**Theory and Practice of Rotor Dynamics**
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**Module - 5**
**Torsional Vibrations**
**Lecture - 14**
**Torsional Vibrations: Simple Rotor System**

Till now we have studied very simple rotor models, mainly we considered with transfer analysis of the rotor system, even we incorporated some complexity in some cases in the bearing in the form of various spring and dampers. Even we considered complexity of the gyroscopic couple and to the rotor, but till now we have not considered the other type of vibrations especially torsional vibrations which occurs in various tubulation rays.

So, today we will see the basic concept of the torsional vibrations and then we will analyze some simple rotor like single mass rotor, and two mass rotor today, and then subsequently analyzing more complex rotor by more systematic methods, transfer matrix methods, finite element method. So, today we will see the basic concept of the torsional vibrations.

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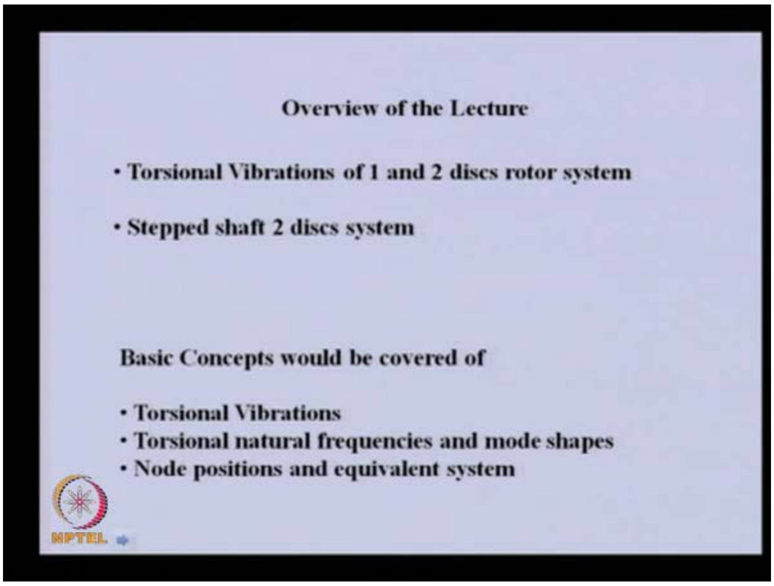


So, this is the over view of the lecture in which I will be introducing torsional vibrations of single and two disc rotor system and even. These discs are connected by stepped shaft then how the torsional vibrations will be analyzed will try to see where we will do some kind of equivalent shaft system. Then we will analyze this particular system the basic concept which will be covering which is torsional vibrations torsional natural frequencies and mode shapes. Even we will be having the concepts of node positions and equivalent system especially for the stepped shaft.

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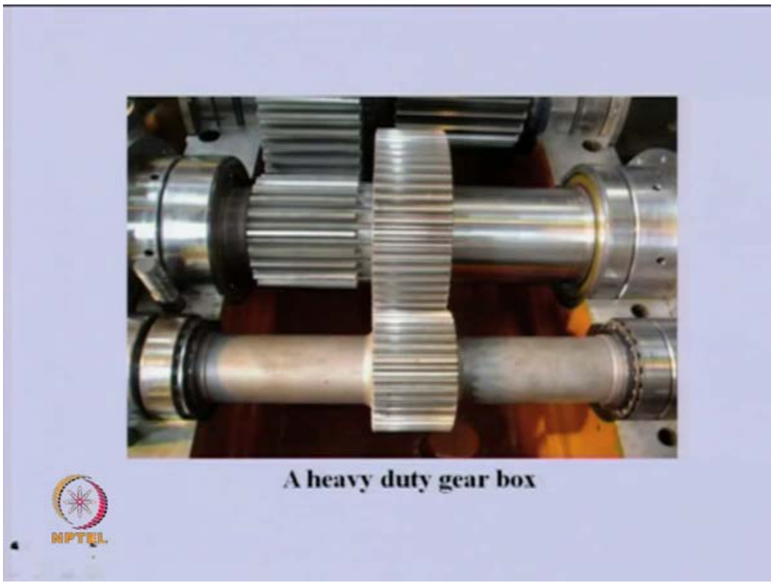


So, this is a typical application where we have a torsional oscillation there is the gear system in which we transmit lots of power and these gears gives a impulses in the form of talk. This may give torsional form torsional excitation in the form of vibration and any of the natural frequency in the torsional mode is coinciding with these gear mess frequency. Then we may have torsional oscillations in the resonance condition in the system similar to the gear system.

We have some other systems in which we can have torsional oscillations like if we have some applications where we like punching of brands in a sheet in that we have in that particular case. We have the sudden change energy of the fly wheel small duration during the punching and remaining time it stores the energy and these. Generally, we use fly wheel and that is mounted on relative flexible shaft and because of the variation in the energy.

It may be in the form of talk can have torsional critical speed or torsional resonance condition in the systems even in the systems in which we have rotating machineries, specially the machine specially the rotating engines. We can have the torsional oscillation in which relation of the tau takes place because of the talk radiation is periodic in nature and because of that we can have torsional resonance condition in reciprocating engine system.

