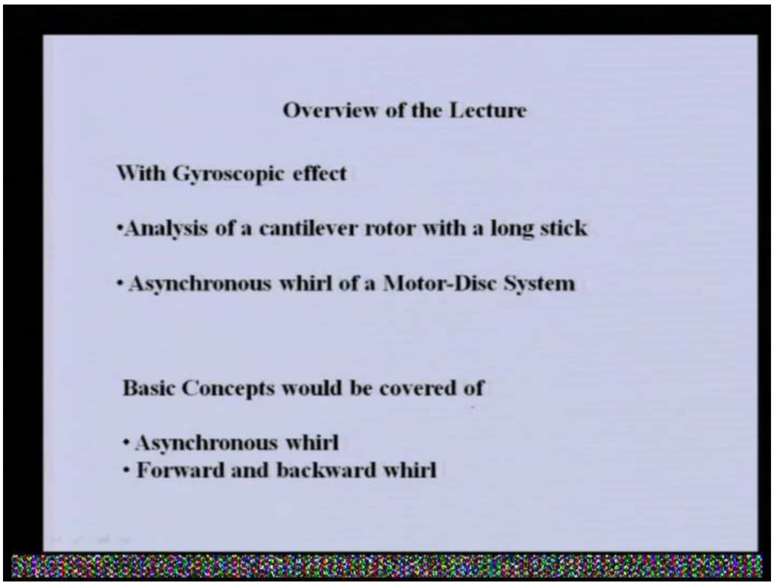
Theory and Practice of Rotor Dynamics

Prof. Dr. Rajiv Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 4
Gyroscopic Effects
Lecture - 11
Synchronous and Asynchronous pure Wobbling Motions

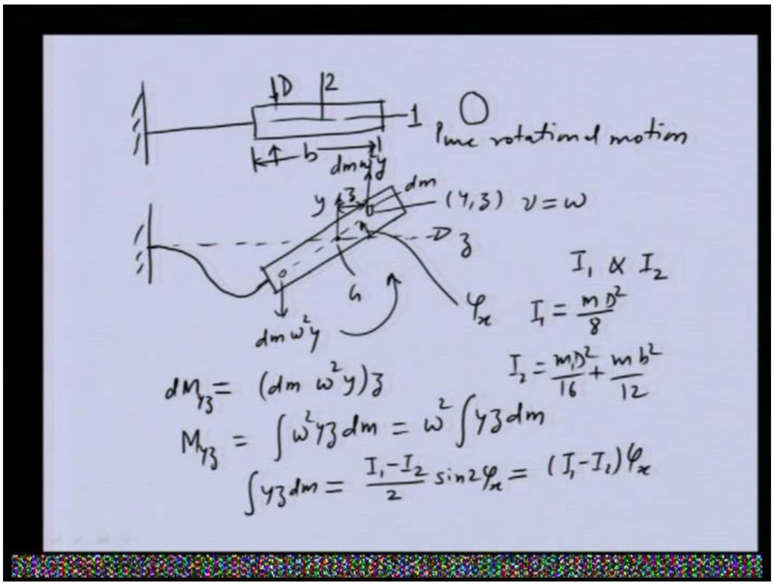
In the previous lecture, we have seen the gyroscopic effect on a thin disc. And in that particular case we considered the synchronous whirl condition; that means, we were trying to aim to find out the critical speed of the system. And we observed that the critical speed increases with the disc effect, that is the diametral mass moment of inertia of the disc is increasing, then the disc the critical speed also increases. Today, we will see another kind of rotor in which the disk will be replaced by a long stick, and we will see that in this particular case, the effect of that particular long stick will be opposite of the thin disk. That means in this particular case the gyroscopic effect will reduce the critical speed of the system.

(Refer Slide Time: 01:21)



So, this is the overview of the lecture in which basically we are dealing with the gyroscopic effect analysis of cantilever rotor with long stick. Apart from this we will see this asynchronous whirl of a motor disc assembly. In this particular case, we will try to see the whirl frequency and then from there we can able to get the critical speed of the system. And here when we will be considering the asynchronous whirl we will observe the splitting of the natural frequency of the system due to the gyroscopic couple. So, these are the concepts which we will be covering asynchronous whirl, even asynchronous whirl will be having the forward and backward whirl frequencies, and corresponding critical speeds also. So, let us start the analysis of a long stick which is attached to a cantilever shaft at the free end.

