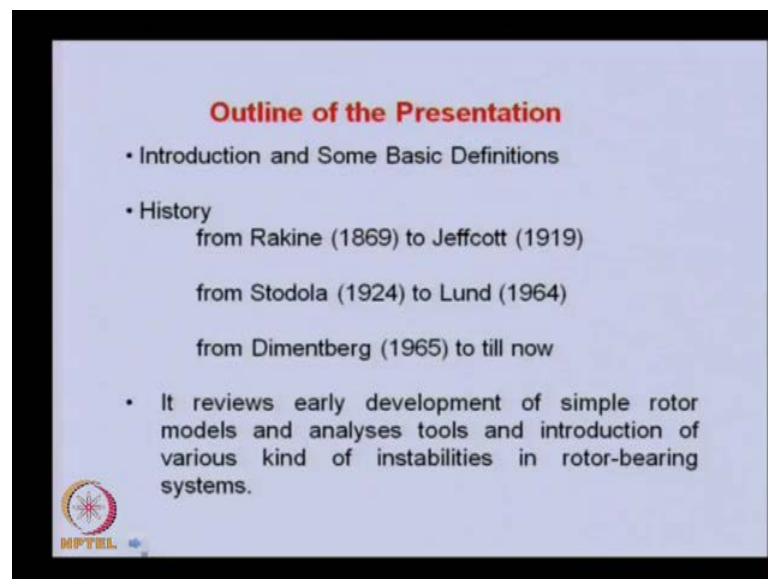


Theory and Practice of Rotor Dynamics
Prof. Rajiv Tiwari
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Indian Institute of Technology, Guwahati

Module - 1
Overview
Lecture - 2
A Brief History of Rotor Dynamics

Last class we studied basic introduction to the rotor dynamics. We saw thus the course content. We saw the very basic definitions of some of the phenomena, which occur in the rotor dynamics like whirling, unbalance. In the present lecture, we continue with those concepts, but we will review some history of the rotor dynamics. So, during that description of the history, I will be introducing various phenomena of which people observed, and then they try to analyze. So, this particular module is concerned with the history of rotor dynamics.

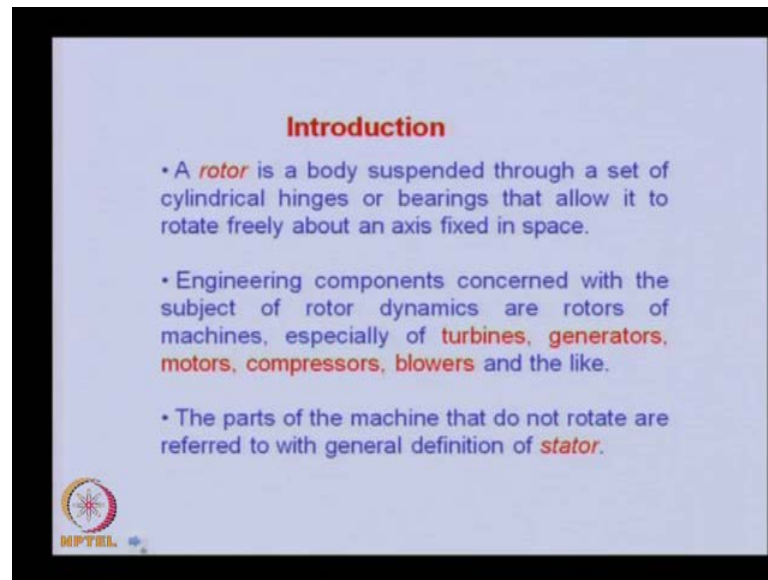
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This is the outline of the course in which we have introduction and some basic definitions, which we studied earlier also. We will review that we will extend the mode explanation to those definitions. In history, I will start with the Rankine who first analyzed on the rotor analyses how to obtain the natural efficiency of the system. This is the first phase in which Rankine to Jeffcott, I will review and then the second stage will be suitable from Stodola to Lund and third stage from Dimentberg till now what are the

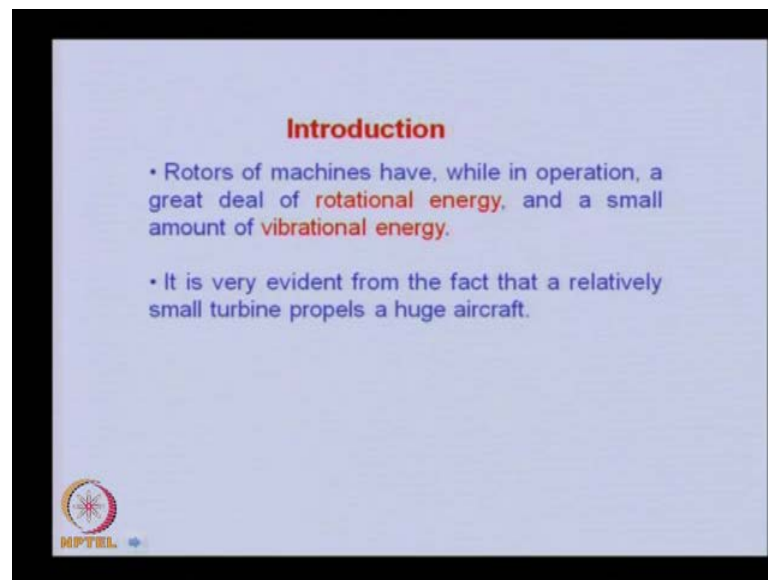
developments have been taken place. You can able to see that this particular lecture will review early development of simple rotor model, analysis of simple tools, introduction to various kinds of instability in rotor bearing systems. Instability is very important in analysis of the rotor bearing systems.

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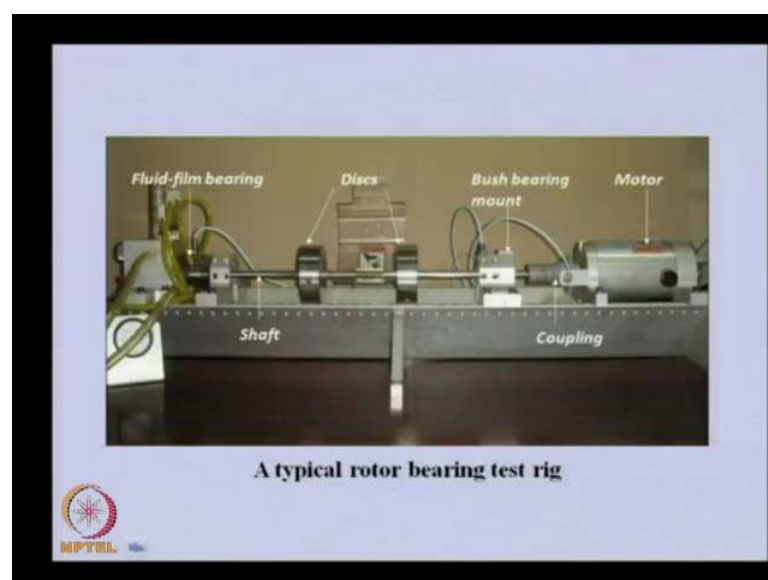
So, let us again see what is the basic definition of the rotor? Rotor is nothing but a body suspended through a set of cylindrical hinges or bearings that allow it to rotate freely about an axis fixed in space. So, it is the abstract definition of the rotor and various engineering components concerned with the subject of rotor dynamics are rotors of machines, especially of turbines, generators, motors, compressors, blowers and the like, even in the household applications or medical applications will find such rotors. The parts of the machine that do not rotate are referred to the general definition of the stator.

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Rotors of machines have, while in operation, a great deal of rotational energy, and a small amount of vibrational energy, which is actually unwanted form. The main aim should be to have as much as have rotational energy minimum vibrational energy. You can able to see that it is very evident from the fact that a relatively small turbine propels a huge aircraft.

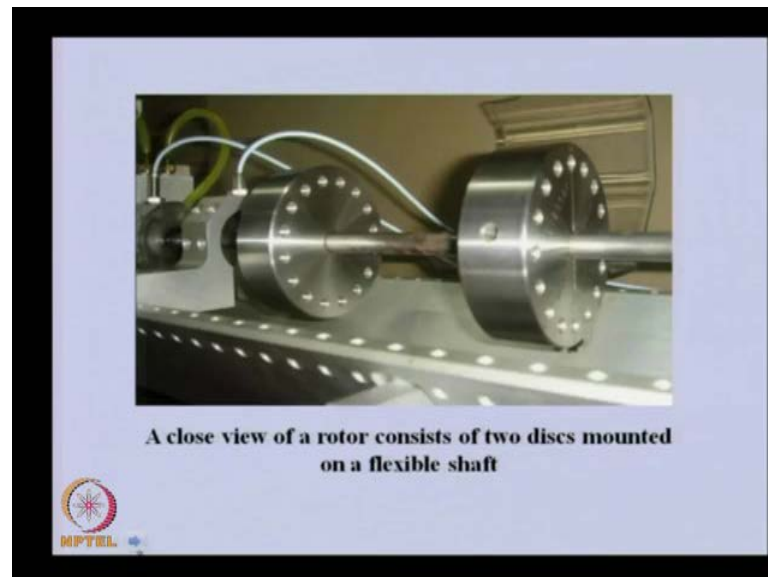
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With this figure, I just want to introduce a basic component of a rotor, especially in the laboratory. So, here is a motor, which drives a shaft through a coupling or the shaft is

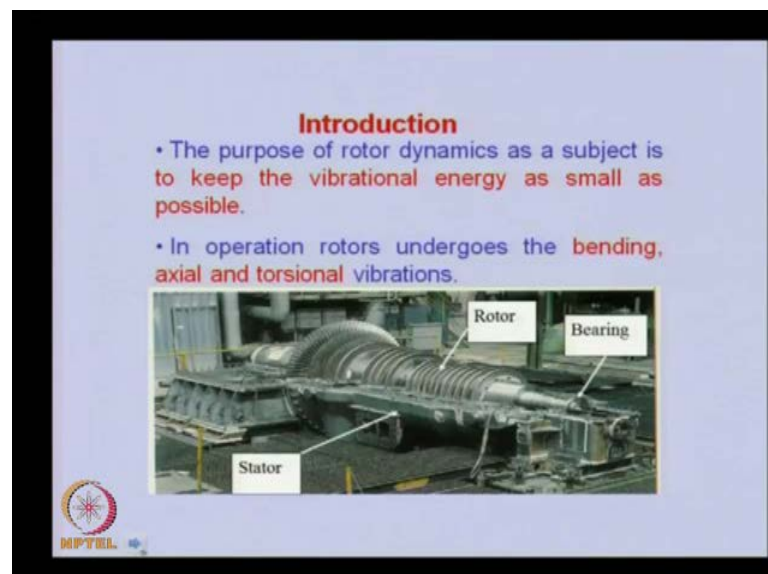
supported on two bearings here. One is bush bearing, another one is the fluid film bearing. On the shaft, there are two mainly discs. Shaft is relatively a mass less or is flexible. There is the rigid base generally, which is fixed to the heavy foundation. So, this is the very basic component of the rotor. So, it is termed in laboratory.

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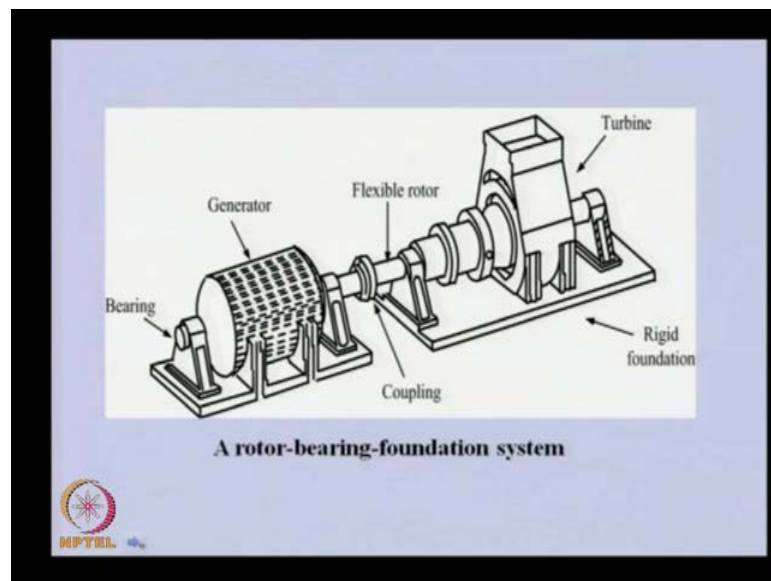
So, here is the close view of the disc. Generally, we provide some kinds of holes in it so that we can be able to provide some kind of unbalanced known and unbalanced to study the unbalanced vibration of the system.

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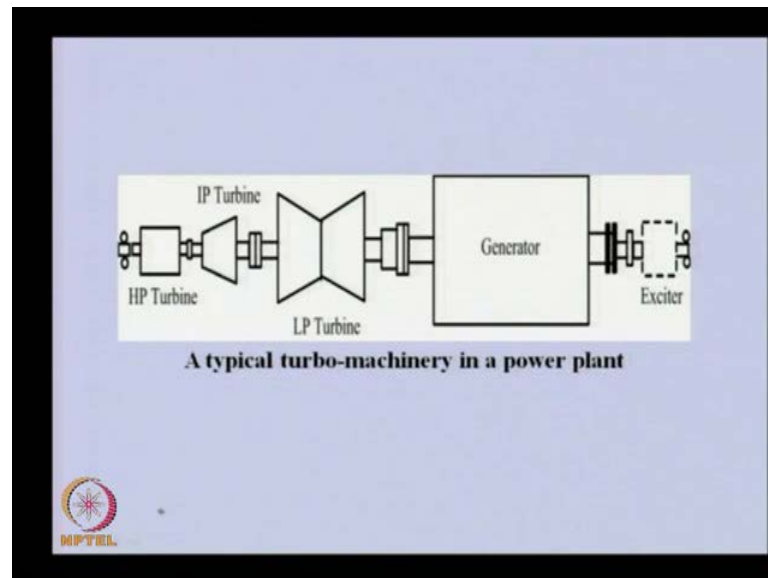
Now, let us see endow steel rotor that is also having similar components, but they are having huge size. You are able to see the bearing here and rotors and various stages of the blades stator foundation which will be mounted on a heavy concrete structure. So, you can able to see that the main purpose of the rotor dynamics as rotor dynamics as a subject is to keep the vibrational energy as small as possible. In operation, rotor undergoes the bending, axial and torsional vibrations.

