Theory and Practice of Rotor Dynamics
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Module - 4
Gyroscopic Effects
Lecture - 10
Synchronous Whirl of a Rotor System with a Thin Disc

Previous lecture we have seen some simple rotor model in which we considered the flexibility of the bearing. Specially, when we considered the hydro dynamic, specially when we considered the hydro dynamic bearing model there is eight linearized coefficient of the stiffness and damping. We showed that those coefficients, we can able to calculate at equilibrium positions and that equilibrium position changes with speed. So, these coefficients in actual practice they will vary with speed, so whatever the analysis we did earlier that can considered the speed dependency of such stiffness and damping coefficients of the bearings.

Today, we will study effect of gyroscopic couple on the natural frequency or the critical speed of the system. This gyroscopic effect generally occurs when the disc is spinning at high speed and it is wobbling. That means when simultaneously tilting of the disc is taking place during the whirling motion and because of this particular gyroscopic effect we will see that even the critical the natural frequency the whirl natural frequency will now no longer will be equal to the spin speed. So, if we are considering a perfectly balance rotor and if we are operating at certain speed the whirl frequency may not be equal to the spin speed.

So, for this particular case we will see that as we are changing the speed the gyroscopic moment changes and that changes the whirl frequency. So, the whirl frequency basically, now it will depend up on the speed of the rotor itself. Two things we will observe specially regarding the natural frequency that in this particular case when gyroscopic effect is there, the splitting of the natural frequency will observed. These particular phenomena, we will see in the subsequent lecture. Today, we will concentrate on a synchronous whirl condition in which we will try to calculate the critical speed rather than obtaining the whirl frequency.

So, with this particular module of the gyroscopic couple now the definition of the whirl frequency, critical speed they will be quite distinct. Then we will observe that we will be having phenomena of forward whirl and backward whirl also because of the splitting of the natural frequency. Corresponding we will be having critical speeds also which will be having forward whirl and backward whirl in which case the spin direction or the spin sense of rotation and the whirl sense of rotation may be same or different.

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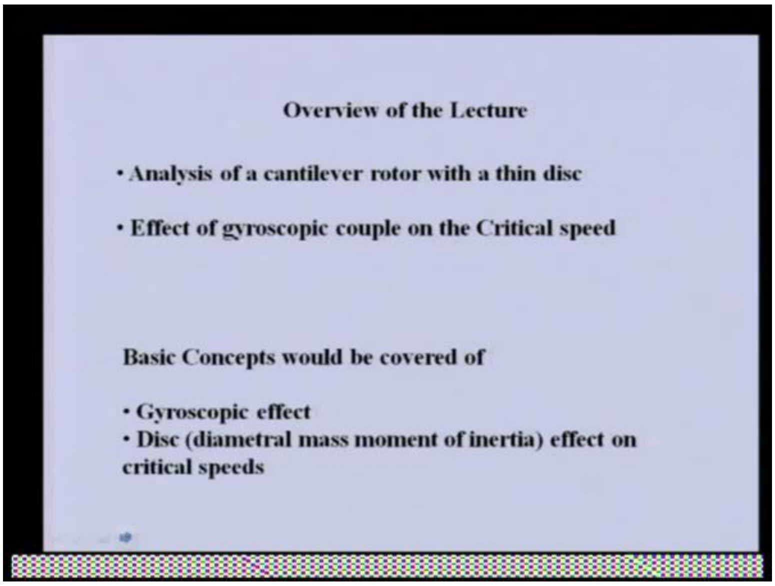

So, for this particular case, this is the overview of the lecture in which we will be doing the analysis of a simple cantilever rotor within a thin disc at the end. We will not consider the mass of the shaft. Already the elasticity of the shaft will be considered and disc will be considering as a rigid, it is having no working of the disc in this particular analysis. In this we will see the gyroscopic couple effect onto the critical speed. Basically, we will be calculating what is the critical speed, because of this gyroscopic couple is… Some concepts which we will be covering in this lecture is the gyroscopic effect disc effect or the diametral mass moment of inertia, how it changes in the critical speed of the rotor system? Now, with some simple example let us see when this gyroscopic couple will be predominant.

