Non-Linear Vibration

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Module - 2
Derivation of Non Linear Equation of Motion
Lecture - 4
Development of Equation of Motion for Continuous System

Welcome to today class of non-linear vibration. So, in today class we are going to study about the development of equation of motion for continuous systems. So, this equation of motion will derived using d'Alembert principle and Extended Hamilton principle. Later I will tell you how to use this generalized Galerkin's method to develop the temporal equation of motion. So, as you know in case of distributed mass system or continuous system, the governing equation of motion will be that of a partial differential equation, unlike incase of the discrete system. In case of discrete system we have this, we have the ordinary differential equation of motion, but in case of this continuous system we will have the partial differential equation of motion or we may have integro differential equation of motion.

So, today class we will derive the equation of motion for the continuous system and this derivation will be carried out by using some examples. So, where the d'Alembert's principle and extended hamilton principle will be used to derive those equations. So, before deriving these equation of motion for the continuous system so, first we will start deriving the equation for a simple beam under bending; so simple bending of beam by using the Euler Bernoulli Beam equation. So, first we will derive this equation of motion and by using this equation of motion then, we will derive the equation motion for the non-linear vibration of some beams.

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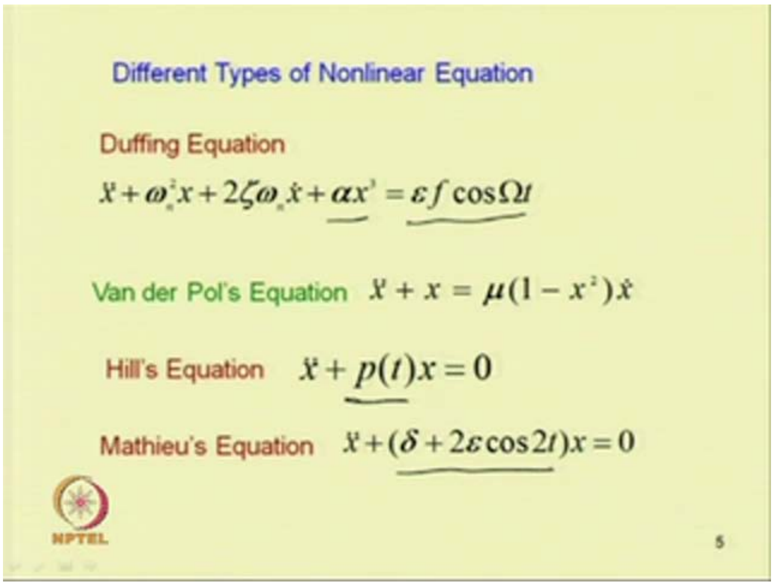

So, till now in case of the discrete systems we have studied different type of non-linear equations or the equation of motion what we have derived till now can be categorized into this types; one is Duffing type of equation, other one is this van der pol's equation and third one is hills equation and Mathieu equation. So, in case of Duffing equation we can have a cubic non-linear term or the non-linearity can be also of higher order so, for free vibration this forcing term can be taken to be 0 and incase of force vibration we can consider a forcing term. Also this forcing term can be considered as a strong forcing in which this epsilon so, will be of the order of 1 and or it can be a weak non-linear system or weak forcing system in which this forcing is of the order of epsilon.

Similarly, in case of the van der pol's equation we can have a equation of this type x double dot plus x equal to mu into 1 minus x square into x dot. And in case of hills equation the simplest type of equation we can have is x double dot plus p t x equal to 0 where, this p t this periodically time varying term is coefficient of the response x. So, if this p t can be written in this form that is delta plus 2 epsilon cos 2 omega cos 2 t then, this equation is known as mathieu equation. So, hills equation will reduce to that of a mathieu equation when p t equal to delta plus 2 epsilon cos 2 t where delta can be written equal to omega n square that is square of the natural frequencies.

So, for discrete systems we have derived these equations where you can see all these equations are that of ordinary differential equation. But, in case of the distributed mass system or continuous system so, we will find the equation which are of partial differential equation of motion because their function the displacement or the space or the state factor are function of both space and time.

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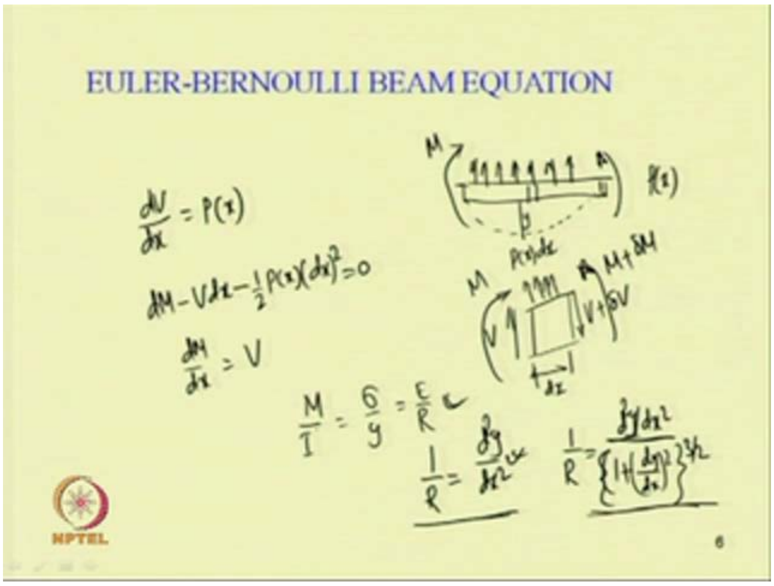

So, let us first derive the equation motion for Euler Bernoulli beam. So, in case of Euler Bernoulli beam let us consider a beam if the beam is subjected to only bending. The beam is subjected to only bending then if it is pure bending then this equation of motion what we will get is known as Euler Bernoulli beam equation. So, let us take a small element so, this beam is subjected to moment M so, if you take a small element let us take a very small element then in this small element so, left side of this let us have sear force V then, this side the sear force is V plus delta V then the moment is M and this side the moment is M plus delta M and if we are considering the loading for unit length equal to P x let P x is the loading for unit length of this on this beam then, so, if the loading is P x for unit length so, here the loading for the small element d x so, this is the small element d x so, the loading can be written as p x into d x. So, in this case if we can take we can do the force balance then.

We can have this relation so, by doing this force balance we can write so, V plus d V minus V will be equal to P x d x. so, we one can write this d V by d x so, we can have this relation d V by d x equal to P x that is, the change of sear force with length equal to the loading for unit length. Similarly, by taking moment about any point in the this side or this side let us take the right side then, one can have so, if you will take the moment about this side then, V into d x, V into d x plus M in M will be equal to P into d x this is the force and it is acting at a distance of d x by 2. So, in this case one can write the equation to be so, d M minus so, in this case the resulting equation will be V M minus V d x minus half P x into d x square equal to 0.

So, for the limiting case one can write so, P M by d x so, d m by d x will be equal to V. so, this means the change of bending moment equal to the sear force, rate of change of moment along the beam is equal to the sear force. So, if one so, from the elementary strength of material one knows for pure bending this M by I equal to sigma by y equal to E by R. So, where M is the bending moment, I is the moment of inertia sigma is the force, sigma is the stress and y is the distance from the neutral access, E is the young's modulus and R is the radius of curvature.

So, one by R, one can write one by r equal to d square so, if you are taking let y is the deflection so, due to this bending let y at any distance x is the deflection of the beam. So, in that case one by R can be written equal to d square y by d x square. So, for large curvature one can write this one by R equal to d square y by d x square by one plus d y by d x whole square to the power 3 by 2. So, one may note that for large curvature if one takes this denominator to the numerator part and expand that thing then, one will get a Non-linear equation. But, for small oscillation or small displacement,

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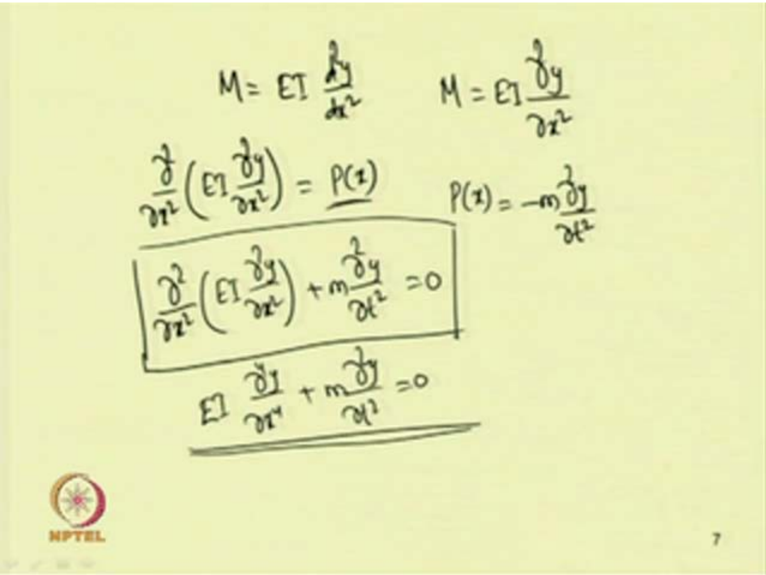

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One can take this radius of curvature or 1 by R equal to d square y by d x square by taking this in this equation so, one can write this m equal to so, one can write M equal to E I d square y by d x square. So, till now we have not considered the time but, in actual case this y is a function of both x and time so, one should write instead of writing m equal to E I d square y by d x square so, one should write m equal to E I del square y by del x square because y is a function of both time and space coordinate x.

So, now by differentiating this M twice already we know this d M by d x equal to v and d B by d x equal to P x so, differentiating M twice, one can get the rate of loading so, differentiating this equation twice so, one can have this d square or del square by del x square E I del square y by del x square equal to P x. Now, for this vibrating beam this rate of loading will be equal to inertia force so, the inertia force equal to mass of that element into the acceleration and it takes place in a direction opposite to that of acceleration. So, this P x can be written as so, one can have this P x equal to minus M so, this is mass for unit length and or mass of that element into d square y by or del square y the other one can write del square y by del t square. So, del square y by del t square is the acceleration, M is the mass and it takes place in a direction opposite to that of the acceleration. So, the equation becomes del square by del x square into E I del square y by del x square plus M del square y by del t square equal to 0 so, this is the Euler Bernoulli beam equation. So, here if the E I term that is product of young’s modulus and moment of inertia or which is known as the flexural rigidity of the system is changing then one can write this equation in this form.