Apart from that if there is fluctuation in the talk in any form rotating machinery has due to the fault in the electric system or may be due to the whirling of the rotor with stator we have torsional oscillation in the system. If these excitation frequency external excitation frequency is in the form of talk coincide with the torsional natural frequency of the system we can have the resonance condition. So, let us see what the torsional oscillation with a simple model is.

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So, in this particular case this is the thin shaft and a relatively rigid disc and the torsional oscillation is nothing but if we give twist to this and leave it to oscillate. So, you can able to see that w coming a small twist the disc oscillates about its mean position about its own axis. So, at frequency at which it is oscillating because this particular shaft is relatively rigid so and lot of damping is there, so the oscillations are 1 at the two, but the damping is less which is inwards practice case. We can have one large oscillations and we can have continuous oscillation of this.

As we know these machineries rotate at high speed and apart from this the nominal speed of the rotation of the shaft it can have oscillation about its nominal speed. It is something like the spin speed of the shaft and these torsional oscillations are super impose on each other. So, we can see the net effect of that the speed variations will be there about its nominal speed because of the torsional oscillations.

So, with this the simple model you have seen how a relatively disc which is having large moment of inertial and the shaft having torsional flexibility can produce torsional oscillations in about its own axis. This particular when we are discussing the system with some kind of initial disturbance and what the frequency of oscillations are takes place without any external excitation. There is the torsional natural frequency and any of the external tau which is coming on to the system coincides with this natural frequency, then we may having resonance.

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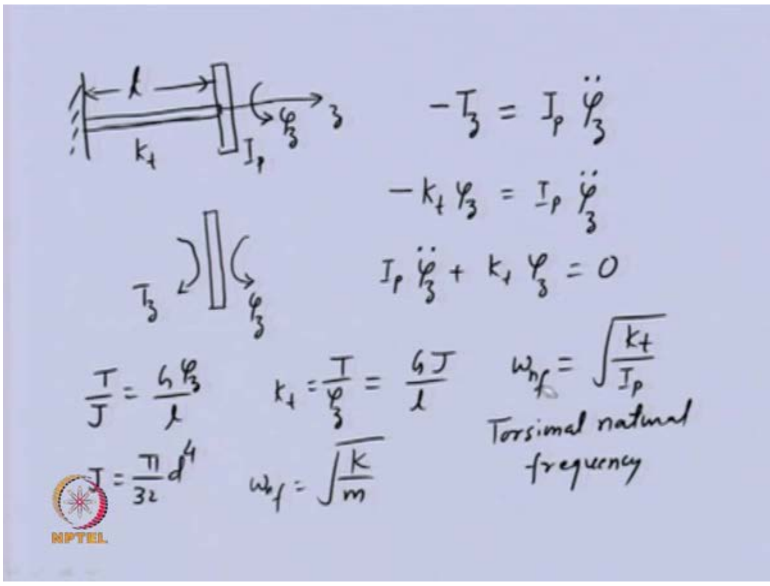


So, let us take very simple rotor model in which we have a relatively thin disc it is having torsional stiffness. Let us say k t this is the relatively thin shaft and the flexible mass or the fuller mass moment of the inertial, we are neglecting this is the thin disc which is at the free end of the particular rotor system and we have oscillation of the disc. Let us say fe z about z axis this is excess of z, so this is the rotational displacement of the disc about its own axis. In this particular case, you can see there is the very simple rotor model in which we can able to obtain the system equations by drawing the free body diagram on the disc.

So, if we remove the disc from the shaft, let us say we have the torsional displacement counter clockwise direction positively signed. If we are giving the displacement in this direction the resistance from the talk or from the shaft will be opposite, let us say this is tau clockwise it will be active let us say that is t z and this is the only reaction tau that is acting on to that disc.

So, we can able to write the equation of the motion of this by newton's law by summing up all the external tau. This the external tau is only t z which is acting negative direction to the displacement towards this reacting shaft from the tau on to the disc and this should be equal to the inertial of the disc. So, this is the rotor inertial of the disc we are equating to this much. Now, this particular torque is coming from this stiffness torsional stiffness, so we can replace this torque with the expression of the k t into the displacement angle displacement which we are given. So, this will be equal to I p, now you can able to write this in more frequent form which is very familiar equation similar to m x double dot k x is equal to 0, only thing is in place of mass.

Now, we have the polar moment of inertial of the disc and you have the stiffness is torsional stiffness this torsional stiffness we can able to obtain from the torsion. We know from strength of material this relation t is the torque c is the certain moment of area that is polar certain moment of area for circular shaft d j is given by this expression g is the modulus of rigidity theta. We can able to replace this theta that is fe z angular displacement and l is the length of the shaft. So, the length of the shaft is l, so torsional stiffness is nothing but torque by the angular displacement similar to the divided by linear displacement because the motion is in angular displacement.