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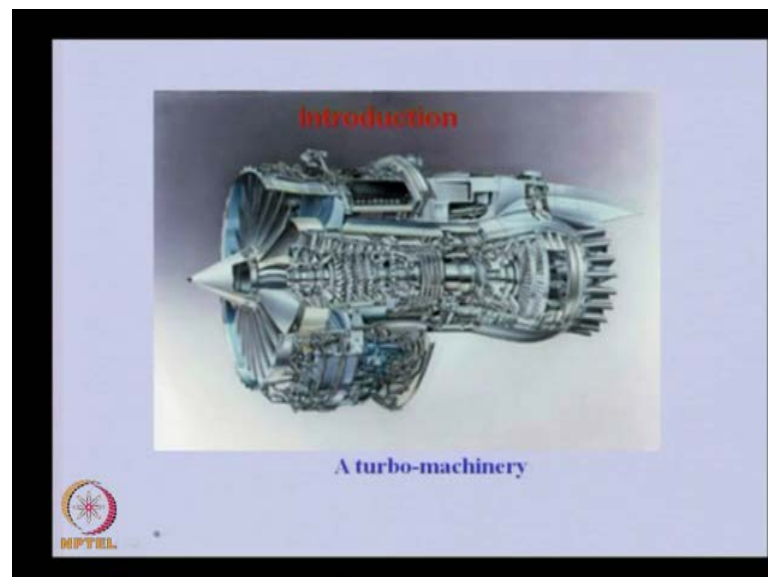
This is very schematic of the same and that is turbine generators at with a coupling in between.

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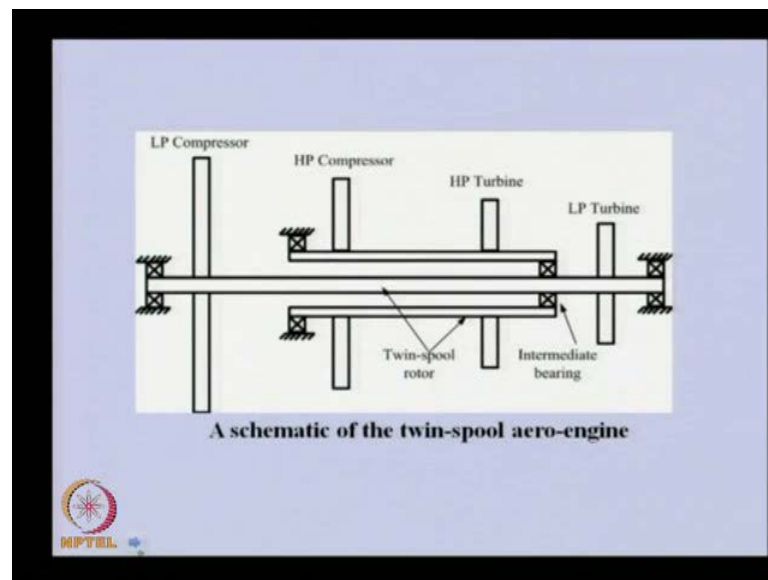
If we have multi staged turbine, then we can have high that is high pressure turbine, low pressure turbine and generator and all are connected by coupling and supported at various bearings here. Even you can be able to see the exciter. So, these all are nothing but rotors, which are mounted on bearings.

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This is another example of the turbo machinery. This is generally used in the air craft.

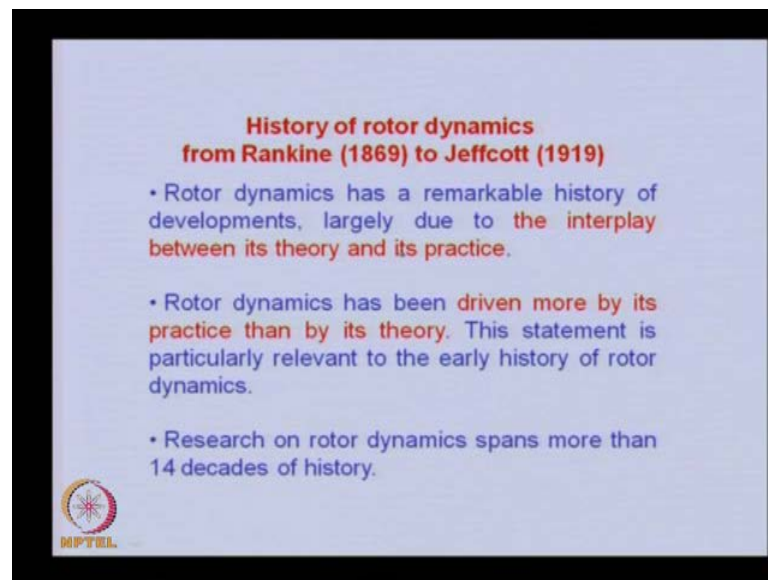
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In this, because of the space constraint, if you see the schematic of this, you can able to see there is a rotor, which is mounting a low pressure compressor and then the other component that is low pressure turbine. The shaft is supported at two bearings. One is at this right hand and another is at stream left hand. You can able to see there is the another rotor, which is concentric with this main rotor on which there is the HP compressor, this one and there is the HP turbine here. They rotate at different speed and they transmit power. You can see that because of this space is constrained, these twin rotors, which are concentric and supported at intermediate bearing, and then they rotate at different speeds.

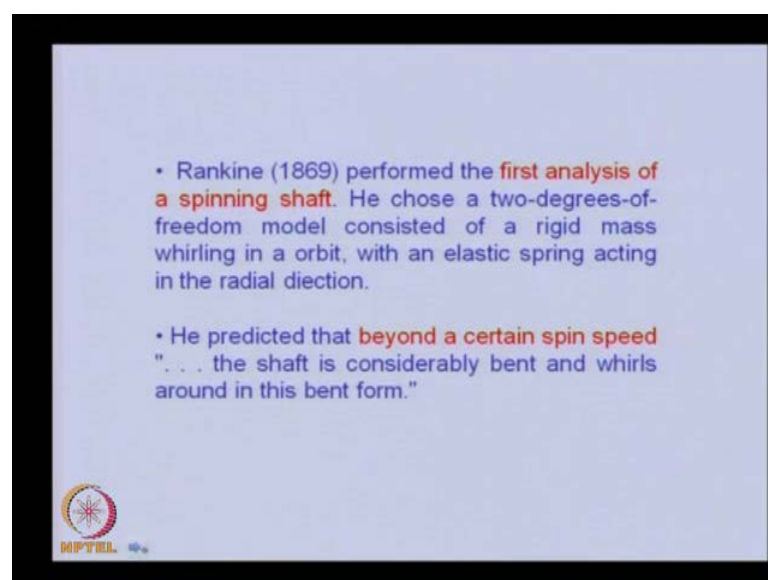
Let us begin with the brief history of rotor dynamics. We will start with rankine and then till now, what are the developments that are taken place, we will study here. You can able to observe during the discussion that is already more than one forty years have passed, when the first rankine analyze the rotor and system. Let us see the history of rotor dynamics.

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Rotor dynamics has a remarkable history of developments, largely due to the interplay between the theory and its practice. So, the practice and theory are in the same type, goes side by side. Rotor dynamics has been driven more by its practice than by its theory. This statement is particularly relevant to the early history of rotor dynamics. Research on this particular rotor dynamics spans more than 14 decades of history.

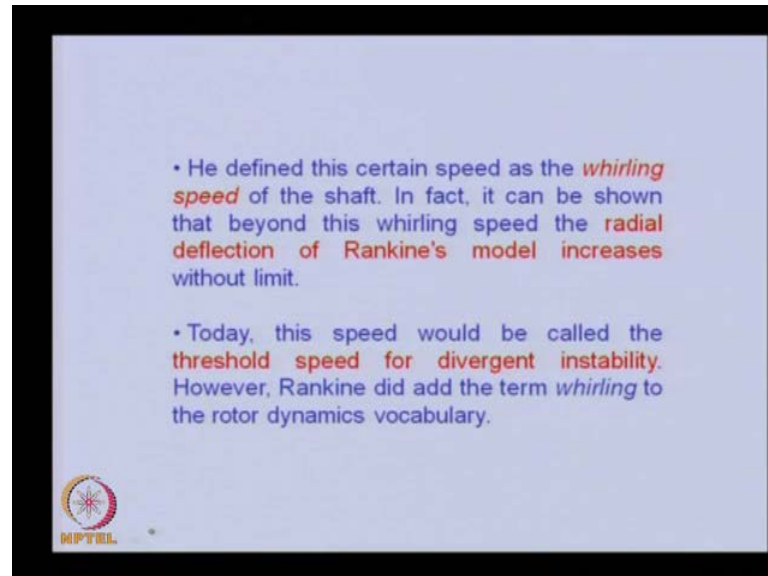
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Rankine in 1869 performed the first analysis of a spinning shaft. He chose a two degrees of freedom model consisted of a rigid mass whirling in an orbit, with elastic spring

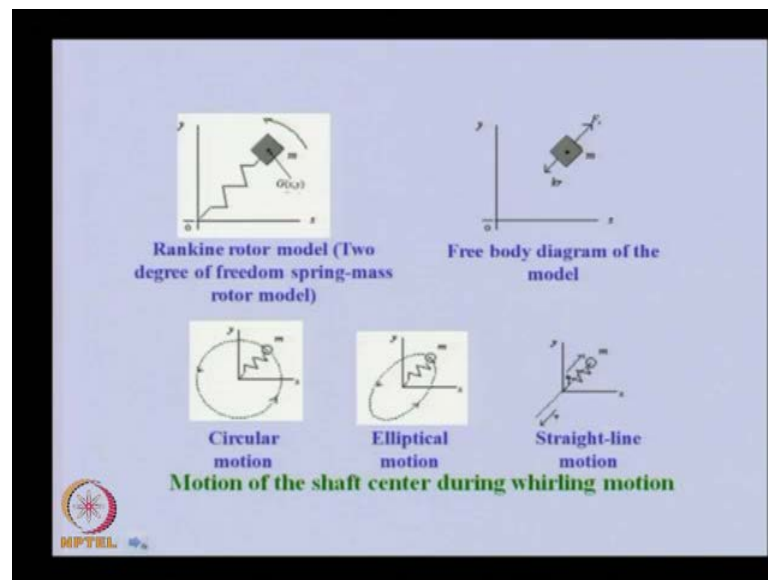
acting in the radial direction. He predicted that beyond a certain spin speed, the shaft is considerably bent and whirls around in this bent form.

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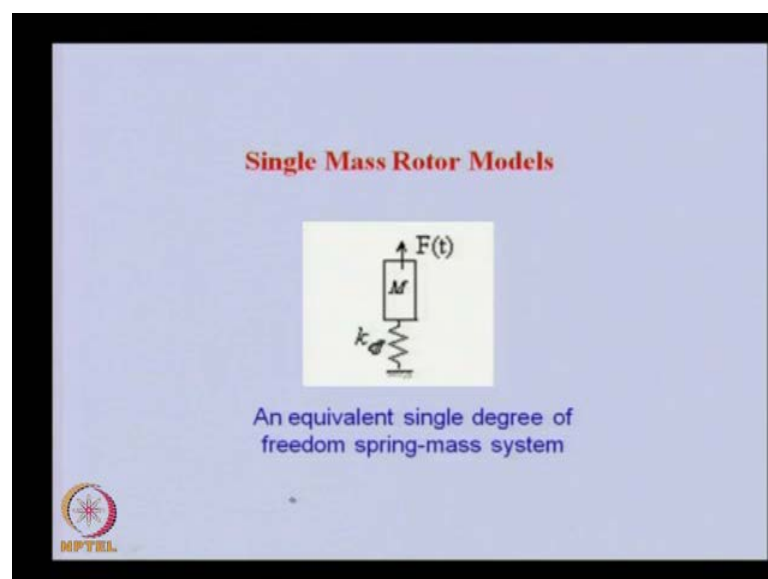
He defined this certain speed as the whirling speed of the shaft. In fact, it can be shown that beyond this whirling speed, the radial deflection of rankine's model increases without limit. So, it goes to an unstable region. Today, this speed would be called the threshold speed for divergent instability. However, rankine did add the term whirling to the rotor dynamics vocabulary. So, whirling was first was used by rankine. His analysis was having some flop as we will see in subsequent style.

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So, this is the subsequent model in which he considered a rotor as the mass and rotor by a spring. Spring has directional change in the thickness. You can able to see that of this particular rotor will be having some orbit. Depending upon the stiffness of this spring, it can have a circular motion or elliptical motion. If there is a cross coupled term and a symmetry in the support, you can have elliptical or in extreme case, it can or elliptical, it can have a straight line motion.


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Basically there is nothing but a single degree of freedom mass system in which that particular model can be analyzed. It is the very basic model of spring and mass temper, system temper is not there in this an external whirling due to the unbalance.