(Refer Slide Time: 02:34)



So, in this particular case; the stick is attached, this is a long stick and which is attached to a mass less shaft. And in this case we are considering the pure rotational motion of the long stick. We are not considering the translation motion at present. So, during whirling let us say this is the bearing axis which is z. So, during whirling this particular disc is tilting and its center of gravity remains at the axis of the bearing axis. So, this point is G and if you see the disc, the long stick from this end basically is having circular cross section to be more precise and this is the shaft during the whirling.

So, this will whirl this particular configuration here, also we are considering the synchronous whirl condition; that means, the whirl frequency and the spin speed is same. And we are looking into the condition, when they will be same and that will give us the critical speed of the system. In this we are not considering the any unbalance in this particular rotor. So, for this particular kind of long stick if we see a small mass D M, let us say this is a small mass D M, because; of this motion it will be having a centrifugal force D M omega square. And let us say the axis in this direction is y and the position of this mass is in let us say the co-ordinate of that is (y, z). So, that means; the centrifugal force will be D M omega square into y.

And, you can able to see that another moss symmetrically placed opposite to the, this previous mass here, this will produce a another centrifugal force. The direction is taken care of the negative side of the axis. So, now you can able to see that this 2 forces, centrifugal force are making a couple, and the couple direction is counter clockwise direction this particular figure.

So, we can able to see that the tilting of the disc which is, if we draw the axis of the disc, this is the tilting that is tilting is phi x about x-axis. So, that tilting is becoming more because of this particular movement. So, it is opposite to the previous case in the thin disc, the this kind of moment due to the centrifugal force was trying to reduce this angular displacement, but; in this particular case for long stick this moment is trying to tilt more and more towards the angular displacement direction.

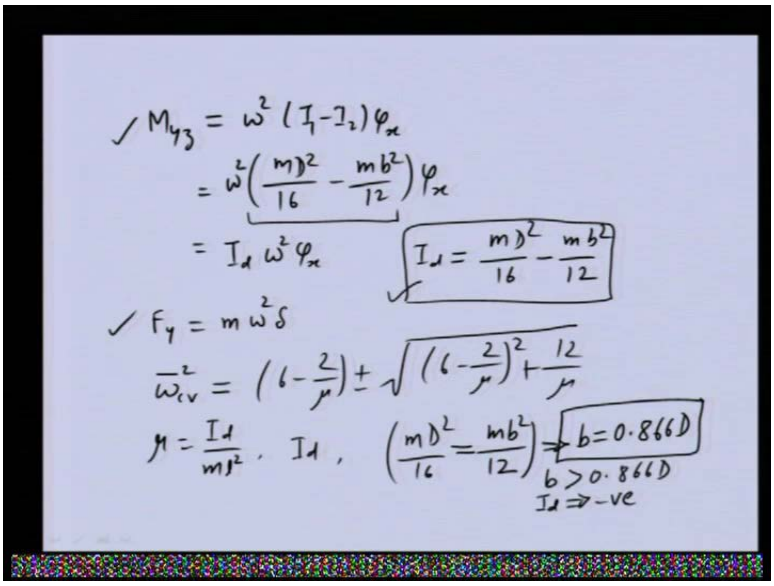
So, that means; effective stiffness of the system is getting reduced, because; more displacement means less stiffness. And we can expect there will be decrease in the critical speed also, because critical speed is defined as root k by n for this particular case effective stiffness by M, because; stiffness is decreasing. So, we expect the critical speed will decrease for this particular case. So, now let us analyze this situation. So, in this particular case; let us say the moment in this plane, y z plane due to one of the small mass D M will be given as the force which is D M omega square y.

And, the moment arm of that about the this. So, this distance is basically z, because the co-ordinate of that particular point is y z. So, this will be the moment produced by this centrifugal force of mass D M. Now, if we want the moment of all such particles of the long stick we can able to integrate this and we would get this expression; omega square we can able to take common. Now, this particular quantity within the integral is we can able to express that that is a product mass moment of inertia, and that can be obtained for this particular long stick.

So, let us see for this long stick, along the direction of the, this along the direction of the axis of the cylinder let us it is a axis 1 and perpendicular to this is axis 2. And let us say I 1 and I 2 are the mass moment of inertia of this long stick along the 2 principle axis direction that is 1 and 2. So, in this particular case the second figure that principle axis direction will be inclined, because now this has tilted by phi x direction. Now, we want this product of product moment of inertia, this will be defined similar to the shear stress concept in the strength of material, that is sin 2 phi x.

