Non-Linear Vibration

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Module - 2
Derivation of Non Linear Equation of Motion
Lecture - 5
Development of Equation of Motion for Continuous Systems andOrdering Techniques

Welcome to today's class of non-linear vibration. So, in today's class we are going to study about this development of equation of motion for continuous systems using extended Hamilton principle, and also I will tell you about these ordering techniques. And in this lecture we are going study, how to develop the equation motion by using extended Hamilton principle, also I will use this generalized Galerkin method to develop the temporal equation motion. After deriving the equation motion we will study how to order this equation using the scaling parameter and book keeping parameters. So, in the last class, we have studied or we have developed the equation of motion of the continuous system using Newton's second law or d'Alembert's principle.

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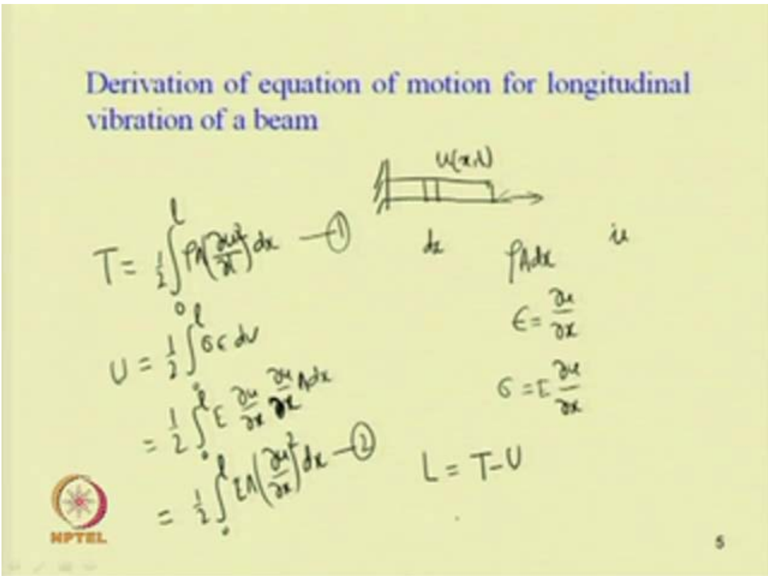

So, in today's class we are going to deriver the equation motion by using this extended Hamilton principle. So, let us take simple example for of linear system before going for the non-linear system. So, we will derive the equation motion for the longitudinal vibration of the beam and then, we will go for the Euler Bernoulli beam then, will make that equation motion non-linear and will take several examples to derive equation motion for non-linear systems. Then, we will see some exercise problems to derive the equation of motion also, will study about the ordering techniques.

So, let us now derive the equation motion of the longitudinal vibration of a beam. So, in case of the longitudinal vibration of a beam so, let us take a beam; for example, this is a cantilever beam, in this beam you want find the equation motion of this beam. So, the difference between a continuous system and a discrete system is that incase of the continuous system it is a distributed mass system unlike incase of the discrete system. And in this continuous system or the distributed mass system we have infinite number of degrees of freedom. So, each point you can consider as a spring and mass system so, in this infinite degrees of freedom system you can have infinite number of infinite number of natural frequencies.

So, let us consider a small element so, this longitudinal vibration of the beam so, the vibration takes place in the longitudinal direction that is in this direction and let us take a small element so, this small element at a distance x from the fixed end so, this small element has length d x. So, let rho is the density of this material so, rho into A into d x is the mass of the small element and we are considering, let u is the displacement at point at this or this element so, if u is the axial displacement of this element then, the velocity is the u by d t or I can write it as u dot is the velocity so, u dot is the velocity and rho A d x is the mass of the small element.

So, the kinetic energy using extended Hamilton principle, first we should write all the energy terms so, let us first write the kinetic energy, kinetic energy T equal to so, it will be for the small element it is equal to rho A d x and it will be half rho A then, it will be del as u is a function, as u is a function of both x and d so, we can write this velocity equal to del u by del t whole square into d x. So, for the small element the kinetic energy can be written as half rho A del u by del t whole square d x and for the whole beam one can integrate that thing to find the kinetic energy. So, the kinetic energy of the whole beam in longitudinal vibration can be written as half rho A del u by del t whole square d x. And similarly, one can find the potential energy that is U so, the potential energy can be written as, it is equal to half integration 0 to l stress into strain into d v.

So, d v is the, v d v is the volume. So, this d v can be written as a into d x if you are taking the uniform cross section and this stress can written in terms of the strain by using this young's modulus. And epsilon can be written epsilon that is your strain so, that is will be equal to del u by del x so, epsilon equal to del u by del x and this sigma equal to so, sigma, stress by strain equal to young's modulus so, stress sigma can be written as young's modulus into so, it will be equal to young's modulus into del u by del x that is the strain. So, substituting these 2 in this equation, I can write this U equal to half integration 0 to l so, this is equal to E del u by del x into del u by del x del u by del x into d v so, for d v one can write this is equal to A into d x. So, the strain energy associated with this longitudinal vibration of the beam can be written as half integration 0 to l E del u by del x into del u by del x into a into d x or in other word one can, one can write this equation equal to half integration 0 to l E A then, del u by del x whole square into d x.

Now, considering known force acting on the system let us first try and find for the free vibration of the system so, in this case the Lagrangian of the system can be written as T minus U. So, the Lagrangian equal to this is expression for T and this is expression for U. So, the Lagrangian can be written in this form T minus U and one can use the Hamilton principle to derive this equation motion. So, as in this case known force is acting on the system then, this extended Hamilton principle reduces to that of the Hamilton principle which is generally applied for a conservative system.

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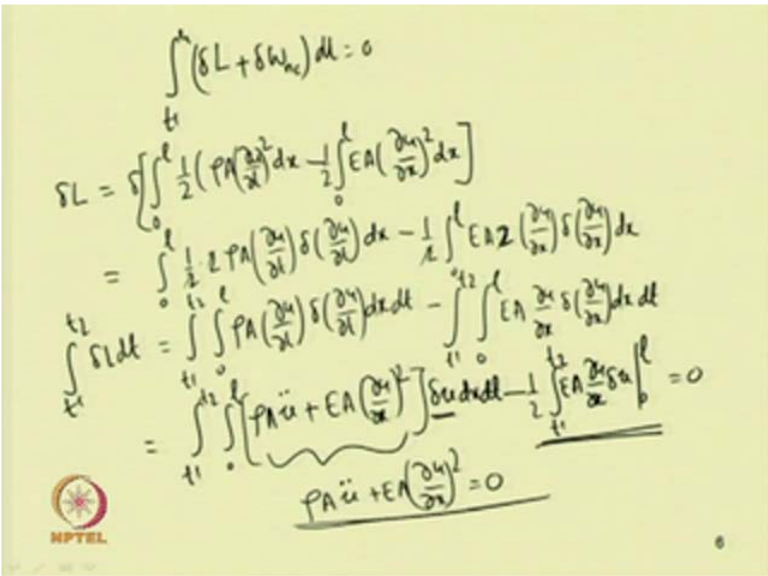

So, in this case using this Hamilton principle one can write the Hamilton principle, Integration t 1 to t 2 then, del L plus del W n c d t equal to 0. So, in this case as known non conservative force is acting on the system one can write integration t1 to t 2 del l d t equal to 0. Now, one can find this del L from the previous expression for this L so, that will be equal to operating this del operator on 0 to l so, this is equal to half rho A then, del u by del t whole square d x so, this is for the kinetic energy minus for the potential energy one can write this is from 0 to l half E A del u by del x whole square d x. Now, this del operator will be acting on this and this can be written as so, this will be equal to integration 0 to l then half into in this case it will be multiplied with 2 then, rho A del u by del t into del u by del t into del of del u by del t into d x minus similarly, it can be half integration 0 to l E A into 2 into del u by del x into del of del u by del x d x.