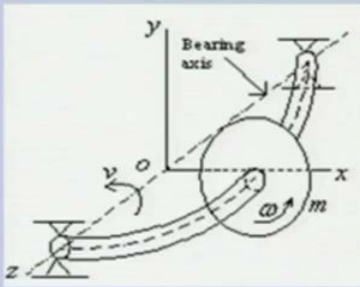
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- **Whirling** refers to the movement of the centre of mass of the rotor in a plane perpendicular to the shaft.
- The **frequency of whirl** depends on the stiffness and damping of the rotor and the **amplitude** is a function of the excitation force's frequency and magnitude.
- A **critical speed** occurs when the excitation frequency coincides with a natural frequency, and can lead to excessive vibration amplitudes.




So, now let us see our whirling. It refers to the movement of the centre of mass of the rotor in a plane perpendicular to the shaft.

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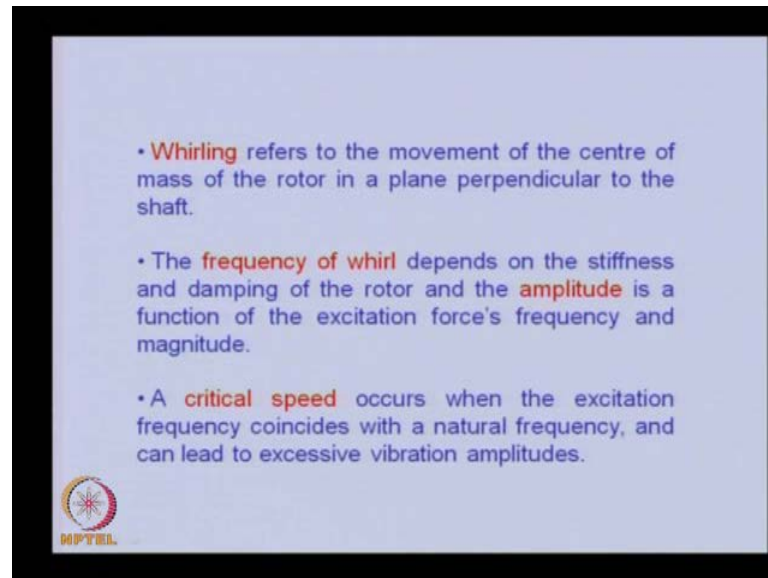
A Jeffcott (or Laval) rotor model in a general whirling motion



To have more understanding of this, let us see this figure in which the bearing axis and there is the shaft, which is bent. Now, this has bent because of its bending on its own axis

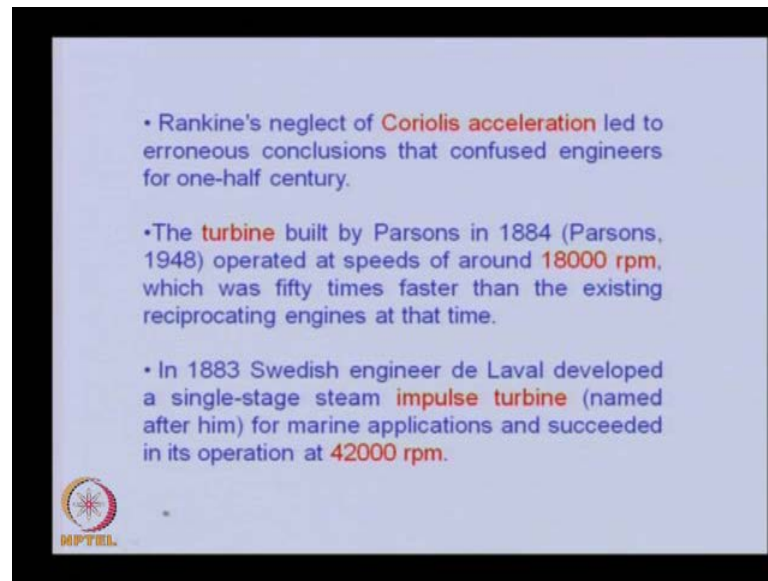
at ω and this prime shaft itself is whirling about the bearing axis there is that frequency is new that is called whirling frequency. You can see that whirling is nothing but the bent shaft revolved about the bearing axis. That motion is called whirling.

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The frequency of whirl depends on the stiffness and damping of the reactor and the amplitude is a function of the excitation force's frequency and magnitude. There is the concept of critical speed also in the rotor. The critical speed occurs when the excitation frequency which comes because of the unbalance coincides with the natural frequency, and can lead to excessive vibration amplitudes.

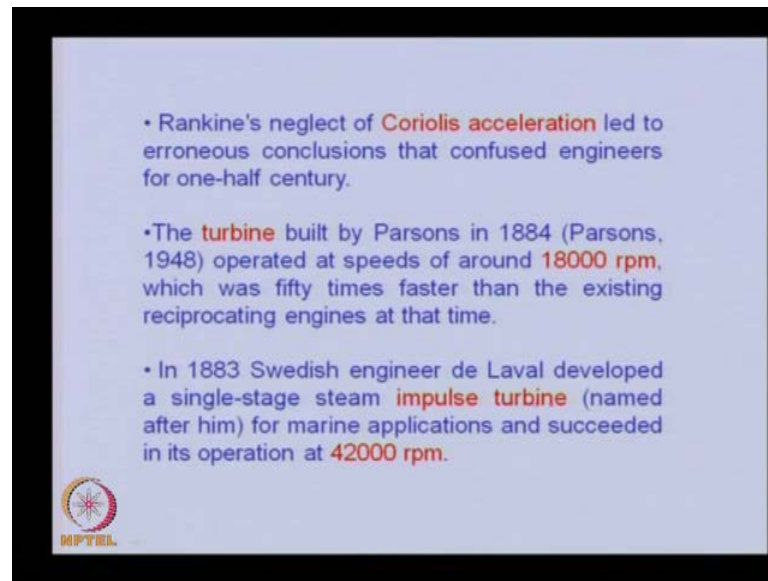
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In this particular Rankine's model, as I pointed out, there were some flops in model. So, Rankine's neglect of Coriolis acceleration led to erroneous conclusions that confused engineers for half, one half century. The turbine built by Parsons in 1884 operated at speeds of around 18000 rpm, which was fifty times faster than the existing reciprocating engines at that time.

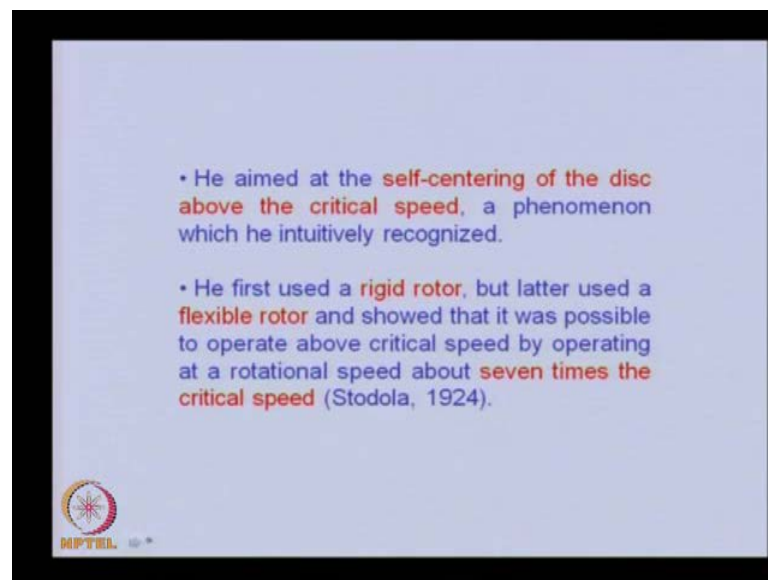
Actually, Rankine, he predicted that there is the certain speed beyond which the rotor cannot be operated because it goes into the unstable region and you cannot able to operate the rotor. That discouraged the industry to design rotors, which can operate at very high speed. Rankine was such very renowned person. So, no one could able to object his projections, but you can able to see his like Parsons, like he actually made rotor and they were operating at very high speed.

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Similarly, in 1883, Swedish engineer de Laval developed his single stage stream turbine that is impulse turbine that is named after him for marine applications and succeeded in its operation at 42000 rpm, which was very high speed at that time.

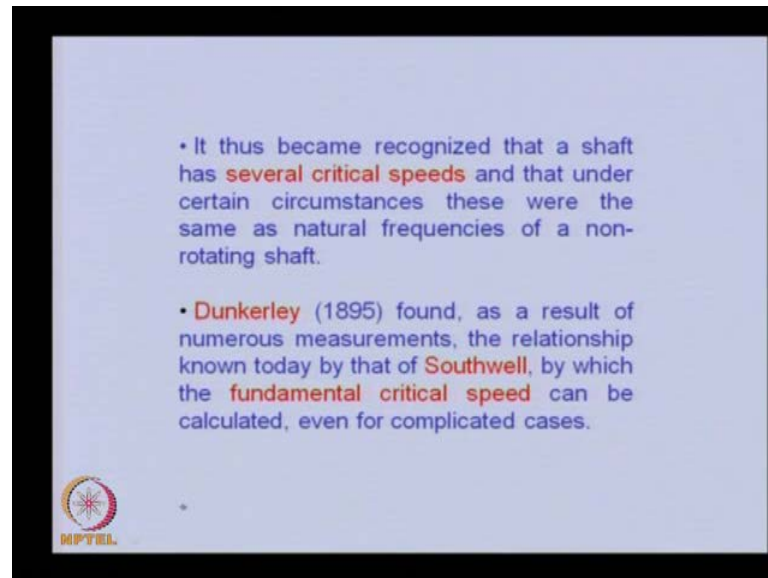
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He aimed at the self centering of the disc above the critical speed, which he intuitively recognized that was actually contradicting the Rankine's prediction that you cannot be able to operate this rotor beyond a certain speed, but he was getting in actual centering, self centering of the rotor when it is operating at a critical speed that is it was operating

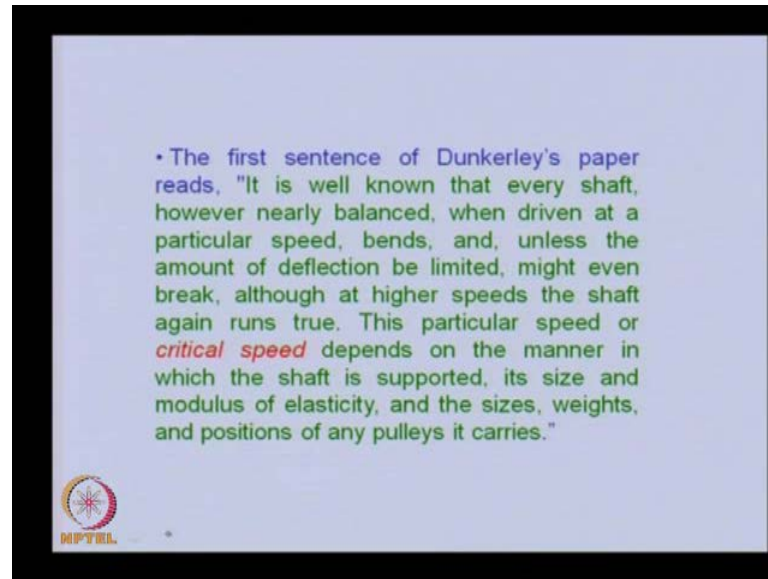
at more of stable or form. He first used the rigid rotor, but later used a flexible rotor and showed that it was possible to operate above critical speed by operating at a rotational speed about seven times the critical speed. So, that was the real experiment.

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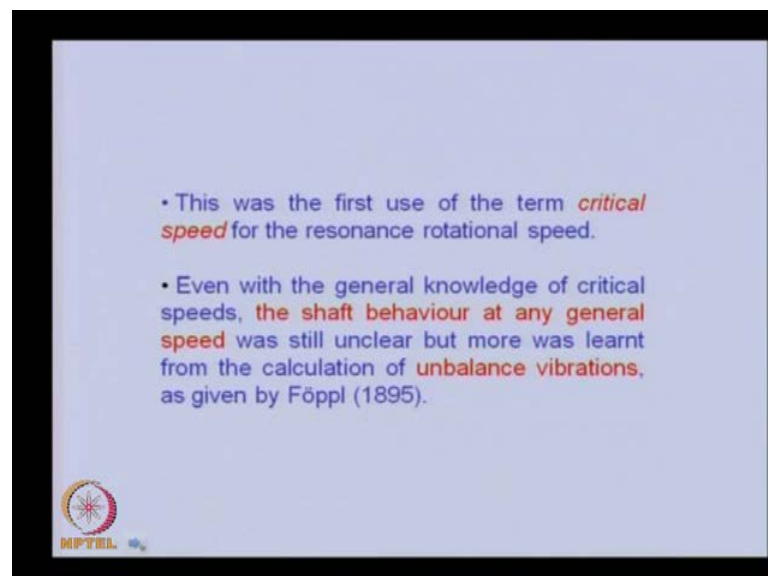
It thus became recognized that a shaft has several critical speeds and that under certain circumstances; these were the same as natural frequencies of a non rotating shaft. Dunkerly in 1895 found, as a result of numerous measurements, the relationship known today by that of Southwell, by which the fundamental critical speed can be calculated, even for complicated cases.

