Theory and practice of rotor dynamics
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Module - 5
Torsional Vibrations
Lecture - 19
Geared And Branched Systems

Today, we will study geared system and branched system, this is special kind of rotor system on which we rotor with single shaft, but multiple shafts are there. They are connected by the gears either reduce the speed or to increase the speed of the shaft. In this particular case, because the shafts are rotating at different speed, we need to take a special care for such system. Today, we will see in detail how we can able to analyze gear system and expansion of branch system in which from one particular point multiple shaftings are connected through gears.

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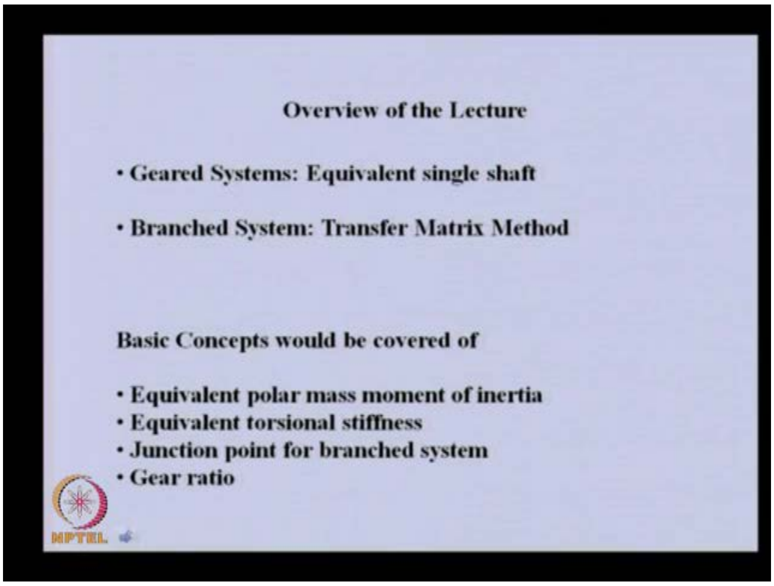

So, let us see what are the things that we will be covering, so for gear system, basically we will try to convert gear system to equivalent single shaft systems. So, that whatever the previous analyses we did we can able to apply for this particular system for branched system, because multiple shafts are connected at one particular junction.

So, for this particular case we will use transfer frequently and subsequently we will be using finite element method in the substituent lecture. So, basic concept which we are introducing here equivalent polar mass movement of inertia equivalent torsion stiffness junction point for branch system gear ratio.

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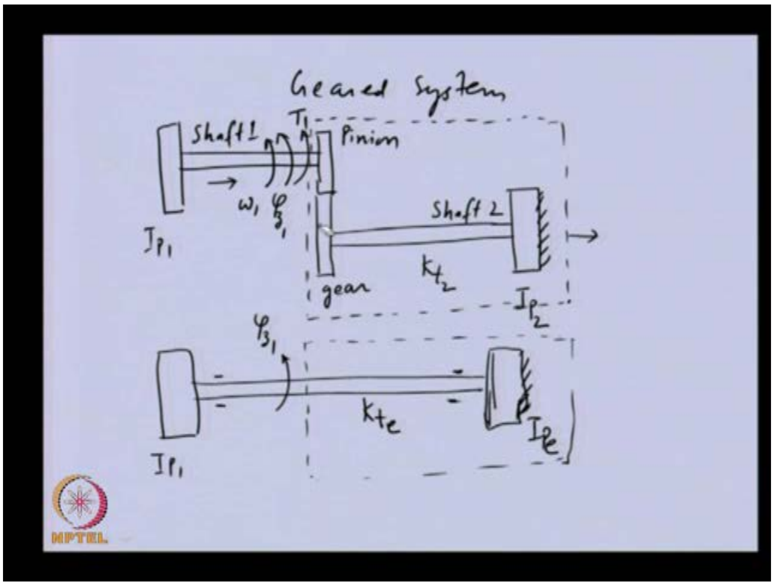

So, let us start with a branch system a gear system. So, in gear system we have one shaft which is connected by thy wheel fly wheel and if we rise as a gear which is connected to another gear and that is gear to another shaft and here another fly wheel is there let us say polar movement of inertia is I P 2. So, basically import is form here and we are getting output from this particular shaft system, let us say this stiffness of this particular shaft which is between gear and this fly wheel is 2. Now, let us say we are mentioning this as shaft 1 and this is shaft 2 and in this particular shaft let us say is having nominal speed of omega 1.

Apart from this because of the polar movement of inertia, it is having some torsional oscillation. So, let us say that fe z 1 and because it is transmuting power, so is having torque T 1, these are just before the this particular gear. Basically, this we call it has premium and this gear and this is gear, now if we to analyze this for caution vibration and we can convert this particular shaft system to single shaft system.

Let us say we have this particular movement of inertia first fly wheel and another fly wheel is here another and they are connected by uniform shaft single shaft. So, we will try to convert the previous actual system to this equaling system in which single shaft is there. If we want to convert we already such system in which this shaft system mounted on stiffness bearing we can able to obtain the energy frequency for torsion for this particular case. So, basically you can able to visualize this as if we enclose this particular gear system including this second shaft and the second fly wheel into a black box kind of thing and same thing is here. So, basically this particular polar movement of inertia will not be same as previous one now we are talking about equivalent system.

So, let us say this is P equivalent I P equivalent and this shaft which was having stiffness kg two let us say that we are mentioning k t e here we have up to this point gear. So, before this whatever the stiffness is the as the same as previous one we are not converting this particular shaft to equivalent system. So, basically we are trying to find out this equivalent type of this particular shaft system and this polar movement of inertia with respect to the output speed of the gear system. In this particular case, we are not considering the polar movement of inertia of this gear we are neglecting them, but if we want to consider then additional polar movement of inertia will be coming here.

So, that will be give us the system as three degree system from the two degree for simplicity we at present ignore this, but that can be added subsequently. So, here our aim is to obtain this equivalent system which we if we can find out what will be the new polar movement of inertia of this fly wheel and what will be the new torsional stiffness in this particular shaft. In this particular equivalent system in which single shaft is there, so basically the principle will be will be equating the strain energy of this two system the actual system and the equivalent system.

The kinetic energy of this actual system of the equivalent system, so this two energy should be same then we can call these two system as equivalent system, so let us begin with strain energy for calculation of strain energy. So, what we will be doing, we will first we assume this particular disc we are giving we are attaching we are giving the 0 displacement means we are fixing this particular movement of inertia also here also in this particular fly wheel. Now, we are giving input to the shaft 1 here and same input to the shaft here also we are giving fe z 1, so you can able to see that because of that the effect of the gear ratio.

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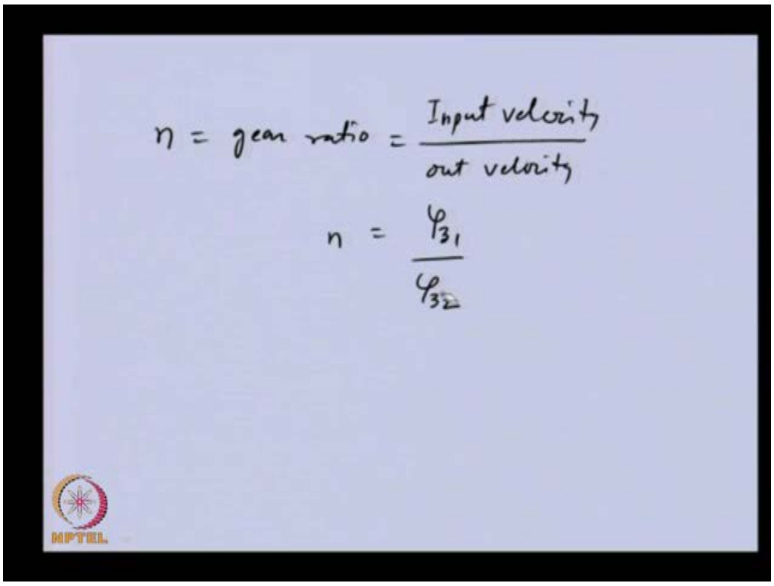

So, the gear ratio is nothing but input velocity divided by the output shaft velocity, so in our case if we are talking about angular displacement. So, this will be fe z 1 divided by fe z 2 or it will be equal to omega 1 nominally speed or the gear. So, let us take the angular displacement as the gear ratio in which the input angular displacement divided by the output angular displacement.

