Non-Linear Vibration
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Module - 2
Derivation of Nonlinear Equation of Motion
Lecture - 1
Force and Moment Based Approach

Welcome to the second module of non-linear vibration. So, in the first model we presented to you about the introduction of non-linear systems. And in this module, we are going to discuss on the free on the derivation of non-linear equation of motion.

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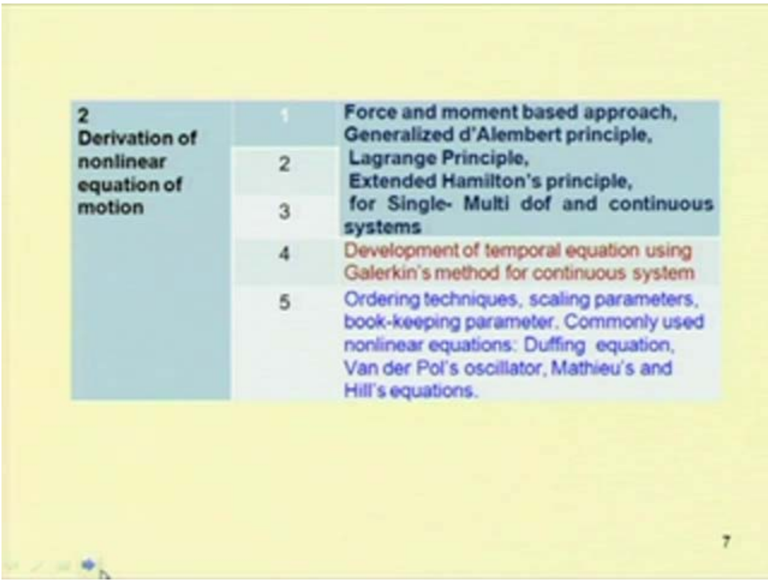

So, in the first three classes, I will tell you about this force and moment based approach generalized d'Alembert principle, Lagrange principle, extended Hamilton principle for single multi degree of freedom and continuous system. In the fourth class, we will see the development of temporal equation of motion using Galerkin method for continuous system. And again on the fifth class, we are going to discuss, re-discuss about this ordering technique, scaling parameters, book keeping parameter and commonly used non-linear equations like Duffing equation, van der pol equation and Mathieu and hills type of equation. So, already we have discussed or we have seen these four different types of equation and a combination of this dump Duffing and van der pol equation or Duffing and Mathieu equation and Mathieu hill's equations.

So, in combination of all these equations, we can have other forms of non-linear governing equations. So, in today's class, we will study about the force and moment based approach. So, before that, we will initially review what we have discussed in the last four lectures.

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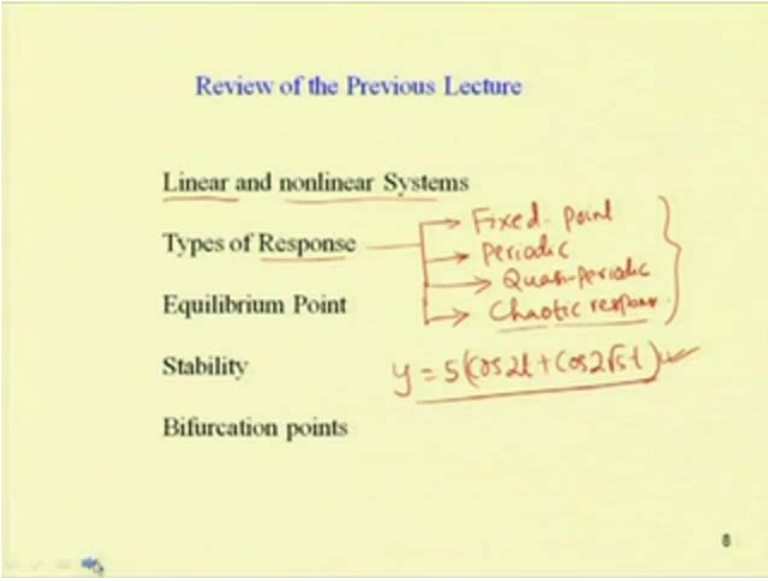

So, in the last four lectures we have discussed about this linear and non-linear systems. So, these linear and non-linear systems can be grouped based on the superposition theory. So, if in the system we can apply superposition theory then, it can be a linear system otherwise, the system can be a non-linear system. And in case of superposition theory, it has to obey the additive rule and the homogenous rule.

So, both homogenous and additive rule can be checked to see whether the system is a linear system or the system is a non-linear system. And in case of a non-linear system, we can order the magnitude or order the non-linear terms by using the scaling parameter or book keeping parameter. And also we have discussed about different type of response observed in case of this non-linear system. So, generally the vibrating response can be the transient response or the steady state response. So, in case of steady state response we have discussed 4 different types of response that is fixed point response, periodic response, quasi periodic response and finally, the chaotic response.