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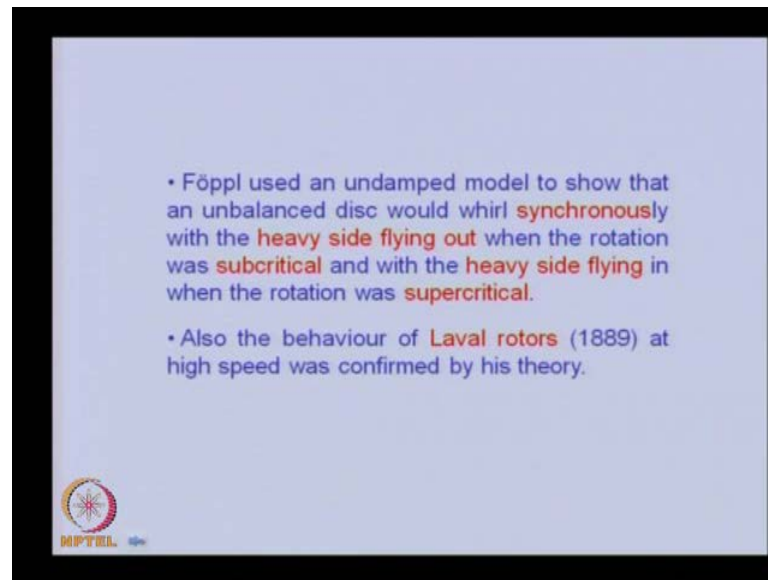
So, this is the first sentence of the Dunkerley's paper, which reads as it is well known that every shaft, however nearly balanced, when driven at a particular speed bends, and unless the amount of deflection be limited, might even break, although at higher speeds the shaft again runs true. This particular speed or critical speed depends on the manner in which the shaft is supported, the size and the modulus of elasticity and the sizes, weights, and positions of any pulleys it carries.

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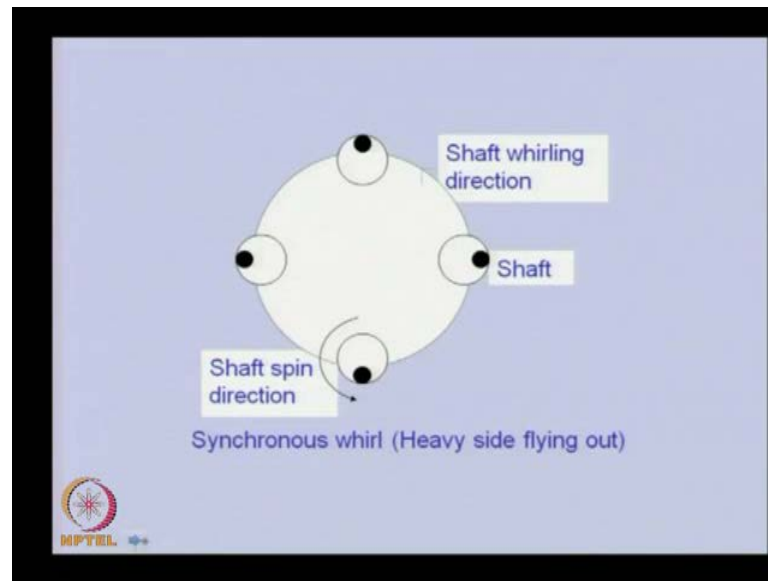
This was for the first time use of the term critical speed for the resonance rotational speed was used. Even with the general knowledge of critical speeds, the shaft was still unclear, but more was learnt from the calculation of unbalance vibrations, as given by Foppl in 1895.

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Foppl used undamped model to show that an unbalanced disc would whirl synchronously with the heavy side flying out when the rotation was subcritical and with the heavy side flying in when the rotation was supercritical. Also, the behavior of the Laval rotors in 1889 at high speed was confirmed by this theory. Now, let us see through the animation what is the synchronous whirl and the heavy side is flying out and when the heavy side is flying in at the supercritical region. So, let us see through.

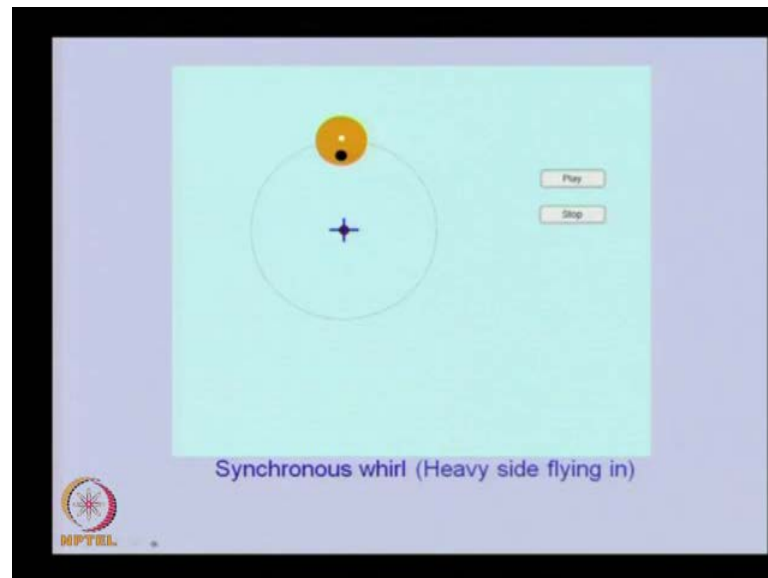
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Here we can be able to see that this is the shaft which is whirling; this is the bearing center and the heavy side that means where the unbalance is there is outside. We are considering the synchronous whirl because of that when it rotates along the orbit, always the heavy side will be outside. You can see that the spin direction is counter clock wise and also the whirling also is counter clockwise in this particular case.

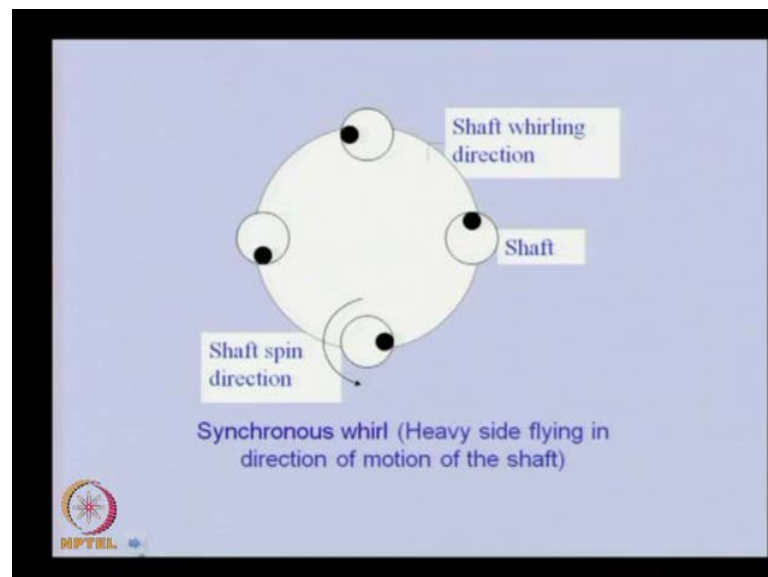
So, this is the motion in which the heavy side is always outside, the same motion as the earth and moon is having. This is the moon and earth is at the centre portion, that kind of motion we can have. Once we cross the critical speed, this heavy side comes inside and you can able to see that always this is inside the whirling.

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This is the animation in that in which the rotor is whirling. Generally, this will occur when we are operating about the critical speed, heavy side will be inside all the time and because of that, you can see that vibrations will be less because the unbalanced force will be towards the bearing access.

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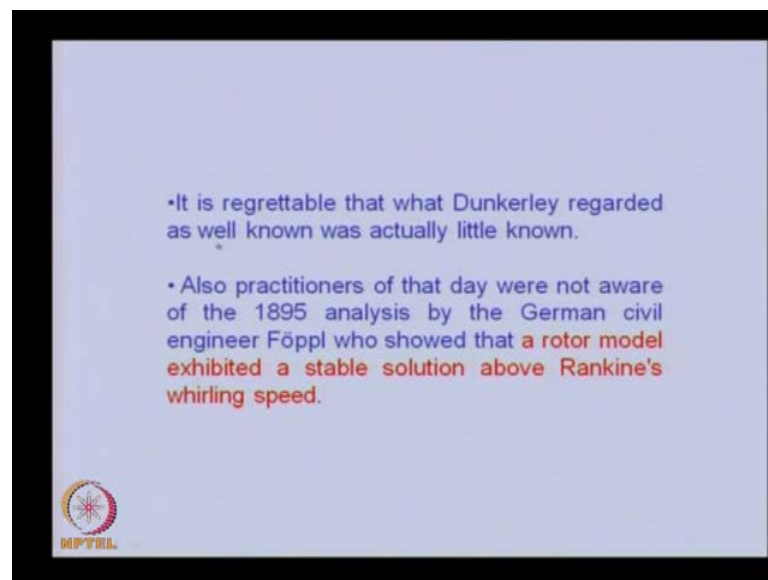


This is another kind of motion in which...actually this is the critical speed. At critical speed, this heavy side will be in the direction of motion. So, it will be tangent to the orbit

motion and as we know at critical speed, large oscillations take place and there response is increases with time and this is a dangerous situation.

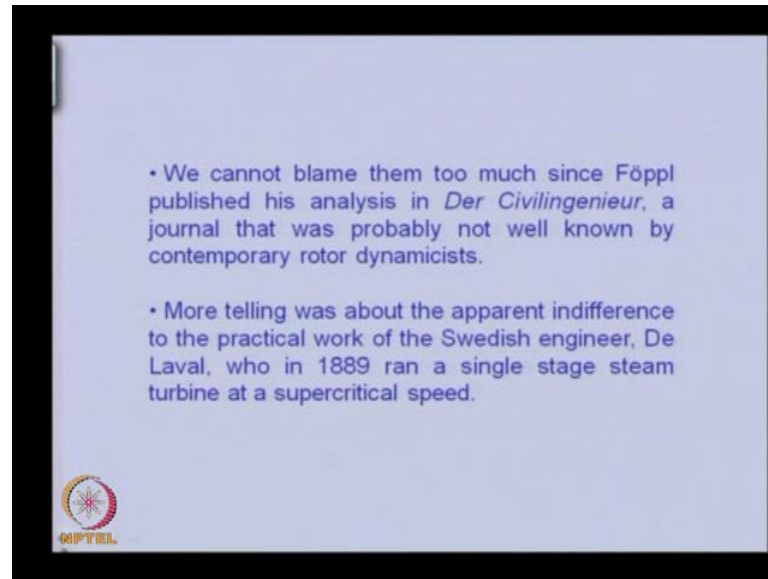
This is another vault in which anti synchronous that means the spinning of the shaft and the whirling of the shaft directions are different. In this particular case, the shaft is rotating in the clockwise direction and the whirling direction is counter clockwise direction. So, in this particular case, the heavy side will be coming in and out at every 90 degree. This is the animation of that particular motion.

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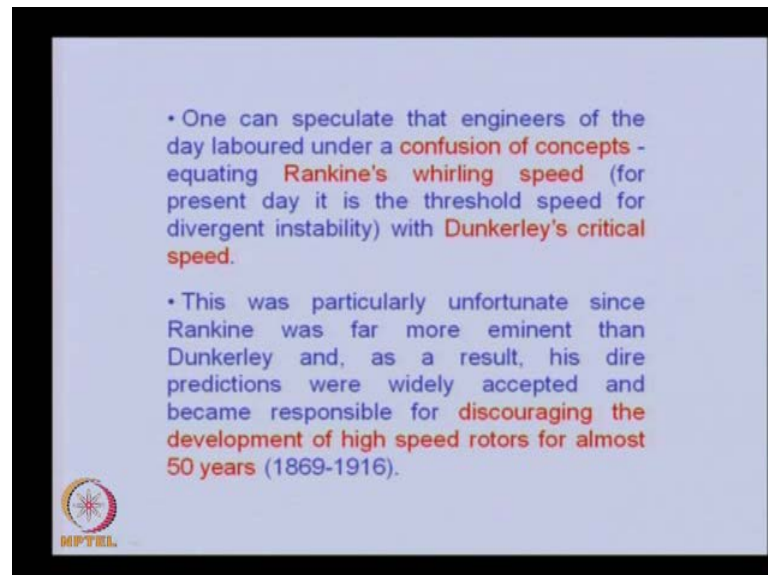
Now, we will see what was the contradiction between the Dunkerley's calculation of lateral frequency and Rankine's model? So, you can able to see that it is regrettable that Dunkerley regarded as well-known was actually little known. Also practitioners of that day were not aware of 1895 analysis by the German civil engineer Foppl who showed that rotor model exhibited a stable solution above Rankine's whirling speed.

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We cannot blame them too much since Föppl published his analysis in a journal that was probably not well known by contemporary rotor dynamicists. More telling was about the apparent indifference to the practical work of the Swedish engineer de Laval who in 1889 ran a single steam turbine at a supercritical speed.

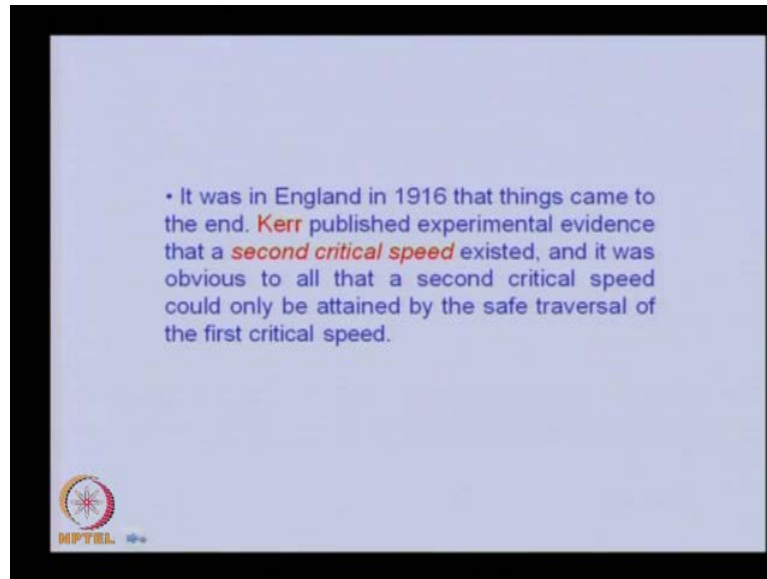
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One can speculate engineers of those days labored under a confusion of concepts, equating Rankine's whirling speed, for present day it is the threshold speed for divergent instability, with the Dunkerley's critical speed. There was not matching and people were

confused what to do with these two calculations, which are not matching. This was particularly unfortunate since Rankine was far more eminent than Dunkerly and as a result, his dire predictions were widely accepted even when it was erroneous and became responsible for discouraging the development of high speed rotors for almost 50 years, up to 1916.

