Non-Linear Vibration
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Module - 2
Derivation of Non-linear Equation of Motion
Lecture - 2
Lagrange Principle and Hamilton's Principle

So, welcome to the second lecture on this module 2 on non-linear vibration. In today's class, we are going to study about the Lagrange principle and Hamilton principle. We will apply these two principles to different mechanical systems and we will derive the equation motion.

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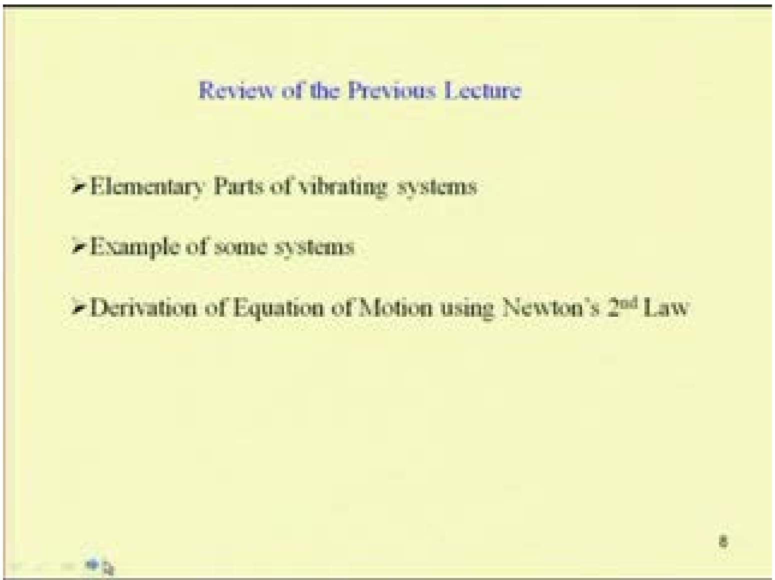

In the last class, we have studied about elementary parts of vibrating systems, then examples of some systems and then derivation of equation motion using Newton's second law.

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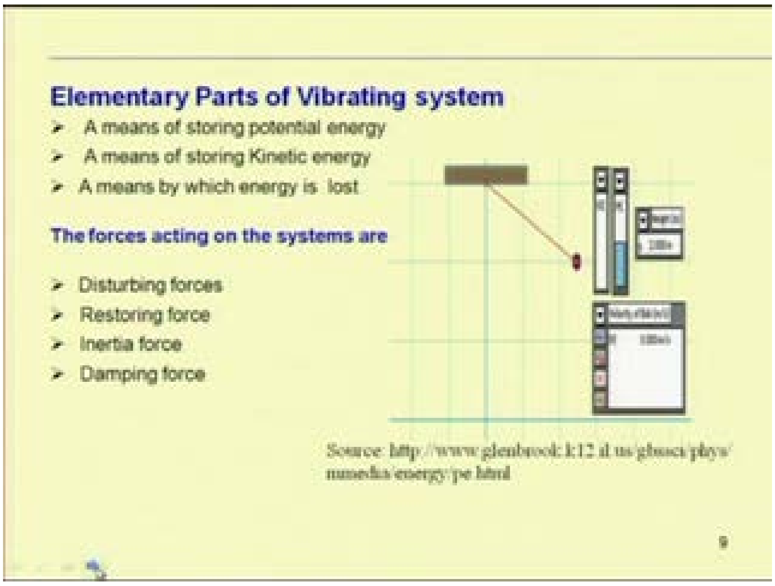

So, if you briefly preview the systems, you know that elementary part of the vibrating systems contains this disturbing force. So, one has to apply a disturbing force and then it will come to a position where all the energy are converted to potential energy. So, here it is subjected to a restoring force and this restoring force when it is coming to this equilibrium position, and here the velocity becomes 0, but due to this inertia, the system is subjected to a force and it comes back to this direction.

So, when you are taking the system or when we are applying a disturbing force, the system moves from this equilibrium position or from some initial position to a particular position, where the velocity is 0. Now, due to the restoring force, it come backs to the equilibrium position where the velocity is maximum and again due to inertia force, it goes back to this side. So, in this way the vibration continues. So, you required a disturbing force, restoring force, inertia force and to damp out the vibration, also you require a damping force. So, restoring force stiffness is responsible for the restoring force. Inertia force corresponds to the mass of the system and dumping force result in or arises from the dumper of a system.

So, you require a means of storing potential energy, a means of storing kinetic energy and also a means by which the energy is lost in this simple example of this simple pendulum. So, one can illustrate the variation of this energy from one form to another form for example. So, at this position, all the energy are kinetic energy and here, it is purely potential energy when it is moving from this position to this position. So, this is a partly kinetic and partly potential energy which one can observe from this diagram.

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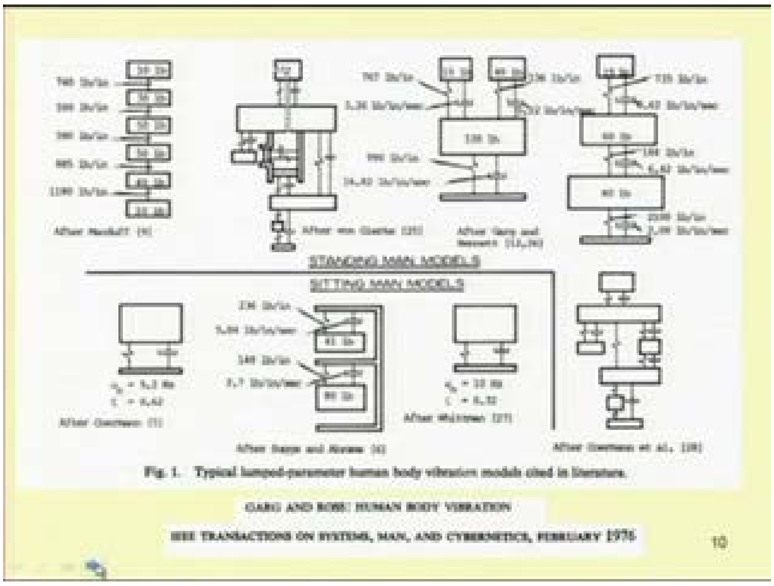

So, we know how to or for a different mechanical systems or different systems. For example, this is a biomechanical system in which the standing position of one person is shown standing and sitting posture of one person is shown. So, here the different parts of the body can be modeled as lump parameter systems. So, it can be modeled as a single system, single degree of freedom system and multi-degree of freedom systems. So, this is a single degree of freedom system when it is in sitting posture. So, one can have a two degree of freedom system also or one can have multi-degree of freedom system also.

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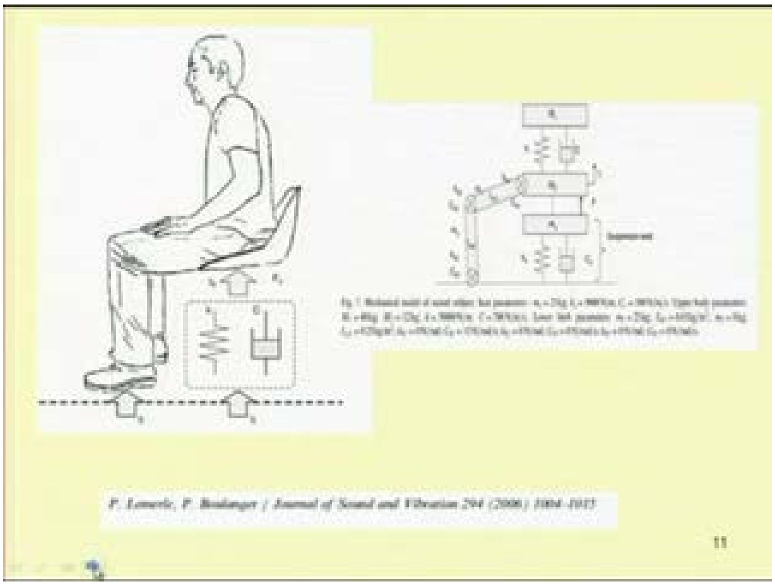

So, this is the sitting position. So, the leg is modeled as tool link. Then different postures and the sitting arrangement is modeled by a spring mass dumper system, and the backbone and head is modeled also by different spring mass dumper systems. So, in this way one can model different system by different lump parameter model system.

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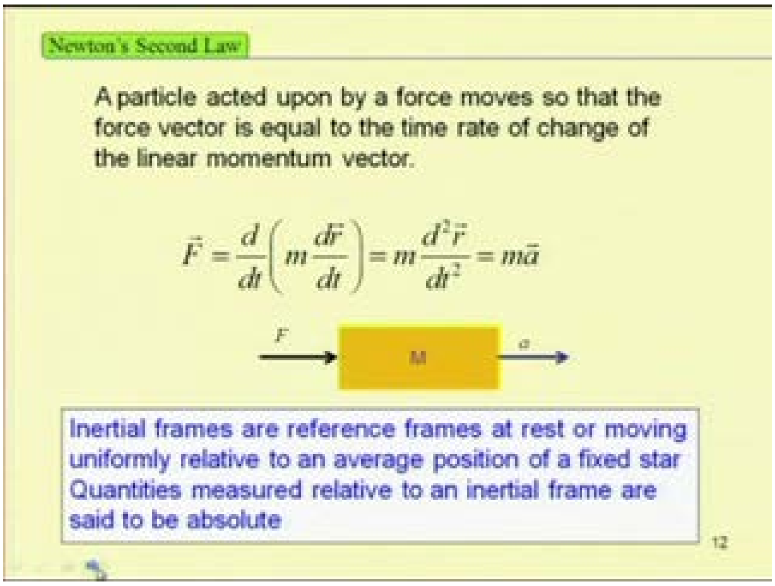

In the last class, we have studied about this Newton's second law. So, in case of Newton's second law, when you are applying a force F to a mass M, it is subjected to one acceleration a, where we can write this F equal to M a. So, in this case of second

Newton’s second law, we are assuming one inertial frame and based on this inertial frame, we are writing the equation motion in vector form. So, one can note that this force and the acceleration, they are vector quantities. So, when we are considering multi-degree of freedom system, this vector quantities representation of the vector quantities which is some problems to eliminate those things.

