Theory and Practice of Rotor Dynamics
Prof Rajiv Tiwari
Department of Mechanical Engineering
Indian Institute Technology, Guwahati

Module - 4
Lecture - 4
Gyroscopic Effects: Asynchronous Whirl Analysis with Dynamic Approach

Previous three lectures, we have studied the gyroscopic effect. Mostly we analyze the effect of the gyroscopic on the old frequency and the critical speed and the approach was cost analytical which we did not have any equation, which were time dependent. Today we will study the gyroscopic effect by dynamic approach in which will be obtaining the governing equation of a single simple rotor system in dynamic form. So, time dependent terms will be there in that and we will be considering the general sequence of the motion of the rotor system.

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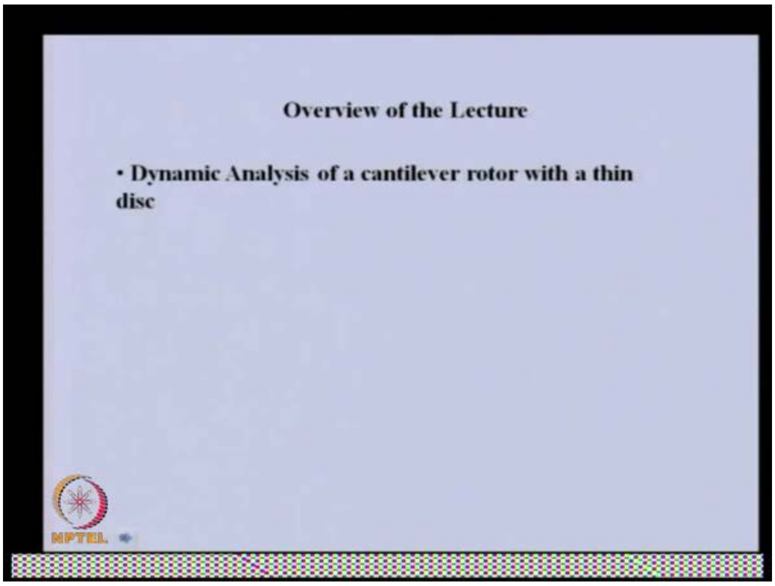

So, basically will be considering the dynamic effect of a cantilever rotor with a thin disc and in this particular case we will start with let us say we have.

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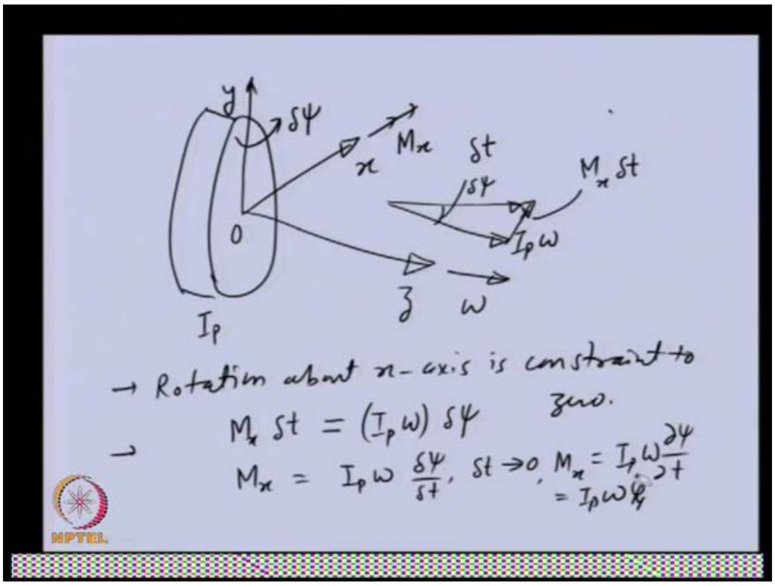

We are considering the disc like this. This particular disc is having this is the axis about which it is spinning the positive action direction, that is z axis. There are two ware axis in this particular disc x and y. This particular disc is having four moment inertia about z axis I p, because of this spinning the angular movement AM from the disc is let us say I p omega. In the positive z direction we restricted the rotation of this particular disc about x axis.

So, rotation about s axis is constant zero, but it is free to dealt about y axis. We are applying the movement in positive x direction M x for a small time duration at sl delta and because of this movement which we applied for small duration time delta t. There will be change in that angular momentum, let us say that is this across change in angular momentum and this particular angular momentum direction could be if the direction of the movement applied. That is this one, this direction which will be along the x axis.

So, to close this vector we drawn this particular line or this particular angle is the small tilt of the about the y axis. So, this is in the positive y direction, this is the tilt of the disc y axis, because of the change in angular momentum. Now, this particular change in angular momentum and so from this figure we can able to relate the angular momentum engine, change momentum due to movement from this vector.

We can able to see that this particular change is nothing but I p omega and that small angle which the disc got deleted because of this particular movement. We can able to write this movement, this is x which is nothing but gyroscopic movement I p omega delta chi delta t and for malamute delta t transfer to 0. We will be having this gyroscopic M as I p omega dealt t and this particular angular velocity we are writing as I p omega fee y dot because it is deleting about y axis.

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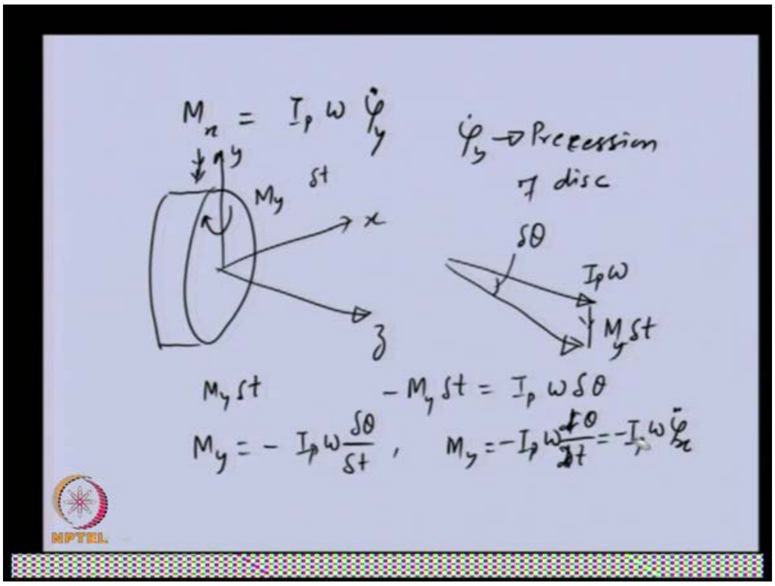

So, let us write that particular gyroscopic movement again here about y axis. So, this is the angular velocity of the disc about y axis, this is called precession of disc. So, we obtain gyroscopic movement when we gave the movement about x axis to a disc spinning about z axis. We need to pressed that particular disc about that third axis, that is y axis. So, in other way, in other words if the particular disc is spinning, if we are having the presses ion to that particular disc about let us say y axis. Then we need to apply the movement in x axis.