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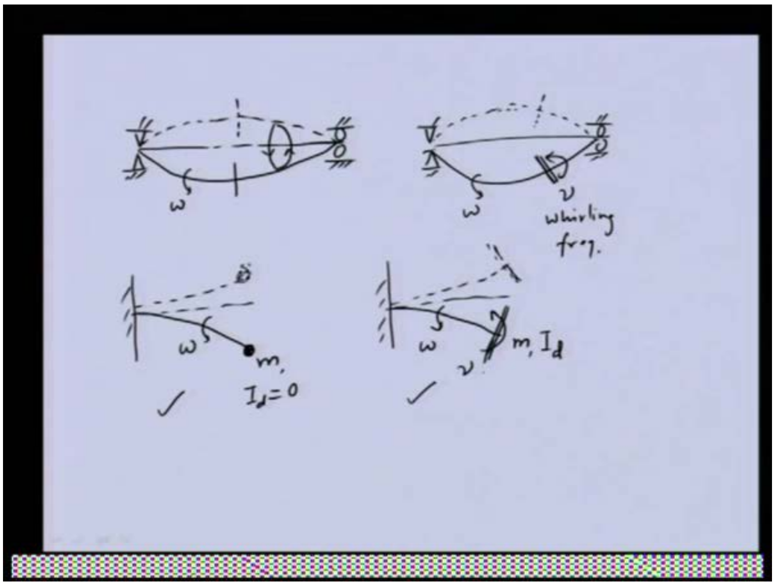

Let us take a simply supported case in which this is the bearing axis and we have a rigid bearing that is simply supported condition and this is the shaft elastic line. Let us say disc is at the center of the mid span of the shaft. So, during whirling we will observe the disc would not tilt. It is spinning about its own axis with omega, but disc tilting will not take place because it is there at their mid span and at mid span the shaft slope is 0.

So, during whirling of this, there will not be any tilting of the disc would take place. So, once this disc is not tilting about it diameter there will not be any gyroscopic couple effect on to this particular system. But if the same system is there with the disc at offset position, so let us say here in this particular case because the disc at any particular position should always be perpendicular to the shaft elastic line.

So, we will find that the disc will tilting about its one of the diameter in this particular plane, so not only we are having spinning of the shaft, but the disc itself is having some kind of precision frequency. That is the whirling frequency of the disc. So, this wobbling motion of the disc along with the spinning of the shaft would give the gyroscopic couple, so we expect this gyroscopic couple effect will be present in this particular model.

Another case if we consider, let us say a cantilever shaft and this shaft is mass less, only elasticity is there. Let us say disc is in the form of a point mass m. If this is spinning at certain frequency during whirling we expect this will occupy this position, so this is one of the cases. If this particular cantilever beam is having a large thin disc of the same mass, but having diametral mass moment of inertia of the disc in this particular because this is point mass the material mass moment of inertia will be nearly 0. When this is spinning we will find that during whirling this disc would be tilting about its diameter. So, in this particular case we expect a gyroscopic, so you can able to see there will be precision frequency of disc would be there.

So, if we consider this particular model of the rotor and in this in which the mass at the free end are same, but only difference is here its point mass here it is disc. So, we expect the critical speed of this system and this system should not be same. Especially when this particular speed is high we will find that the gyroscopic couple will make the critical speed of the system and this system is different.

So, with this example it is very clear that when the gyroscopic couple will be present in the system and when it will not be present. So, when point mass is there in a cantilever beam we cannot expect a gyroscopic couple in that because its diametral mass momentum of inertia is 0 now. Now, we will consider a simple cantilever beam we will not be considering any unbalance of that in that particular disc.

We are assuming that there is no unbalance in the disc and we are operating near the critical speed of the system, so that the natural frequency or the whirl frequency is equal to the spin speed as I mentioned. When gyroscopic couple is there the whirl frequency and the spin speed may not be same. In this particular case we are considering a very special case in which we are trying to find out at what whirl frequency the spin speed at what speed it is the whirl frequency is equal to the spin speed. So, that means we are trying to find out directly the critical speed of the system and in this particular case let us try to analyze a cantilever beam with a large disc at the free end.

