

Theory and practice of rotor dynamics
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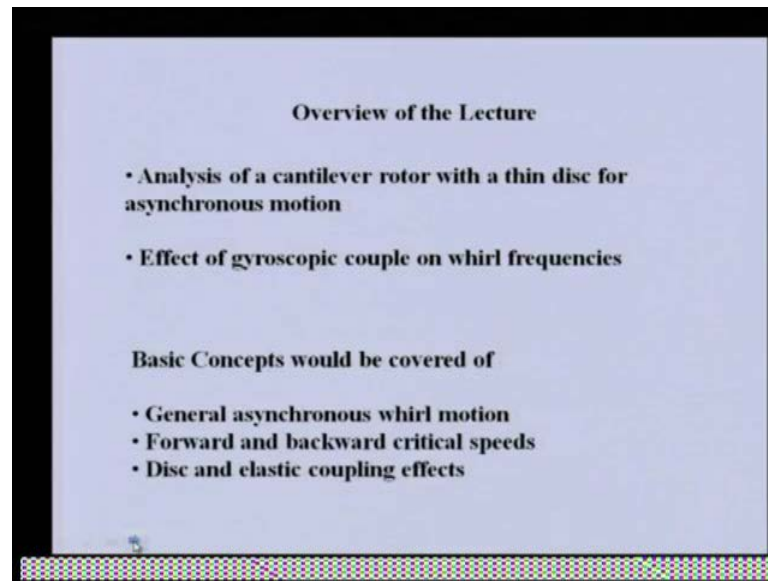
Module - 4
Lecture - 3
Gyroscopic effects: asynchronous whirl of
A rotor system with a thin disc

Last two lectures, we studied gyroscopic effects on the critical speed or the whirl frequency of the rotor system. In fact, those two lectures were slightly special case like we consider a cantilever on or a rotor system that with a thin disc or long stick. We consider a very special case of whirl motion; that is incrudness whirl condition. In that particular condition we were the definition of the critical speed.

So, basically we were looking for such analysis to find out the critical speed, as such the wall frequency we did not consider. During that lecture also we considered in some restrictive way the pure tilting motion or wobbling motion of rotor shaft discussed early. In that particular case we relax this particular condition of synchrony as one and we obtain the whirl frequency variation with respect to speed.

So, in previous lecture basically did not consider a general motion of a disc. That means both translatory as well as the wobbling motion or the tilting motion along with a asynchronous whirl. So, that particular case we did not consider till now and today we will consider that particular case with which we will take an example of a cantilever shaft attached with a thin disc at a free end. We will analyze asynchronous whirl condition and we will try to find out how the whirl frequency changes with the spin speed of the shaft, and also with Campbell diagram with we can able to obtain the critical degree of the system.

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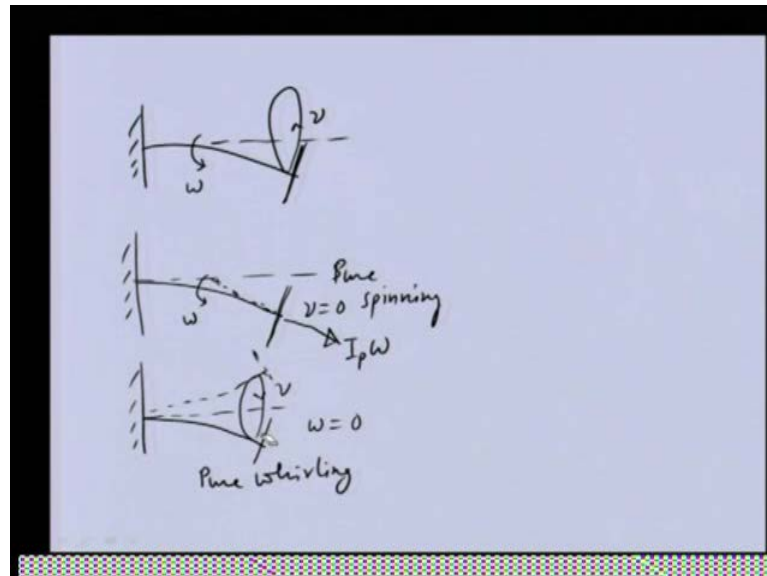
So, this is overview of the present lecture in which we are doing analysis of a cantilever rotor with disc for asynchronous motion. In this particular case we will be considering both translator and the wobbling motion of the disc, effect of the gyroscopic couple on whirl frequencies. We will analyze some of the concepts which we will be covering in this particular lecture is general asynchronous whirl motion, forward and backward critical speed, disc and elastic coupling effects on the whirl frequency. These are the analysis we will be doing.

For the present case, we will be considering a cantilever of beam like this and this beam not only it will be spinning on its own axis. The shaft will be spinning and also it will meet whirling and this particular whirling motion spinning and whirling both takes place and this particular case it will be spins. And the whirl frequency different and will try to see how this whirl frequency changes with the speed. In this particular case we will not consider the that is unbalance in the rotor.

So, this will be free vibration analysis and we will in this particular case, we will asynchronous motion is not there which asynchronous motion is not there in that part. In previous case we considered that and again we saw that of bents and whirls in asynchronous condition, the fibers of this shaft which is intention always of the main intension, which is in compression, which remains in compression for this particular case

that will not be there for the fiber wheel during the whirling. It will be changing its tension or compression such as...