So, that we can able to this particular the system is in equilibrium, that means the angular momentum need to be balanced and because of that we need to apply movement about x axis. So, both the ways that is valued, now let us consider this disc, the same disc. Now, we applying the movement about the other axis. So, let us say that this is spinning about z axis, we have axel y axis horizontal and vertical axis, we are applying the movement positive y direction. This particular movement is not positive m direction, this is from top if we look clock wise. That means this particular movement direction is towards the down go direction about the y axis.

Now, the original angular momentum is this one which is along the z axis and this particular movement which is applied about y axis, let us say small duration. So, this my into dy dt is the nothing but change in angular momentum in the direction of the movement applied. So, this is the change in angular momentum direction of the this particular movement applied and we have equilibriums. We have to close the particular angle of momentum and this angle is the pressperson of tilt of the disc about x axis. So, let us say this is delta theta. In this particular case we have restricted the rotation of the disc about y axis of this only we are allowing the rotation of the x axis.

So, from this we can able to relate this particular vector, that is in y direction to the change in angular momentum due to this tilt and this we can write. In this particular case sign is opposite because this particular displacement is in the positive direction and we have I am repeating from this. So, from this vector diagram we can able to see that the direction of the angular momentum change in angular momentum from the negative y direction, but the tilt is in the positive x direction.

So, when we balance this particular change in momentum, it should be negative because in negative direction should be equal to I p omega delta theta. This can be written as by minus as I p omega delta theta by theta y and in limit this can be written as minus I p omega delta that by delta t. This I am expressing that p omega delta dot about x axis. So, this the angular velocity about the x axis. So, this is the gyroscopic movement which will be acting or about x axis, if we are giving a press ion about x axis by delta x. In this particular analyses we took a special case when we restricted the motion in one of the direction.

That is deleting in one of the cases in deleting direction, in one of the direction, but in actual part is both the axis the axel y, this particular axis will have precision. So, spontaneously we will be having angular velocity of this two axis. Apart from this we will be having the angular acceleration of this particular disc about two axis. It is the movement which we are talking about and which we are applying about the x axis or the y axis. Actually that movement will be summation of not only the procession, but also due to the angle acceleration. So, we need to write this acceleration terms also in this particular expression.

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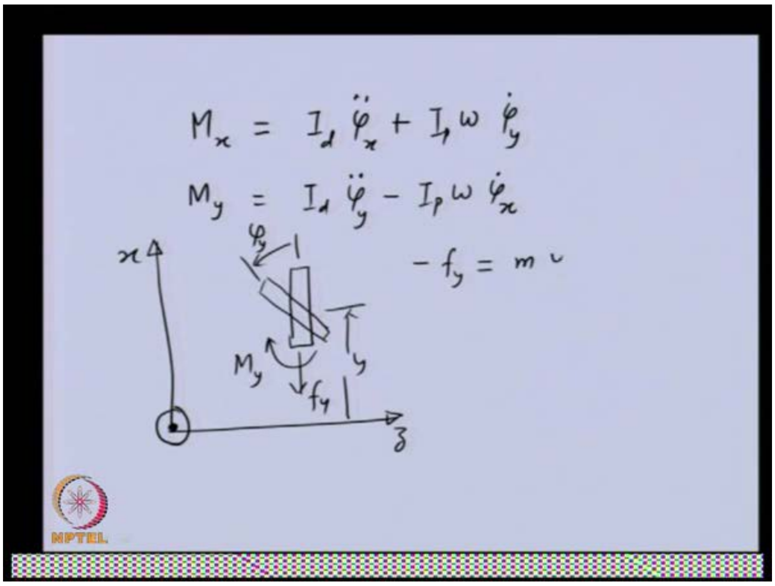

We can able to write like the movement about x axis, total movement which should be equal to I d. This is the angle acceleration term plus whatever the gyroscopic movement we obtain earlier. Similarly, in about y axis this will be I d, this is the due to the angle acceleration. So, basically if you see this particular equation is similar to the Newton law which we are equating the sum of on the acceleration forces acting on to the disc to the inertia term, because this particular term and both are inertia term. One is coming from the because of the diameter and the masses calculation of the disc, another due to the molar mass ratio of the disc and while obtaining the motion we need to consider this two inertia term.

Now, once we obtain the movement in inertia terms, now we can able to take the free body diagram of the disc, not only for the rotation, but also for the translator motion. Then we can able to get the free motion. So, if I am taking the free body diagram of the disc in the wall of the plain, that is z x plane in which y axis is outside the screen. That will perpendicular to the screen and this is coming outside the screen disc is somewhere here or it is having tilt t in according to the positive convention. That is this is about the y axis because y axis is perpendicular to the screen and outside the screen.

So, according to the right hand rule, screw rule, this particular direction will be positive. Now, we can able to see that we will be having because this particular disc is attached to the shaft and we have draw the free bird diagram. So, there will be elastic force and there will be movement. Also, this let us say is about y axis, this will distinct the motion of the telting t and this is opposite to the y direction, f y is opposite to the y displacement. So, we can able to write the equilibrium expression, all of this particular disc in f y direction force balance.

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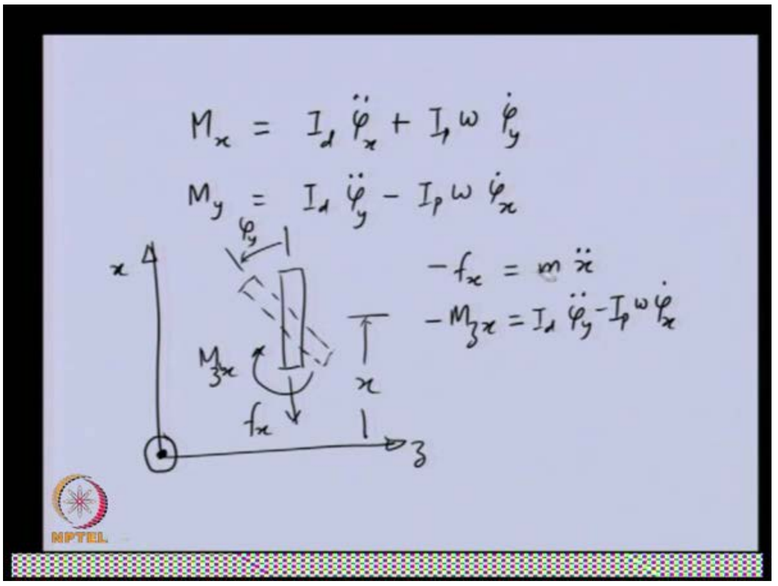

So, this will be I am drawing the free body diagram of the disc z x plane. So, this the disc this is the x displacement of the disc translator displacement, apart from this it is telting in positive y direction. That is f e y y axis is perpendicular to the screen and it is coming outside the screen. So, this particular point presenting the tip of the arrow of the y axis and because of this displacements we will be having f x in downward direction opposite to the displacement.