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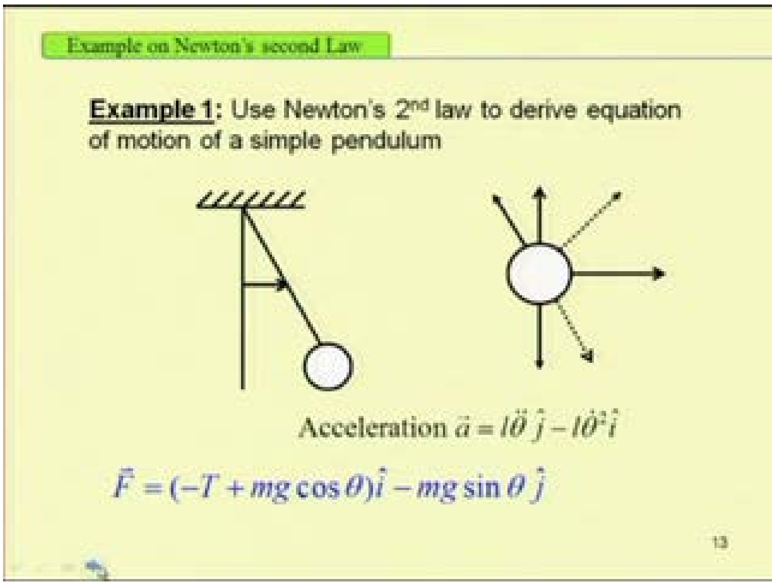

So, one can go for using scalar quantities or one can use this energy based principle which derive the equation motion. So, in the last class, we have derived this equation motion of this simple pendulum using Newton’s second law. So, first we have written the force, but this external force acting on the system and we have written the acceleration expression. So, acceleration can be written as l theta double dot minus l theta dot square. So, one can write using this i and j co-ordinate system, one can write this force and equating this force with mass into acceleration.

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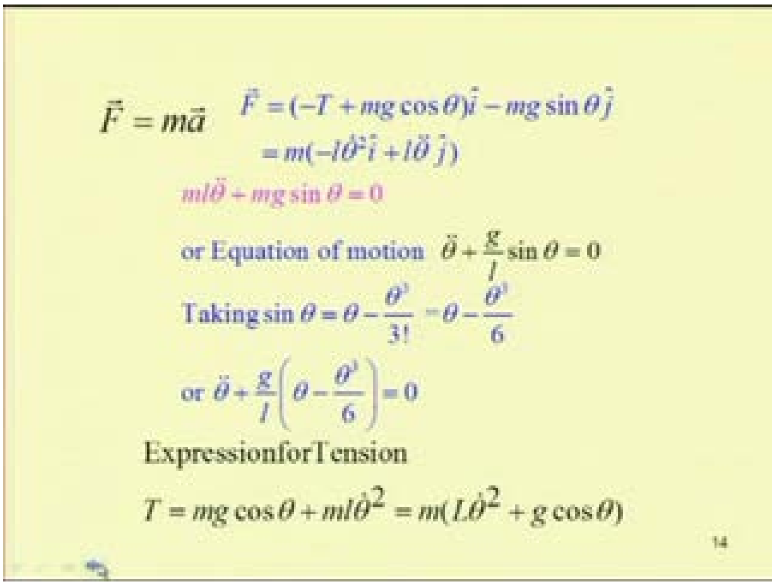

So, one can find the expression for the equation motion. So, the equation motion becomes theta double dot plus g by l sin theta equal to 0 or taking the sin theta or assuming the sin theta to be moderately large. So, one can expand this thing by using Taylor series, and one can write this sin theta will be equal to theta minus theta cube by factorial 3 or it will be equal to theta minus theta cube by 6, or the equation motion can be written theta double dot plus g by l theta minus theta cube by 6 equal to 0.

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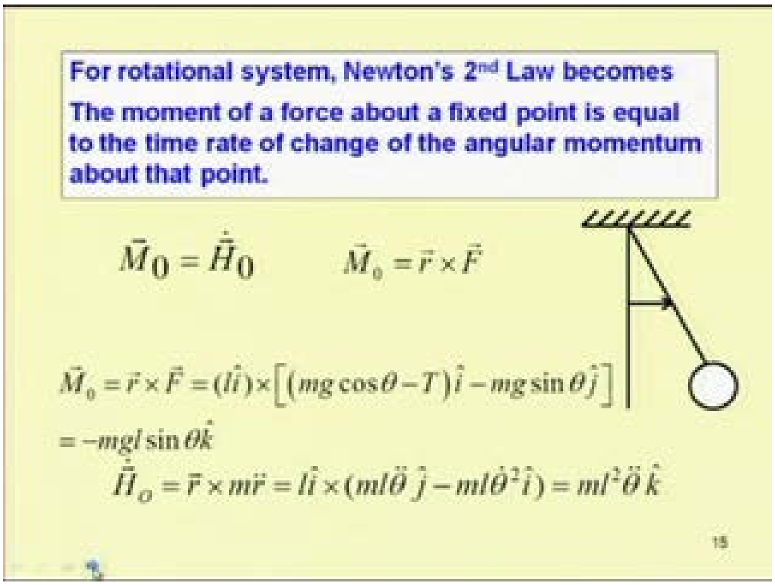

Similarly, one can take the moment also. So, that is using this force also, one can take the moment about this point and find the equation motion where this applied moment will be equal to, so due to this inertial force moment, due to this inertia force. So, by taking movement about this, already we know the expression for this force. So, r cross F will be equal to H 0 dot.

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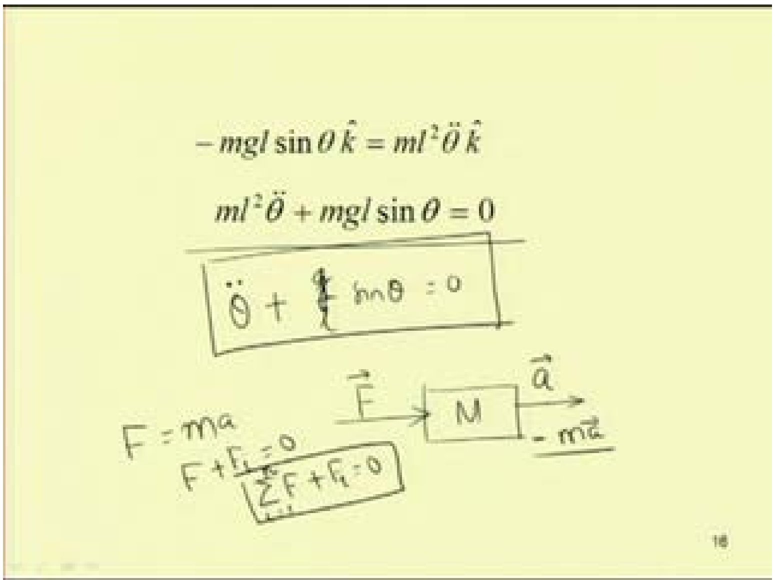

So, one can write or one can find the equation motion of the system as m l square theta double dot plus mgl sin theta equal to 0. So, by dividing this m l square, one can write this equation motion in this form. So, theta double dot plus l by or g by l sin theta equal to 0. So, this is using Newton second law, one can derive the equation motion. So, last class also we have taken some continuous system to derive the equation motion. So, in today class, we are going to study about the Lagrange principle and Hamilton principle. So, before going for this Lagrange and Hamilton principle, briefly I will tell you how we can derive this equation using Lagrange and Hamilton principle.

So, already I told you about D'Alembert's principle. So, in case of D'Alembert's principle, the main advantage of using D'Alembert's principle which states that if we have a mass M and subjected to a force F, so using Newton's second law, we are writing F equal to ma, but this D'Alembert's principle, the same thing can be written that F minus or F plus F i equal to 0 or summation of external force plus the inertial force. So, if you have a multi degree of freedom system, so you can write for this multi degree of freedom system, F plus this F i will be equal to 0. So, that means, we are converting a dynamical system to its equivalent static system by applying this D'Alembert's principal.

So, here we are adding this inertia force which is equal to mass into acceleration and it acts in a direction opposite to the direction of acceleration. So, by using this D'Alembert's principle, we are converting this dynamical system to a static system. The advantage is that in case of static system, we can apply this virtual work principle. So, by applying this virtual principle, we can write or we can find or we can convert this vector based equation motion to a SCLR based equation motion, or we can use these scalar quantities like potential energy and kinetic energy of the system.

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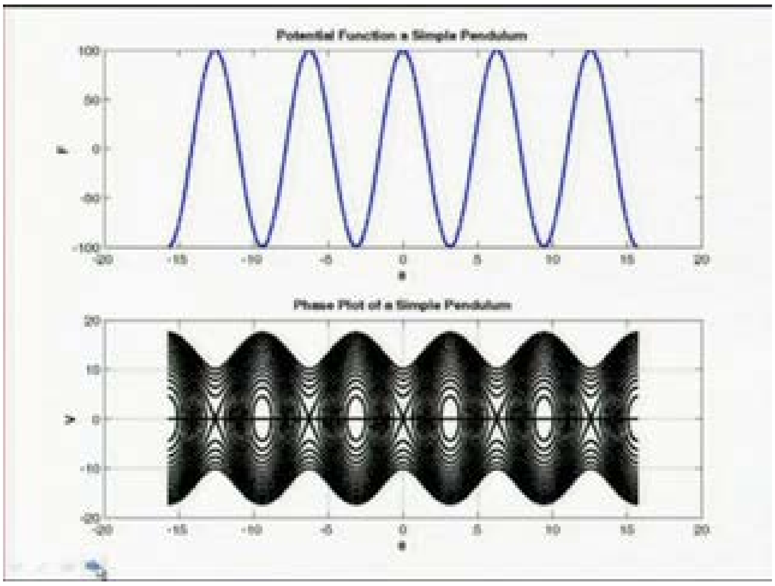

In the last class also, we have seen this qualitative analysis for finding this face portrait of a simple pendulum.

