Theory and Practice of Rotor Dynamics
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Module - 5
Torsional Vibrations
Lecture - 10
Finite Element Analysis-III

Today, we will extend the torsional vibration using finite element method, today especially we will be dealing with the geared system. In previous lectures, we have covered the geared pair using the equivalent system approach or even with the transfer matrix method. So, today will expand the gear element will develop the gear element using finite element method. We will see that how this gear element can be used for either geared system or for even branch system. In this particular torsional vibration of the geared system will extend even if there is a flexibility at the gear tooth how it can be incorporated in the finite element formulation.

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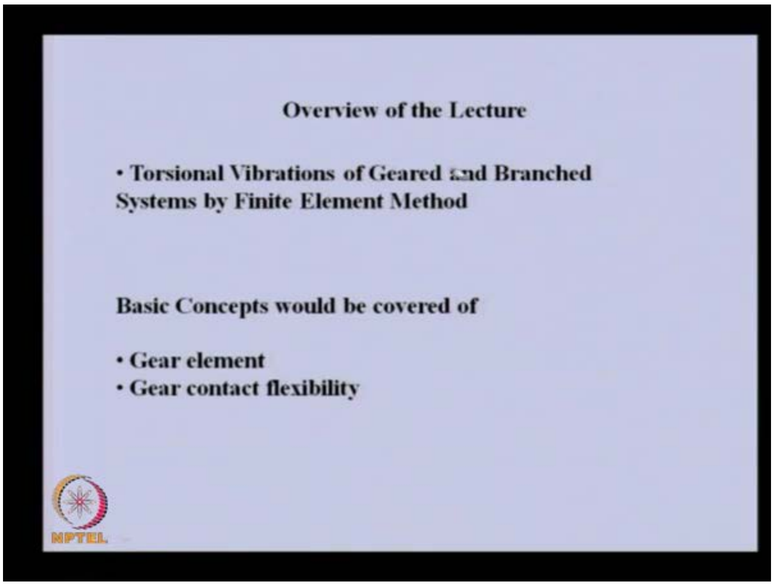

So, basically this is the overall overview of the lecture in which in which we will be developing, will be doing the torsional vibration of geared and branched system using finite element method. We will do the development of the gear element and will see if gear contact is having flexibility or that can how we can able to incorporate in the geared element.

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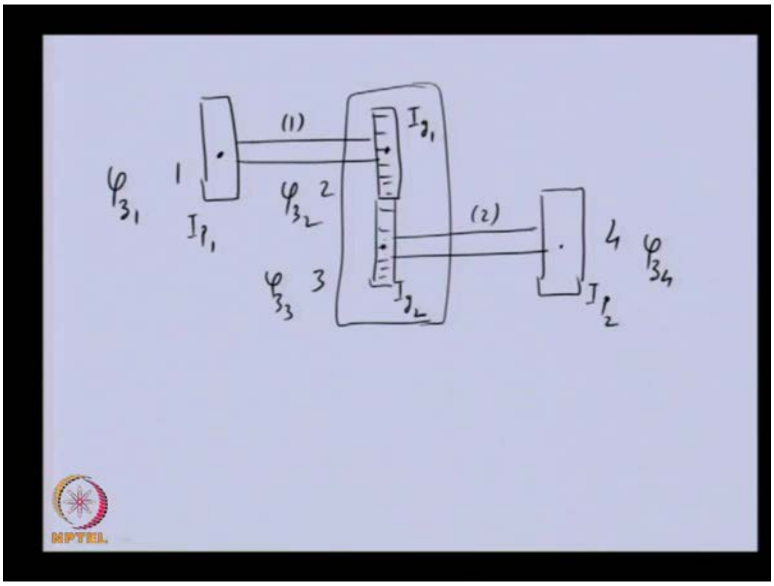

So, let us take one a gear system this is a fly wheel and this is a shaft and we have one gear and it is connected to another gear and we have another fly wheel. So, these fly wheel are having large polar movement of inertia these gears can have polar movement of inertia, but they will be comparatively smaller as compared to the fly wheel. So, we will try to develop a gear element, so that for this particular gear, so we will be considering this particular gear pair and because the angular displacement of the second gear is related with the first gear with the gear ratio.

So, when will be discretizing this particular system, let us say if we are discretizing the upper shaft with the gear as one element one and the second shaft with gear and fly wheel as second element. That various positions like this is a node one node two node three node four so we will see that at each node we need to assign one angular displacements and there are angular displacements, but out of this four, these two are related by gear ratio. So, will try to eliminate or replace the angular displacement at node 3 by the angular displacement at node 2 and that will give us the elemental equation. So, let us see how we can be able to develop this element equation for the gear pair in the subsequent slide.

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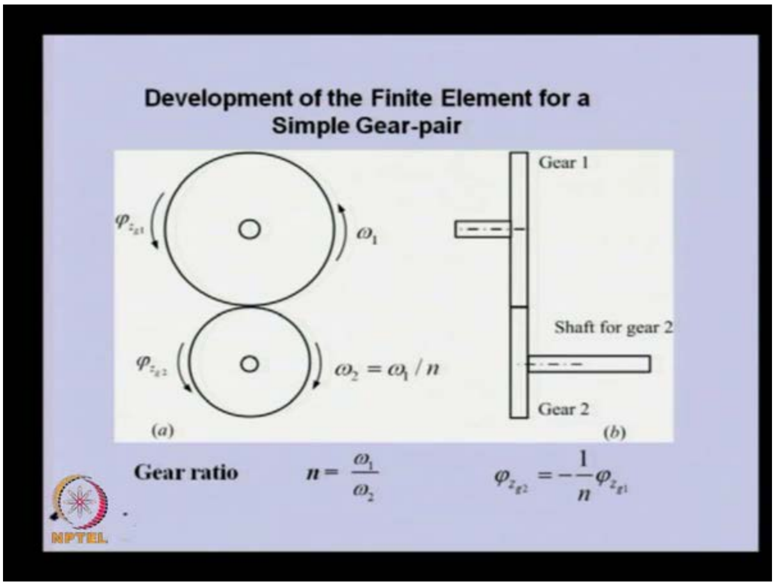

So, we a have gear pair and they have rotational speed omega 1 and this will be having opposite to that, but this angular speed is related with previous one with the gear ratio. I am taking the torsional oscillation of this these are the nominal speed, these are the torsional oscillation or angular displacement. I am taking both as counter clockwise direction as positive convention and the sine negative sine will take care of the their direction.

Now, this has to rotate opposite to this, but according to this sine convention then they will be related with the negative sign. So, the gear 1 and gear 2, they have angular displacement these two and these two are related by the gear ratio. Now, we will see the movement of inertia polar movement of inertia fast movement of inertia of this gear corresponding to the angular displacement or the angular speed of the first gear. We already had seen that this can be obtained using equivalent system like this.

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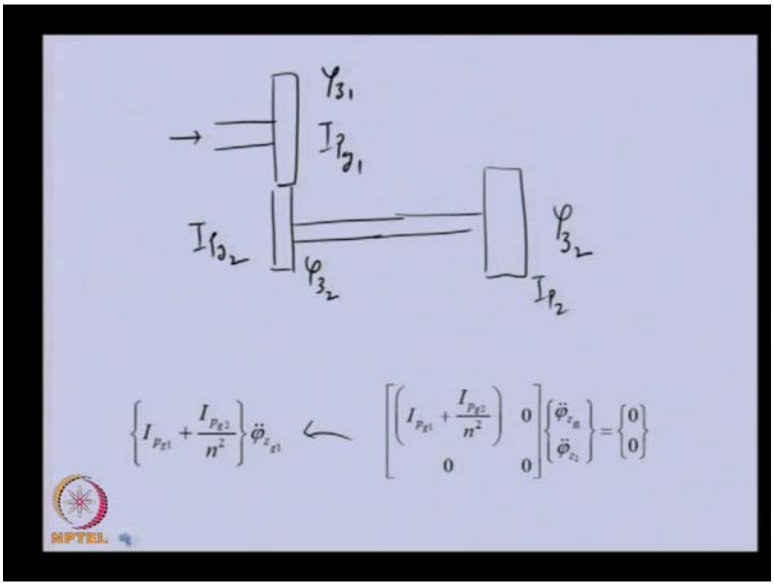

So, we have first gear which is having polar movement of inertia on this one and we have second gear and polar movement of inertia is this, and these two angular displacements as we have seen are related. So, at this position corresponding to the speed of the input shaft, let us say this is the input shaft this movement of inertia can be replaced by equivalent movement of inertia corresponding to the angular displacement at this node.

So, we can able to see this is the angular displace this is the polar movement of inertia of this gear and this is the equivalent polar movement of inertia of this second gear corresponding to the angular displacement, here this is the gear ratio. So, this is the total inertia of both the gears would be at a node at this node position. This can be replaced can written in this form corresponding to the angular displacement at 2, one this is corresponding to the angular displacement at gear here and this is the second is corresponding to the if we have this is I P 2.

So, this particular angular displacement or this this one which is which is a fly wheel, now because if we expand this equation I will get back this one. So, this is 0, 0 and 3 is not making any difference as compare to what we got the in the equivalent polar movement of inertia of the both the gear. Now, we will see the strain energy stored in the shaft so this particular shaft is the second shaft this shaft. We are trying to see what is the what is the strain energy stored in this and what is the work done by the torque at the ends, so we can able to see this is the shaft.