Similarly, will be having this is elastic force which is coming from the shaft. Apart from that we will be having movement that will be about y axis opposite to the angle displacement. And this particular elastic movement which will be same as that will be basically exchange due to all the forces and movement which are acting to the particular disc. Now, we can able to write force balance faster in x direction. So, this is the summation of all the external direction should be equal to mass and acceleration of the disc in that direction. Then regarding movement we have this is let us say y z. I am writing this as z x plain movement, which is this one z x movement. This is the external movement.

Now, we once we have done force balance in x direction and the movement balance will be z x and this is equal to the inertia of the, which is second one and e phi y and it is y p omega pi x dot. So, this is the equation motion in x direction, that is movement balance in y z plane. On the same line we can able to obtain equation of motion.

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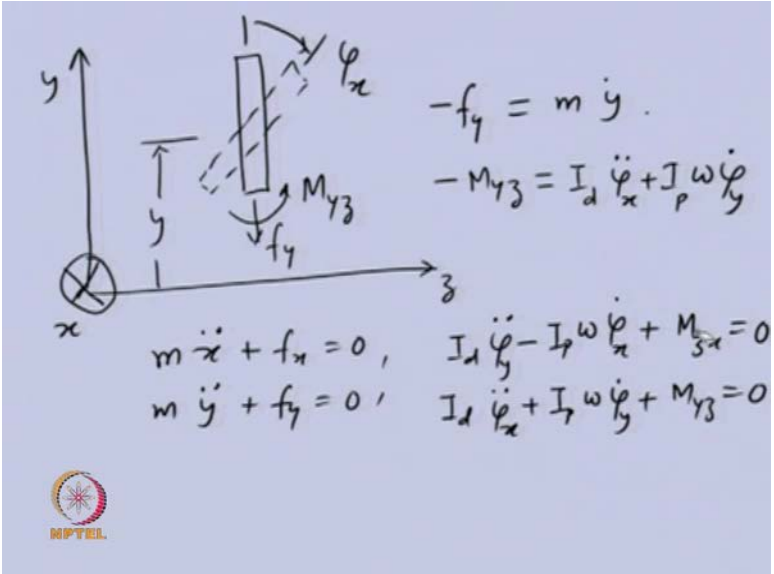

In z y plane for this let us consider the list having this y displacement and titling is in positive x direction. Then it is positive x, x axis is perpendicular to the screen and going into the screen cross is representing. That we are looking into the telt of the x which is inside the screen because of this displacement from the shaft will be having elastic force reaction force. Apart from that reaction movement from the shaft that will be y z.

Now, we can able to write the force balance. So, external balance is x 1 should be equal to inertia of the y direction. This is the inertia in y direction this and the movement balance this will be equal to the first due to the angular direction. In this particular plane that is pi x double dot and the gyroscopic couple I p omega pi dot y. Now, we can able to see that you have got four equations of motions, they are x double dot plus f x is equal to 0 and y double dot f y is equal to 0.

Similarly, the movement the movement balance I d pi double dot y I p omega pi x plus is equal to z x 0, previous equation. Then we have fourth equation these are the velocity plus m y z is equal to 0. Now, we need to relate this elastic force the shaft displacement that is linear and angular displacement to the influence coefficient.

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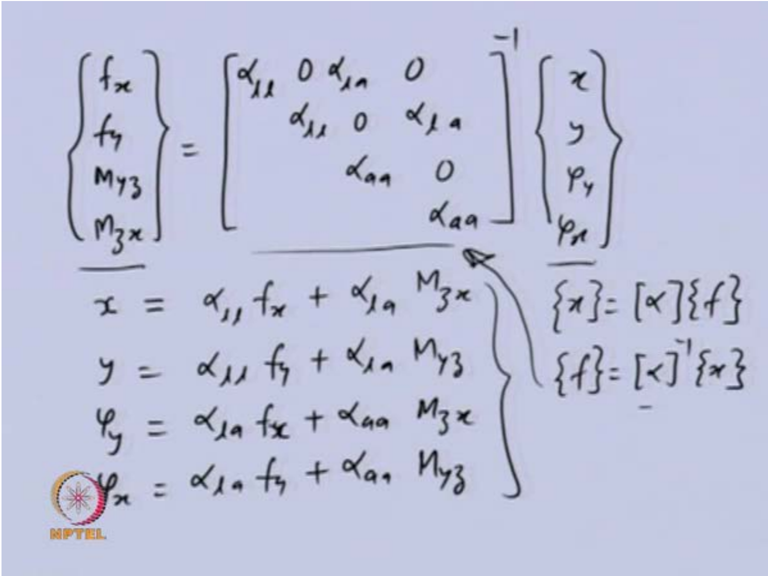

So, these we can able to relay this forces and movements. We can able to relate like this here, we have influence coefficient the linear. Once linear angular, I will be defying this coefficient alpha 0 and this is. So, earlier we related the x as alpha linear, linear f y plus that is alpha angular z x. So, basically all such relation earlier we have put in matrix form. So, if this I am again repeating this one related, the linear displacement x as alpha linear. Linear f x plus alpha linear angle m, that is z x and y as alpha linear, linear fy plus alpha linear, linear m y z.

Similarly, other two angle that is f e y, we have linear angle f that is f x plus alpha angle angle m z x and this is last one angular linear ,linear f y plus alpha angular y z. So, basically these we can able to put in a matrix like displacement influence coefficient and the forces then this force you can able to get by influence co efficient matrix. So, basically this equation is the above on in which we own this is force this is m y influent coefficient and this is the displacement factor.