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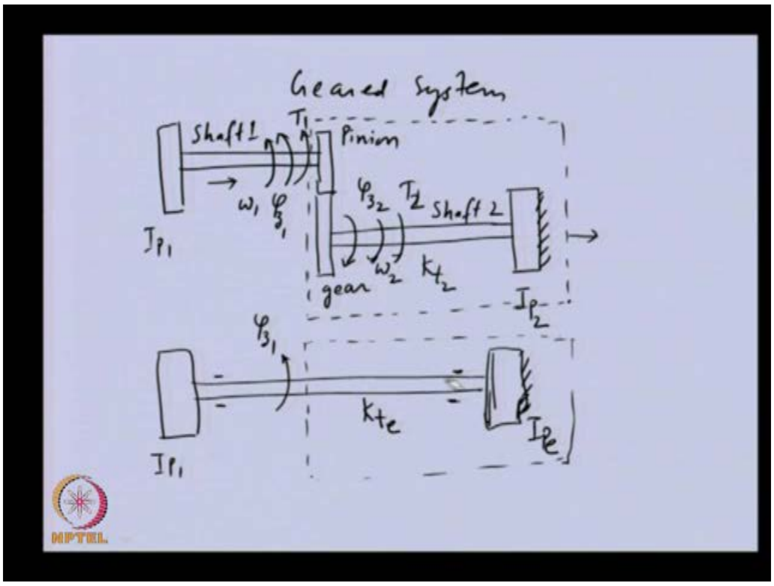

So, we can be able to see that when we are giving fe z 1 input to this particular gear will be having opposite motion and the front angular displacement. Similarly, if we want the nominal speed they will be different top will also be different, so these are all at the location of the gear that is fe, that is the gear. So, you can able to see that if we want to equate the two in this system for a particular input, so we giving a fe z input, let us say we are giving a twist of fe z 1 s in the both the system and we are trying to find out the strain system in the energy.

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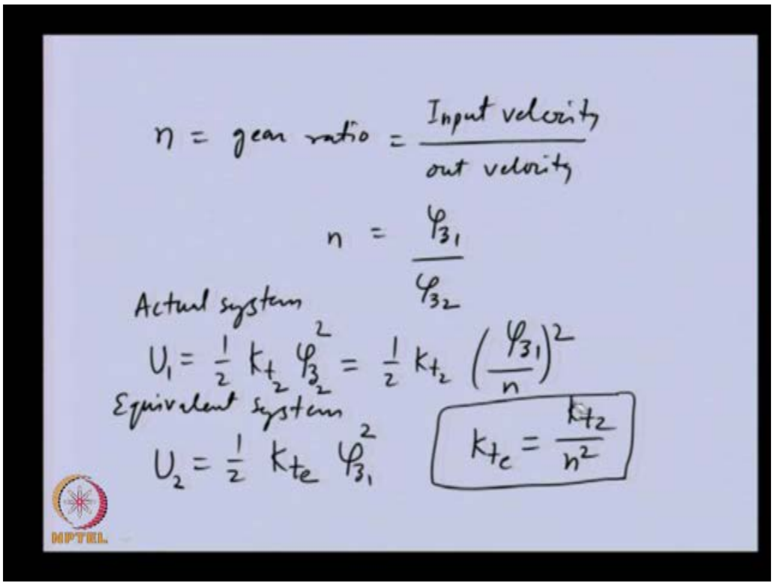

So, for first actual system we will be having strain energy that will be because the second shaft is having the stiffness 52 and its angular displacement will be this square. So, this strain energy system in the actual system fe z 1 by 1 full square, this is for the actual system for equivalent system similar strain energy we can obtain that is because the tertian stiffness of the equivalent system is k t e.

Here, if we again see whatever the angular displacement giving at this point the same displacement this particular shaft is getting. So, here we will be having strain energy corresponding to the same displacement, now this two energy because these two systems are same these two energies. We quit because these two energy are same, so you can able to see if we equate this two we will get k t e after the cancelation of angular displacements as k t 2 divided by n square. So, this is the equivalent stiffness of the shaft new shaft which is given as k t 2 divide by gear ratio square.

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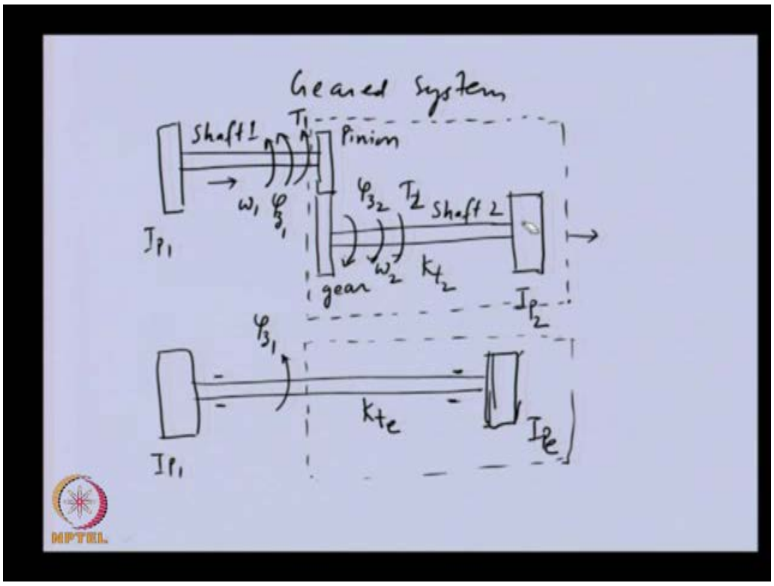

So, you can be able to see that this particular torsional stiffness we can obtain by using this. Now, in second case will release this fly wheel as we did earlier we fixed it, now we are assuming they not fixed, but now we are considering the shaft as a recite for second case, these two shaft are recite. That means whatever the displacement we are giving here this particular fly wheel will be having same displacement because now we are considering the shaft as recite.

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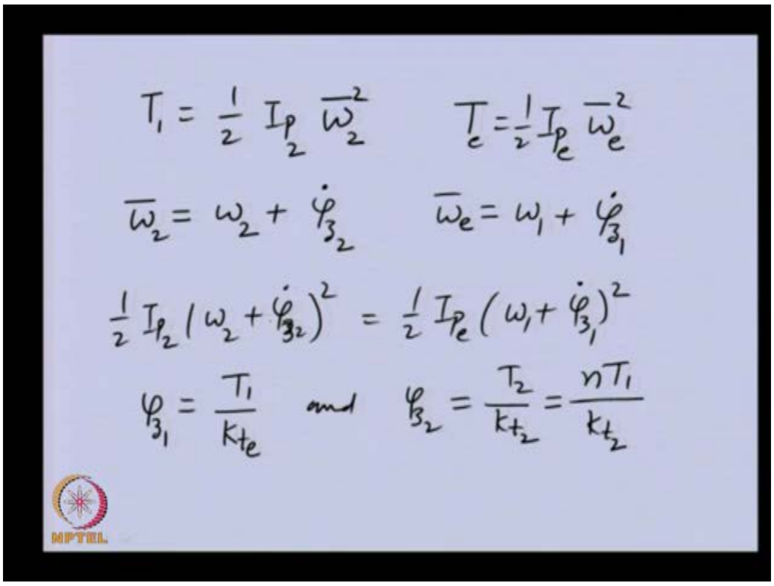

Now, we will be obtaining the kinetic energy these two systems, so let us say for actual system we have kinetic energy I P 2. The angular velocity, let us say this and for the equivalent system the kinetic energy is I P equivalent, let us say the angular velocity of this. Now, this angular velocity which we have it is having two component 1 is corresponding to the nominal speed omega 2 of the second shaft plus angular displacement due to the tertian angular velocity to the torsion. Similarly, the equivalent angular velocity or the angular velocity of the equivalent system will be omega 1 because there is no gear in this.

So, whatever the input velocity will be transmitted to this particular shaft and then cautioner displacement or cautioner velocity that will be also be same. Now, this particular angular velocity we can able to we can able to substitute this in this equation and we substitute. So, we will be having basically these two kinetic energy are the same, so we can equate them also, half P 2 omega 2 plus fe z whole square is equal to half I P e omega 1 plus fe z 1 of whole square.

Now, this displacement we can able to write for pi z 1, we can able to write it as in terms of torque and the this for first system in which torque this is the torque and the divided by the torsion stiffness. Then, angular displacement then for second system and the actual system T 2 divided by k T 2 and using the gear ratio, we can able to write this as in terms as 182, this displacement we can able to substitute in this expression.

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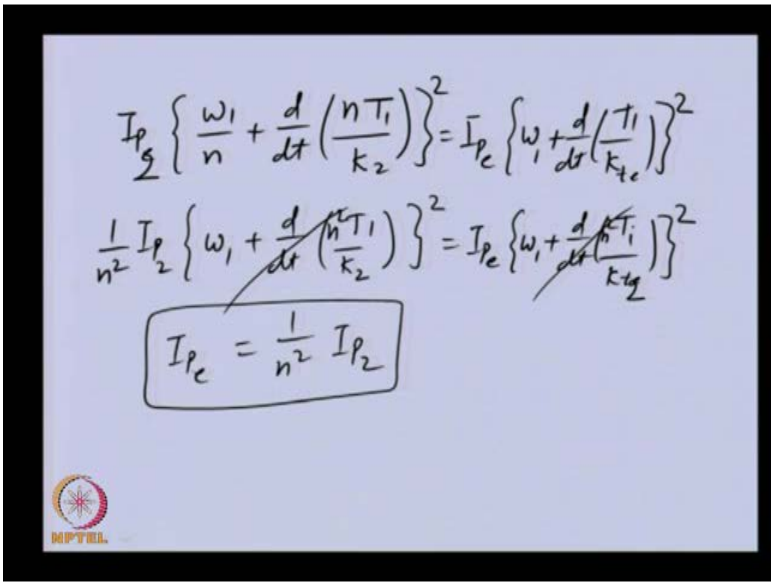

So, once we substitute this expression in kinetic energy of both the system which we equated earlier, we will get omega 1 by n time derivate to of the angular displacement which we are replacing by the torque divided by this. For the actual system, this is actual system, this is the actual system I P 2 and this is I P e, this is for the equivalent system whole square. So, basically we have done here we have substitute this and this quantity here and we use the gear ratio to convert everything to convert omega 1 fe z 1.