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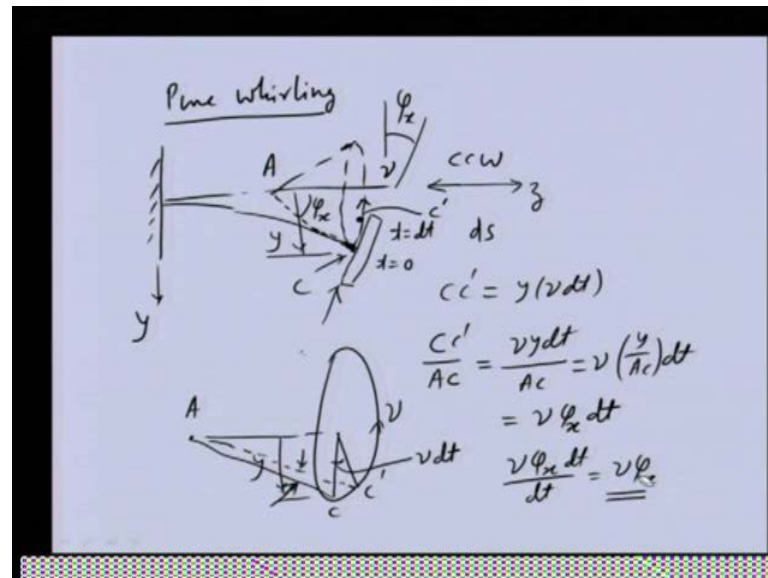
So, let us consider the cantilever shaft with a fluid disc at the end, because in this particular case we have spin of the shaft depending on its own axis and the bearing axis and this itself is having whirling with different frequency. So, if you want to analyze the motion of the disc, if both the spinning and whirling is there it is very difficult to visualize the motion of the disc. So, in this particular case we will consider two cases, one in which we will be considering pure spinning of the shaft.

So, let us see this extreme position of the end of the shaft that is spinning, but there is no whirling. So, whirling is not there, so I am putting this 0, another case will be pure whirling. So, in this particular case shaft is not spinning, only the pure whirling is taking place. So, it is moving along with this path with frequency in it, but as such in there is no spinning, there is no 0. So, this is the regular case, we will call it as pure whirling and this particular case is pure spinning.

So, in this particular case the first one the pure spinning case was spinning is there. You can see the angular momentum of the disc will be in this direction and value will be $I_p \omega$. So, for the pure spinning case angular momentum is straight forward to tension to the shaft at this location. So, it is free to draw a tangent to a shaft, it will be something like this, but for pure whirling case the calculation of the angular momentum is slightly a

difficult. So, try to see the motion of the disc during whirling, how it tildes. Then we will calculate the angular momentum of this particular disc and then we come for pure whirling.

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So, for pure whirling let us see the motion of the disc how it take place, so this is the bearing axis and let us say there is the shaft and a disc here. Now, as I told now the motion of the disc is slightly difficult at this case. So, we will take a small shaft segment here, let us take length ds which is attached to the disc, very close to the disc and we will try to see the motion of this particular shaft segment. This one, this particular shaft segment if we extend that is nothing but a tangent to the shaft at the bearing location.

So, we will see that this particular shaft segment make, will make a conical shape about the enter, let us say A. So, this is the whirling direction μh , if you want to see the motion of the disc and when it is doing from the locums. Let us say the bottom most time t is equal to 0. So, at the time t is equal to after sometime Δt , where this particular disc will go. So, in this particular case, let us visualize the motion of the disc. So, it is not spinning, only the whirling we are considering.

So, this is at the bottom most position. Now, if I am considering the motion of the disc that are the whirling, you can see that here you can able to see only the line of the disc, but when it is whirling, when it is inside towards me, you can able to see the inside particle of the disc. But once it will come to the top again you will see the disc as a

straight line, but when it comes out towards the u, then you will see the outside of that plane of the disc. So, earlier when it was here we could be able to see the inside side of the disc and here we assume this particular side of the disc.

So, basically this particular disc is tilting about its diameter during its whirling, and now again we will consider we will see this, now bottom most position. So, when it is coming this side, so you can be able to see basically it is trying to see the tilt about this particular diameter which is a vertical diameter. So, this is tilting in this particular direction for the diameter about this diameter. So, let us say when we are looking at this side, the whirling direction is counter clock wise direction. So, when after sometime the disc will reach on this bottom position which is C to let us say the position which is C prime.

So, that will be inside the screen. So, this C prime will be inside the screen and u, this particular disc will be tilting about its diameter or here and if we see the tilting direction. Tilting direction will be a vector which will be this side, we use the right hand rule. The tilting direction will be this side. So, this we can be able to visualize better from here. So, when it is going this side you are able to see tilting about its diameter this. So, upward direction, so basically we can be able to show this particular motion which we are considering is for very small time variation from 0 time to Δt time. And for that particular instant of time this disk will be tilting about its diameter here.

Now, we can see how we can be able to get the angular momentum because of this tilt of the disc. So, again I am drawing the motion of the disc. So, initially it was here and then after 20 it reaches to c time, because the whirl frequency is c, the angular frequency is μ . So, this particular angle will be given as because we know that if we see here we have this as y displacement and the tilt of the disc is axis z. When we have axis system as z and this as y the tilting of the disc is given by $\frac{y}{A} \frac{dx}{dt}$ and the translator e displacement is of that disc is y. So, we will be having this particular angle will be $\mu \Delta t$.