So, the tau divided by this is the torsional stiffness from this expression, we can be able to write this as gj by l. So, this particular notional stiffness is given b y this and which is depend upon the length of the shaft or the diameter of the shaft or the dimension of the shaft. The material property and from this is very clear from the previous understanding of the single degree of freedom system we can able to say the natural frequency will be given by the stiffness divided by the polar moment of the inertial is k by m.

So, this is the single degree of the rotor system for partial oscillation this is torsional natural frequency this frequency will obviously different as compared to the transfer frequency which we are obtained earlier. In that particular case, we wrote this particular natural frequency for transpose vibrations as k by m where m is the mass of the shaft k is the stiffness of the shaft in transpose direction. So, that expression will be different as compared to the decimal of and so from here that natural frequency and the transverse natural frequency will be different, they will not be same for the different system. So, from simple analysis of simple system or torsional oscillation that is for single degree of freedom is clear that we have torsional natural frequency similar to the natural frequency that we are studied, now we will take some more complex system.

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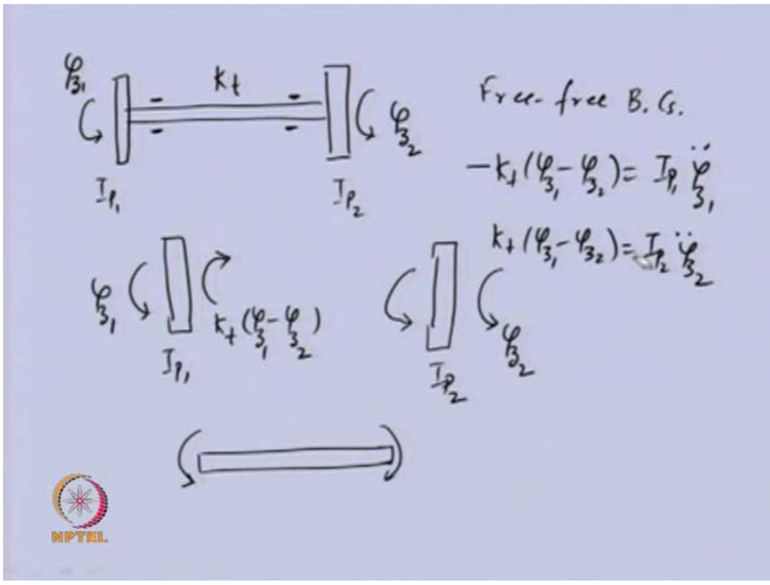


Now, let us take two disc system 2 disc that is connected by flexible mass less shaft this discs I p 1, I p 2 the shaft is having torsional stiffness k t, the shaft is having torsional stiffness k t and this shaft is supported, let us say on friction as bearing. So, that this disc I have angular displacement I am taking both as counter clockwise direction as positive, so this bearings are not provided as friction.

So, basically when this particular system is under torsional oscillation when this system will be under torsional oscillation the bounding conditions will be something like free bounding condition. There is no support which is restricting the motion of this two discs, so bounding conditions free bounding conditions and to obtain the vibration of equation of motion of this.

Obviously, our aim is first to obtain the natural frequency of the system for that he need to obtain the equation of motion in this system. So, for this we will draw the free body diagram on both the disc and from there we can able to obtain the equation of motion. So, let us see that first disc I p 1 in this we have taken counter clockwise direction for the angle displacement is positive and because of this the torque will be opposite that will be clock wise. So, this torque is k t and related twist because the torque which is exerted on the shaft disc will be relative displacement between these two discs. So, basically this shaft will be getting twisted by the relative displacement on these tools so that is this 1 minus 2.

So, this is the free body diagram of the disc 1, similarly we are able to draw a diagram of disc two in this we are taking the angular displacement as obviously counter clock direction. This particular torque which is the reaction torque from the shaft this particular shaft will be acting here that will be same, because of this is the torque which is transmitting from the shaft. It will be acting opposite to that means if we have a frequency body diagram on this shaft. So, obviously this torque will produce a reaction here, so this is the clock wise direction, so here will be having same torque, but opposite to that here.

Again, you will be having opposite to this one and the torque here is opposite to this that means this is clock wise and that is again counter clockwise and the value will be the same, this one is clock wise and this should be anticlockwise, so value will be same. Now, we have free body diagram of the disc 1 and 2 and from this the equation of the motion by equating the external torque to the inertial.

So, for first disc we can able to write k t this is the external torque which is acting on to the disc should be equal to the inertial of the first disc this one. In this particular case the second disc will be having this is negative, because this is clockwise this is here both are in the same direction counter clockwise. So, we will be having k t fe z 1 fe z 2 is equal to I p 2 fe double dot z 2, so these are the two equation of motion which can able to write in more standard form, in the next side we can write that.