Now, you can able see that once we take one by n common we will get I P 2, if we take 1 by n common, this will be because n by square. We will get one by n square omega n plus n by 1, 2 here will be having T 1 by k 2 m square is equal to I P e omega 1 plus d 1 by d 2 T 1 n square by k t e. Here, we have converted this also into k T 2, k t e is nothing but k 2 divided by n square n 2, here now you can able to see this quantity is getting cancelled. We are getting equivalent polar movement as this one, so just we need to divide the gear ratio square to the actual movement polar movement inertia to get this.

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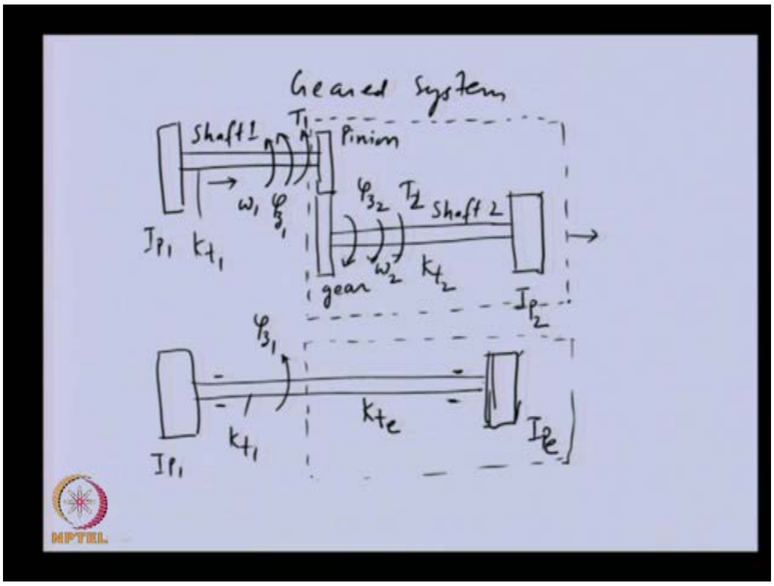

So, once we have done this, we can be able to see that, now we know this and this and this particular shaft if we reserving the stiffness, this is k T 1, this is the two shaft segment. Basically, they are connected in series, so we can able to find out the total stiffness of this and then we can able to as we did earlier as for single shaft system. We can be able to analyze for torsion vibration, so I am not repeating that particular things here, only things will be molded sheet.

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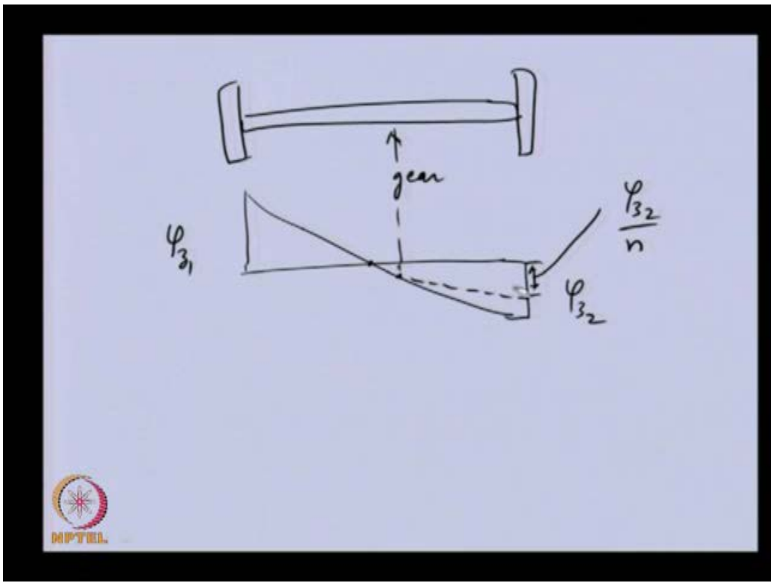

So, this the equivalent system two rotor system which we obtain, so if we are getting more shape let us say one of the mode shape will be having one node here. So, let us say this gear possession gear position is here and the node is here, so this is for the equivalent system if we want to obtain the node shape for the system. So, what were the displacement are there, angular displacement because this is nothing but angular displacement of z 1 fe z 2, so angular displacement should be divide by the gear ratio. So, we can see that after the gear this is the gear location or to the displacement will be divided by n, so again the respect will be change in the slope of the line and this particular thing is nothing but fe z 2 by n.

So, the mode shape will be solid line up to the gear and this dotted line will be the modified node shape for the n shape all the analyses will remain same as the previous one. Now, we will analyze the branch system which is an extension of the gear system in this particular case we will find that at one particular junction there will be more than two B s are attached. So, the power which is getting transmitted from one particular shaft it is transmitted to are three many shaft generally such application. We find when we are having one main motor and it is driving several other shaft and this motions are given by the various gear.

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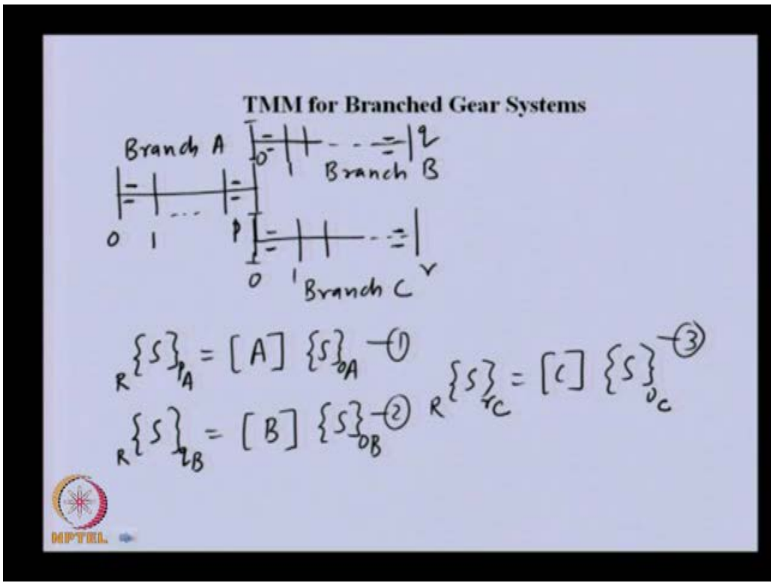

So, let us see how this particular system can be analyze, let us see first what is the branch system, so we have one shaft system in which we can have multiple disc. So, let us say we have one 0, 1 several disc are that is fe number of term in a particular system, so this is the single shaft system, now we are having another branch here. So, this gear basically which is connecting another shaft system, so let us say this particular branch we are giving the name as branch A this is and this is branch C and in top there is another branch which is let us say branch B.

So, basically this is pinion and is driving this gear as well as this gear, so these are various fly wheels this or a disc in a rotor system and we can have like for branch B, we can have q number of disc. Similarly, for branch C we have the branch system of let us say R number of disc so there are so many number of disc particular system, but main thing is this branching. We can able to see that from one particular shaft the power is getting transmitted into two shaft we can have more also, but for illustration we are showing two because this is a this is a multi rotor system.

So, will be using the transfer this method and for this we have already obtain like state factor, let us say P th station of branch a right side of the disc, so basically particularly this state factor is here. This is the branch B and fe th station right side of the this particular gear is this particular state factor we are relating this state factor with state factor at zero station, let us say overall transform matrix this is A. So, this is a state factor at zero station at branch A, so basically this equation representing the whole branch a similar expression we can able to write for branch B.

In this particular case, we have q number of things, so this is equal to let us say overall translation of this is B and this is zero station and branch B. Similarly, we can able to write for branch C, so write of R th disc of branch C is equal to let us say that C is the overall transfer matrix of trans C this is the state factor of zero station so here is the state factor this is right of R th.

So, you should write it correctly so this state factor is right of R th disc, so these equations basically representing the bond equation for the three branches, but they are not been connected as they are connected by gear. Now, we will be taking one at the time these equations and we will try to relate them basically we try to solve the bond three conditions which is there here or here or here or at the junction point. These rotors basically we are assuming them as we did earlier, they are mounted on frictionless support, so we can have the torsional motion like this.