Now, using this integration t 1 to t 2 del L d t will gives us integration t 1 to t 2 integration 0 to l. So, this 2, 2 cancel so, this becomes rho A del u by del t into del of del u by del t into d x so, one can write this thing del of del u by del t into d x d t minus so, this 2 and half cancel one can write again integration t 1 to t 2 integration 0 to l E A del u by del x into del of del u by del x into d x into d t. Now, one can simplify this equation so, to simplify this equation one can write so, one can use this integration by parts, by using this integration by parts one can write this equation equal to so, it will be equal to integration.

Now, one can write this part rho A u dot. So, one can write this equation in this form integration t 1 to t 2 integration 0 to l rho A u double dot by using this integration by parts one can have rho A u double dot plus E A del u by del x whole square into del x d t del, one can have del u into del u d x d t equal to or plus one can have plus or minus sin one can one can expand this thing so, it will be minus half t 1 to t 2 E A del u by del x into del u 0 to l so, this will be equal to 0. So, this part is the boundary condition and as del u is arbitrary so, this represent the equation motion of the system. So, the equation motion becomes rho A u double dot plus E A del u by del x whole square equal to 0. So, this is the equation motion at the system. Similarly, one can derive the equation motion for similar other equations.

And the advantage of using this Hamilton principle over Lagrange principle or Newton or D'Alembert principle is that so, in this case in addition to getting this equation motion one can get the boundary conditions also. So, this gives the boundary condition this E A del u by del x into del u 0 to l so, that gives the boundary condition that means u will be either u will be 0 or del u by del x will be 0 at either x equal to 0 or l. So, in this way one can derive the equation motion for the longitudinal vibration of a beam. So, to be more precise so, one can so, let us derive this term again.

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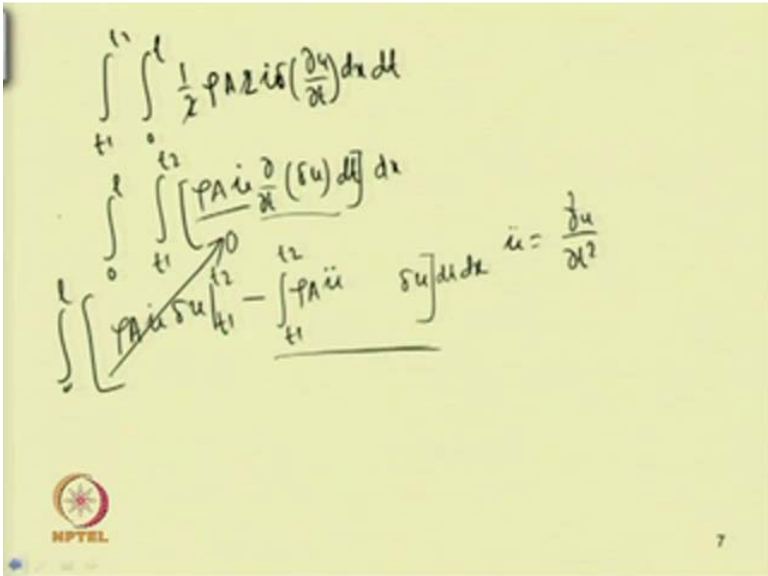

So, that is integration t 1 to t 2 in 0 to l half rho A 2 u dot then, it becomes rho A del u by del t into del of del u del t d x d t del u by del u by del t into d x d t. So, in this case one can write this term so, this term can be written in this form by changing the integral so, one can write 0 to l t 1 to t 2 so, this 2, 2 cancel so, this becomes rho A u dot into so, one can so, here we have this delta root so, u dot so, one can interchange between this del by del t into del u so, one can write this way into d t and d x. So, this equation can be written in this form rho A u dot then changing between this del and del by del t so, one can write del by del t of del u d t. Now, one can use this integration by part so, to use this integration by parts so, first function remain as it is so, rho A u dot so, you keep the first function as it is.

Now, integration of the second so, integration of this del by del t of del u so, this becomes del u so, this is from t 1 to t 2 so, one can have this 0 to l outside and so, rho A u dot del u so, this minus then, one can have integration t 1 to t 2 then, this del u into derivative of this first term. So, derivative of this first term becomes rho A so, u dot derivative becomes u double dot so, rho A u double dot u double dot is this del square u by del t square. So, u double dot is nothing but, del square u by del t square so, del rho A u double dot into del u. So, outside we have this d t and d x. So, this term this del u at so, according to our Hamilton principle this term del u vanishes at this 2 time that is t 1 and t 2.

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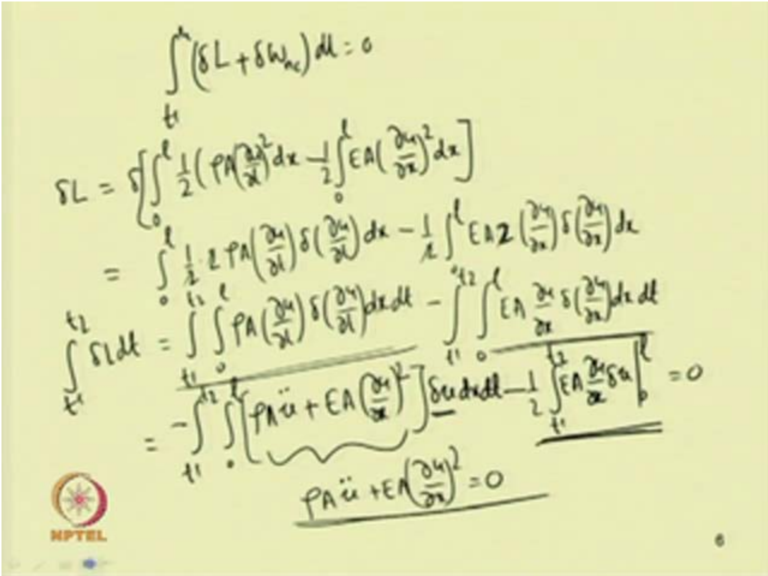

That is why this term becomes 0 and so, for this purpose one can get this equation so, you will have a minus so, minus term here t 1 to t 2 0 to l rho A u double dot E A del u by del x whole square e u del u d x d t minus half t 1 to t 2 E A del u by del x del u 0 to l. So, to derive this term one can easily find this term so, this term will reduce to minus E A del u by del x whole square del u d x d t minus half t 1 to 2 e a del u by del x del u 0 to l. So, in this way one can derive the equation motion by using extended Hamilton principle.

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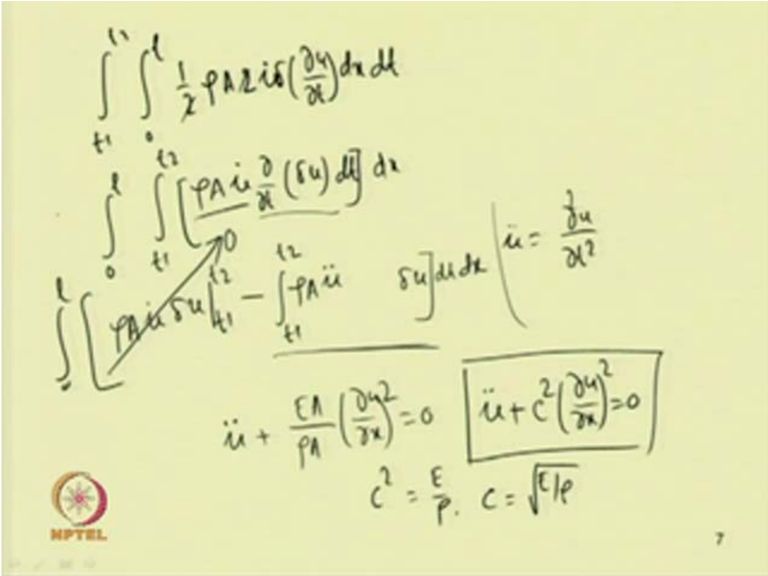

So, while taking the kinetic energy term one has to change the integral. So, in this case one can change this thing from 0 to l and t 1 to t 2 and then integrated by parts so, by integrating it by parts so, the first term becomes 0 and the remaining term one can take it in equation motion. So, the final equation motion becomes rho A u double dot plus E A del u by del x whole square equal to 0 or one can write this equation in this form so, E U double dot plus E A by rho A del u by del x whole square equal to 0. So, here A can be cancelled or one can write this equation in this form u double dot plus C square del u by del x whole square equal to 0 so, where C square equal to so where c square equal to E by rho or C equal to root over or one can write C equal to root over E by rho. So, this is the equation for a longitudinal vibration of a beam when we have taken a linear system so, we have not consider any non-linearity in this but, the method shows how one can derive the equation motion by using Hamilton principle. Similarly, one can derive the equation motion for Euler Bernoulli beam.