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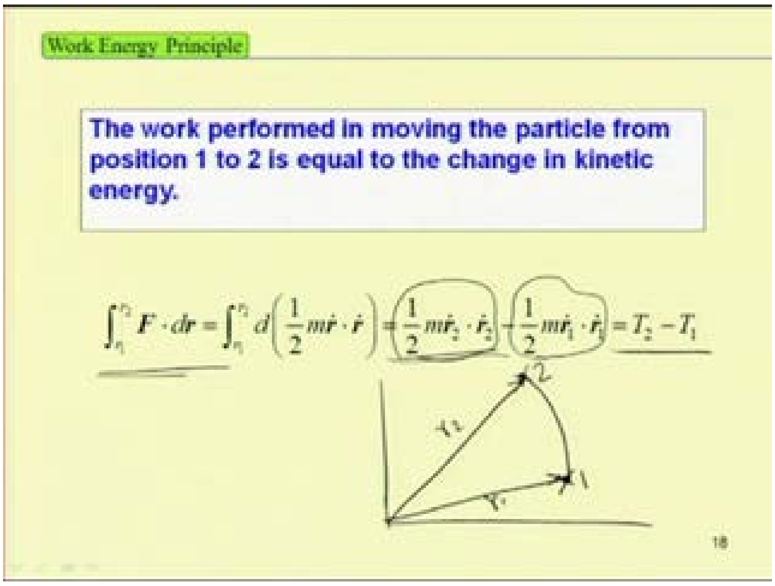

So, now let us see what this work energy principle is. So, the work performed in moving a particle from position 1 to 2 is equal to change in its kinetic energy from engineering mechanics. So, you know that the work performed in moving a particle, let a particle is at time TT equal to 0. It is at this position and it has to be moved to this position. So, the work performed in moving a particle from position 1 to 2 is equal to its change in kinetic energy. So, work can be written at this scalar product of the force and this displacement. So, from position r 1 to r 2. So, this r 1 and r 2 we can write the position vectors. So, this is position vector r 1 and this is position vector r 2.

So, by moving the body from position this to this is equal to, so one can find. So, already we know that F equal to F can be written in terms of change in momentum. So, from that one can write this expression. So, this can be written like d of half m r dot m r dot this r dot. So, this thing equal to half m, this minus this or this is the kinetic energy at position 2 and this is the kinetic energy at position 1. So, this is equal to this. So, one can write this work energy principle that is work done in moving a particle from position 1 to 2 equal to change in its kinetic energy.

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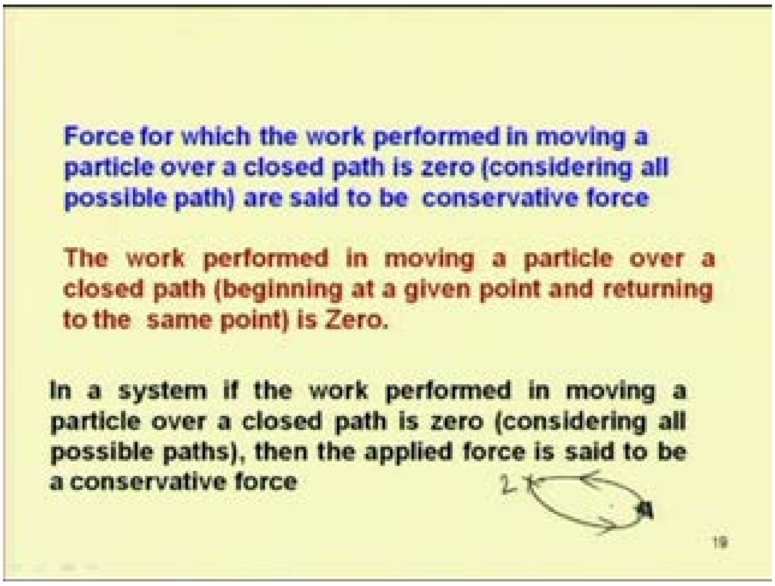

Similarly, these principles can be derived from that work energy principle that is force for which the work performed in moving a particle over a closed path is 0. So, if the work is performed in a closed path; that means this is position 1 and this is position 2. So, if it is coming back to this original point, that is starting and position 1, it has come to position 2 and it has come back to its original position. That means, force for which the work performed in moving a particle over a closed path is 0 considering all possible paths are said to be conservative forces. The work performed in moving a particle over a close path beginning at the given point and returning to the same point is also 0. So, work performed in moving a particle over a closed path is 0 also in a system if the work performed in moving a particle over a closed path is 0, then the applied force is said to be a conservative force.

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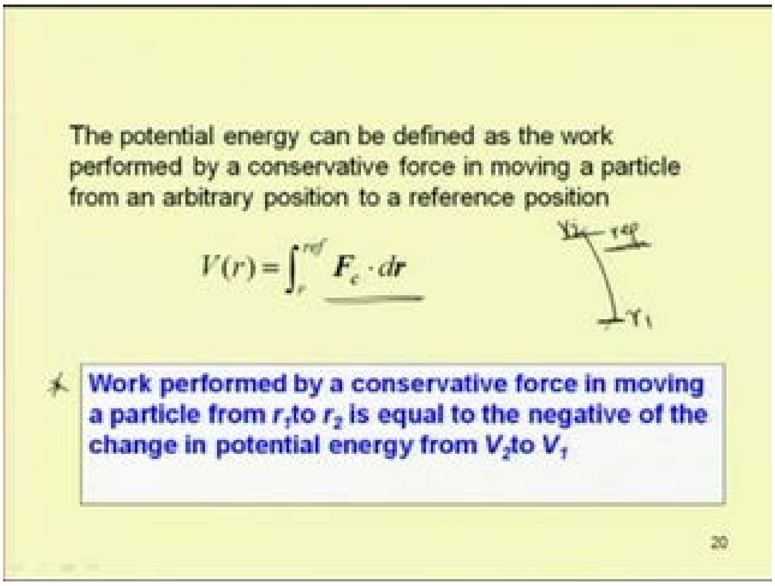

The potential energy can be defined as the work performed by a conservative force in moving a particle from an arbitrary position to a reference position, from arbitrary position to a reference position. So, let this be the arbitrary position from which it has to be taken to this reference position. So, the work done by moving the particle from this arbitrary position to this reference position is known as its potential energy. So, work done is force, this is force dot d r. So, from this, one can find or using this expression one can find the potential energy of a body. So, work performed by a conservative force in moving a particle from r 1 to r 2 is equal to the negative of the change in potential energy from V 2 to V 1.

So, one can note this thing. So, from this expression, it can be found that the work performed by a conservative force in moving a particle from R 1 to R 2. So, let this position be R 1 to R 2. So, in moving a particle from R 1 to R 2, the work will be equal to the negative of the change in potential energy from V 2 to V 1.

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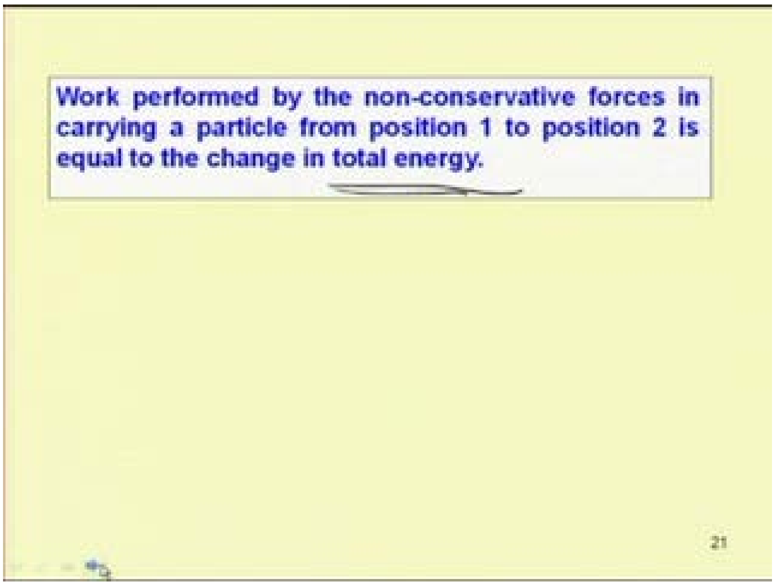

Also, the work performed by non-conservative forces in carrying a particle from position 1 to 2 is equal to change in total energy. So, the change in total energy work performed by non-conservative forces in carrying a particle from position 1 to 2, but the change in total energy. So, the work performed by conservative forces in carrying a particle from position 1 to 2. So, already we have seen that this is equal to negative of the change in potential energy. So, using these principles, one can find the potential energy and kinetic energy of a given system and using those energy expressions.

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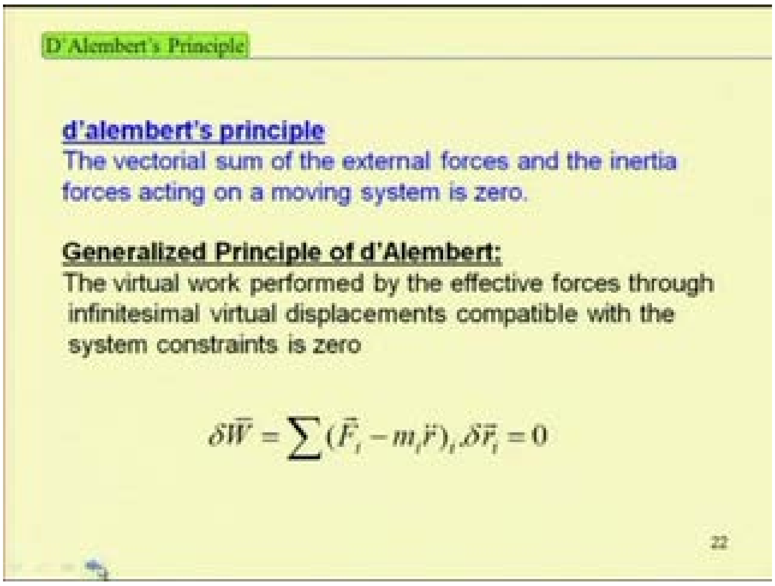

One can use either Hamilton principle or this Lagrange principle to derive the equation motion. So, already we are familiar with this D'Alembert's principle. So, the vectorial sum of the external forces and inertial force acting on a moving system is 0. So, by using this D'Alembert's principle, we are converting the dynamical system to static system.