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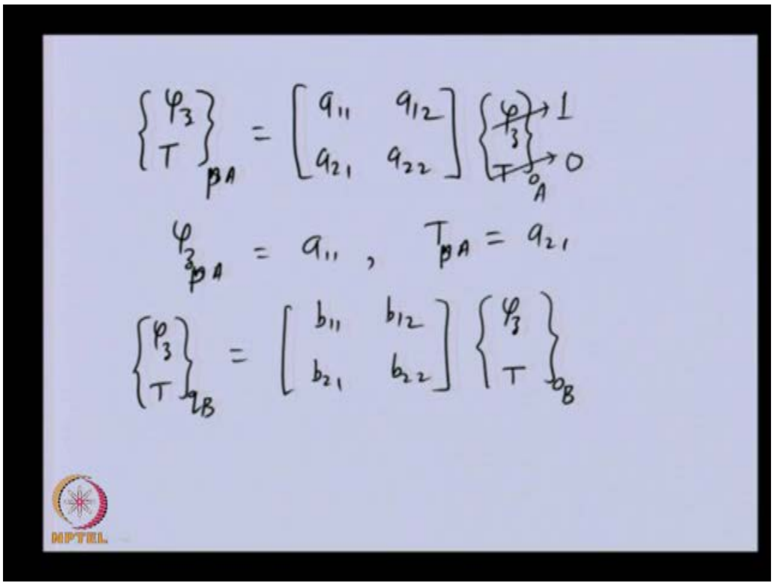

So, let us start with the for branch A we can able to expand this as like this, let us say overall transfer matrix for one I am representing as a 1 1 and a 1 2, a 2 1 and a 2 2 fe z and torque at zero station of branch A. Now, the bonding condition at zero station is that this particular torque is 0 because particular and of the shape of the pi torque will be 0. Here, this placement will not be 0 and for simplicity because we are analyzing the free vibration we are taking this particular station angular displacement as unity and obtain this related what are the displacement of the others.

So, we will equate this equation one and this according to bonding condition bond zero, so you can able to see that if we expand this we will get fe z and A as a 1 1 and second equation a 1 1 as a 2 1. Now, we will take the branch B, sorry this is fe, these are fe is equal to the fe number of discs are there in the branch A and in this we have q number disc of b 1 1, b 1 2. These are the overall translation for branch B here torque is there and here angular displacement is the illustration of branch B at junction point between gear A and gear B.

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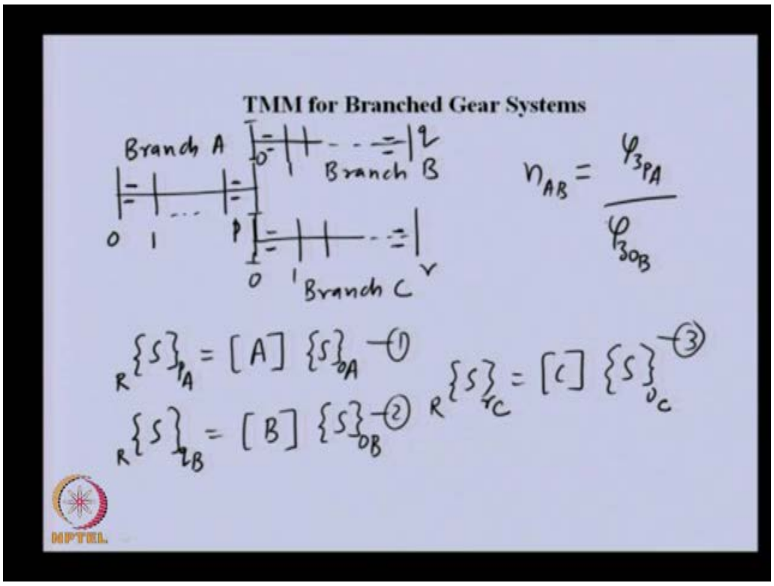

Let us say that gear ratio is n A B, so that is nothing but fe z 1 that is P th station of branch A divided by fe station. This is fe z of branch B, so this input this is output, so you can able to see that.

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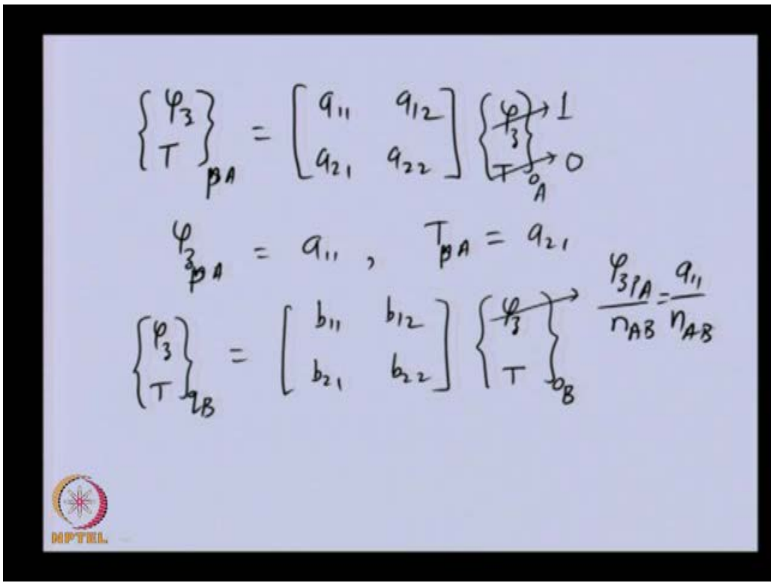

You can able to replace this particular displacement is pi z divide by gear ratio that is A to B and this we already obtain earlier that a 1 1. So, this can be written a 1 1 divide by the n A B, if we see the other bonding condition of the branch B at this station we have torque 0.

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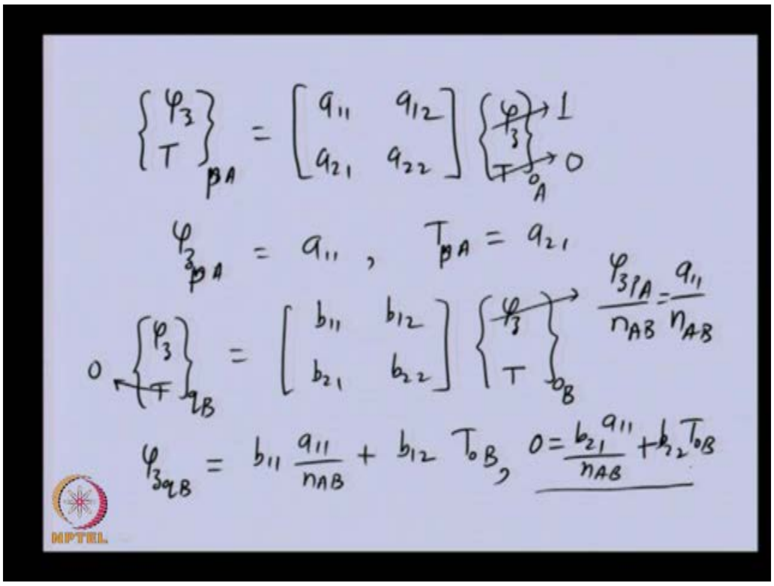

So, that means here torque is 0 and is we expand we can able to get pi z q B, this is the state angular displacement of branch B at q th station is equal to B 1 1 first equation A 1 1 divide by and A B plus B 1 2 into T naught B. So, this is the first equation second equation will give us 0 is equal to a 2 1 a 1 1 by n A B plus a 2 2. So, we have two equations, now we can able to see that we can able to this particular equation we can solve for T naught B and then you can able to substitute this in the first equation.

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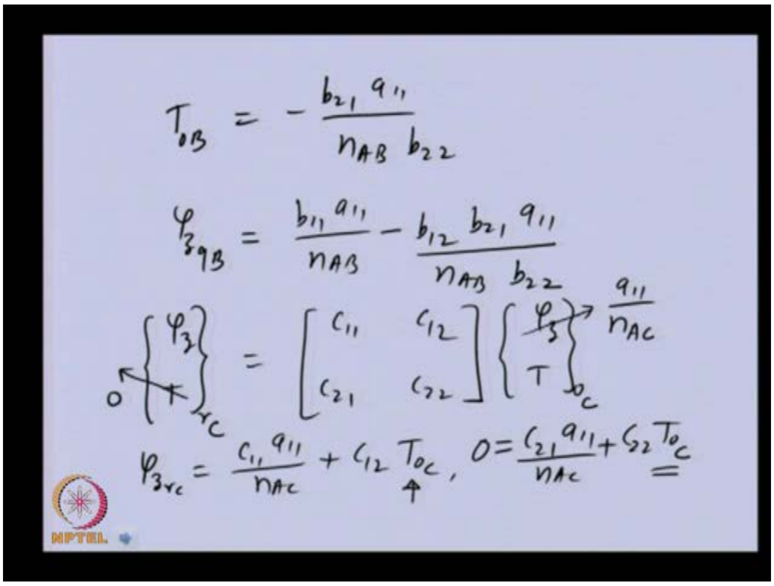

So, we are solving for T naught B from previous equation, so we can able to see we will be having minus 2 b 1 a 1 1 by n A B and then this will go in denominator b 2 2, this is we got from second equation we substitute here. So, we can able to get fe z q B s B as 1 1 a 1 1 divided by 1 A B, then b 1 2 and this T naught B will give us minus b 2 1 a 1 1 divided by n A b 2 2. We could able to get angular displacement at the branch system at q th station that is the far end of B of this one. Now, we can able to take up the third branch for third branch we can able to write the expanded form that is C branch.