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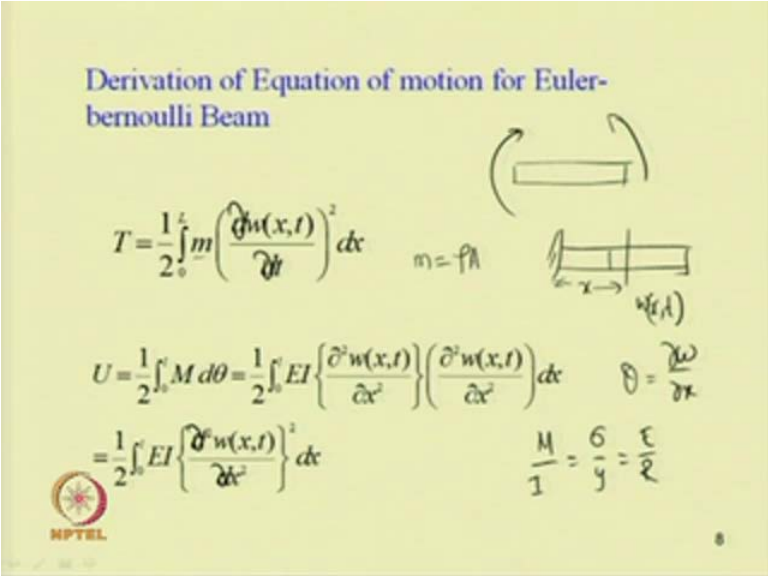

So, in case of the Euler Bernoulli beam, the beam is subjected to pure moment pure bending moment. So, in this case as the beam is subjected to pure bending one can derive the equation motion for the system by taking the kinetic energy and potential energy of the system.

So, the kinetic energy of the system if one can consider so, let us consider a system cantilever beam or any beam one can consider so, at a distance x at a distance x let w is the transverse direction vibration. The displacement in the transverse direction is w so, if w is the displacement in the transverse direction which is a function of both x and t, x is it is at a distance x that is the phase coordinate and at time t one can find the equation motion by using this extended Hamilton principle or by using the simple Hamilton principle incase where there is no non conservative force acting on the system. So, here the kinetic energy can written as half let m is the mass for unit length then at m into d x is the mass of the small element and into velocity square so, velocity equal to d w del w by del t.

So, in this case it will be equal to del w by del t as w is a function of both x and t. So, one can write this kinetic energy for the whole beam as the integration of half m del w by del t whole square d x where, m is the mass for unit length m is the mass for unit length or it can be equal to the density of the system into a density into a will give the mass for unit length. So, mass for unit length into d x that is the mass of the element then its velocity

velocity equal to del w by d t so, the kinetic energy equal to half integration 0 to l m del w by del t whole square into d x. And one can find the potential energy similar, to the previous case one can find the potential energy equal to half stress into strain into d v or one can write this by using this bending moment into d theta so, it is equal to half integration 0 to l M d theta and here theta is the slope so, theta can be written equal to theta, one can write theta equal to del w by del x.

So, as this bending moment one can write so, if one take small deflection so, in that case this bending moment can be written as E I del square w by del x square. By using this formula one can find this thing so, M by I so, pure bending equation M by I equal to sigma by y equal to E by R so, here M will be equal to E I by R so, 1 by R can be written that is the curvature can be written as del square w by del x square. So, one can write this M equal to E I del square w by del x square and this d theta will be equal to del square w by del x square again. So, one can write this potential energy or the strain energy associated with this vibration of the transverse direction equal to half integration 0 to l E I del square w so, it is del square w by del x square into d x. Now, one can proceed in the similar way and derive the equation motion in this case which already we have seen as the Euler Bernoulli beam equation. And if, one considers large amplitude displacement so, in that case this M can be written in this form of large amplitude large curvature.

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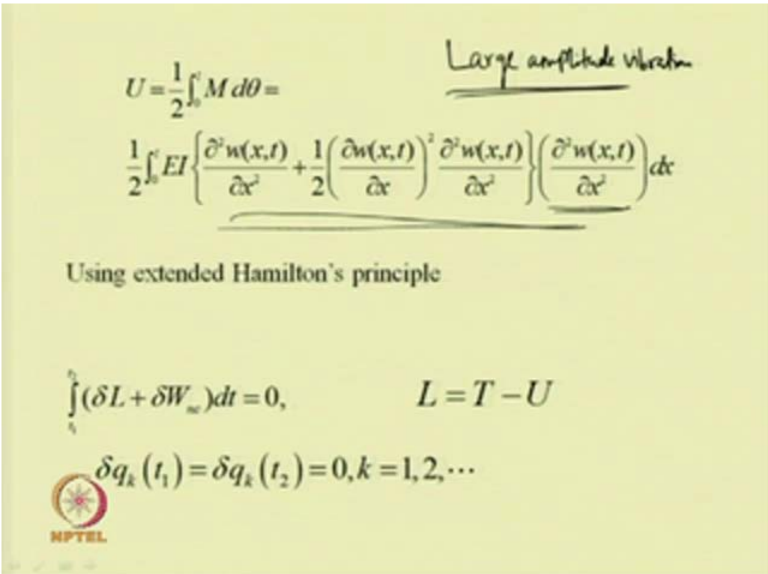

So, in case of large curvature one can or incase of large amplitude vibration, one can write large amplitude vibration, one can write this M equal to del square w by del x square plus half del w by del x whole square into del square w by del x square. And d theta can be written in the same way as before that is equal to del square w by del x square. So, in this case one can have this U equal to half E I del forth del square w by del x square whole square or one can take this del square w by del x square common so if one takes common then it becomes 1 plus half del w by del x whole square into del square w by del x square.

So, one additional term one can get in this case so, that is equal to half del w by del x whole square so, this term will lead to the non-linear terms if one derive the equation motion so, one can find the equation motion by using this formula that is del l plus del W n c d t equal to 0 where, l equal to l is the Lagrangian of the system so, that is equal to T minus U. So, while deriving this equation one can take this del q k where, q k is the generalized coordinate so, here W is the generalized coordinate so, here one can consider this del w at t 1 will be equal to del w at t 2 equal to 0. Now, this W that is the transverse direction displacement which is s a function of both space and time can be written by using the scaling parameter.

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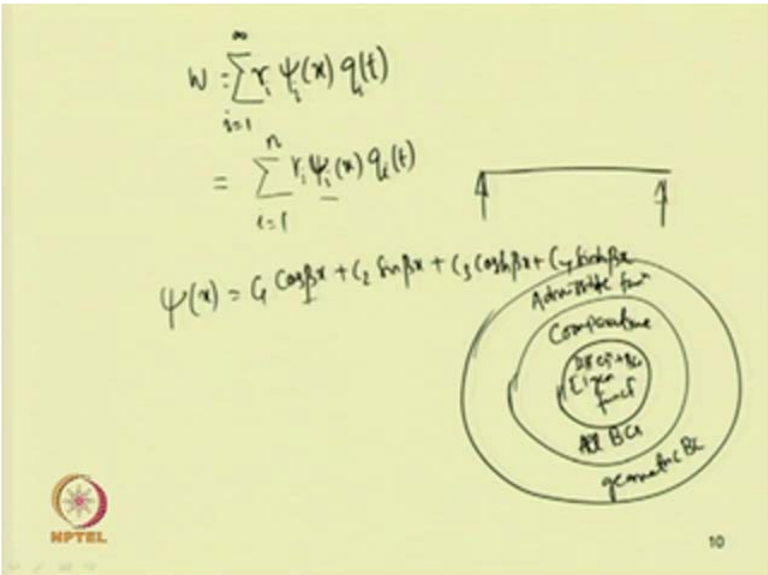

And time modulation and safe function so, one can write this W equal to r psi x into q t. Where, r is the scaling factor, psi is the safe function and q t is the time modulation. So, by using this equation in kinetic energy potential energy and finally, in this Hamilton principle one can derive the temporal equation of motion by applying the Galerkin principle.