But, if it is constant then, this equation can be reduced to this form so, it is E I del forth y by del x forth plus M del square y by del t square equal to 0. So, this is the Euler Bernoulli beam equation one can obtain.

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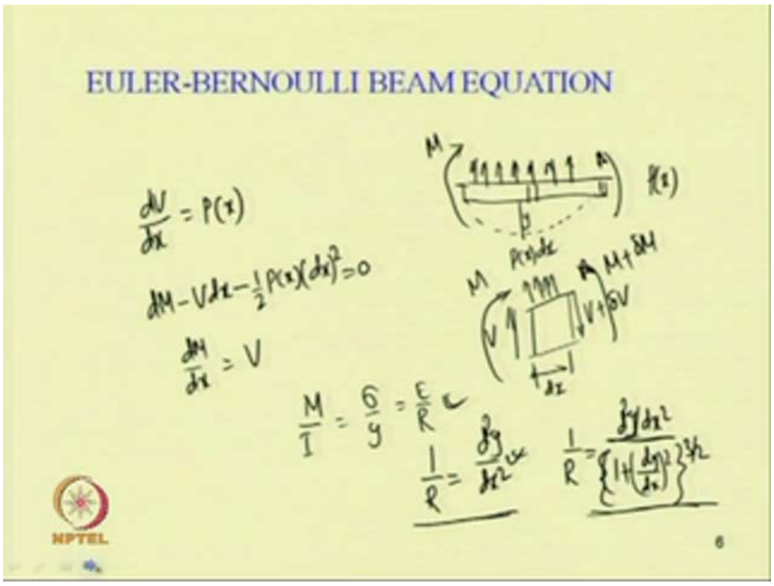

So, if one takes large deflection so, in that case this 1 by R will be replaced by del square y by del x square by 1 plus del y by del x whole square to the power 3 by 2.

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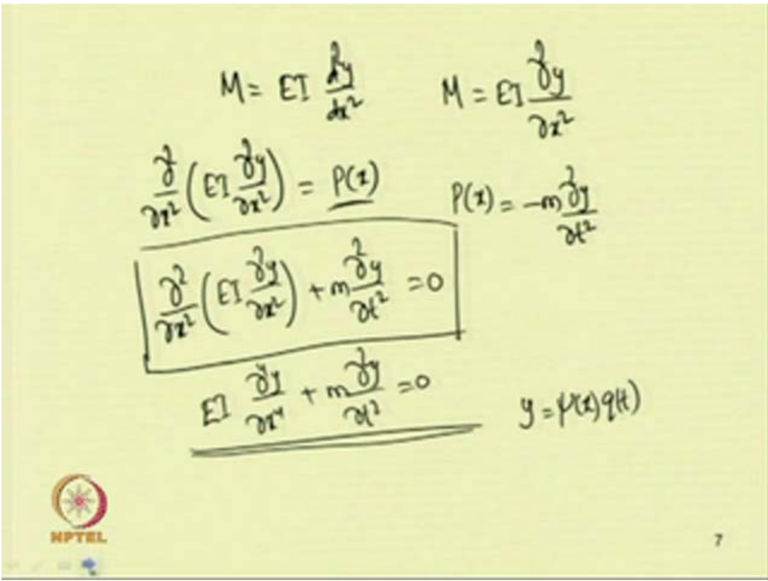

And one can expand this term by taking thing to numerator and have a non-linear equation. So, let us first solve this linear equation or let us see what the importance of this equation is or how it can be solved. So, one can take the variable separation method by considering y equal to let one consider this y equal to psi x into q t so, where psi x is the safe function and q t is the time modulation.

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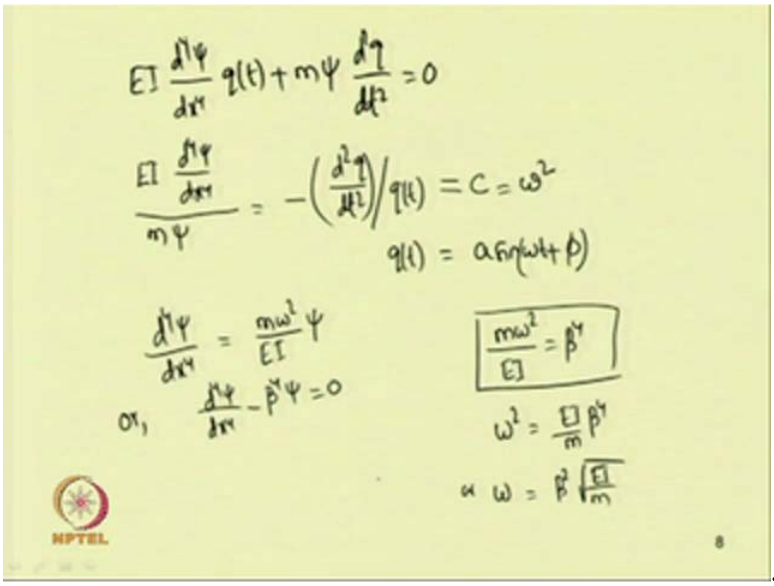

So, if one substitute in this equation then this equation can be reduced to so, it is E I d forth psi so, it will be d forth psi by d x forth into q t. So, for this is a function of psi only x is a function of psi so, differentiation psi into q t plus M so, this is a function of time this del square y by del t square so, q will be differentiated with respect to time and psi will like that so, it will be M del square q by del d square q by M psi into d square q by d t square equal to 0.

Now, separating the terms or time terms into one side and space term to the other side, one can write so, one can write this way so, it will be E I do forth psi by d x forth divided by this M psi it will be equal to minus d square q by d t square divided by q. So, one can see that the left side is a function of space co ordinates and right side is a function of time so, as they are equal so, they should be equal to a constant and this constant will be nothing but, equal to omega square because this d square y by d t square is acceleration term and q is the displacement and in this case acceleration is proportional to displacement and this constant of proportionality is nothing but, the square of the natural frequency.

So, one can write this d square q by d t square by q t equal to omega square or this equation by taking only this part one can write or the motion is simple harmonic so, from this one can write this q t equal to or q t can be written in this form a sin omega t. So, one can put a sin omega t plus pi where, a and pi can be obtained from the initial condition.

Now, from the other part so, one can write this d forth psi by d x forth equal to M omega square by E I, M omega square by E I into psi or it can be written as d forth psi by d x forth minus beta forth psi equal to 0. So, where one can take this M omega square so, one can take this m omega square by E I equal to beta forth and from this one can find this omega square equal to E I by M into beta forth or omega equal to beta square into root over E I by M.

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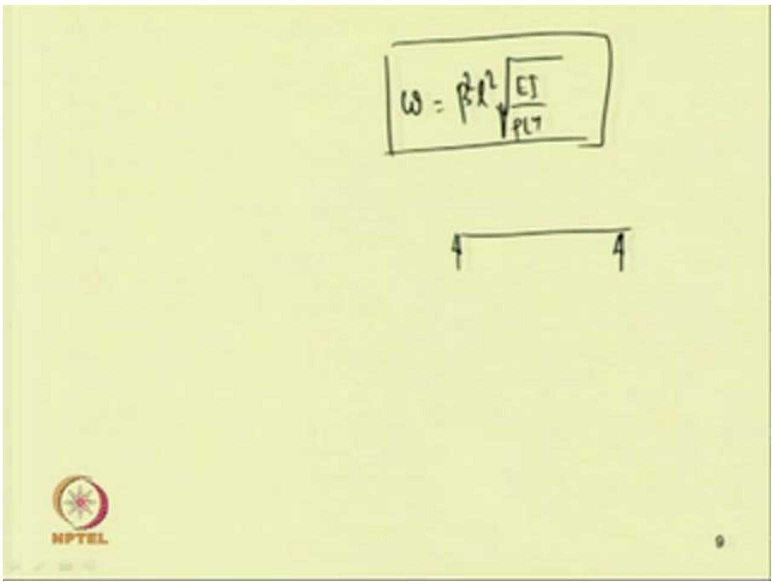

Or if one multiply l so, this omega can be written as so, omega can be written as beta square l square root over E I by rho l forth and one can so, from this equation one can find the expression for psi that is the safe function for different boundary conditions. For example, let us take the simply supported case.

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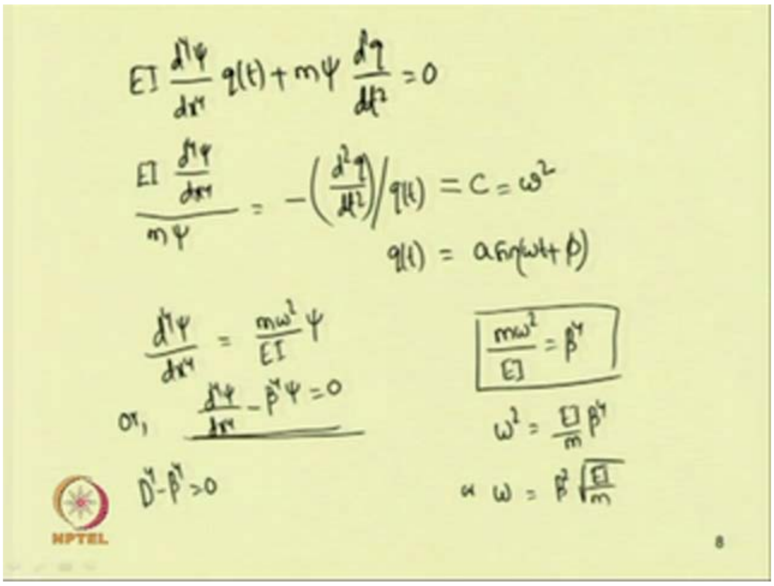

So, in simply supported case the boundary conditions are the displacement and slope are 0 at x equal to 0 and x equal to l. So, in this case the general solution will be so, the auxiliary equation so, for d forth psi by d x forth minus beta forth psi equal to 0,