Here, we have R C is equal to c 1 1 c 2 2 c 2 2 fe z and T and this is the zero station of the branch C. So, here the bonding condition is because this end is free and this is 0 and gear ratio between branch A and C, we can able to write this as has we did earlier for this here the gear ratio will change A 1 1 divided by A to C.

Earlier it was A to B, now we can able to see that is A to C can able to express again the expressions fe z or C is equal to c 1 1 A 1 by n A C plus c 1 2 T naught c is the first equation. Second equation will be zero c 1 1, a 1 2 n A C plus c 1 1 T not C, so these are the two equations again we can able to see that we can able to solve for these and you can element T naught C from here. Now, let us see apart from this at present we could able to relate the branch A, branch B and branch B and C using the gear ratio.

We apply the bonding condition, now another condition we are not exploited that is the junction point whatever the work will done we will be doing at the shaft a will be equal to the work done at shaft a B and C. So, let us see this particular bonding condition or this particular condition how we can able satisfy and once will satisfy this we will get the frequency equation from the formulation.

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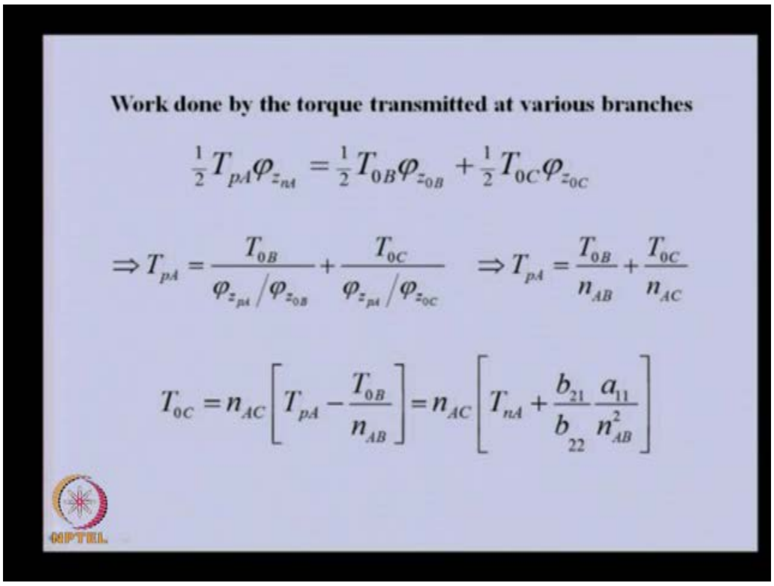

So, this the last condition we need to satisfy at the junction, so this the work done at branch A should be equal to the work done in branch B and C and you can able to solve this for this particular torque. So, I have divide by the angular displacement of this and this angular displacement ratio are nothing but the gear ratio so we can able to write like this. Now, I am solving for T not C this expression T not C is multiplication of this quantity and if we take in other side the quantity will be negative earlier we obtain T naught B, so I am substituting this we obtain earlier T not B we obtain earlier.

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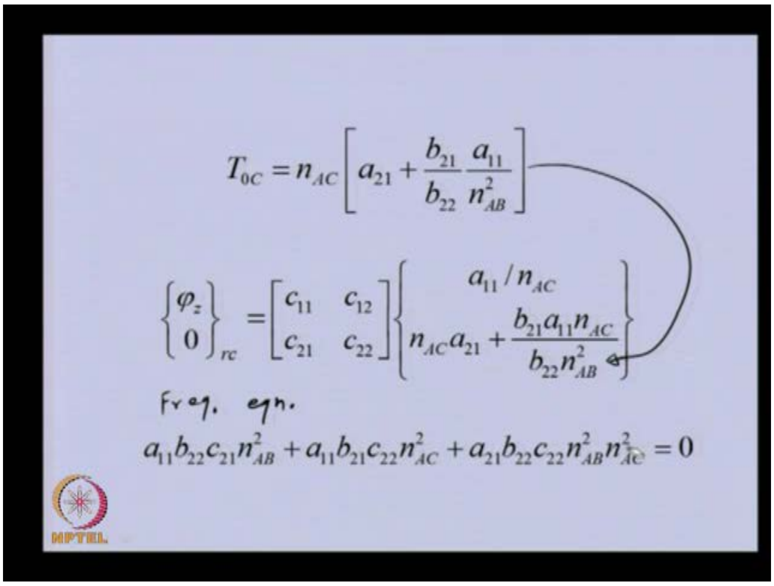

So, substitute that expression well and T naught this particular we obtain earlier, so that I substitute as a 2 1, so this A to C. This is the equation for branch C which we earlier wrote this was the bonding condition this using gear ratio we converted and this was T naught. So, basically this expression we are writing and now you can able see from the second equation if we expand we will get this expression which is not containing any other state factor like fe z or torque.

They are containing the terms related to the overall transformation of branch a B and the gear ratio so basically this transform ties officiate they are the function of omega. So, this frequency equation basically and we need to solve this using some kind of empirical technique which we have already explained earlier frequency equation and this particular case we will be having many natural frequency.

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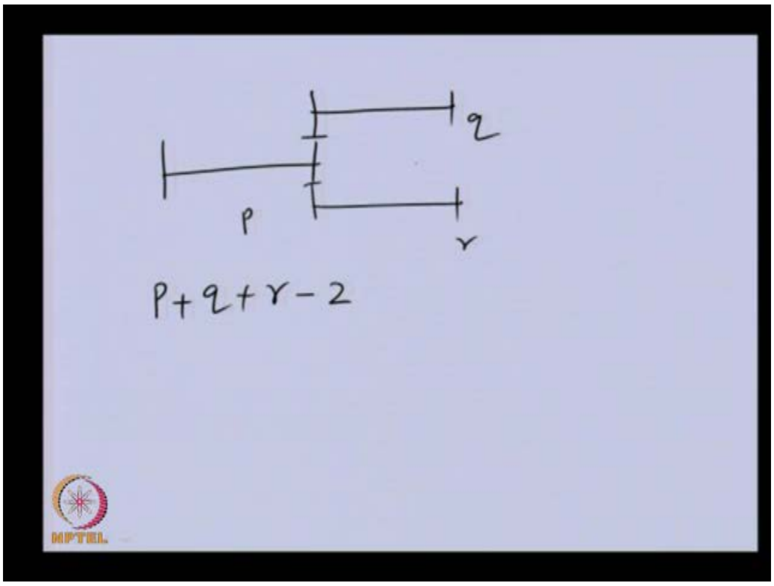

So, in the branch we had P number of disc branch B we and q number of disc branch C we had R number of disc. So, total pinup of degree x, but the angular displacement of the branch B at this point is related by gear ratio. Similarly, for branch C is related with this so that means two angular displacement are related, so the degree of film will be that much less because we have two additional constant.

So, these many natural frequencies we expect from the previous frequency equation and once we get the frequency equation we can use transformative to get the relative displacements. This place mode shape for the corresponding to each of the natural frequencies, let us take one example for branch system in which whatever the method we explain will be more clear.

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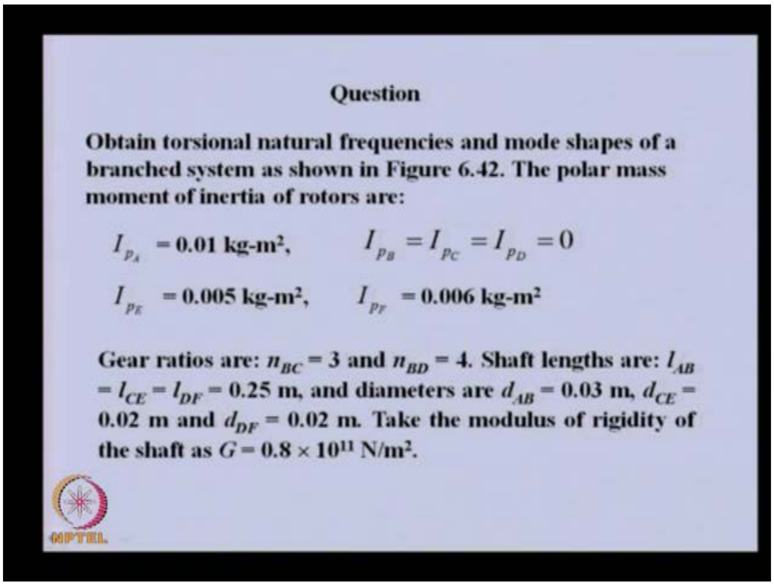

So, here we are taking a branch.