Now, by applying this virtual principle, we can have this generalized principle of D'Alembert's which states the virtual work are found by the effective forces through infinite decimal virtual displacement compatible with the system constraint is 0. So, the virtual work. So, now after we convert the system, the dynamical system to a static system we can apply this virtual principle which is then known as this generalized principle of D' Alembert which states that the work done by the effective force. So, effective force is nothing but the summation of the external force and the inertia force. So, this term is the inertia force. So, the work done by this or virtual work done by this effective force is equal to 0. So, by applying this principle, one can convert this vector expression of the force and acceleration in to this energy, that is the work done and energy of the system. So, now we can see what you mean by the configuration space.

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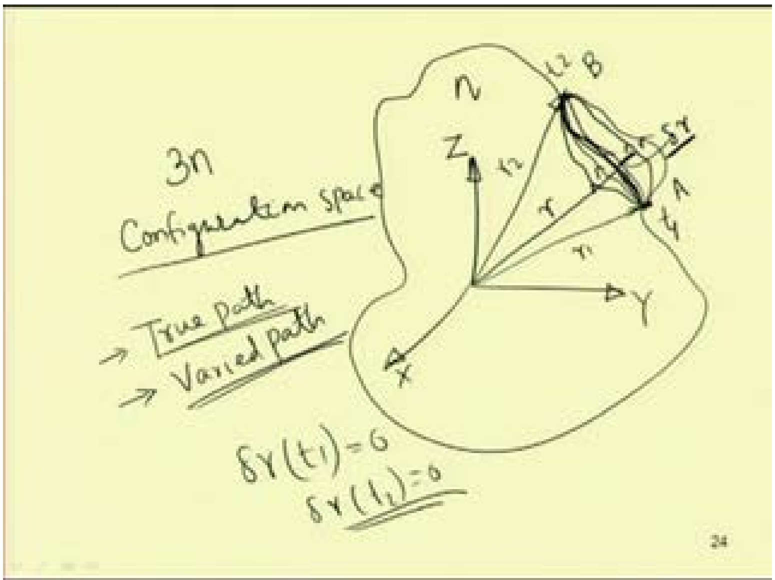

So, let we have n number of particles in a body. So, we can represent the position of a body by using this x y. We can represent this by x y and this Cartesian coordinate system x y z. So, for each particle we can have x 1. For first particle, it is x 1 y 1 z 1. So, for n particle, we can have 3n configuration space. So, we can imagine a configuration space in which all these n particles moves from one position to another position. So, we can, so particle has moved from this position to this position at time t equal to t 1, it is at this position. So, this position vector can be written as r 1. Similarly, at this position, the position vector can be written as r 2. So, to move from this position to the second position B or position A to position B, it can take infinite number of paths.

So, either it can move this way or it may move this way or it can take several paths to reach. So, either it may take this path also to reach to this position. So, it can take many paths from position A to B to reach at that position. So, let us assume that or let us assume this one is the actual path or the dynamic path which it must take to reach from this position to this position. So, other paths can be considered as the varied path. So, let this is the actual path r. So, if we are considering this, so this is our delta r. So, this is the variation in path or this is the displacement or this is the virtual displacement if we consider a varied path. So, we can have two different paths. One is the true path or actual path what the dynamic system has performed, and the other path we can assume that these are the varied path or the virtual path. So, actually these are which have been covered by the body, but actual path it has moved in this actual path. So, this delta r is the variation between the varied and the actual path. So, this delta r. So, this varied path will be equal to the true path if delta r equal to 0. So, using this concept, we can derive or we can write the Hamilton principle.

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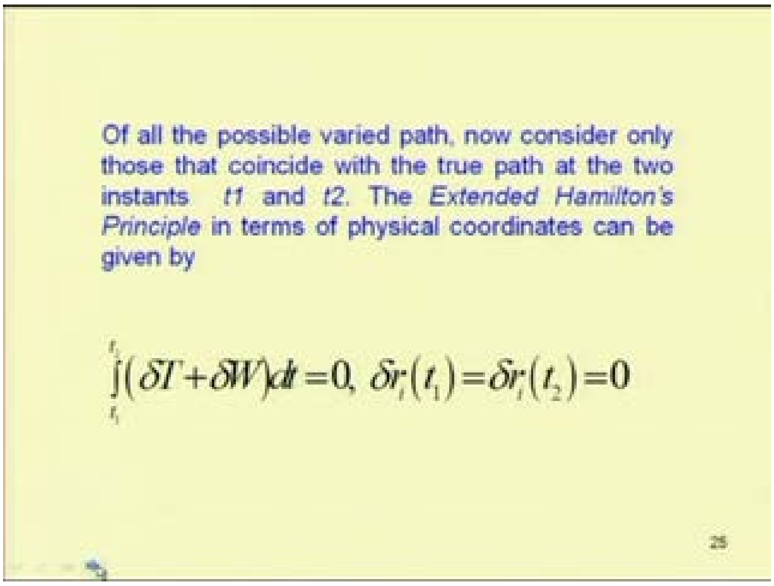

So, in this Hamilton principle, we are assuming all those paths, all those varied paths for which at time t 1 and t 2, the true path and varied path coincide. So, we are considering this delta r at t 1 equal to 0 or delta r at t 2 also equal to 0. So, by considering this thing, we can find the extended Hamilton principle in this way. So, already we have seen this work done equal to change in kinetic energy. So, from that by using that principle, one can find this expression that is the extended Hamilton principle.

So, according to an extended Hamilton principle, the delta T plus delta W d t equal to 0, where delta T is the change in kinetic energy and this is the change in work done. So, delta r i at t 1 delta r i at t 2 equal to 0, where i will be equal to 1, 2. It depends on the number of particles present in the system or it depends on the number of degrees of freedom present in the system. So, this is written in terms of the physical coordinate. This r represent the physical coordinate of the system. So, one can write the same thing using generalized coordinates. Already we are familiar with the generalized coordinates and physical coordinates. For example, in case of this simple pendulum.

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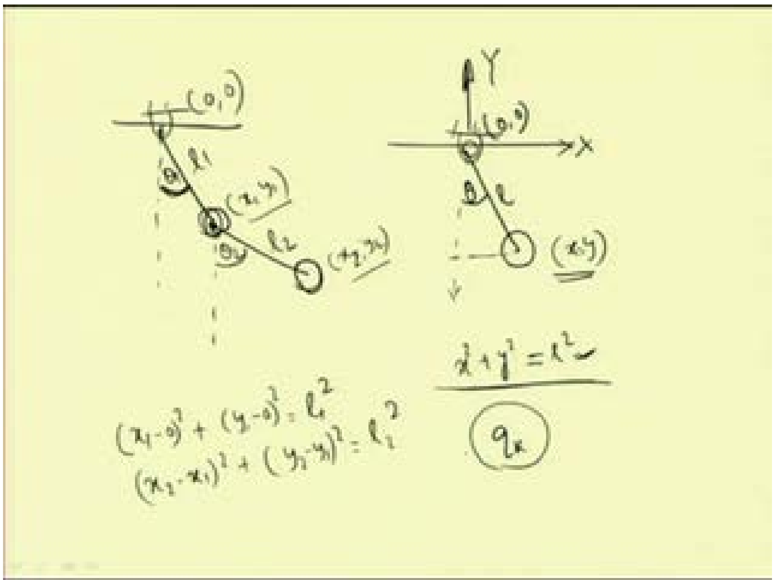

So, in case of this simple pendulum, already we know that one can express the position of this by using this X coordinate and Y coordinate. So, one can put X coordinate and Y coordinate and write this coordinate of the system by using this XY. So, here you require two physical parameter, that is X and Y to represent the motion of this (()), but this X and Y, they are related by this constant equation. So, if this position is 0, they are related by this constant equation that is x square plus y square will be equal to if this length is l. So, this is equal to l square. So, this is the constant equation. So, out of these two physical parameters, one parameter depends on the other parameter. So, out of these two, one will be independent and other will be dependent.

So, one can use a single parameter to represent the position of this by either using X or using Y or in other words, one can use another parameter that is theta. So, this theta is the generalized coordinate considered in the system. So, this position, this X coordinate, one can write this X coordinate and Y coordinate. So, this X coordinate is nothing but this is the X coordinate and this is the Y coordinate of the system as this angle is theta. So, X coordinate will be equal to l cos theta and this y coordinate equal to l sin theta. So, l cos theta square plus l sin theta square, it gives l square which is the constant equation or if one take a double pendulum, in a similar way one can take this double pendulum also and instead of representing the position of this 2 by x 1 y 1 x 2 y 2 by four physical parameters.

So, one can use only two generalized coordinates that is theta 1 and theta 2 to represent the motion of this double pendulum. So, in this case, one can find this theta 1 and theta 2. So, one can eliminate out of this four physical parameters. One can eliminate two parameters by using the constraint equation that is x 1 minus 0. So, the constraint equation will be x 1 minus 0 square plus y 1 minus 0 square equal to l 2 square, and second equation becomes this x 2 minus x 1 whole square plus y 2 minus y 1 whole square equal to l 2 square.

So, by using this, we can represent the position of this mass of the system. So, these are the generalized coordinates and x 1 x 2 what we have represented before, those are the physical coordinates. So, one can have a correspondence between the physical coordinate and the generalize coordinates. So, generalized coordinates are the minimum number of coordinates required to express the position of the system. So, here theta 1 and theta 2 are the generalized coordinates. So, representing the generalized coordinates, we can now write this Hamilton principle using generalized coordinates.