So, we have discussed about the equilibrium points in case of the steady state response. As in case of steady state response, the response is independent of time then; we can find the equilibrium position by putting this time dependent on equal to 0 and finding the response of the system. So, the response may be fix point periodic, quasi periodic and chaotic. In case of periodic response also the response may be harmonic or other different types of periodic response. But other periodic response can be reduced to that of the harmonic response. And in case of quasi periodic response it may be a periodic response in which the different frequencies can bear irrational ratios.

For example, I can write this response y equal to 5 cos 2 t plus cos 2 root 5 t. So, the response resulting response will be quasi periodic response with frequency 2 and 2 root 5 and the ratio between these 2 that is, 2 root 5 and 2 that is root 5 is an irrational number.
So, the resulting response is quasi periodic. Similarly, we have seen when the response is neither fixed point periodic or quasi periodic then, the response is chaotic response. Also we have seen the period doubling root to chaotic response. And after studying this equilibrium point, we have briefly discussed about the stability and bifurcation of the responses.

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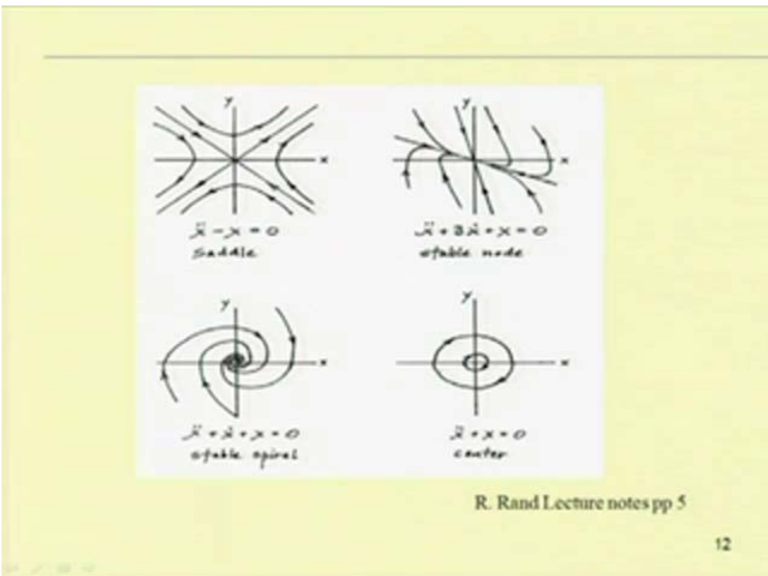

So, one can observe different types of response or response type or bifurcation type also. So, this example represent the saddle point, the equilibrium point is a saddle point. Here, the equilibrium point is x equal 0. So, this point is x equal to 0, this is the equilibrium point and the flow around this equilibrium point is shown here. So, this is a saddle point. So, clearly the solution x double dot minus x equal to 0. So, one can find the solution, one can write the auxiliary equation in this case. For example, this is x double dot minus x equal to 0.

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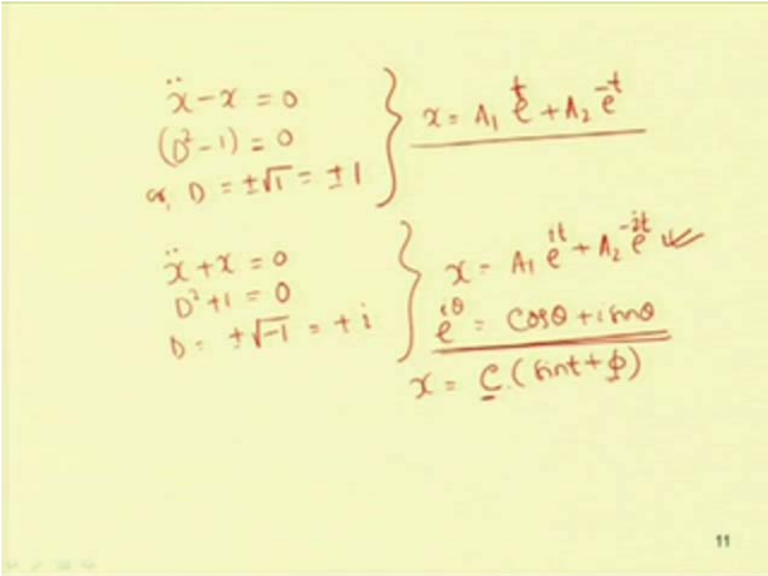

So, the auxiliary equation can be written as D square minus 1 equal to 0 or D equal to plus minus root over 1 this is equal to plus minus 1. So, as the roots are plus minus 1 the solution becomes x will be equal to A 1 e to the power e to the power t plus A 2 e to the power minus t. So, due to the presence of e to the power t, which is exponentially growing and e to the power minus t which is decaying exponentially, one can find a response like this around the equilibrium point that is x equal to 0.

And one can have these are the asymptotes. So, these are the separatrices dividing these points. Similarly, one can have a response x double dot plus x equal to 0 which will give a center. So, for example, for this case x double dot plus x equal to 0. So, in this case the auxiliary equation equal to D square plus 1 equal to 0 or D equal to plus minus root over minus 1 that is plus minus i. So, its solution will be x equal to A 1 e to the power i t plus A 2 e to the power minus i t where, t is the time.