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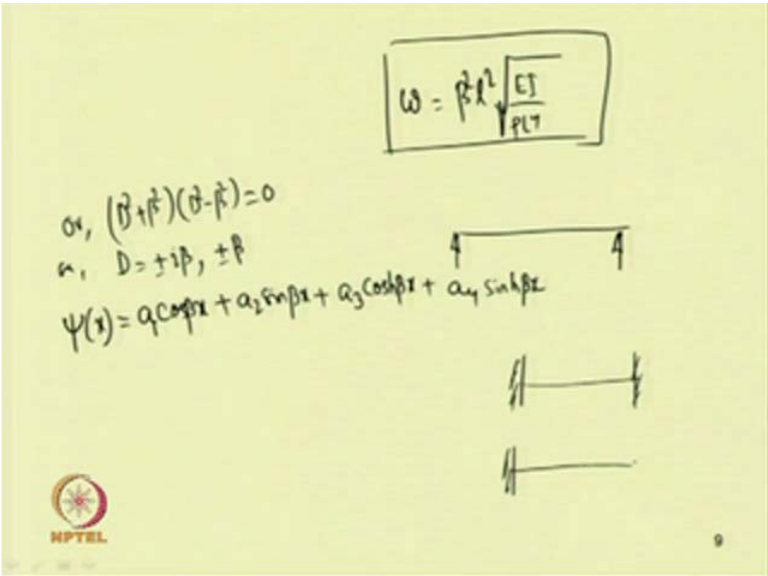

The auxiliary equation becomes d forth minus beta forth equal to 0 or d square plus beta square into d square minus beta square equal to 0 or d will be equal to plus minus i beta and so, this will give so, it will be or it will be plus minus beta so, one can 4 root and from this 4 roots one can find the general solution for psi x. So, general solution for psi x can be written so, when it is plus minus i beta so, the solution will be harmonic and when it is plus minus beta the solution will be hyperbolic.

So, one can write the solution, psi x will be equal to a cos beta x plus or a 1 cos beta plus x a 2 sin beta x plus a 3 cos hyperbolic beta x plus a 4 sin hyperbolic beta x. So, this is the general solution and one can find the specific solution for different boundary conditions. For example, in case of this simply supported case so, the boundary conditions are at x equal to 0, psi x will be equal to 0 and the bending moment also will be equal to 0. So, in case of fixed fix boundary condition, in case of fixed fix boundary condition the slope and displacement will be 0 for both the ends and incase of the cantilever beam so, in case of a cantilever beam the left side fixed condition, fixed side, the displacement and slopes are 0 and incase of the right side which is free the bending moment and sear force are 0. So, by using these boundary conditions so, one can find the solution or the general solution for the safe function.

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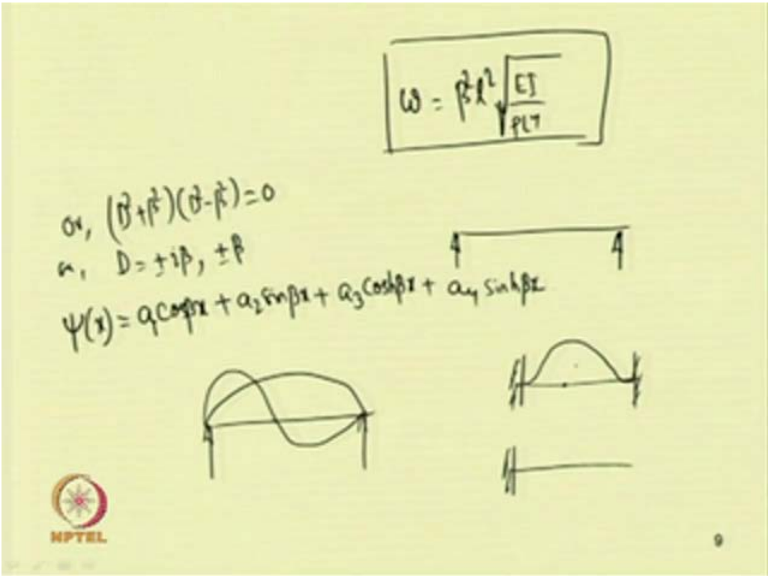

So, for example in case of the simply supported case so, one can obtain the first mode like this so, displacement will be 0 at both the ends. So, similarly, for the second mode so, there will be node formation here so, the displacement will be like this and similarly, for second node second mode there will be 2 nodes and incase of a and incase of fixed fix the first mode will be so there will be slope and displacement 0 at both the ends so, it will be of this type and incase of the second mode so, there will be node here and for third mode there will be 2 nodes.

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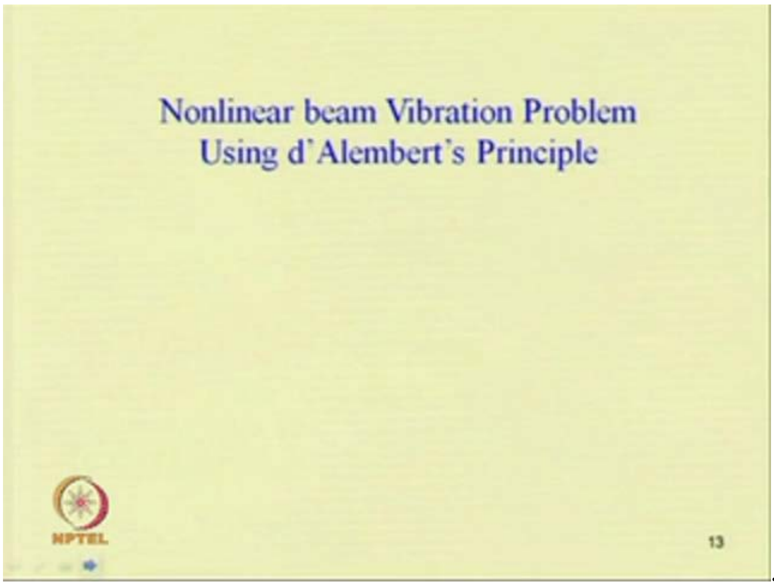

So, the purpose of studying this linear equation for the beam vibration is to apply this concept to the non-linear vibration or for deriving the non-linear governing equation for the non-linear systems non-linear beam systems. So, let us consider a non-linear vibration problem now, we will use this d'Alembert principle to derive the equation motion.

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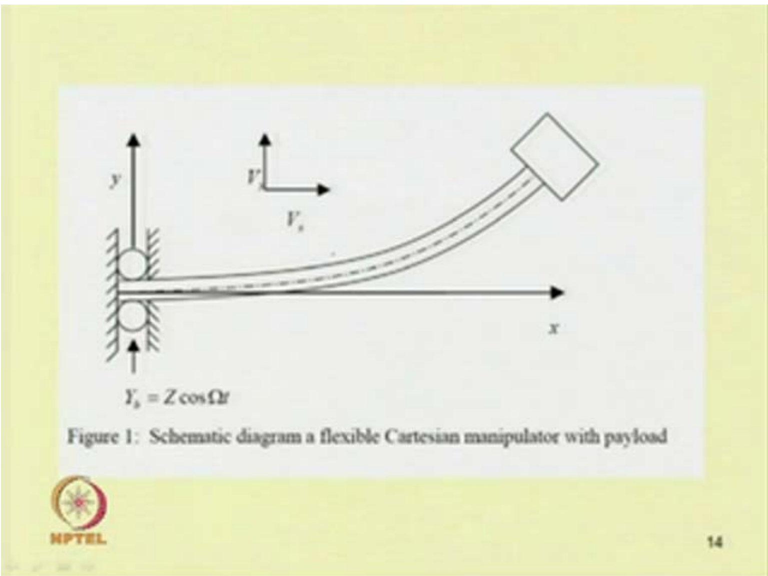

So, in this case so, let us take this example so which so a Cartesian manipulator with a payload or which can be modeled as a cantilevered beam where, the left end is roller supported and which can move up and down and let us consider the large transverse vibration of the beam. So, as we have seen before so, in case of the large vibration so, this M can be written instead of writing M equal to E I del square y by del x square, one can write that equation in this form.

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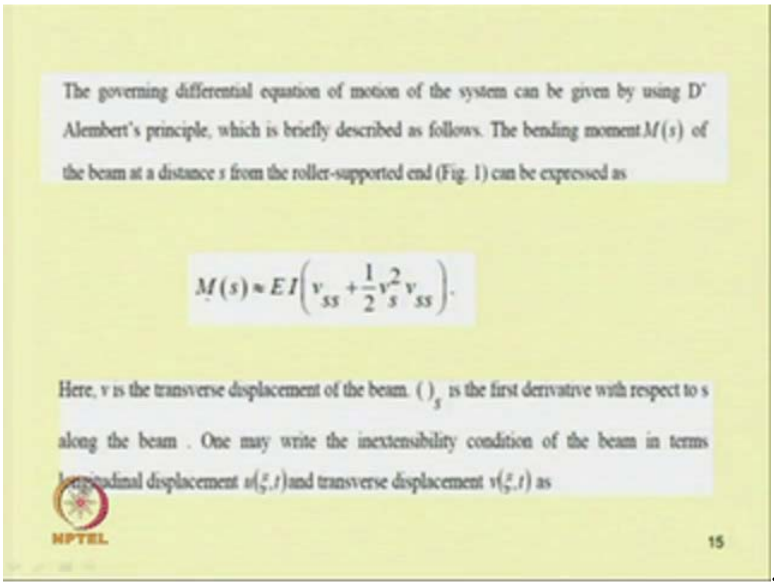

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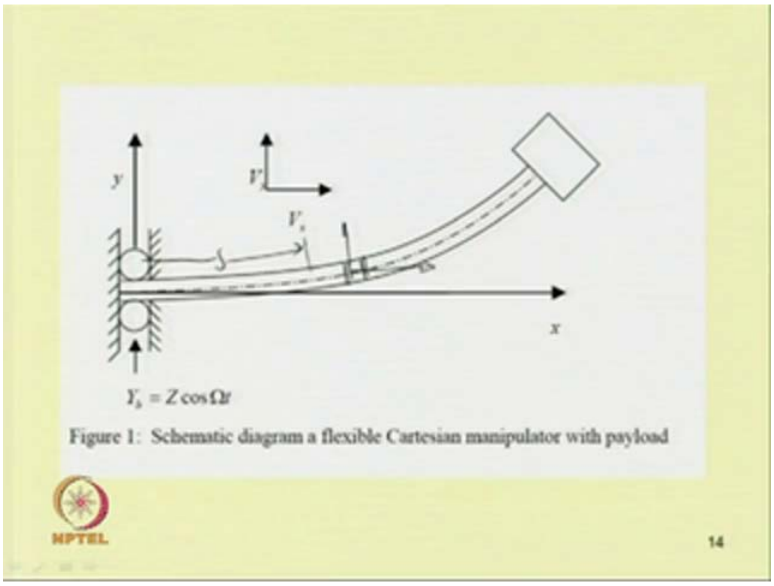

So, this moment at any section s so, if you take the moment at a distance x let us take a take it at a distance S. So, then so this will be written as E I del square v by del x square plus half del v by del s whole square into del square v by del s square. So, here some terms are been neglected and one can get this equation now this bending moment at any section also can be written in terms of the force.