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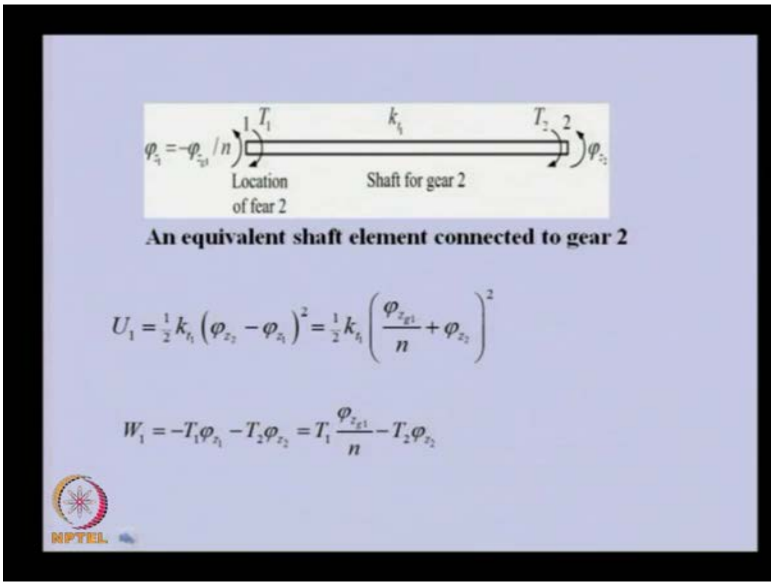

At this, let us say is node one this is node two corresponding to this I am assuming phi z 1 and phi z 2 as the angular displacement, but this phi z 1 is with which is at the gear 2 is related to the angular displacement at gear 1 by gear ratio and negative sine. The sine convention we have chosen, so we can able to see that, now the angular displacement at in the second shaft where that gear 2 is there. We need to will be representing the angular displacement corresponding to the gear 1; here this is the angular displacement corresponding to the fly wheel 2.

So, here we are not referring to the first shaft speed here whatever the angular displacement of the second gear is taking fly wheel is taking place that itself, we are taking it. So, as such in finite element method, now we are not converting the whole shaft into the equivalent shaft system. So, this is the original stiffness of the second shaft, now the strain energy stored in the shaft will be the torsional stiffness.

The relative twist between the two ends of the shaft, so phi z 2 minus phi z 1, now phi z 1, we can able to replace with the gear 1 angular displacement. So, this expression we can substitute here, so this will become positive so this is the term and this is phi z 2. Similarly, the work done by these two torques the reaction torques because of these angular displacement will be given by this and this angular displacement. Again we can able to replace with respect to the gear one angular displacement now we got the energy stored in the system.

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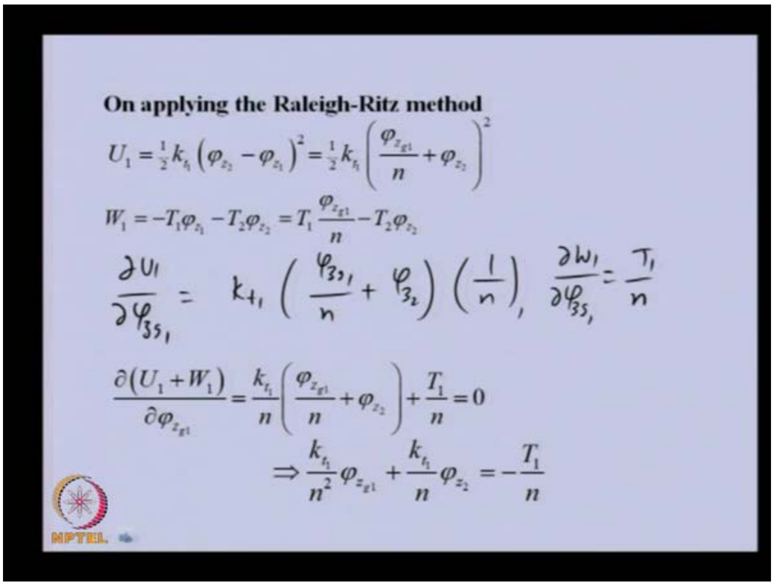

Now, we can apply Rayleigh Ritz method, basically it minimizes the energy these two energy, so we can able to see we are minimizing these two energy with respect to the hmm first with respect to the angular displacement corresponding to the gear 1. So, if we differentiate this, we can able to see the first one if we differentiate del u 1 by del phi z g 1, so we will get or two will get cancelled and phi z g 1 by n plus phi z 2 and then once we again differentiate we will get this.

Similarly, we can able to differentiate W 1 with respect to phi z g 1, so only one term is there. So, we will get t 1 by n, so these two expressions are here and they can be rearranged like this I am taking the torque in right side and you can able to see we are minimizing the energy. So, we are after partial differentiation of this we are equating this term equal to 0 and now I am rearranging this equation in this form where torque I am taking this side phi z g 1 and phi z 2 terms are in the left side.

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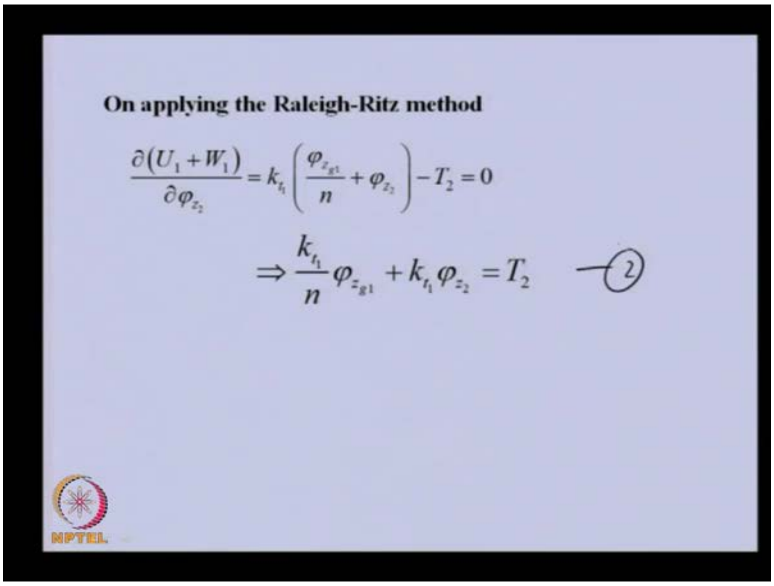

Similarly, we can minimize this energy with respect to phi z 2 second variable of the element and once you differentiate we will get this expression this is corresponding to the strain energy. This is corresponding to the work done and this again can be rearranged in this form in which we are putting the torque in other side, now this equation in the previous equation this equation we can combine.

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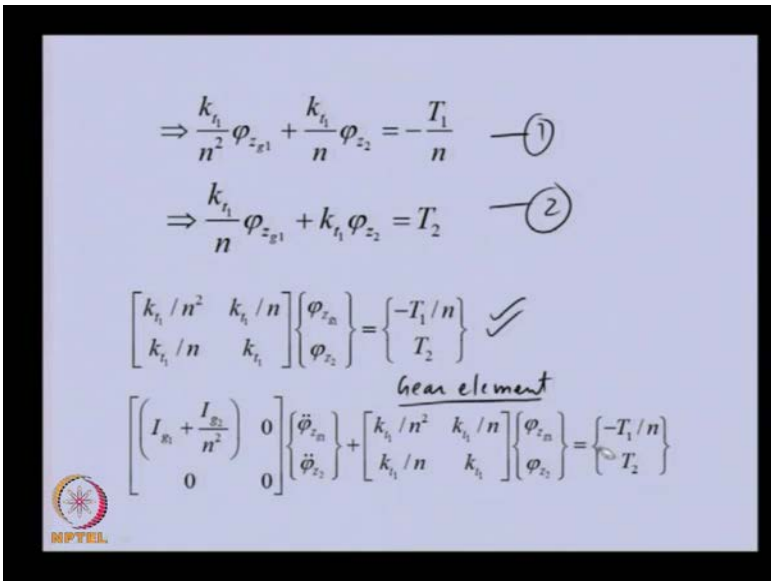

So, these are the two equations which we got at now I am combining these two equations in a matrix form. So, you can able to see this is a vector and these four elements are coming in the matrix and right hand side is a vector containing these two terms. So, this equation we got it from the strain energy and the work done earlier we got the term corresponding to the inertia that was this one, so that we can add it here to get the overall elemental equation for the gear pair. So, this is the inertia term which we obtained earlier and this is the stiffness term and this is the reactive torque at the end of the shaft.

So, this is the gear element equation and this is very important relation using this we can able to analyze the branch system and geared system very easily. Now, once we have developed the gear element we have seen that we got the with the gear element one the mass matrix, the stiffness matrix which are having a different form as compared to the conventional finite element of the torsional oscillation, torsional vibration of a simple shaft element. Now, gear ratios are also appearing now with this gear element, let us see how we can able to analyze a simple geared system.

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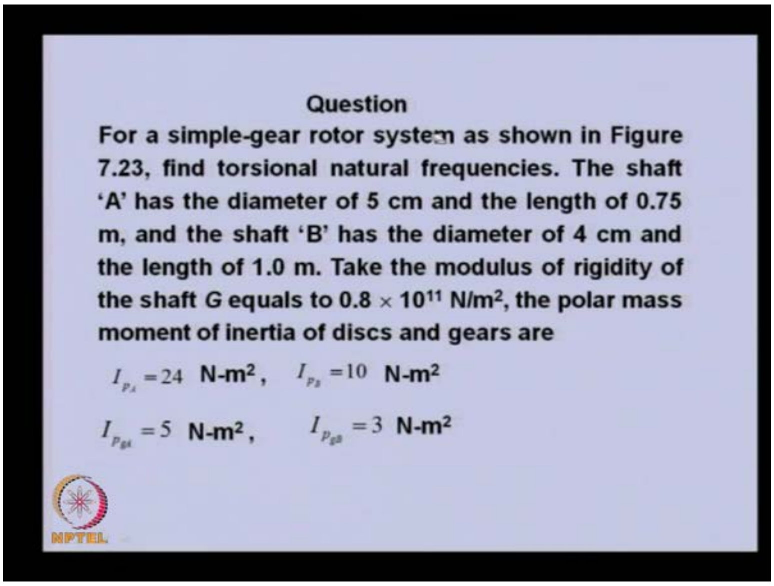

So, we are taking a simple HMM gear rotor system.