So, one can write this term as it is known that this e to the power i theta equal to cos theta plus i sin theta. So, by substituting this thing one can write this equation in this for x will be equal to c sin t plus pi so, where, this pi and c can be obtained from the initial condition. So, one can have the center in this case. So, this will give a periodic response whose phase portrait can be look like this. Similarly, one can have a stable spiral in this case and stable node in this case. So, this is the second order equation with a damping term here, the damping term is damping factor is 3. So, one can find the solution and one can see this is a stable node.

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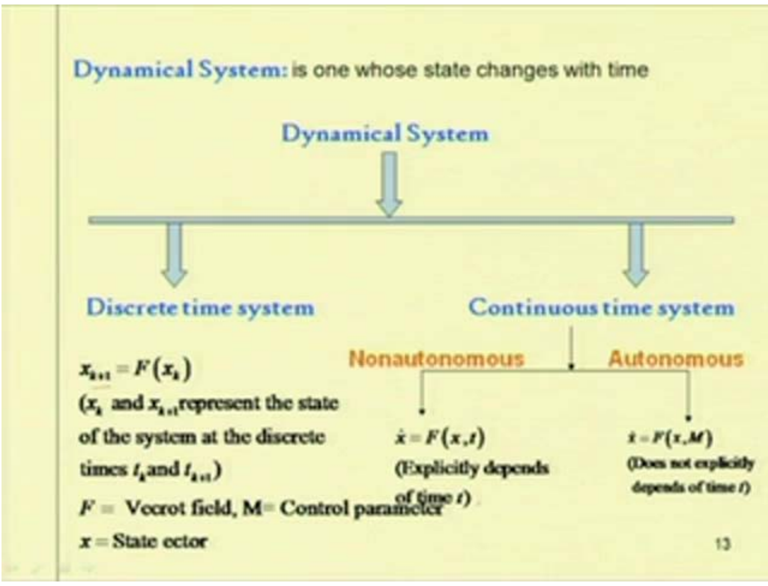

So, we have divided the dynamical system into this discrete time system and continuous a time system. So, in case of the discrete time system we are dividing the system or we are finding say, system response are k plus oneth time as a function of kth time and incase of this continuous system we are dividing this thing to autonomous and non autonomous system. So, in case of non autonomous system, x dot depends explicitly on the time parameter and incase of autonomous system it does not depend explicitly on time.

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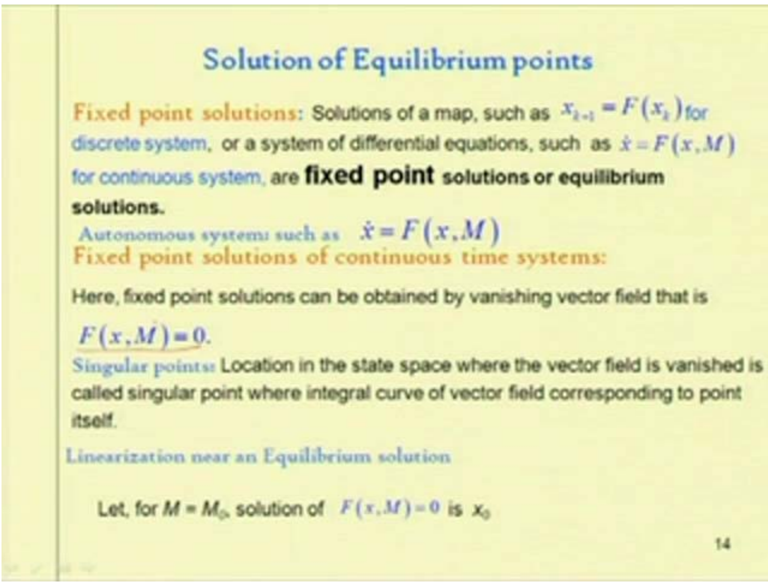

So, for these cases already we have discussed about the fix point by substituting this x dot equal to 0. So, we can write the fix point equal to F x m equal to 0. So, here m is the control parameter so either by linearization near the equilibrium position or by perturbing we can find or we can study the stability of the system.

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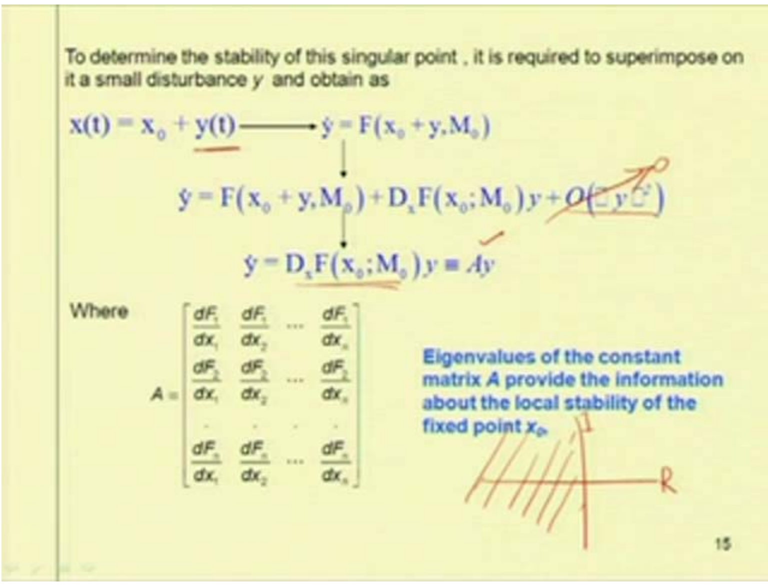