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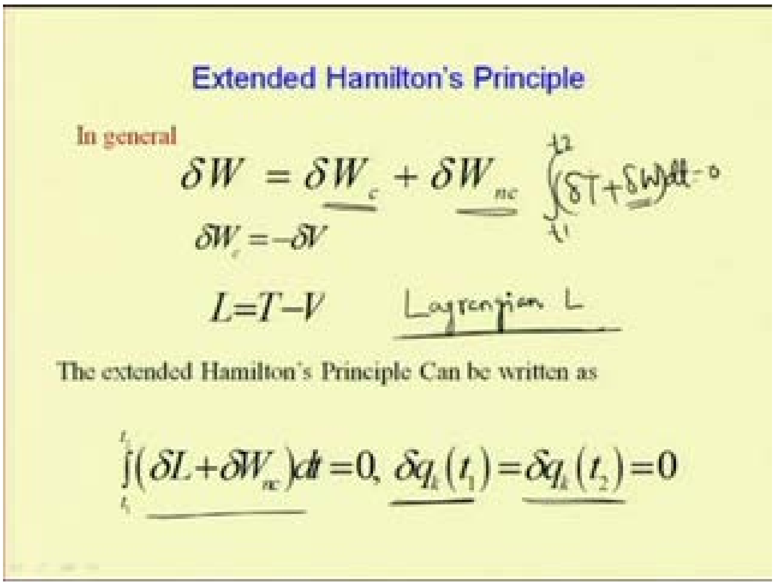

So, before that we already know the total work done can be written as the work done by the conservative forces plus the work done by the non-conservative forces, and we know that work done by these conservative forces can be written as the negative of the change in potential energy. So, delta W c can be written as minus delta V using this Lagrangian. So, L is the Lagrangian of the system. So, using the Lagrangian L which is equal to that is the kinetic energy minus potential energy, the previous equation can be written. So, the previous equation was delta T plus delta W integration t 1 to t 2 d t equal to 0. So, this delta W can be written as minus delta V plus delta W nc. So, this becomes t. Delta of T minus V, that is delta l plus delta W nc d t equal to 0.

So, in this way one can derive this extended Hamilton principle from the physical coordinates to the general coordinates also. So, in case of general coordinates, one can use instead of delta r i1 can use delta q k. So, previously it was written in terms of the total kinetic energy plus the work done and here, it is written in terms of the Lagrangian of the system and the non-conservative work done. So, these extended Hamilton principle can be used to derive the equation motion for dynamical system, and as we are interested to find the equation for the non-linear systems. So, this principle one can conveniently also use. Generally, this extended Hamilton principle will be very useful for finding the equation of motion for continuous systems and in case of continuous system in addition to finding the equation motion, one can find the boundary conditions also.

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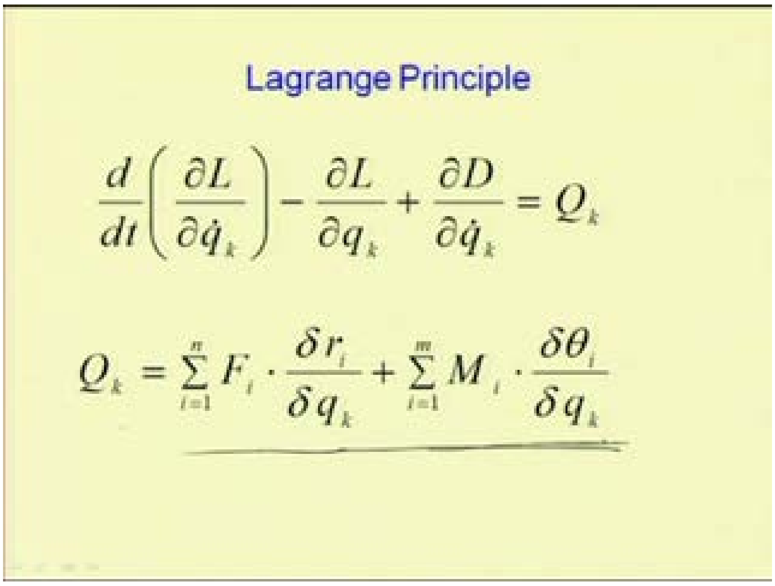

So, the Lagrange principle. So, from Hamilton principle, extended Hamilton principle, one can derive that Lagrange principle which is given by d by d t by del L by del q k dot minus del L by del q k plus del D by del q k dot equal to 0. So, here we are not considering in case of extended Hamilton principle, we have considered friction in the system, but that can be added in the non-conservative work done. So, here this d is the dissipation energy, L is the Lagrangian of the system and q k is the generalized coordinates and this capital Q k is the generalized force which can be written as or which can be found by using this expression. So, if n number forces of external forces are acting on the system, then the work done due to that can be found F i dot del r i, but this generalized force can be found by adding i equal to 1 to n f i dot delta r i by delta q k plus if some moment is acting on the system, then it can be this momentum also can be added to the equation to find the generalized force acting on the system.

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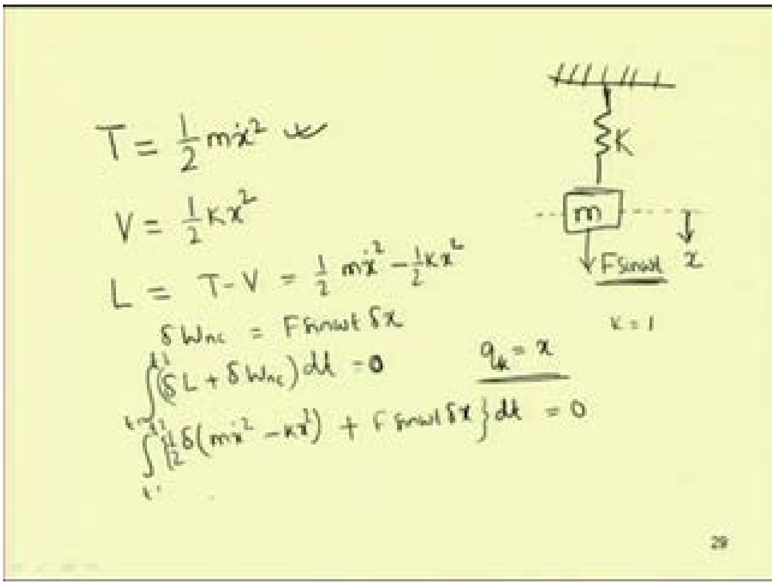

So, let us take the simple example of a spring mass damper system and derive the equation motion using Hamilton principle. Let us first derive for a simple spring and mass system. So, the spring may be non-linear also. So, let us first derive for a linear spring and then we can extend that thing for a non-linear spring also. So, this is the system.

So, we have a spring with stiffness k mass equal to m and a force acting on this that is equal to F sin omega t. So, I can write the kinetic energy of the system t equal to, so it has been given a displacement or it has been displaced by x, so let x represent the displacement from the equilibrium position. So, the kinetic energy can be written as half mass into x dot square. So, x dot represent the velocity of the mass and then potential energy of the system can be written as V will be equal to as we are writing about the equilibrium position. So, it will be due to the motion of the spring only. So, it will be equal to half k into x square.

So, the kinetic energy of the system equal to half m x dot square and potential energy equal to half k x square. So, one can write the Lagrangian of the system will be equal to T minus V. So, this is equal to half m x dot square minus half k x square. So, if you apply this extended Hamilton principle, we can write the equation motion in this form. So, it will be del l plus del W nc d t will be equal to 0, where here q k equal to 1. Only this is a single degree of freedom system k equal to 1. So, q k equal to x and delta W nc.

So, you can find this delta W nc. So, this delta W nc can be found from this force. So, now let us find this equation motion. So, this force will be equal to del W nc, that is this force direction is this and the displacement direction is the same direction. So, it can be written as F sin omega t del x. So, now I can write this equation motion in this form. To derive this equation motion, I can substitute this. So, this will be t 1 to t 2. So, half m x dot square minus k x square. So, this represent del l. So, I can use this del operator here plus F sin omega t delta x into d t. So, this would be equal to 0. So, now, I can use this del operator here. So, if I will use this del operator, so this will give rise to this or this equation I can write it again.

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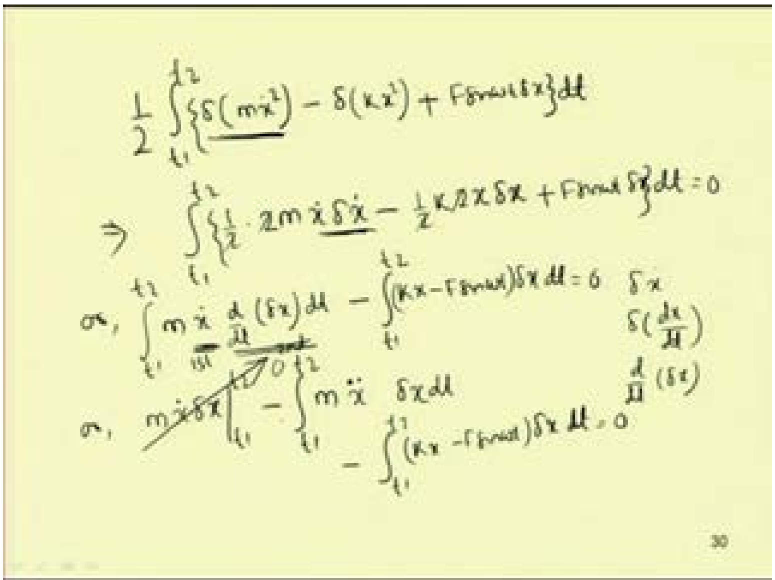

So, this I can take this half outside. So, integration t 1 to t 2 if I will separate this thing, so this is del of m x dot square minus del of k x square plus F sin omega t into del x d t. So, this can be written as this is equal to if I will use this del operator here. So, this is equal to t1 to t2. So, if I will do this first term only, then this half outside half into. So, this becomes 2 m x dot del x dot.