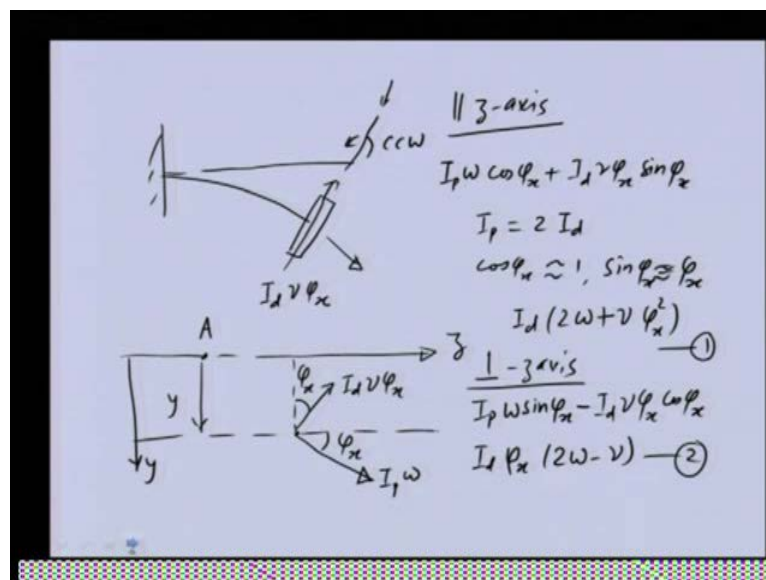
This is the angle which will be traversing that is from c to c prime. So, this is the center of the bearing and this is the distance y. So, the c c prime will be y and the angle $\mu \Delta t$, if we want the c c prime divided by the AC, AC is the apex of the cone. So, A into C, so this can be written as $\mu y \Delta t$ which is this is coming from the above expression and this I can be written as $\mu y \frac{y}{AC} \Delta t$. Now, $y \frac{y}{AC}$ is y is this distance and AC is

this. So, we have this particular angle which is basically tilt of the disc ϕ e x. So, y by AC is ϕ e x μ ϕ x dt. So, this ratio becomes μ ϕ ϕ e x dt.

So, if we see this particular a point here, there is the line AC, this one and there is the another line AC prim. So, AC prime t in this particular delta time t, the change of the angle of the AC and AC prime and this particular angle is basically C C prime by AC. This is the angle which we were obtained. There are CC CC prime and AC, this is the angled, this particular AC is traversing of AC, here AC is the next one which is at bottom most position and it is traversing some angle, that is cone. We consider the small sharp segment here and that sharp segment is traversing a cone or like this.

So, this particular this point is A. So, this line is making angle that is we obtain here that is C C prime by AC. Now, this is the angle, now the rate of change of this angle that will be μ , this divided by the time taken. So, μ into ϕ e x, so this is the rate of change of that particular angle. So, this is the velocity of the line AC where it is the motion of the disc. So, AC is having this particular angular velocity μ into ϕ e x and the direction of, so we have obtained the angular velocity of the line AC, that is μ ϕ e x.

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Now, we are able to get the angular momentum of the disc because this particular disc is tilting about its diameter. This diameter or so we need to consider the four angular momentum calculation for I_d diameter mass momentum equation and will be

multiplying with the diameter we obtained. So, the angular momentum of this particular disc will be.

So, again I am drawing that figure. So, disc is here, disc is tilting about an axis which is the diameter of the disc and will be having the angular momentum in this direction as $I_d \mu \omega$. If we again look into the angle of the momentum direction, this particular disc is tilting about the diameter. It is at the axis, we are looking from the top, the tilting direction is counter clock wise of the disc and because of this the angular momentum is in the direction of the ω . So, again I am repeating the tilt of the disc. When this particular disc is moving from time 0 to time, dt is moving inside the screen and because of that the tilt about its diameter is taking place.

If you look from the top, that particular disc is tilting in counter clock wise direction. So, this particular angular momentum is in the upward direction. So, once we obtain the angular momentum because of pure spin motion and because of the pure tilt whirling motion. Now, we are able to obtain what is the change in the angular momentum because of tow motion w can combine these two motion to get the general motion of this particular disc.

So, we have this is the bearing axis which is z axis. Let us say A, the cone apex is this one and let us say this is the disc center line and we had a angular momentum in this direction which was in this particular direction and there is the another angular momentum this one which is having magnitude $I_d \mu \omega$ angular momentum. This one having magnitude $I_p \omega$. So, here you can able to see there is the angular momentum due to the spin and angular momentum. This is due to the whirling which is whirling frequency μ .

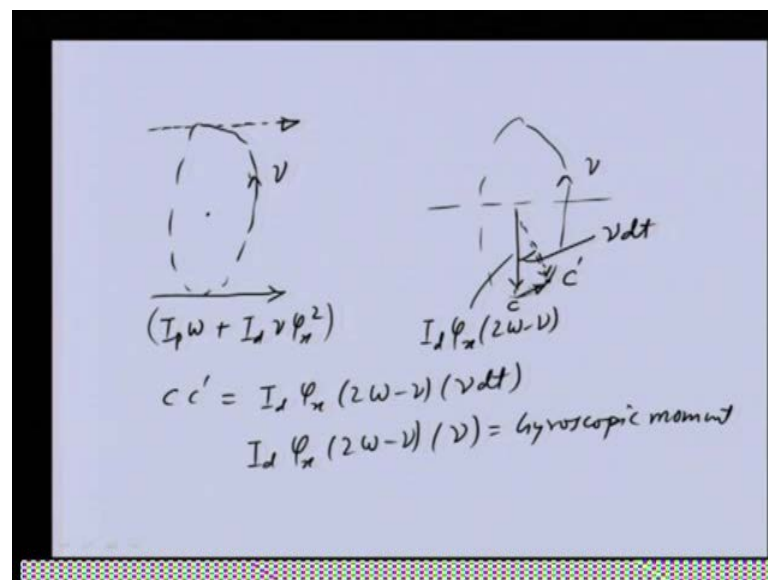
Now, we will take the component of these vector parallel to the bear axis and perpendicular to it. So, we know that this particular distance is y and in various angles we know because of the disc the tilt is ϕ this angle is also ϕ . So, we have all this, now let us say we have the component first parallel to z axis of this angular momentum. So, $I_p \omega \cos \phi$ plus $I_d \mu \omega \sin \phi$, this because for thin disc case we know I_p is equal to twice of the I_d .