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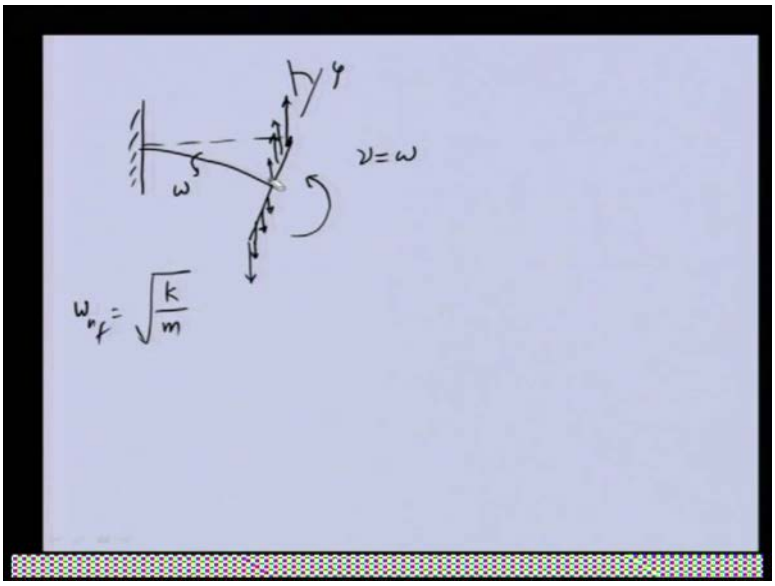

So, if this is the disc this shaft is spinning and disc is also spinning with the same speed and because of this we will see that for this particular configuration. Let us say when the shaft is at the bottom most position during whirling, if you try to see for this particular instant how the centrifugal force of various masses of the discs are acting. For this particular instant we will see that the mass element which is away from the spins axis will be having large centrifugal force because this is spinning about this. As we come toward the spin axis, the centrifugal forces will be lesser and lesser like this. Now, if you see the effect of this centrifugal force what they are trying to do to the disc? They are trying to apply a couple which is acting in counter clockwise direction.

So, it is trying to orient the disc to its original vertical position because the tilt of the disc during whirling is this angle and this particular moment is trying to oppose that tilting which is coming from the centrifugal force. So, we will see that because of this moment, effective stiffness of the shaft against the rotation is becoming higher because this particular moment is trying to prevent the, it is making this particular angular displacement lesser and lesser because of the this moment. You can expect that when the centrifugal force is high at if we are increasing the speed as we are increasing the speed this will be more and more and we expect this moment will be more and because of that effective stiffness of the system increases.

As we know from fundamental that their natural frequency is defined as root square k by m, so if stiffness is increasing mass is same. So, we expect that the natural frequency of the system should be increasing. So, we will find that in this particular case when we are rotating the rotor at certain speed. We are trying to find out the condition for at what speed the whirl frequency is equal to spin speed for a balanced rotor. The effective stiffness is increasing and because of that the natural frequency is also increasing.

So, this particular phenomena which I have illustrated through the centrifugal force concept is we will see in more detail through the concept of the quasi static analysis. We will be finding out what is the effective the force and moment this particular centrifugal force are applying at the free end of the shaft. From there we will try to get the frequency equation and calculate the critical speed of the system. Now, we will analyze how the whirl frequency of the system increases with the spring speed of the shaft because of the centrifugal force.

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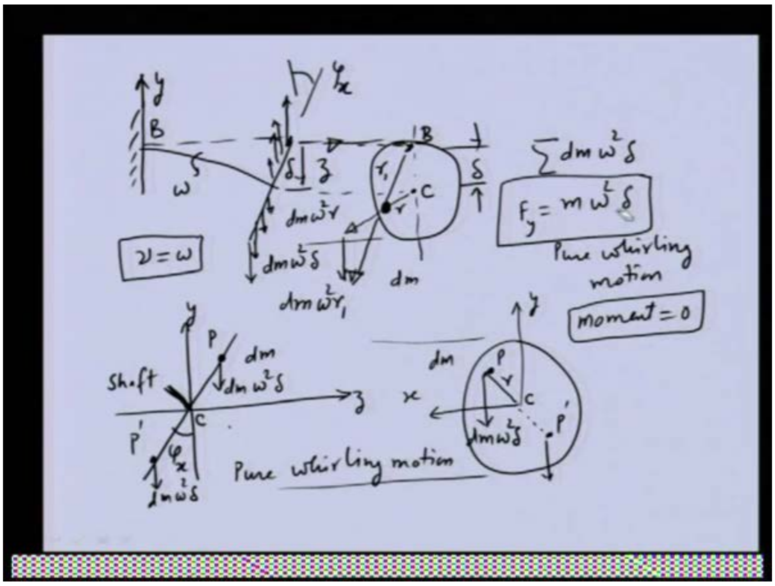