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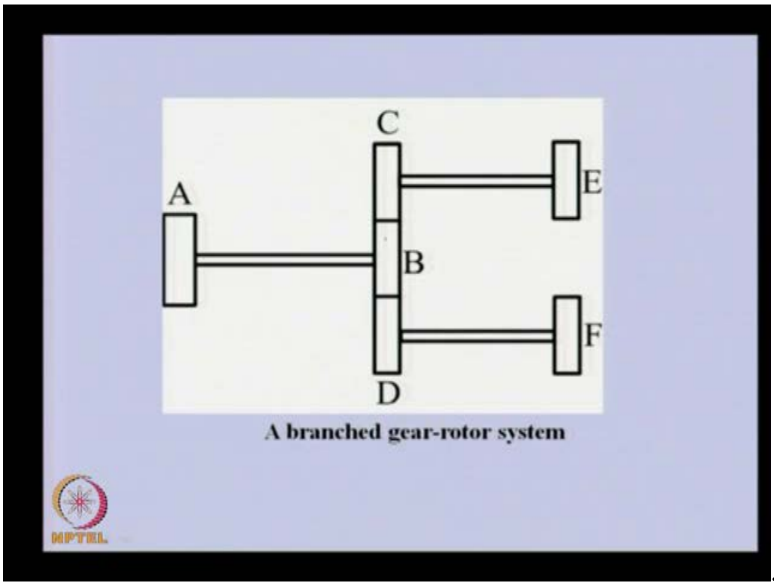

The branch system is similar to the previous one only thing for the analysis only, we have taken the only the one disc in branch A, similarly another disc branch this particular branch. So, we are not using the multiple disc only single disc we are consider each of them and in this particular case these are the polar movement of inertia various disc is this polar movement of inertia gear. We are neglecting and the gear ratio between branch B and C or between the B and C three that means B and C. Similarly, between B and D gear is four various length are given of the previous of the figure diameter of the shafts are given at the material quantity of the shaft is given.

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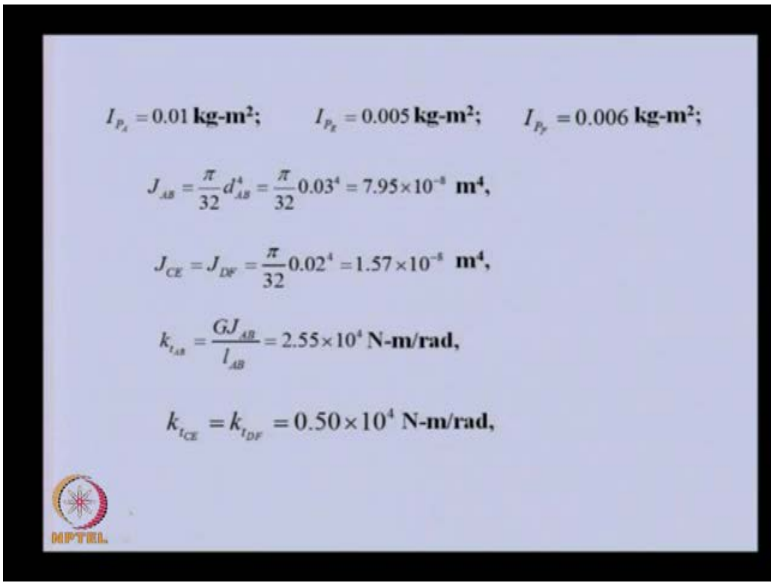

Now, we can able to obtain the various the property of this particular problem, so this are the polar movement of inertia three fly wheel which is given to us second movement of area can be calculated or to the torsional stiffness of each of the shaft segment. We can able to obtain this two shaft segment having same property so they are stiffness is same.

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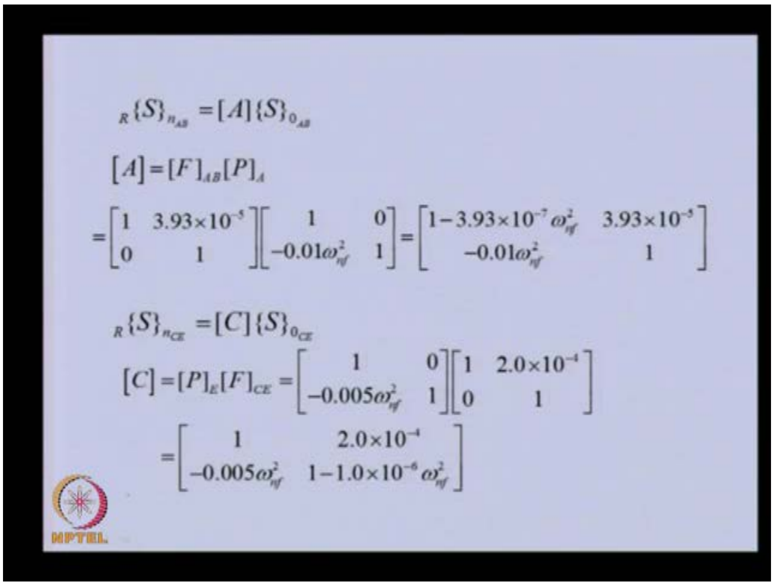

Now, we can able to write the vector branch a in this particular case we can able to see that overall transmuting two component what is the odd matrix and the field matrix and we know the property of this. So, we can able to get the A matrix you can able to see that this contain unknown as natural frequency, this is for branch A similarly for this particular shaft third here this particular shaft segment.

So, this state factor is the right of the that particular shaft segment this is the overall transition for that branch this also only point and filled matrix because we are not considering the inertia of the gear. They can be obtained, so this is the for that particular branch overall transfer matrix for second for third branch the lower one that means if we see the figure this particular branch again we are writing overall transfer matrix.

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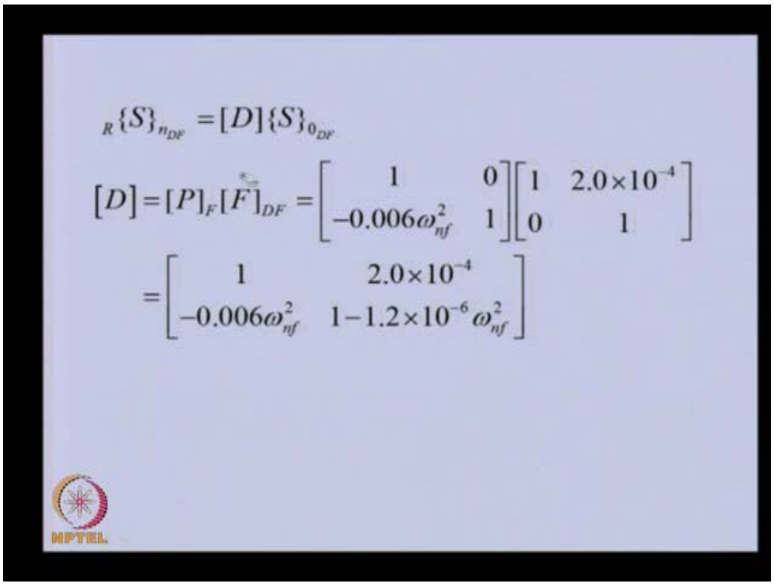

So, this is also having one matrix and one field matrix and they can be multiple and we can now obtain all the branches, overall transfer matrix.

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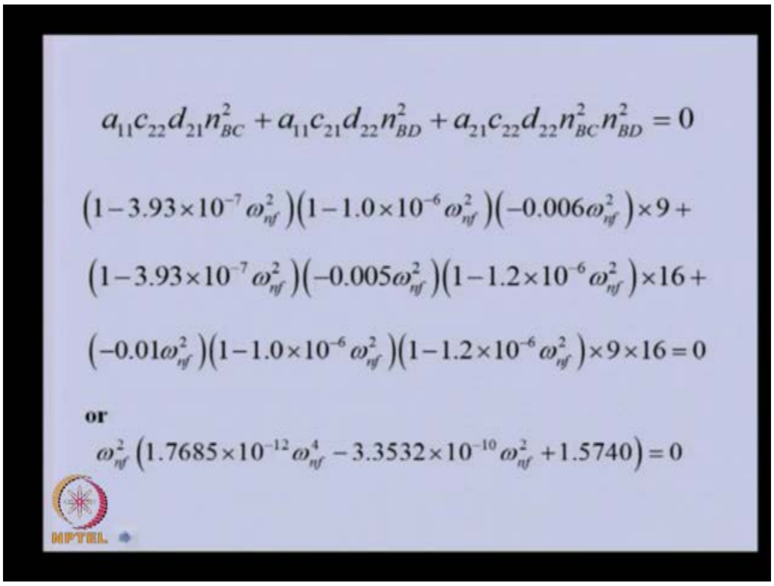

Now, the frequency equation derived value was of this form of three branches and these are various components of the matrix. So, this form a matrix first row first column this is C matrix second one second column like this, so we can able to substitute this various storms from three matrix which we obtain like this so we will get expression like this. If we simplify, we get a polynomial of this form and you can able to see that one of the natural frequency is 0 and other two natural frequencies will be getting quadratic part.

