Theory and Practice of Rotor Dynamics

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Module - 5
Torsional Vibrations
Lecture - 18
Transfer Matrix Approach

So, in today's lecture, we will see how this transfer matrix method can be extended when damping is also there in the torsional system. The damping in torsional system, as we have seen that it is having basically two elements; one is having the shaft. So, we can have the damping in the shaft. You can able to see that this particular damping will be dependent upon what is the relative twist of the shaft. Another kind of damping is at the disc. May be that is due to what is the working fluid or lubricant or the coolant, which is the surrounding the disc.

In that particular case, we may have some kind of dissipation of the energy. When this particular fluid will interact with the disc in actual sense, this disc can be fan or turbine blade or similar to components in the rotating parts. Apart from this, we can able to see how we can analyze reciprocating engine specially, for torsional oscillation analysis. In this particular case, the main aim of this particular analysis will be that somehow we can able to convert this particular reciprocating engine, whether it is single cylinder or if it is multi cylinder engine to a equivalent rotating components. Once we can able to obtain the equivalent rotating components, we can able to analyze with the methods. We already discussed for multi disc rotor system. We are able to obtain the free vibration analysis or force vibration analysis of such torsional system. So, let us see the objective of the present lecture.

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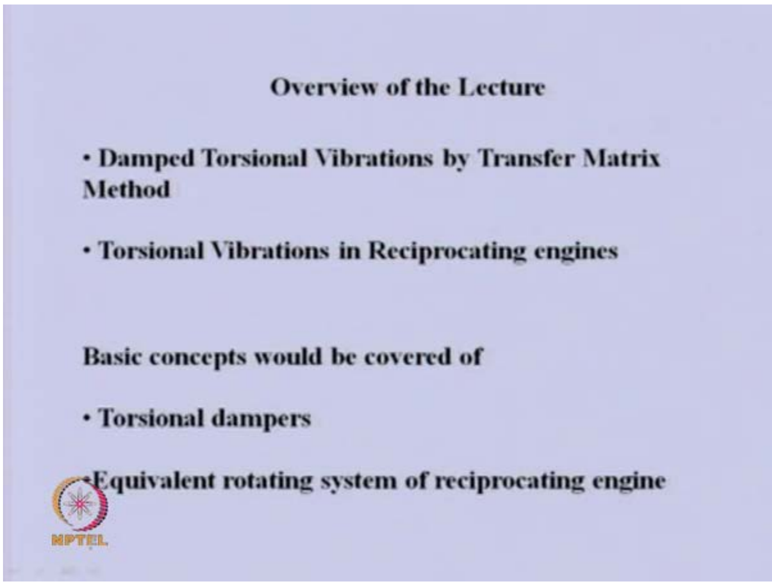

We will be analyzing the damped torsional vibrations. We will be extending the transfer matrix for this and another torsional vibrations of reciprocating engines. This will be introducing torsional dampers. They will be basically proportional to the relative velocity between two components. These whatever the torsional damping forces are there, they will be depending upon the rotating discs; either upon the relative twist of the stepped shaft. Even we will be introducing the concept of equivalent rotating system of reciprocating engine system. So, let us begin with the case in which we have some kind of damper in the shaft.

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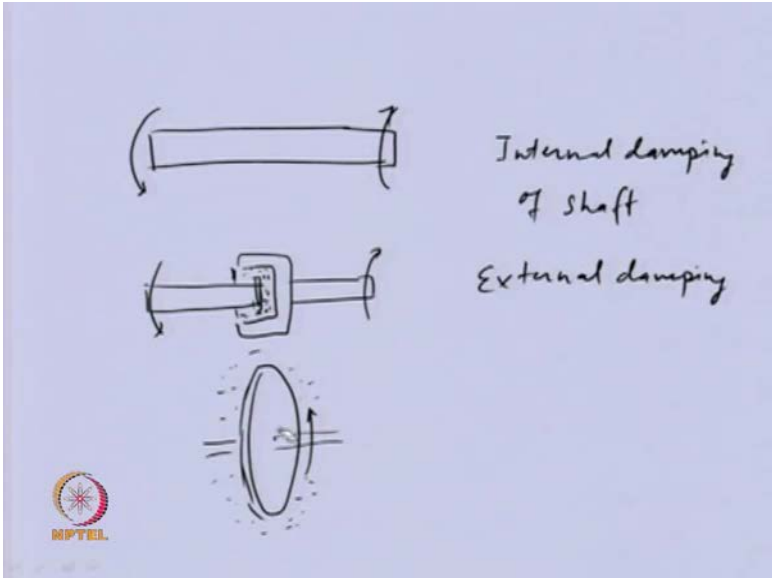

You can able to see. This is the particular shaft. When we are applying the torque at either end, this particular shaft, the material inside the shaft will go under tensional and cooperation. So, we expect that inside the shaft what are the there will be intermolecular interaction between various particles. That will generate heat. That particular heat will be dissipated in the form of; that will be coming, that will be resuming the temperature of this particular shaft. That will be dissipated to the surrounding. So, you can able to see that some form of energy is getting dissipated. Because of this, sometimes we provide some kind of external damping in the shaft.

So, we join this particular shaft with some kind of external damper like this. So, when we are applying, when this particular shaft is undergoing torsional oscillation, what are the fluid is there between these two, they will turned out. Because of this, there will be heat generation. Some of the energy will dissipate as heat. So, there is some kind of external damping. This is the first one; is internal damping of the shaft. This is something like external damping, which is in this particular case. We can able to design the damper. We can change the damping to the level, which we want. So, this is the external damping.

Apart from this, we have external disc in the system, which is rotating at very high speed. So, you can able to see that when this particular disc is rotating in fluid media, which is either a working fluid or some kind of lubricant or coolant then, there will be interaction of the solid and fluid. Because of that, we will be having some kind of damping effect because energy will be having dissipated from this particular motion. That is another form of damping, which we can have in the torsional system and we can see how we can analyze such system.

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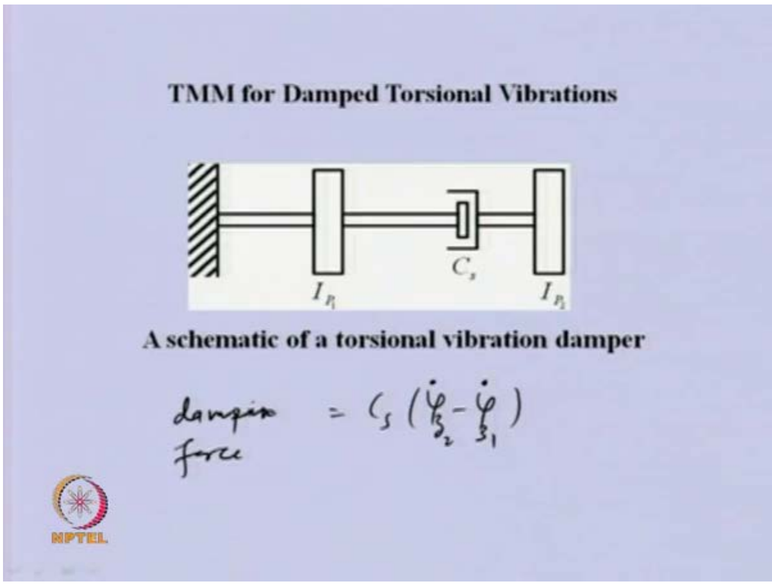

This is a picture of external damper, which is added between two discs and the shaft. Generally here, we this is the damping coefficient. This coefficient multiplied with relative velocity between with this particular shaft will give s the damping force. So, C s and the relative angular displacement at two ends of the velocities, not representing the velocity. So, this will be the damping force of this particular kind of external damper; is a damping force.

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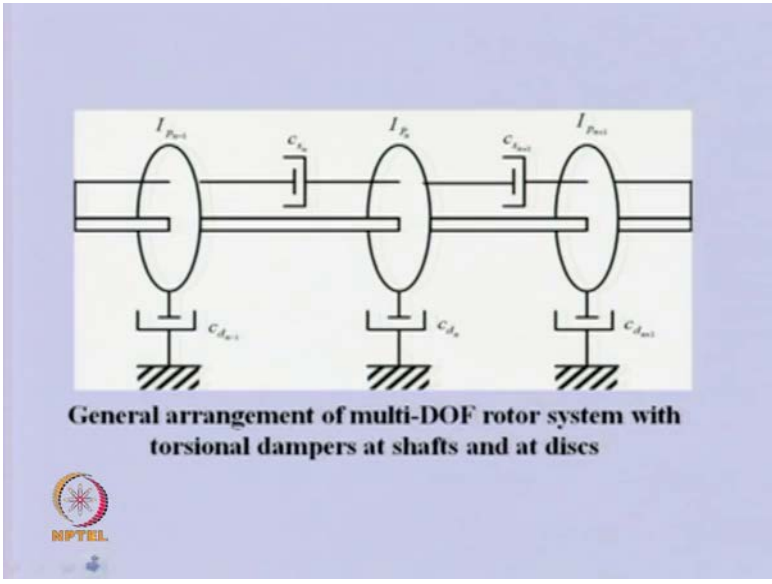

Now, let us see a multi disc rotor system in which number of discs are there and disc because of the working fluid. It is having some kind of damping so that, every place we have added a damper. Damping coefficient of this disc, we are writing as C d, C subscript d. These are for various discs. n minus 1 disc n th disc n plus 1 disc. So, each discs are having some dampers. Apart from that, we have damping in the shaft either in the form of material damping or external damper.