Let us for that particular case, we will take the same example and now we will see this particular disc from side how this particular system is giving us the. So, let us say this is bearing axis B to and this particular axis bearing axis we are representing as z this angle is or let us say this direction is y. So, this angle we will represent as phi x this is the static deflection let us say delta of the centre of the shaft on the disc. So, here let us say the intersection of this axis to the disc is I am writing as B. This is the shaft centre which is coming from here and this B to C is distance that is translating displacement of the shaft from its equilibrium position of the bearing axis.

Now, if we consider a small mass let us say this is at reduce r 1 from the bearing axis. So, if we see the motion of this particular disc and as I told we are looking into the condition of synchronous whirl. So, let us see this particular motion what it represent, so in the synchronous whirl condition the spin speed that is the spinning of the disc and the whirling frequency. Let us say this is the cantilever beam, so this whirling frequency and the spin should be same now when we are considering this synchronous whirl condition.

So, what will happen for this particular case the shaft will bend? Once it is bent, it will be rotating about bearing axis in the rigid body mount and without bending of the shaft. So, bending of the shaft if you see, the fibers in this particular shaft which is in tension will be always be in tension during the whirling and which is compression will always be in compression during whirling for the synchronous whirl condition. So, it will be as a rigid body, it will be rotating about the bearing axis.

So, that is the synchronous whirl condition and in this particular case we see that each and every particle on to the shaft will be making a, if shaft is symmetry again circular path. It will be making a circular path about the bearing axis. Also on the disc, if you take any element or the mass that also will be making a circular path may be the radius of the circular path will be different for each of them. So, in this particular figure the r 1 which I have shown here, so this particular disc during the synchronous whirl will be making a circle with r 1 as a radius and bearing axis as the centre. So, it will be making a circular motion, so we can expect that because of this the centrifugal force which will be acting on this mass.

Let us say this mass is small d m, so this will be along the direction of the r 1 only. So, let us say this is centrifugal force and d m omega square r 1; this is acting in along the direction of r 1, now this particular motion of the whirling again. Let us see this particular whirling motion, so this whirling motion, which I told for synchronous whirl is taking as a rigid body about the bearing axis, so and it is spinning also. Both are taking place because of that synchronous whirl is taking place.

If we consider this motion in two parts, one is pure spinning at omega RPM or radians per second and then without rotation only whirling, so basically in actual case both spinning and whirling take place simultaneously. But now, I am considering spinning separately and whirling separately, so in this particular case the disc is not spinning only it is whirling. So, in this particular case we will see that the two motions which was spinning and whirling is simultaneously which was taking place beyond split in two parts, so when we are considering the spinning.

So, we can able to see that this particular mass for that particular rotation when we are considering pure spinning, it is spinning about the centre of the shaft, so that means we can able to take the centrifugal force because of this. Let us say radius of the disc is r, this will be d m omega square r and another case is this one, the pure whirling. In that particular case we expect this particular mass and this particular configuration will be again.

If I can able to see here this when pure whirling is taking place, so when disc is coming from let us say top to bottom. So, when it has reached at the bottom the direction of the centrifugal force of that mass due to this whirling will be toward the downward direction. It is coming, so at this position the centrifugal force should be downward direction that means vertically downward. So, this component is that particular direction and this will be d m omega square delta.

Now, when the disc has reached at the bottom, each and every particle of the shaft is having delta displacement. So, we will be having this particular force centrifugal force, so you can able to see that the total displacement which is total centrifugal force which is d m omega square r 1. Summation of these two components due to pure spinning and pure whirling this particular forces which now we have got now we will take the forces component of this in two planes.