So, if we use this in the above expression and if we for small angle ϕ , we can assume this as 1 and $\sin \phi$ is equal to ϕ . As you substitute these value, the above equation

we can able write the equation as $I_d 2\omega + \mu f e x^2$, if you get this angular momentum which is acting along the direction of the bearing z axis.

Now, perpendicular to the z axis what is the these angular momentum? So, this will be $I_p \omega \sin f e x$. In this particular case, let us say y is perpendicular to y downward direction. So, we have the component of this is in the positive y direction component of this will be negative direction. So, this will be $I_d \mu f e x$. Then $\cos f e x$ again if we apply this simplification of dynamics angle and we substitute the I_p is equal to $2 I_p$. This can be written as $\omega \sin \mu$. So, this is one equation, this is another. So, these are the angular momentum, which are acting on the direction of z axis and perpendicular to this. So, let us see these vectors separately.

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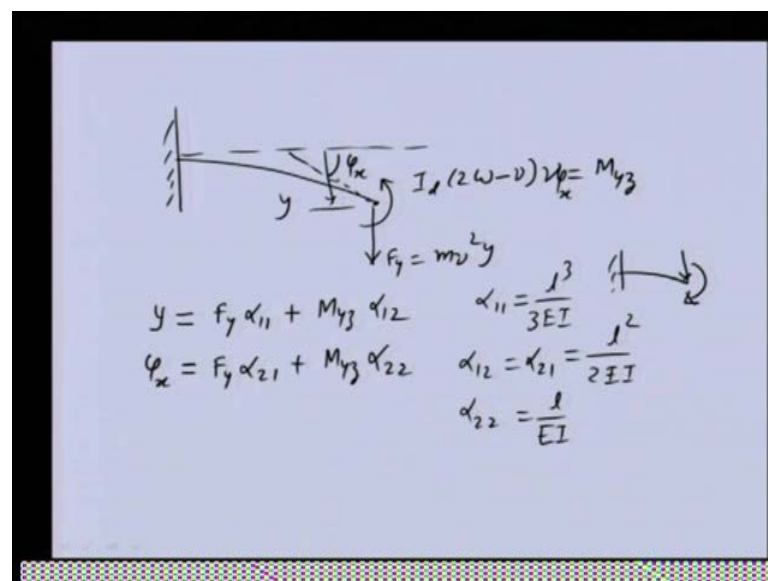
So, the first vector which is let us say this is a shaft center motion which is μ . So, in this particular case we have angular momentum which was parallel to the axis, it is this one. So, during motion this particular vector will always be on the z axis. So, let us say if you consider other extreme, here again it will be in the same direction. Magnitude of these are not changing and for this particular motion even the direction of this particular motion will not change, always will be in the direction of z. If we see the second angular momentum which was perpendicular to the bearing axis plus 1, this particular vector during motion it will change position like this.

So, if this I c c point and this is after delta time t c dash prime, then the direction of this particular angular momentum is changing and because e o f this. There will be change in the angular momentum which will be C C prime and we know that this angle is mu into dt because the whirling frequency is mu d. We have at delta time t, this particular angle will be mu into delta t. You can able to see the change in the angular momentum which is C C prime will be given as I d f e x 2 omega minus mu into this angle mu dt.

So, this is the change in the angular momentum and tangent of angular momentum. If we get we can divide this expression by delta t. So, that will be this, so basically this is nothing but the gyroscopic moment. So, this gyroscopic moment will be obtain on the disc and the direction of this gyroscopic moment we can able to get from this figure. That will be the direction of C C prime, that means in this particular case it will be into the screen inside the screen. That means it will be acting c on the screen, that will be acting clock clockwise direction on to this.

It will be clockwise direction according the direction c to c prime. So, this particular gyroscopic couple which we obtained is acting, that is active couple which this particular acts and as we obtain that is the direction of the gyroscopic moment is c to c prime. That means into the screen and this is the active couple. This is if we want to know the couple acting on the shaft due to the direction, it will be opposite to that. It means it will be acting to the counter clock direction on to the shaft.

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So, if we summarize we have now on to the shaft, if we remove the disc gyroscopic moment which we this the moment on to the shaft. So, this is opposite to what we got on to the disc. So, this will be outside the screen direction to this angular momentum and well U is $I d^2 \omega$ minus μ and this is μ we have seen earlier because of the linear motion that means displacement y . We had net resultant center figure force F_y , it was acting downward this is the gyroscopic moment.

So, this the gyroscopic moment and this is center fluid force and if we remember the one of the asynchronous couple, we had invisible ω . That was the first lecture and if we substitute this here we will get the same expression as we got earlier. Only thing is in this one expression $f e x$ is missing. We will get same expression, we will get the gyroscopic moment as $\mu^2 f e x$. So, this is the gyroscopic moment which we got the asynchronous whirling condition.