In this particular case also, whatever the damped force that will be there that will proportional to relative twist of this two discs. In this both cases, we are considering this particular model as proportional damping. So, this will be proportional to the either relative proportion of the disc or the actual velocity of the disc, depending upon what crank damping we are considering. So, to develop the transfer matrix, that is the point matrix and the field matrix as we did earlier.

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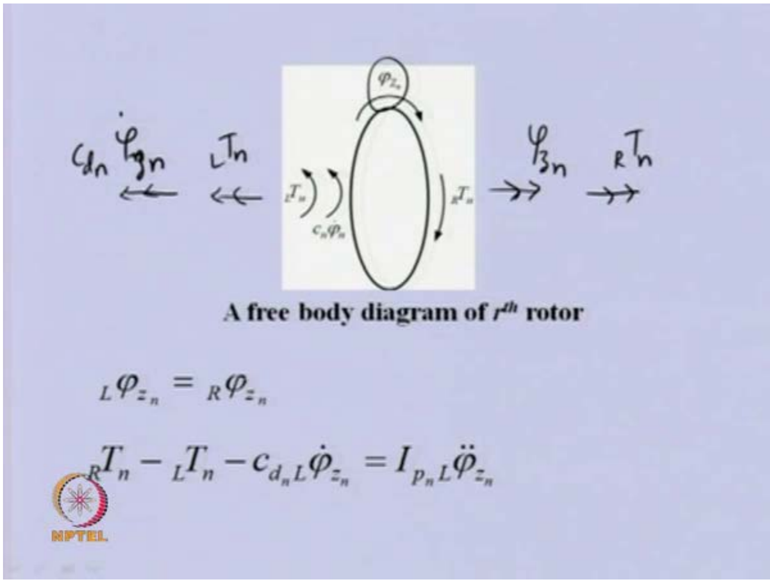

You will be finding the free body diagram that is the first of the disc. So, this is the one particular disc, n th disc. Let us say, this particular direction is positive direction for the angular displacement. This particular displacement, which we have written here, will be having some kind of reaction at either end of the disc from the shaft. So, this particular reaction is from left hand side of the disc. There is another one. This one I am representing that as the right side of the nth disc. So, these are the reactions which are coming from the shaft. Apart from this, there is the damping that is coming from the vibration of the disc damping.

So, that means, we can able to see is this will be opposite to the motion. So, that will be opposite to the motion because, motion direction we have taken right side positive. So, this will be C n or C d n and angular velocity. So, you can able to see that various torque that are presented here. Now, we can able to take the relations of these various variables. Here so, we can this particular disc is very thin. So, you can able to see that either end of the disc, particular disc will be having same angular disc. So, this left of disc nth and this is right of the nth disc displacement. They are same because they are thin disc apart.

From this, if we take the torque balance, we can able to see, this particular disc is in positive direction; so, the first torque. Then, the first side, one is in the negative direction and the damping torque also in the negative direction. These are the external torque, which is acting on to this particular disc. It should be equal to the rotary inertial of the disc. That means the polar moment of inertial of the disc and the angular acceleration.

So, you can able to see that similar to the previous case, we could able to relate various stationed state vectors on a term. This particular damping term is coming here extra as compared to the previous one. Now, we can able to rearrange this equation. So, because this particular case, we are dealing with let us say free vibration. So, whatever the torque which is there that will be having some amplitude.

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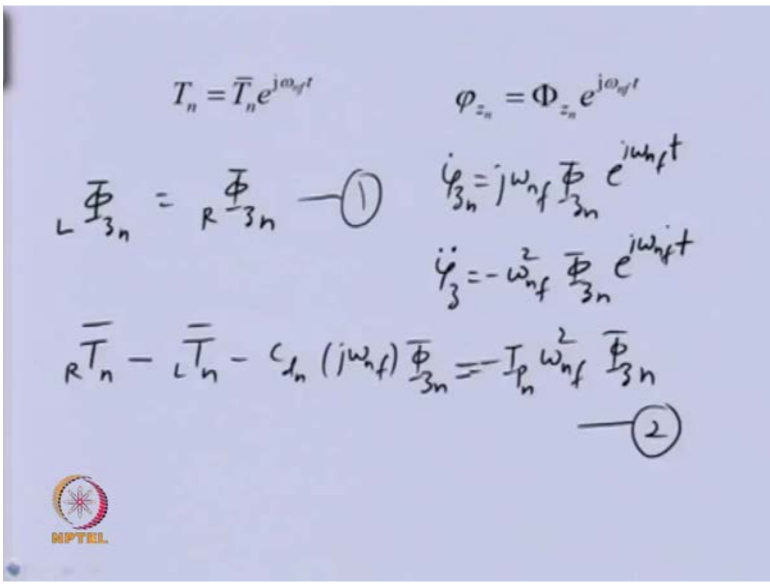

This amplitude, this particular, this is basically complex amplitude. The frequency of that particular torque is equal to one of the natural frequency of the system; torsional natural frequency of the system. Because of this, obviously, we can able to see whatever the response will be; that are also having same form. It will be having some complex displacement, angular displacement and the frequency. This complex displacement will be not only contain the amplitude and also the phase because there will be the damping of the system. So, there will be the phase difference between the torque and the displacement.

So, this particular complex will take care of not only the amplitude but also the phase. So, these assumed solution if we substitute in the previous expression. So, you can able to see that angular displacement is having singe derivative and the double derivative also. If we take derivative of this with respect to time, we will get because this is independent on time only. This time is dependent on time. So, that will get derivative. This is the Phi and e j omega n ft. So, you can able to see that if we take another derivative of this, we will get minus of omega n f square phi.

This is the amplitude part, e j omega n f t. So, these expressions, we can able to substitute in the previous equation. So, if we substitute, you can able to see that the first term because it is not containing any derivatives. That actually of the same form of this will contain now, complex displacement. This is form first equation second equation contains so many torque, so we can have t or n. This j omega term I am taking out because that will be cancelled out throughout the equation. This is second expression. Then, we have damping expression, so minus C d n j omega n f Phi 3 j omega t e rays to j omega t. I am taking out. So, this term will not be there.

So, you can able to see that you express this equal to the inertial term. So, here will be having so, you can able to see. This is the first equation. This is the second equation. Now, we can rearrange these equations or so that the left side and the right side terms are in one side and of the equality. The other part is in the other so; we can able to write some kind of transfer matrix. So, you can able to see here, I have arranged the previous equations, this equation in this particular matrix form.

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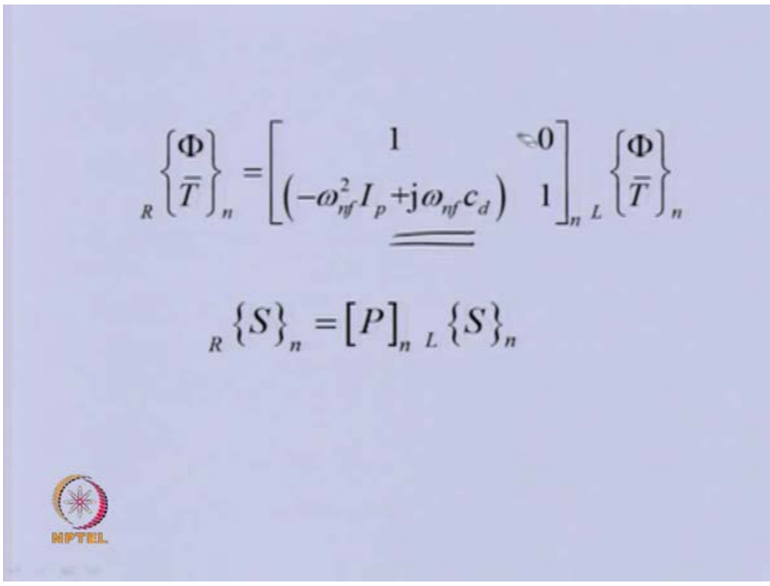

As we did earlier, this is in the state vector right on the nth disc. This is in the state vector left of nth disc. I have arranged all those terms here. So, you can able to see the first equation is multiplication of this one will be this. So, this is the first equation with this equation. The second equation is the second equation. Expand this particular matrix. We get that particular second equation and in more compared form. We are writing this as state vector right of nth disc is equal to point matrix state vector in the matrix nth disc. So, you can able to see that now the point matrix is a complex. As against the previous case, it was the real quantity.

This is the extra term, which is coming. The present software like matlab; they can handle like matrix complexes. So, those can be used easily. Now, coming to the shaft element. So, we developed a point matrix of a disc in which we considered the damping. Now, will come to the development of the shaft element; shaft the field matrix in that also, we will be considering damping.