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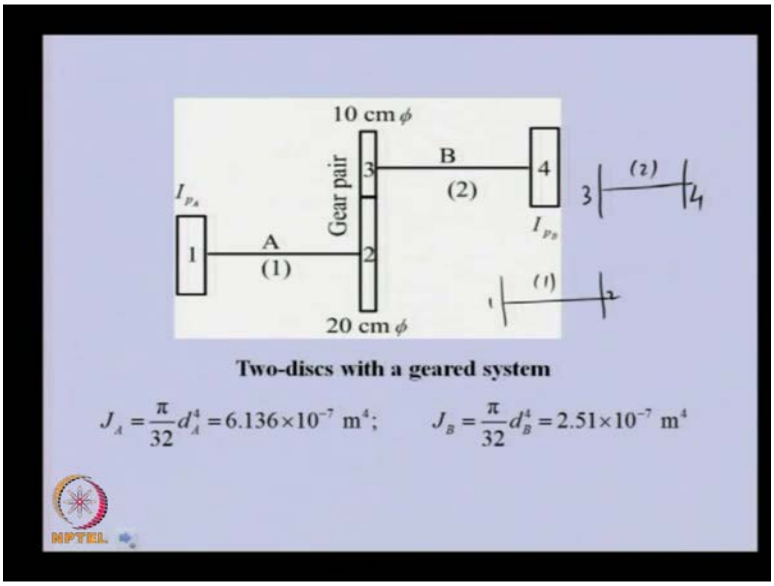

In this we have we gear pair like this we have two fly wheels at the end and this two shaft A and B are connected by gear pair various property of the system like diameter of the shaft A length of the shaft A also shaft for shaft B they have different dimensions. So, they are given modulus of rigidity of the both the shafts are given or even polar movement of inertia of the two fly wheel on shaft A and B are given. Now, we are considering the polar movement of inertia mass movement of inertia of the gears also.

So, gear A and B they have some polar mass movement of inertia now with this we can able to obtain second polar movement of area is in these relations, we know the diameter of the shaft this will be using in the elemental equation. Now, we can able to see we have divided this whole system into two elements, so is one element in which node 1 and 2 are there and this is the element 1. Similarly, this is the second element in which this is node 3 and 4, so basically we have we have divided this into two parts element one which is coming here node 1 and 2 and second element is like this in which we have node 3 and 4.

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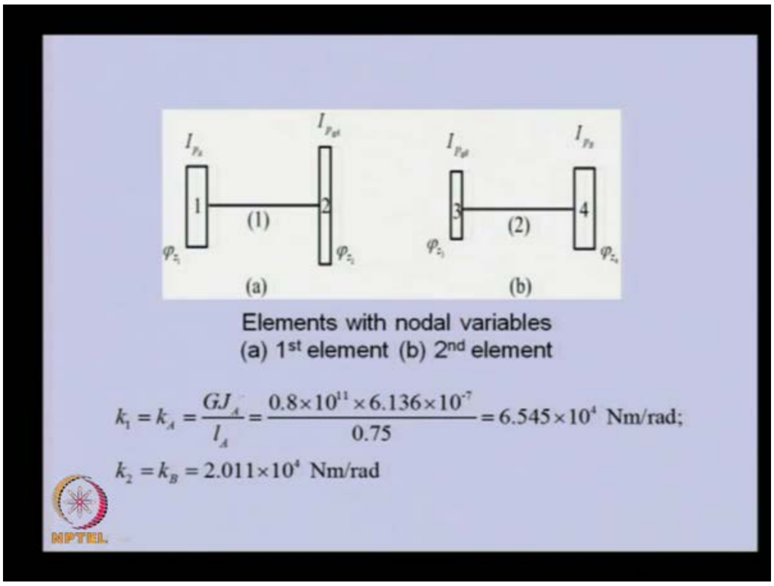

Now, once we have divided into two elements we can able to like this we can able to write the elemental equation for each of them and then we can able to see how they can be combined some more property like the stiffness. We can able to calculate of the shaft 1 and 2 using these relations for the data given in the problem.

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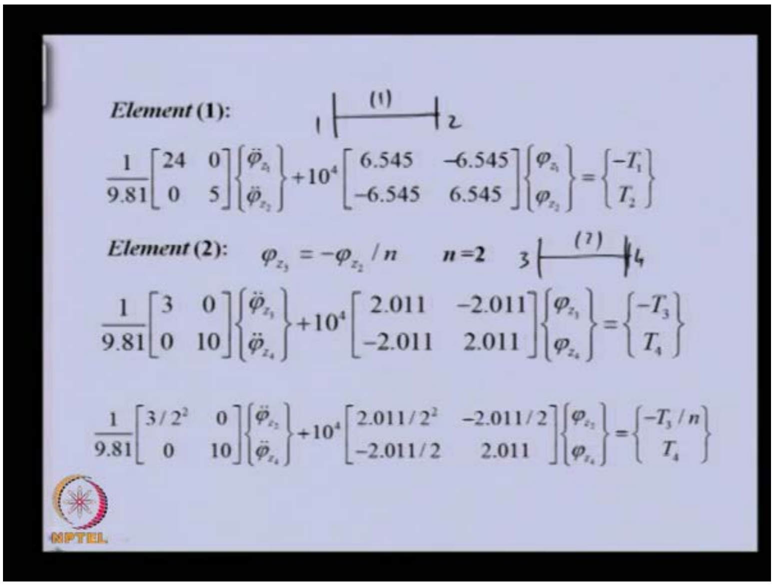

Now, this is the elemental equation for element 1 so in element 1, we had node 1 and 2, so I am again drawing that element 1 node 1 and 2. So, correspondingly we will be having the phi z 1 and phi z 2 here we are not considering the shaft inertia property only whatever the fly wheel and the gear polar movement of inertias are there that we have considered. So, in this particular case we are not considering the shaft to inertia only whatever the lump masses are there at the end that has been considered. This is the stiffness matrix conventional stiffness matrix and this is same vector as this only thing is here this is a displacement this is inertia.

These are the reaction torque at node 1 and 2 t 1 and t 2 for gear element to, sorry for the element 2 we have node 3 and 4. So, in this if you want to write exactly the same elemental equation for this we will be having phi z 3 and phi z 4 and these are the gear and fly wheel which are there at node 3 A 3 and 4. This is the stiffness matrix and these are the reaction torque are at the ends, till now we have not used the gear element because now you can see that node 2 and 3 angular displacements are related.

So, as such we should eliminate this angular displacement at node 3 with the node 2 because we know they are related with the gear ratio like this. If you want to do that, so obviously we need to convert this elemental equation according to the geared element which we developed. So, you can able to see this particular movement of inertia mass movement of inertia which is there at gear node 3 need to be divided by square of the gear ratio. This we need not divide because this particular angular displacement is same as the as that of the node 4 this is not related with the gear ratio.

So, we are keeping this as it is here if we see this term will be divided by gear ratio square and these two by gear ratio as if we refer back the elemental equation. This was divided by n square the gear mass movement of inertia and these stiffness terms are divided by gear ratio square and gear ratio. This will not be changing and here also torque will be divided by gear ratio, so we have divided that also by gear ratio. Now, we will be assembling this equation and this equation and you can able to see in this only 3, variables are there corresponding to node 1 2 and 4.

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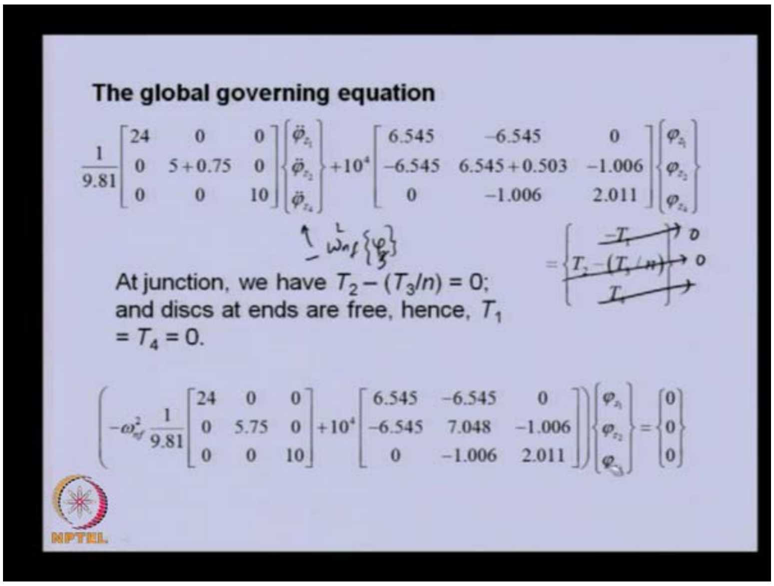

So, this is the assembled equation because now we have enough background how to assemble such equation. So, you can able to see that three variables are there and the first two are corresponding to the element 1 and these two are from corresponding to second element. Similarly, here this is corresponding to the second element and these four are from the first element and these are the torque which are this is at the common node that is node two and at junction.

We know this torque will cancel because they are reactive torque when once we assemble these elements they should cancel at the junction and apart from this because the ends of this both the shafts are free. So, at node one and four there will not be torque because they are free end where the fly wheels are there corresponding torques will be 0. So, this will be 0 this will be 0 this any way 0 because of this is at the junction point, now once these are 0 this equation we can able to write in this form after simplifying.

In this particular case, we have replaced this with simple harmonic motion this minus of this, so this is minus of omega n f square and this is displacement vector I have taken outside. So, basically this if you take the determinant of this equal to 0 will get a polynomial and that will give us the natural frequency of the system because that is a homogeneous equation.