So, for example, if you take a section here the section is subjected to inertia force. So, let us consider if U is the displacement along the axial direction and V is the displacement along the transverse direction then for the section it will be mass of this element into U double dot that will be inertia force in axial direction and so, inertia force for this element we can have this inertia force in this axial direction. Similarly, we can have the inertia force in the vertical direction that will be m into del square v by del t square

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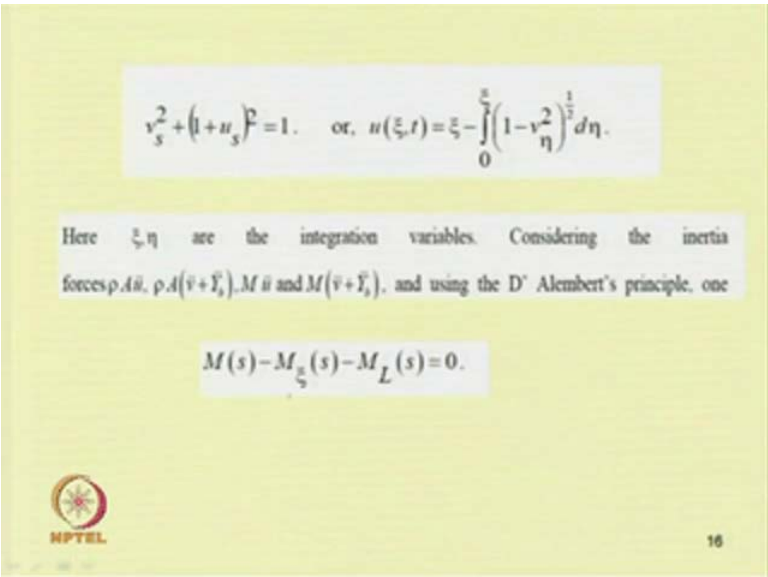

So, taking those inertia forces so, we can find the moment or we can find we can so, at a distance zeta so, one can write this one can find this bending moment also.

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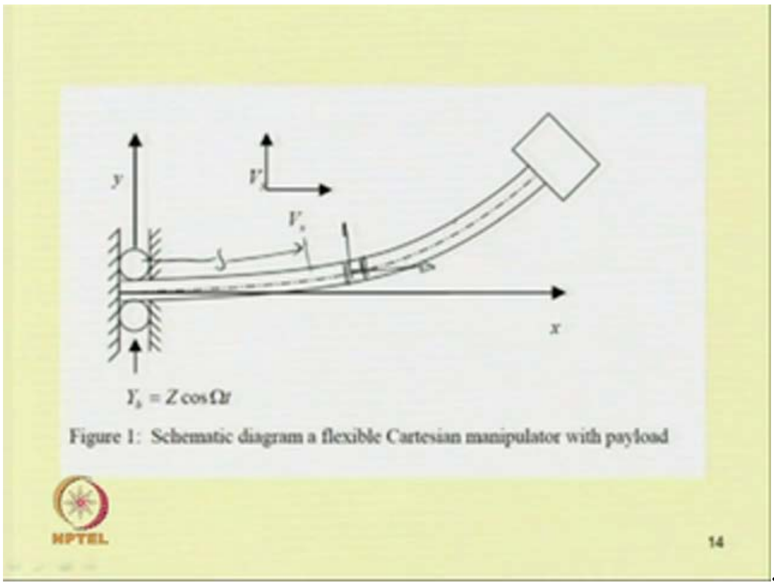

For this first we can find for a single element then we can find for the entire beam by integrating that also one can find the moment due to this end load. So, let m is the mass for this n load so, in that case one can consider the inertia force due to this end mass and one can find the bending moment so, that is represented that by m l. Now, one can write this M s equal to M zeta plus M l or M s minus M zeta minus M l equal to 0. So, here zeta, ita so, if one consider this in extensibility condition so, that that means the beam is not extendable during its vibration or the length remain constant.

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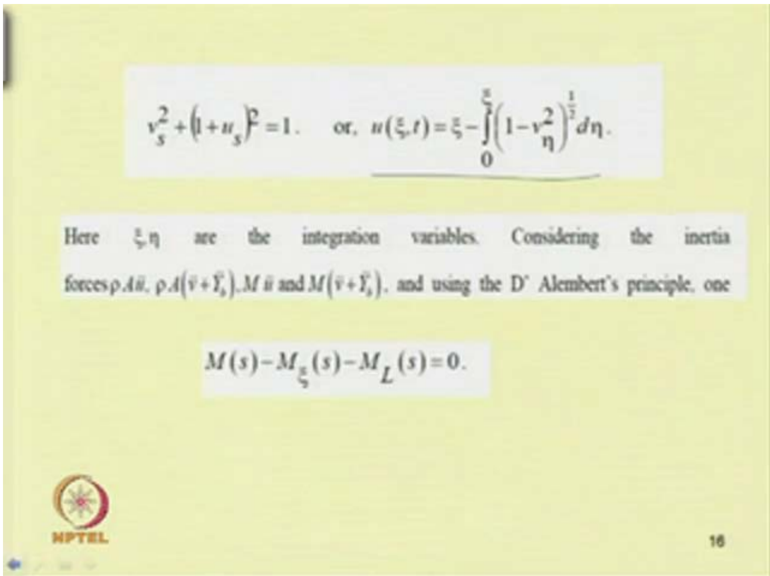

So, one can achieve this relation where del square del v by del s whole square plus 1 plus del u by del s whole square equal to 1. That means, the length remain constant if you are considering the length to be constant then one can have this in extensibility condition. Or in other word one can write this longitudinal displacement in terms of the transverse displacement using this relation.

Now, taking the inertia force so, inertia force will be due to this axial displacement, due to vertical displacement and also due to the displacement of the support. So, here the support is moving with a excitation y b equal to z cos omega t. So, this is an example just we are taking. So, in this case then the inertia force will be rho u double dot where rho is the length or m where rho is the mass for length, a is the area of cross section, u is the axial displacement so, u double dot is the acceleration in the axial direction.

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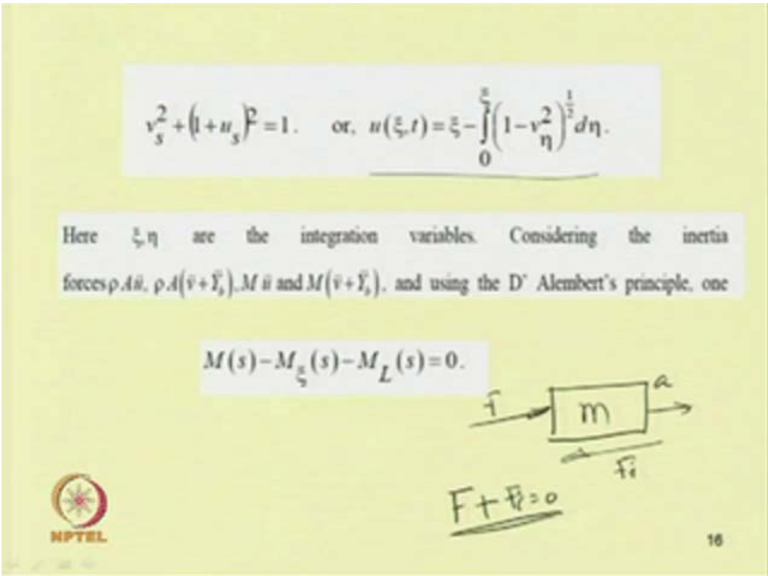

And in vertical direction we have 2 accelerations so, one is due to this base motion and other one is due to the displacement of the beam in vertical direction that is, v from the initial position. So, this acceleration will be rho a v double dot plus y b double dot. And now using d'Alembert principle, one can write this expression, M s minus M zeta s minus M l s equal to 0. So, to recall this d'Alembert principle so, one can write so, let M is the mass of the system so, it is subjected to an external force f and there is acceleration a. So, in this case a d'Alembert principle can be written as the summation of the external forces plus the inertia forces equal to 0. So, here external force is f and inertia force equal to mass into acceleration which acts in a direction opposite to that of acceleration. So, in this case so, this is the direction if this is the direction of acceleration so, inertia force act in this direction. So, that is m into a and this is this can be written as F i.

So, one can have the summation of this force plus a pi will be equal to 0 so, this is d'Alembert's principle. So, external force plus inertia force equal to 0 so, by taking all these forces so one can write the equation motion.

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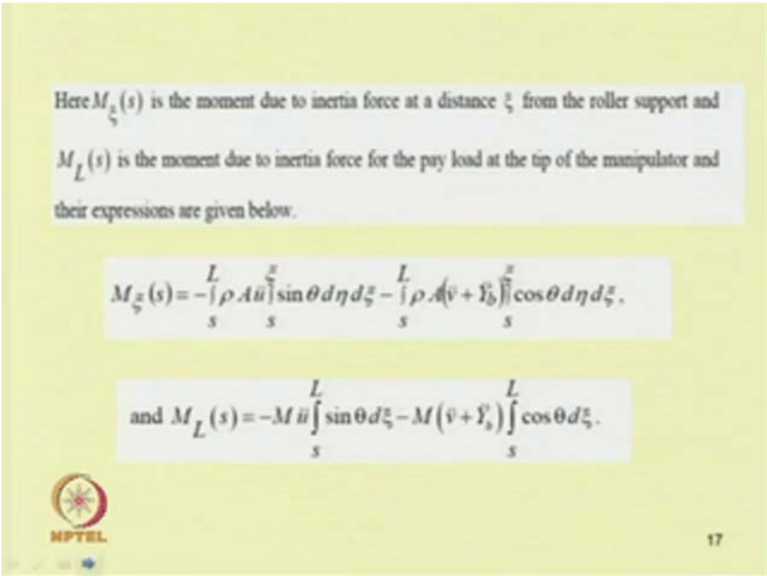

Now, this m zeta so at a section zeta at a dummy distance zeta from base so, by taking the moment one can write m zeta s equal to so, first one can find for the small element so, for the element one can find this and then for the whole beam it will be s 2 L integration s 2 L so, then this m zeta s equal to minus integration s 2 L rho a u double dot integration s 2 zeta sin theta d eta d zeta where, eta zeta are dumb variable so, this is due to axial load or axial force and due to this vertical force one can find similarly, it will be equal to integration s 2 L rho v double dot plus y v double dot integration s 2 zeta cos theta d eta d zeta. So, one can easily find that thing.