We will try to find out because of these forces what are the resultant force and moment which is actually coming on to the rotor system for which I am drawing, let us say in one of the plane. Let us say this is z axis, bearing axis and y vertical axis and disc is this one. So, this is the disc already I am showing it and let us say there is a particle here having mass d m. Let us say that point is P and this is the shaft which is attached to the disc. This is the shaft now, this is the centre of the disc, C here tilt is about x axis of phi x. Now, this is the case for a pure whirling case, pure whirling motion, there is no spinning.

So, if you see from side of the spin of the disc let us say this is y axis and this is x axis. Particle P is here is at a radius r and this is the centre of the disc, now if you see this particular force which is acting downward direction. So, for this particular configuration not only this mass, each and every mass will be having same inertia force, because each and every particle of the disc is having delta displacement.

So, this particular mass d m which is there at p will be having a force m omega square or d m omega square delta. So, each and every particle will be having same force. This particular force on this plane is in this direction, so this is d m omega square delta. If we considered another point vertically opposite to this here P prime, this will also be having same direction force same magnitude, because that particle is d n here P prime. If we consider it will be having downward direction d n omega square delta.

Now, you can able to see that if we consider all the masses in the system they will add up and they are not cancelling each other. Basically, all forces are downward, so if we add all such forces, this is for the elemental mass. If we add for all such masses of the system will get in net force and the direction of that force will be in the y direction. So, this is the net force which will be acting due to the pure modeling case pure modeling motion regarding the in this particular case there are adding up to give the force.

In this, what about the moment you can able to see this particular particle and this particular will produce about the center moment, which will cancel each other because they are opposite to each other, So there will not be any net moment due to this or here also, if we try to take moment about this point this two will cancel each other. So, the net moment due to the whirl motion will be 0, so moment will not be there due to the whirling motion. Only this particular total mass of the disk in to omega square delta force will be acting on this particular disk. Now, let us see the pure rolling motion, pure spinning motion, now will consider the pure spinning motion of the disk.

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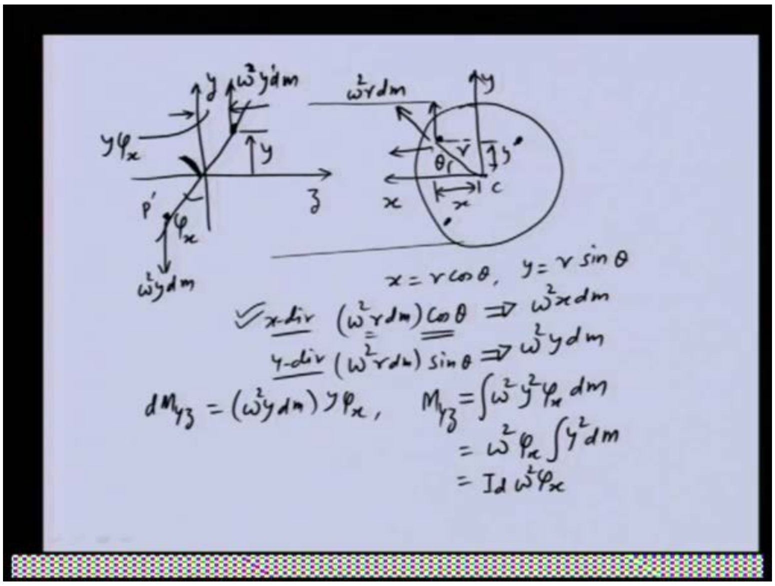

So, in this particular case I am again drawing the free body diagram of the disk in, let us say z y plane. z is the bearing axis, disk is here, shaft is this one. Now, if we take the diagram in other plane will see the disk like, this is center of the disk C, these are the y axis and x axis. P particle is let us say here and radius of that is r. Let us say this inclination with respect to axis theta here. Because of the pure spinning we have centrifugal force omega square or d m, which is acting readily out ward.

This particular force we can able to take component in two directions, this particular form here we can able to relate let us say this distance is y and this distance position from the position of the mass is x. So, we know that x is equal to r cos theta and y is equal to r sin theta, so the component which is there in the horizontal direction here will be having value omega square r d m cos theta, this will be acting in the x direction. If we combine the r in cos theta this can be written as omega x d m.