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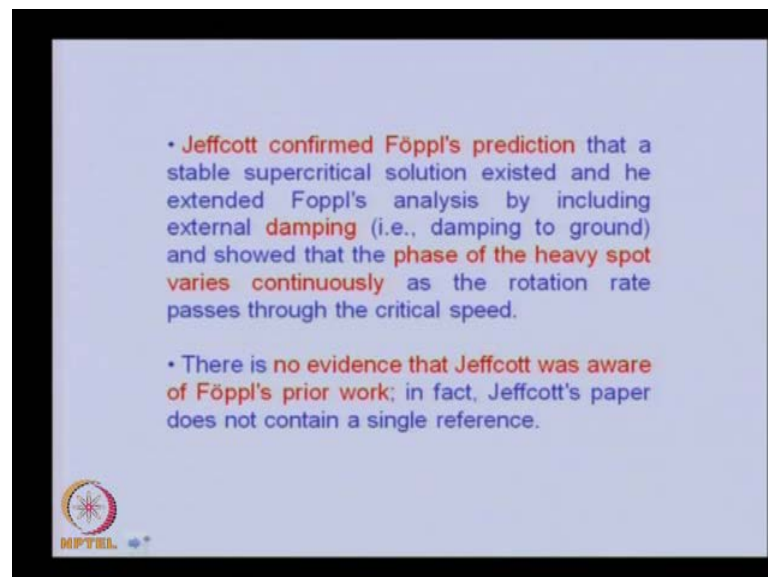
It was in England in 1916 that things became things came to the end. Kerr published experimental evidence that a second critical speed existed, and it was obvious to all that a second critical speed could only the attained by the safe traversal of the first critical speed.

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Then, the royal society of London commissioned Jeffcott to resolve this conflict between Rankine's theory and the practice of Kerr and de Laval. The first recorded fundamental theory of rotors dynamics can be found in a classical paper of Jeffcott in 1919, in a place where it was more likely to read by those interested in rotor dynamics.

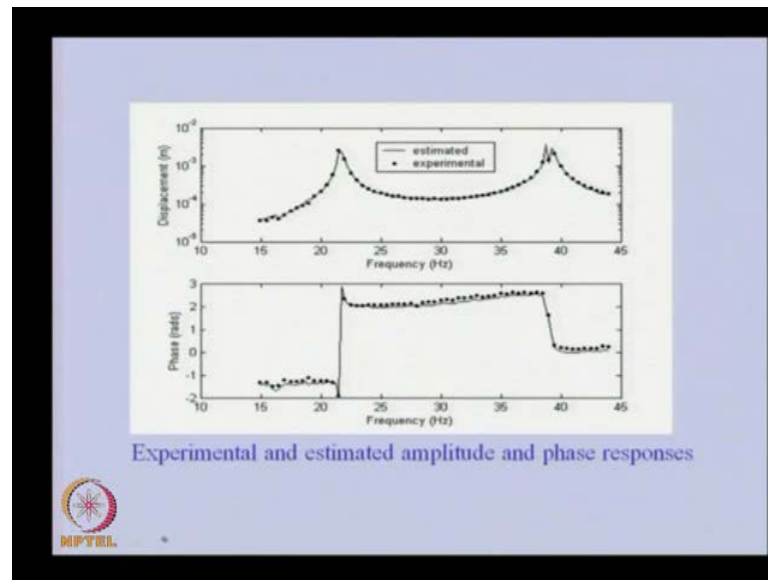
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Jeffcott confirmed Föppl's prediction that a stable supercritical solution existed and he extended Föppl's analysis by including external damping that is damping to the ground and showed that phase of heavy spot varies continuously as the rotation rate passes

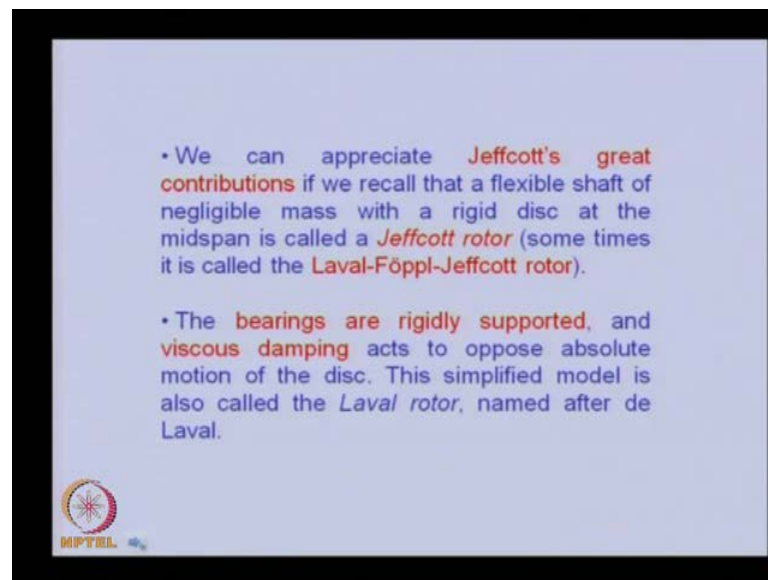
through the critical speed. There is no evidence that Jeffcott was aware of Foppl's prior work. In fact, Jeffcott's paper does not contain a single reference. So, there is no way we can be able to be sure whether the Jeffcott knew the Foppl's work or the similar work by Foppl's.

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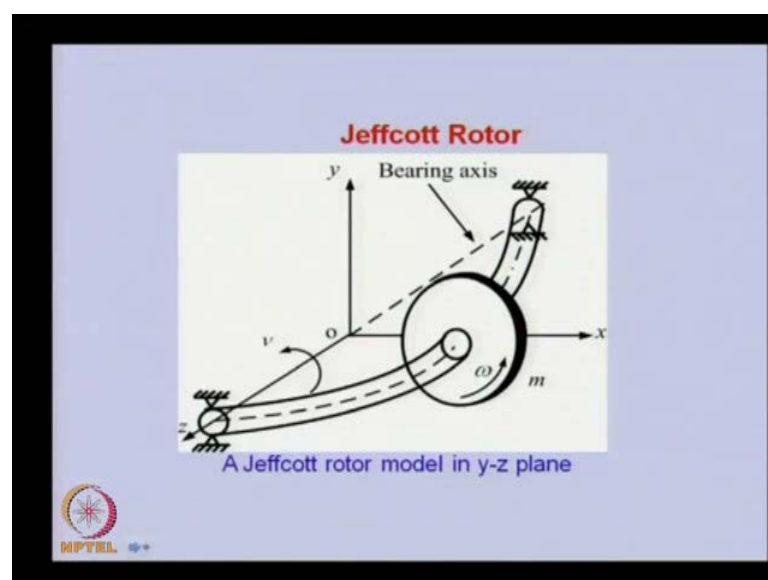
So, this is the typical plot for rotor. You can able to see that this is the spinning speed of the shaft. It is increasing from some from 15 to let us say 45 hertz. This is the displacement. So, whenever the spin speeds was close to critical speeds, there was high amplitude. You can able to see the second plot is a phase plot there is a change in the phase of around pi radians and even at the second critical speed, there was a change in the phase.

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We can appreciate Jeffcott's great contributions if we recall that a flexible shaft of negligible mass with a rigid disc at the mid span is called a Jeffcott rotor in rotors dynamics community. Sometimes it is also called the Laval Foppl Jeffcott rotor. The bearings are rigidly supported, and viscous damping acts to oppose absolute motion of the disc. This simplified model is also called the Laval rotor, named after de Laval.

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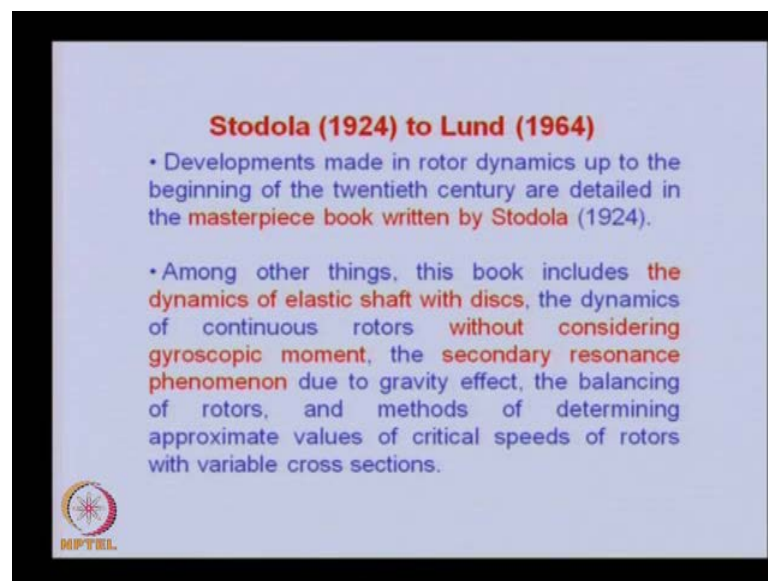


This is a diagram of Jeffcott rotor in which there is a flexible rotor, which is a mass less rigid disc and the bearings are rigid. They allow only the rotational motion of the shaft

and the disc is at the mid span of the shaft. This particular model is Jeffcott model. In rotor dynamics, this Jeffcott rotor model is very popular. We will see that this particular model can be use to predict various kind of phenomena in rotors dynamics. There is a very famous and very important rotor model in rotor dynamics as we will see in the subsequent lectures.

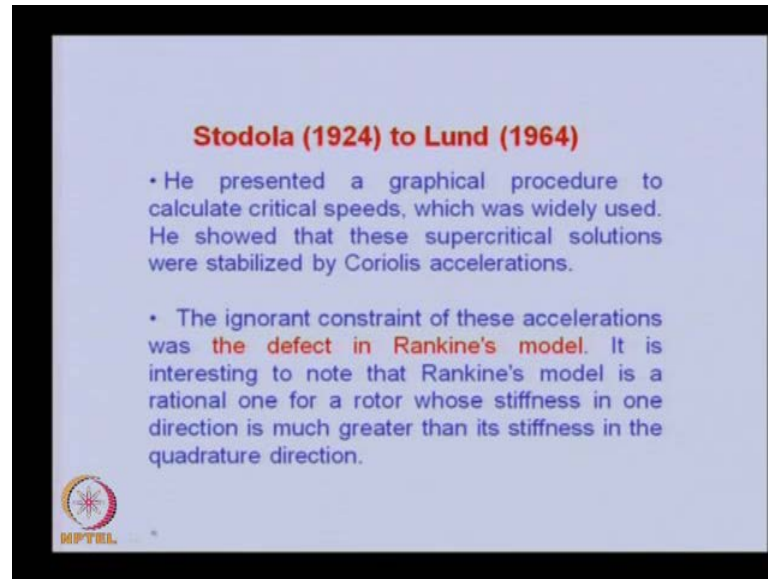
So, after Jeffcott's publication, lot of confusion which where there in the calculation of critical speed was clear and then so many other developments started, specially engineers, they added making high speeds rotors, but they had some others problems. So, in the second phase of this review, we will see what are the various problems industrialist they face and then what was the other phenomena people could able to, they could able to analyze.

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So, in this particular case, second phase is in which I will be starting from Stodola, it was from 1924 to Lund up to 1964. So, developments made in rotor dynamics up to the beginning of the twenty eight century are detailed in the master piece book written by Stodola. Among other things, this book includes the dynamics of elastic shaft with discs, the dynamics of continuous rotor without considering gyroscopic moment, the secondary resonance phenomenon due to gravity effect, the balancing of rotors, which is of very much practical importance and methods of determining approximate values of critical speeds of rotors with variable crosses sections.

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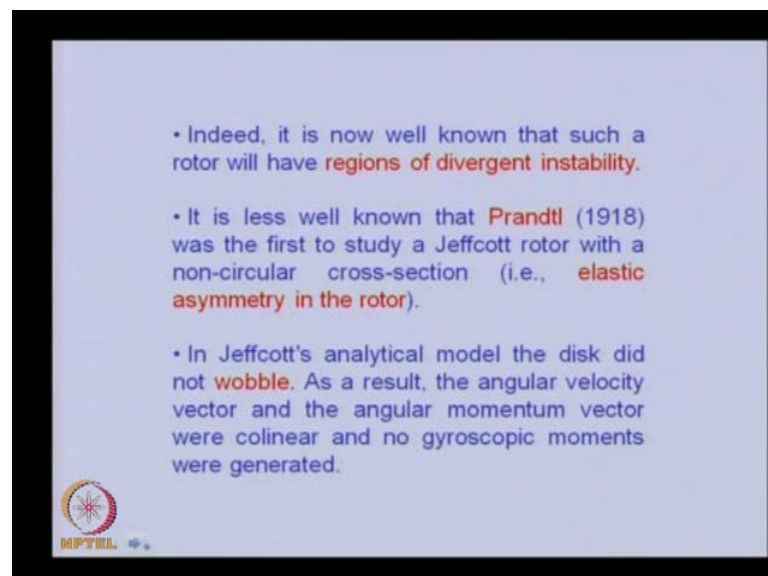
Stodola (1924) to Lund (1964)

- He presented a graphical procedure to calculate critical speeds, which was widely used. He showed that these supercritical solutions were stabilized by Coriolis accelerations.
- The ignorant constraint of these accelerations was **the defect in Rankine's model**. It is interesting to note that Rankine's model is a rational one for a rotor whose stiffness in one direction is much greater than its stiffness in the quadrature direction.