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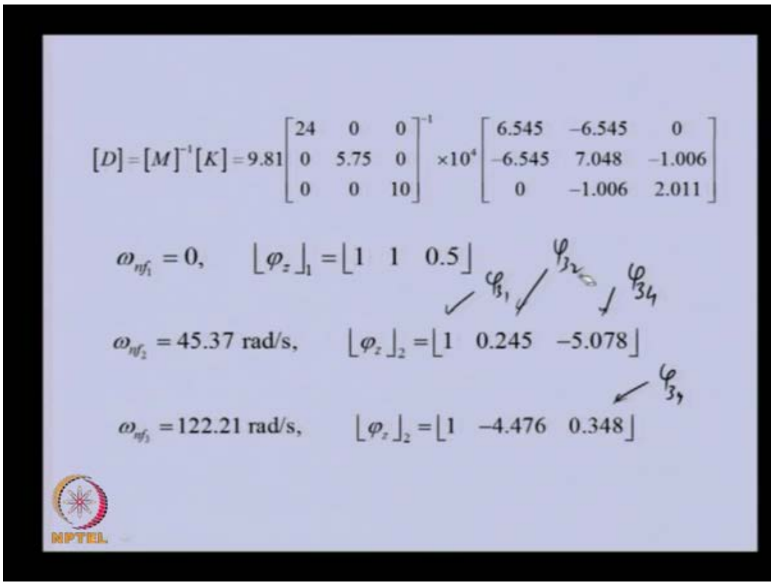

Alternatively, what we can able to do we can develop the Eigen value as we discussed in the previous lecture. So, we will take the inverse of the mass matrix and multiply with k matrix and if we obtain the Eigen value of this from, this we can able to get the natural frequency from Eigen values. Eigen vector will give us the how various nodes are having relative displacements, so this is corresponding to the first natural frequency we can able to see the node 1 and 2 are having same displacement. This is a corresponding node for which is because of the gear ratio become half otherwise as such this is a rigid body mode but, these are flexible modes.

So, we have three natural frequencies out of that two are flexible mode and one is rigid body mode only this particular this is corresponding to the 4 and this is phi z 1 and phi z 2 phi z 3 is related with this. So, if we divide this by gear ratio we can get the phi z 3 displacement, now we will take up another example for branch system in the same elemental equation. For gear we can use for analyzing the branch system also, let us see that particular example.

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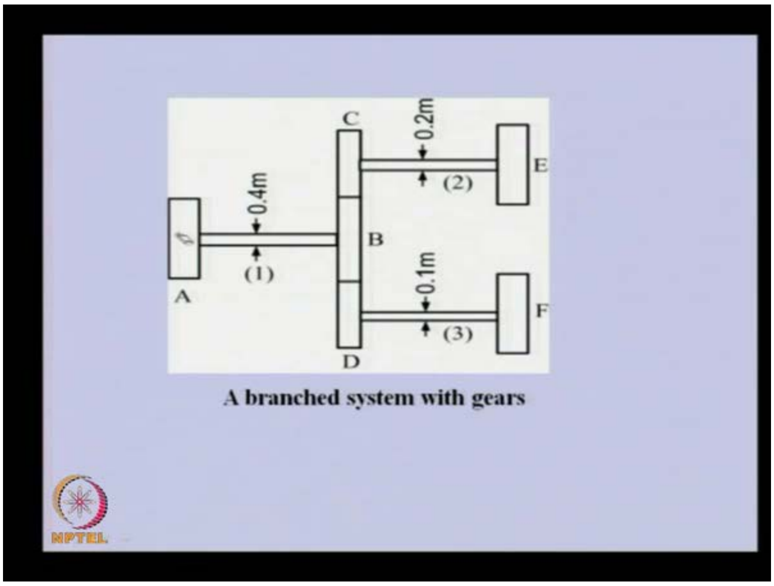

So, basically this is a branch system in which one fly wheel is here and we have shaft and we are having two shafts connected with this at two gear. So, gear b is connected to gear C and B and which is giving power to the fly wheel E and F. For simplicity of illustration we have not taken intermediate this, but that also can be taken.

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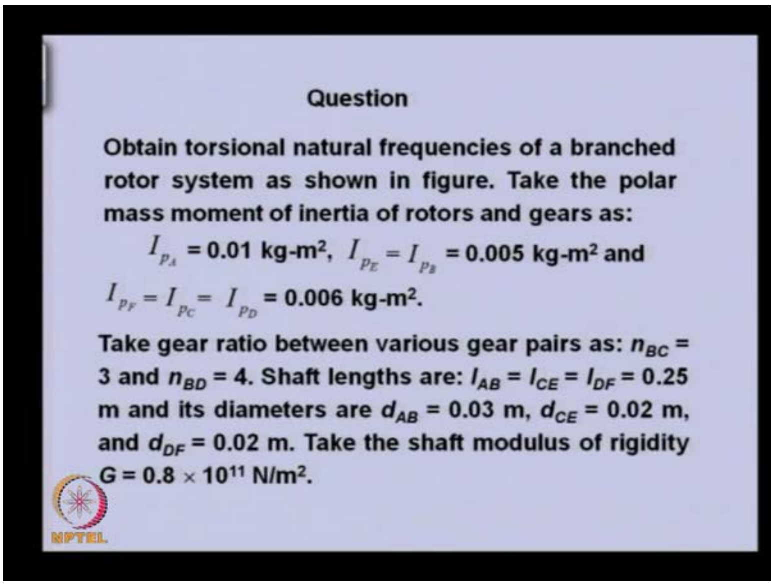

So, basically various properties of these discs are given for flywheels we have this property and gear also have polar movement of inertia. They are not negligible, whereas gear ratio like for between gear B and C is 3 between gear B and D is 4 lengths of the shafts are given diameter is also given and shaft modulus of rigidity is also given. So, with this property now you can able to see if you want to analyze this we need to take this particular branch as element 1 this we can take as element 2 and third branch as element 3. We can able to write the elemental equation for each of them and then we can assemble them for applying the boundary conditions.

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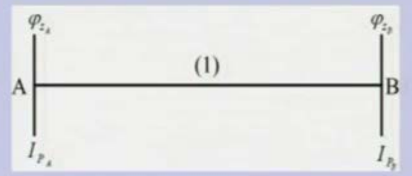

$$\begin{bmatrix} I_{p_A} & 0 \\ 0 & I_{p_B} \end{bmatrix} \begin{Bmatrix} \ddot{\varphi}_{z_A} \\ \ddot{\varphi}_{z_B} \end{Bmatrix} + \begin{bmatrix} k_{t_1} & -k_{t_1} \\ -k_{t_1} & k_{t_1} \end{bmatrix} \begin{Bmatrix} \varphi_{z_A} \\ \varphi_{z_B} \end{Bmatrix} = \begin{Bmatrix} -T_A \\ T_B \end{Bmatrix}$$

So, let us see the first element which is branch a here we have one flywheel and this is corresponding to gear, so as such in this whatever the torsional elemental equation we developed will be valid for this. So, we can able to see these are the lumped masses here again we have not considered the inertia of the shaft only masses which are lumped at the ends are considered. So, they are coming at the diagonal and these are the angular displacements corresponding to this node A and B and this is the stiffness of the shaft element 1. These are the reaction torque at and A and B, similarly we can able to write elemental equation for element 2.

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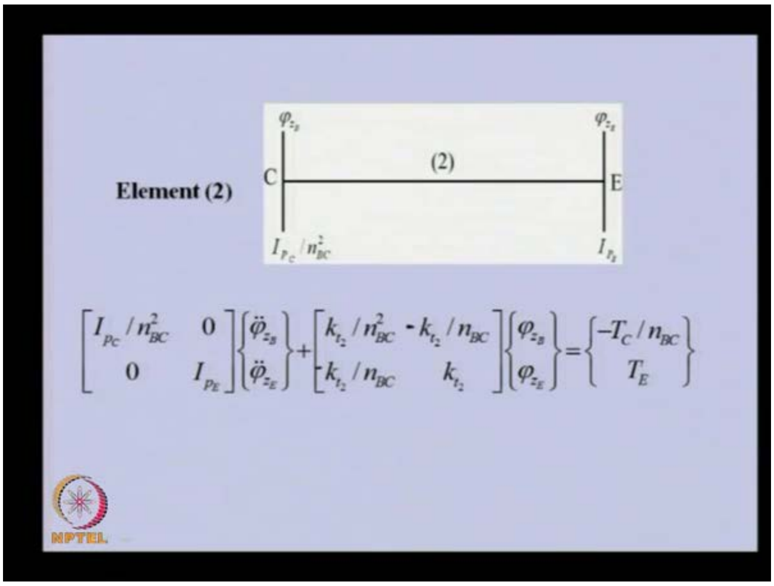

Here, we are considering instead of phi z C, we are considering the phi z B that is A because this gear C and B are connected by and they are related by gear ratio. So, directly we are replacing that particular angular displacement this will remain same as of the p. So, you can able to see this we need to divide by gear ratio square similarly, in the stiffness matrix this term will be divided by gear ratio square and half diagonal terms will be divided by gear ratio only. These will be minus and this torque also we need to divide by gear ratio as we have seen earlier, but this will be unaffected.

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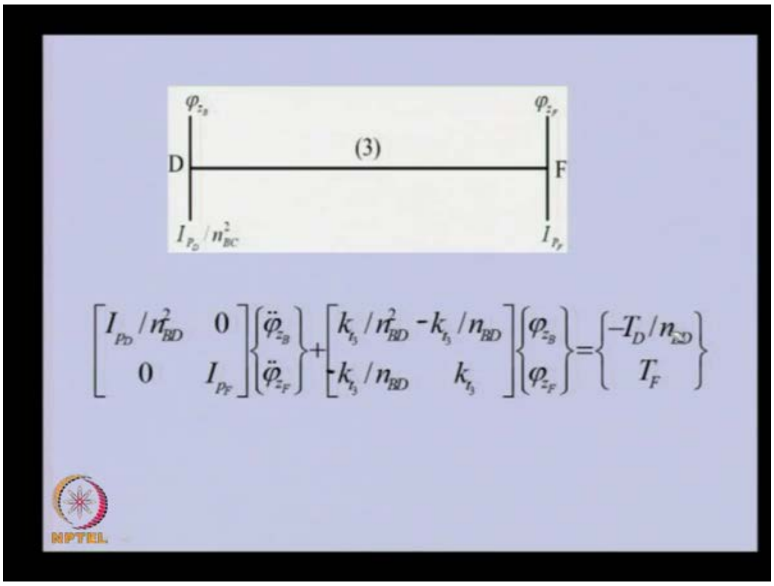

Similarly, we can able to develop elemental equation for third branch that is and this is the element 3 here we should have angular displacement phi z B, but we have replaced that by phi z B the gear which is driving this. So, we need to use the gear ratio B to D here in the inertia term also in the stiffness term they will also be negative here, so you can able to see torque also we need to divided by the gear ratio.