So, the Galerkin principle is the use of Galerkin principle has been discussed in the last class. So, one can use that principle to derive the equation motion. So, either one can apply the Galerkin principle or one can apply this equation that is W equal to r psi x q t in this u and v t and u and then apply this Hamilton principle or after applying the Hamilton principle and getting the differential equation so, one can apply this equation there to find the temporal equation. So, both the method will yield the same equation. So, in this case one can take only single mode but, instead of taking a single mode one can consider multi mode analysis. So, here one can take psi i or r i into psi i x and q i t where, psi i x is the safe function of the i th mode so, already we are familiar with, so we are familiar with the continuous system, linear continuous system in which we know that it has infinite number of modes so, by taking different modes one can find or one can write the transverse displacement equal to r i into s i x into q i t where i equal to 1 to infinity.

But, in actual case as the higher modes will not be effective or not be useful for our study so, one can limit this number of modes to few lower terms. So, it can be written as i equal to 1 to n so r i psi i x into q i t, this psi terms can be obtained so, this psi terms can be admissible function or it can be the Eigen function of the system. So, when one can consider Eigen function then, the resulting equation can be reduced to a simpler form but, if one consider the admissible function which are not the Eigen function so, then model interaction will be there in the equation motion. So, for example, one can consider the psi i for a simple supported beam.

For example, for a simply supported beam one can has tried the general solution for the Euler Bernoulli beam. So, one can write psi x equal to c 1 cos beta x plus c 2 sin beta x plus c 3 cos hyperbolic beta x plus c 4 sin hyperbolic beta x and now, apply the boundary conditions to get psi x. So, where one can get this characteristic constant beta also from frequency equation and applying this boundary condition one can find psi x so, that will give the Eigen function though Eigen function satisfy both the differential equation. So, Eigen function satisfies both the differential equation motion and all the boundary conditions.

And one can have another set of functions also, that is comparative function so, that comparative function satisfies the differential equation and also the geometric boundary conditions. So, Eigen function satisfy both differential equation and all boundary condition this comparative function satisfy only the boundary condition so, it has not satisfy the governing equation. And another set of functions also one can use that is admissible function so, this admissible function satisfy only the geometric boundary condition of the system. So, one can use admissible function so, in case of admissible function it satisfy only geometric boundary condition. So, in case of comparative function it satisfies all boundary conditions. And in case of Eigen function it satisfies both differential equation plus all boundary conditions.

So, if one takes this Eigen function of the system which is satisfying the differential equation motion and all the boundary conditions so, one can get orthogonal functions and these orthogonal functions one can use to reduce this multi degree of or this continuous system into a set of multi degree of freedom systems. If one use this Eigen function then most probably one get an equation which are decoupled but, if one use this admissible function where it satisfy only the geometric boundary condition sometimes one may get the coupled equation motion. Now, for diff let us consider different cases so, how one can find what are the boundary conditions associated with this. So, in this case we have just discussed about 2 cases so, in one case if one take the potential energy in this form that is half integration 0 to l E I del square w by del x square whole square. So, one can get the linear Euler Bernoulli beam equation. But, if one can take the strain energy in this term by considering this additional term so, one can obtain the non-linear equation motion.

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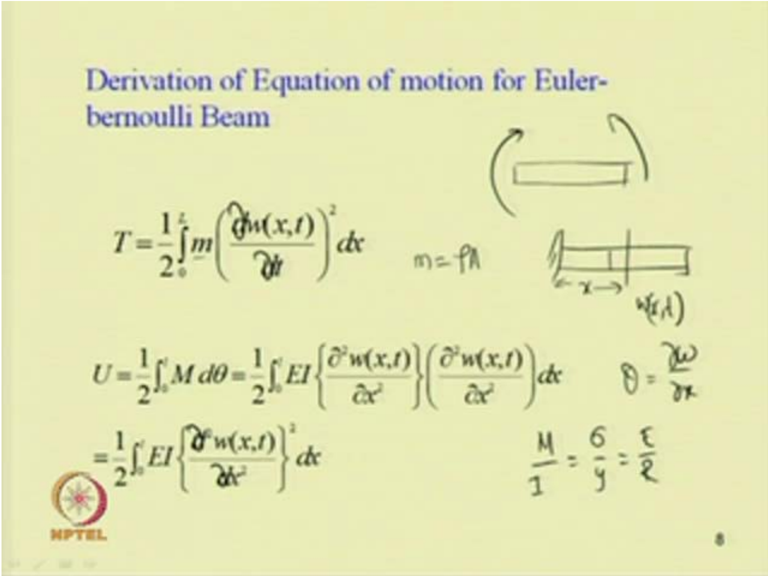

So, with examples we can study about this system after a few minutes. So, let us now discuss about some of the linear boundary conditions or some of the boundary condition in the transverse vibration of the beam.

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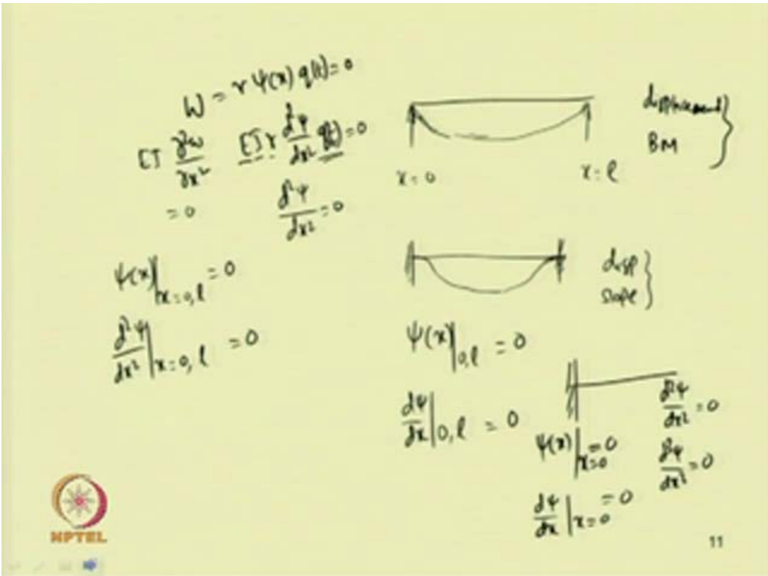

So, for example in case of this simply supported beam, the boundary conditions are so, in this at x equal to 0 and x equal to l boundary conditions are the both displacement and slope equal to 0 so, both displacement and slope equal to 0 here and in case of a no so, in case of the simply supported beam. So, here slope is not equal to 0 one can see that slope is not equal to 0 displacement equal to 0 at this end displacement equal to 0 at this end but, slope is not 0 in case of a fixed fix beam one can find both displacement and slopes are 0. So, here up to this one can see the displacement is 0 that means slope is 0 here also both displacements and slopes are equal to 0. So, in case of the simply supported beam so, displacement at this end is 0 displacement at other end is also 0 so, along with that one can have the bending moment equal to 0.

So, both bending moment and displacements are 0 at both the end. So, this displacement is the geometric boundary condition and bending moment is the natural boundary condition or the force boundary condition. So, if one can find the one can take one admissible function so, and then it will satisfy only the displacement is 0 at both the ends, one may not take a function which will satisfy this bending moment equal to 0 at both the ends. Similarly, here for the clamped beam for this fixed fix beam one can have both displacement equal to 0 and so both displacement and slope are 0 in this case so, in this case displacement and bending moment are 0.