So, you just note that if you are using this del operator here, so this becomes this x dot square becomes 2 into x dot into del x dot. Then minus I can write this thing equal to half. So, this half is here, then k into 2 x into delta x plus F sin omega t delta x d t. So, this will be equal to 0 or I can write this thing, so these two cancels. So, this becomes m integration t 1 to t 2 m x dot and then this delta x dot I can write as I. So, delta x dot can be written as this is d x by d t. So, I can interchange. So, I can write this as d by d t of del x. So, if I will write that way. So, this is m x dot into d by d t of delta x into d t, then minus I these two cancels. So, integration t 1 to t 2. So, this becomes k x minus F sin omega t delta x d t. This is equal to 0 or from this part I can write this equal to. So, I can use this as the first function and this as the second function and use integration by parts. So, integration it will become m.

So, this is the first function and this is the second function. So, first function remain as it is integration of the second. So, integration of this becomes del x. So, this is t 1 to t 2 minus integration t 1 to t 2. So, this is m del x and derivate of the first one. So, this x dot derivate if you make, so this becomes x double dot. So, m x double dot del x d t minus integration t 1 to t 2 k x minus F sin omega t del x d t equal to 0. So, already I told you this del x vanishes at t 1 and t 2. So, this term becomes 0. So, now, we can write taking these things.

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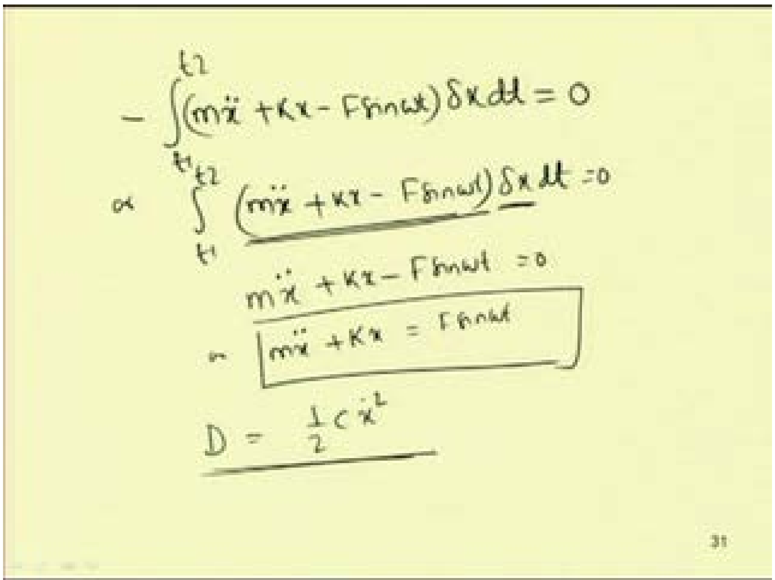

So, we can write minus m x double dot plus k x minus F sin omega t del x d t. So, this is equal to 0. So, this is the equation we got or we can write this is t 1 to t 2 or integration t 1 to t 2 m x double dot plus k x minus F sin omega t del x d t. So, this is equal to 0 as this is the virtual displacement. So, it can take any arbitrary value. So, this will be 0 only if coefficient of this will be equal to 0. So, the equation motion, one can obtain from this. So, m x double dot plus k x minus F sin omega t equal 0 or m x double dot plus k x equal F sin omega t. So, this way one can derive this equation motion using extended Hamilton principle.

So, here this point has to be taken care. This delta x dot I have written in this form and by taking care this thing, so one can easily derive the equation motion for the system. So, this here what I did is delta x dot I have written equal to del of d x by d t. Then I have interchanged between these two. So, this becomes d by d t of del x and then by using integration by parts, we have arrived the final expression of the system. So, instead of this if we will have a damper present in the system, so one can use this dissipation energy D equal half c x dot square and derive the equation motion of the system using Lagrange principle.

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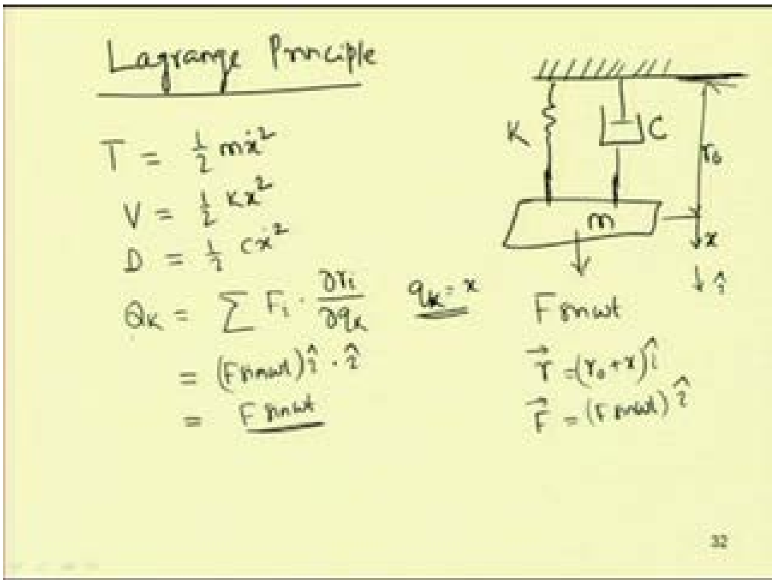

So, by using Lagrange principle, let us derive the equation motion for the same spring and mass damper system. So, let us take the dumper here and derive the equation motion. This is mass m. So, this is the force F sin omega t. So, this is the force acting on this. It has a stiffness of k damping c. So, already we have derived or we have written the expression for kinetic energy. So, that is equal to half. So, this is x. So, half m x dot square potential energy, V equal to half k x square and also this damping or dissipation energy, D equal to half c x dot square and we have to find this expression for Q k. So, here k equal 1. So, Q k equal to, so we have. So, no moment is acting. So, according to our definition, it will be equal to F i dot del r i by del q k.

So, let us take the physical coordinate system here. So, let this to this distance becomes r 0. So, if it is r 0, then one can write this physical coordinates. So, from this, one can write this r equal to if I am taking this direction as the positive i direction, then it will be r will be equal to r 0 plus x i. So, this is the position vector and then this force already we have this force also in the same direction. So, this force equal to F sin omega t. It is also in the same direction, i direction. So, this is the force and this is the r. So, we have only one force. So, this Q k will be equal to F sin omega t f sin omega t I dot. So, del r i by del q k. So, here q k is nothing but our generalized coordinate that is x. So, we can differentiate this with respective to x. So, that will give us this. So, it will give us i only. So, because r equal to r 0 plus x i, so if I will differentiate with respective to x, so this will be nothing but this will be 1 into i. So, this gives rise to F sin omega t. So, our Q k that is the generalize force equal to F sin omega t, now by using this extend this Lagrangian Lagrange principle.

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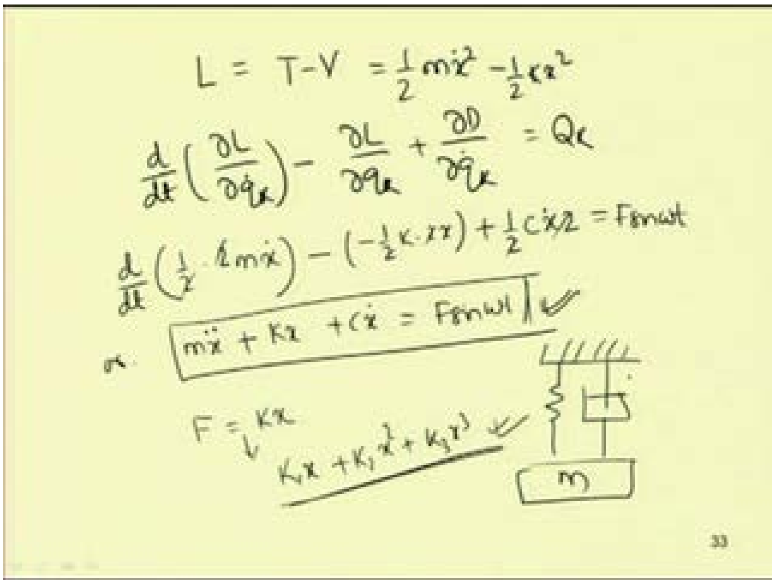

So, we can write L equal to T minus V. So, that is equal to half m x dot square minus half k x square. So, this langrage principle tells us d by d t of del L by del q k dot minus del L by q k plus del d by del q k dot will be equal to Q k. So, del L by del q k dot. So, if I will differentiate with respective to x dot, so this expression will become d by d t. So, this is half into 2 into mx, so half into 2 into half into 2 into m into x dot. So, x dot square we are differentiating with respective to x dot. So, this becomes m x dot. So, as this is not a function of x dot, so this becomes 0. So, this is this, then del L by del x will be minus of minus half k into 2 x and del d by del q k dot also becomes half c x dot into 2. So, it will be equal F sin omega t or this becomes this two cancel. So, this becomes m x double dot plus this minus plus k x plus c x dot equal to F sin omega t.

So, this is the same equation we have derived before by using Newton second law. So, by using Newton second law or by using this Lagrange principle or Hamilton principle. So, for a simple spring mass system, you can derive the equation motion in this form. So, now, you can make the spring. So, here we have taken one linear spring. The spring can be made non-linear for the case of a non-linear system. So, this spring force which was previously F equal to k x 1 can replace this by k 1 x plus k 2 x square plus x 3 x cube or using other non-linear terms. So, by using this extended Hamilton principle or by using this Lagrange principle, also taking this spring force equal to this one can derive the equation motion which can be of this form.