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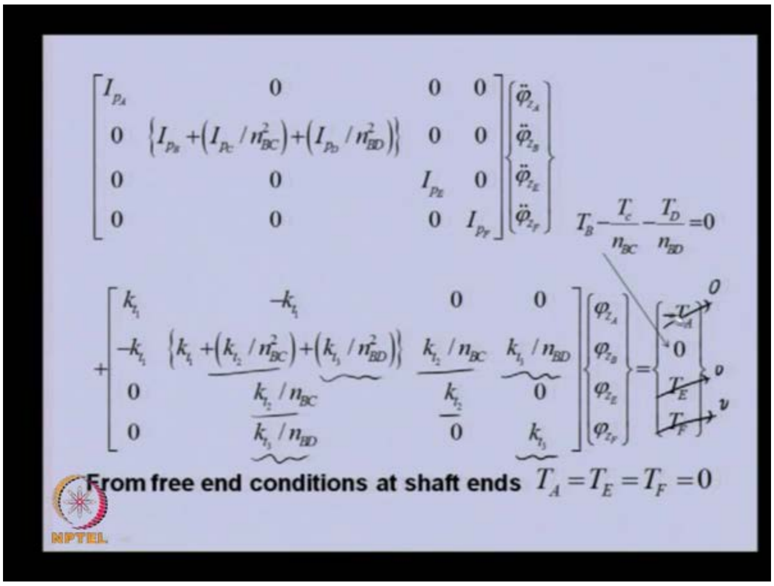

Now, we can combine these you can able to see we will be having only four angular displacement at node A B E F at the junction point C and D gear displacements we have already replaced by the gear ratio gear displacement at B. So, you can able to see that because of that during the assembly at this position we have equivalent polar movement of inertia of gear C and D also added to the gear B polar movement of inertia.

Similarly, this is the stiffness term you can able to see the first four terms are corresponding to element 1 then these terms are corresponding to second branch and these are corresponding to third branch and at junction point. This torques will cancel each other and now in this particular case also and A E and f they are free ends. So, torque at those locations will be 0, so will be having this torques also as 0 and now you can able to see in the right hand side all terms are 0.

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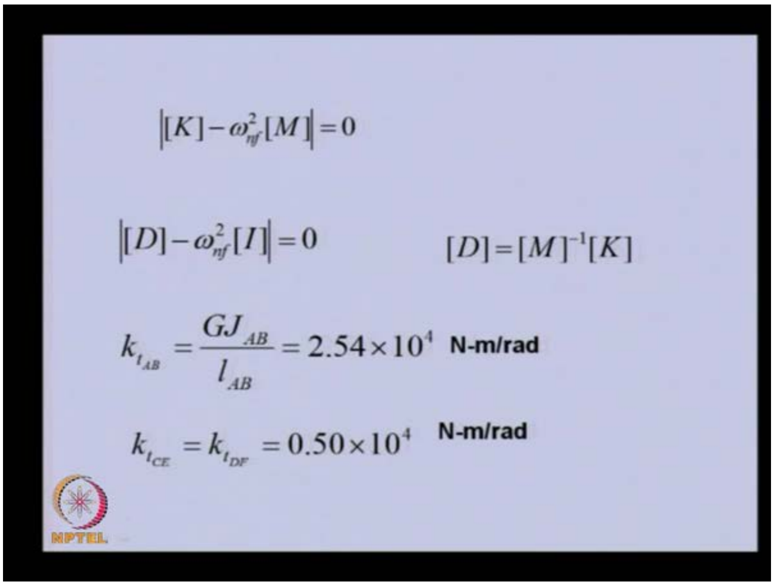

So, this equation can be simplified in this form k minus this m this is conventionally we have been doing this converting the for free vibration this equation of motion in this particular form. So, if we take the determinant of this equal to 0, we can get the frequency equation or we can go for the Eigen value problem in which we can this d in this particular case defined as inverse of m into k. So, if we obtain the Eigen value of d we can able to get the natural frequency and the mode shape also, for given data various stiffness terms we can able to calculate.

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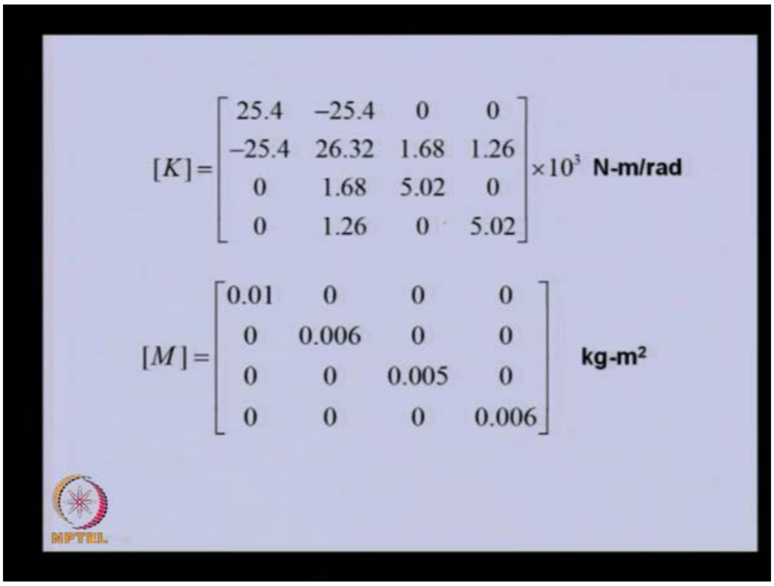

This is the stiffness matrix and this is the mass matrix corresponding to the various property of the branch system.

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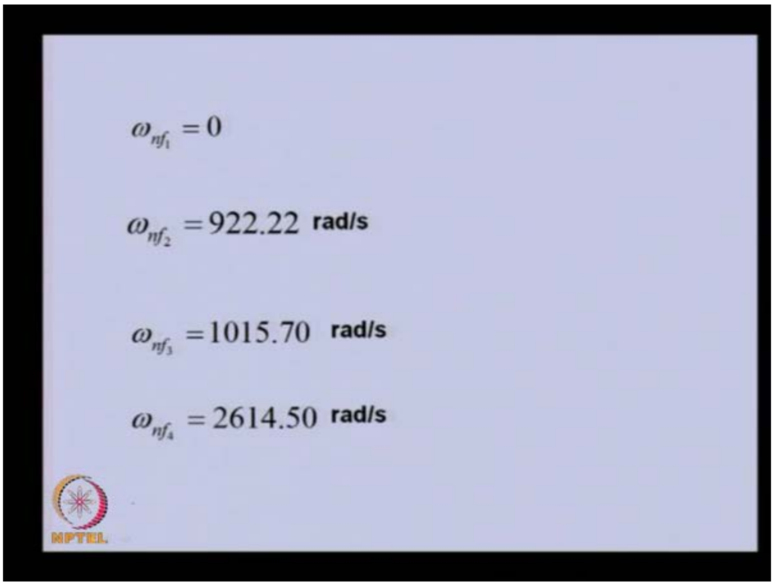

If we solve the Eigen value problem we will get four natural frequencies, one of them is 0 because this free end conditions are there, so we expect one rigid body mode, but these are flexible modes.

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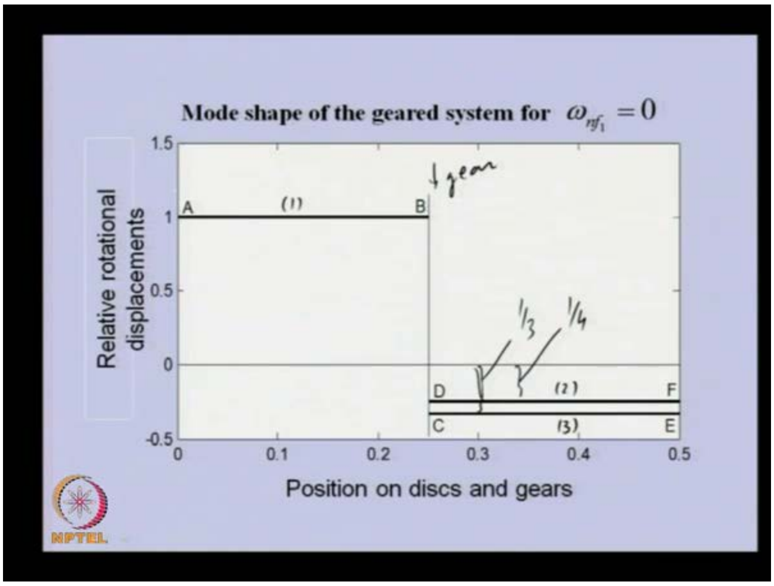

Even we can able to obtain the Eigen vectors and we have drawn this, so basically this is branch 1 and so this is corresponding to element one this is corresponding to element 2 this corresponding to element 3. This is the position where the gear is connected, so for unit this is for zero natural frequency, we expect a rigid body mode. So, node 1 and 2 or node A and B will be having same displacement one we expect this two have same displacement, but because of the gear ratio this will be divided 1 will be divided by 2.

So, this, sorry this is 3 gear ratio is we have two sets of gear ratio, so one is the 3, another is the 4, so we can able to see that the difference. So, this is the this is 1 by 3, this is nothing but 1 by 3 or this one is 1 by 3 and this is 1 by 4, so this is the gear ratio 3 and gear ratio 4. So, you can able to see that whole the second element and third element is having same as such there is no relative twist between the ends.