So, if we are writing displacement equal to w so, this bending moment equal to E I del square w by del x square as we have written w equal to r psi x into q t so, it can be reduced to E I r del or by removing this del one can write this d square psi by d x square into q t so, as this E I r and this q is a function of t time modulation. So, for a at a particular distance one can take this constant so, one can have this del square or at this for all times, for all times as these bending moment will be equal to 0. So, one can write this E I r d square psi by d x square into q t equal to 0 so, for all time as q t will not be equal to 0 so, this reduces that this becomes d square psi by d x square equal to 0. That means, in case of the simply supported beam so, one can take this w that in this is equal to r psi x into q t equal to 0 so, for all time as q t will not be equal to 0 r also will not be equal to 0 so, in this case psi x will be 0 and d square psi by d x square also will be 0.

So, for simply supported beam one can write psi x at x equal to 0 and l so, this will be equal to 0. Similarly, d square psi by d x square at x equal to 0 and l will be equal to 0. So, this is for the simply supported beam and incase of the fixed fix beam so, one can have both displacement and slope equal to 0 here so, in that case one can show that psi x at 0 l equal to 0. Similarly, d psi by d x so, who is correspond to the slope at x equal to n will be equal to 0. So, in a similar way one can find the boundary condition for a cantilever beam the left end is similar to that of this fixed fix beam that means so, here psi x or psi 0 equal to 0 psi x at x equal to 0. So, in this case one can write this way at x equal to 0 equal to 0 also the slope will be equal to 0 this means d psi by d x at x equal to 0 equal to 0 but, at the free end one can have both bending moment and sear force equal to 0 already we have seen that this bending moment is proportional to d square psi by d x square so, one can write d square psi by d x square equal to 0 and sear force proportional to d q psi by d x q equal to 0. So, in this way one can find all the boundary conditions.

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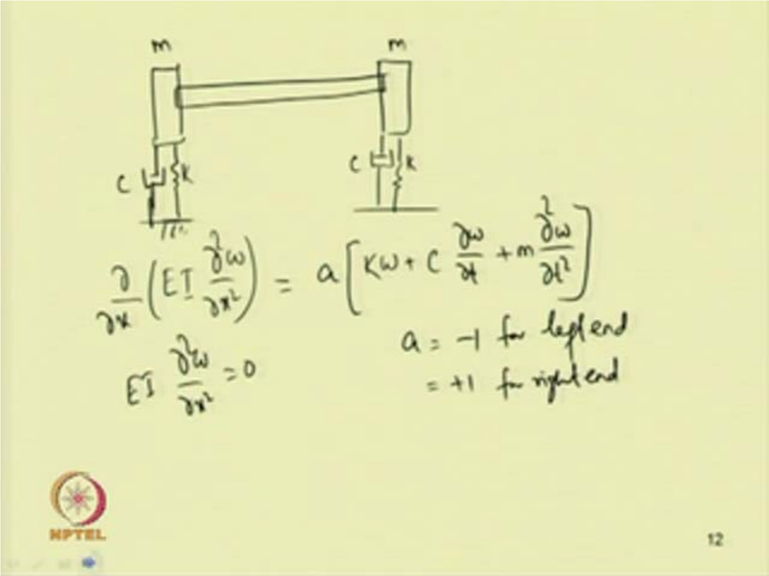

So, let us see some more complicated boundary condition so, let us take one beam let at this end one have a mass also let us have mass at both the ends and this is also supported by some spring and damper. So, if it is supported by some spring and damper then one can write let this is k and this is c this is stiffness and this is damping. And for this beam let this is mass m then, at this end so, or one can write so, the sear force will be equal to the inertia force plus the damping force and plus the stiffness force. So, one can write this del by del x of E I del square w by del x square so, this is the sear force that is rate of change of this bending moment equal to sear force. So, this will be equal to a k w plus c del w by del x del w by del t plus m del square w by del t square. So, here a equal to minus 1 for left end and equal to plus 1 for the right end. So, if one take the free body diagram of the side so, one can show that the sear force will be equal to sear force will be equal to the inertia force so, which is equal to m del square w by del t square then, plus the damping force that is c into del w by d t and plus the spring force that is k w.

So, if one take this right side then the sear force will be positive, if one take to the left side the sear force will be negative. Also in addition to this the bending moment will be equal to 0 so, one can write this E I del square w by del x square equal to 0. So, instead of taking this linear spring and damper one can take the torsional spring and damper also.

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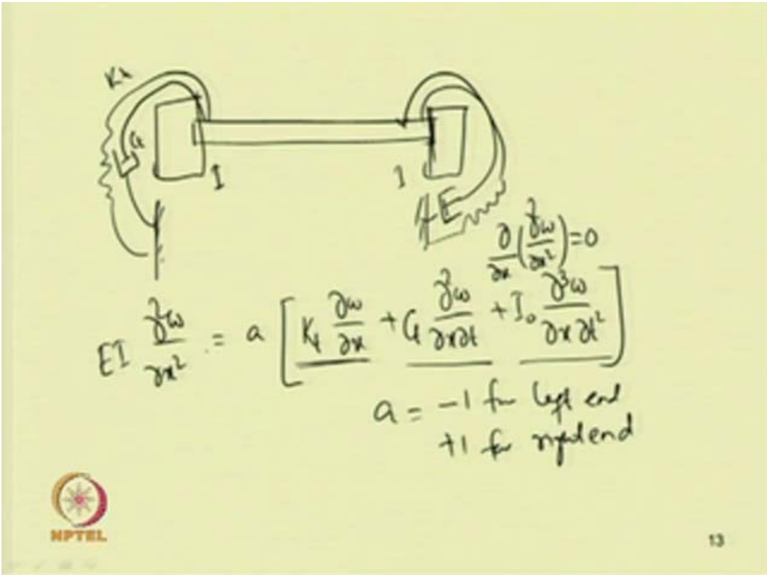

By taking a torsional spring and damper in this case one can have let us take a rotary mass also, a mass with moment of inertia I and let us take a torsional spring and torsional damper. So, in case of this torsional spring that of this is a torsional spring and let us take a damper this way similarly, this side also one can take a spring and damper and let I is the moment of inertia then, this is the damping element and one can have a spring element also torsional spring element.

So, in this case this torsional spring element k t and damping element let us take c t. So, in this case the sear force will be equal to 0 that means del by del x of del square w by del x square will be equal to 0 but, this bending moment will be equal to E I del square w by del x square will be equal to a into k t into del w by del x plus c t into del square w by del x del t plus i 0 into del q w by del x del t square it may be noted that this del w by del x is the slope that is theta square theta by del t square that is the inertia due to this torsional mass that is i theta dot square then this is due to damping and this is due to stiffness.

So, here a will be equal to minus one for left So, one can have this equal to i 0 into del end and it will be equal to plus 1 for right end. So, one can similarly, derive the equation motion or temporal equation motion of the system by taking a safe function which depends on the boundary conditions. So, for different boundary conditions by using these expressions one can derive the safe function and after using those safe functions one can reduce the governing equation to that of a temporal form. So, in all these cases till now we have considered the cross section of the beam to be uniform.

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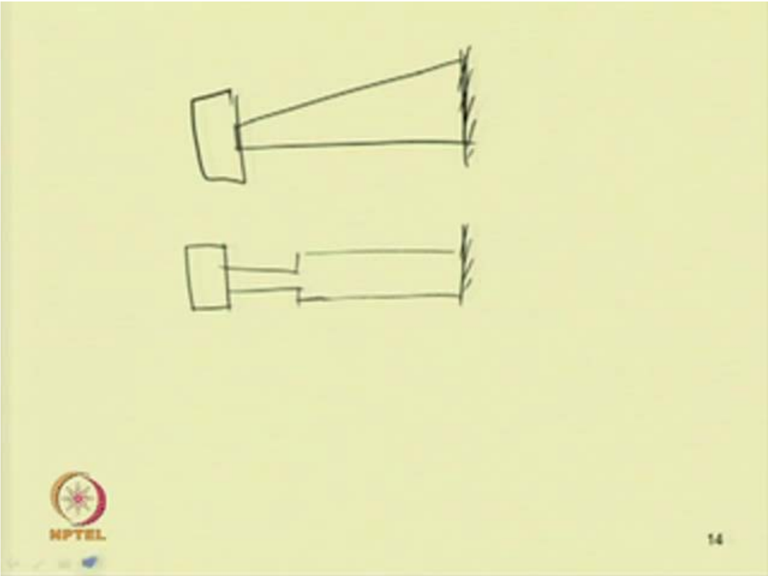

So, instead of taking uniform cross section one can take also non uniform cross section. In that case let us take one non uniform cross section so, for a cantilever beam we can let us consider this system so, one can write the boundary condition for this but, while writing the equation motion so, if one consider the mass to be of a homogenous material then, only one can consider the variation in the i term. So, while deriving this equation so, this i term will be different and it can be so, while doing the integration one can take this i as a function of x. Similarly, in case of similarly, one can consider a cantilever beam with let us consider one more example so, here also one can derive the equation motion of the system this way so, here up to this one can use i 1 and after that one can use i 2 and use appropriate boundary condition to derive the equation motion.