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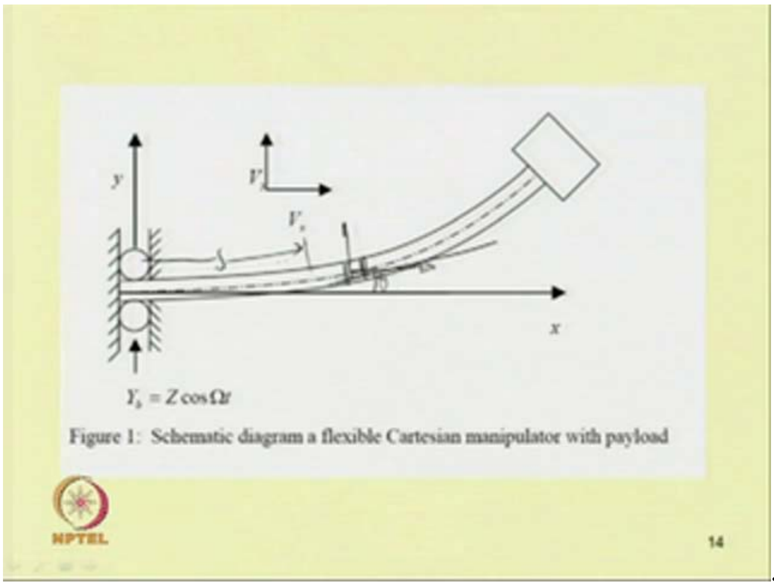

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So, where theta is the angle between so, this is this angle is theta. So, when taking the moment so, one can take the moment about this point so, one can have this cos component and sin component. So, for axial and for vertical between one takes the moment so, then it can be written in this form. Similarly, this M L s that is due to the tip load so, one can write this equation M L s will be equal to this is the bending moment due to the tip load.

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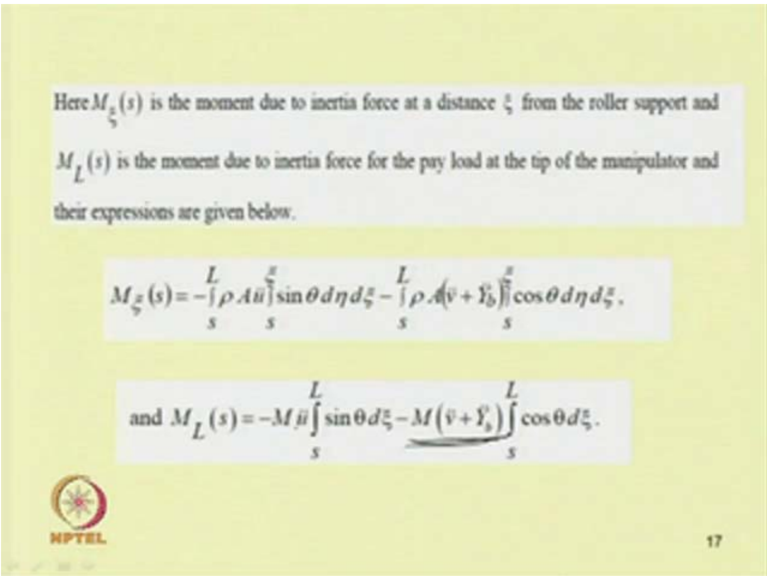

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So this will be equal to minus M u double dot integration s 2 L sin theta d zeta minus M. So, this is due to the axial this is due the vertical direction inertia force and this is due to the axial direction inertia force.

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$$EI\left(v_{ssss}+\frac{1}{2}v_s^2 v_{ssss}+3v_s v_{ss} v_{sss}+v_{ss}^3\right)+\rho A v_s\left(\int_0^s\left(\dot{v}_\xi^2+v_\xi \ddot{v}_\xi\right)d\xi\right)+v_s v_{ss}$$
$$\left(\int_s^L \rho A \ddot{v}\, d\eta+\rho A \ddot{Y}_b(L-s)+M\left(\ddot{v}+\ddot{Y}_b\right)\right)-v_{ss}\left(\int_s^L\left(\rho A\int_0^\xi\left(\dot{v}_\xi^2+v_\xi \ddot{v}_\xi\right)d\eta\right)d\xi+M\int_0^s\left(\dot{v}_\xi^2+v_\xi \ddot{v}_\xi\right)d\xi\right)$$
$$\left(1-\frac{1}{2}v_s^2\right)\left(\rho A\left(\ddot{v}+\ddot{Y}_b\right)\right)=0. \qquad (1)$$

And finally, one can write this equation in this form so here one may note that this u has been replaced by this u term has been replaced by this v term using this in extensibility condition. So, if one use this in extensibility condition and differentiate this equation twice differentiate the equation this twice where M s equal to where M s equal to M zeta s plus M L s. So, one can obtain this equation so, where this E I del forth v by del s forth plus half del square plus half del A del v by del s whole square into del square del forth v by del s forth plus 3 into so, this becomes v s that is differentiation once, differentiation with respect to space coordinate s then v s s v triple s.

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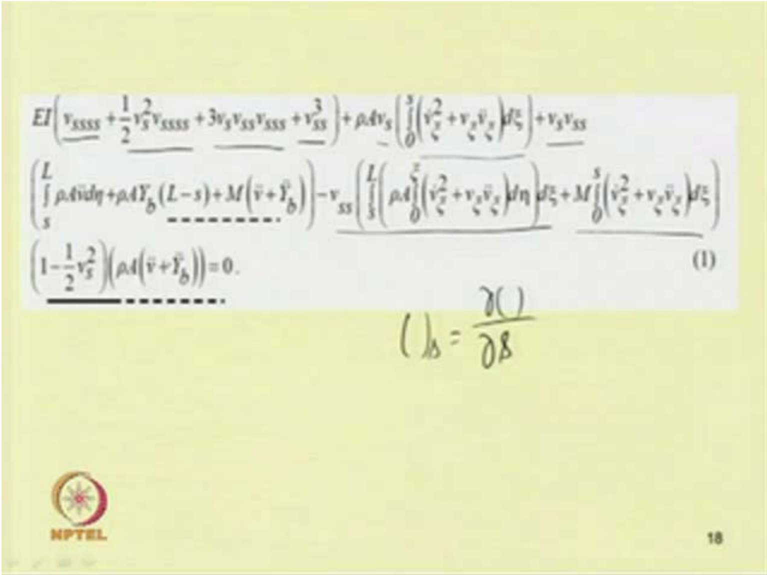

So, where this A is represent del by del s. So, one can note in case of the Euler beam equation we have only the simple term E I del forth v by del s forth. This term plus m v double dot equal to 0. Now, by taking this large displacement or large curvature into account in this vibration one can obtain this equation in this form where many non-linear terms are coming into picture. So, these terms are non-linear terms so, these are non-linear terms where this product of 2 displacement terms are there. So, all these terms are non-linear terms. So, this is also a non-linear term this term is non-linear and these terms are also non-linear but, one may note this term so, where one can find this coefficient of v s square equal to v double dot plus y v double dot y v double dot this y v is written that is the base acceleration of the base motion.

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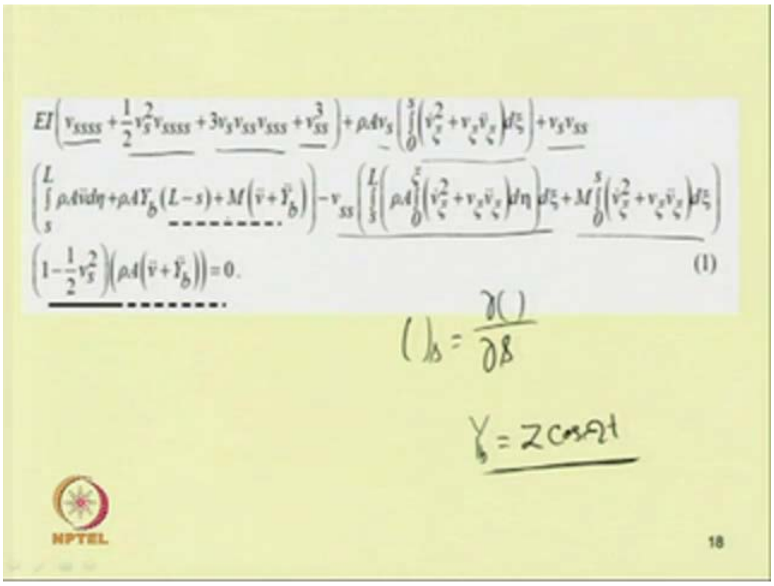

So, this y v if one represent this y v by z cos omega t that means if the base is moving periodically then, in this case so, this periodic motion is a coefficient of the displacement. As this term is coefficient of the displacement then this equation is known as a parametrically that of a parametrically excited system. So, in addition to that one can see this term also, in which this term so, this will give rise to a forcing term so, one can have a force and parametrically excited system in this case.