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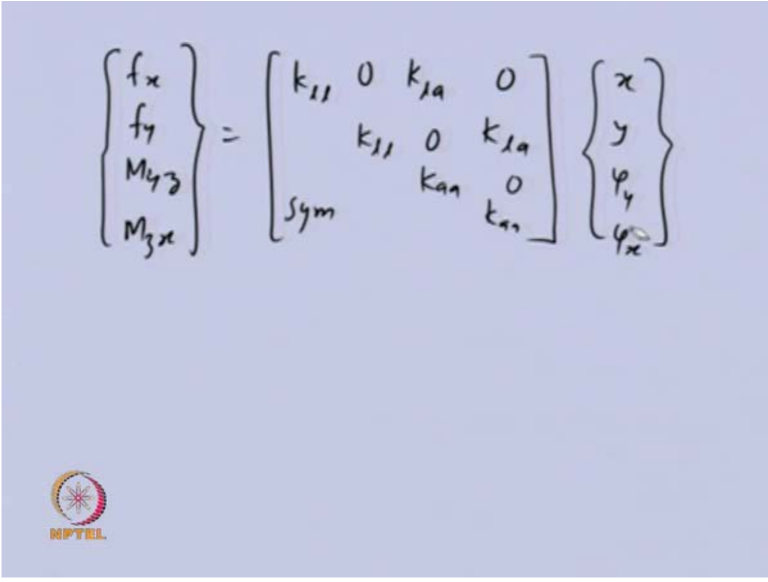

This particular equation can be simplified in terms of once we take the influence of this coefficients that will be something like stiffness term. So, I am writing in those form, so this will be k l l and for linear displacement for angular displacement I am writing half of the matrix because this is symmetric. Now, we are related to this particular vector through stiffness coefficient and the displacement factor. Now, we can able to this equation motion. These four motion of equation, we can put in the matrix from and that particular matrix form.

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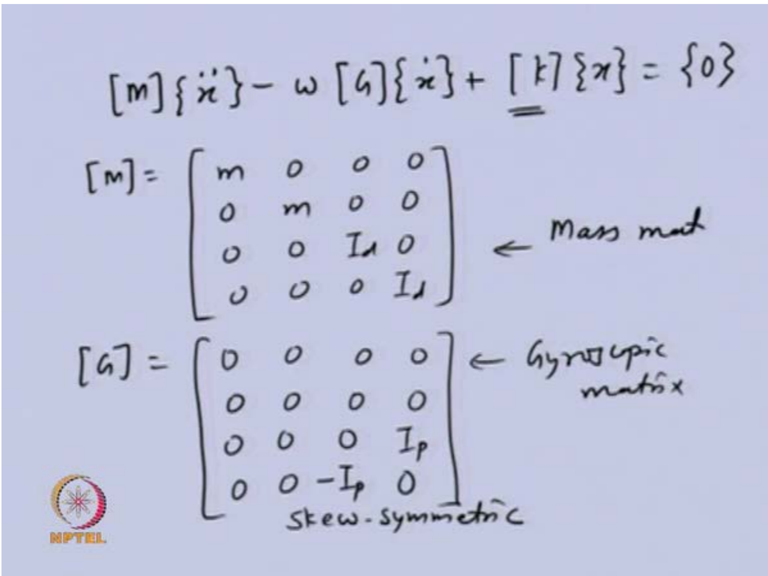

Let us say of those four equation of motion would be we have the sobriquet for elastic movement. Also I will be expanding this matrixes in more clarity, this k matrix is same as the previous one. This the k matrix and m matrix is the mass matrix, m based on the ordering of the displacement factor. This will be having this form this is diameter of mass movement of inertia, two terms. This G is the gyroscopic matrix which is having most of the terms 0, which is non-zero. This is I p 0 0 minus I p 0.

So, basically this matrix is skew symmetric, this is gyroscopic matrix. This is mass matrix, this is stiffness matrix. Now, we can able to assume this because at present we are not considering the external force, we can able to solve for free vibration by assuming the solution of this.

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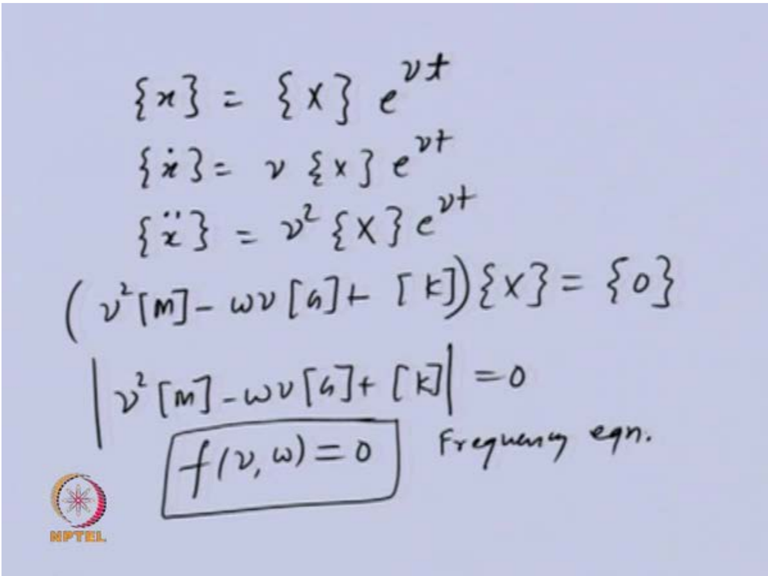

Let us say some amplitude and the frequencies, let us see frequencies is new. So, it is evt, so we can take the derivate of this once. So, we will get exact expression because our equation of motion contain derivate up to second. So, derivate up to second and here this are the amplitude. So, this are not independent, this are time dependent. So, if we substitute this equation of motion we will get v square M minus omega new G plus k, X is common, capital X is equal to 0.

So, this have come from previous of motion after substituting the assume solution. Now, in this particular this particular cases one solution is, when X is 0 we can able to satisfy the equation. But for non-elemental solution we have to determine this equal to 0 and if we take this determine as 0 for we get a polynomial atoms of mole frequencies new and the routes of the polynomial will give the natural frequency of the system. So, in this particular system you can able to see this particular determine M is function of new and omega we can able to write like this. So, when omega rotor is changing, the spin speed of the rotor is changing. The wall frequency depends on that. So, various well of that wall frequencies spin speed, we need to obtain this particular word frequencies and this is nothing but the frequencies equation if you want the critical speed directly.

