative side. So, today we have seen the influence coefficient method. This method, we can able to apply for multi rotor system through simple example of one rotor, one disk and two disk keys.

We have illustrated the method, but this method can be extended for more number of disk. Even the calculation of the influence coefficient can be done using tabular method in which by just choosing the variable where the force is acting and where the displacement to be measured. We can able to calculate the influence coefficients quite easily and in this particular case, the example we have seen that even a single disk rotor can have two natural frequency, because the in transverse vibration we have linear motion.

The angular motion in the second example, we consider only the linear motion; that is why for two mass rotor system, we got two natural frequencies. But if we, if we incorporate the tilting of the disks also, then we will see that we expect the whatever the homogenous equation we wrote, will be having 4 by 4 size and we expect four natural frequency or four critical speed out of this. In this particular case, we have not considered the gyroscopic movement, but that can also be included only change will be in the movement equation. Because gyroscopic movement generally affects the movement equation, so that we are not considering in this in this method, but in the subsequent methods like transfer matrix method and finite element method, which are more powerful will be considering the gyroscopic effects also.