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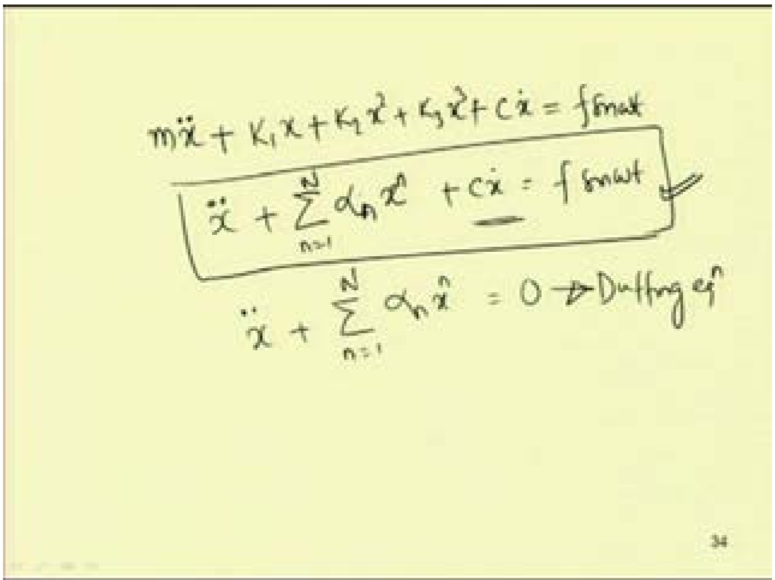

So, the equation, final equation for the non-linear system may be in this form, m x double dot plus alpha 1 or m x double dot or k 1 x plus k 2 x square plus k 3 x cube plus using this damping, one can write this c x dot will be equal to F sin omega t or this equation can be written also in the generalize form x double dot plus summation alpha into alpha n into x to the power n plus c x dot equal to F sin omega t without using this damping term.

So, this is well known Duffing equation, x double dot plus n equal to 1 to capital N equal to 1 2 3 4 up to capital N. So, this is alpha n x to the power n equal to 0 for free vibration, so Duffing equation for free vibration. Similarly, one can find this is the Duffing equation with damping for the force vibration.

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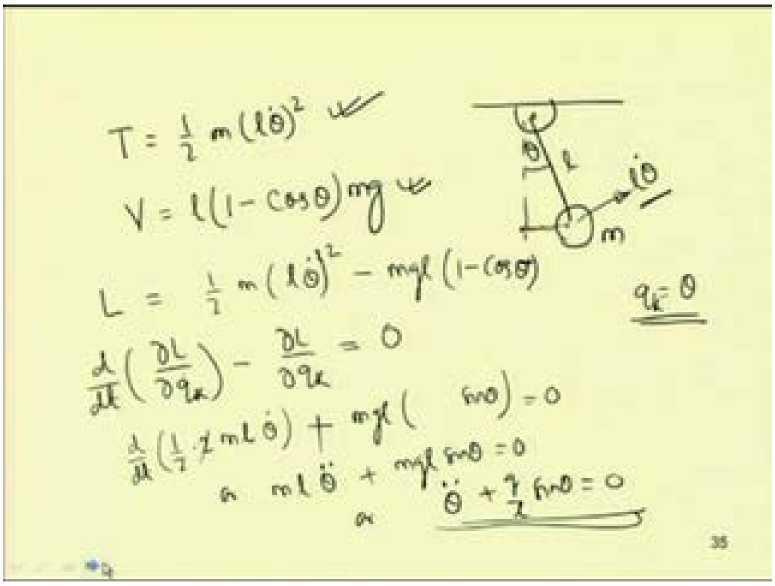

So, we can take some other system also. So, for this simple pendulum, let us take the simple pendulum. So, in this case, the velocity at this position if it has moved by angle theta, this one then this is l theta dot. So, this is perpendicular to this link length. So, this is the velocity at this position. If the mass is m, the kinetic energy will be equal to half m into l theta dot square and potential energy will be, so this is the change in position. Potential energy can be obtained by finding the change in position. So, initial position is here and now, it has come to this position. So, the potential energy V can written as l minus l cos theta or taking l common. So, this will be l into 1 minus cos theta into m g. So, this is the change in position into mass and gravity acceleration, due to gravity will give the potential energy. So, potential energy equal to m g l into 1 minus cos theta.

So, this is the kinetic energy of the system, this is the potential energy of the system. So, Lagrangian of the system can be written as half m l theta dot square theta dot whole square minus m g l into 1 minus theta. So, either one can use this extended Hamilton principle or Lagrange principle to find the equation motion. So, using Lagrange principle, I can write d by d t of del l by del q k dot. So, here del here q k equal to theta.

So, del q k dot that is theta dot minus del l by del q k as no damping we are considering in the system. So, this will be equal to 0. Also, we are not considering any external force. So, in this way, this equation will reduce to del l by del theta dot will give us half into 2 into m l m l theta dot, so d by d t of this. So, d by d t of this. So, 2 cancel minus and then you differentiate this thing with respect to theta. So, this will become minus m g l 1. So, this gives rise to cos theta differentiation of cos theta equal to sin theta into del l by del. So, we are differentiating with respect to theta and we have a negative sign here. So, minus plus… So, this becomes 0 or the equation becomes m l theta double dot plus m g l. So, this is differentiation 1 equal to 0. So, you are differentiating with respective to theta. So, this becomes theta. So, this is m g l sin theta. So, this is equal to 0 or the equation become theta double dot plus g by l sin theta equal to 0.

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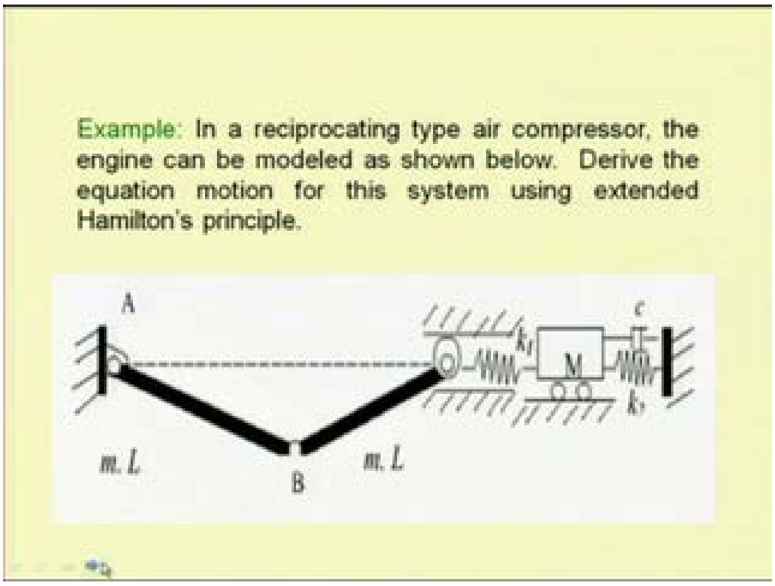

So, let us consider one more example, slightly complicated example. So, when I will take a two degree of freedom system. So, this is a slider plank mechanism consisting of two links and at this end, it is connected to a spring and then we have a mass and we have a spring and damper system. So, in this case up to this, if you consider up to this, if you consider this is a spring mass damp. So, this is a slider plank mechanism which is single degree of freedom system. So, you can represent motion by using this angle theta. So, if I will consider this angle theta and this angle delta, I can write the position vector of all these points. So, to find the kinetic energy, first I should know what the velocity at these points is. So, at this center of mass of this, I should find the velocity. So, I can write the position vector of points ABC and other points also.

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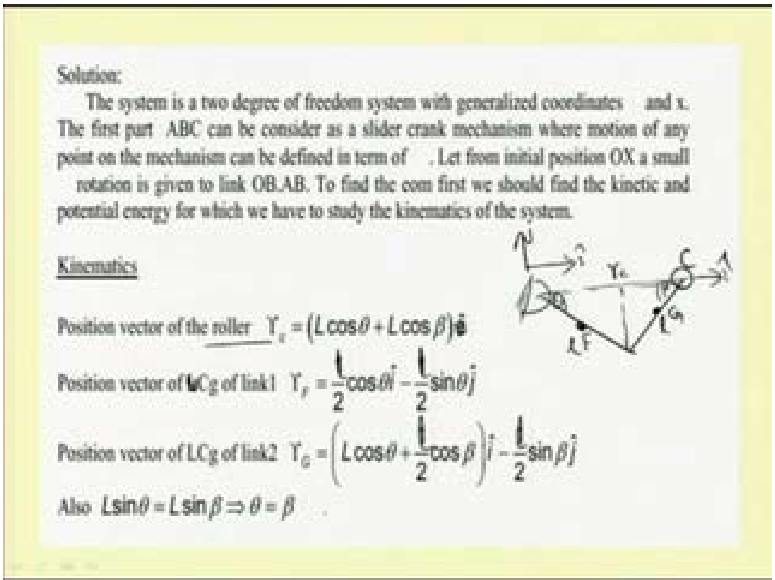

So, position vector of point C will be equal to l cos theta, this is the link. So, position vector of this will be l cos theta plus l cos beta into i and then position vector of the C g of link 1, position vector of C g of length 1, this is the position vector of, so this is the roller C. So, position vector of this point from this. So, if you are considering a coordinate system here, so this is at C. So, this is equal to this is l, this is also l. So, this angle is theta, this angle is beta. So, the position vector of the roller C equal to l cos theta plus l cos beta. So, this direction I am taking i. So, this is i similar position vector of the C g of this link 1. So, this is point F. So, it can be l by 2. So, this will be l by 2 cos theta i minus. So, this is i direction and this is j. So, it will be minus l by 2 sin theta j. Similarly, position vector of point this g can be written. So, this will be this plus this distance. So, this is l cos theta plus l by 2 cos beta by 2 i minus l by 2 sin beta j. So, we know this l sin theta equal to n sin beta. So, theta equal to beta.