So, by taking a small parameter near the equilibrium position x 0 one can write the governing equation y dot equal to F x 0 plus y controlling parameter M 0. And by expanding this thing using Taylor series one can write this y dot equal to F x 0 plus y M

0 plus D x F x 0 M 0 y plus higher order terms. So, neglecting these higher order terms, one can write y dot equal to D x F x 0 M y or these term one can write as a. So, 'a' is the Jacobean matrix of the system, j a is the Jacobean matrix which is function of first derivative of the system with respect to different parameters.

So, by finding the Eigen values of this Jacobean matrix one can study the stability of the system. So, this will give the local stability of the system. So, if one plot the Eigen values in the real and imaginary plane, if all the roots are on the negative side of the s plane then, the system are stable otherwise, the system becomes unstable.

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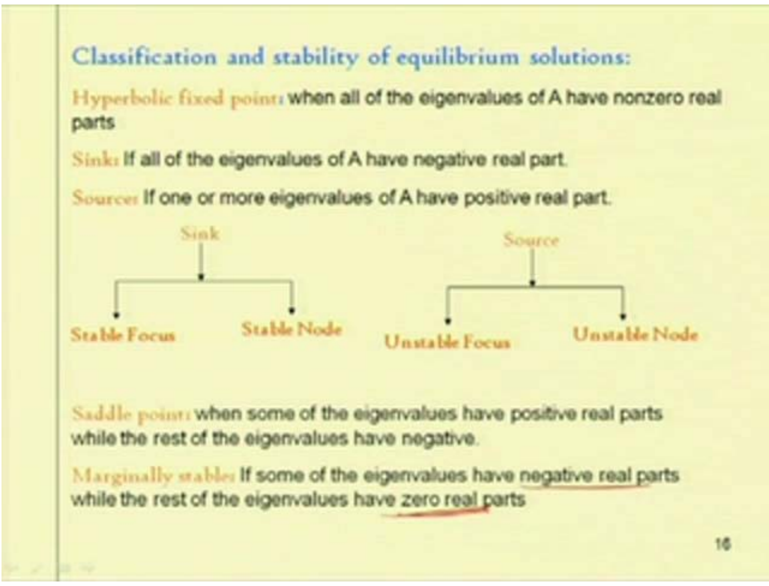

So, one can classify this stability of equilibrium point by hyperbolic fix point when all of the Eigen values of a have non 0 real parts, then it is hyperbolic fix point. And if all the Eigen values of a have negative real part then it is known as sink and if one or more Eigen values of a have positive real parts then, it is source. That means, if all the real parts are negative that is there on the left side of the s plane then, the system is stable and if some of the roots are on the right hand side of the s plane then, the system is unstable. So, due to this nature the response of the system grow and that is why it is known as source. But when it is in the left hand side of the s plane then, the system is stable that is y it is known as sink. So, the sink can be a stable focus or a stable node also. Similarly, the source can be an unstable focus or unstable node. And when some of the Eigen values have positive real parts while rest of the Eigen values have negative then, the point is known as a saddle point.

Similarly, if some of the Eigen values have negative real parts while the rest of the Eigen values have 0 real parts then, the system is marginally stable. So, in case of marginally stable some of the Eigen values have 0 real part and most of the Eigen values have negative real part. So, in this case the system is marginally stable.

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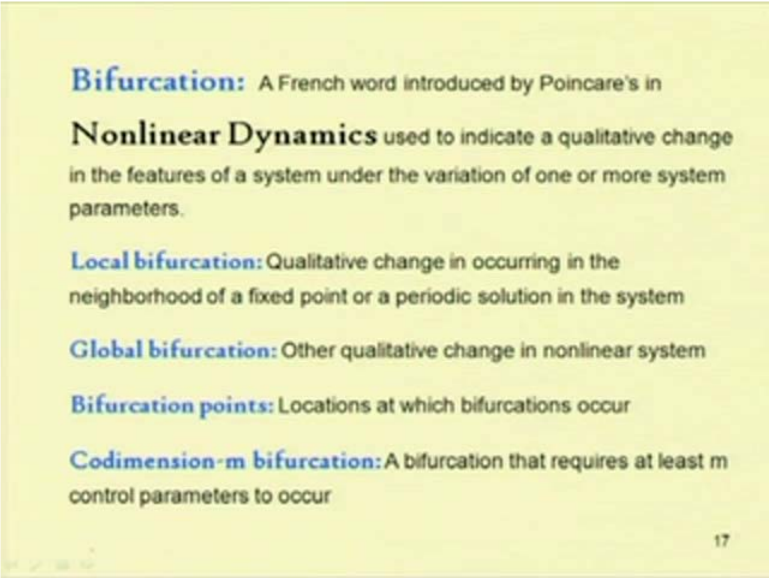

And we have also briefly reviewed about the bifurcation of the system. So, this is a French word introduced by Poincare. So, we can have local bifurcation, global bifurcation and bifurcation points also and a codimension-m bifurcation. So, a bifurcation that required at least m control parameter to occur is known as codimension-m bifurcation and the location of the bifurcation point is the bifurcation. The location at which the bifurcation occurs that is the bifurcation point.