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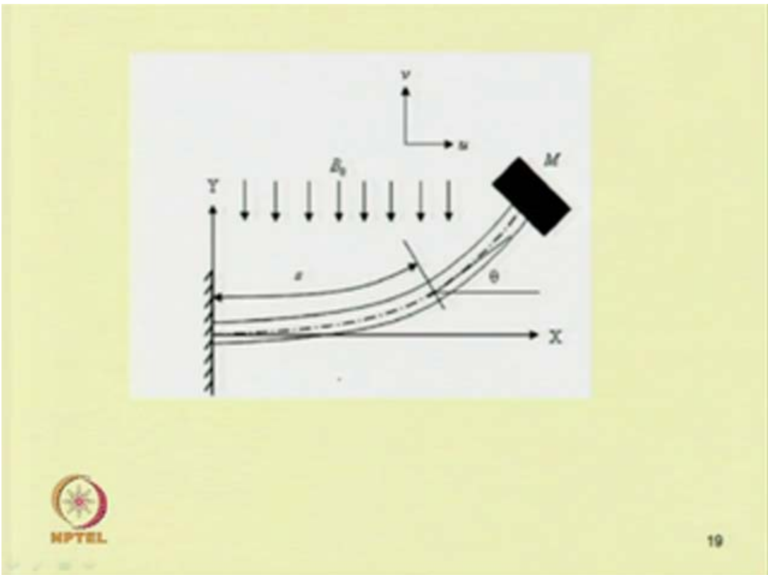

So, as an assignment one can take the system so, let the same cantilever beam is subjected to and magnetic field so, in this case how to determine the equation motion. So, in this case also one can consider the effect due to this magnetic field, the if one take a small element and this beam if this beam is a conducting that of a conducting material or magneto elastic material then, it will be subjected to a magnetic load in axial direction and in addition to that it is experiencing a couple so, one has to take this force.

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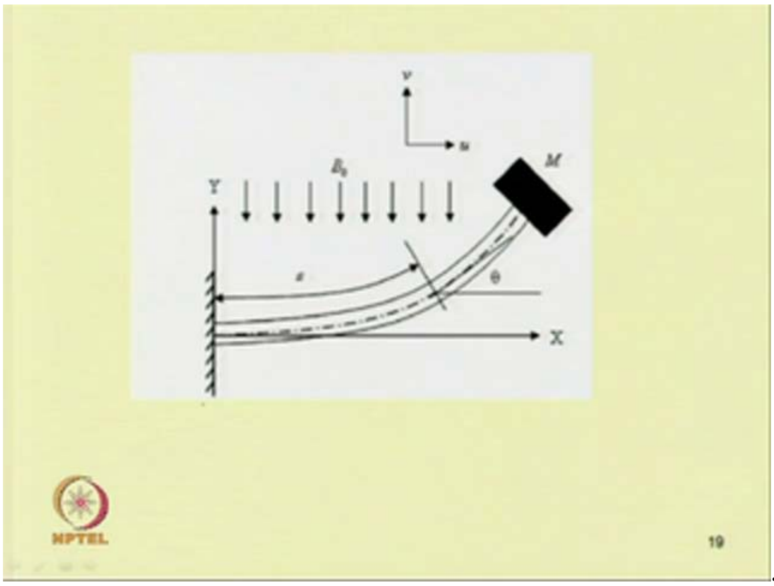

And couple in addition to the inertia force acting at a distance s from the base of the beam. So, by taking all those forces into account and by applying this d'Alembert principle one can obtain the equation motion.

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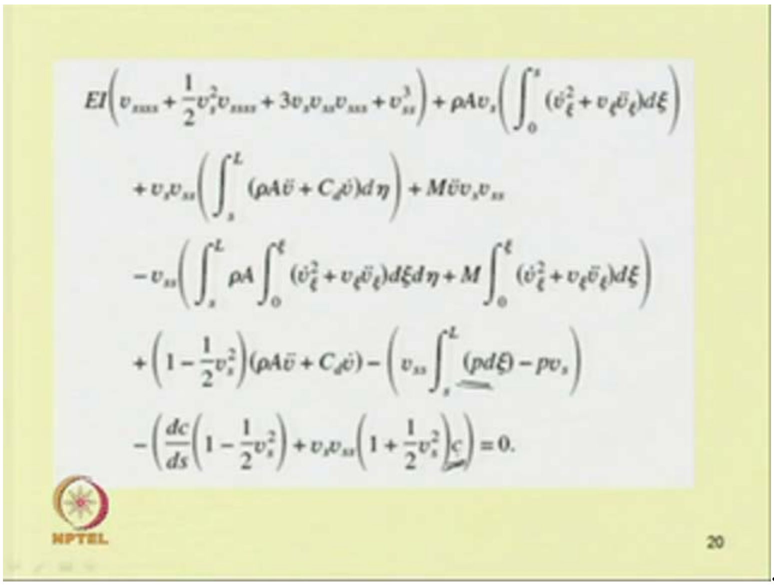

So, in this case the equation motion will be of this type in which only 2 terms can one can observe that there are two terms one is this p one contain this p term. So, p is due to the loading due to this magnetic field, axial loading due to magnetic field and one can find another term this c. This is the couple due to this magnetic field one can have a force and a couple. So, including the inertia forces acting at a element one can derive the equation motion in a similar way.

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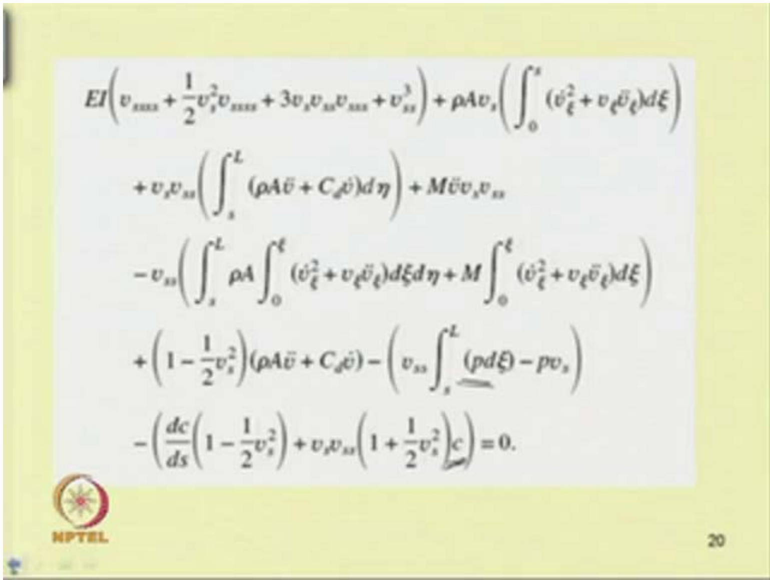

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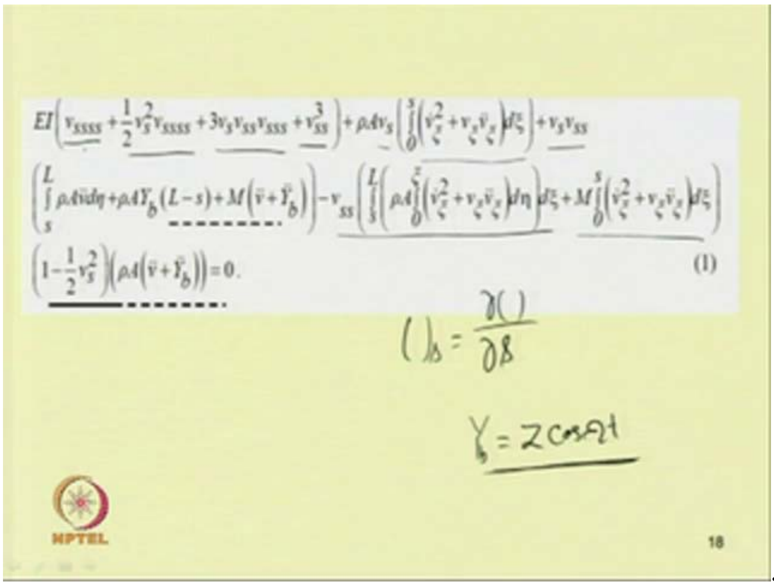

So, now this equation or the previous equation what we have derived so, in this case this v is a function of both space and time. So, like incase of our simple Euler Bernoulli beam so, we have separated the variable by using this variable separation method, here also similarly, one can separate the variable that is the time and the space coordinates by using this generalized Galerkin principle. So, in this generalized Galerkin principle so, one can take this v, one can take this v as a function of a scaling factor r into psi i s and q i t so, where i equal to 1 to n.

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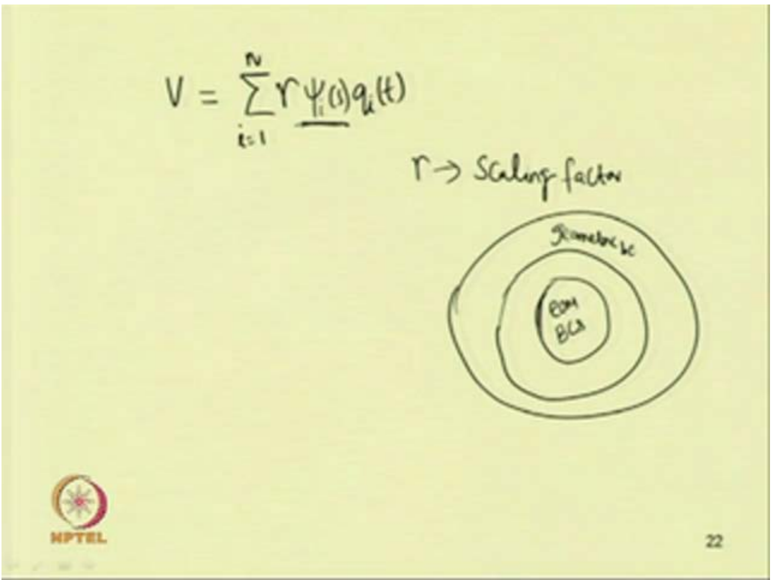

So, instead so, as a continuous system has infinite degrees of freedom so, instead of considering all the degrees of freedom one can truncate or one can consider only few modes for this analysis purpose. So one can limit this n to few modes and then perform this analysis. So, by taking this n let for example, taking n equal to 1, one can take the single mode approximation.

And if one takes n equal to 2 then, it will be 2 mode approximations and if one takes n equal to 3 then, it will be 3 mode approximations. So, here r is known as the scaling factor. So, one can use a scaling factor r so, generally the scaling factor is used to order the coefficient of the equation motion and psi s is the safe function and q t is the time modulation of the ith mode so, this i represent the ith mode. So, if one considers the psi as the Eigen function of the system then, this Eigen functions are orthogonal. So, one can take the psi as either Eigen function or comparison function or an admissible function. So, in case of the Eigen function the safe function satisfy both the boundary condition and the governing differential equation. So, this is Eigen function so, Eigen function satisfies both equation motion and differential equation motion and the boundary.