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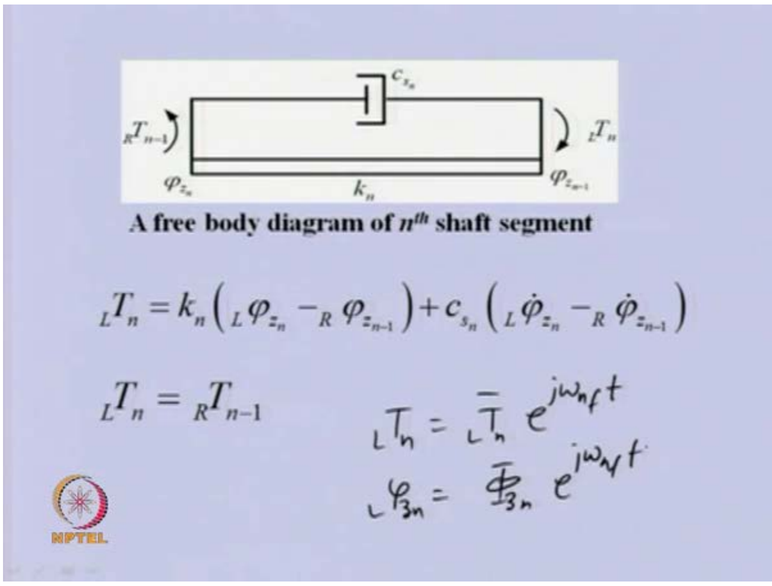

So, let us see. This is shaft element, shaft segment in which not only it has the stiffness property but also the damping. This particular damping; for this particular damping factor is multiplied with relative angular end of the shaft. We will be getting the damping force. So, you can able to see that now we can able to express the field variable like this. We can able to express the field variable like the torque at left of the length nth disc. This side, we can able to express as the stiffness of the n th shaft. The relative displacement, angular displacement of two ends of the shaft plus the damping and the relative velocity and the angular relative velocity, earlier we had only this restoring force.

But now, we have the damping force also in the torque. Because, there is no inertial, we are considering the shaft. So, either end of the torque will be equal. Now, let us see how we can able to rearrange the equation. So, first equation again; once we take the solution of these all, the torque and response if we express force free vibration like this. Even the angular displacements when we express them like this. Once we substitute this in this equation, we can able to get the torque.

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$$ {}_R\bar{T}_{n-1} = k_n\left({}_L\Phi_{z_n} - {}_R\Phi_{z_{n-1}}\right) + j\omega_{nf}c_{s_n}\left({}_L\Phi_{z_n} - {}_R\Phi_{z_{n-1}}\right) $$

or

$$ \left(k_{n\,L} + j\omega_{nf}c_{s_n}\right)_L \Phi_{z_n} = \left(k_n + j\omega_{nf}c_{s_n}\right)_R \Phi_{z_{n-1}} + {}_R\bar{T}_{n-1} $$

and

$$ {}_L\bar{T}_n = {}_R\bar{T}_{n-1} $$

$$ \begin{bmatrix} k + j\omega_{nf}c_s & 0 \\ 0 & 1 \end{bmatrix}_{n\,L} \begin{Bmatrix} \Phi \\ \bar{T} \end{Bmatrix}_n = \begin{bmatrix} k + j\omega_{nf}c_s & 1 \\ 0 & 1 \end{bmatrix}_{n-1\ R} \begin{Bmatrix} \Phi \\ \bar{T} \end{Bmatrix}_{n-1} $$

$$ [L]_{n\,L}\{S\}_n = [M]_{n\,R}\{S\}_{n-1} $$

It is, we can able to see that these are the complex amplitude or terms. Now, they are relative angular displacement. This particular equation, we can able to separate. You can able to see here. We can keep all the terms, which is left hand side of the disc terms. This side we are keeping all the terms, which is there in the right side of the disc so that, we can able write the transfer variable for such fields. So, here also, the left and right side with this two equation for the first and the second equation, we can able to combine in this form.

We can able to see if we expand the first term we get this term and second equation you will get this one. You can able to see that this particular matrix, which is complex matrix I am representing as L for n nth shaft. This is the state vector left of n. This particular matrix, I will call n for nth shaft. This is the state vector of right of n minus L station. Now, we can able to see that this particular equation is having not the same form as we had earlier for the without damping case. So, to make this equal, we need to inward this matrix. We need to multiply both by inwards of L matrix.

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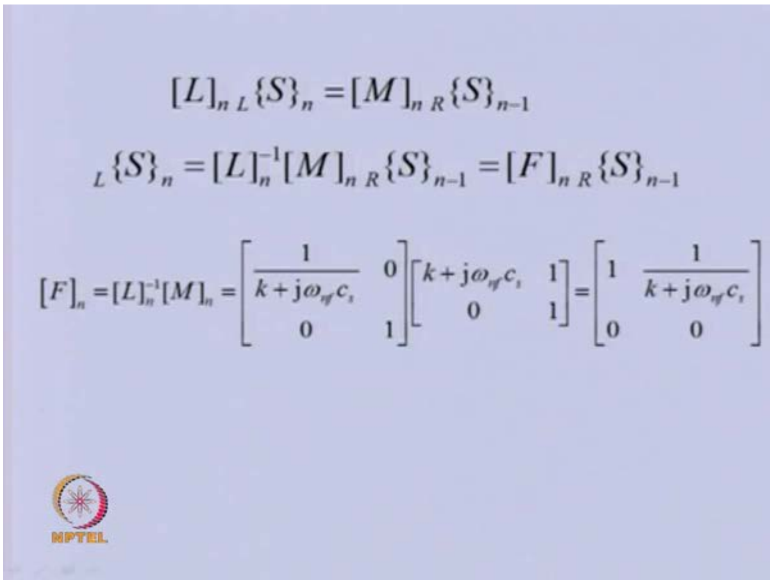

So, you can able to see. Now, we are inverting this matrix to get this inverse of to get L and M. I am calling as field matrix. The field matrix is the multiplication of this. If you multiply them, we can able to see the field matrix is simpler. It is having simpler form. So, it is simpler matrix. So, this is field matrix, which is having damping on the shaft. Also, you can able to see the field matrix is relating the state vector left of nth disc and the state vector right of n minus 1 disc. Now, it is having similar form. Only thing is the F is damping. Only thing is, if you neglect the damping that is having the exactly the same form, which is having the same damping case.

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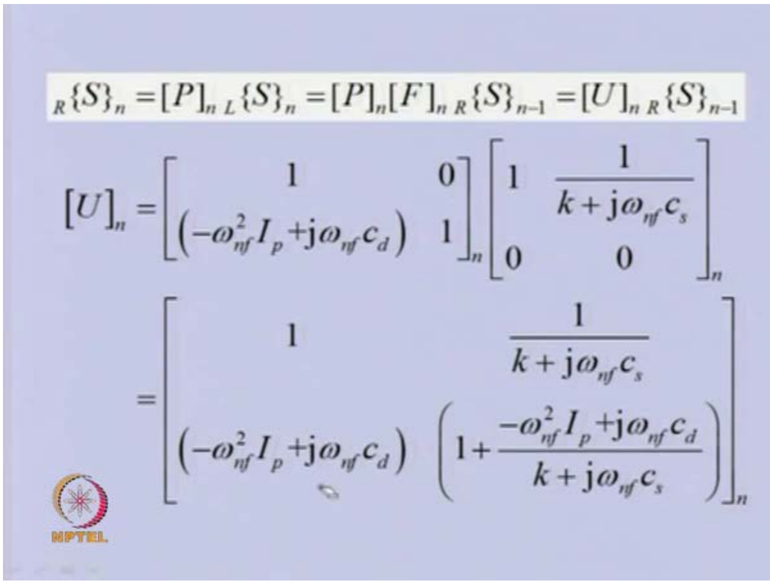

So, once we have obtained this particular field variable and point matrix field variable and point matrix, we can able to see that we can able to get the transfer matrix. So, this is the point matrix relation. If we substitute the previous expression, which includes the field variable; so, you can able to see that we can able to relate this state vector right of nth disc and the right of n minus 1 disc. This P into F is the transfer matrix which relates we can able to see the right n minus 1 disc to the right of nth disc. This transfer matrix we can able to get by multiplying P with F. This is the form of this transfer matrix.

So, you can able to see this is the complex. But, still the size is 2. So, once we obtain the transfer matrix which relates the state vector from right of n minus 1 disc to the state vector in the nth disc then, you can able to see that whatever the analysis we did for un damped case is valid here. We can able to write for a whole rotor system. We can able to relate the state vector from may be extreme left to the extreme right. Once we relate them, we can able to get the transfer matrix T matrix. Then, we can apply the bounding condition as we earlier applied.

We have seen several example of as such application of bounding conditions. From there, we can able to get the frequency equation that can be solved to get the natural frequency. So, the overall procedure of solution remains the same. There will not be change in that. So, I am not repeating the same thing. Only thing here is the only difference is the form of what are. The matrix is a complex form. As such, there is not much change in the overall procedure. Now, we will see how we can able to model the reciprocating engine in the system especially, if we have the multi cylinder engine. How we can able to analyze the torsional vibration behavior of that?

How will you can able to obtain the torsional natural frequency and any be torsional force vibration for that particular case. Here, as I explained earlier, the main idea would be to convert the all the reciprocating parts to equivalent rotating parts in the main shaft system. Once we can able to convert shaft into equivalent revolving parts then, what are the analysis? We have done on the torsional vibration can be applied in that. So, let us the modeling of the reciprocating engine system.