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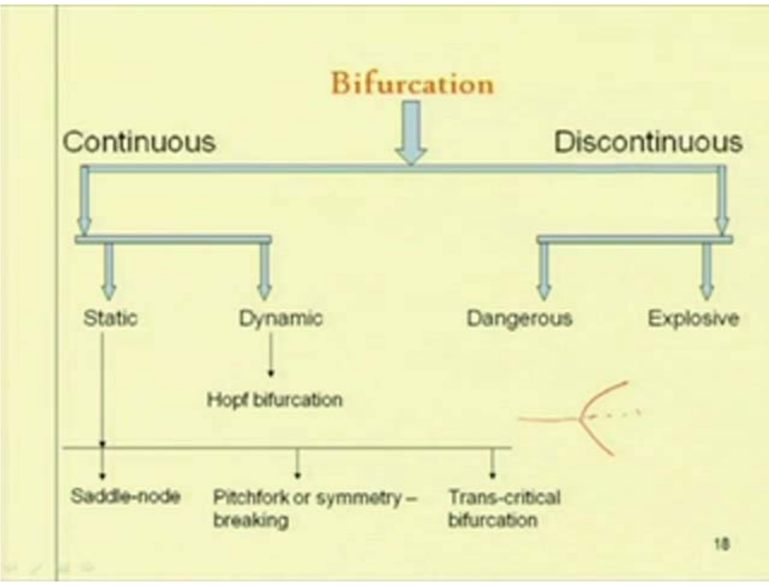

Similarly, we can classify the bifurcation into continuous and discontinuous. So, in case of continuous it is static bifurcation and dynamic bifurcation and in discontinuous it is dangerous or explosive. In case of static bifurcation, one can have these three different types of bifurcation that is saddle nodes pitchfork or symmetry breaking and trans-critical bifurcation. And in case of dynamic bifurcation, it is hopf bifurcation. So, in case of pitchfork or symmetry breaking the response may be super critical or this is super critical or it may be sub critical also. Similarly, in case of hopf bifurcation one can have super critical and sub critical bifurcation.

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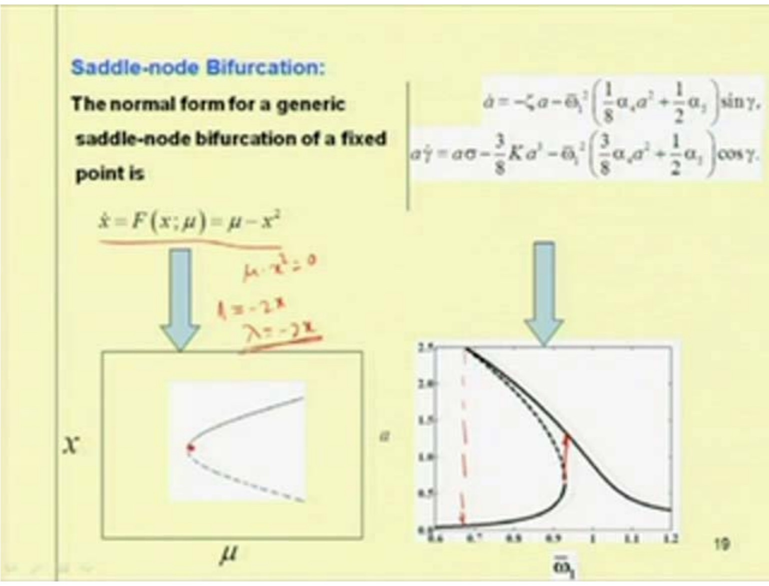

So, these examples already we have seen. So, for example, in this case x dot equal to mu minus x square the equilibrium point becomes mu minus x square equal to 0. So, one can find the equilibrium position mu minus x square equal to 0 and by plotting that thing one can have these 2 lines. So, for example, for its Eigen value we can find the Jacobean matrix by differentiating this thing. So, by differentiating this one can get the I a equal to minus 2 x so, a equal to minus 2 x, a minus lambda I becomes minus 2 x minus lambda. So, lambda becomes minus 2 x. So, as lambda becomes minus 2 x. So, when x equal to plus root mu then, the system has lambda negative that is why this branch is stable and this branch is unstable. So, this point is saddle node bifurcation point. And similarly here for A 2 dimensional equation that is in terms of when the equations are written in terms of a dot and gamma dot where, 'a' is the amplitude and gamma is the phase response of the system.