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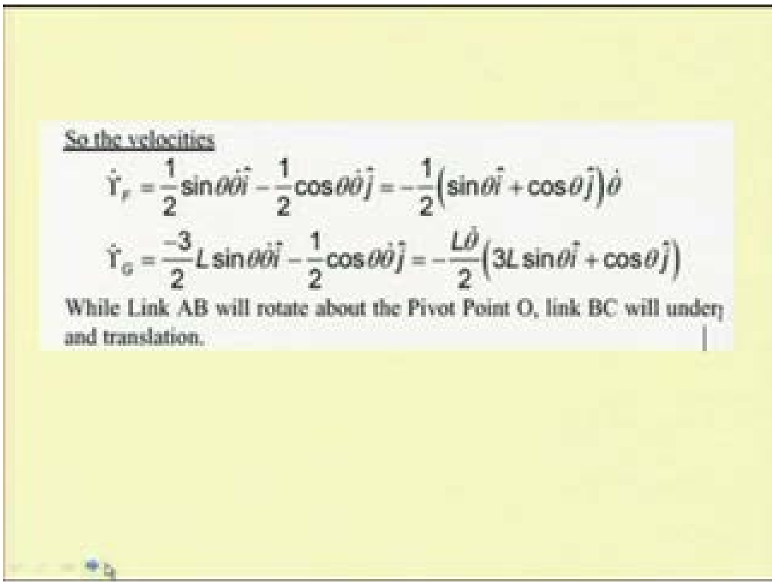

So, in this way one can find the position vector. So, differentiating the position vector, one can find the velocity and getting this velocity, one can find the kinetic energy of the system. So, kinetic energy of the system consists of kinetic energy of two links.

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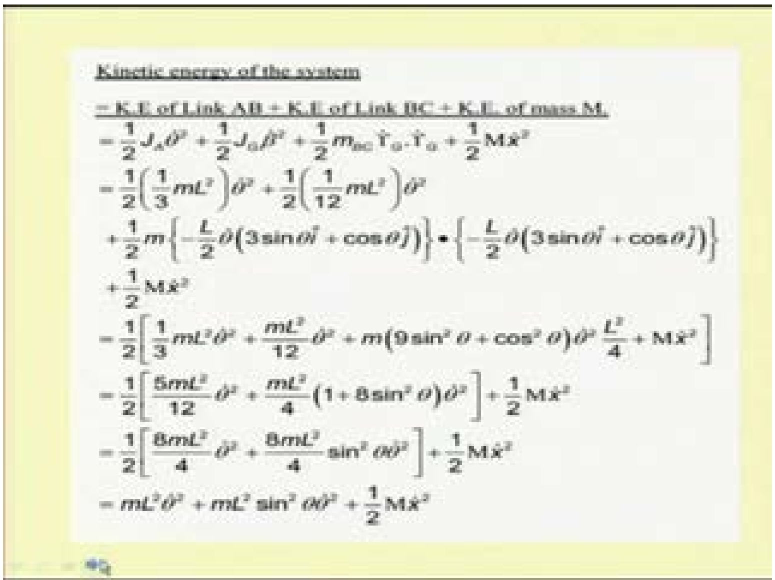

So, this is the kinetic energy of the system. So, considering the rotation and translation, one can find the kinetic energy of a system.

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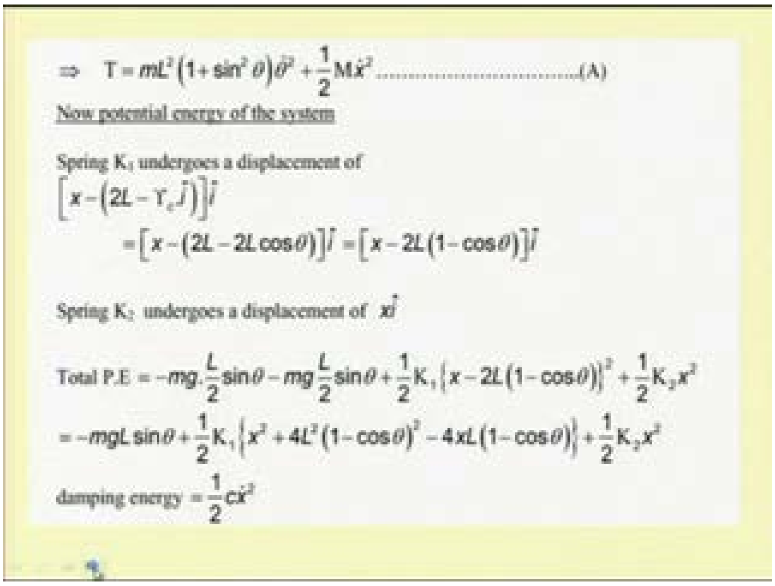

Similarly, one can find the potential energy of the system and then Lagrangian of the system damping energy equal to half c x dot square. By using Hamilton principle, one can find the equation motion of the system which can be written as this.

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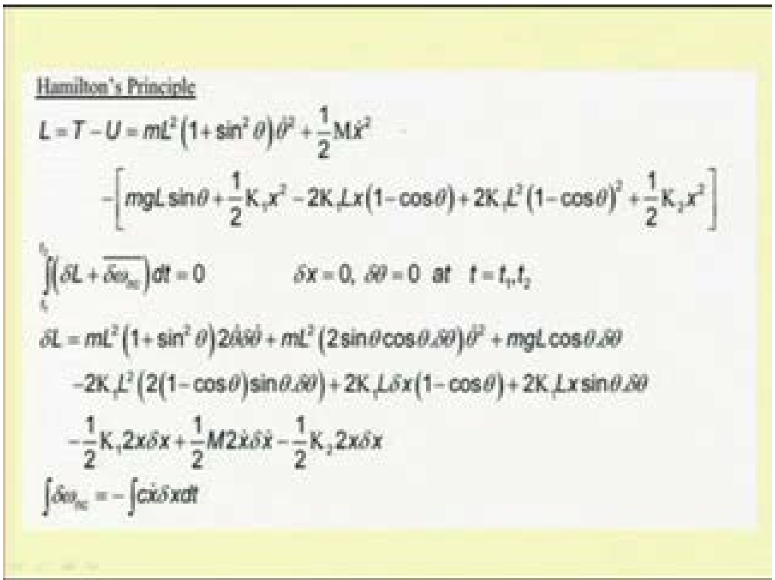

So, one can write as this is a two degree of freedom system. One can get two equations. Similarly, by using Lagrange principle also, one can find the same equation motion.

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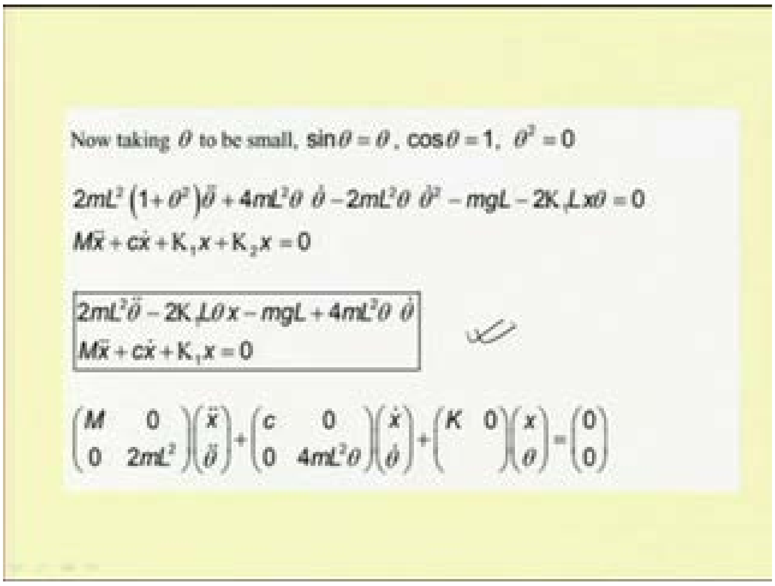

So, in this class, we have studied about how to use this Lagrange principle and Hamilton principle to derive the equation motion. So, as exercise problem, one can find the equation motion of the system by using Hamilton principle or Lagrange principle.

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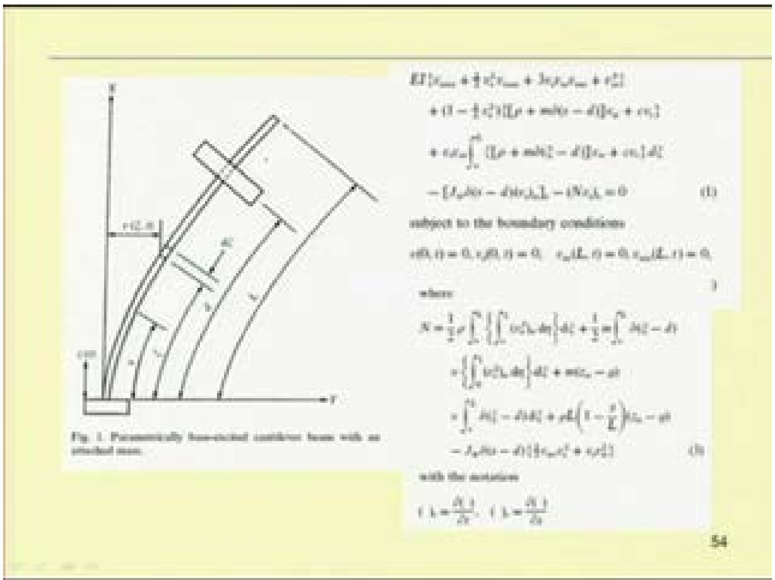

So, this is a continuous system of the example what I have solved before are discrete system, but one can find the equation motion of this continuous system also by writing the potential energy and kinetic energy of the system. The equation motion using Newton second law or D’Alembert’s principle is written here. So, one can find the same equation by using Lagrange principle or Hamilton principle. So, this is exercise problem or one can use this to find the equation motion of the system.

So, in the next class, we will solve some more problems on continuous system using this extended Hamilton principle and Lagrange principle unlike in case of the discrete system where you are finding this differential equation. So, in case of continuous system, we find integral differential equation and by using this Galerkin principle, we will convert that thing to temporal equation.

Thank you.