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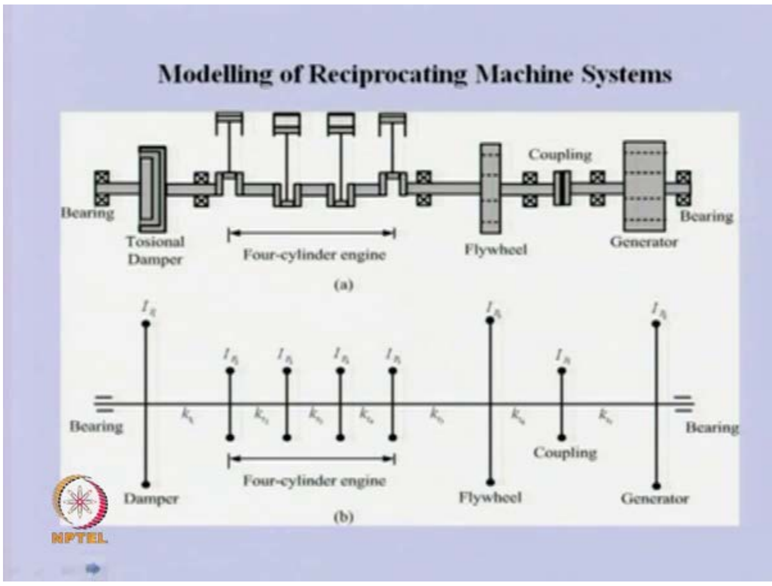

So, in this particular case, we can have let us say, four stoke four cylinder engine. So, these are the four engines reciprocating engine. Apart from this, we will be having like other accessories, like we can have some kind of torsional damper or some fly wheel because the fluctuation of the energy takes place. Because of the fly wheel model of the engine are different so, to make the supply of the energy uniform we provide flywheel. Then, the coupling may be generator or similar such parts will be there. But, other parts like flywheel coupling generator or damper, they are something like revolving parts. But, these engines are not only the revolving parts.

But, some parts are oscillating or some are pure reciprocating motion. Now, our aim would be to convert these particular engines to equivalent revolving parts. So, you can able to see the converting the torsional damper to revolving part is easier. Flywheel is easier coupling and generators are easier. But, this particular thing, how we can able to do it? It will be using very approximate crude method, for this particular conversion of reciprocating parts to the revolving parts.

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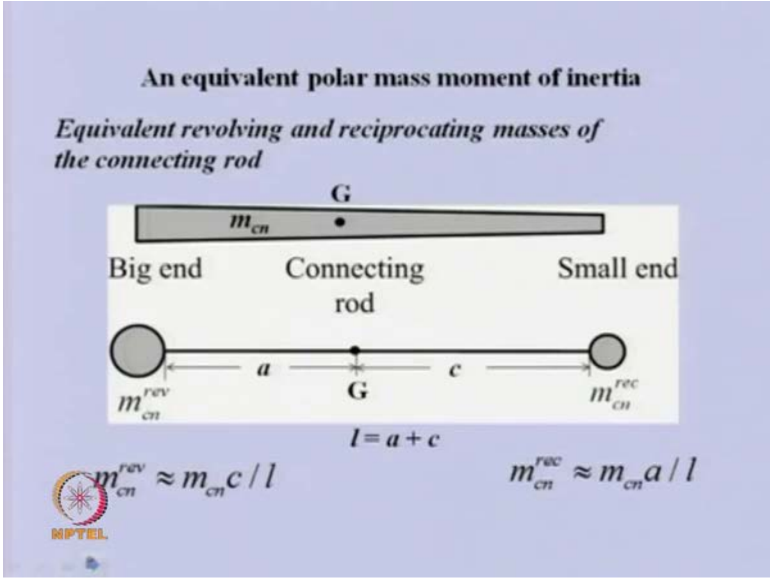

So, we will start with the crank or connecting rod. We can able to see. This is the connecting rod, in which one end is big end and another end is small end. As such, this particular connecting rod will be a rigid body. Mass will be continuously varying from one end to another end. We would like to convert this connecting to two mass system. One mass will be this. Generally, big end, it is connected to the crank and the small end to the piston. If we connect this to system, two mass system, two mass dynamic system so, you can able to see that this will be attached to the crank. It will be revolving.

So, I am calling, this is the mass. So, this is revolving with the crank. This one is mass of the crank, which will be attached with the piston of the reciprocating component of the system. So, this two masses can be obtained by simple order system by centre order gravity is at G of this two system. It is the distance from one end to another end. You can see this distance; total distance is l that means a plus c is l so approximately. This particular mass will be given as total mass of the connecting rod into c the distance divided by l.

This particular mass will be given by the total mass of the connecting rod into this distance by l. So, this approximate relation. So, basically we this, we are obtaining by ensuring that the center of gravity of these two systems are same. Even if you want more accurate calculations of this, we can have the moment of inertial to be same. Then, we can able to get more accurate result. But, these are quite good approximations. You can able to use such approximation.

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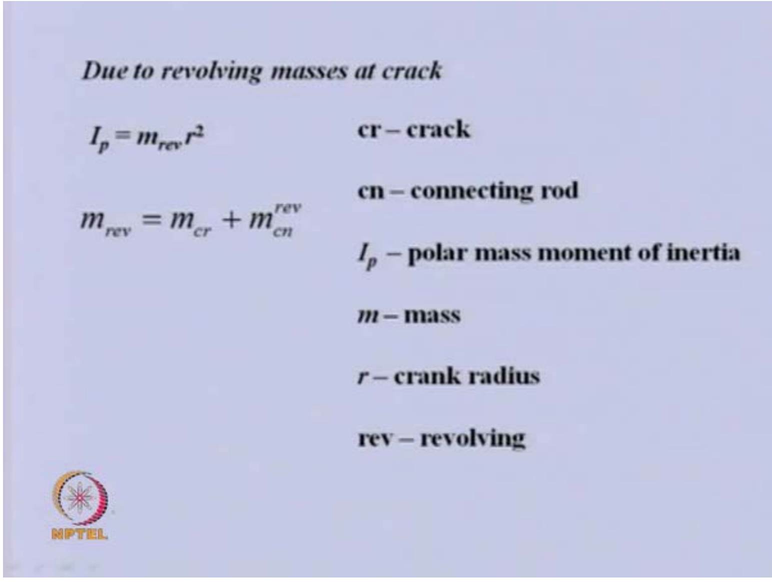

Now, coming to the revolving masses at crank. So, in the polar moment of inertial will be given by total revolving mass of the crank. r is the crank radius. This revolving mass is the revolving mass of the crank itself as well as the connecting rod. So, in the previous line, we obtain this particular mass. What will be the equivalent of the connecting rod which will be having the revolving part? That means this one because this is connected with the crank. This is connected with crank. So, this is the total revolving mass at the radius, which is coming from crank itself and connecting rod two parts.

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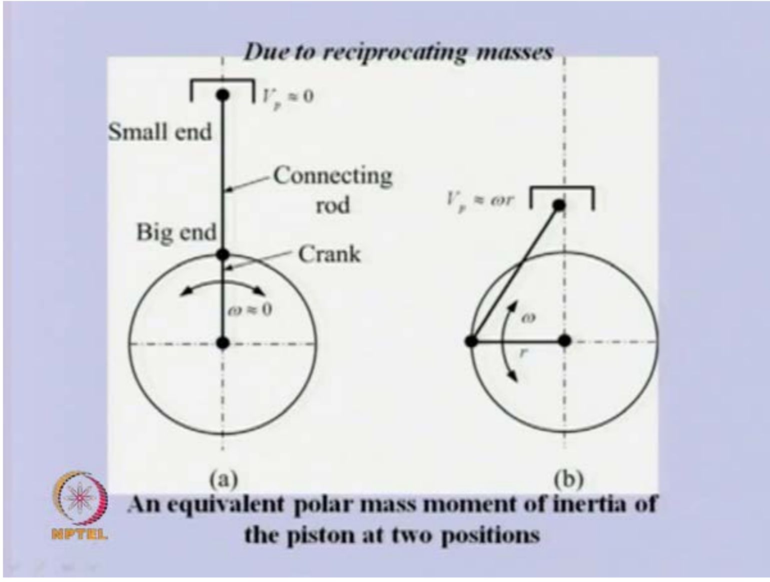

Now, coming to reciprocating mass. Let us see the particular engine that is nothing but crank and connecting rod. Here piston is there. So, you can able to see. In this particular configuration, it is the one of the dead end of this piston. This is another configuration, in which that is crank is making 90 degree with the previous one. So, if we compare this two; so, in this particular configuration, if try to give some kind of oscillation to this particular crank because this is dead end. So, hardly any motion of piston would take place. So, velocity of the system will be nearly 0.

If we give a small some kind of oscillation to this crank, the piston will hardly move. That means for this particular configuration as such piston will not be having any inertial for this particular configuration. When it is 90 degree, we can able to see because crank connecting rod length is generally very large. So, if we give some kind of oscillation to this particular crank here in small amount, this piston will be having same velocity as it is. Same velocity, this crank connecting rod; will be very high.

So, that means what are the motion of the connecting crank is taking place? Piston will be having same amount of velocity. So, as if this particular system, mass is attached to the cranking itself so, that much inertial it will impart. So, you can able to see that in the first case it is imparting 0 inertial. Second case as it is same in the reciprocating part. So, to be to approximate, we can take average of this two as the rotor inertial of the piston. So, the reciprocating part when we want to analyze for the polar moment of inertial.