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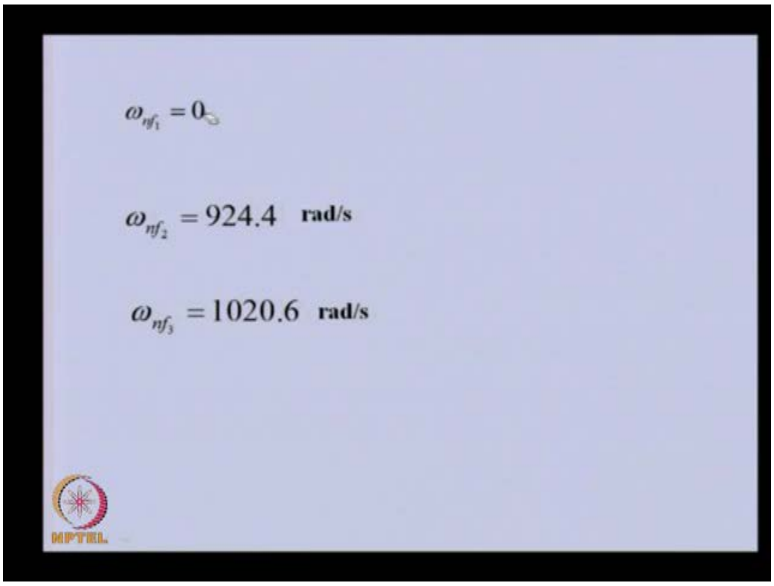

So, one natural frequency is 0 another two are two natural frequencies corresponding to the flexible mode which we are getting from this. Basically, we have three natural frequencies, one of them is 0, natural frequency which is corresponding to recite body mode and these are flexible mode we take up another example of a simple gear system in which two gear are there. In this particular case we will obtain the frequency equation natural frequency system of the torsion natural frequency of the system.

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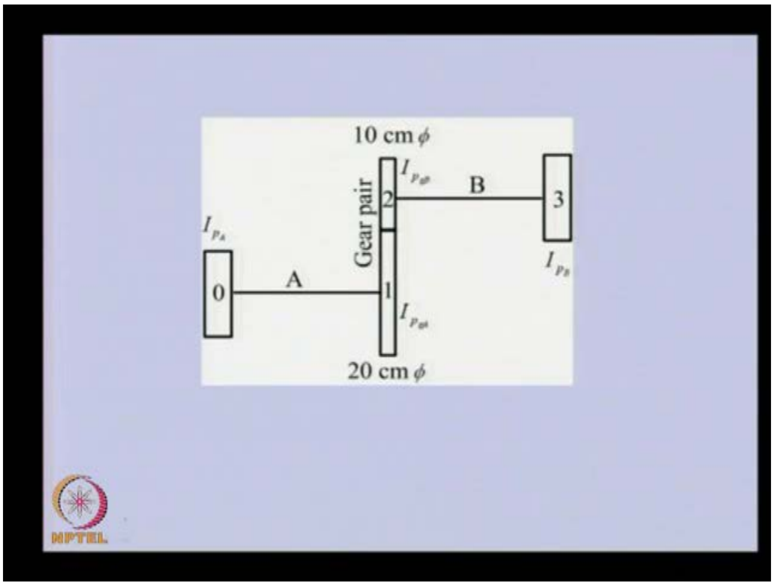

In this case also you can able to see the gear system in which this is the one of the fly wheel and this connected by this particular gear to this gear this is the another gear.

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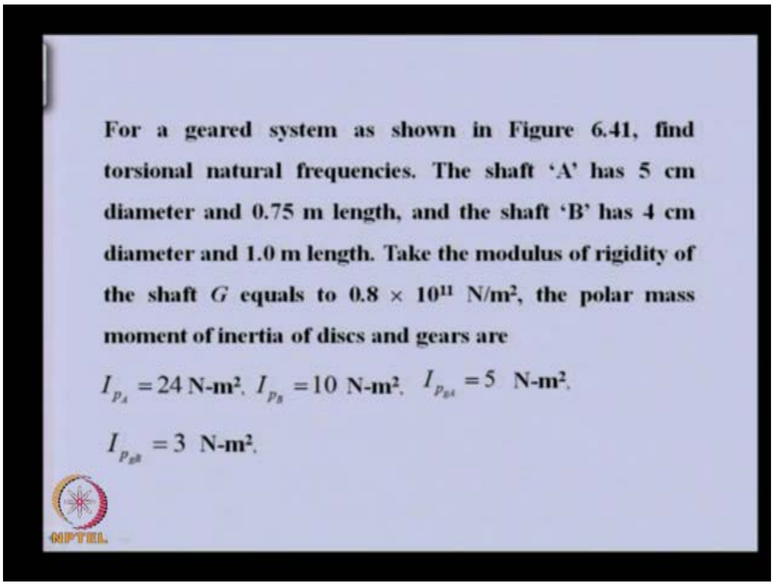

The various property of this a gear and in this particular case these are the fly wheel and in this particular space, we consider the inertia of the gear also and other property of the shaft A and B are given here and the metallic company of the shafts are given.

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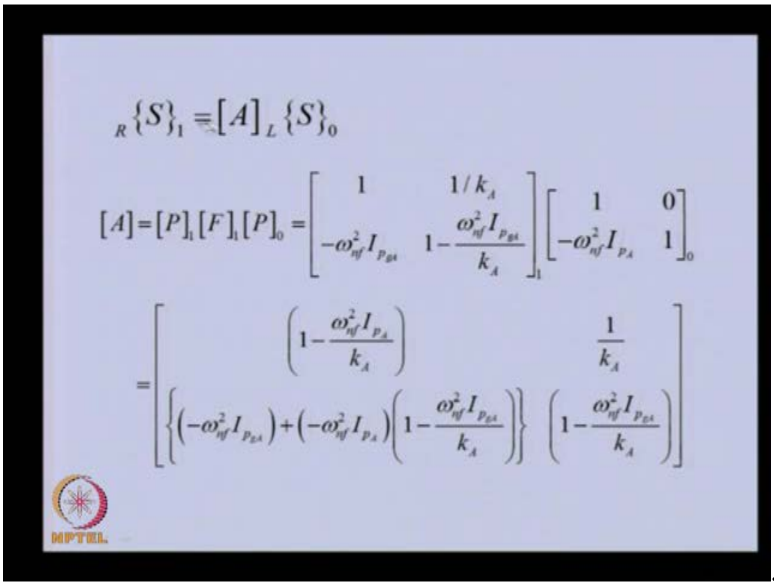

Now, in this property we can be able to now I am writing the transfer matrix that is can able to see is the from shaft A 0 th station of shaft a and is the right of one station.

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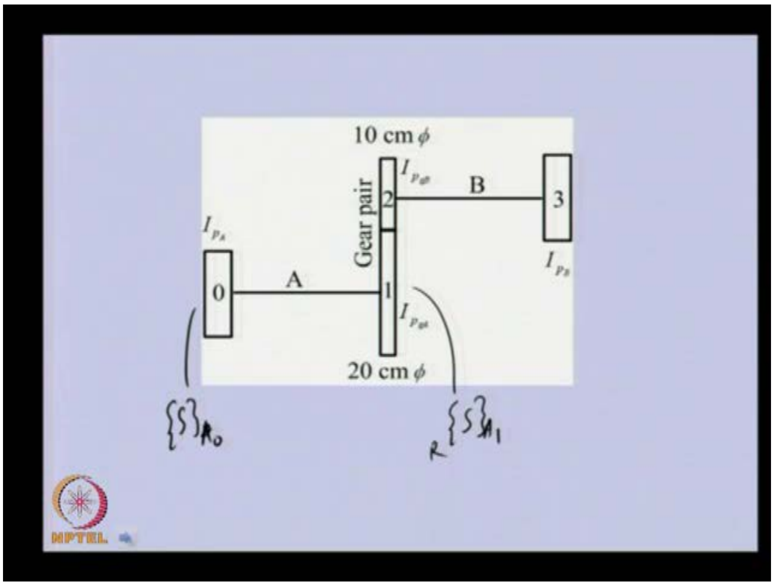

This is the zero station of shaft A this is the 1, so we had related the state factor which is here right of A 1 and this is the 0 station of branch A. So, we are related by a matrix were a matrix is multiplication of three on matrix fill matrix odd matrix. In this particular case, we can able to if we see that if we multiple this we will get this expression condition for branch A for other shaft soon, we can able to overall transfer matrix multiplication of this will give us this expression.