Now, this was ω , now we are considering the asynchronous whirling condition. So, whirl frequency and the spin feed is different. Now, apart from this angle will be project the tangent at the free end of the shaft. This is the $f e x$, that means that we need to relate. Basically this linear displacement or translator displacement, the force and emplacement coefficient which we define earlier $M y z \alpha_{12}$ and similarly angular displace will be given by this force with influence coefficients.

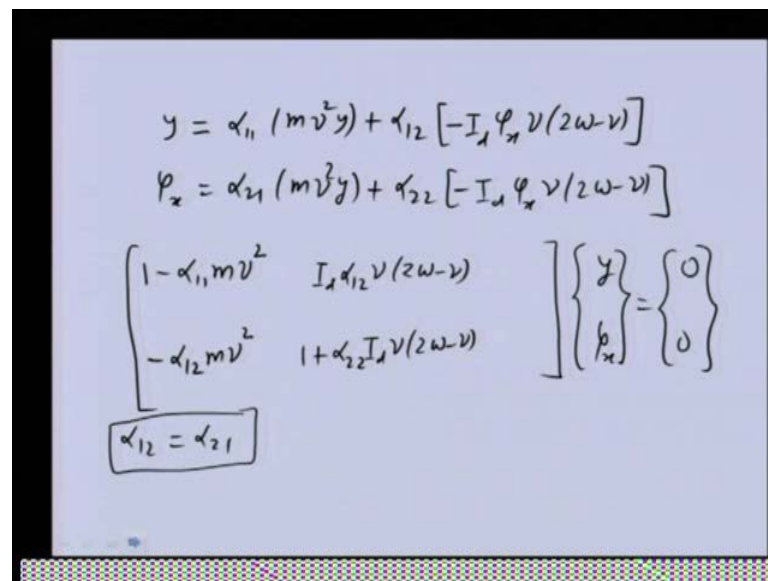
So, basically if we want $O T$, see the definition of this particular influence coefficients α_{11} is nothing but $A \alpha_{11}$ is a unit, sorry α_{11} is that only the displacement for the unit force keeping all other moment 0. So, that if we keep all other moment 0 α_{11} is equal to y . If you are taking the F_1 , that is 1. Similarly, α_{12} is a linear displacement for a unit moment when we keep all other forces 0 on the same line. We can define up to a α_{21} . That is a angular displacement for a unit force keeping moment 0 and is very clear that α_{22} is defined as angular displacement for unit moment and keeping all other forces equal to 0.

So, these are definition of the influence coefficients and these can be obtained from simple binding up these formulae's little for cantilever beam case. We have these influence coefficients as $\frac{1}{3} E I$ and $A \alpha_{12}$ and α_{21} are same. That is $\frac{1}{2} I$ and α_{22} is $\frac{1}{E I}$. So, these influence coefficients, this is for the cantilever beam in which the direction of the force is this one and the moment is

clockwise. So, these have been derived based on this sign convention you can see that the force is the same direction as this one, but the moment is opposite.

So, that when we use these expressions here. We need to keep these moments with minus sign. So, that they are in line with the sign convention. So, basically we are now substituting this amplitude force and gyroscopic moment here and you will be getting a homogeneous equation.

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$$y = \alpha_{11} (m v^2 y) + \alpha_{12} [-I_A \dot{\phi}_x v / (2\omega - v)]$$

$$\dot{\phi}_x = \alpha_{21} (m v^2 y) + \alpha_{22} [-I_A \dot{\phi}_x v / (2\omega - v)]$$

$$\begin{bmatrix} 1 - \alpha_{11} m v^2 & I_A \alpha_{12} v / (2\omega - v) \\ -\alpha_{12} m v^2 & 1 + \alpha_{22} I_A v / (2\omega - v) \end{bmatrix} \begin{Bmatrix} y \\ \dot{\phi}_x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\boxed{\alpha_{12} = \alpha_{21}}$$

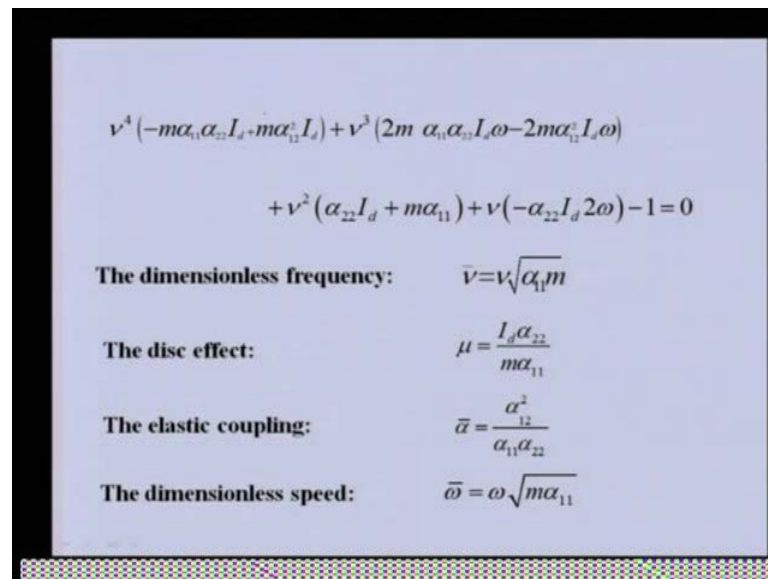
So, I am substituting the amplitude figure in this $m\omega$, sorry in this particular case if we go back here this amplitudinal force is now because due to the pure whirling case. So, here there is the correction, this will be μ , instead of ω this will be μ because in this particular case again if we relocate this. This is coming when we have pure rolling case pure whirling. So, when the disc is coming here its angular velocity is ω and this is the y displacement, because of this the centre figure is this particular center figure is down.