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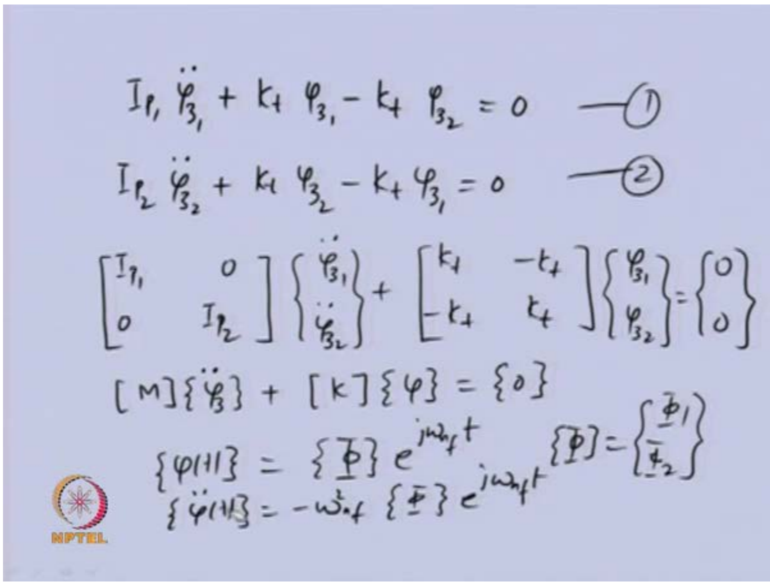


So, these equation are again if we write here this is the first equation second equation n is k t whole is not there y z 2 3 minus, so its k t, only single shaft is there. So, these are the two equation of motion of this particular system which we have two disc connected by flexible shaft and there is no as such support which is giving a torque external torque to this system. So, these two equation we can able to now write in matrix form that will be convenient to handle, so I am writing them in the matrix form.

So, here we will be having I p 1, I p two plus this is stiffness terms, there is no external force, so will be having k t minus k t. So, this you can able to see this is we expand this matrices or we can get back this equations. We can able to write this in more standard form like matrix and mass matrix and angular displacement that is the acceleration stiffness matrix and angular displacement vector is equal to 0. Now, we can able to choose angle displacement which is function of time as the amplitude part of the displacement.

In the harmonic part omega fez is the natural frequency of this system torsional natural frequency of this system and this is the amplitude, and phase in the functions will be here, amplitude will be formed both the discs like this. Now, we can able to take the derivative of this once more double derivative, so you will get minus omega n f square phi e j omega n f t, now this assumed solution the derivative of the import in the equation of motion.

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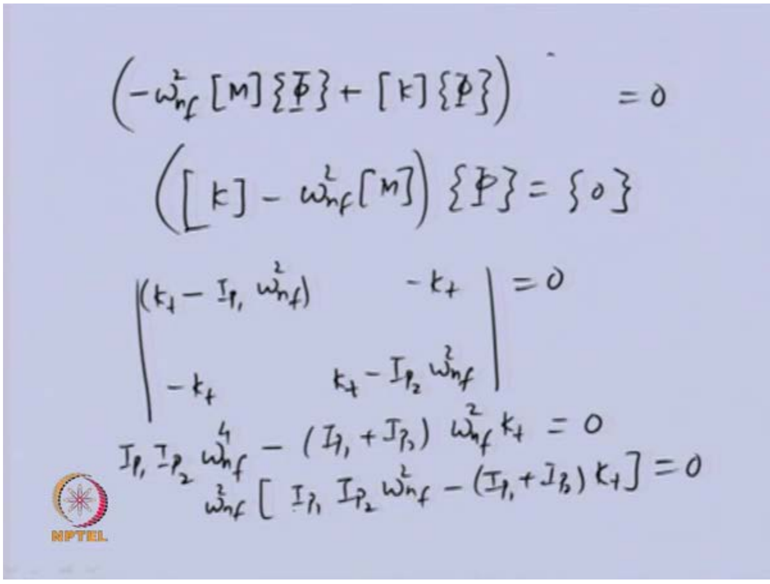


So, we will get M matrix then We have capital five then e j omega t time dependent I am taking it out as a common, so because we can able to this cannot be 0. So, we need to eliminate this here also we can able t see we can able to simplify this as k minus omega square n f m then phi vector we can take it as common is equal to 0. So, this is the typical homogenous equation or ideal value problem one of the solution of this is that this particular displacements as 0, that means both the discs are having displacement.

So, we are not entering that in which basically we are talking about the system is not vibrating. So, to have a non trivial solution of this obviously we need to take the determinant of this to be 0 and that will in this particular matrix matting is a determinant. So, we had k t minus I p 1 omega n f, so this is first term of disc two minus k t minus k t and then k t I p 2 omega n f. So, this particular determinant is the solution of this two, so when we take the determinant of this we will get a polar nominal in terms of omega n f square that is the natural frequency of the system which we want to obtain.

So, we will get that after simplifying the determinant this term I p 1 plus I p 2 omega n f square k t is equal to 0. So, you can able to see that omega n f square is common if we take out that also, will be having one I p 1, I p 2 omega n f square I p 1 plus I p 2 k t is equal to 0. So, from here from last equation it is clear that one of the natural frequency is becoming 0 and the term within the bracket is also we need to equate it to 0 independently, so from there we are getting another frequency two natural frequency.