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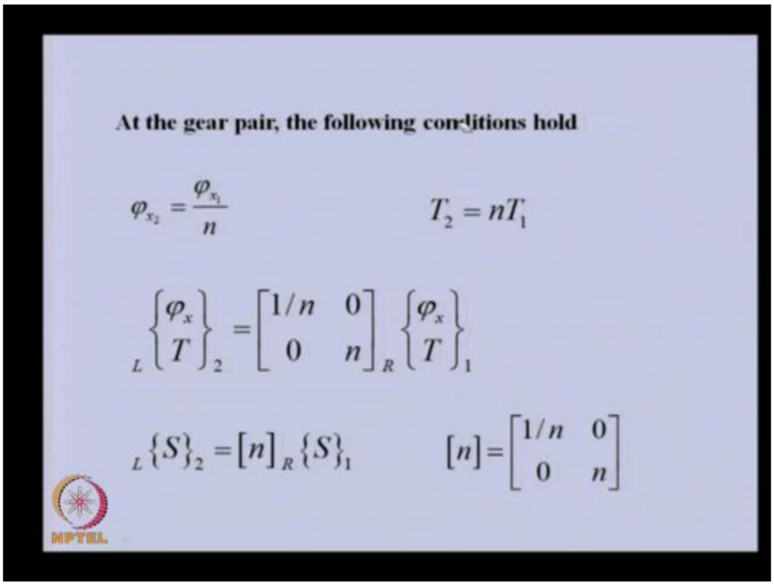

At gear location, we have this angular displacement these in the gear ratio and the torque are related like this for and these two can be combined like this. So, basically this particular matrix is nothing but gear ratio, so we can able to see that using m matrix we can transfer the state vector from one shaft to another shaft in between gear is there having gear ratio is n.

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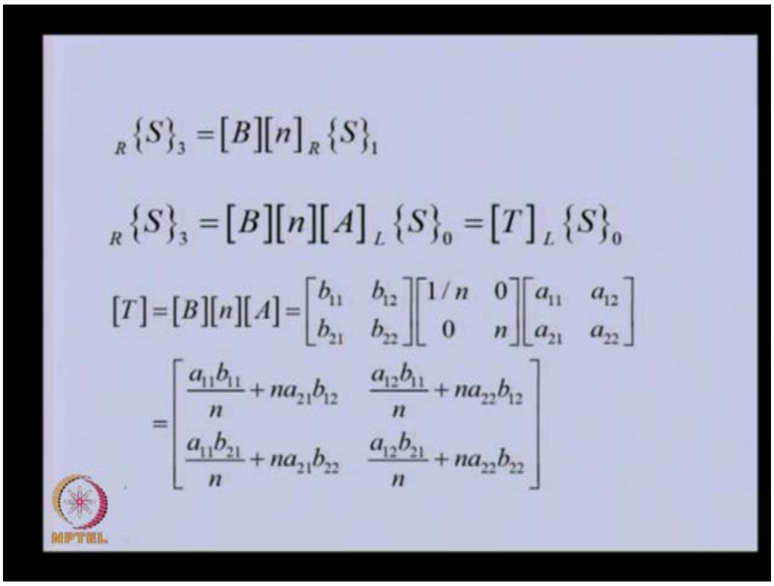

We can able to this m matrix to connect two equation like this, so now we can able to see that we are related the state factor of branch a 2 branch B directly with the gear ratio this is overall transfer matrix and this transfer multiplication is given here.

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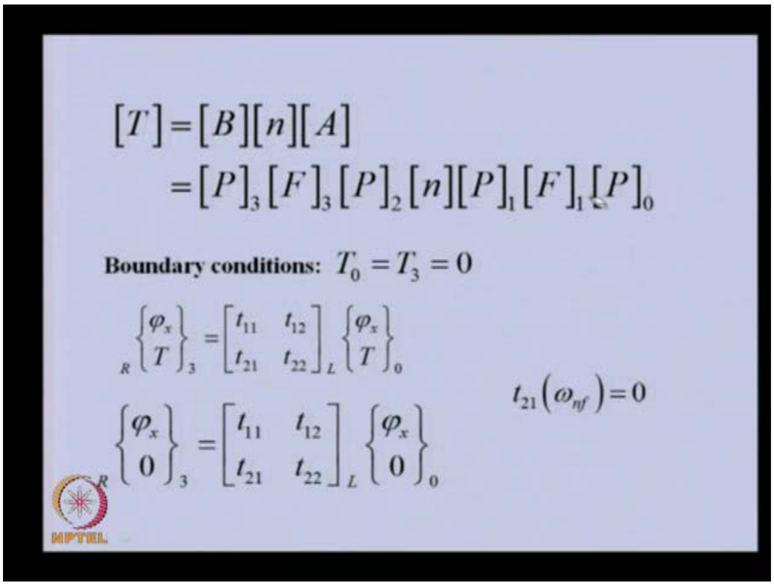

So, if we multiple them will get this transfer matrix and for the condition free condition we already seen that if we apply abounded the condition equation, we get this as frequency equation, so this particular form will be the frequency equation.

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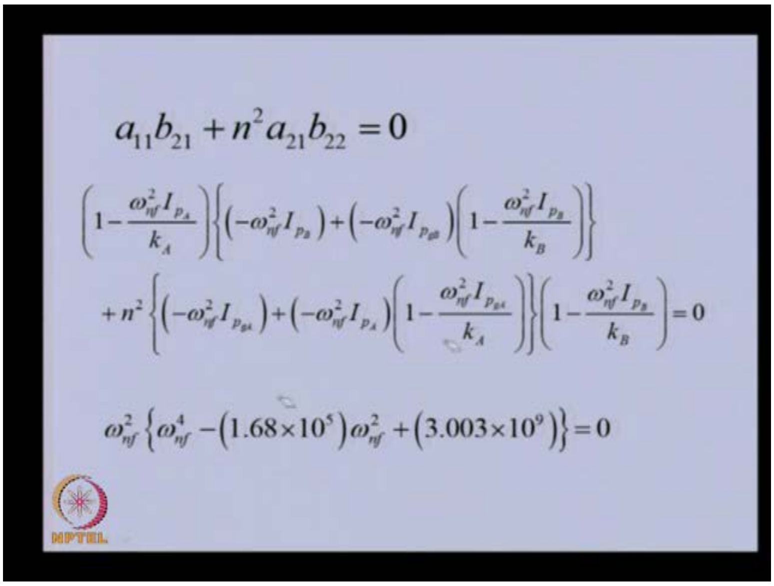

So, after multiplication of this we can get this expression, so this is the particular frequency equation. So, if we expand this and substitute the value of this parameter, we will get the polynomial of this form.

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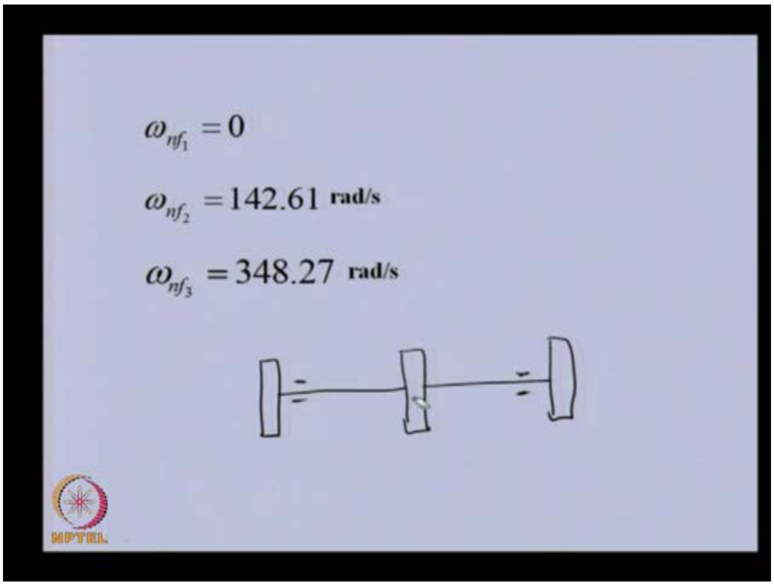

You can be able to see that we are getting frequencies which are 1 is 0 natural frequency and another two are corresponding to the flexible mode just continue. In this particular case consider the gear polar movement of inertia also, when we convert it when we transfer matrix basically we got three degree of epidermis of this form because this is corresponding to the gear system. So, that is why we are getting three natural frequency one of them is 0 because free boundary condition, I will conclude now. So, today we have seen the analysis of the gear system and branch system for tort analysis vibration case.

We obtain the frequency equation for both of cases for branch for gear system we did some kind of equaling we converted the gear system to single shaft system. That we can able to analyze with the method describe previously for shaft single shaft system which is much easier to analyze. With transform method Trans system which we have multiple degree of freedom the T m method is more advantages. We can able to get the recite body mode and flexible mode depending upon of the bonding condition of the problem and in substituent class; we will be studying the gear system and branch system analysis with finite element method also.