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$$\left| \nu^2[\alpha][M] - \omega\nu[\alpha][G] + [I] \right| = 0$$

$$[\alpha] = \begin{bmatrix} \alpha_{ll} & 0 & \alpha_{la} & 0 \\ & \alpha_{ll} & 0 & \alpha_{la} \\ & & \alpha_{aa} & 0 \\ \text{sym} & & & \alpha_{aa} \end{bmatrix} \qquad [M] = \begin{bmatrix} m & 0 & 0 & 0 \\ & m & 0 & 0 \\ & & I_d & 0 \\ \text{sym} & & & I_d \end{bmatrix}$$

$$[G] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p \\ 0 & 0 & -I_p & 0 \end{bmatrix}$$

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$$\begin{vmatrix} \nu^2 m\alpha_{ll} + 1 & 0 & \nu^2 I_d \alpha_{la} & -\omega\nu I_p \alpha_{la} \\ 0 & \nu^2 m\alpha_{ll} + 1 & \omega\nu I_p \alpha_{la} & \nu^2 I_d \alpha_{la} \\ \nu^2 m\alpha_{la} & 0 & \nu^2 I_d \alpha_{aa} + 1 & -\omega\nu I_p \alpha_{aa} \\ 0 & \nu^2 m\alpha_{la} & \omega\nu I_p \alpha_{aa} & \nu^2 I_d \alpha_{aa} + 1 \end{vmatrix} = 0$$

So, this particular equation we wrote in the previous slide as the functional form. If we substitute various matrix in this we can able to get determinant over like this. So, we can able to see in this particular equation expressive form of thee new and omega evident here. So, both new and omega are appearing, we can take the polynomial in term of a mole frequency of the order of the a degree and the roots of the mole frequency are the and for each omega spin speed is need to find out. So, we have seen how to get hold frequencies using the outline frequency in the, this particular way of obtaining frequency is the crude way of finding.

In this particular way you need to find the roots of that polynomial. In the sequent lecture we will see that when the equation of motion having that is velocity terms like exhaust in this particular speed. It is convent if we transform these equations in the state space form and then we can able to convert the second order differential equation to first order differential equation. In that particular space that will be diagonal problem and that is more easy to solve.

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$$\nu = \omega = \omega_{cr}^{F}$$

$$\left(\left(\omega_{cr}^{F} \right)^{2} \left([M] - [G] \right) + [K] \right) \{X\} = \{0\}$$

$$\left[M_{eff} \right] = [M] - [G]$$

$$\nu = -\omega = \omega_{cr}^{B}$$

$$\left(\left(\omega_{cr}^{F} \right)^{2} \left([M] + [G] \right) + [K] \right) \{X\} = \{0\}$$

$$\left[M_{eff} \right] = [M] + [G]$$

So, particular state form will be seen once we go multi degree of problem system of such rotor bearing system. Again I am repeating this particular thing, this state space form again will relook into this particular method. We analyze the rotor bearing system using the finite element method and now we will see if we want to calculate the critical speed directly instead of obtaining the old frequencies with respected to omega. If we are interested in obtaining directly the critical, then because the critical speed condition is nothing but once the mole frequencies into the spin speed we able to get the critical speed directly.

Now, let us see how that can be obtain from this equations. So, this is the mole frequencies, which is equal to spin speed and we are equating that to forward critical speed because we have taken the positive speed, that is hold direction and spin direction are same. So, if we substitute this equation in the previous equation that can be arranged like this because m was having new omega square.

So, we are now having the omega critical where G was having to omega. They will make the term of new omega square that we have made common and this K matrix is equal to this. Now, in this particular case you can able to see that this is something mass of the system and so because of gyroscopic couple the effecting mass is getting decreased. As we know the critical speed is nothing but stiffness of mass, effective mass is getting decreased.

So, we are expecting the critical speed will be increasing and that is why are able to see the forward critical speed in basis as come closer to the backward critical speed. So, this is the second case where we have moiled frequencies minus of omega. That is backward critical speed we are defining the heat. So, if we are substituting the heat, in this equation we will see that this sing will change to positive and the effective mass will be now m plus G.

So, here the effecting mass is increasing. So, if we accept the backward critical speed this is v, the backward critical speed will be decreasing. This is the particular trend we have seen in the previous cosmic analyses. In case now we will take some example. So, that what were the method we discussed and the causes and dynamic method that will be more clear. How you can able to get the mole frequencies or critical speed especially you will see the Cambell diagram through this example.

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A long rigid symmetric rotor is supported at ends by two identical bearings. The shaft has the diameter of 0.2 m, the length of 1 m, and the material mass density equal to 7800 kg/m^3. The bearing dynamic characteristics are as follows: $k_{xx} = k_{yy} = k = 1$ kN/mm with other stiffness and damping terms equal to zero. By considering the gyroscopic effect, obtain whirl natural frequencies of the system, if rotor is rotating at 10, 000 rpm.

So, one of the example is in which a long rigid region supported on two identical bearing. The shaft has the diameter 0.2, length of 1 meter, the density of the mass is mass of the density of the shaft is given the support spring are having the direct stiffness and this is given by this expression or the damping we are considering. We are considering the gyroscopic effect in the particular case and we are interested in finding the mole natural frequency at particular heat.

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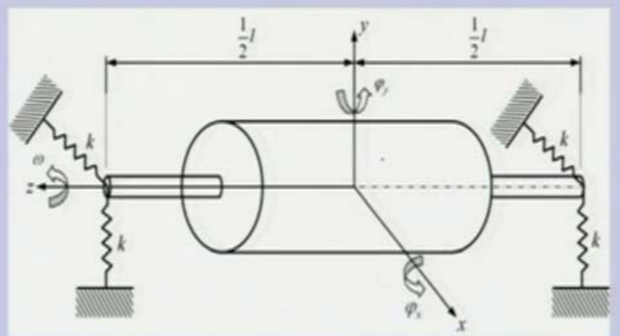

A long symmetrical rotor mounted on identical flexible bearings

So, this is the rotor system which is supported on buy springs on two directions. Now, we have given the value of this particular case and especially we are interested in this particular case coupling between the translator motion and the. So, we could have able to analyze this in the translator motion tilting motion separately without any problem and based on our experience now we can able to write the equation of motion system like this.