And this comparison function satisfy only the so, it satisfy only the boundary conditions so, all the boundary conditions it satisfy but, it does not satisfy the equation motion and this admissible function satisfy only the geometric boundary condition so, it satisfy only geometric boundary condition. So, in case of this continuous system we have 2 different types of boundary conditions; one is the geometric boundary condition and other one is the force boundary condition or natural boundary condition.

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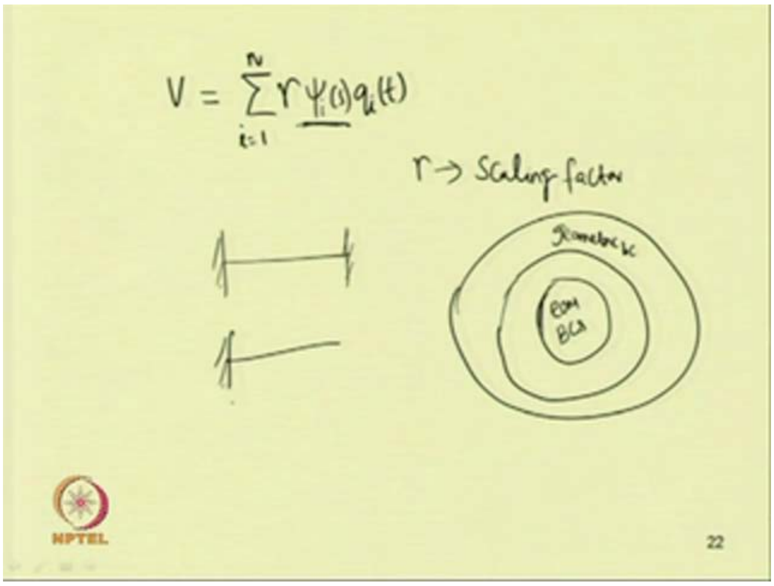

For example, in case of this fixed fix beam so, we have both geometric boundary conditions as at both ends the displacements and slopes which are geometric term are 0. But, in case of this cantilever beam the left end we have this geometric boundary condition but, in the right end we have the force boundary condition. So, in case of right end we have this sear force and bending moment equal to 0.

So, both sear force and bending moments are force boundary conditions. So, in case of the cantilever beam so, if one take the admissible function so, one has to take a function which satisfy only the geometric boundary condition that is, it has to satisfy the displacement and slope at x equal to 0 equal to 0.

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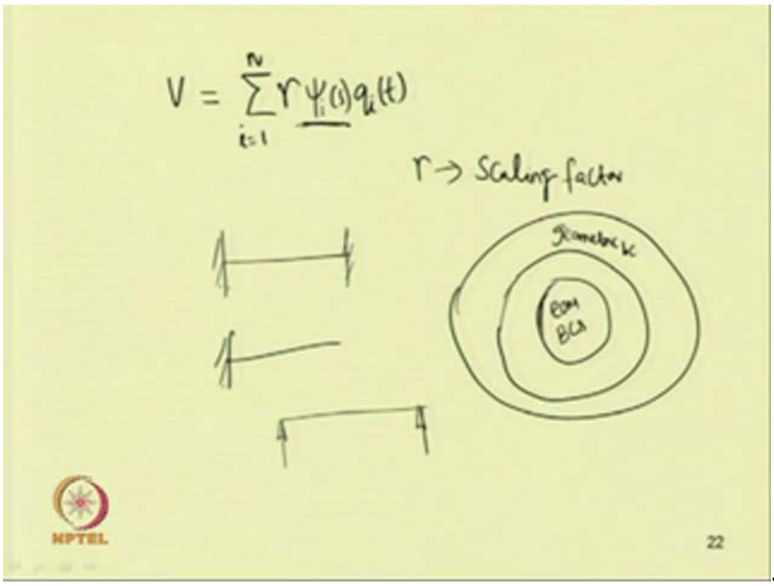

Similarly, in case of a fixed fix beam so, one has to consider a function where both displacement and slope are 0 at x equal to 0 and at x equal to l. But, in case of a simply supported beam so, in case of a simply supported beam so, one can consider the geometric boundary condition only so, in that case the displacement at both the ends equal to 0, the force boundary conditions becomes the bending moment equal to 0 at the left end and also at the right end. But, for simpler geometric conditions like the simply supported, clamped free or cantilever or fixed fix as the Eigen functions are easily available. So, one can take those Eigen functions easily for the analysis purpose and also one can use the orthogonality property of this Eigen function to find the Governing temporal equation motion of the system.

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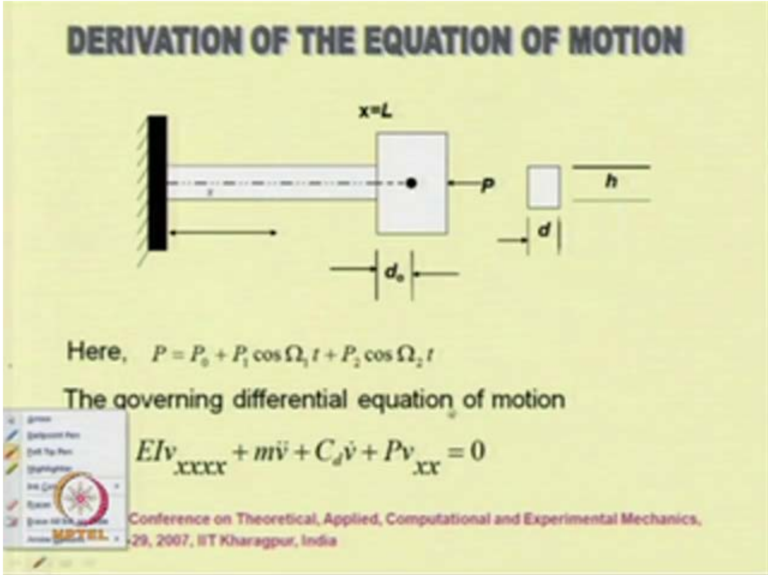

So, for example in this case by substituting v equal to r psi s and q i (t). So, if will apply the Galerkin method.

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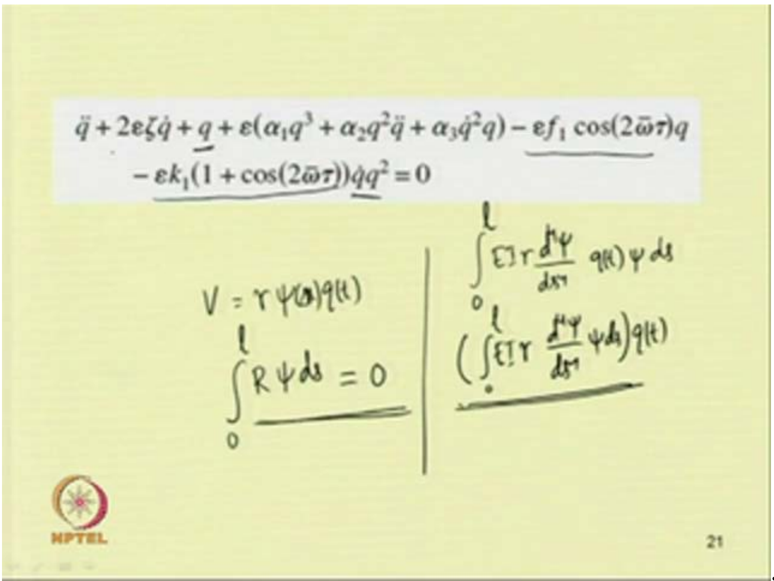

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So, one can apply the galerkin method and one can reduce this equation to that of the temporal form where the equation in space and time is written only in terms of the time coordinate that is q or time modulation and the space part becomes the coefficient of that term. So, by taking a known safe function or by taking the known Eigen function of the system so, one can easily find these coefficients and one can convert these equations in space and time to that of temporal form. So, by to apply the galerkin method so, let us consider this equation so, this equation in this equation by substituting v equal to so, if one substitute v equal to let us take only a single mode so if one substitute v equal to psi x and q t and if psi is not a function which is satisfying the differential equation of motion then, there will be some residue left in the equation. So, the by substituting that equation will not get this left side equal to 0 so, there will be certain residue in this equation and our aim should be to minimize that residue. So, to minimize that residue so, we can minimize that residue by integrating over the whole length so, this is the residue and let us multiply the psi i th or psi and integrate it over the domain.

Now, you equate to 0 so, this will reduce this equation this will reduce this equation in space and time to that of time domain. For example, let us take only one term and see how it can be reduced so, let us take this first term that is E I del forth v by del s forth so, if you integrate that thing by substituting v equal to r psi s and q t. So, then this becomes so, E I so, we have differentiate it forth time so, this becomes r so, this becomes del forth or now, this d forth psi so, this becomes d forth psi by d s forth into u t into so, you have multiply the psi into d s. So, if you are taking psi is a function of s then, it will be d s instead of d s it can be of t s now by integrating this term so, one can write integration 0 to l, E I r d forth psi by d s forth into psi d s into q t.

So, this term become the coefficient of q t so, here one can divide this term throughout the terms and get the coefficient equal to 1. So, in this way by taking the terms term by term one can convert this spatio temporal equation to that of a temporal equation. And one can note that in this temporal equation so, this time varying term is a function of is coefficient of q so, this is a parametrically excited system so, this is a linear parametric excited term. But, here this time varying term is a function is coefficient of a non-linear term that is q dot into q square. So, this is a non-linear parametric term so, in this way one can derive the governing temporal equation motion of the system so, by using in this class I told you how one can use this temporal equation, how one can use this D’alembert principle to derive the equation of motion and then, apply this galerkin method to convert that governing differential equation or partial differential equation to its temporal form.