So, this is for a typically used non-linear system. So, in this case one can have the saddle node bifurcation point at these 2 points. So, if one plots the Eigen values, one can study that this to this portion the response is stable and after that the system is unstable. So, at this point the system may experience a jump of phenomena. Similarly, at this point I will show the system may experience a jump down phenomena. So, while reducing this control parameter from this position the system response may jump down to this stable solution. Similarly, here when we are sweeping of the frequency at this point the system will have a tendency to jump of as this branch is unstable. So, similarly we can study the pitchfork bifurcation.

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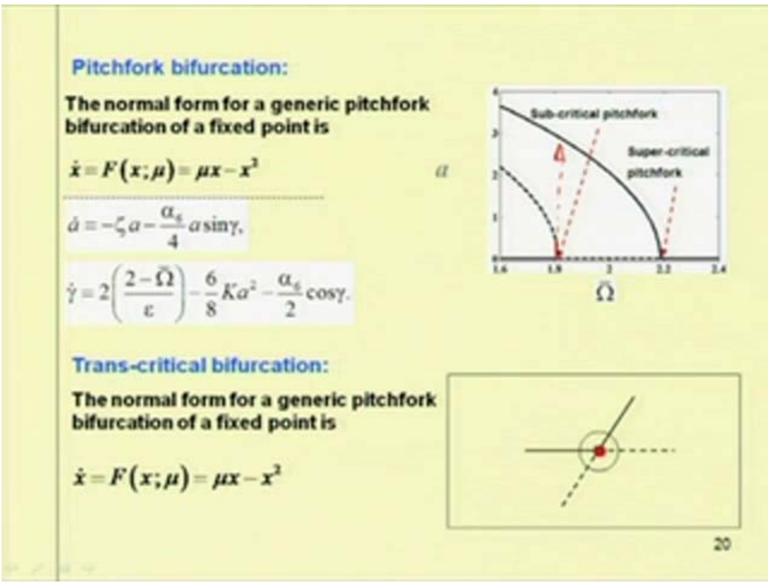

So, in case of pitchfork bifurcation, these 2 are pitchfork bifurcation. So, this is super critical pitchfork and this point is super critical as from when you are decreasing the response. So, from a positive response, from a stable response it goes to a stable non trivial braches. So, this is the trivial branch, this is the non trivial branch. So, the trivial branch becomes unstable but, the non trivial branch becomes stable. So, this point is a super critical pitchfork bifurcation point. But, this point so, initially we have 1 stable trivial state but, after this bifurcation point we have both unstable, trivial and non trivial response. So, as both the trivial and non trivial response are unstable. So, from a stable branch we are getting unstable branch that is why this is sub critical pitchfork bifurcation point. So, this sub critical bifurcation point is a danger or catastrophic failure point. So, at this point if one increase this control this parameter omega bar then, the system will have a tendency to jump up. So, here it will jump up to the upper stable branches.

So, in case of this non-linear system one can observe that, one can observe that at a particular control parameter one will have multiple type of response or multiple solutions present in the system. So, in this case before this bifurcation point the system has a by stable region and one unstable region, after this bifurcation point the system has only one stable region and between these two super critical and sub critical bifurcation point the system have 1 stable and 1 unstable response fix point response. So, these responses are fix point response.

In case of trans-critical bifurcation so, for example, one can take some. So, in this case of trans-critical bifurcation this point so, this is the trivial branch this is the non trivial branch, the trivial and non trivial branch the change their stability. So, the trivial branch becomes unstable and the non trivial branch which was unstable before becomes stable. So, this is a trans-critical bifurcation point.

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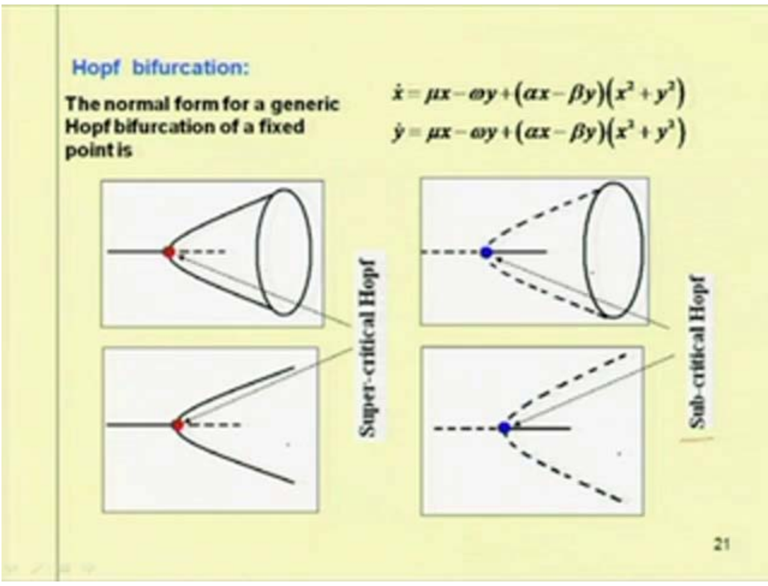

Similarly, one can have this Hopf bifurcation in which the fix point response becomes periodic response after the bifurcation. So, if it becomes a stable periodic response from a stable trivial response, it becomes a stable periodic response then this is known as super critical Hopf bifurcation. But if from an unstable trivial branch it becomes again unstable or from a stable trivial branch the periodic response becomes unstable or from a periodic stable periodic response the resulting response become unstable the resulting response becomes unstable then, this becomes sub critical hopf bifurcation point.