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$$m\ddot{x} = \sum f_x$$

$$m\ddot{y} = \sum f_y$$

$$I_d\ddot{\varphi}_x + I_p\omega\dot{\varphi}_y = \sum M_{yz}$$

$$I_d\ddot{\varphi}_y - I_p\omega\dot{\varphi}_x = \sum M_{zx}$$

So, these two are basically transmitter motion because gyroscopic effect is not present in this. So, this will be simpler. So, we will not consider this particular thing. So, the gyroscopic motion, so you can able to see the extra external movement in y z plane and this is inertia term. One to the angular acceleration and another is to the gyroscopic couple. This expression in today’s lecture we derived. Also basically we need to obtain what is the effective stiffness, the torsnal of this bending stiffness, the beading are providing. If we can obtain that then the equation of motion can be obtained.

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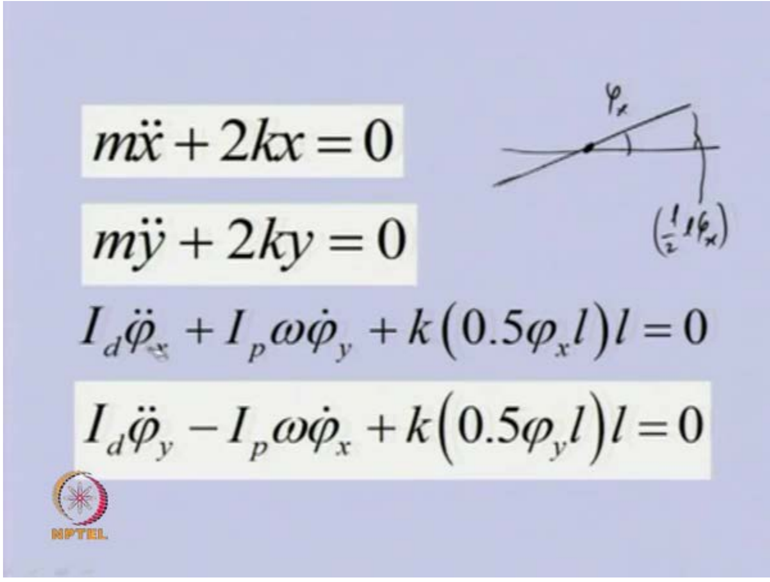

So, let us see so because these two equation we can able to get because if we are getting x displacement the in the x direction, two spin will match the elastic force. Similarly, in the y direction this will be elastic force, the movement from the two bearings will be we can able to get from this because if you see tilting of the disc by one of the let us say the tilting of the centre of the gravity. So, if let us say that this is pi x, so total.

So, the total length as we have seen here is l. So, we will be having this particular equation or the extension of the steal will be 1 by 2 l pi x and because of this k into this much will be the force which will be exacting here. If we take the movement I, we will get the movement to the springs at the bearing on this particular plane. Similarly, so basically these are the movements handling movement, which are coming, the couple coming on to the rotor bearing this two.

Now, we can able to this particular thing, this particular equation if you see carefully this equation are coupled. Unlike the first equation they are coupled. The first equation, second contain y only angle displacement pi x and pi y is the similarly pi y and pi x are two equation we need to solve simultaneously.

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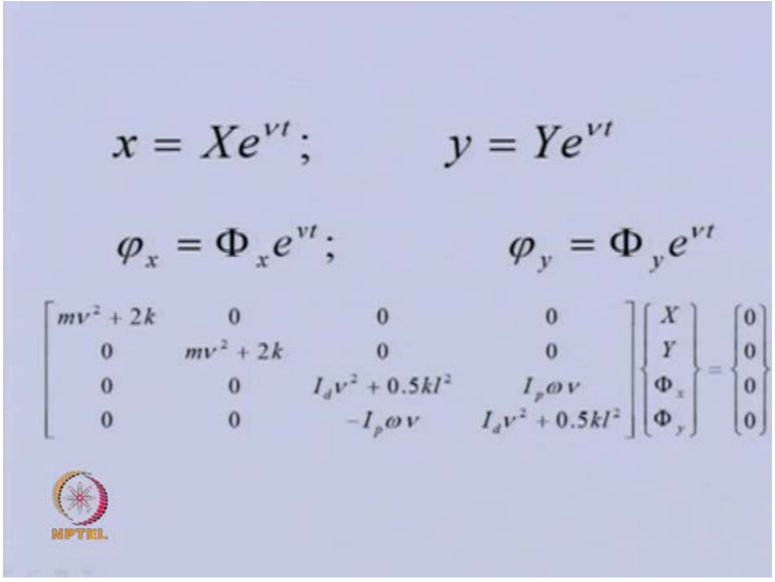

So, let us if we assume this solution in this form we can able to basically write. We can substitute in the previous direction and we can able to get same line in the previous case, these particular homogeneous expression. If we need non primer solution, this particular determine we need to keep it 0. So, if we keep that particular 0.

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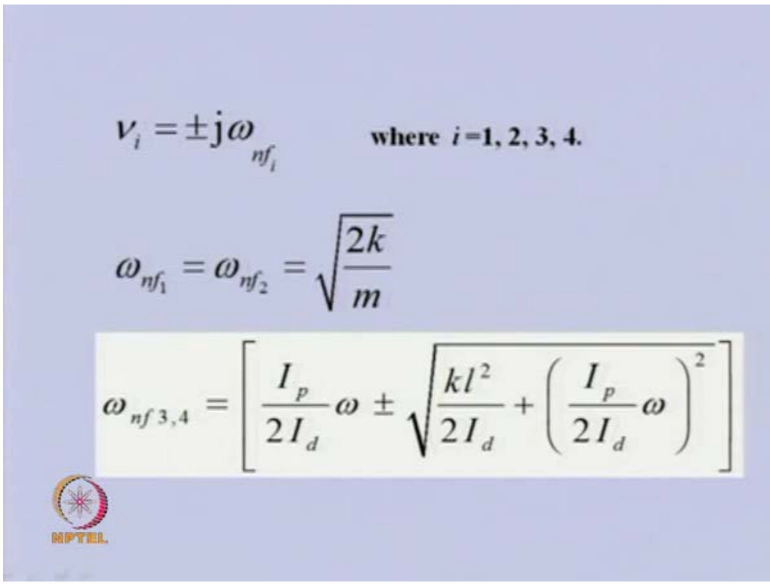

Because first two equation were uncoupled, so we get the natural frequency from them directly these one, but second two equation, second and third equation will give as the mole frequencies like this which will depend upon the spin speed, also that you can able to see that mole frequencies not dependent on the spin speed due to the gyroscopic couple.