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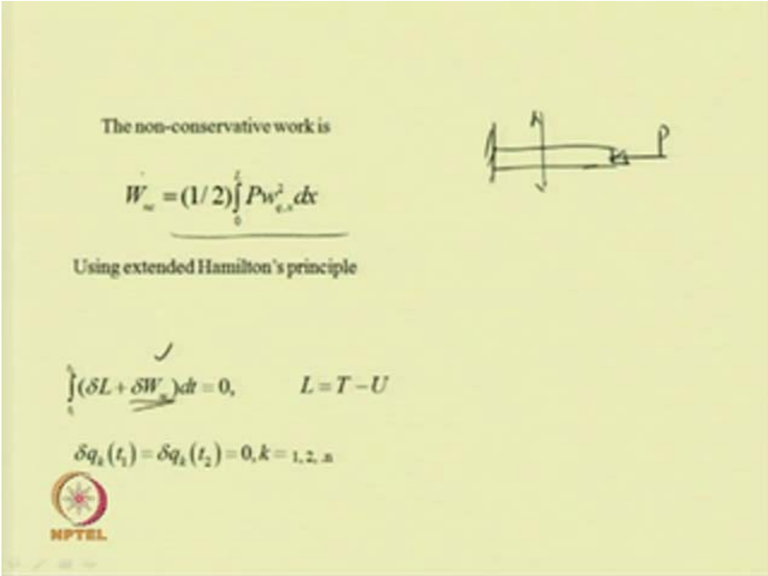

So, if let us consider a case when some force is acting on the system. So, a beam subjected to a axile force p so, in that case so, one can find the work done due to this axile force in this transverse direction in this way so, half 0 to l p into del q x that is del w by del x whole square into d x and then one can use this extended Hamilton principle by taking this non conservative force into account and find the equation motion.

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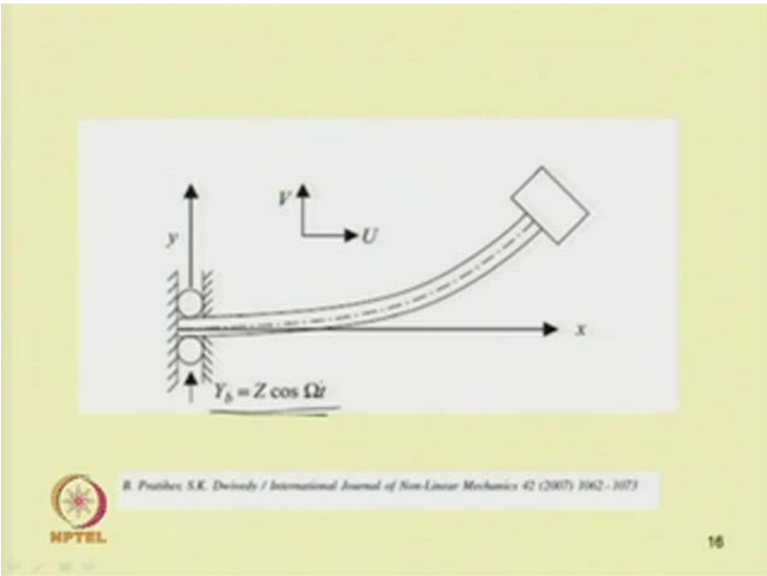

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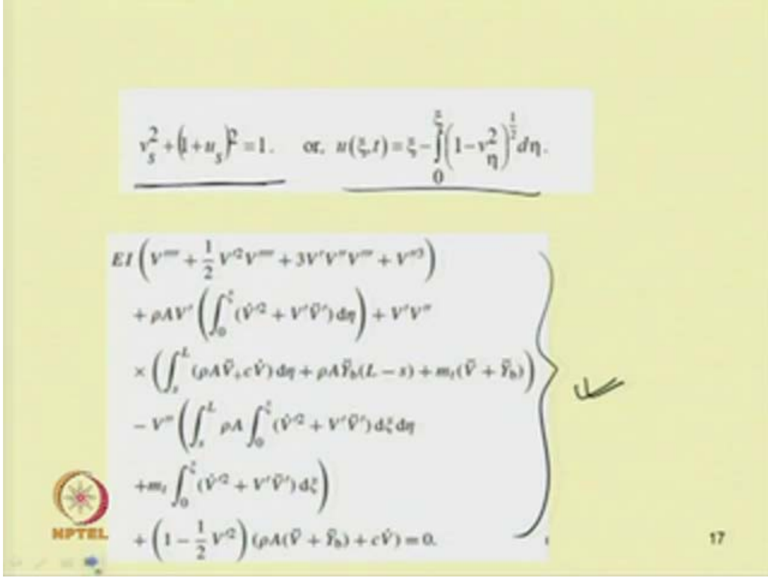

So, let us consider one more example which was published in international journal of non-linear mechanics by Pratiharan Dwivedy. So, here a roller supported beam is considered so, in this case we have to find the equation motion. Now, considering a small section one can write the potential energy, strain energy of the system and in addition to that so, it is subjected a vertical force in vertical direction. So, one can find this non conservative work done also in this way so, after using all these and using these in extensibility condition so, in inextensibility this is the condition for inextensibility one can find this and then one can find the governing equation in spatio temporal form in this way.

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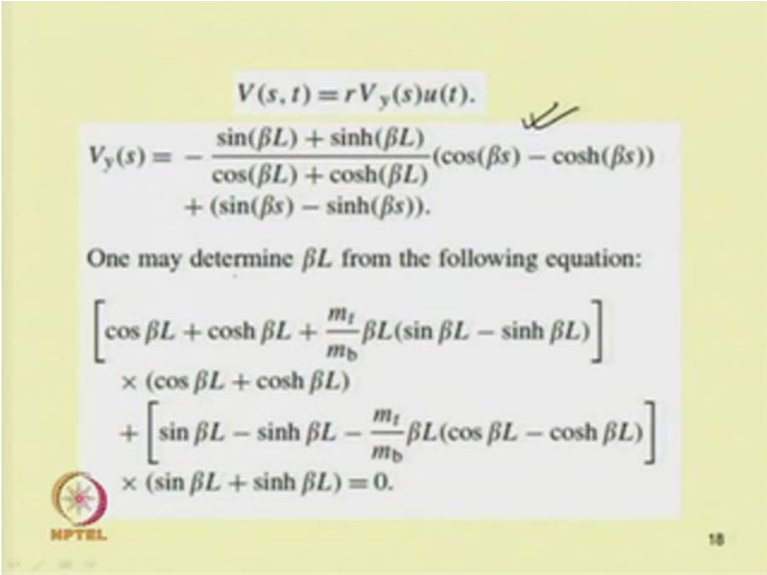

So, after getting the equation motion in spatio temporal form now, by substituting this safe function one can find the temporal equation motion.