So, this will be a $r d$ at stream and the magnitude will be at $1 \mu \omega^2 y$ plus α_{12} . We will be substituting the gyroscopic moment with minus sign to take care with the sign convention this. Then $f e x$ tow $1 m \mu^2 y$ plus α_{22} , here also we need to keep the minus sign. Now, we can able to see y term is here y term is $f e x$ is here $f e x$ is here. Now, we can able to rearrange in a equation in a matrix form like this, where y is $p x$, we can able to write. So, here we will be having one $\alpha_{11} 1 m \mu$

square then here I d 2 alpha 1 2 mu 2 omega minus mu. Here I am having, I am keeping 1 2 same because they always will be same. 1 plus alpha 2 2 I d mu 2 omega minus mu.

So, here I have kept alpha 1 2 is equal to alpha 2 1 because they are symmetric, they are equal. So, this is homogenous equation, the one solution could be when the both displacements equal to 0, but that is a trivial solution, but for a non trivial solution. We need to take the determinant this equal to 0, form this we will get polynomial in term of mu and that polynomial can be written like this and this polynomial can be written like this.

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$$v^4 (-m\alpha_{11}\alpha_{22}I_d + m\alpha_{12}^2 I_d) + v^3 (2m\alpha_{11}\alpha_{22}I_d\omega - 2m\alpha_{12}^2 I_d\omega) + v^2 (\alpha_{22}I_d + m\alpha_{11}) + v(-\alpha_{22}I_d 2\omega) - 1 = 0$$

The dimensionless frequency: $\bar{v} = v\sqrt{\alpha_{11}m}$

The disc effect: $\mu = \frac{I_d\alpha_{22}}{m\alpha_{11}}$

The elastic coupling: $\bar{\alpha} = \frac{\alpha_{12}^2}{\alpha_{11}\alpha_{22}}$

The dimensionless speed: $\bar{\omega} = \omega\sqrt{m\alpha_{11}}$

This will be fourth degree polynomial in terms of mu which is whirl frequency and in general, obviously we expect general routes of these four solution of these. But here so many parameters are involved. So, we will define some non dimensional parameter like dimensionless frequency mu as mu bar as mu and square route alpha 1 1 m. Disc effect over like this elastic coupling between the linear motion and the angular motion. That is translator motion and the angular motion like this, then dimensionless speed I am defining like this. So, if we use this dimensionless parameter, this frequency equation will become relatively simpler here.

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$$\bar{v}^4 - 2\bar{\omega}\bar{v}^3 + \frac{\mu+1}{\mu(\bar{\alpha}-1)}\bar{v}^2 - \frac{2\bar{\omega}}{\bar{\alpha}-1}\bar{v} - \frac{1}{\mu(\bar{\alpha}-1)} = 0$$

Case I $I_d = 0, \mu = 0$

$$\frac{0+1}{\bar{\alpha}-1} \cdot \bar{v}^2 - \frac{1}{(\bar{\alpha}-1)} = 0$$

$$\bar{v} = \pm 1$$

$$v = \pm \sqrt{\frac{1}{\alpha_{1,m}}} = \sqrt{\frac{k}{m}}$$

$$\bar{v} = v \sqrt{\alpha_{1,\omega}}$$

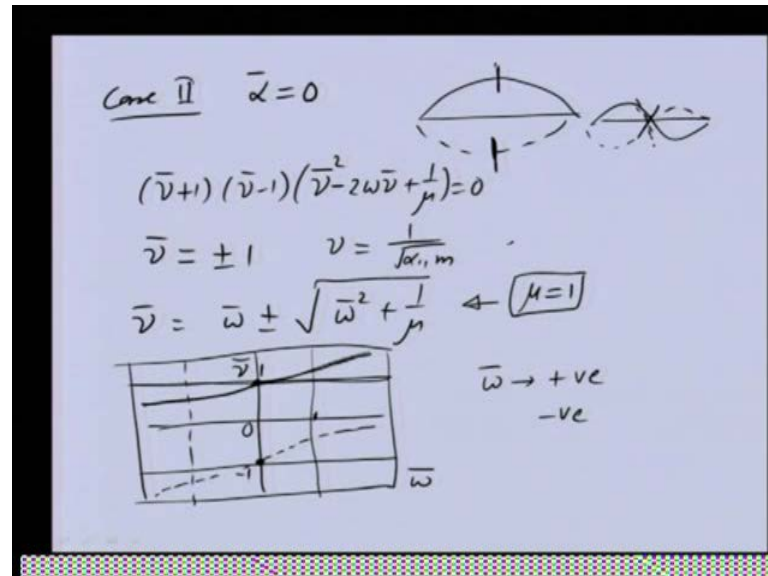
Apart from the μ bar, we have ω , there is the spin, there is the speed. Speed parameter coupling parameter α bar and the disc parameter. So, we can able to solve this for and we can get four routes, but let us take some special cases in which we can able to simplify these expressions. So, this case one in which we are considering, let us say I_d is 0. That means point mass we are considering for that μ will be 0, because I_d is related with μ because if you see substitute μ is equal to 0. Here you can see that μ is here.

So, first we need to multiply μ to whole expression. So, this and this term will vanish from sub vacant term. We will get 0 plus 1, third term will get 0 minus α 1 bar μ bar square and form this expression this will vanish because this will get multiplied by this μ . So, the last term will give us α bar minus 1 is equal to 0. So, you can able to see the solution of this will be μ bar is equal to plus minus 1 and we have defined the μ bar as $\mu \alpha$ 1 1 ω .