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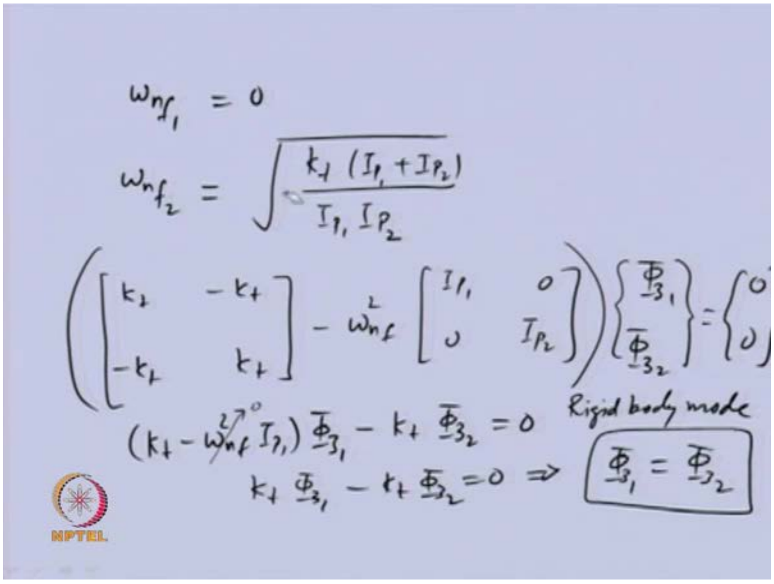


One is the 0 natural frequency and the second term within the bracket, if we equate we will get this expression or the second natural frequency of the system. So, in this particular case you can see this is zero natural frequency this particular natural frequency we will see what is the physical interpretation of this. For that, we need to obtain the more shape of the system in the mode shape actually this is nothing but how the two discs are oscillating with respect to each other relating to each other. So, in this particular case we will see that when the natural frequency is 0, both the discs will be having identical displacements that means both the discs will not be having any relative twist between them and because of that shaft is not twisted.

So, we will refer this particular distillation as oscillation body mode and how to get this let us see how we can able to get these particular mocked shapes, so if we go back to the equation this equation. So, as we know these are two equation I am writing this in the expanded form again for the clarity, so we have k t minus k t, this has stiffness matrix minus natural frequency I p 1 I p 2. Outside we have displacements amplitude complex to the displacements amplitude, so if we take first equation let us say that is k t minus omega n f I p 1 into this displacement first term then minus k t phi z 2, so I have taken the first equation of this all matrices.

So, if we put the one of the natural frequency which we are obtained let us say first natural frequency in this particular expression this term will vanish and we will left k t f e z 1 minus k t fe z 2 is equal to 0. This gives us z when the system is oscillating at 0 frequency their displacement must be equal and we can able to now make out when the both the discs are having, both the displacement the shaft will not be having any twist. So, we call this as the rigid body mode and this there is no stresses in the motion shaft is not of practical importance. Now, the second natural frequency if we substitute in the same equation or we can able to get from this natural frequency, what will be the relative displacement between the two disc, now in the next slide I will substitute this one on the same equation.

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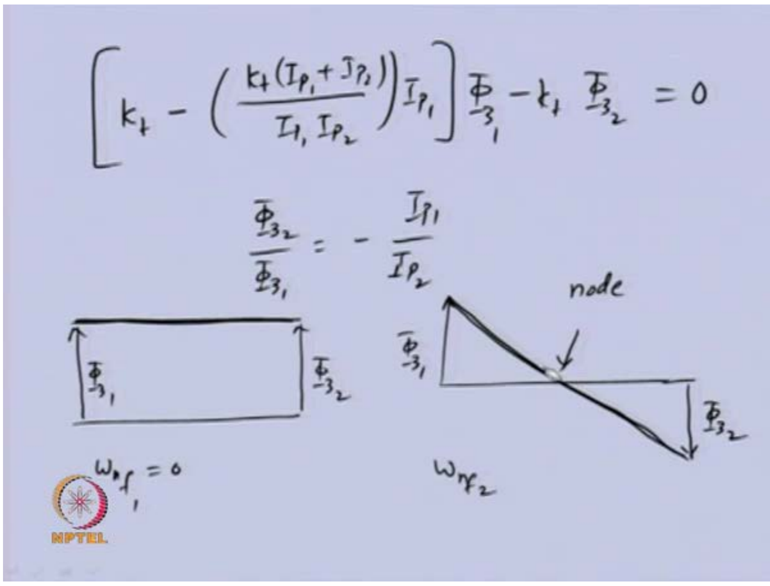


So, we have k t minus omega square is k t I p 1 plus I p 2 divided by I p 1, I p 2, so this the expression that gets into I p 1 and whole is multiplied by phi z 1 and the previous expression is this one minus k t fe z 2 is equal to 0. If we simplify this expression we will z 2 by z fe z 2 by fe z 1 is minus of I p 1 by I p 2, so here you can able to see that the negative displacement between this two discs will be inversely proportional to the mass inertial for the mass moment of inertial. We expect that in this particular case when if one disc is lighter and another is heavier the disc which is heavier will be having less displacement as compared to the disc which is lighter and in this particular case the negative sign is there.