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He presented a graphical procedure to calculate critical speeds, which was widely used. In industry, he showed that these supercritical speeds solutions were stabilized by Coriolis accelerations. The ignorant constraint of these accelerations was the defect in Rankine's model. It is interesting to note that Rankine's model is a rational one for a rotor whose stiffness in one direction is much greater than its stiffness in the quadrature direction.

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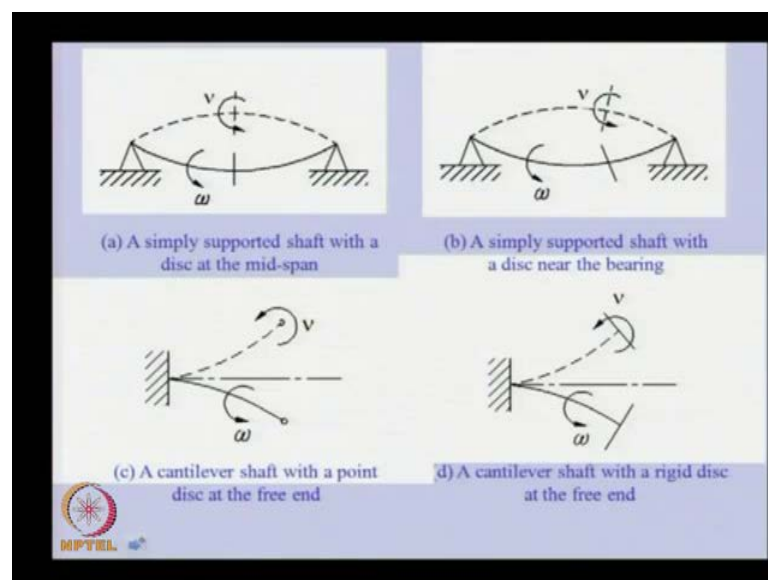


- Indeed, it is now well known that such a rotor will have **regions of divergent instability**.
- It is less well known that **Prandtl (1918)** was the first to study a Jeffcott rotor with a non-circular cross-section (i.e., **elastic asymmetry in the rotor**).
- In Jeffcott's analytical model the disk did not **wobble**. As a result, the angular velocity vector and the angular momentum vector were colinear and no gyroscopic moments were generated.

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Indeed, it is now well known that such a rotor will have regions of divergent instability. It is less well known that Prandtl in 1918 was the first to study a Jeffcott rotor with a non circular cross section that is elastic asymmetry in the rotor. In Jeffcott's analytical model, the disc did not wobble because this particular phenomenon was not occurring because in the case of the Jeffcott rotor, the disc was at a mid span. When the whirling was taking place, the disc was not having any tilting and because of that, it was not wobbling. So, in the Jeffcott analytical model, the disc did not wobble. As a result, the angular velocity vector and the angular momentum vector were collinear and non gyroscopic moments were generated.

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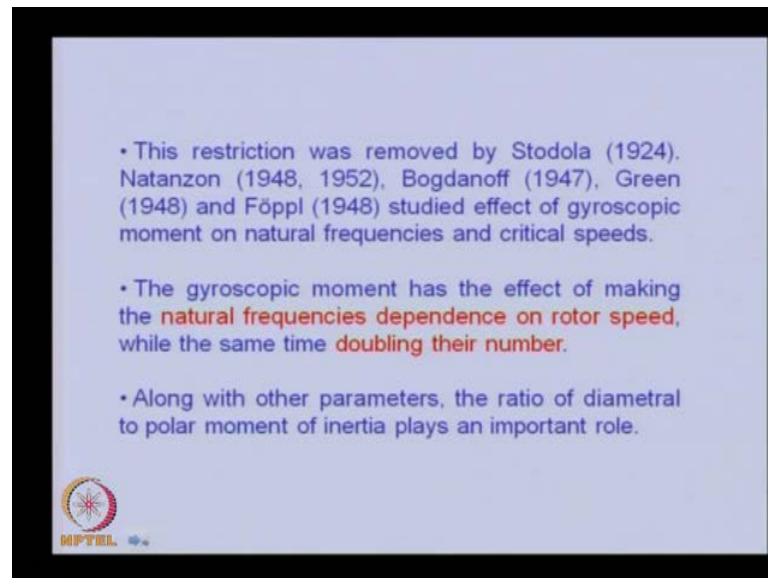


So, you can able see this is the Jeffcott rotor model in which the disk is at the mid span. So, during whirling, this disc will remain vertical. It will not tilt because it is at the mid span. It is actually tangent to the elastic line of the shaft, but in the second case, the disc is the slightly offset. Then during whirling, obviously this disc will tilt and because it is spinning about its own axis and tilting about its diameter, there will be a gyroscopic couple. So, these two cases, you can able to expect that there will be different natural frequency of this system because of this tilting motions and because of this tilting motion, there is a gyroscopic couple.

There is another model. It is a cantilever beam. You can able to see this. Here, we have a point mass. This is spinning about its own axis ω and the whirling at ν . This is

another one in which there is a disc instead of a point mass. It is spinning about its own axis with the ω and the whirling frequency is ν . So, you can expect that the figures c and d, they will be having different natural frequency. They will be having different particles speeds because of gyroscopic moment because here point mass is there. So, here there will not be any gyroscopic moment, but here gyroscopic moment will be there.

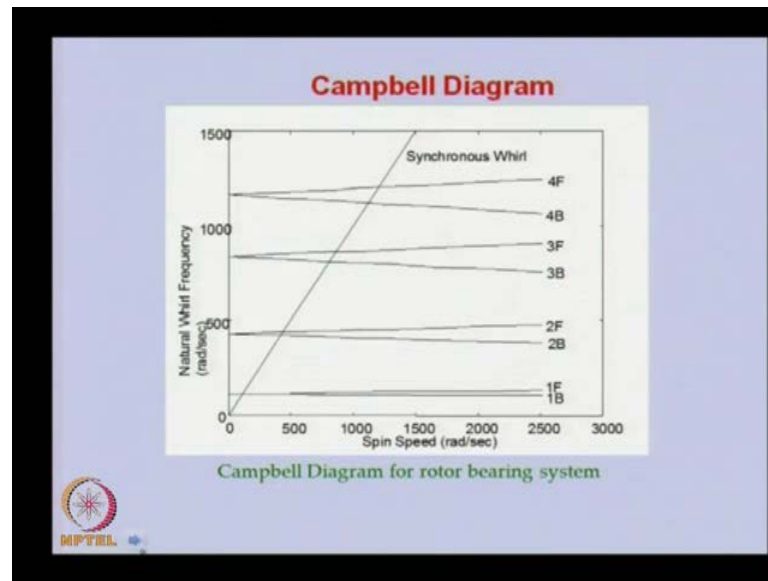
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So, the restriction was removed by Stodola and others, the restrictions of the Jeffcott rotor in which the disc was at the middle and because of that, the gyroscopic couple was not there, so these researches the study, the effect of the gyroscopic moments on natural frequencies and critical speeds. The gyroscopic moments has the effect of making the natural frequencies dependence on the rotor speed. This is very important phenomenon and also not only natural frequencies depend upon the rotor speed, also at the same time, doubling of their number.

The natural frequency becomes twice. This we will see in subsequent plot how the natural frequency becomes twice as compared to non gyroscopic couple case. Along with the other parameters, the ratio of diametral to polar moment term of inertia plays an important role in this particular gyroscopic effect.

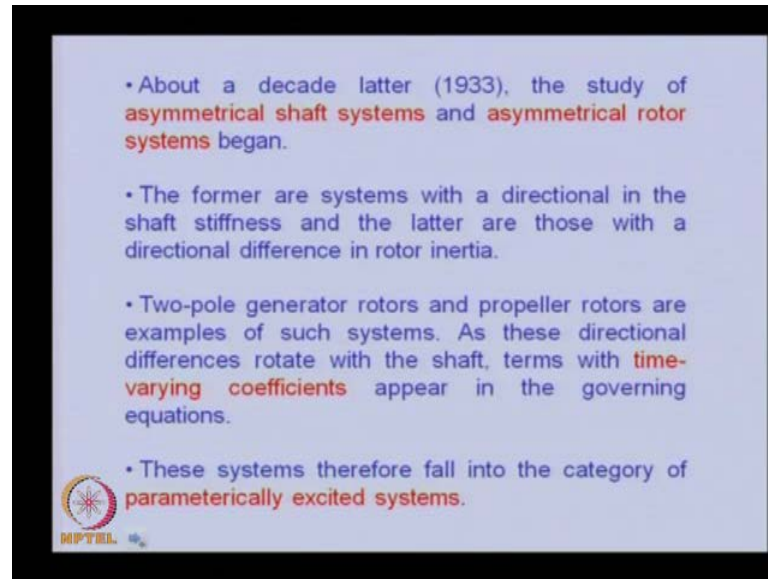
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So, this is a famous diagram in rotor dynamics that is called Campbell diagram. In this diagram, the horizontal axis is the spin speed of the shaft; the vertical axis is the natural whirling frequency. At zero speed, you can able to see that these are the first natural frequency, this first natural frequency, third, fourth, several frequencies are there and because in this particular case, the gyroscopic couple has been considered, so as we will increase the spin speed, the splitting of the natural frequency takes place.

They belong to forward whirl and backward whirl. You can able to see even this got split. As we are going on higher and higher, the splitting is more because of the gyroscopic couple, and especially when we are operating at high speed, the splitting is more. This is the line, which represents forty five degree line. Along this line, the spin speed and the whirl frequencies are same. Wherever they are intersecting the natural frequencies line, those are critical speed. This concept we will be seeing in more detail as we go into the subject.

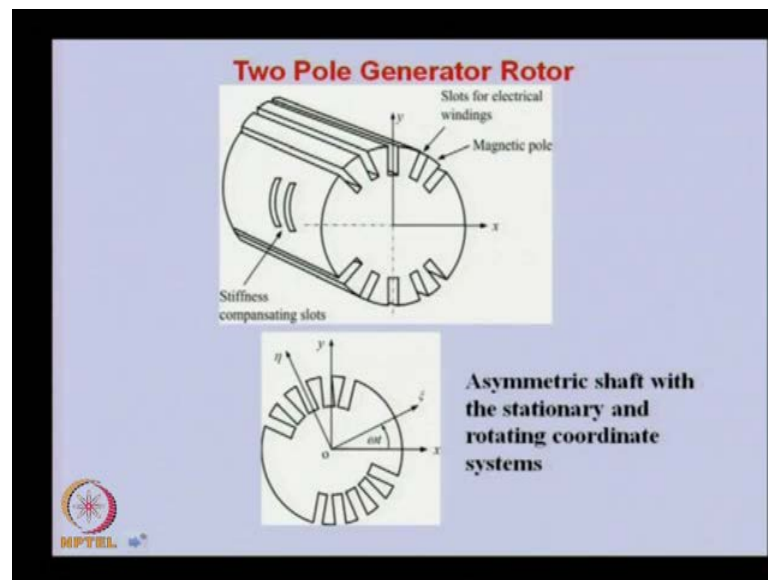
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So, after the Jeffcott's analysis about a decade later in 1933, the study of shaft system and asymmetrical rotor system began. These two rotor systems, we will see what are the differences between them? The former are systems with a directional in the shaft stiffness. The symmetrical shaft system is in which there is the directional in shaft stiffness. That is the shaft stiffness changes with the direction as it rotates.

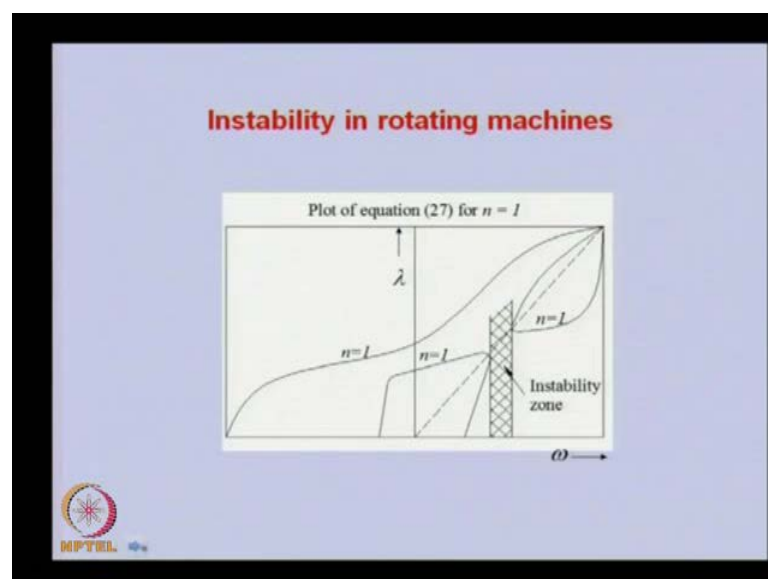
The latter one is those in which there is directional difference in the rotor inertia. With some figures, we will see these two kinds of symmetry systems and rotor system like there are two pole generator rotors and propeller rotors are the examples of such systems. Two pole generator systems is the example of asymmetrical shaft system. Propeller rotors are the example of symmetrical rotor systems. As this directional system rotates with the shaft, the terms with the time varying coefficients appears in the governing equations. These analysis systems therefore fall into the category of parametrically excited systems.