So, from a stable fix point if you are getting stable periodic then, it is super critical and from a stable fix point if you are getting unstable fix point and unstable periodic response the resulting response, the resulting response is a resulting bifurcation is sub critical Hopf bifurcation point. So, in this way we know the different bifurcation point that is static and dynamic bifurcation points for the fix point response. So, these things will be used in our later analysis.

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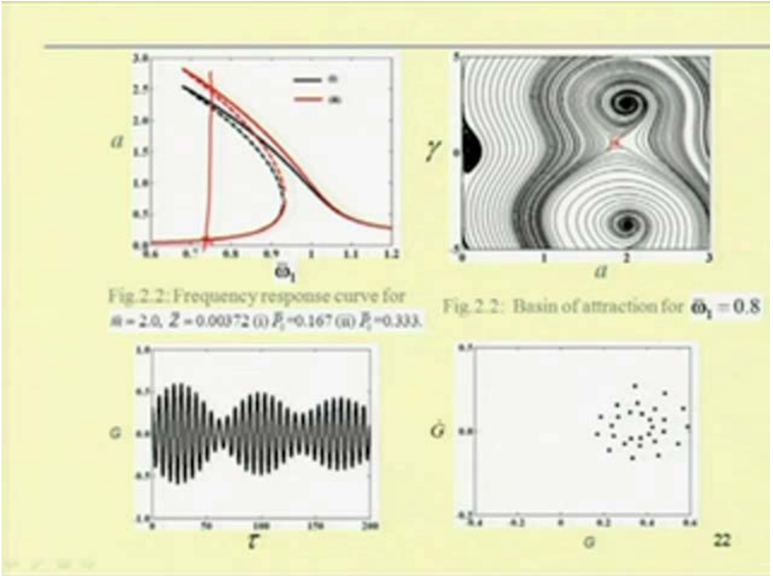

So, already we have discussed about different type of multi stability region. And in this multi stable region so, one should know so, one should know to which response one has to. So, at a particular control parameter one can have multiple. So, this is a fix point this is another fix point so, it has 2. So, these two fixed points are stable but, it has one unstable. So, to know to which stable branch the system response will move from an initial point so, one can plot the basin of attraction.

So, here the basin of attraction has been plotted by taking different initial conditions. So, these initial conditions one can take by considering one can make different reads and by taking different initial conditions one can plot the responses. So, here one can clearly observe 2 stable responses. So, this corresponds to. So, one can clearly observer 2 stable response and a saddle node here. So, saddle node correspond to saddle node correspond to the unstable response of the system. So, this is unstable response of the system and one can have a stable response also. So, near 0, this point is, this point corresponds to this and this correspond to the stable branch and this point correspond to the unstable branch. Also in previous classes, we have reviewed about different type of periodic response phase portrait and how to analyze this responses using Poincare section.

So, in case of a periodic response we have seen that the Poincare response can be shown as a single point and incase of a quasi periodic response the Poincare section can be a closed curve and incase of a chaotic response the Poincare section will filled up the phase portrait.

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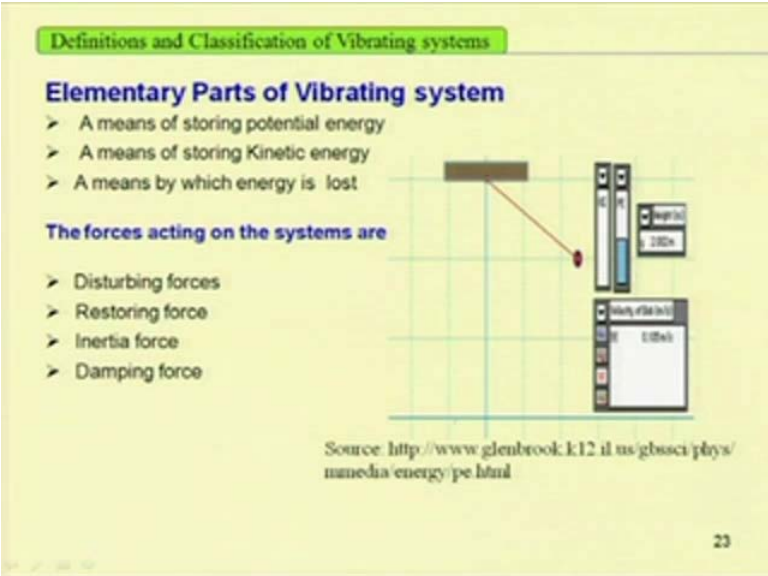

So, we have discussed about different elementary parts of a vibrating system. Today's class we are going to derive the equation motion by using this Newton's method and d'Alemberts principle, which are based on the force and moment equations. So, already we know different elementary parts of the vibrating system.