So, if you substitute this here we see that the ν is plus minus 1 by α 1 1 m and α 1 1 is similar to the inverse of stiffness for cantilever beam case. So, this will be route k by m . So, we can able to see that this is standard stiffness divide by m at the frequency or when there is no disc effect. There is point mass, we will be getting the natural frequency which will be independent of the speed in this particular polynomial. We can able to see

the whirl frequency is depends on speeds some of the speed terms are there, but this disc effect is not there. Then this whirl frequency is independent of the speed.

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So, the same expression if we take another case 2 in which we have coupling parameter, that means the coupling parameter, the linear and the angular motion is not there. In that particular case can able to simplify the polynomial this one by putting, take a second case in which we are having α bar is equal to 0. The coupling parameter is equal to 0. That means in this particular case we have in this we are having the linear displacement. The angular displacement is not taking place and when we are giving a angular displacement, linear displacement is not taking place. There is the generally we can say simply supported in beam and disc is at the middle.

So, this during whirling will be having pure translator motion. It will not tilt and the same we can able to tilt without giving the linear displacement. So, other extreme will be displaced, other extreme will be this one in which this will be tilting with other direction. So, in this particular case the disc will be tilting thee is not translatory motion. In this particular case there is the translator motion, there is no tilt.

So, that is the particular special case. So, if we substitute the previous polynomial and simplify we will get a polynomial in the factorize form like this after substituting α not equal to 0. This can be simplified like this. So, this is giving us first two u is equal to plus or minus 1 and the second one is giving us expression, this one. So, you can able to

see this is similar to the previous one in which we have, earlier the critical speed was the whirl frequency was like this, but in this particular case the whirl frequency was frequency depends on the speed of the shaft.

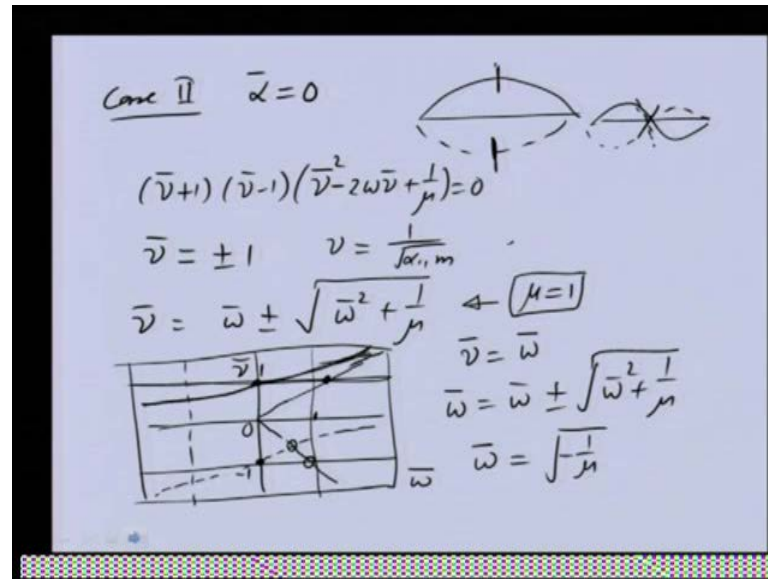
Now, we can plot this, if we plot let us say ω here and μ here and this is the 0. So, we will get corresponding to this or gentle line because the μ bar is this is plus 1 and here is minus 1 corresponding to the first two routes. In the second equation it is the another parameter μ or plotting purpose, we are assuming that μ is equal to let us say 1. So, that means if this is equal to 1 at the when this speed is 0, then the μ bar is at this location.

So, basically there is the, so for this particular value it is coincide here and if you increase the speed the frequency plot will be something like this or if you decrease it, it will go through like this and similarly, here because there are positive and negative routes. So, we will be getting this kind of curves. So, this is one line, this is horizontal line and this is the curve frequency curve, this is also frequency curve.

So, you can able to see that we can get ω . Let us say apart from the zero spin speed draw a vertical line, where you are getting four to intercept 1, 2, 3, 4, but these two intercepts or so there are four whirl frequency, but you can see this is 1. Basically they are representing same 1. So, for ω , when we are using the positive are getting this in four intercept if we use the negative. Let us say here again you will get four intercepts, but those intercepts will be symmetric.

That means this intercepts will be same as these one and these will be same as this one and that is resulting that when we are rotating a rotor in clockwise direction we expect four critical speed, but if we rotate the same rotor in the anti clock direction. We should get the same critical speed that is representing, either we other rotation of the critical speed as the system or the whirl frequency of the system should not change. So, in this particular case when we consider the coupling to 0, we obtain the whirl frequency variation with respect to speed. In this particular case if you want the critical speed. So, for that particular case we as critical condition is if we remove this whirl frequency should be equal to the spin speed.

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So, that means if we substitute this into this expression μ bar is equal to ω bar and we will solve for ω bar. Then we will get ω bar is equal to ω bar plus or minus ω bar square plus 1 μ . If we solve for ω bar we get the critical speed and this particular case we will see that one of the critical speed. If we solve this for ω bar will be minus 1 by μ .

So, that means in this particular plot, this μ is equal to ω 9 draw. So, that is nothing but a 45 degree line. So, if you draw this particular line here, this will intersect here that will correspond to on this critical speed and similarly, if we draw this side we will be having intercept here and this solution which I am talking about is the intersection of this curve and this line.