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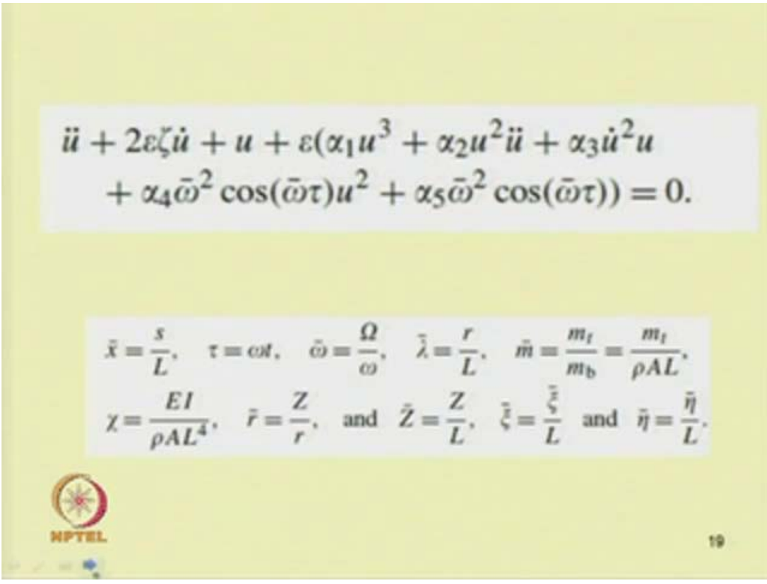

So, in this case one has used this time modulation u t V y is the safe function so, this is the safe function of a cantilever beam with tip mass.

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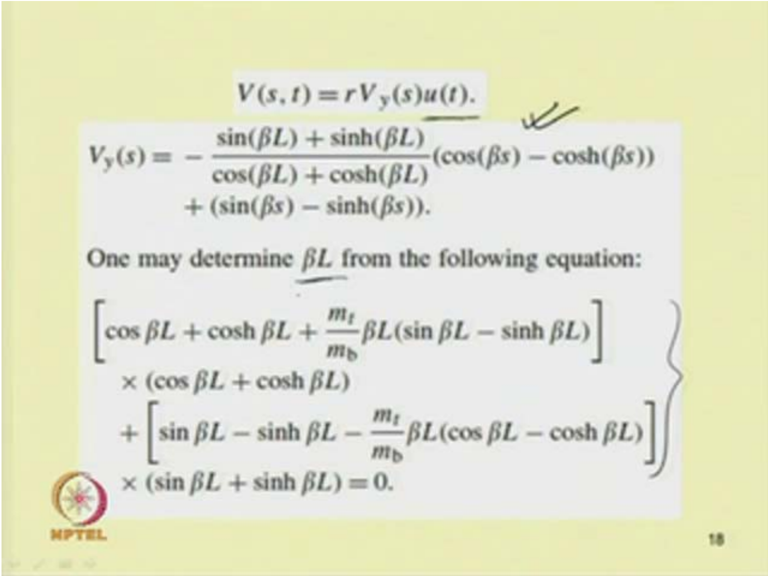

As a tip mass is there so, one can consider the safe function of a cantilever beam with tip mass and this expression gives the frequency equation for beta l. So, one can find this temporal equation in this way so, after finding this temporal equation where one can see the coefficients can be written in this form.

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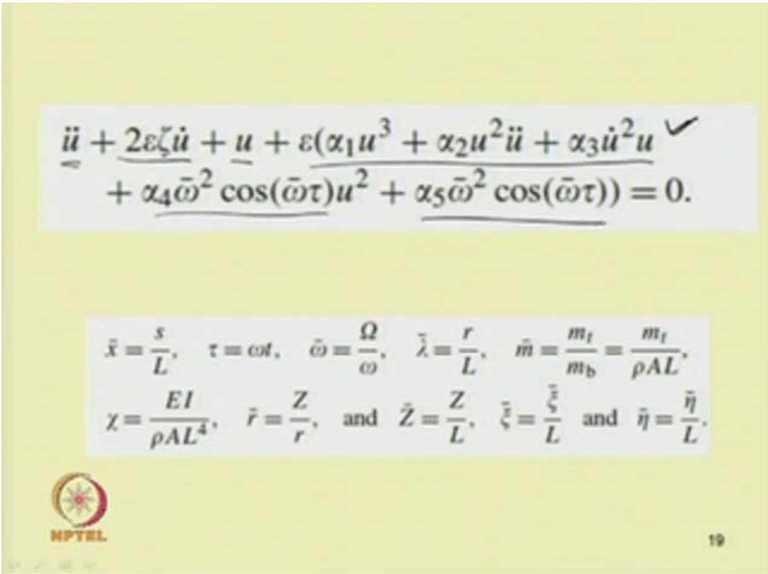

For example, in this case one has the linear term u double dot so, this is the linear term this is another linear term but, all these terms are non-linear terms. So, the coefficient of the non-linear terms can be obtained from these expressions.

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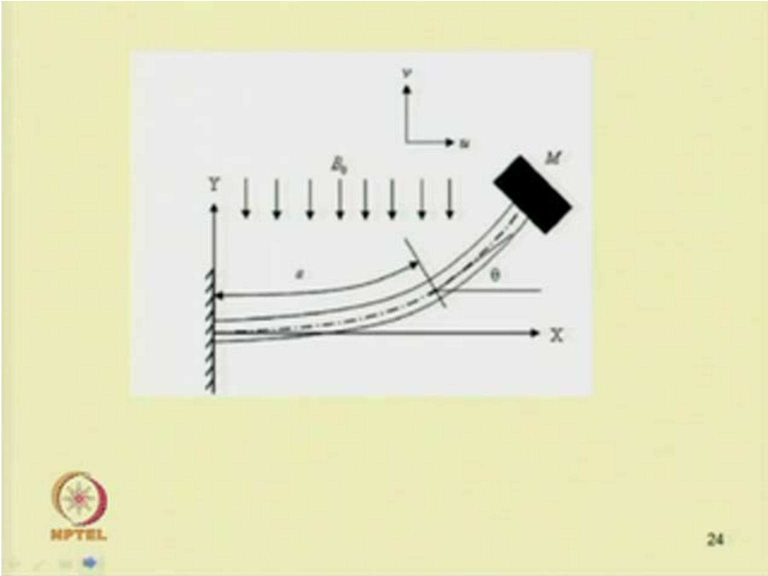

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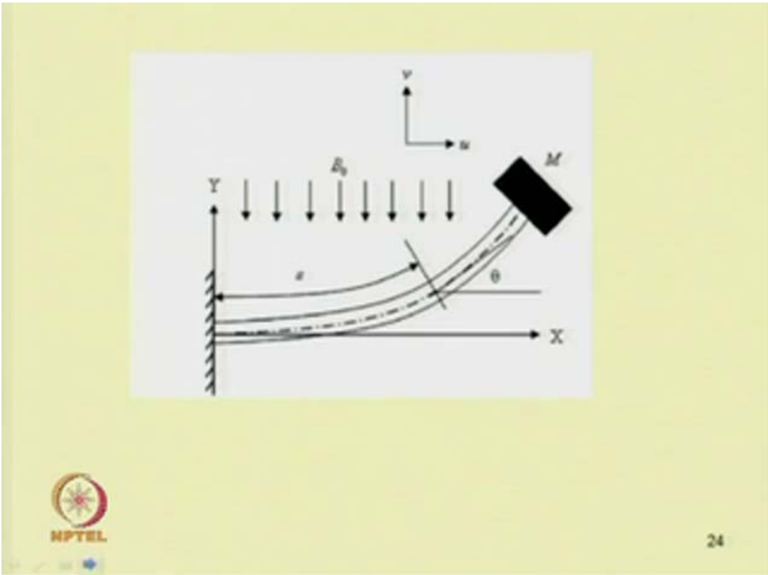

Where, one can find the coefficient alpha 1, alpha 2, alpha 3 by integrating these terms. So, these integrations are function of the safe functions so, by using this integration one can find the coefficients. So, after finding the coefficient one can use this ordering technique to order the non-linear equation motion. So, next class we are going to study about how to order the equation motion or some systems so, also will see different some more different systems where some magnetic field is applied.