And, because this angle displacement is small, we can approximate sin 2 phi x as 2 phi x and this will get simplified to I 1 minus I 2 phi x. For cylindrical shape; this I 1 is given as M D square by 8, where D is the diameter of this particular cylinder. D is the diameter is the diameter and b is the total length of the cylinder. So, and I 2 is given as M D square by 16 plus m b square by 12, where b is the length of the stick and d is the diameter. So, once we have obtained the moment which is coming from the various particles of the long stick now, we can able to substitute the moment of inertia. Now, you can able to substitute the moment of inertia in 2 principle direction to get the moment.

(Refer Slide Time: 10:57)



So, the moment we can able to write it as once, we are substituting the these 2 inertia moment of inertia which we defined earlier, we will get m D square by 16 minus m b square by 12. So, I have simplified the expression after substituting the I 1 and I 2. So, we can able to see this, we can able to write it as, let us say I am expressing this term within the bracket as I d omega square phi x. So, I d the equivalent I d for long stick is this much, because if you remember for thin disc is gyroscopic moment we got exactly same expression, I d omega square phi x, but; I d was different for that. But now, we are defining the I d like this.

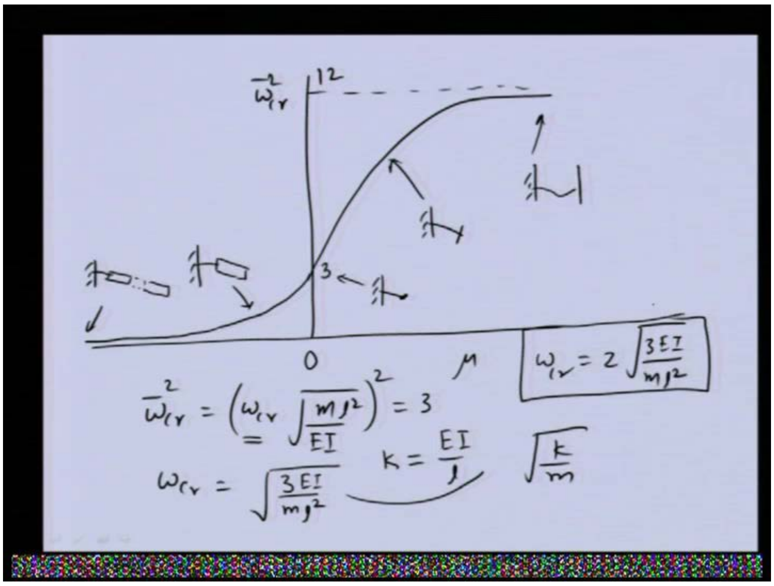
So, as such this particular gyroscopic moment is identical to the previous analysis, previous case. In this particular case; while deriving the, this gyroscopic moment we considered that the center of gravity of the long stick is at the bearing axis. So, as such there is no net centrifugal force acting which was there for the thin disc is, but; now once we have obtained the gyroscopic moment, if we considered the translatory motion also. So, as for the thin disc is will be having the gyroscopic moment, gyroscopic force F y as m omega square into y, because in this particular case sorry, this will be delta. Delta is the linear for translatory motion of the disc the long stick.

So, as such these 2 expressions are similar, as for the thin disc. So, that means; whatever the analysis we did in the previous case is valid here also. That means, the natural frequency which we, the critical speed which we obtained the thin disc is equally valid here. So, this was the expression we derived in the previous lecture. So, this is still valid here, but only modification will be the definition of the mu. Here, mu is similar to the previous one, but I d is now, defined like this. So, this is the difference. In fact, this particular expression is more general. So, it define, not only for thin disc also for the long stick depending upon the relative value of the D, the diameter of the stick.

Now, you can able to see that in this particular case; this gyroscopic moment can become 0. If we have some condition like, if we have m D square by 16 of is equal to m b square by 12. If we have this condition of the long stick then we will see that I d of that particular system will be 0. So, there will not be any gyroscopic moment. So, you can able to see that the gyroscopic moment becomes 0, if we are having this condition. And if you simplify this, we will get this as b is equal to 0.866 D, this is the length of the long stick, and this is the diameter. So, this condition is there we will be having the 0 gyroscopic moment.

And, if we have b greater than 0.866 D we will see that this quantity will be more than this. So, I d will be negative, for this particular case I d will be negative. So, in this particular case; we have seen that when there is b which is length of the long stick is greater than 0.66 into diameter of the, this particular stick. The mu becomes that is I d become negative and because of that mu the disc effect also will be negative.