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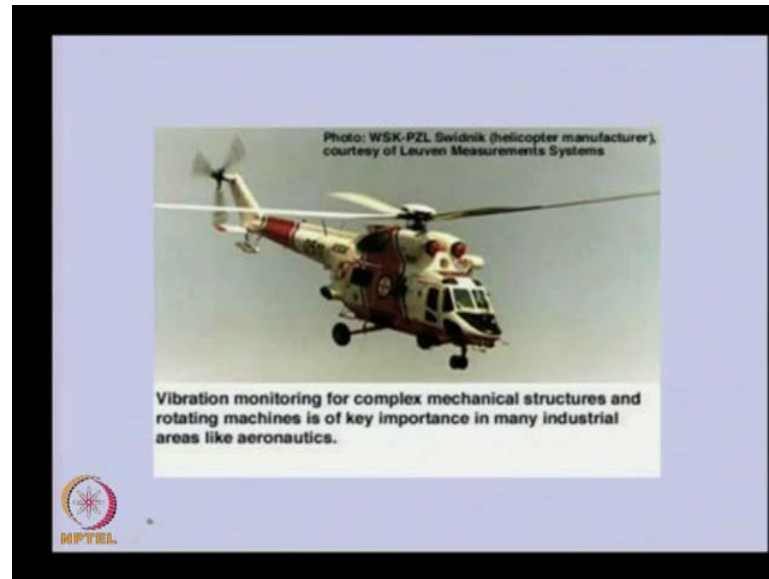
So, let us see with diagram what is two pole rotor. We can able to see this rotor in this generator. Here some slots are there for winding and these are the holes for because of this shaft, the stiffness in one of the plane will be less as compared to another one. To compensate that, some slots are made in other direction also, but still, when such rotor rotates, we can expect there will be change in stiffness with respect to time and that gives parametric excitation to the system.

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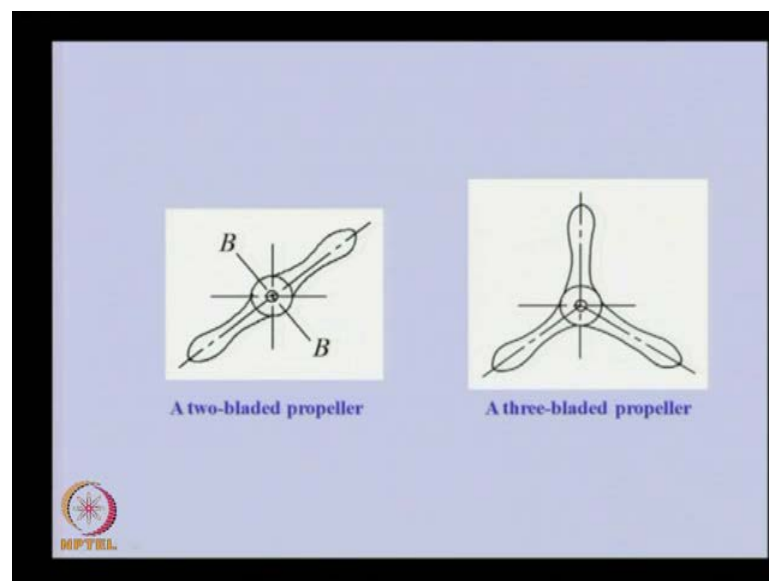
The Campbell diagram, which we drew earlier, this was the spin speed, this was the whirl frequency. We will find that at certain regions, the natural frequency will be complex. These are nothing but instability zones. These analysis, we will study in the subsequently during the study of instability of the rotors. The rotors will be highly unstable in these regions.

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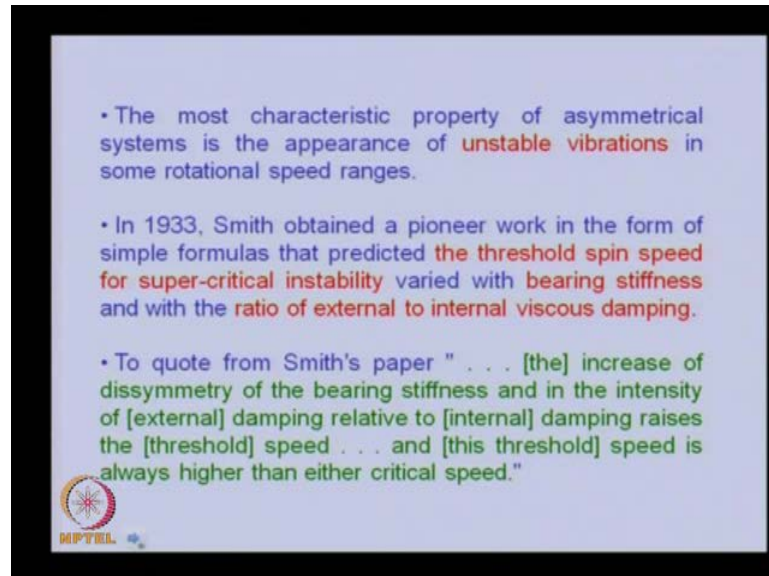
This is another example in which the rotor asymmetry is there.

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So, we can able to see as this propeller rotates about the fixed axis, their diameter mass movement would change and because of that, there will be the rotor mass asymmetry.

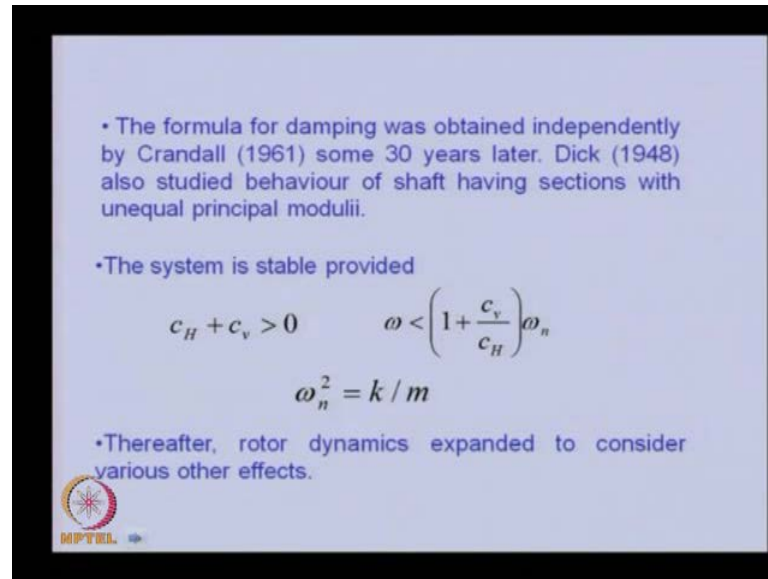
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The most characteristic of the asymmetrical systems is the appearance of the unstable vibrations as we had seen in the Campbell diagram in some rotational speed ranges. In 1933, Smith obtained a pioneer work in the in the form of simple formulation that predicted the threshold spin speed for super critical instability with the ratio of the external to internal viscous damping. This was a very simple formula which Smith gave. This is the quote from Smith's paper that the increase of the dissymmetry of the bearing stiffness and in the intensity of the external damping relative to the internal damping raises the threshold speed. This threshold speed is always higher than either critical speed.

In rotor dynamics, there are two type of damping that we talk about. One is external damping, another is internal damping. External damping comes from the bearings or if there is an introduction of the working fluid with the rotor. Internal damping is of the rotor material itself since, because of the whirling of the shaft, there is inter nuclear interaction and because of that, there will be the tension, compression of the rotor will take place. Flexible vibration of the rotor will take place and that will generate heat. So, there is internal damping. So, generally the internal damping uses stability, but the external damping stabilizes that.

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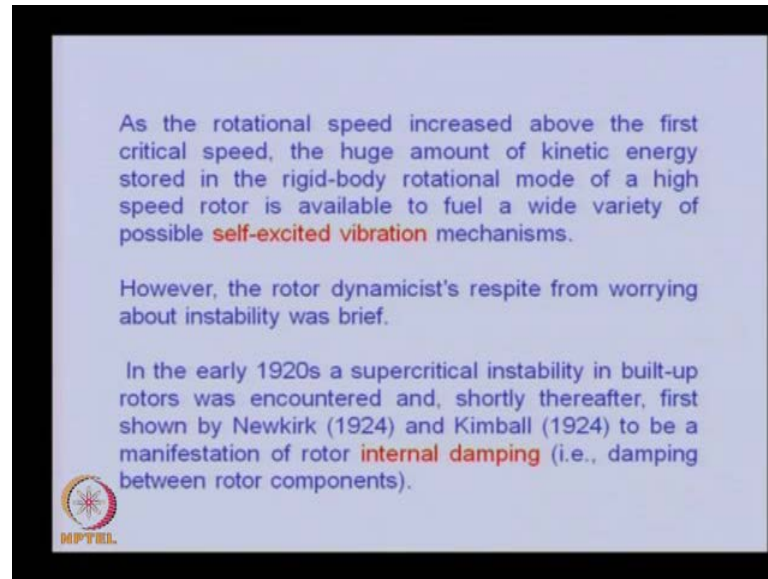
- The formula for damping was obtained independently by Crandall (1961) some 30 years later. Dick (1948) also studied behaviour of shaft having sections with unequal principal moduli.
- The system is stable provided
$$c_H + c_v > 0 \quad \omega < \left(1 + \frac{c_v}{c_H}\right) \omega_n$$
$$\omega_n^2 = k / m$$
- Thereafter, rotor dynamics expanded to consider various other effects.



So, that is the concept we are looking into the formula for damping obtained independently by Crandall in 1961, some 30 years later. Dick in 1948 also studied the behavior of shaft having sections with unequal principal moduli. The system is stable provided, you can able to see the first is the external damping, and then is the viscous damping, this is the internal damping and the second one is the external damping. This is the formula by which we can predict the whirling, the speed beyond which the system is unstable.

So, if the speed is below this range, ω is the natural frequency of the system, and then the rotor system will be stable. If it crosses this particular speed, then there is a possibility of unstable region in the operating speed of the rotor. How much time? So, we are seeing that what are the problems which rotor dynamics engineers were seeing when the rotors were operating with high speed? One of the phenomena we were seeing due to the internal damping of the system, there were some regions of unstable operating speeds. Now, let us see some more phenomena, which were taking place because of various other regions.

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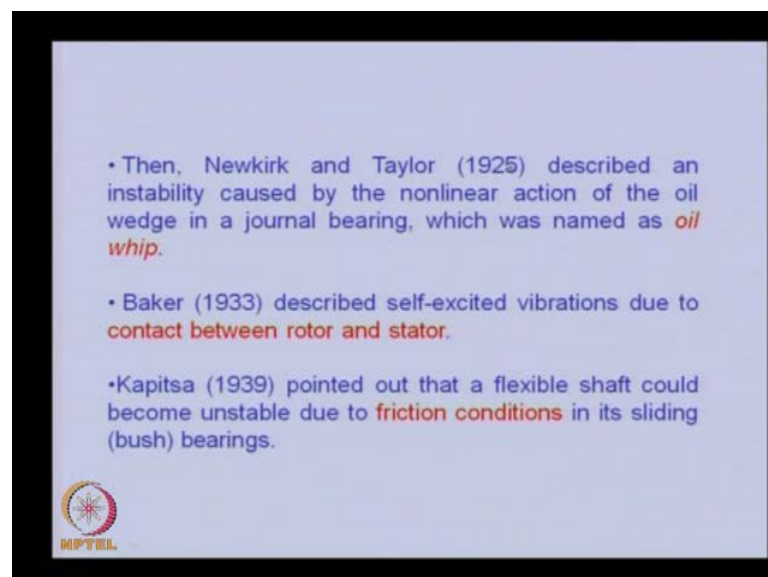
As the rotational speed increased above the first critical speed, the huge amount of kinetic energy stored in the rigid-body rotational mode of a high speed rotor is available to fuel a wide variety of possible self-excited vibration mechanisms.

However, the rotor dynamicist's respite from worrying about instability was brief.

In the early 1920s a supercritical instability in built-up rotors was encountered and, shortly thereafter, first shown by Newkirk (1924) and Kimball (1924) to be a manifestation of rotor internal damping (i.e., damping between rotor components).

As the rotational speed increased above the first critical speed, the huge amount of the kinetic energy is stored in the rigid body rotational mode of a high speed rotor is available to fuel a wide variety of possible self excited vibration mechanisms. However, rotor dynamicist's respite from worrying about stability was brief. In the early 1920s, a super critical sensitized in built up rotors was encountered and shortly thereafter, first shown by Newkirk in 1924 and Kimball to be a manifestation of rotor internal damping, the damping between rotor components or sometimes due to rubbing of the rotor also, this internal damping occurs.

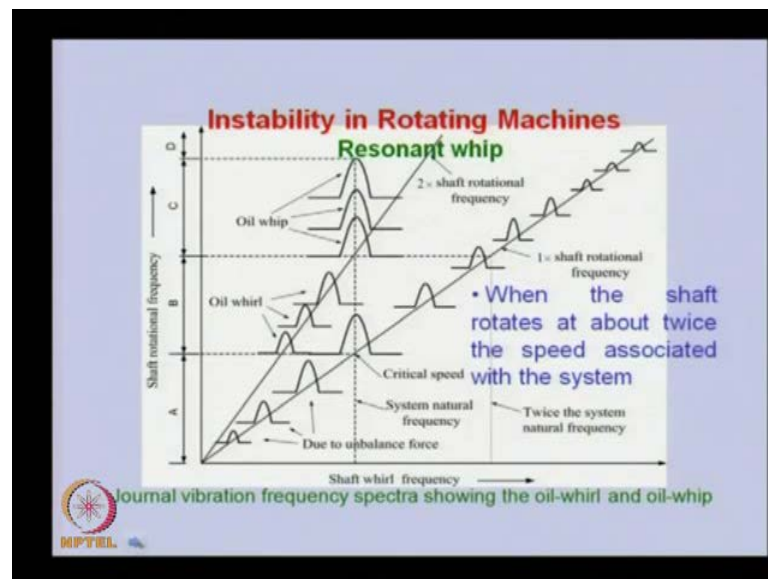
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- Then, Newkirk and Taylor (1925) described an instability caused by the nonlinear action of the oil wedge in a journal bearing, which was named as oil whip.
- Baker (1933) described self-excited vibrations due to contact between rotor and stator.
- Kapitsa (1939) pointed out that a flexible shaft could become unstable due to friction conditions in its sliding (bush) bearings.