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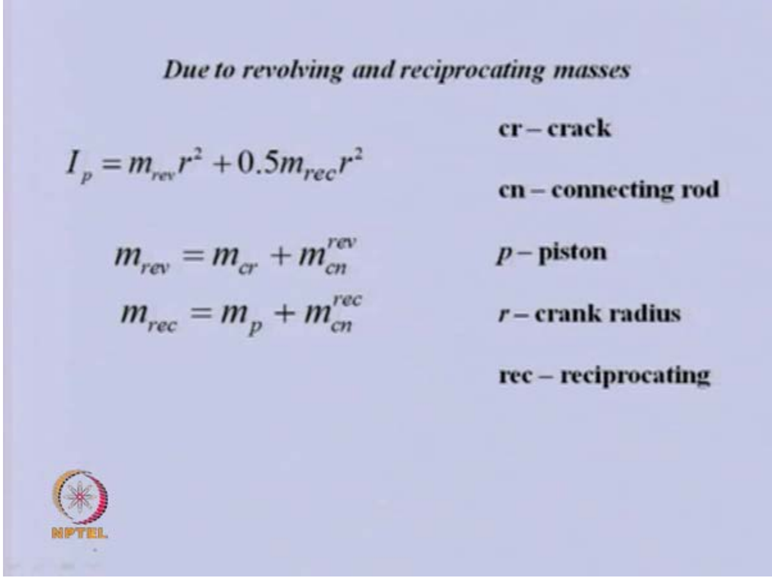

So, we will be taking half of the reciprocating part as it is attached to crank radius. So, this was the previous one, which we obtained this one. From piston, we are getting this. So, you can see that contains from crank itself and connecting rod. This is containing from piston and the connecting rod because earlier we converted the connecting rod to purely rotating and purely reciprocating part. So, this particular is mass corresponding to this particular component because this is attached to the piston.

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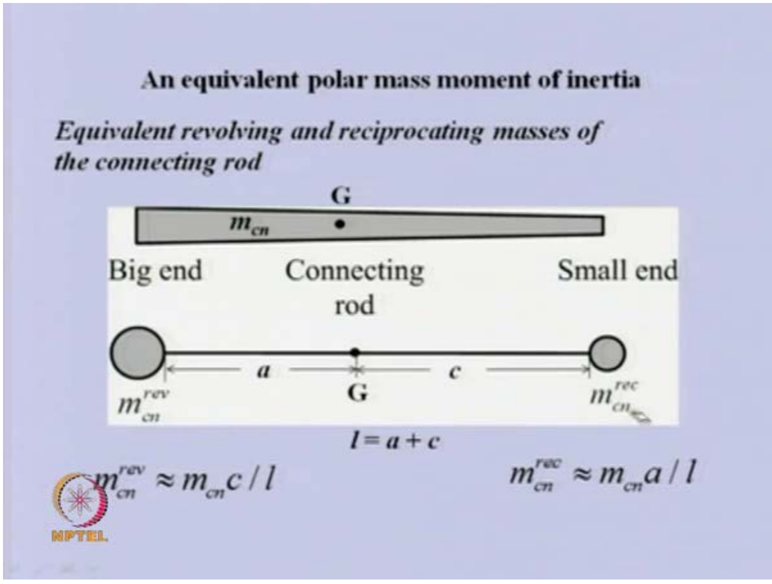

So, this mass will add up to the piston mass. So, you can able to see that.

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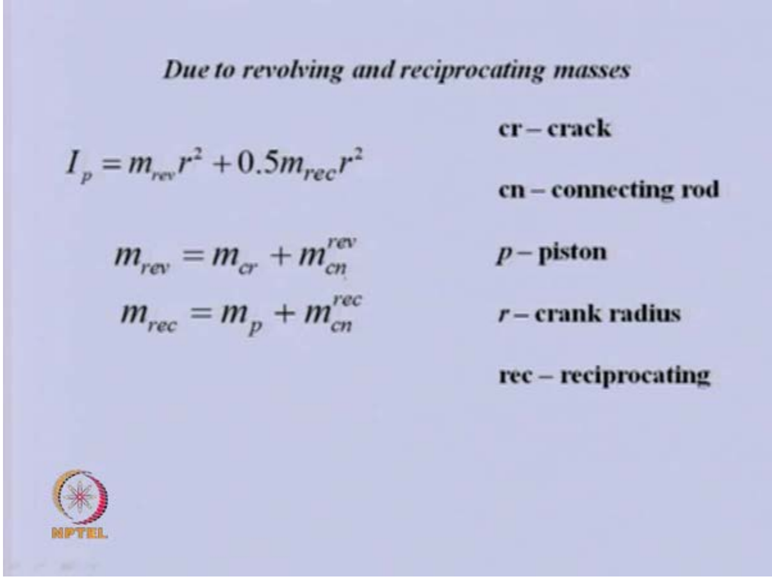

Now, you could able to get what is the equivalent polar moment of inertial of the whole crank connecting rod and the piston.

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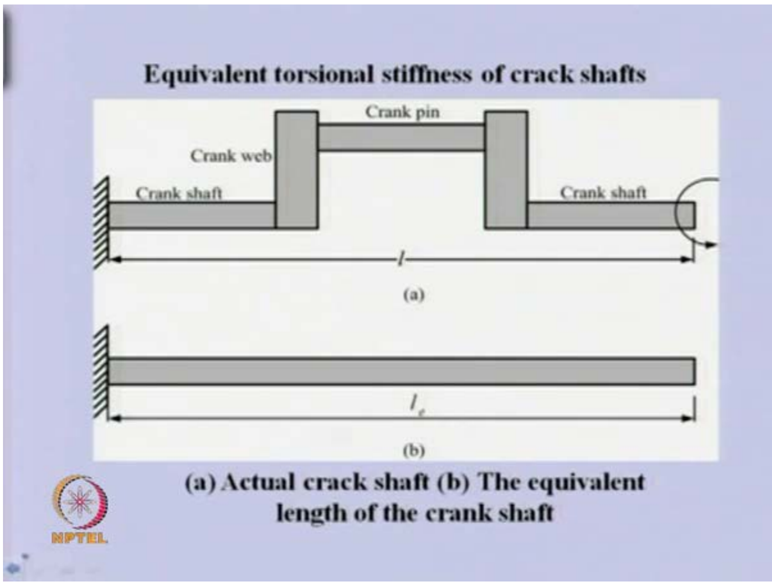

Once we have done it, we can able to convert the reciprocating mass and the revolving mass is purely revolving mass system. Now, let us see the torsional stiffness. Because the crank is having variable geometry so, if we want the uniform shaft so obviously, a equivalent shaft. So, torsional vibration of this system. This system should be same so, we can able to see rather we can do experimentally. We can able to give some twist.

From there, we can get what is the partial stiffness of this particular crank. We can able to design or we can able to get the equivalent uniform shaft from that. Using finite element or packages, we can able to model this and obtain the torsional stiffness. Then, we can then get equivalent shaft out of that so that we can able to convert the whole reciprocating engine system to uniform shaft system like this.

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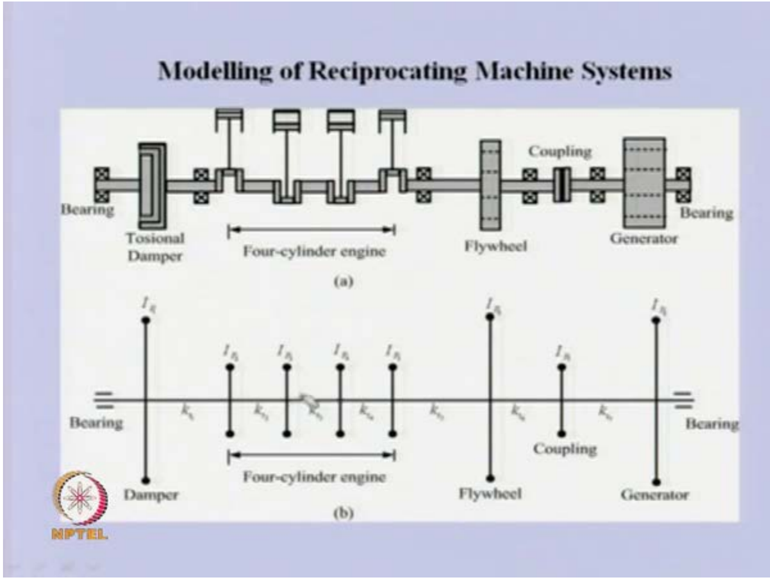

So, these are revolving mass of this engine. What are the shafts is there? We can able to obtain equivalent uniform shaft using the analysis of this cranks; either by experiment or by effluent packages. So, once we obtained this kind of equivalent revolving system then, we already know the transfer matrix method. It will be helpful for analyzing such system. In the subsequent class lecture, will be dealing with the finite element method that can be used in reciprocating engine. As we know as because we have compression store expansion store like it is four stoke engine or two stroke engine, we have different cycle.

Out of these cycles, only one is the power store I, which we get the power. Remaining period engine takes the energy from the fly wheel, which flows the energy or other rotating parts. Now, you can able to see that there will be variation of the torque that take place. These torques are some kind of periodic in nature. Even if it is in multi cylinder engine, we will be having some periodic variation of the torque and these torque once it is periodic.

So, we can able to see that by using polar series, we can expect there will be fundamental frequency and their multiples of harmonics, which will be there in the system. So now, let us see if we know the taw variation which is periodic in nature. How we can able to get the various components of the higher harmonics using some kind of square fit?