Similarly, in the y direction force component will give us omega square r d m sin theta that will give us omega square y d m because half n sin theta is y. Now, this particular force in this plane if we want to see the y component is acting in the direction of y x. So, if the component is acting in the direction of x axis, so if a particle is here the force one is there in this direction. The magnitude is omega square y d m other force the x direction, x direction is that is perpendicular to the screen because according to the right hand rule I will be inside the screen, x will be inside the screen.

So, this particular force component which is in x direction will be in the plane of the disk, so we cannot able to see that particular force here that will be on the plane of the disk, but the y component is along the y direction. So, we can able to see that this particular component is not on the plane of the disk, now this particular distance we can able to see that this is y distance this angle is phi x.

So, this distance will be y into phi x because this is y and the angle is phi x, so this distance will be phi x, so the component of this particular force will be making y phi x distance from the x y axis. Now, if we see this particular particle x component which is this direction if you take a particle here opposite here, so this particular force and this particular mass force will balance each because they are in the same plane.

If we consider another point, let us say below this particular force in the y direction and this particular force if we consider here P prime let us say which is below this will act because spinning, so it will act in this direction. The magnitude of this will be same as the upper one, so you can able to see that they will make a couple, they will balance each other, but they will make a couple and the magnitude of the couple if you can able to find out will be. So, let us say for single mass d m the couple about the bearings the shaft centre will be that is in y z plane, omega square y d m this is the force and momentum is y phi x this is the momentum this one.

So, this is a couple due to mass d m, if we sum such moments we can able to get the total moment due to the various forces and that will be giving us this and because omega is constant it will come out this is also independent of this integration. So, we will be having y square d m. We know y square d m is nothing but diametral mass moment of inertia of the shaft, sorry of the disc about its diameter, because in this particular case when we are considering the pure spinning I am let us speak on here.

So, for the case of pure spinning we found that the x component forces are cancelling each other whereas, the forces in the y direction they are giving a pure moment and that moment is the gyroscopic couple moment. In the previous case, the pure whirling case we saw that the net force centrifugal force, we were getting so effectively, because of the pure spinning and pure whirling. We got one force and one moment and now we will be using this to get the frequency equation.

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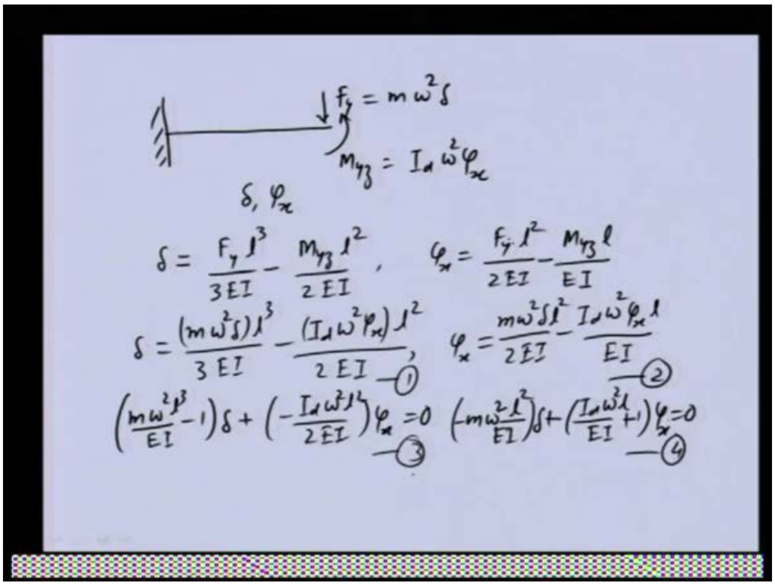

So, from the previous analysis of the pure whirling and pure spinning if we remove the disc on to the shaft, we will be having inertia force and in moment and direction of moment. In the previous slide, we can able to see is this particular force is producing a counter clockwise in y z plane. So, that particular direction we have retained here now effectively. Now, our problem is to, you have converted the dynamic forces into a static force, so that means basically we have converted a dynamic system to a quasi static system. Now, we will be obtaining what is the deflection and angle because of these particular forces.