(Refer Slide Time: 16:22)



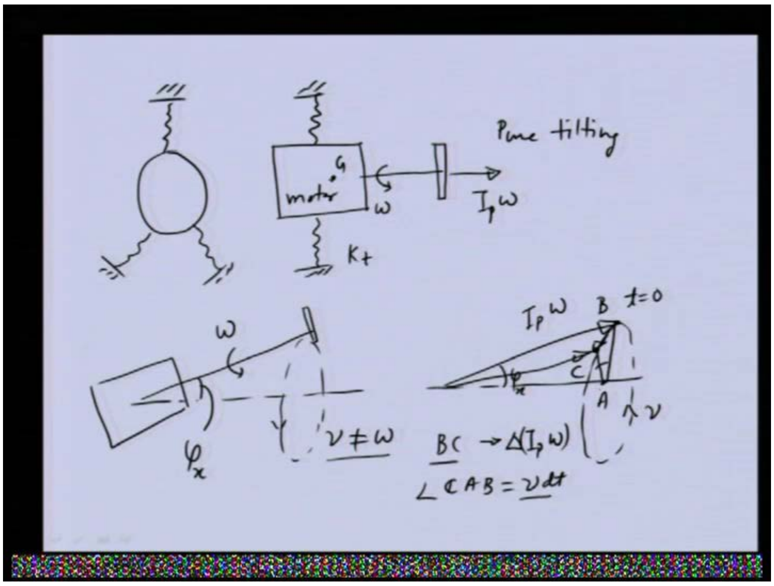
So, now we will be plotting the similar graph as we drew for the thin disc case earlier, because; the critical speed square and mu and in this case because mu can be negative also. So, we have 2 axes now, you can able to see in the previous. So, we are plotting this particular expression with respect to mu, and mu we are varying not only from positive direction, but; also negative direction. So, earlier we had a curve something like this, it was starting with value 3 and asymptotically it was going towards the value of 12, for omega critical square. Now, if we take the negative value of the mu, we will get a curve which will be asymptotically it will be going towards the 0, omega square critical. So, that means; if we recall this was a point mass, this was a thin disc space, and this was in case in which we had the very large this diametric mass moment of inertia of the disc.

Here, this will correspond to a long stick, and when we are approaching towards 0 natural frequency, we will see that basically this stick is very long. And for a small disturbance also this is having large displacement, because; of gyroscopic couple is acting towards the angular displacement of the rotor. So, the natural frequency becomes 0 for this particular case. So, you can able to see that whatever we derived now, this equation represent not only for thin disc case also for the long stick case. Now, we will see from the non dimensional critical speed parameter how we can able to get the critical speed.

So, earlier we defined the non dimensional critical speed parameter like this. This is critical speed and m l square that is the length of the shaft. So, we can able to get like, if we want the critical speed at this position. So, we have the square of this and at this position for point mass we have is equal to 3. So, from this we can able to get the critical speed. Critical speed will be 3 E I by m l square. And if you see the stiffness of the cantilever beam for point mass is basically E I by l. So, basically this is representing as k by m for point mass. For this particular case; we got the value of this is 12. So, if we use here, we will get the critical speed.

So, instead of 3 if we use 12 we will critical speed as the twice of the critical speed, which we obtained with point mass. So, we can able to see that the increase in the critical speed due to the disc effect could be as high as twice. In the previous 2 cases we considered a synchronous condition; that means, we are trying to find out at what speed the whirl frequency will be equal to the spin speed. So, that means; we were trying to find out the critical speed directly. So, now we will relax that condition and now, we will be considering the whirl frequency which is not equal to the spin speed; that means, we will try to obtain the natural frequency of the system. And in this particular case; we will see that the natural frequency of the system depends upon the spin speed of the shaft.