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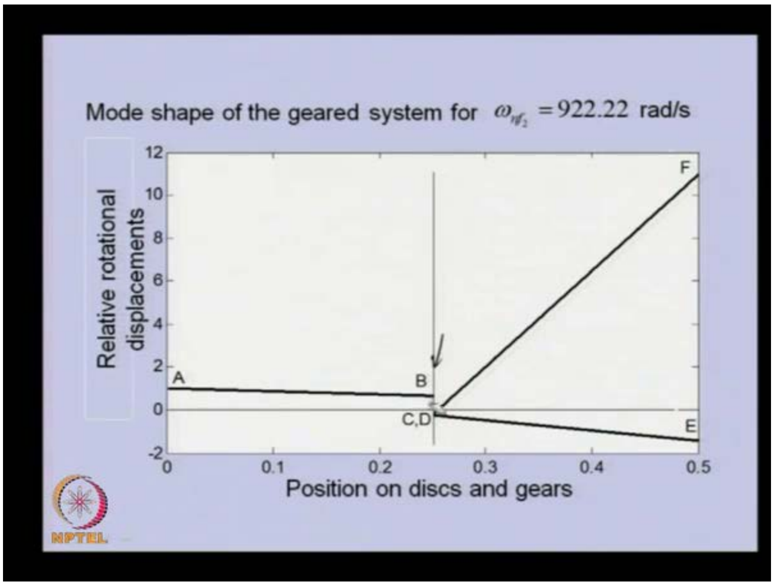

Similarly, we can able to plot the mode shape for flexible mode that is second mode, so you can able to see this is now they have relative twist at end A and B C and D will be 1 by 3 and 1 by 4 times of the this particular height from here to here. They are very close, so it looks at the same point, but they are not at the same point, then and E and F will be having different displacement. So, they can be connected, so this is the mode shape corresponding to this this natural frequency and this is the point where the gears are connected to each other.

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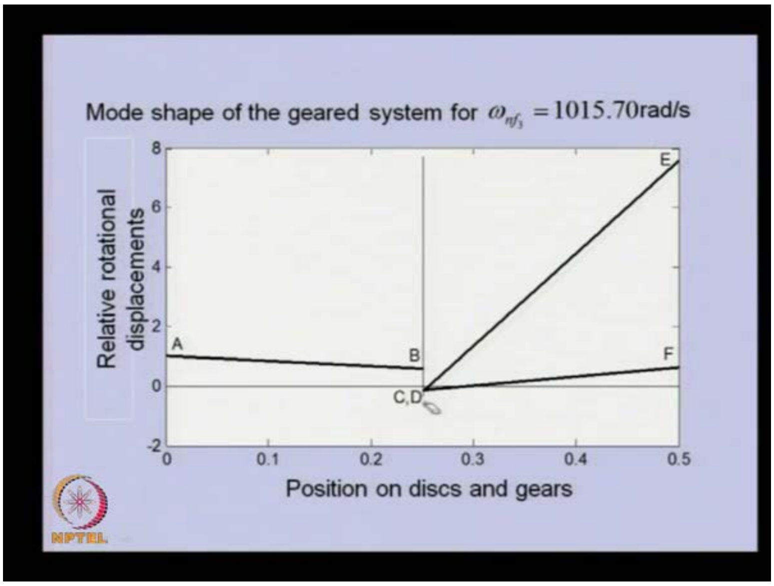

Similarly, other modes can plotted you can able to see the relative positions are changing here because this amplitude is very less. So, C D appears to be at the same position, but they are not at the same position.

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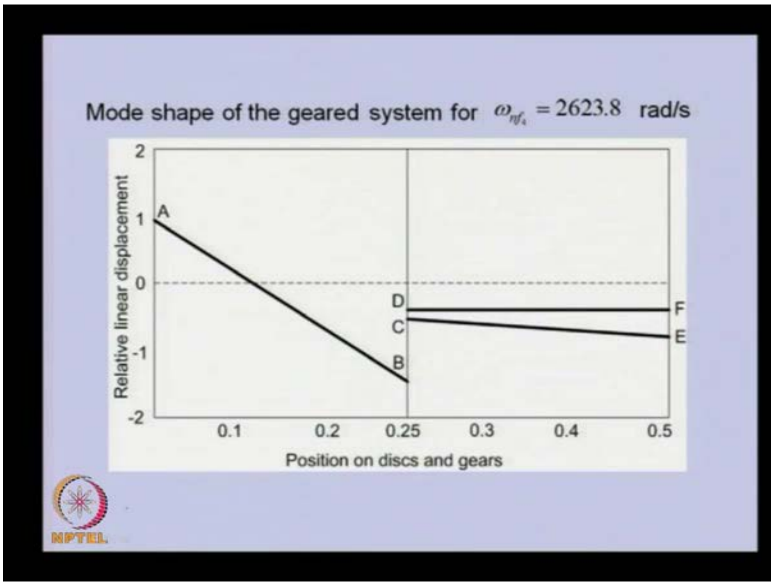

Similarly, this is the fourth mode here, now they are distinct you can able to see now B is having opposite motion as compared to A and now we have when we are dividing by 1 by 3 of this C is coming here and 1 by 4. Then, we are getting A D point here and these are the other angular displacement at and E and F, so this is the mode shape corresponding to the forth natural frequency which is flexible mode. Now, we will take the case when the gears pair when the two teeth when they are in contact because at that point, now we have direct contact between two teeth.

Generally, in gears we have a line contact, but when it is transmitting power that particular line contact becomes area contact and that particular stresses is governed by the Hessian stress theory. Generally, they are non linear in nature, that is the force and deformation relations are non linear relation, but this particular case we are considering the linear stiffness at the contact point. If there is a flexibility which is linear how we can able to develop the governing equation for such gear pair, we will try to illustrate and we will try to obtain the equation of motion for such cases.

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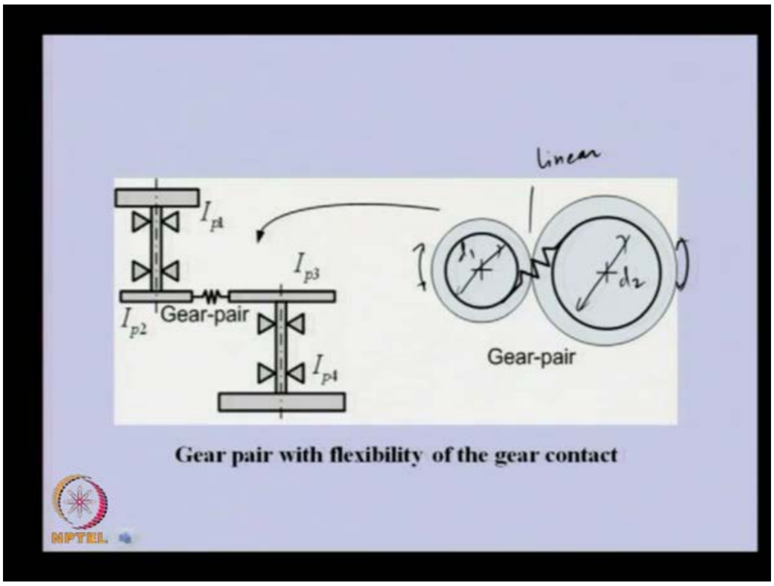

In this particular case we are taking basically this kind of gear pair you can able to see this is gear 1 and gear 3 these are flywheel these are two flywheel and in between the gear pair. We are considering the flexibility due to the contact between two teeth and that we can able to model like this. So, this is the pitch circle of the gear one and this is of gear two and whatever the flexibility is there between the gear 2 during the contact we are representing that as a linear spring.

So, this is basically a linear spring that means the extension or compression of this would take place during the motion because these two are oscillation the torsional oscillations like this and because of these we can able to expect. This particular spring will get extension or compression depending upon the relative angular displacements of these. That will not only will depend upon the relative twist, but also the diameter of these pitch diameter of the gears, so if pitch diameter is D 1 and D 2, then that particular extension whatever the will be governed by these diameters also.

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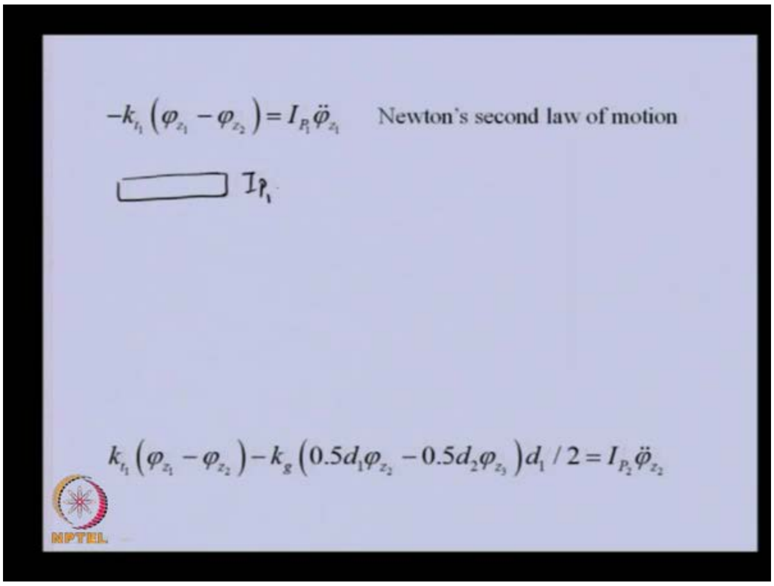

So, now let us see how we can able to analyze such system, so I am first considering the first flywheel which is here.

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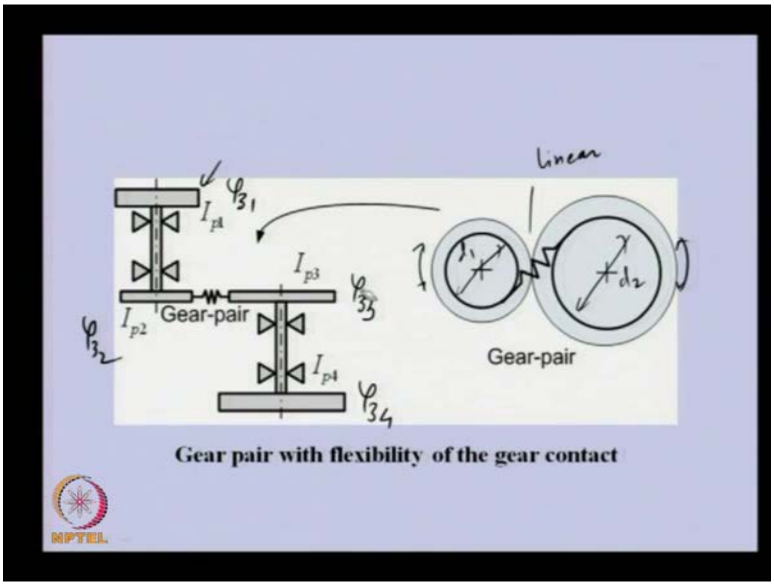

This particular flywheel and I am trying to attach phi z 1 is the angular displacement of this phi z 2 is angular displacement of this phi z 3 is angular displacement of this phi z 4 is angular displacement of this. Now, we can able to see that these two angular displacements are as such not related with the gear ratio. Now, there is a flexible element in between this, so basically these two are now independent angular displacements.