So, even he motion of this two will be opposite if we have two disc, one is rotating in clockwise and another one is rotating at anti clock wise. If the motion reverses because of the free vibration then both the disc will be having inverse opposite motion.

So, if we want to draw this particular motion so you can able to see that in the first case where the natural frequency was 0, if you want to plot the relative displacement between un this is phi z 1 and this is phi z two because both are same. So, the shaft will be having as such no relative twist is a rigid body mode and for second natural frequency as we are seen in this particular expression. If we have displacement in the positive direction the other disc should have the negative displacement and the shaft displacement will be varying linearly.

So, you can able to see that the shaft is having maximum displacement here and here and their amplitude will be inversely proportional to the mass moment of inertial. In between, you can see that there is small position where the displacement angular displacement is 0. So, this particular position is called node and you can able to see that there is no angular displacement here and either side of this the angular displacement are opposite to each other because they are this is positive and this is negative. So, as such this particular node we are able to think that of as a rigid support, so what we can able to analyze there is 2 mass rotor times the two single degree of rotor system, let us see that particular equivalent concept.

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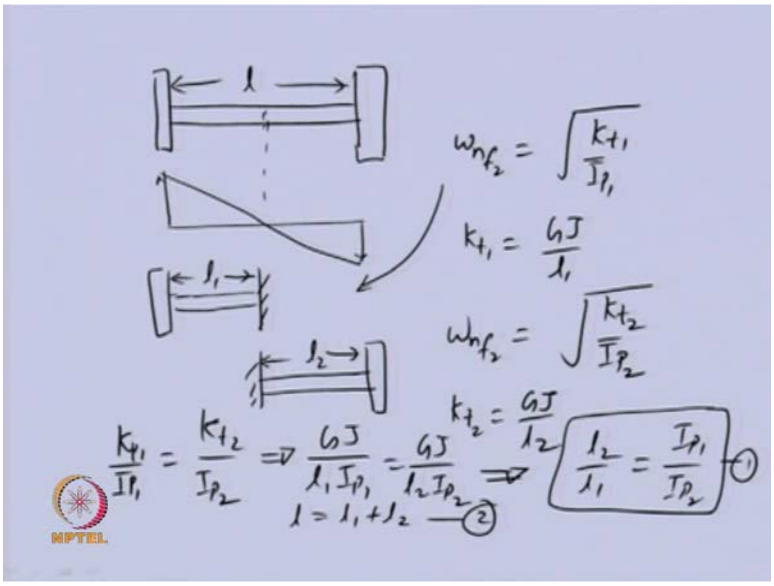

The two disc rotor system which is there with us in this particular case when we are drawing the mocked shape depending upon the stiffness and the polar moment of the inertial of the disc. We will be having node at some intermediate position necessary at the midpoint at the shaft, so this is no position what I am saying because here displacement is 0. So, corresponding to the left hand side the system, which is there the shaft and the disc, and in the right side which is there, because at the node position there is no displacement.

So, two rotor systems which is having two degree of freedom system can be analyzed by two system single degree of freedom system and in this particular case they are cantilever rotor system. This is both the system shows they are the same natural frequency because when we are considering the motion of this. Obviously for second mode both the disc are having same angular displacements or the same angular frequencies, so this two system which are equivalent to this dynamically.

They will be same only when we the natural frequency of this system and this system are equal, so that means if we have the length of the shaft is less l here. So, the node position from the left disc is l 1 and here it is l 2, so this will help us in finding out the note position. So, for first case we can able to write the natural frequency of the system as stiffness k t 1 let us say I p 1 k t 1 is g j by l 1 of our second system.

We have this is correspondence to this one and for second system we have the same natural frequency because it cannot be different and they are corresponding to the second shaft k t 2 will be g j by l 3. Now, we can able to see that these natural frequency is same we can able to equate them, so if we equate them we will get k t 1 by I p 1 is equal to k p 2 by I p 2. Now, you can able to see if you substitute k t 1 and k t 2 in this expression we will get because most of the things are common l 2 by l 1 is equal to k p 2 by, sorry I p 1.

I am repeating this when we substitute I p 1 k 1 and k t 2 from this expressions we will be getting g j l 1 I p 1 is equal to g j, l 2, I p 2 is equal to this will give us because we have some terms common I p 1 will get cancelled. So, will be having l 2 by l 1 as I p 1 by I p 2, so you can able to see that the node position will be inversely proportional to the mass momentum equation. This is one of the expression and we know second expression that of total length is l 1 plus l 2, so we obtain the equations for l 1 and l 2 we can able to solve this equation to get the l 1 and l 2.