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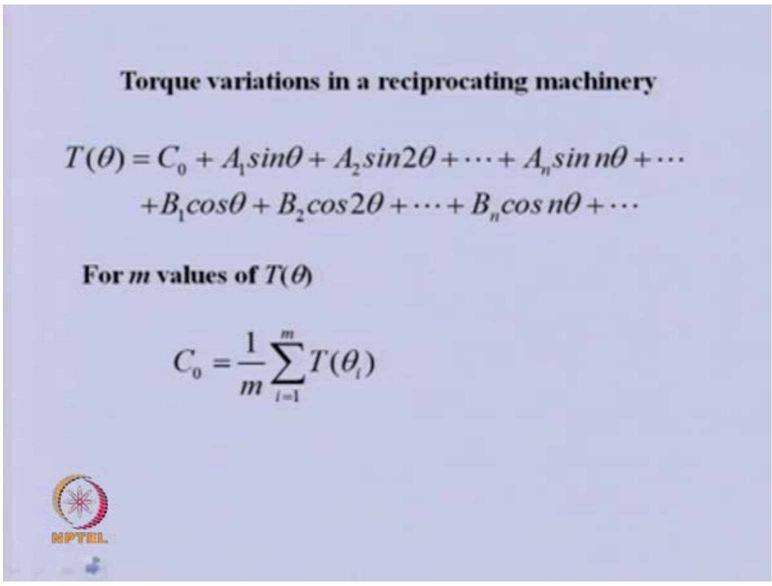

So, that particular for case, let us say the torque variation in the reciprocating engine, let us say this is the torque variation. Let us say with angular or displacement of the crank that torque will vary. This is the periodic. We can able to write this in the form of Oreo series, in which we will be having some constant term or sin and cosine terms. Depending upon the accuracy, we want, we can take n of any order but, n of tenth or twenty will be good enough for any practice purpose. As we know this particular is constant. But, the mean of this particular torque is expressed like this. Once we know the variation of the torque; how the torque varying with theta or varying with 0 degree, 10 degree, 15 degree like this, how what are the torque. This kind of data if we have, we can able to get the mean of that particular taw. This A 1, A 2, A 3 and A n, B 1, B 2, B 3, B n. These we can able to obtain by following procedure.

(Refer Slide Time: 39:33)

$$T(\theta_1) - C_0 = A_1 sin\theta_1 + A_2 sin2\theta_1 + \cdots + A_n sin\, n\theta_1 + \cdots$$
$$+ B_1 cos\theta_1 + B_2 cos2\theta_1 + \cdots + B_n cos\, n\theta_1 + \cdots$$

$$T(\theta_2) - C_0 = A_1 sin\theta_2 + A_2 sin2\theta_2 + \cdots + A_n sin\, n\theta_2 + \cdots$$
$$+ B_1 cos\theta_2 + B_2 cos2\theta_2 + \cdots + B_n cos\, n\theta_2 + \cdots$$
$$\vdots$$
$$T(\theta_m) - C_0 = A_1 sin\theta_m + A_2 sin2\theta_m + \cdots + A_n sin\, n\theta_m + \cdots$$
$$+ B_1 cos\theta_m + B_2 cos2\theta_m + \cdots + B_n cos\, n\theta_m + \cdots$$

$$m \geq 2n$$

So, let us say, we are expressing the previous expression for theta for one of the value because c naught. Once we know the T variation, we can able to get the mean of this. So, I have taken this in the left hand side. I am expressing these terms. So, you can able to see instead of naught, theta in this place. I am writing theta 1 in first equation because I am expressing T at theta. Similarly, I can able to write this for theta 2 same equation. I am writing for theta 2. So, here also, you can able to see all theta will be replaced by all theta 2.

All these constants will remain same. They will not change with the angle. Then, for theta m, you can able to write here. Theta will be replaced by theta m. I am saying that m is greater than 2 n because, in this particular case, we are truncating these for a series after nth terms. So, all the higher order term, I am neglecting it. So, if we neglect this, you can able to see that we have for A and n terms for B, another n terms so, two n terms are there. So, the total number of equation, which we should have; that means the total number of information of T theta we should have should be equal to 2 n or more than 2 n. Then, we can able to get these constants.

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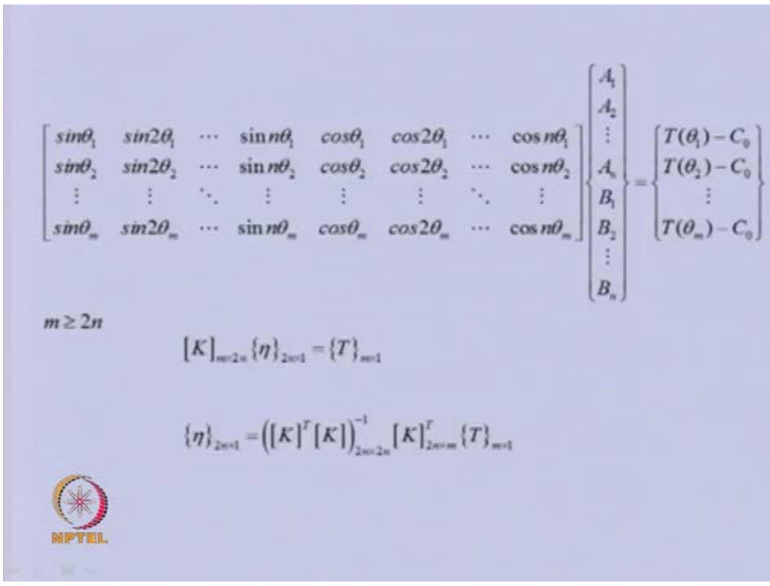

So, all those previous equation, I have rearranged in a matrix form. So, you can able to see that there are 2 n are unknowns in the form of A and B s. I have put in this vector. This is all the terms, so if you see previous equation.

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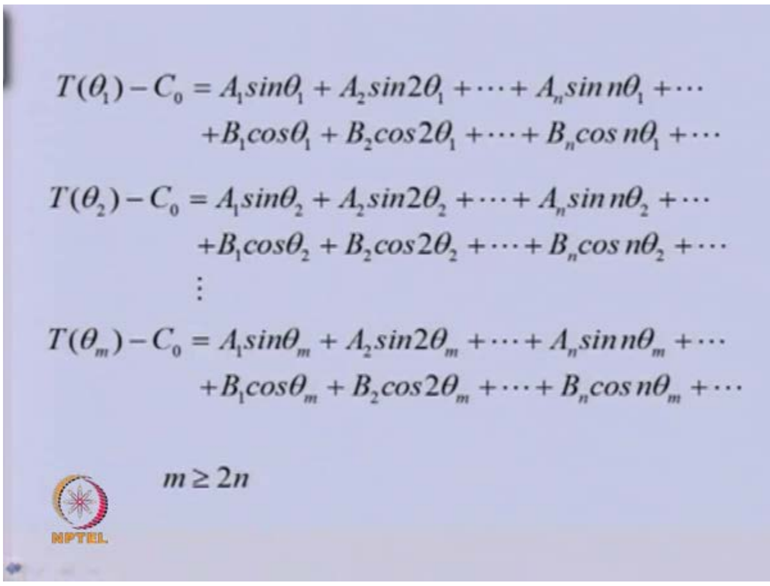

So, these equations in the left and right hand side have arranged.

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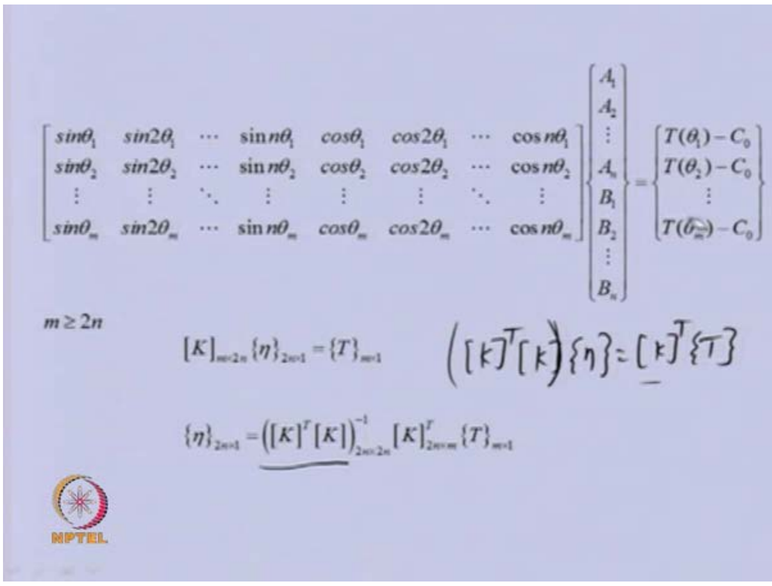

In a here, in the left hand side, in this particular form. All the terms, which were there in the left hand side, I have bought here in right hand side. So, you can able to see that these are all total n equations, which is either equal to 2 n or more than 2 n. So, if it is so here, this particular matrix I am writing as K matrix. This unknown vector of A and B, I am writing as mu. This is the torque variation. We have information. I am writing as T matrix. We able to see these subscripts are representing the size of the matrixes. If n is not equal to 2, n it is greater than 2 n, then this K matrix will be rectangle matrix.