From the strength of material deflection theory we have such relations like for displacement, the linear displacement is given as because of the force and because of moment. So, this is the linear displacement and force and moment if we are applying, we will get this relation. Similarly, for angular displacement we have relation F y l square to E I then due to moment because we are dealing with the linear system. So, because of this moment and force displacements can be added up to get these relations. So, this is coming from the deflection theory of the strength of material, so any strength of material book can be referred for such relations.

Now, we need to substitute the centrifugal force and gyroscopic moment in this expression, so we can able to substitute like this gyroscopic momentum. Hence, similarly in the deflection rotational displacement. Now, we will see that these two equation this equation one and two basically each and every term. They contain either linear displacement or translatory displacement or angular displacement.

Here also delta is there, here also phi x is there, so all terms containing so basically, they are homogenous equation. We can able to rearrange these equations like this m omega square l cube by E I minus one delta. So, I have taken this in one side and I have taken the delta common plus minus I d omega square l square by 2 a phi x is equal to 0, so this have to obtain from equation one. Similarly, from equation two we can able to get delta plus I d omega square l by E I minus plus one phi x is equal to 0. Now, express this series equation three and four, so equation three and four now you can able to put any matrix form.

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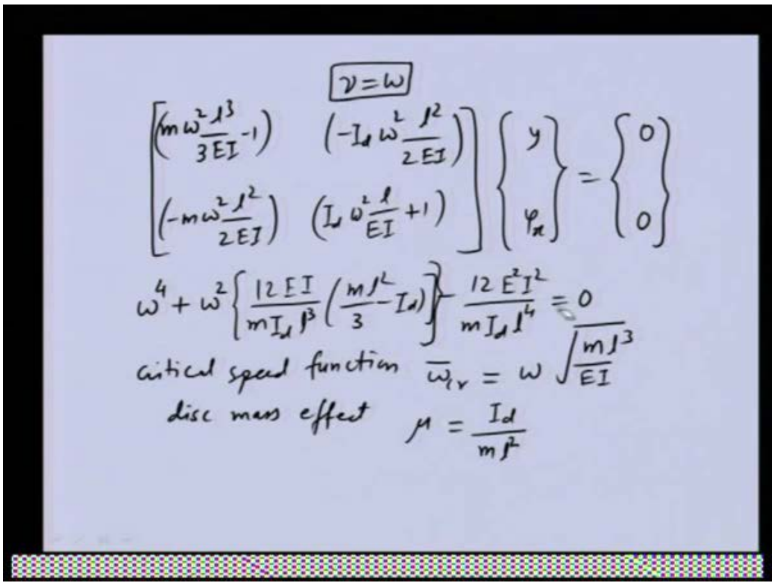

So, we are able to write this as y phi x and because this homogeneous equation right hand side is 0, so here we have m omega square l cube by 3 E I minus 1. First term then minus I d omega square l square by 2 E I and here minus omega square l square by 2 E I and here I d omega square l by E I plus 1. Now, this homogeneous equation one solution is when both displacements are 0. So, that is the case when we are not considering the motion at all, so for non trivial solution for non zero displacement, the determinant of this matrix should be 0. If we put the determinant of this matrix 0 will get a polynomial in terms of the omega which is the critical speed of the system because we started with whirl frequency is equal to omega, so we were looking for the critical speed of the system directly.

So, we will get a polynomial in terms of omega four basically it is a quadratic n omega square, so this equation will be after rearrangement of this equation this is square term then a constant term for equal to 0. So, this is a polynomial and we can close the bracket we can able to solve this for omega square the polynomial which we have got is basically, a frequency equation. It could able to solve in terms of omega square, but in this particular expression several variables are there.

Now, we will try to reduce the variables, so that we can able to do better interpretation of this particular equation, so I am defining two terms non dimensional terms. So, first is the critical speed effect critical speed function, so that is non dimensional term defined as spin speed m l cube by E I. Another is a mass effect disk, mass effect this is mu that I am defining is diametral mass moment of inertia divided by m l square.