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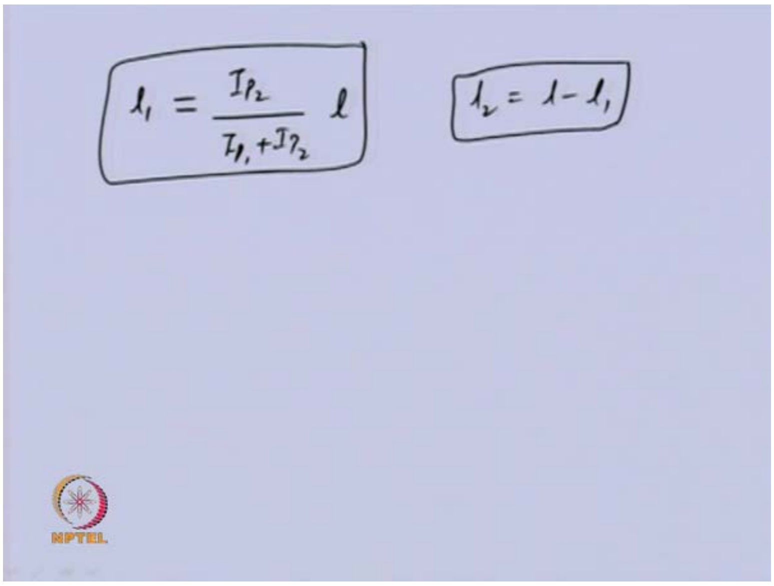


If we solve it we can able to get the nodes position, so we if we solve this we get this as and once we know the l 1 that is the l position that is the note 2 can be obtained l 2 will be l minus l 1. With this procedure, we can able to get the note procedure this is very important because here the angular displacement is not taking place, but stresses will be maximum because either side of the node. We have angular displacement opposite to each other maximum stresses will be more here. So, we considered two disc system in this particular case we were connecting these two disc where we found. Now, we consider a case in which the shaft is connecting these two disc are having different procession in between and in that case how to analyze such system Equivalent concept.

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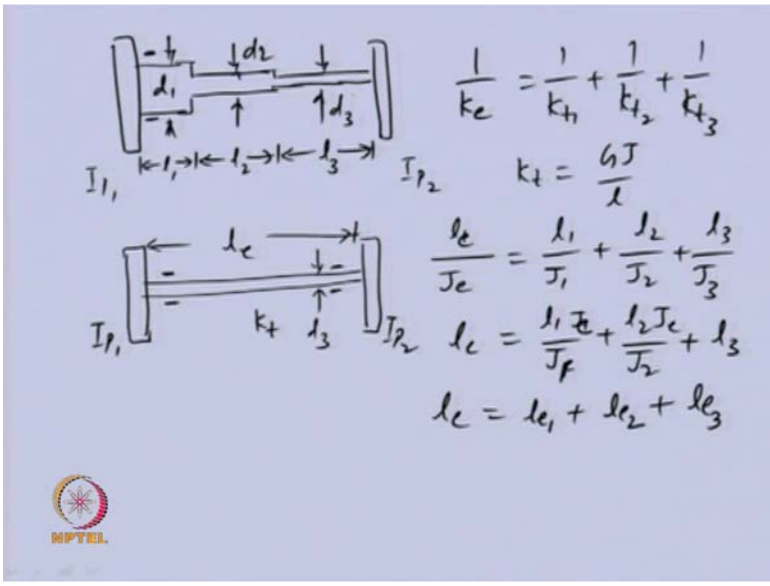

We have one disc and second disc and then disc connected by shaft having different dimensions in between like this, so this is I p 1 this is I p 2 length of various stepped shaft are l 1, l 2, l 3 even their diameter are different d 1, d 2, d 3. So, if we may get the mass moment of inertial moment of the shaft only if we consider the disc. So, this particular system is in some way is identical to the previous rotor system in which we had two disc, but connected by uniform shaft. So, if by some means we can convert this particular system to this one, then what were the analysis we presented earlier will be valid.

So, in this particular case this two system will be equivalent when we talk the partial stiffness of the system and this system same. So, if they are same and polar moment of inertial are same this two system can be identical, so if this is the stiffness of the equivalent system. Let us say this is one of the diameter of the original shaft d 3, so the length of the shaft of this will be we need to find out what should be equivalent of this.

So, the stiffness of this and if we can find out the previous analysis we can get here, so basically we can able equate the stiffness torsional stiffness of this system. You can able to see that the stiffness of this because these are connected in the series, so in equivalent torsional value of this system will be given as, these are the stiffness of the first shaft segment second shaft segment and third shaft segment. We already know in generally the k t s given by g j by l, so this k t can take value of 1, 2, and 3.

So, depending upon that this j 1 and l one expressions will come because this j depends upon the diameter. So, if we substitute this we can able to see that will be getting l equivalent in this divided by j equivalent l one by j 1, l 2, by j 2, l 3 by j 3 and basically j 3 and e are same because we are converting the equivalent system into the diameter of the mass segment. If we multiply with je we will get this as je plus l 2, so this is e and this is one this is j e by j 2 and l 3 because j 3 and l 3 are same.

So, you can able to see that this can be written as equivalent length of first segment equivalent length of second shaft segment equivalent length of the third shaft segment. So, this is the total length of the shaft segment, so with this we can able to obtain the equivalent shaft segment length. Now, we can able to apply the previous analysis to get the natural frequency of the system in which we had the zero natural frequency and one flexible mode frequency, so all the analysis will be similar to the previous one. So, in this case again to analyze that we need to assume that this need to mounted on the friction fort which is s naught according any friction, so in this particular case we have converted the equivalent stepped shaft system of uniform diameter.