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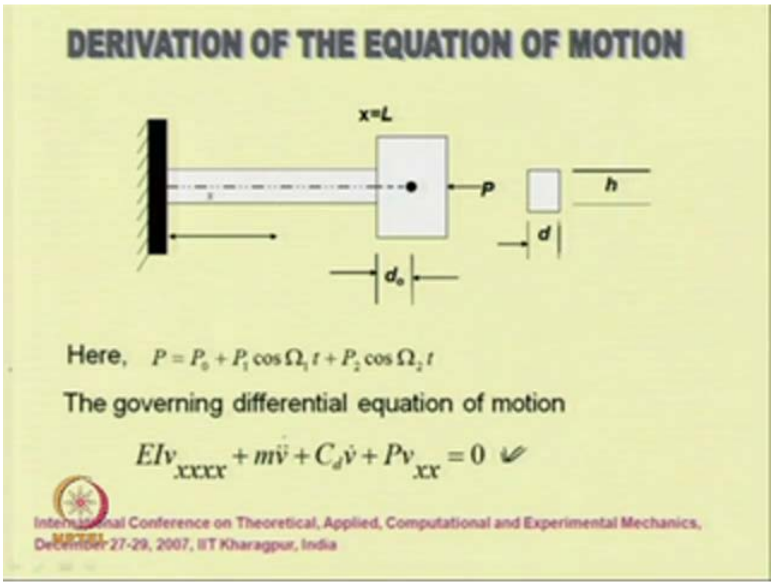

So, we can see another example also let this is a simple cantilever beam with a attached mass where the, where the attach mass has the point mass is concentrated, the mass is concentrated at this point at a distance v 0 from this end. So, in that case let us apply a force p equal to p 0 plus p 1 omega one t plus p 2 omega 2 t so, simply using this Euler Bernoulli equation one can have these 2 terms. Then, by adding this damping term one can have this c d v dot term and one can add this additional term, additional term due to axial loading. So, one can find this governing equation so, this equation can be this spatio temporal equation, later can be converted to its temporal form by applying the galerkin.

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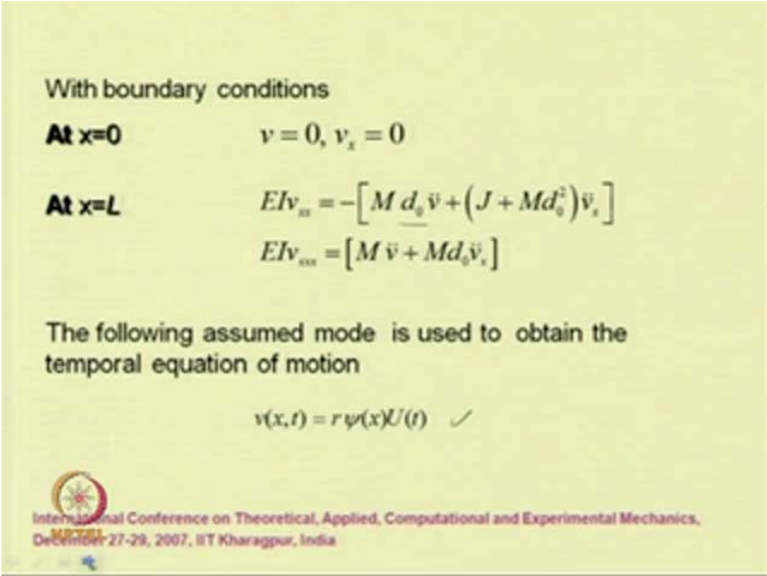

So, one can so, in this case these are the boundary condition so, at the left end at is fixed so v equal to 0 and the slope also equal to 0. At the right end so we will have both bending moment and sear force equal to 0 as the bending moment equal to 0 so, this E I del square v by del x square will be equal to at the mass, the mass is concentrated at a distance d 0 so, one can take the moment and one can find so, this will be equal to this and by differentiating. So, one can find the sear force equal to this so, these are the boundary conditions and taking this v x t equal to this.

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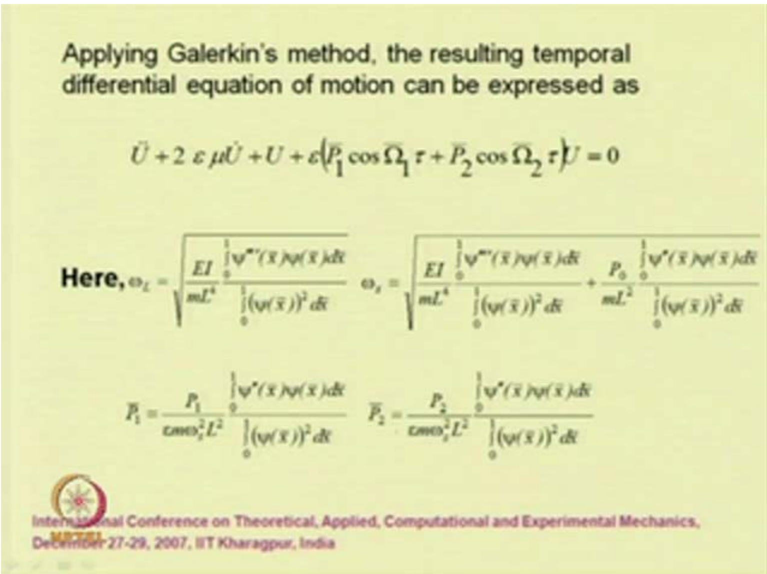

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And using this galerkin method so, one can find the governing temporal equation motion. So, this is the governing temporal equation of motion where this omega l and omega s terms can be written in this way and these are the coefficients, these coefficients can be written in this way. So, today class we have studied about how to derive the equation motion using d'Alembert principle and by applying this galerkin method, how we can convert this spatio temporal equation of motion to that of the temporal equation motion.

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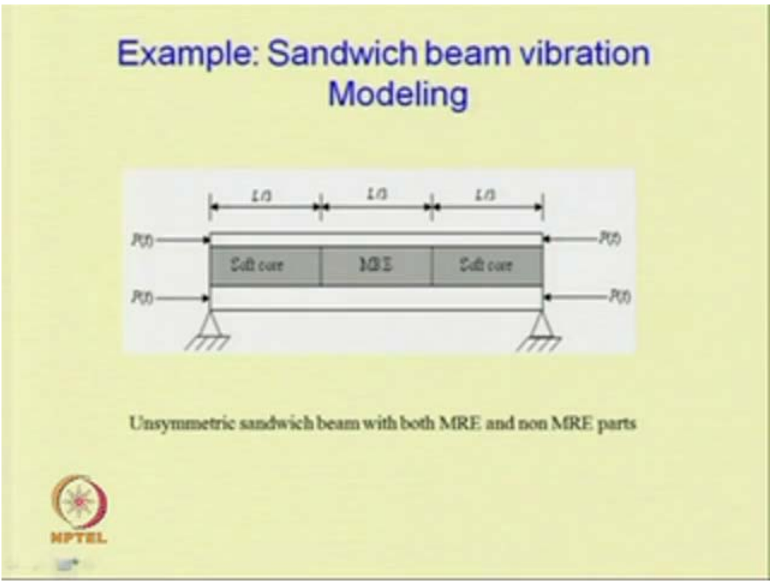

So, you can take some exercise problems so, in this exercise problem, so, you can derive the equation motion for a sandwich beam or you can derive the equation motion of some other systems like a cantilever beam.

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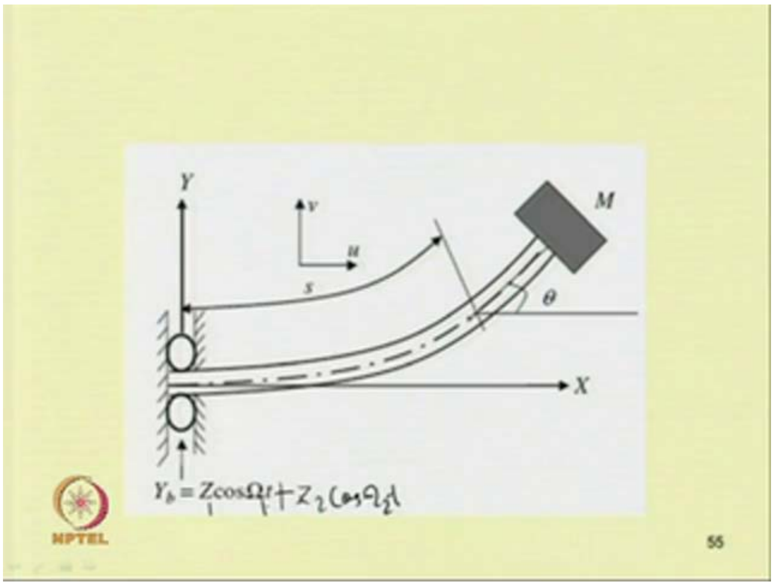

Base excited cantilever beam where the base motion can be given by z 1 cos omega t and z 2 cos omega 2 t. So, in this case it will be subjected to 2 frequency excitation you can take multi frequency excitation also.

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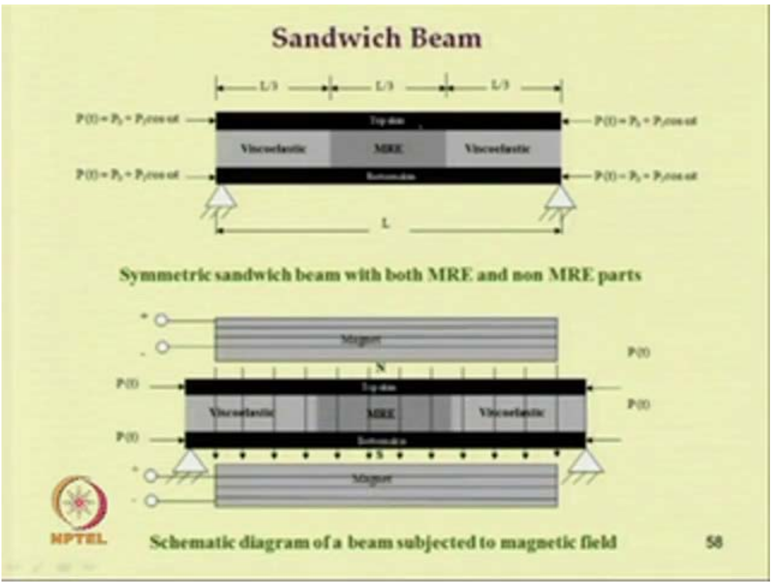

And you can derive the equation motion and also you can derive the equation motion of a sandwich beam, where this middle layer, middle visco elastic layer you can take to content a MRE part that is magneto rheological elastomers part. So, by applying this magnetic field so, you can have this force due to magnetic field and moment due to magnetic field, and taking those force and moment you can find the equation motion by considering the inertia force in longitudinal and transverse direction for both top and bottom skins. So, next class, we will study how to derive this equation motion using energy principles.

Thank you.