So, for that particular case obviously, if you want the mu inward the K matrix. But, mu rectangle matrix cannot inverted directly. So, will be pre multiplying the K matrix, both the sides of this equation, pre multiplying the transpose of the K matrix. Now, we can able to see that this particular matrix will be square matrix. This can be inverted. So, this is thing we have done here. So, this we inverted and this is this particular matrix. This is called C D inverse. But basically, this is nothing but the least square feet. So, we can able to get.

Now, you can able to get all these constants, which is able to representing what the various sin and the cosines harmonics are contributing to the T. So, you can able to see that torque we have expressed in terms of sin and cosines. They are higher in harmonics. So obviously, while designing these frequencies and the natural frequency systems should not coincide otherwise will be having torsional resonance condition.

Now, let us take one example especially, of the multi rotor system because the today's lecture we have seen that a multi cylinder engine can have a multi rotor system or revolving system. So, with the transfer matrix method, we try to analyze a bigger system. We will see the capability of the transfer matrix method, how powerful it is for large system. So, for this particular case, you can able to see, should show first may be this drawing.

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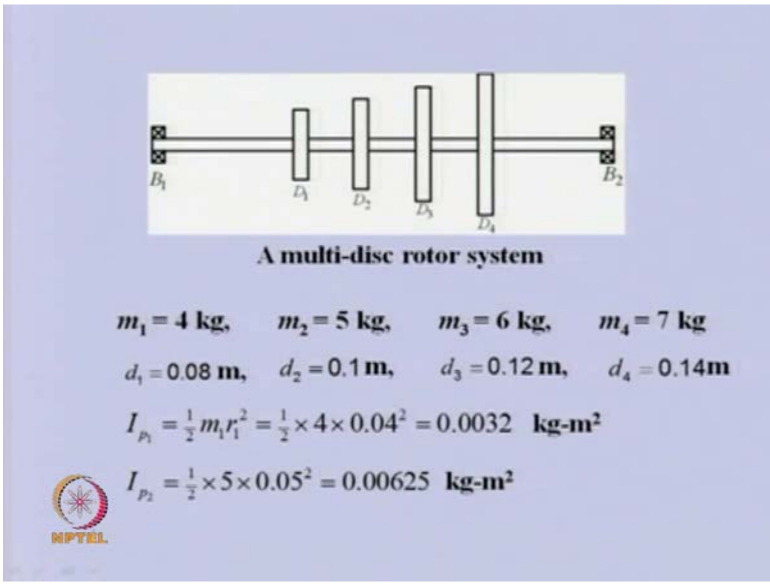

You can able to see. We have four discs. They are connected by uniform shaft. Shaft is supported at ends, frictions is bearing and various properties of this shaft.

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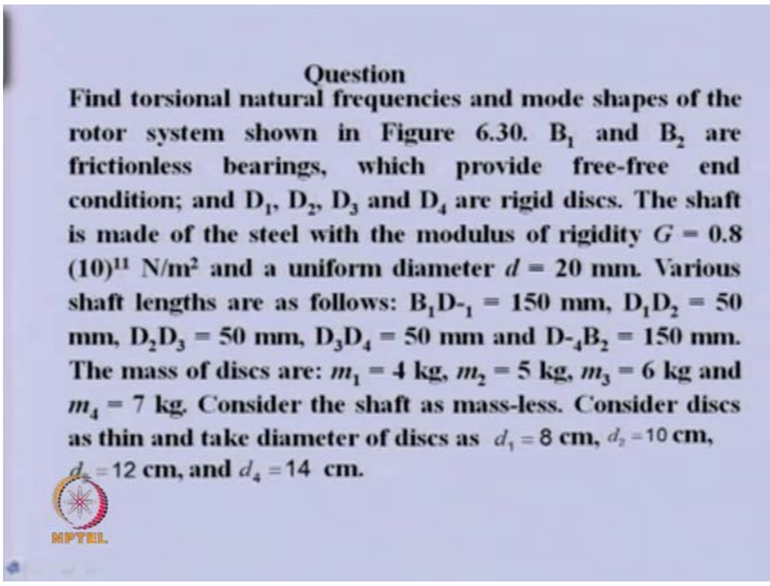

See this various distances and various properties of the mass. Their size of the disc like here, we are considering the thin disc. We can obtain the shaft of the disc. We will be using this information. So, all the geometrical mass and various distances are provided here of the disc. Even the models of rigidity is given. So, you, we can able to use this to obtain polar moment of inertial of the various disc.

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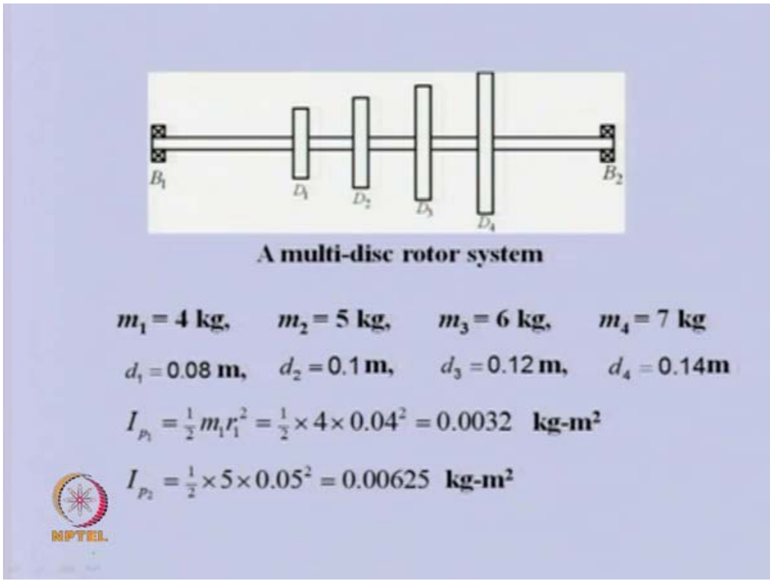

So, you can able to see. There are four discs in different masses. These are thin disc. They have different diameters. So, we are calculated their polar moment of inertial. These are the polar moment of inertial U H.

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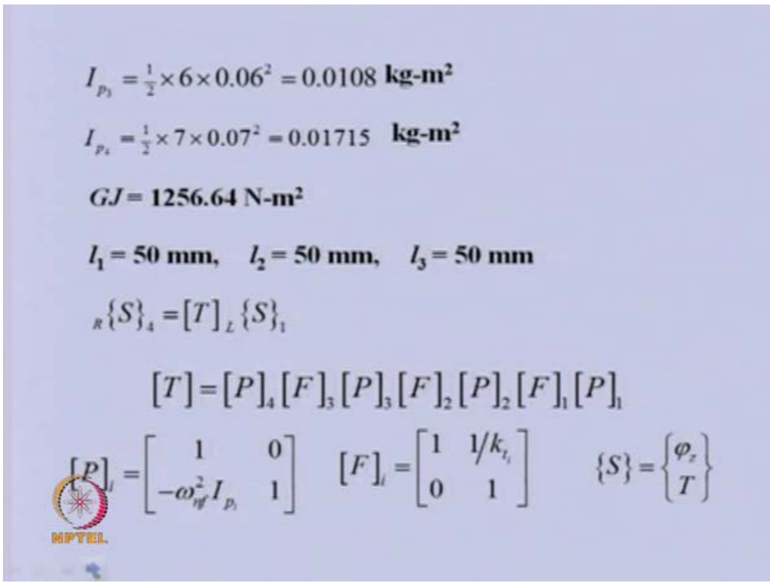

Other geometrical parameter can be obtained like J pie by 32 d 4 diameter. We know various shaft segment lengths are. They are given here. So basically, if we see because this free supported.

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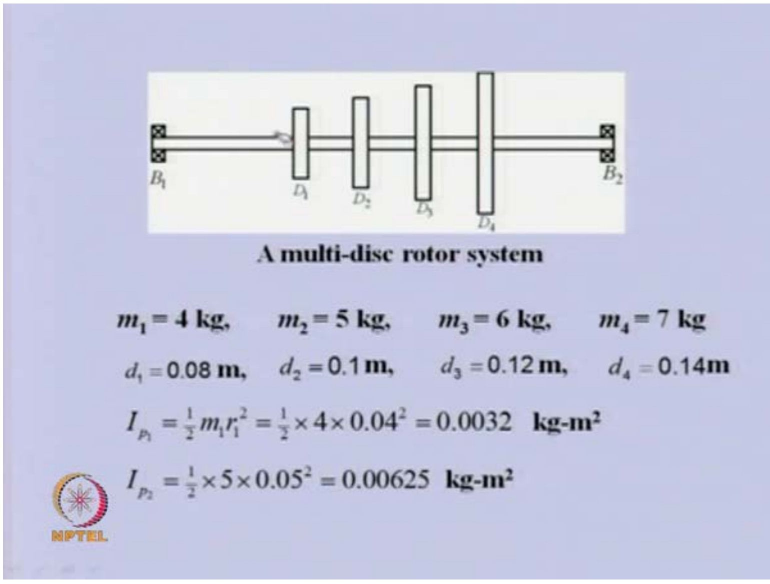

We are neglected in the mass of the shaft inertial of the shaft. So, the shaft segment, which is here and this end as such during torsional oscillation. Then, be moving along with this disc that this particular shaft segment move along with this. This will move along with this as a rigid body. So, there is such they are not contribute into the transfer torsional vibrations of this system because they will not get twisted. There is no inertia attach to they. So, only these segments will be of relevance. So, you can able to relate the state vector like you can able to.