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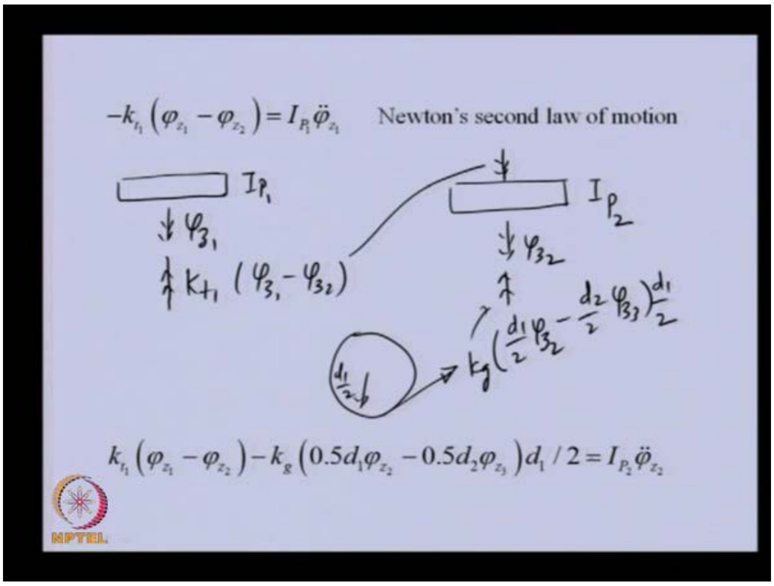

So, in this particular case, if I am considering the angular displacement of this as positive in this direction or there will be torque from the shaft reaction torque which will be coming which will be given as this. Now, you can able to see this is the only torque which is acting on this, so using Newton's law we can able to equate this toque which will be in the opposite direction to the displacement to the inertia. So, this is one of the equation of motion, similarly we can able to write for gear one there is I P 2.

So, in this particular case again the displacement direction is in this positive direction this torque will act in opposite direction to the previous one. Here, this torque will be acting here and apart from this there is a flexibility of the linear spring. So, for that case you can able to see this is the gear 1 having radius D 1 by 2 and this k g is the stiffness of the linear spring and relative extension of the spring will be phi z 2 minus D 2 by 2 phi z 3. So, relative extension of the spring will be if we go back here become C relative extension of this will be how much angular displacement of this is taking place and how much angular displacement of this is taking place.

This diameter will give basically what is the actual extension of the spring of this end and this end, so that is why we have used diameter there. So, that we can able to get the actual extension, so basically within the bracket term is the actual extension of the spring the at the contact point. So, this particular force this particular torque this is a force and the torque will be again if we divide multiply by D 1 by 2, so this will give us the torque which will be active on the gear 1. So this is the force of the linear spring into D 1 by two will give the torque, so this particular torque will be acting opposite to the. So, this torque will be acting opposite to the angular displacement and this free body diagram of this we have written here.

So, you can able to see this the governing equation this is the torque which is in this direction positive direction this particular torque is opposite to their angular displacement should be equal to the inertia of the this gear 2. So, once we obtain this, we can able to obtain the free body diagram of the second gear that is having I P 3. In this particular case also, I am taking angular displacement onward is positive direction the torque from the form the linear spring will be given by this so you can able to see this is the force.

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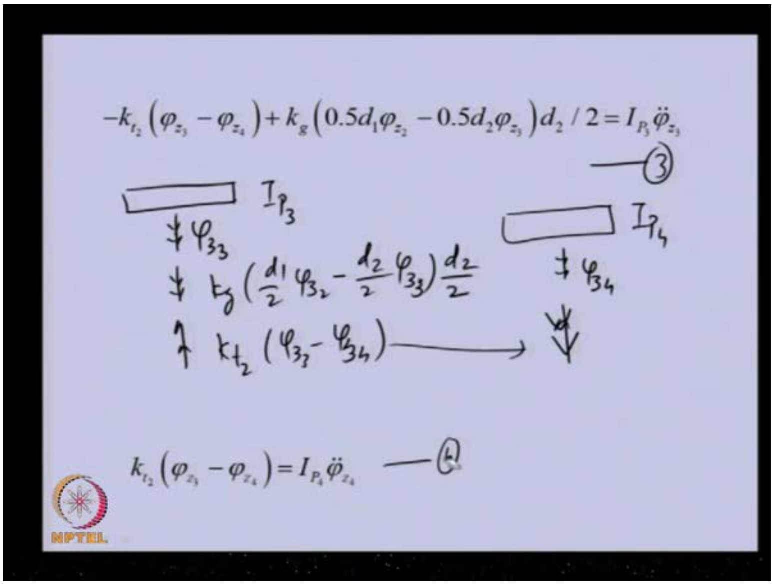

Now, we can able to write the free body diagram for the second gear having polar movement of inertia of I P 3 we are taking the angular displacement direction positive. The downward direction from the linear spring at the contact point, we will get this particular torque, this is basically first this term is force. Now, for torque we need to now multiply by D 2 by two earlier we divide multiplied by D 1 by 2.

Now, we need to multiply by D 2 by 2 and then from the second shaft that means we have taken care this, but this shaft is connected with the second gear. So, this will also give the reaction torque to this because there will be relative twist between these two ends of the shaft. So, this will be opposite to the displacement, so k t 2 phi z 3 minus phi z four, so this is reaction torque from the shaft this is the reaction torque from the contact point linear spring and you can able to see. Now, this is the equation for this opposite to the angular displacement this is in same direction as the angular displacement should be equal to the inertia.

So, this is the third equation of motion then the last disc having I P 4 in this only one torque will be acting and that will be in the value of the torque will be this, but direction will be opposite, but this case they will be in the same direction. So, this is the torque equilibrium of this particular flywheel, so this is the fourth equation. So, basically all these equation first equation second equation third equation and fourth equation are the governing equation of this particular whole system they can be combined in the matrix form like this.

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$$\begin{bmatrix} I_{P_1} & 0 & 0 & 0 \\ 0 & I_{P_2} & 0 & 0 \\ 0 & 0 & I_{P_3} & 0 \\ 0 & 0 & 0 & I_{P_4} \end{bmatrix} \begin{Bmatrix} \ddot{\varphi}_{z_1} \\ \ddot{\varphi}_{z_2} \\ \ddot{\varphi}_{z_3} \\ \ddot{\varphi}_{z_4} \end{Bmatrix} + \begin{bmatrix} k_{t_1} & -k_{t_1} & 0 & 0 \\ -k_{t_1} & k_{t_1} + 0.25 d_1^2 k_g & -0.25 d_1 d_2 k_g & 0 \\ 0 & -0.25 d_1 d_2 k_g & k_{t_2} + 0.25 d_2^2 k_g & -k_{t_2} \\ 0 & 0 & -k_{t_2} & k_{t_2} \end{bmatrix} \begin{Bmatrix} \varphi_{z_1} \\ \varphi_{z_2} \\ \varphi_{z_3} \\ \varphi_{z_4} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

So, you can able to see this is the mass matrix this is the stiffness matrix a solution of such system we already done earlier. So, I will not illustrate the solution of this, but how the equation of motion can be obtained I have illustrated here using the Newtons law, now the same problem we will see using the transfer matrix method.

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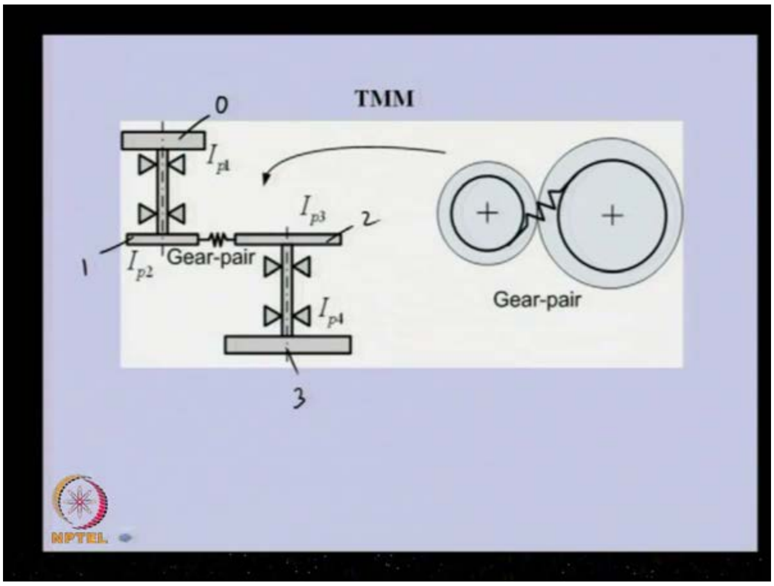

So, in transfer matrix method we need to let us say this is the station 0, station 1, station 2, station 3, so we have 3, 4 stations from 0 to 3.

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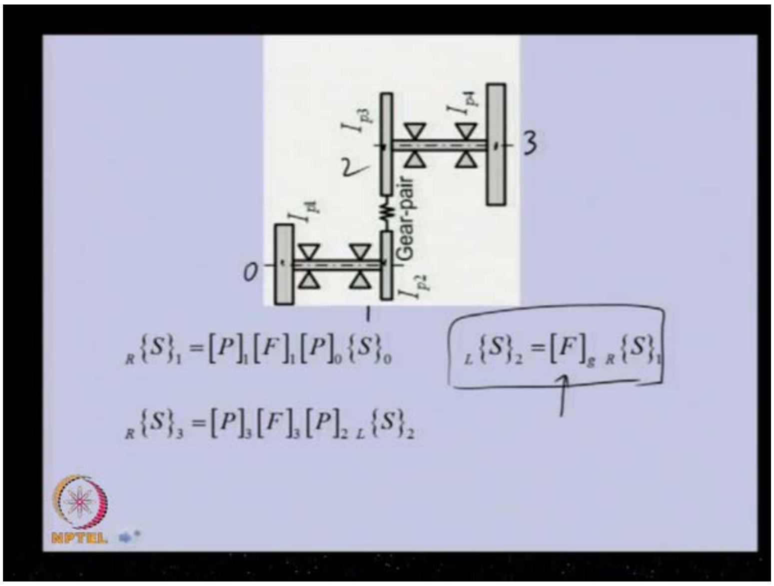

Now, we can able to see I have tilted the same thing and here we have 0 station, 1 station, 2 station and 3 station, now between station 1 and 2 we can able to write the straight vector. So, right of station 1 and station 0 is related with point matrix field matrix and again point matrix. So, this we already conversion with how to relate the straight vector at two ends of this, similarly at the right of three and right of 2 we can able to relate right of 3 and left of 2.