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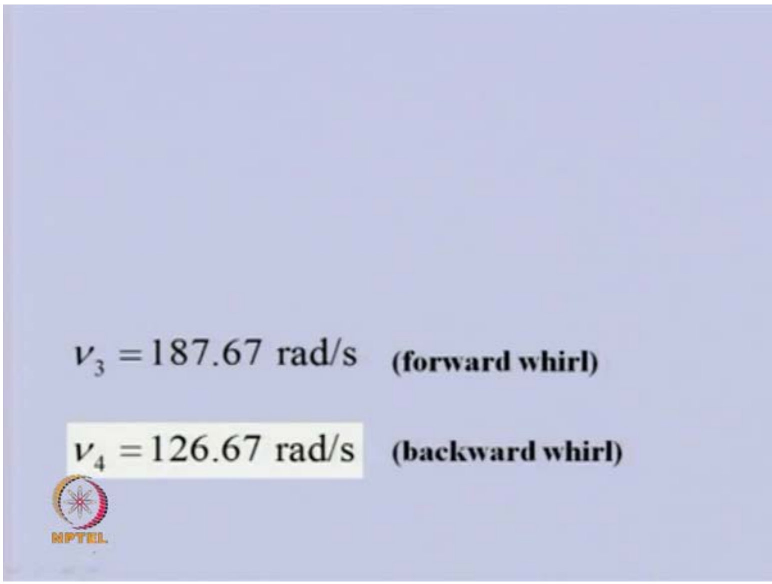

So, if you substitute specially second and third equation, this one.

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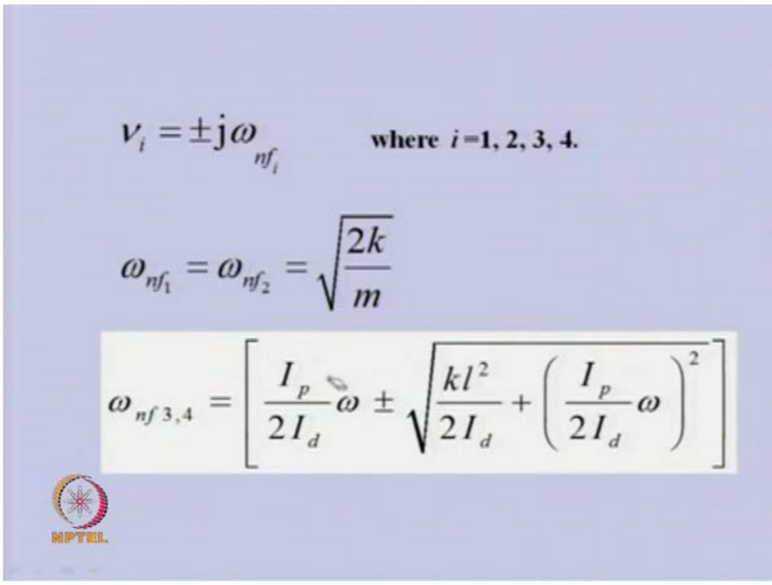

Various value and thousand rpm, we can able to get the third and fourth frequencies like this. Now, we will take another example in which consider coupling in more general form.

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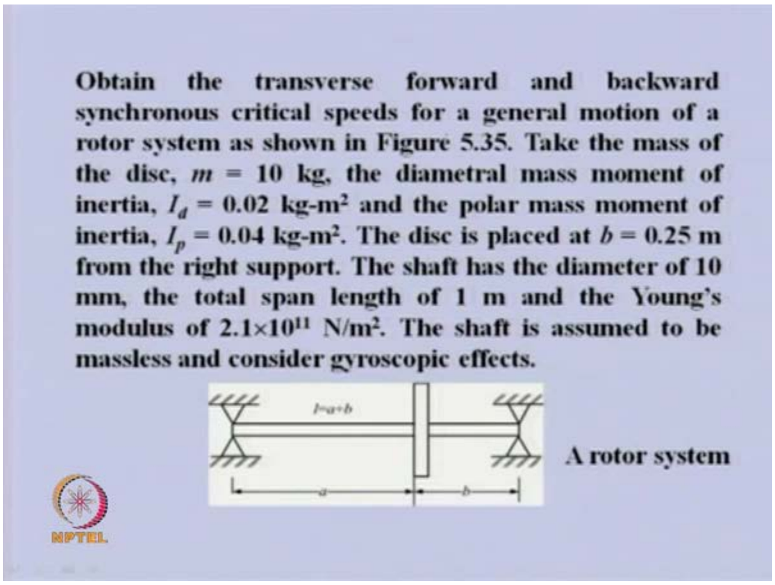

So, let us say we have one rotor like this which we have half side disc and various parameter are given here, mass I d Ip of the disc span of the disc. The particular, this span of the shaft young span life the position of the disc of the dimension are given here.

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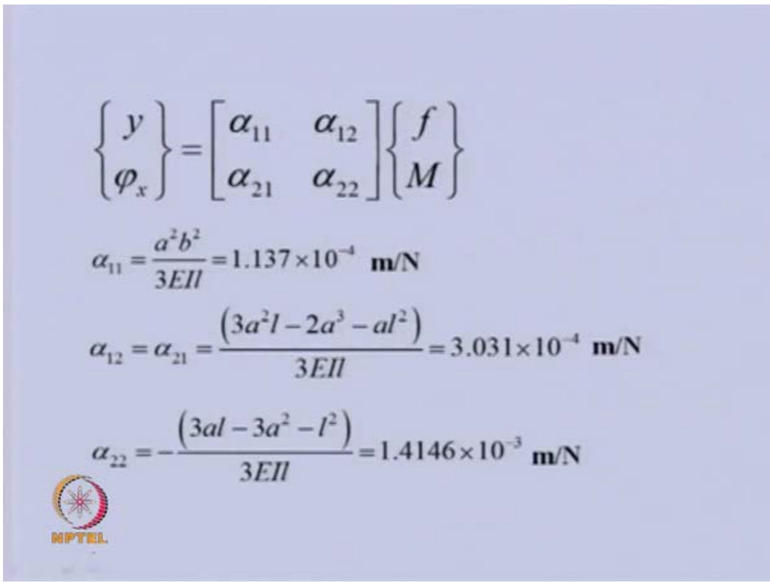

The influence coefficient for the simply supported beam with offside disc are defined like this. So, for a given value we can able to obtain them.