(Refer Slide Time: 21:18)



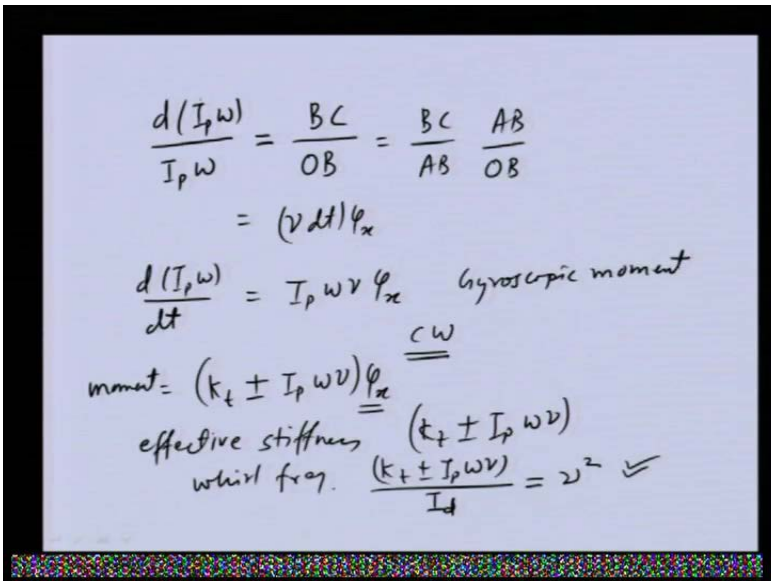
For this particular case; we will let us say there is a motor, and there is a rigid shaft, and there is a rigid disc attached to this. So, this whole assembly motor, the shaft and the disc is we are assuming as a single unit and there the rigid body. So, they do not have relative motion among themselves. This is supported on some flexible support, if we see from side this supported on springs. So, that the effective stiffness of the spring against the tilting of the whole assembly. Let us say is K t. So, in this particular case we are considering the pure rotational tilting motion of the motor shaft and disc.

So, pure tilting we are considering; that means, the center of this whole assembly is if this is the center of gravity is tilting about its center of gravity. So, there is no translatory motion only tilting motion is there, in this particular case we can able to see that when this shaft is spinning which is an high speed. There is a wobbling of the motor shaft assembly, and that will be making a conical shape during the whirling. So, if I draw this is the bearing axis, let us say this is the disc and this motor. So, during whirling it is going in this direction, the whirling frequency is mu, which is not equal to omega. Spin speed is the omega.

So, in this particular case we can expect that the shaft and the disc is rotating at relatively high speed, but; whirling will be taking place slowly. So, it will not be that much fast. So, that is why these 2 are not same. Now, if we consider the angular momentum of the, this particular disc. So, this will be I p polar mass moment of inertia of the disc into the spin speed. If we look into this particular angular momentum vector here, so let us say this is the angular momentum; I p omega, this is the motion of the disc. So, this angular momentum will be moving around with the disc the whirling direction. So, let us say whirling direction is counter clockwise, when we are looking from the right side. This is the frequency of the whirl. So, and tilting of this we are giving as a phi x angle. So, this angle is phi x.

So, let us say at the f x this disc is there at time t is 0 where, after sometime it is reaching let us say this point is B. So, it will reach some where here, because whirling direction is counter clockwise direction. After some time t, let us say this point is C this point is C. And the angular momentum will occupy this position. So, B to C is the change in angular momentum, this B to C is like in the change in the angular momentum. Now, this particular angle which is collecting the center of rotation of the disc these all the bearing axis let us say point A. So, this angle there is C A B that will be nothing but ; nu into delta t time. So, delta t time because this angle is the, because the frequency of whirl is nu and time taken from moving from B to C is delta t. So, this will be the angle nu d t this B A C will be the angle this much.

(Refer Slide Time: 26:30)



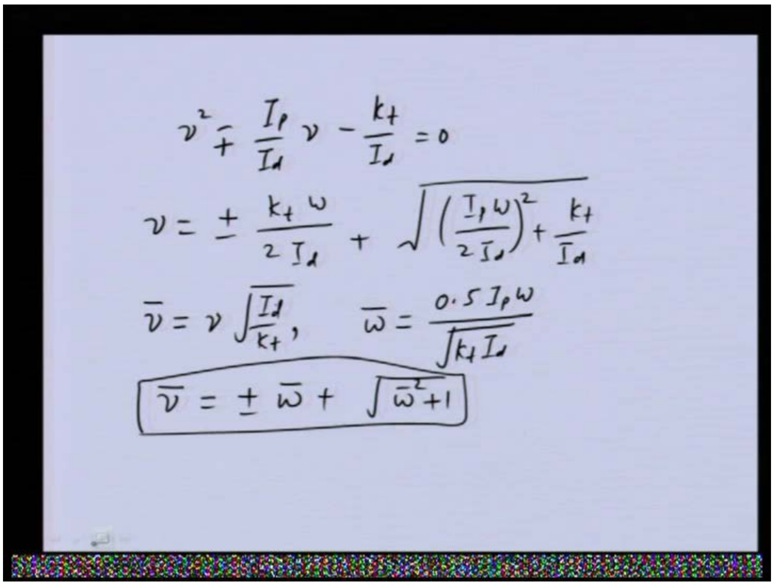
Let us say this change in the angular momentum; we are writing with respect to the angular momentum. So, basically in the previous figure change in the angular momentum was B C and the angular momentum was O B. So, O B let us go to the figure O is this one. So, from here to B is the angular momentum and B C is the change in angular momentum. So, this can be written as B C by A B and A B by O B.