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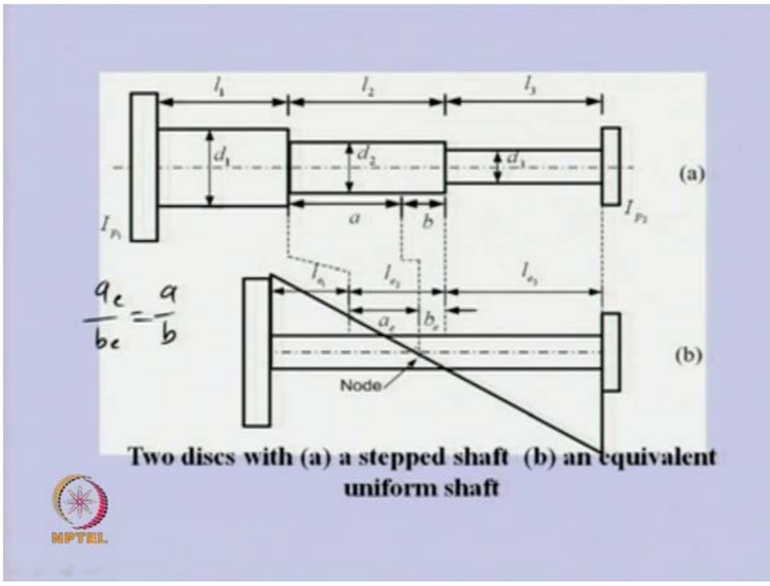


So, let us see what is the difference especially if we obtain the previous node position I in this particular case when we want the node position in the stepped shaft. So, this is the node position in the equivalent shaft corresponding to the second shaft segment. We will see that you can able to see that this dotted line is representing that from here to here is that the equivalent of the second shaft segment. If node is here that means the position of this end is a and b from that end and the ratio of that a by b will be same in the original shaft segment this a and b. So, this node will be somewhere here which will be in the same proportion as in the equivalent system.

To summarize today, we saw the very fundamental aspect of the torsional oscillation how it takes place in a rotor system which itself is the taking at some normal speed, but above that there is a some variation of the angle displacement. That results in some changes in the angular velocity would be taking place and this particular analysis we done for single mass rotor system and two mass rotor system.

We have seen that for two molar system one of the natural frequency system is 0 and another it was non 0 that was flexible mode. We had the concept of the load portioning in that two rotor system where the displacement was 0, but we will see at that position where the node is there then stresses will be maximum at its side of the particular node will be related.

So, it will take place in the next class we will extend this particular analysis for three rotor system and then we find that what were the procedure. We were finding up using the Newton second law which is obtained in the governing equations will be more commenced on especially when we have more degree of freedom system. Then we take up some more frequency methods like transfer methods and other element method.

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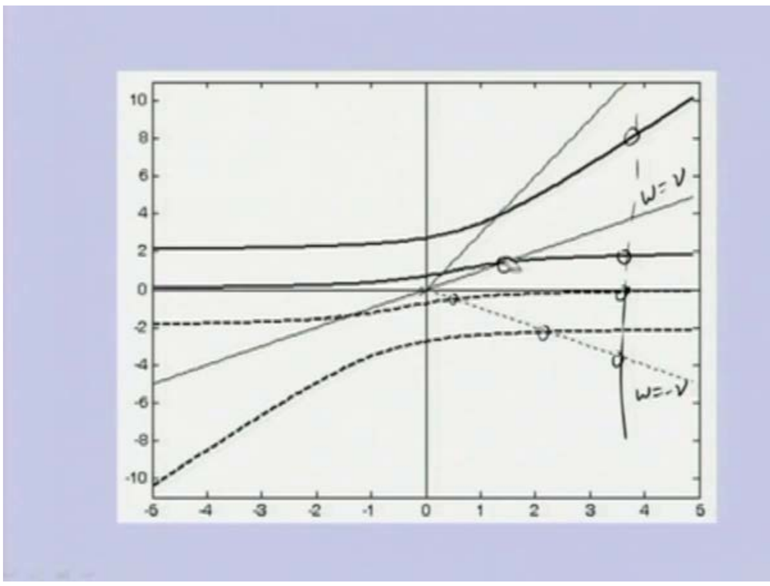

Now, we will see this Campbell diagram in clear picture so you can able to see that we have drawn dot in this there are four lines solid line you can able to see. So, first one is this one second is this one third is this dotted line third is another dotted line and at one particular speed if we got to see want to see the intersections are these are the natural frequency.

These are corresponding to the forward whirl these are corresponding to the backward whirl regarding the critical speed this line is the omega equal to nu line and intersection of this will give the first critical here and this line is not intersecting the second line. So, there is no critical speed corresponding to this, similarly if you draw another line omega equals to minus nu this intersects at here. This is first critical speed and this is the second critical we can able to see that we got three critical one forward and three backward, but another forward was absent.