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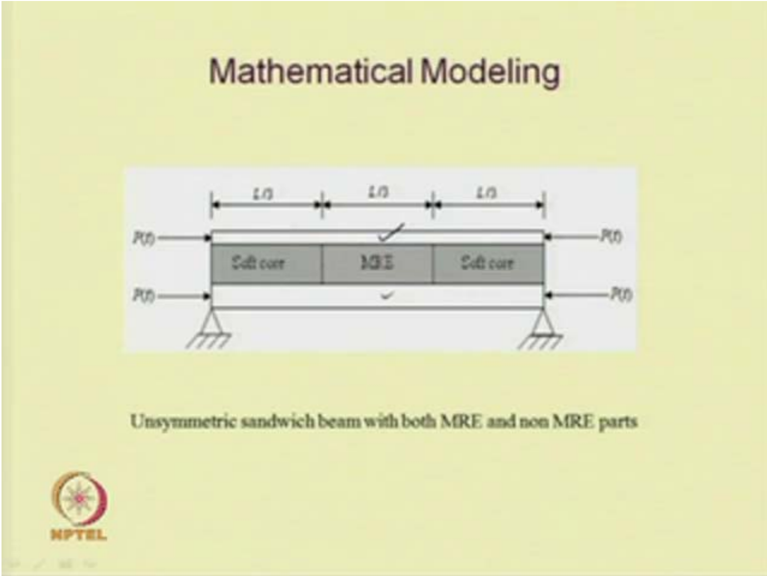

And you can take some exercise problems also where you can derive the equation motion for a sandwich beam so, in the sandwich beam 3 so, this is the core element both are both are skin and this is core so, in this core one can use this Visco-elastic or elastic material also one can use this magnetorheological elastomer also, by taking different property and taking this force so, one can derive the equation motion.

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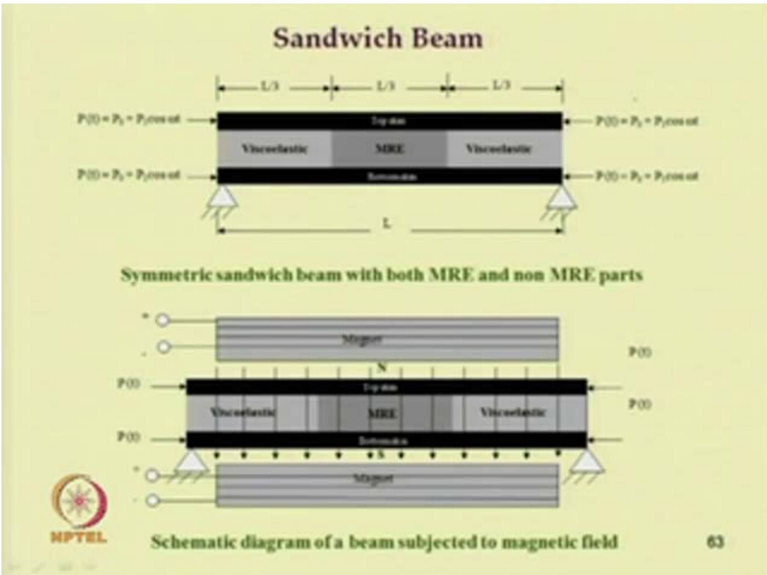

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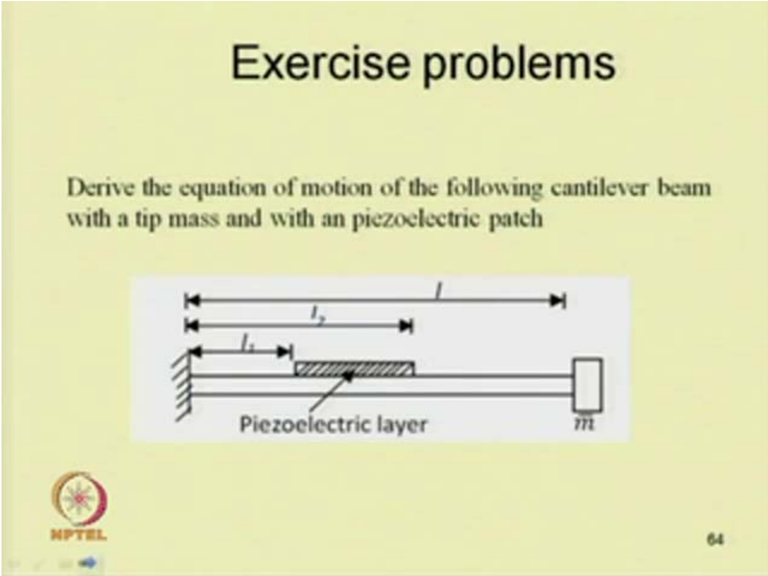

So, one can take this as one exercise problem and some other exercise problem also you can take to derive the equation motion. So, already you have seen this example so, also one can derive the equation motion for the sandwich beam when this magnetic field is applied one can find the equation motion by using the force due to magnetic field. Also one can derive the equation motion for a cantilever beam by taking piezoelectric layer.

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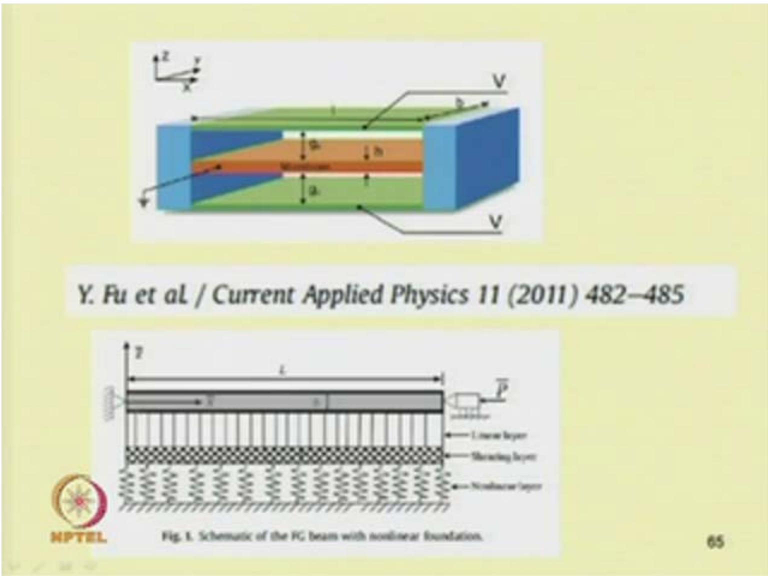

In a similar way one can take a micro beam, also one can take a functionally graded material supported by non-linear springs or by taking a linear or non-linear vibration observer and with different boundary conditions.

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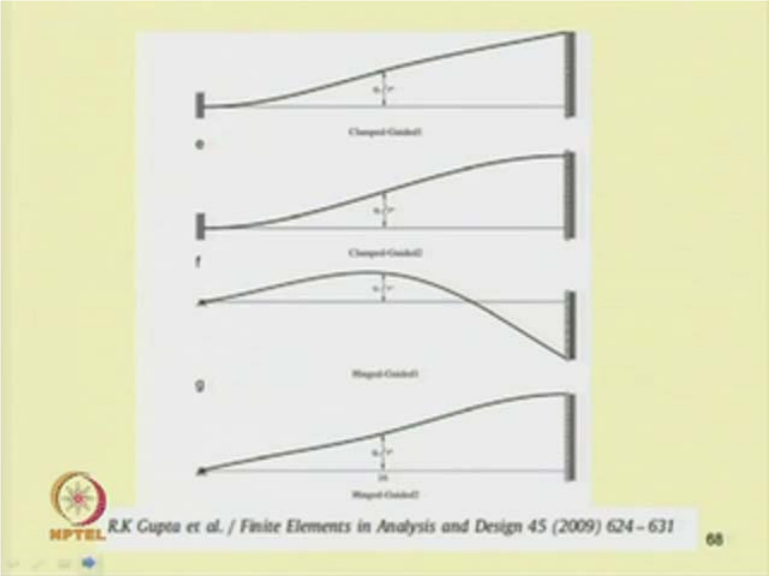

So, taking these exercise problems one can derive the equation motion for the continuous systems and after getting the equation motion by using the safe functions one can derive the temporal equation. So, after deriving the temporal equation one can use this ordering technique, use the ordering technique to find the temporal equation motion. So, next class we are going to study about the ordering technique for commonly used non-linear equation motion like Duffing equation, van der pol equation, Mathieu equation or Mathieu hill type of equations.

Thank you.