Then, Newkirk and Taylor in 1925 described an instability caused by the non linear action of the oil wedge in a journal bearing, which was named as oil whip. Baker in 1933 described self excited vibrations due to contact between rotor and stator. This was another phenomenon, which Baker pointed out that rubbing between the stator describes self excited vibration. Kapitsa pointed out that a flexible shaft could become unstable due to friction conditions in its sliding bush or bearings.

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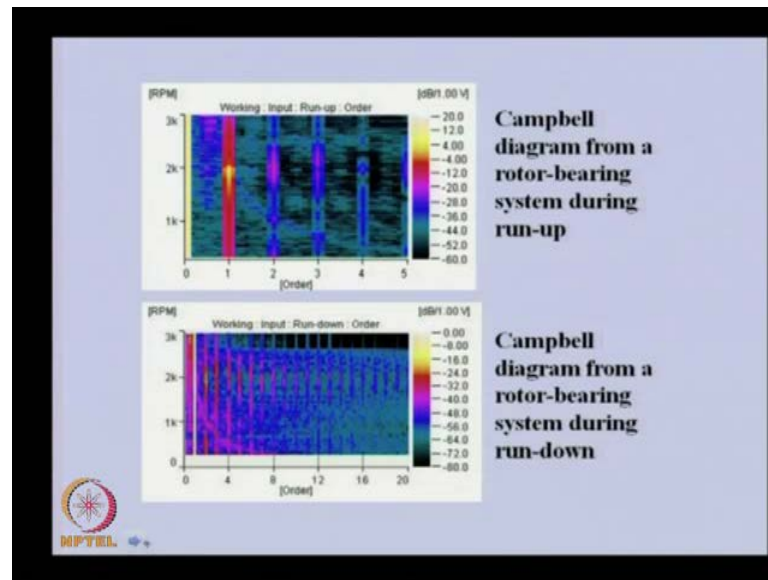
This is the typical Campbell diagram in which there is the spin speed of the shaft. This is shaft natural frequency. We can able to see. As this particular, I am correcting this one again. This is the technical plot of run up or run down the system in which this is the shaft whirl frequency and this is the shaft rotational speed. You can able to see this particular line. As the rotor speed is increasing, the amplitude of the vibrations which are indicated here is increasing up to this. This is the critical speed, but above critical speed, there is a new basis.

So, that is due to the unbalance, this particular response will be occurring. Only the critical speed will be more. These are the frequency components, which is twice the speed of the rotor. Generally, it begins above the first critical speed. You can able to see this is the corresponding first critical speed.

Here, you can able to see that this is corresponding to the first critical speed and basically at this movement, the rotor is acutely rotating twice the natural frequency. The whirling

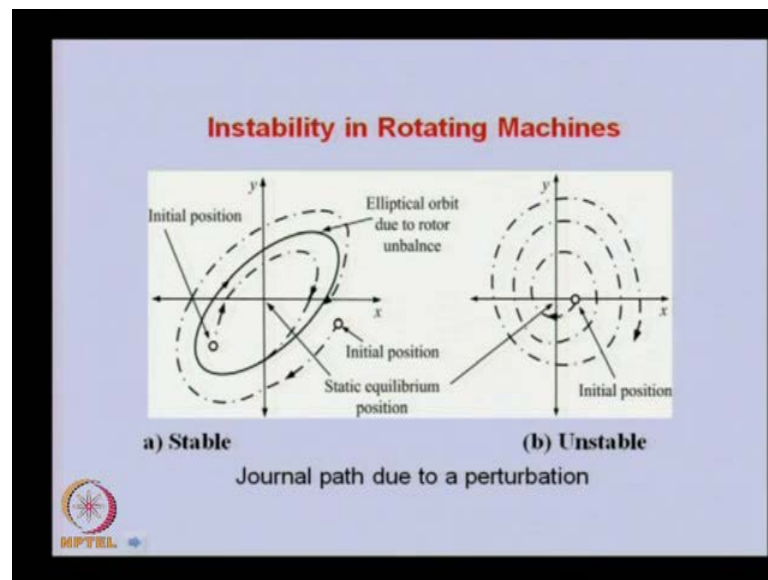
is taking place in the natural frequency. This particular phenomenon is nothing but oil whip. This we will study in great detail as we go into the subject. So, this particular oil whip when the shaft rotates twice the speed associated with the system, and then it occurs.

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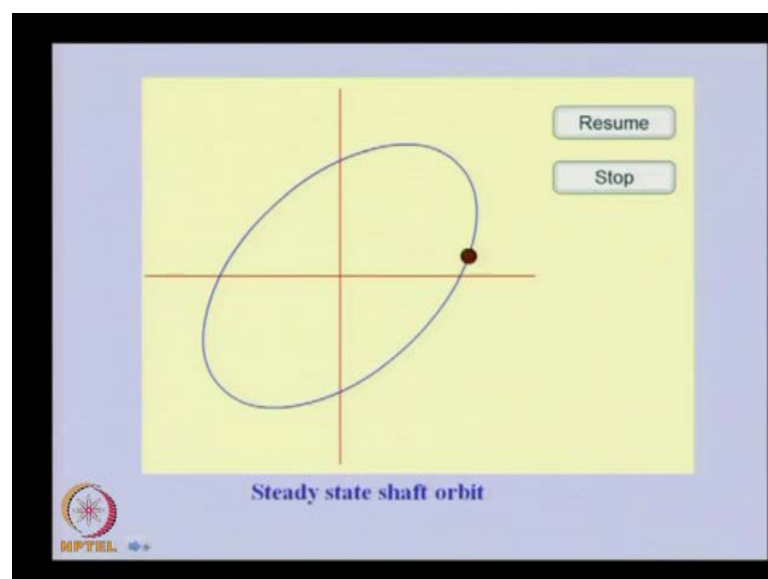
This is the typical experimental plot of the run up and this is for run down for the similar thing. You can able to see that corresponding to the first critical speed, there are large oscillations here. There are some sub critical phenomena also observed in the first plot. The second plot is for run down. There more unstable regions are there. Now, let us see what is the physically instability in the rotor system is there, if rotor is having or operating at unstable region, how the amplitude increases, let us see through some animations.

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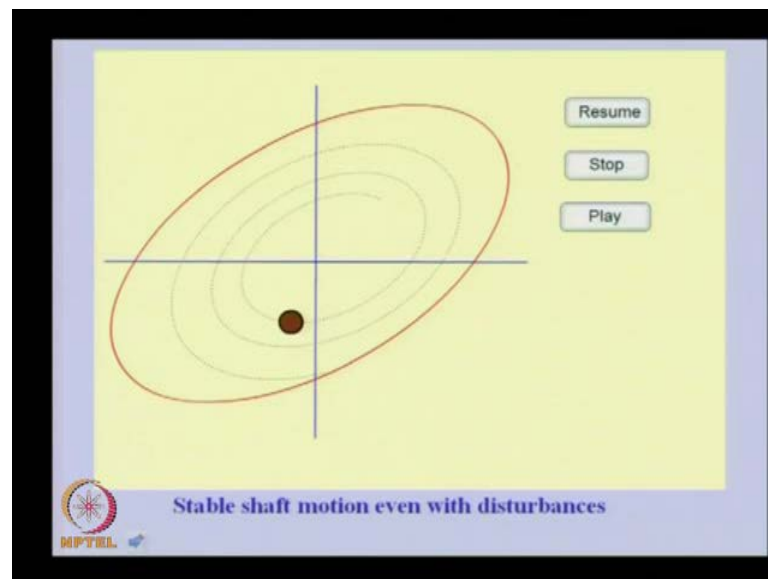
So, this is the plot in which you can able to see that this is the orbit of the shaft and this is x and y axis. This is the orbit of the shaft as it rotates. If shaft rotor is stable, if we give some disturbance, let us say disturb the rotor here. After some time, it will stabilize and give the same orbit. Even if you take of the rotor position somewhere outside also, it will stabilize like this, but in the unstable case, once you disturb, the amplitude will grow gradually and perturbations will be taking place. So, this is the unstable case.

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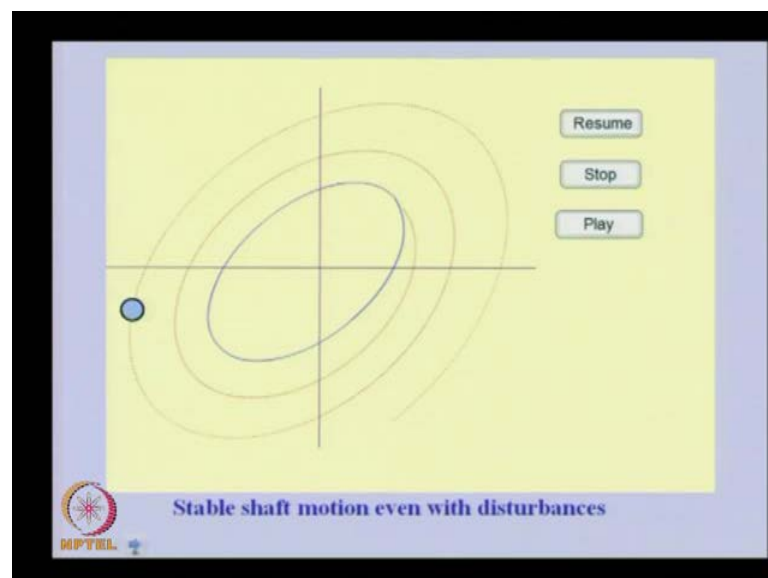
Let us see some animations. This is a perfectly steady state shaft orbit.

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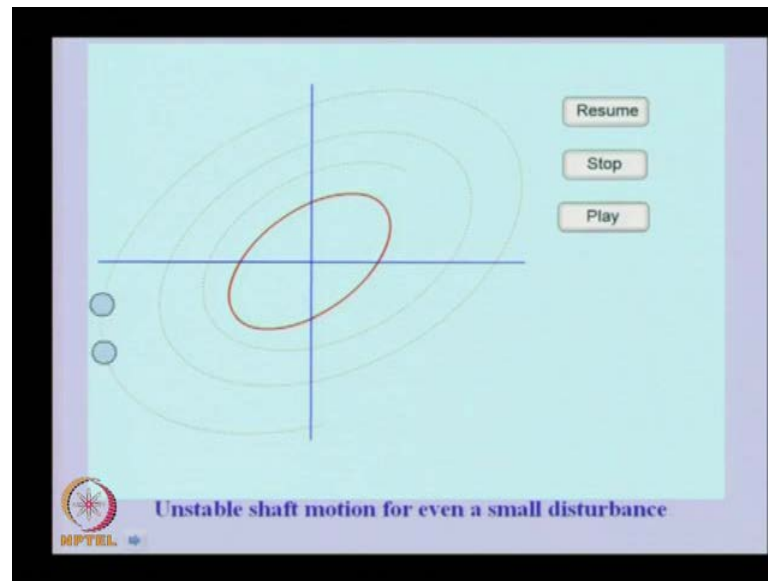
Now, let us see a stable rotor, but we have given some disturbance. So, after some time, again it comes to its original orbit.

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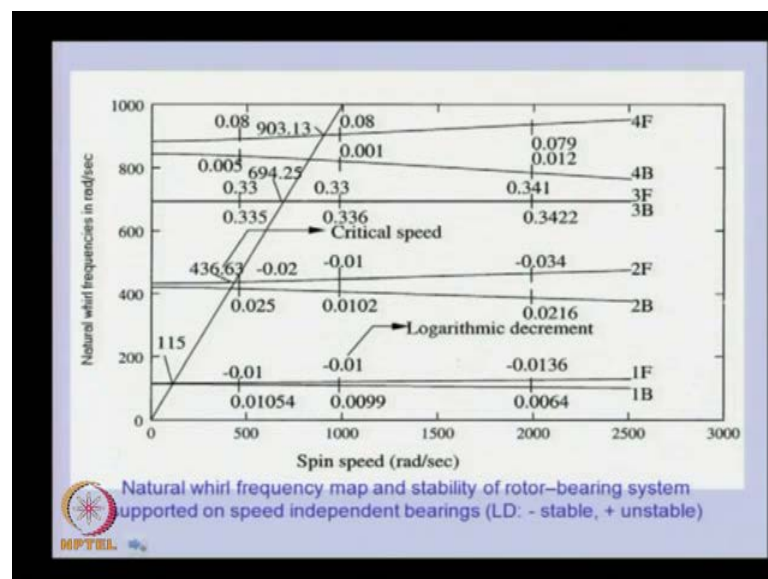
There is another given disturbance outside and it is coming to actual orbit. So, this is a stable system.

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This is unstable system. Once we disturb, the amplitude increases. You can able to see.

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Sometime the Campbell diagram along with the frequency meets at the logarithmic decrements. The size of the logarithmic decrement determines whether the stable or unstable. So, generally whenever it is indicating, the system is stable. If they are positive, the system is unstable. So, with this, we can able to see various modes, so like this, first mode, second mode, which is unstable, after what speed they are unstable, we can able to study.

Today, in this particular lecture, we started with some introduction with the rotor and even some basic phenomena we have seen. We started with the history of the rotor dynamics in which the first published work in this particular field was in 1869 by Rankine, but there was some flaw in that particular paper. That discouraged the development of high speed rotor for almost fifty years. But, the test reading in this period was made and they were operated above the critical speed, which were against the predication of Rankine.

So, in 1919, Jeffcott clarified this conclusion and he gave a very basic model for prediction of the response, unbalance response above the critical speed. He said that it is possible to rotate the rotor above the critical speed and because of that, the development of the high speed rotor began, but that led to the problem a kind of the unstable behavior of the rotor. Then people then started to working on finding the solutions for that instability. We will continue this particular state of the rotor dynamics, the state of the art of the rotor dynamics in the subsequent lecture.