So, because this is coming in engineering t equal you will find that it never intersects. So, that means we will be having four critical speed 1 this one, 2, 3 and forth will be not visible ones. This will be very clear once we plot the frequency for general case. So, you have seen the two cases, the third case is general case. We are not assuming any particular special case of that particular case. This frequency equation will be plotting it, no frequency equation will be rearranged like this.

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$$\bar{\alpha} = \frac{\alpha_{11}^2}{\alpha_{11}\alpha_{22}} = \frac{3}{4}, \quad \mu = 1$$

$$\bar{v}^4 - 2\bar{\omega}\bar{v}^3 - 8\bar{v}^2 + 8\bar{\omega}\bar{v} + 4 = 0$$

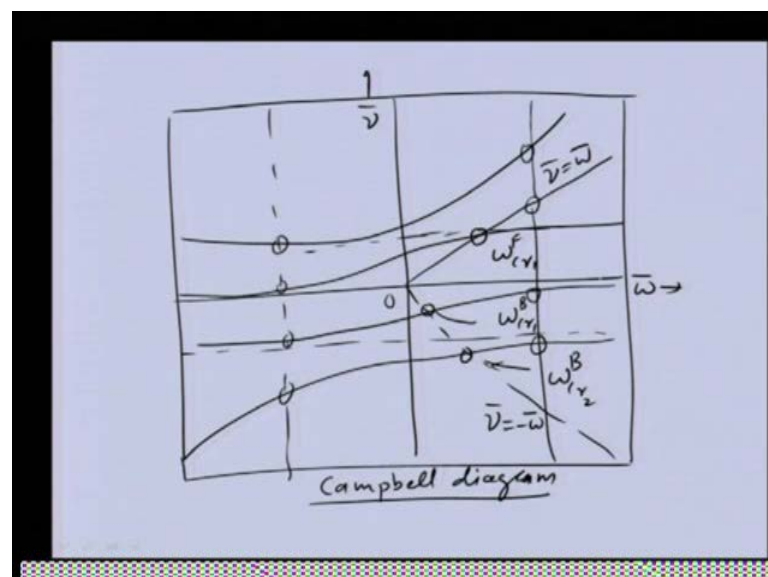
$$\bar{\omega} = \frac{\bar{v}^4 - 8\bar{v}^2 + 4}{2\bar{v}^3 - 8\bar{v}}$$

\bar{v}
 \downarrow
 \vdots

$\bar{\omega}$
 \downarrow
 \vdots

So far in this case we take let us say cantilever beam for that case alpha bar which we define as alpha 1 1, alpha 1 2 and alpha 1 1, alpha 2 2. If you substitute the influents coefficient with which we exercised earlier will be nice number and let us say we are assuming mu is equal to 1 for this. The previous polynomial will be taking this particular form and this can be solved for omega because omega is term.

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So, this will give us expression like this. Now, we can generate for the various values of omega bar, sorry we can generate for the various values mu bar. So, the value of omega

bar, so we can choose some value of this from. This expression we can able to get the omega bar. So, there I no need to solve the polynomial directly for the various value of omega bar. We can able to get the mu bar, we can able to get omega bar and these data we can able to plot it.

So, this plot will look like this in which this is the zero omega bar is in the horizontal direction and the mu bar is in the vertical direction and we will be having four curves as we had previously, but they will be not horizontal will be having this kinds of variation. Basically if you draw a horizontal line these two line will be assumptive to each other. Similarly, here if there is the horizontal line will be having like this and so these are the four line 1, 2, 3, 4 lines which will be there and if now draw mu is equal to omega line. This line will be something like this.

This is the mu we have interested in the spins speed critical speed. So, we have the intersection in this is the critical speed or of the critical speed and the upper curves are belonging to the formal work. So, this is one of the forward critical speed, there is no intersection with the second curve. So, there will not be second forward critical speed, but if you draw a another line which is mu bar is minus omega bar. Then will be having two more critical speed. This will be first critical speed, backward whirl this will be second critical speed backward whirl.

So, this particular case we are getting two backward whirl critical speed, but only one forward critical speed. Other one the second forward critical speed is not feasible and if you see the frequencies these are the frequency lines and that particular omega. That means if we draw a vertical line at any position we are getting four intercepts with these frequency levels.

So, they are the, because we had four degree polynomial. So, because of that these four values are coming, if we draw a identical line at negative omega we will get four intercepts again and they will be same as this side, because if you rotate the rotor in clockwise or anti clockwise we should get the same whirl frequency. In this particular case this is called Campbell diagram. So, this is famous diagram for obtaining the critical speed of the rotor system in which we plot the whirl frequency with spinning speed, what is the whirl frequency variation with respect to the spin speed.

So, I will conclude now. So, today we saw a general motion of a cantilever beam with a rigid disc. So, the motion was quite complicated because apart from that we took the complexity of the synchronous whirl that is and the whirl frequency is different and because of that we had difficulty in learning. The especially in motion and we followed the angular momentum approach to obtain the frequency approach and we found that for this particular case. For establishment of single disc case we had four whirl frequency for particular speed.

So, as we are changing the speed, the whirling frequency are changing and we try to find out at what speed these whirl frequencies are become equal to the spin speed. We could able to get three categories speed one for the forward direction and two for the backward direction. The second forward was not feasible that we observed till now, the gyroscopic motion we analyzed using the pass static analysis. In the subsequent lecture we will be analyzing this particular phenomena by a dynamic approach in which will be writing the governing equation in which time dependent. So, till now we have not considered the gyroscopic effect based on the dynamic approach.