Now, B C by A B is in the previous figure B C by A B is nothing but this particular angle nu d t. So, this is nu d t, and A B by O B in previous figure, A B by O B is the phi x angle. So, this expression we can able to write as d change in angular with respect to time is equal to I p omega nu phi x. So, rate of change of angular momentum is nothing but the moment, gyroscopic moment. So, you can able to see this is the gyroscopic moment which is producing, because of the whirling and spinning of the shaft. So, this is the gyroscopic moment. And the direction of this will be in the direction of the B C. So, B C direction is that is outside the screen, vector will be towards the outside the screen B to C.

So, this particular gyroscopic moment will the disc will experience from the motor frame. So, the reaction of that all to the motor frame will be opposite; that means, in this particular figure, it will be from C to B that will be into the screen direction. So, according to the right hand rule; it will be, if we see into the screen that will be clockwise direction. So, on to the motor frame it will be acting in clockwise direction. So, if we see the, this particular figure when moment is acting in the clockwise direction, this figure that moment will try to reduce this particular angle. That means, it is trying to reduce the tilting of the, this particular motor shaft assembly, and because of that basically the effective stiffness is increasing.

Apart from this particular moment the moment due to the support and the support the effective stiffness is K t that will also be acting opposite to the angular displacement. So, we will be having basically couple moment not only from the support screen, but also from the gyroscopic moment is this much. So, the stiffness into the angular displacement will give the support moment and this is the gyroscopic moment. Now, this is the moment so the stiffness will be moment divided by this angle. So, that means; the effective stiffness on the system will be K t plus minus I p omega square nu. And the whirl frequency will be the effective stiffness this particular stiffness k t plus minus I p omega nu divided by I d, because the tilting is about diameter. So, the stiffness divided by the diametral mass moment of inertia and that is actually the whirl frequency square. So, we know that the natural frequency square is stiffness by mass.

(Refer Slide Time: 31:48)

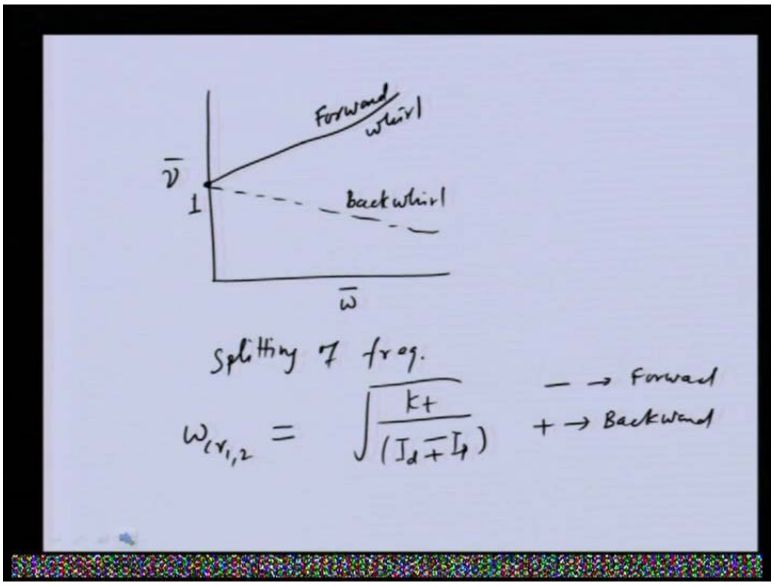


So, this particular equation will give us a quadratic equation in terms of nu, and this can be solved in close form. So, this will give us the whirl frequency expression plus k t by I

d. So, this is the whirl frequency equation, and we will see that this whirl frequency which is less in frequency of the system now, depends upon the spin speed of the shaft. And in this particular case basically 4 roots are visible, because; there are plus minus signs. But if you take the negative outside the square root we will get 2 frequencies which both will be negative. Because negative frequency is having no meaning so we can able to ignore that. And basically those 2 negative values of the frequency exactly same value we will be getting when we are considering the positive sign.