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$$\bar{v}^4 - 2\bar{\omega}\bar{v}^3 + \frac{\mu+1}{\mu(\bar{\alpha}-1)}\bar{v}^2 - \frac{2\bar{\omega}}{\bar{\alpha}-1}\bar{v} - \frac{1}{\mu(\bar{\alpha}-1)} = 0$$

$$\mu = \frac{I_d \alpha_{22}}{m\alpha_{11}} = \frac{0.02 \times 1.4146 \times 10^{-3}}{10 \times 1.137 \times 10^{-4}} = 0.0249$$

$$\bar{\alpha} = \frac{\alpha_{12}^2}{\alpha_{11}\alpha_{22}} = \frac{(3.031 \times 10^{-4})^2}{1.137 \times 10^{-4} \times 1.4146 \times 10^{-3}} = 0.5712$$

$$\bar{v}^4 - 2\bar{\omega}\bar{v}^3 - 95.99\bar{v}^2 + 4.664\bar{\omega}\bar{v} + 93.658 = 0$$

If we recall from the third lecture, from we consider the general motion of the disc. This was the frequency equation we got it in the spin, the non-dimension spin ratio coupling effect. This can be calculated for given value and if we substitute this frequency equation will get a polynomial like this which is a frequency equation.

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$$\bar{v} = +\bar{\omega}$$

$$\bar{\omega}^4 + 91.33\bar{\omega}^2 - 93.66 = 0$$

$$\bar{\omega}_{1,2}^2 = -92.35 \text{ and } 1.015$$

$$\bar{\omega} = \omega_{cr_1}^F \sqrt{\alpha_{11} m} = \omega_{cr_1}^F \sqrt{1.137 \times 10^{-4} \times 10} = 1.0075$$

$$\Rightarrow \omega_{cr_1}^F = 29.88 \text{ rad/s}$$

For forward fold we have the condition this one that the mole frequency is equal to the spin speed ratio. If we substitute this to get the critical speed can able to see the both. This will reduce the quadric equation in terms of omega where they can be solved, but here one of the root is coming negative. Or this we absorbed earlier also that one of the forward critical speed is not visible only the on is visible another is not visible.

So, visible critical speed we can able to see because critical omega bar like this. So, the forward critical speed, one of the can be obtain from this expression. It was this value is this one, this is not visible. So, from this we will get the first critical speed like this, but second is not visible.

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$$\bar{\nu} = -\bar{\omega}$$

$$\bar{\omega}^4 - 33.55\bar{\omega}^2 + 31.22 = 0$$

$$\bar{\omega}_{1,2} = \omega^B_{cr_{1,2}}\sqrt{\alpha_{11}m} = 0.98 \text{ and } 5.71$$

$$\omega^B_{cr_1} = 0.98 \times 29.66 = 28.47$$

$$\omega^B_{cr_2} = 5.71 \times 29.66 = 169.34$$

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$$\bar{\nu}^4 - 2\bar{\omega}\bar{\nu}^3 - 95.99\bar{\nu}^2 + 4.664\bar{\omega}\bar{\nu} + 93.658 = 0$$

$$\bar{\omega} = \frac{\bar{\nu}^4 - 95.99\bar{\nu}^2 + 93.658}{2\bar{\nu}^3 - 4.664\bar{\nu}}$$

So, we are not obtaining that, but from backward whirl if we substitute this expression we will see that in the following mere will reduced to this. In this particular case we will be having two positives roots and we will be getting correspondingly cross backward critical speed, and the second the code critical speed both are in radius per second.

Now, let us see if we want to plot the frequency equation as it is without solving the critical speed. So, you can see this that particular expression can be rearrange and we can able to write like this, omega bar is equal to this expression. Now, we can take varies early of mu and we can able to get the omega if we plot this for various values of mu omega.

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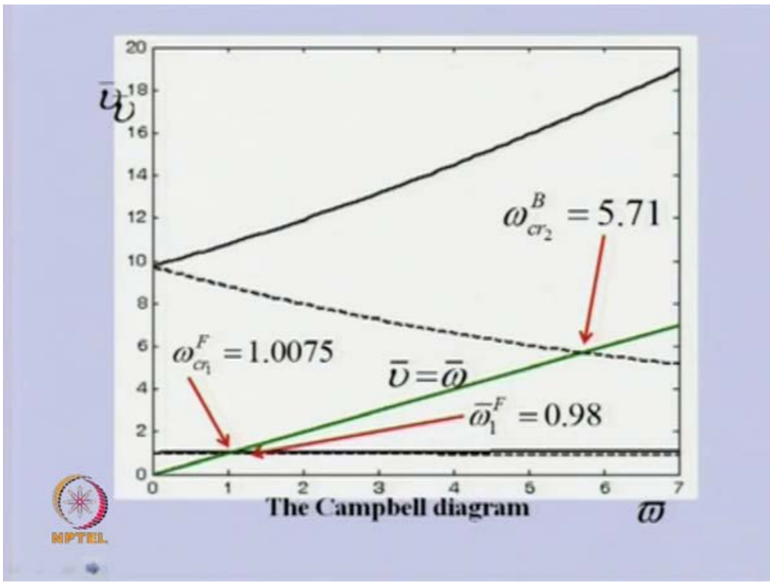

We will get a diagram like this is called Campbell diagram. So, horizontal axis is omega bar and what vertical axis is mu bar. So, you can able to see that we have this two lines, this splitting of the natural frequency is taking place, what is splitting is less for the first whirl. You can able to see this green line is nothing but mu is equal to omega line. There is a to critical split line, wherever this critical line is intersecting the frequency goes there, the critical speed.

So, upper part is this one which we have seen, the one of the root was we got this for the forward whirl. Similarly, for the backward whirl. So, there are two roots here one for forward and lower one is for backward. For second, this dotted one is second, backward that most visible, but this line is not intersecting. The second forward whirl and this is evident from here not get the roots because they are not in the setting. So, that is not visible.

So, this particular thing other thing you can able to see at 0 speed, both the frequency which got splitter because of gyroscopic couples is converging to the same place. So, they are at the this two values are at the same place was 0 speed because gyroscopic couple is not there. So, this splitting is due to the gyroscopic couple. Today we saw derivation of the equation motion of a simple disc due to the gyroscopic couple. How we could able to get the specially the inertia in a rotary inertia, and again I am repeating the last sentence we are seen that how the gyroscopic couple affords the equation a motion.

Specially, the moment equations which we obtain where containing the gyroscopic couple and this gyroscopic couple were coupling this two equation, because the as we know when we apply moment to spelling in this one plane. The precision take place in the third axis. So, that way all the three axis is all the three planes of the rotor system get coupled. Specially, in the moment equations and some example we are seeing, how to obtain the critical speed a whirl frequency, specially we are seeing in the Campbell diagram and how we can able to get the critical speeds to the intersection of the line which is 45 degree to the axel whirl.