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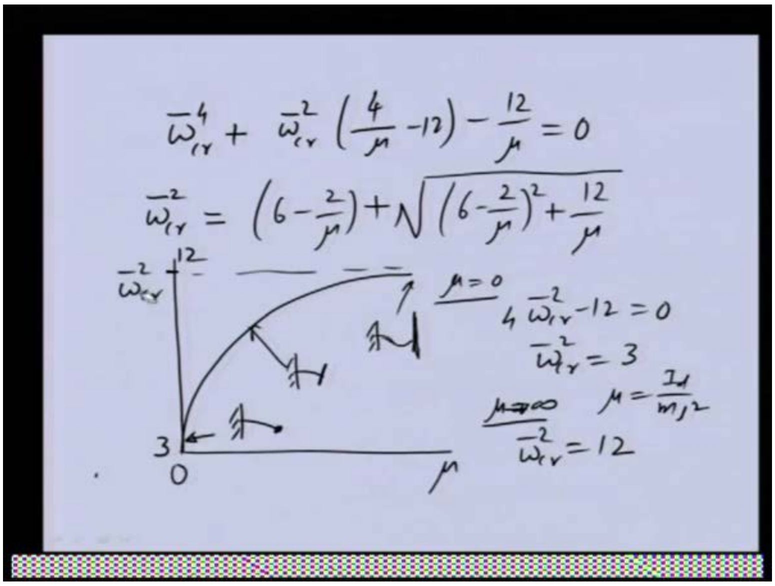

So, this also this two are the non dimensional terms we can able to use this to rearrange this polynomial and if we do it will get a much simpler expression in terms of the non dimensional parameter. We defined this, so this is much simpler and in this you can able to see there is only single disk parameter is there, this are included in the non dimensional terms. Now, in the solution of this, we can able to get the disk can be solved in close form, so the solution will be of this form. We are getting two roots at this, but if we see carefully this expression within the bracket term, with in the square root term this particular disk effect is positive, so this will be positive. This is a positive, so we can able to see that this term, which is within square root is always greater than this one.

Now, if we take the negative sign, we will get this particular critical speed as imaginary quantity, so that is not feasible one. So, we can neglect the negative one and will consider only the positive sign here and with this a phi plot the omega critical verse mu at mu is equal to 0. Let us say, so for mu is equal to 0 we cannot able to solve from here we need to go here, we need to multiply each quantity by mu, so you can able to see that this term will vanish, this term will give us for mu is equal to 0.

From the frequency equation will get 4 omega bar critical square and we get minus 12 is equal to 0. So, omega critical square is equal to 3, so here for 0 mu we are getting value of this is 3. Another case when mu is infinity tends to infinity, we can able to see that this terms will vanish and for infinity. If you simplify will get this as equal to 12, so that is here let us say this is twelve, so mu is equal to infinity means, very large disk or the disk with large mass moment of inertia, because the material mass moment of inertia because mu defined as I d by m l square.

So, if this is large then this should be very large, so for that particular case if we plot this particular equation, we will get a curve like this. Asymptotically, it will disk critical speed will be equal to infinity even will be equal to 12 when this particular frequency ratio this disk effect very large. So, you can able to see if we see this one when mu is 0 what is this case mu is 0 means if you see here I d is 0. So, there is no mass moment of inertia of the disk that means that particular disk is a point mass, so this is representing is a point mass disk is when is having infinite diametral mass whirl of inertia.

So, inertia will be so high the disk will not tilt in that particular case and this the general case of critical speed in which it is varying with the disk effect. So, if disk effect is more and more if diametral mass moment of inertia is more and more will be seeing the critical speed is increasing continuously, but it is having some maximum value is equal to 12. The critical speed is square is 12, this non dimensional parameter.

Today’s lecture we have seen that how a critical speed for synchronize whirl condition of a cantilever been, we obtained with this particular case we have consider the a very special case, in which the not only the motion is the synchronize fault that is the spin speed and the wall frequencies are same they are direction is also same. We have seen that as we are increasing the disk effect we are increasing the gyroscopic moment and because of that we are having increase in the critical speed. So, as we have seen at the beginning of the lecture the centrifugal force tries to kill the disk to its original position. So, it effectively increases the stiffness of the system and because of that the critical speed should increase. So, finally we have seen that that particular effect as we are increasing the disk effect the gyroscopic moment will be more and more and will be having increase in the critical speed of the system due to gyroscopic effect.