So, basically the negative sign we are omitting here. So, now we have this equation and this can be simplified by defining non dimensional parameter. Let us say nu bar I am defining as this whirl frequency ratio as I d by k t, and spin speed to ratio, I am defining as k t into I d. Now, with these 2 non dimensional terms, if we use in this equation, this equation will get simplified to omega bar plus, omega bar square plus one. So, this is a simple equation now, this we can able to plot.

(Refer Slide Time: 34:25)



So, this equation I am plotting it, with respect to nu and the spin speed ratio. So, at omega is equal to 0, we expect there will be a single value. If we put omega is equal to 0 basically, we are getting one only. So, that is this point one. And as will increase the omega bar, we will find that there will be 2 values; one will be increasing in this direction and another will be decreasing. So, this is basically the spitting of the natural frequency is taking place, and this particular frequency is corresponding to the forward whirl; that means, the whirl direction will be same as the spin speed. This is the backward whirl in which the frequency of whirl will be opposite to the spin speed. And if we want the critical speed; obviously, in this particular expression critical speed for that we need to put nu is equal to omega bar that is the critical speed condition. Sorry, this conditions not the bars because we have defined these 2 differently.

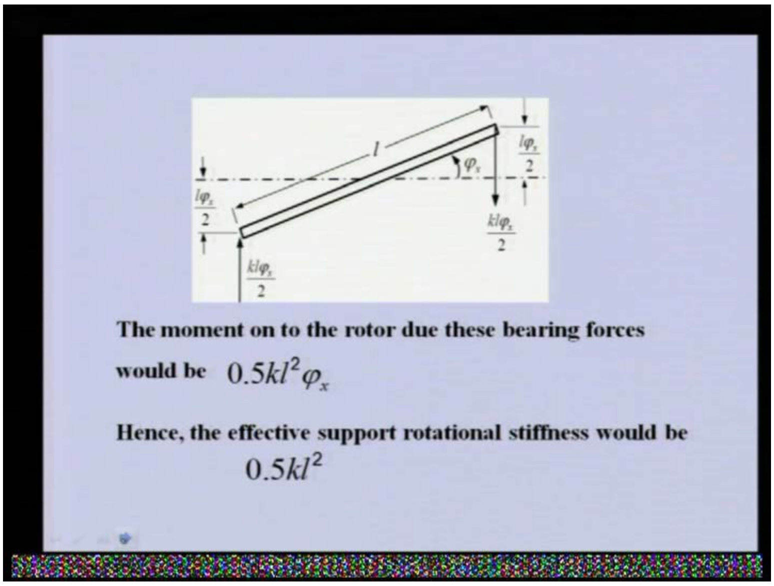
So, we need to go to this expression, and we need to put nu is equal to omega, and we need to solve for omega that will be omega critical. Where, nu is equal to omega here, we will able to get the critical speed. We will be getting 2 values of this first and second and the expression will be k t divided by I d minus plus I p. So, this minus is corresponding to the forward whirl, and plus is corresponding to the backward whirl. Because when the denominator is minus; obviously, this quantity will be smaller, and we will be getting the higher critical speed that will be corresponding to the forward whirl natural frequency. And when it is positive we will be getting denominator higher as compared to previous one. So, we will be getting critical speed lesser that will be corresponding to the backward whirl critical speed.

So, we have seen that how the whirl frequency depends upon the spin speed of the shaft when, we are considering the asynchronous whirl. Now, through one example; we will see how a rigid rotor which is supported on a spring can have gyroscopic effect and how the whirl frequency changes for that case. So, we have a long symmetric rotor which is supported at ends by 2 identical bearings, and these bearings will be modeling by only direct stiffness terms. The shaft is having diameter of 0.2 meter, length 1 meter, and material properties are given here we have to get at the damping in this particular case.

We are considering pure tilting motion of the, this particular rotor and gyroscopic couple we are including in this. So, we want the whirl frequency at one particular r p m and also we can able to calculate the critical speed. Because whirl frequency is changing with speed and we need to track at what speed this frequencies are equal and that is nothing but the critical speed. So, for given parameter, we can able to calculate the rotor polar moment of inertia, its diametral, moment of inertia and this is the spin speed at which we are interested to obtain the whirl frequency. So, in this particular case; we have considered the pure tilting motion of the, this particular shaft.