We can able to relate using this and in between the right of 1 and left of 2, let us say we have this relation where f is the field matrix corresponding to the linear spring. Now, we need to develop this one or those are we know now will see how we can able to develop the field matrix for that linear spring corresponding to the flexibility of the gear pair.

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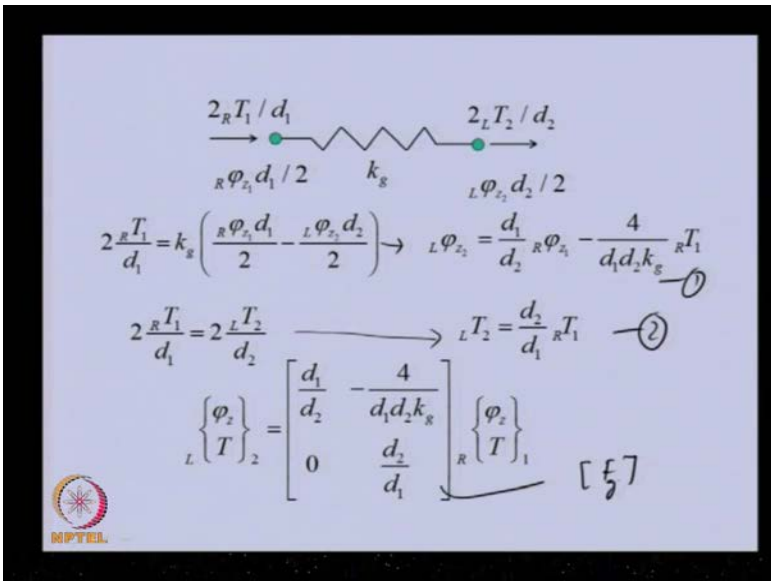

So, this is the linear spring having stiffness this now at two ends we have the linear displacement these because this is the angular displacement if we multiply the radius we can get the linear displacements of the two ends of the spring. If we divide torque by the this respective diameter of the two gears we can able to get the force, so basically we converted linear displacement and force of for this linear spring. Now, we can able to see that we this particular torque this particular force is coming from the stiffness of the spring and relative extension of the spring at two ends.

So, these are the relative extension so this is one of the relation and this relation we can able to write in terms of this we can able to see we have written this expression. We have written phi z 2 left of that and all other terms I have taken this side apart from this that because there is no other force in between the two ends of the force should be same. So, this is the two forces are same at two ends and this we have written in this form, so basically now you can able to see we have related the straight vector left of 2 left of 2 and other is right of 1. So, this two equation 1 and 2 we can able to combine in this form in which this is the straight vector left of 2 and this is straight vector right of 1. This is the required f matrix for the gear which we were we were interested to obtain.

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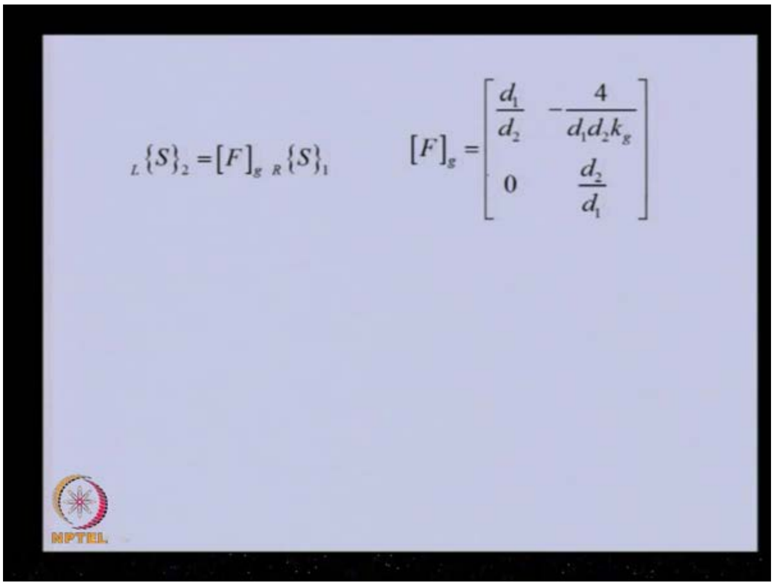

So, once we have this we can able to obtain the governed equation for the whole system because now we can able to combine these because now you can able to see we can able to substitute this here. So, we will get and even we can able to substitute this here, so we will get overall transfer matrix and then we can able to apply boundary conditions those things we already we illustrated previously. So, main focus of this is this presentation is to how to develop this f matrix for the gear pair flexibility element. Now, the same gear element we will develop using the finite element method and let us see how we can able to develop the finite element method for this finite element for this pair.

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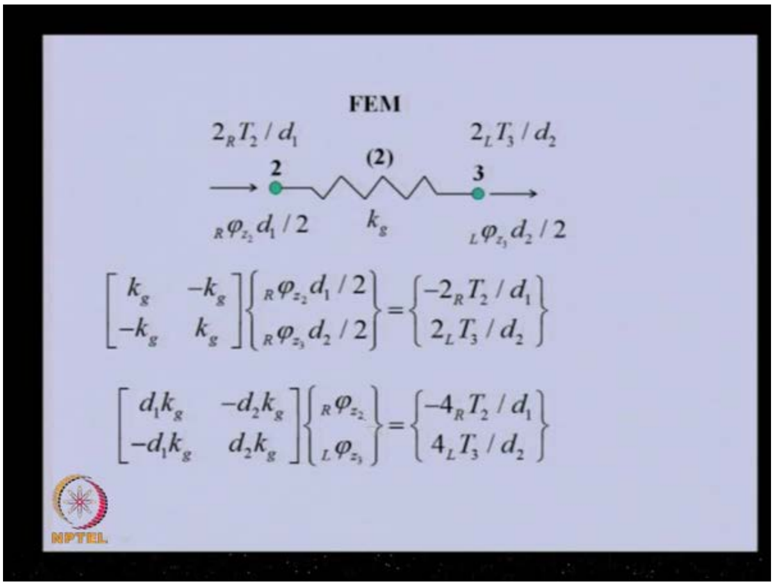

So, same this is a linear displacement and these are the forces at two ends, now you can able to see that for the spring simple linear spring the elemental equation. We can able to write it as this is the stiffness and this is the linear displacements and these are the forces and these we can able to rearrange in this form. So, we have angular displacements here and so just we have rearrange this like this there is now only 2 we have taken this side.

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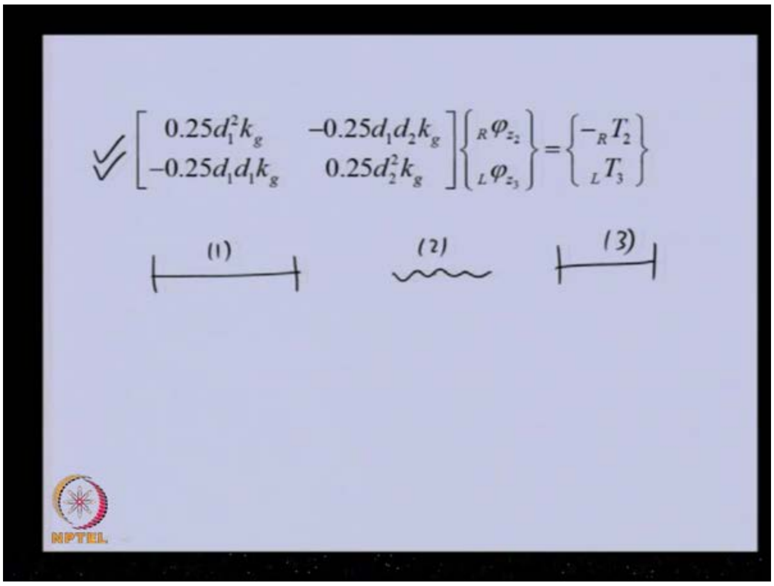

The same this equation we can able to rearrange in the further like this in which even we can able to write this in the torque form. So, angular displacement and torque form, so once we have the elemental equation for that spring element we can able to develop the elemental equation for whole system. That means we have developed for this and the spring and this, so element one two three they can be combined and then we can able to obtain the boundary condition.

So, the main focus of this illustration is that this is the elemental equation for the spring which is there between the two gears because of its flexibility. So, once we know this we can able to apply whatever the procedure we developed earlier to analyze this kind of system. So, today we developed the gear element initially with the finite element method and through illustration, we have seen that how we can able to analyze a simple gear pair or even a branch system using the finite element method. Then, we took up another interesting case in which when there is a flexibility at the gear pair contact point, then how we can able to analyze such system.

So, for such systems we have developed the equations first using the Newton’s second law of motion then we developed the transfer matrix method for such flexibility and even we have developed the finite element for such flexibility of the gear pair. Once we have these elements or the field matrix in the form of matrices, then we can able to easily analyze such system easily. So, with this we can able to see that especially the finite element method is very powerful tool for analyzing a rotor system we have seen this for torsional case, but in future we will be applying similar approaches for transverse vibration.

Also, in the subsequent class, now we will start with the transverse vibration especially for the multi degree of freedom system because till now we have considered for transverse vibration only the single mass rotor system. We have not considered multi mass rotor system, so in this subsequent class will we will be dealing with transverse vibration for multi degree of freedom system.