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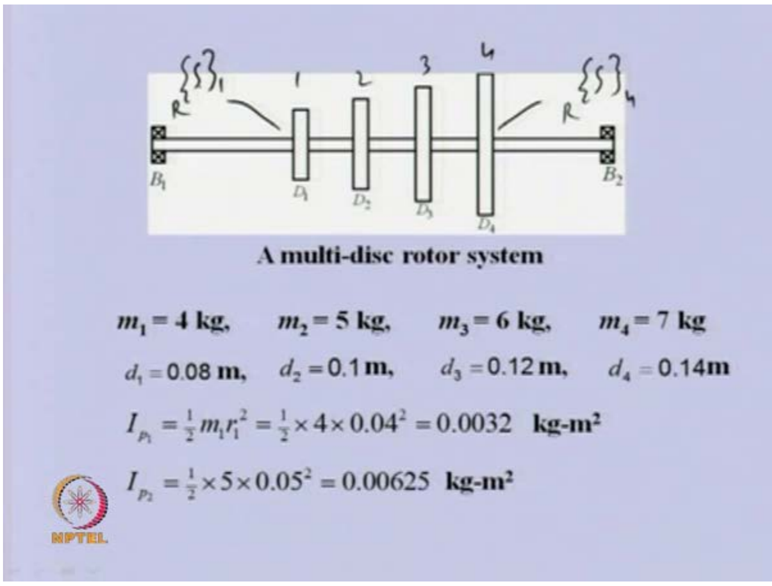

If giving the stations number, station number 1, 2, 3, 4, the state vector which is here right of fourth disc and state vector here right of one disc; they can be related like this.

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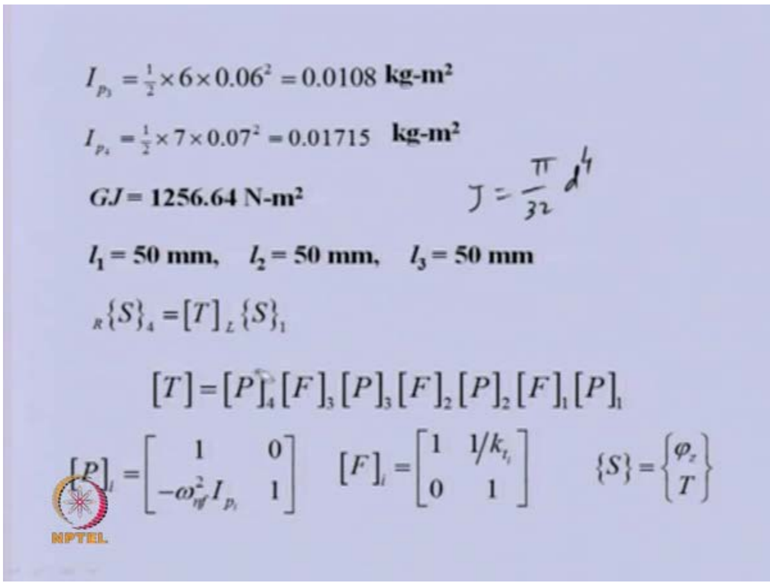

We already know how to obtain such relations. So, this particular field, you can able to see so many point and field matrixes multiplied and point matrix, field matrix as here. We are not consider damping. But, only the stiffness property and the inertia property are there. So, you can able to see that this is that overall transfer matrix and free bounding conditions are there.

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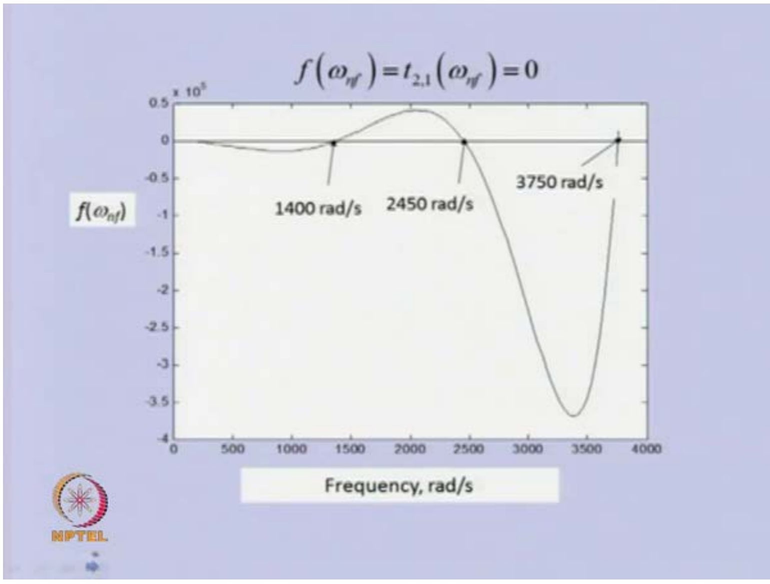

So, for that case, we have once the overall transfer matrix. So, second row and a first column component will give this as a frequency equation. If you expose that as a function of omega n f, if you plot this particular function with the respect to omega n f for various steps; so, you can able to see that bearing functions will vary. Wherever it will intersect with this 0 or line, they will be the root of the equation. So, at 0, we expect one cut. These are the frequency of this system. So, just by plotting this particular function, you will get the natural frequency of the system. Here, you can able to see we have summarized the natural frequency.

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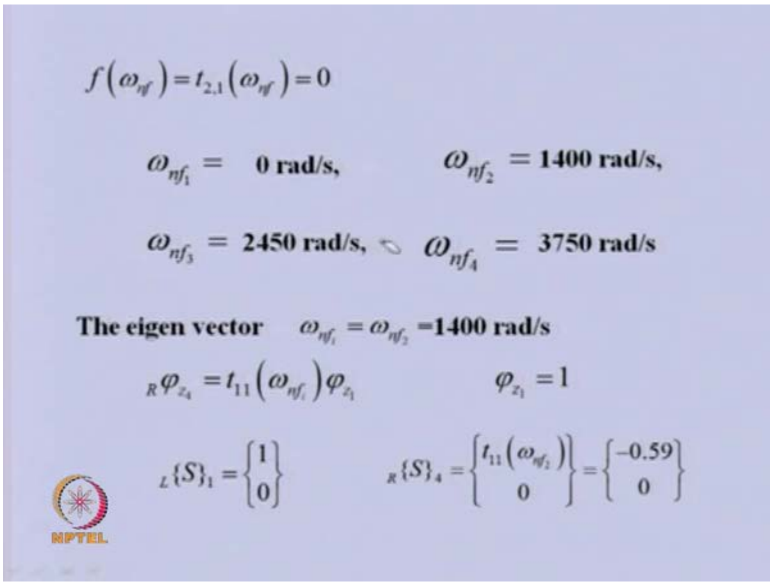

We got the intercept. Once we have obtained the natural frequency, we can able to again the diagonal vector. Let us say for one of the natural frequency per 1400 radius per second. This particular equation will give let us say, if we are resuming one of the angular displacement as unity, we can able to get the angular displacement at right of disc 1. Using these relations, these are all matrixes, which we related earlier. So, we know the state vector the left of 1. Then, you can able to obtain in the right of fourth or using this because we know natural frequency. You can able to substitute to get this.

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$$_R\{S\}_1 = [P]_1 \, _L\{S\}_1 = \begin{Bmatrix} 1 \\ -6272 \end{Bmatrix}$$

$$_L\{S\}_2 = [F]_1 \, _R\{S\}_1 = \begin{Bmatrix} 0.75 \\ -6272 \end{Bmatrix}$$

$$_R\{S\}_2 = [P]_2 \, _L\{S\}_2 = \begin{Bmatrix} 0.75 \\ -15465 \end{Bmatrix}$$

$$_L\{S\}_3 = [F]_2 \, _R\{S\}_2 = \begin{Bmatrix} 0.14 \\ -15465 \end{Bmatrix}$$

$$_R\{S\}_3 = [P]_3 \, _L\{S\}_3 = \begin{Bmatrix} 0.14 \\ -18325 \end{Bmatrix}$$

$$_L\{S\}_4 = [F]_3 \, _R\{S\}_3 = \begin{Bmatrix} -0.59 \\ -18325 \end{Bmatrix}$$

We can march back to get state vector at various locations like this. At left of 2 or right of 2, right of 3, intermediate transfer matrix can used to obtain the relative displacement. So, first term is the displacements. So, they can be expected to obtain what the relative to displacement of the various discs. That can be plotted as a mocked shape. In the present lecture, we started with damped torsional vibration. We did with the transfer matrix for such system that means point matrix. In the field matrix, we developed. Then, we saw how we can able to convert multi cylinder reciprocating engine to an equivalent revolving mass system. Once we have converted that also, if there is the variation in geometry in the form of current, that also can be converted to equivalent uniform shaft.

We can able to analyze the conventional revolving mass piston case. At the end, we have seen four discs for torsional vibration case through transfer matrix method. How numerical method can be used to obtain the natural frequency and its mocked shapes. Once we get the natural frequency, various intermediate matrix can be used to obtain the relative angular displacement of various disc. That is nothing but the mocked shapes. So, in the next lecture, we will explain the same torsional vibration for more advanced technique; that is finite element method.