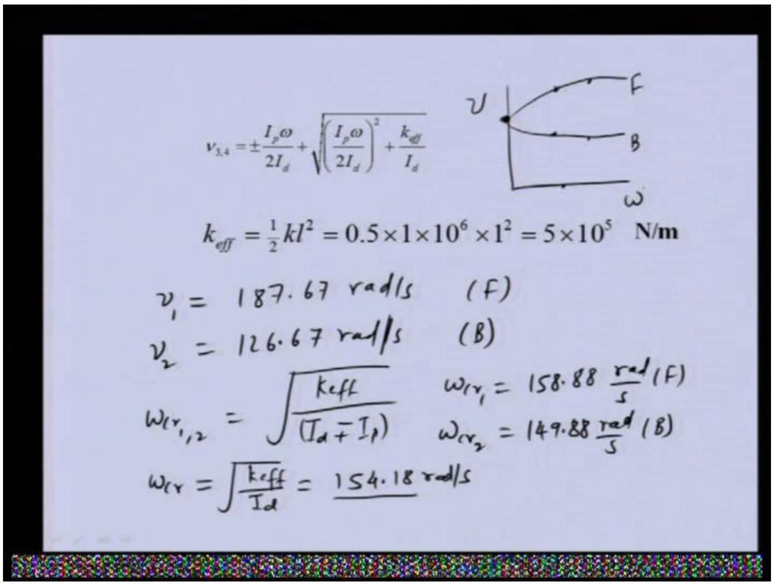
Now, because this tilting about its center of gravity, the tilting let us say we are considering in one of the plane phi x. And because of this the extension of the spring which are here, will be this much, because; this is the half l and this angle. So, this will be the extension of the spring here. So, this will exert this particular force from the spring in the downward direction and this is getting same amount of completion. So, here we will be having a spring force which will be upward. Now, these forces are making coupled.

(Refer Slide Time: 40:38)



So, basically they are providing a couple and these are moment and that moment will be nothing but ; this force value into the length of the shaft. So, this is the moment which is acting. And if you divide this moment by the angular displacement, we will get the effective stiffness of the support or the rotor stiffness of the support, moment divided by angular displacement will give us the effective support stiffness that is the rotation.

(Refer Slide Time: 41:10)



And the expression which we derive for present case; the motor shaft disassembly this particular case is identical to that one. So, whatever the, we had expressions for the whirl frequency will be valid here, and in this particular case; if we obtain the effective stiffness k t will be this much. If we substitute the previous moment of inertia and spin speed and this effective stiffness in this expression, we will get the forward whirl frequency as 187.67 radians per second. This forward whirl frequency and backward whirl frequency which will be less than this, radians per second 126.67 radians per second, this is backward whirl frequency. So, you can able to see that as the spin speed was 1047 radians per second, but; whirl frequencies are these. So, obviously; we are well about the critical speed.

If we want the critical speeds; we can able to use the expression which we derived earlier that is effective stiffness divided by I d plus minus plus I p. So, this will give us the 2 critical speeds corresponding to forward whirl, we will be getting critical speed is 158.88 radians per second. And second one is 149.88. So, this is corresponding to forward whirl critical speed, and this is backward whirl frequency. Now, we want to compare the critical speed of the system when there is no gyroscopic couple. So, for that particular case we will be having the critical speed of the natural frequency. In that particular case; will be same as the critical speed will be k effective divided by I d. So, if we substitute the value in this, we will get 154.18 radians per second.

So, we expect this to be in between the forward whirl and backward whirl, because as we have seen in the previous case, when we varied the frequency with the spin speed at 0 speed when there is no gyroscopic couple, this was having single value. So, that is corresponding to this one and as we are increasing the speed let us say; here we are getting 2 values. Similarly, we can able to get for other speeds another 2 values; if it is random we will get the whirl frequency variation for forward and backward whirl with respect to spin speed.

So, in the present lecture, we have considered 2 cases; one was the pure rotation of the long stick, and another case we consider the whole assembly of the motor and shaft disk for pure rotation. And in this particular case, we considered the asynchronous whirl; that means, we considered the whirling frequency and the spin speed different. And we found that because of this the whirl frequency is getting split into and correspondingly we have a forward whirl, and backward whirl. And in this particular case corresponding critical speeds also, we had 2 in numbers corresponding to the forward critical speed, and backward critical speed. So, in this particular lecture we have seen one interesting phenomena; the splitting of the whirl frequency and the forward whirl and backward whirl of the this critical speeds.