So, one required a means of storing potential energy. That means, when it is at its extreme position it has only potential energy when it is coming down then, this potential energy is converted to kinetic energy. At this point at the equilibrium point due to the inertia the system moves of and the motion continues. So, one required a disturbing force, restoring force, inertia force and damping force for finding the equation motion for the system. So, one can have different types of or different methods to find the equation of motion of a system.

So, these are inertia based approach. So, one is Newton's method, Newton's second law one can apply Newton's second law or d'Alemberts principle to find this equation motion, also energy based approach in which one can apply this Lagrange principle and extended Hamilton principle to find the equation motion.

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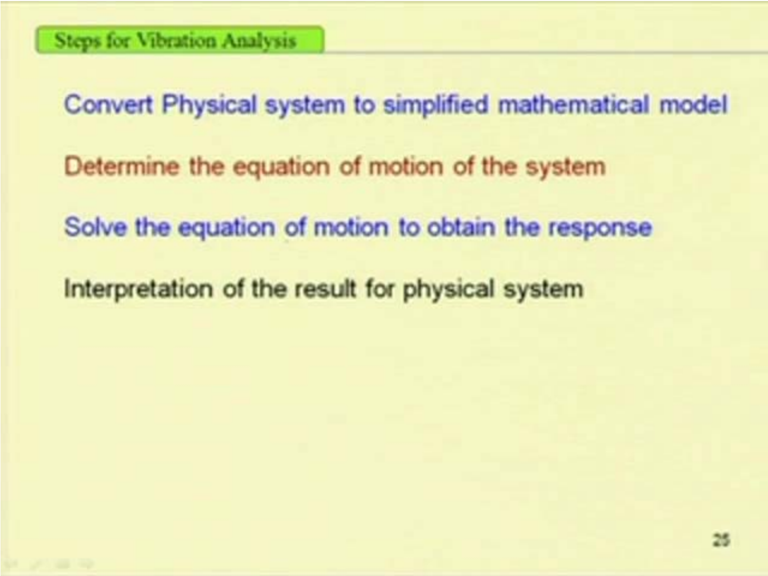

So, even one physical system. So, these are the steps one can follow to find equation motion or to study the system. So, first has to convert the physical system to simplified mathematical model then, determine the equation motion of the systems then, solve this equation motion to obtain the response and finally, interpretation of the result for the physical meaning of the system is required during this course of non-linear vibration. So, we will carry out all these 4 steps to study different systems.

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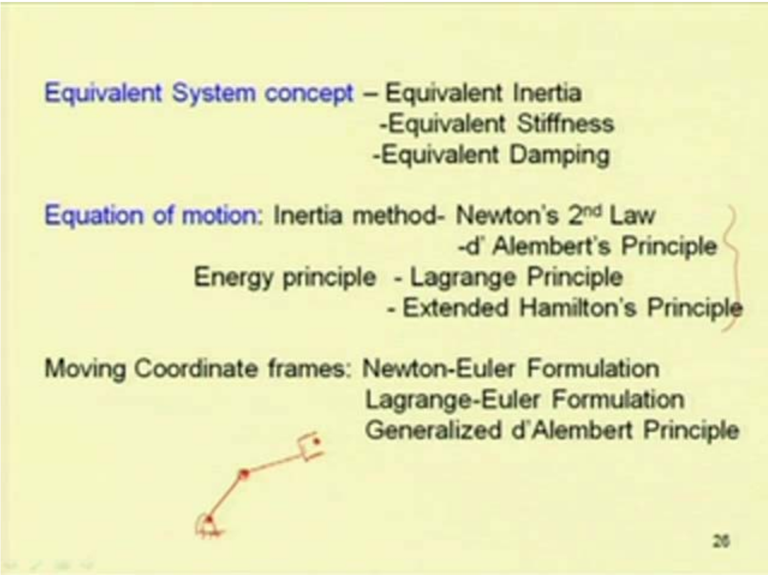

So, for example, let us see one system. So, we can convert a physical system by using this equivalent inertia, equivalent stiffness and equivalent damping. And for finding the equation motion we may use this inertia based method that is Newton’s second law d’Alemberts principle or energy based principle like Lagrange principle and extended Hamilton’s principle. So, if we have a moving coordinate frame, this Newton’s second law or d’Alemberts principle and this you can apply for bodies with fix coordinate frame. So, if we have moving coordinate frames then, we can go for Newton Euler formulation, Lagrange Euler formulation or generalized d’Alemberts principle in this moving coordinate frame.

So, moving coordinate frame problems will come. So, for example in case of a robotic manipulator, let us take 2 line robotic manipulators. So, in this case the end effectors position and orientation can be represented from with respect to this base by using different moving coordinate frame. So, one can attach coordinate frame to this base and at this link point also and at the end effectors point also. So, one can write the equation motion in terms of all these coordinate frames. So, in that case one can use this Newton Euler formulation, Lagrange Euler formulation or generalized d’Alembert principle based on this moving coordinate frame.

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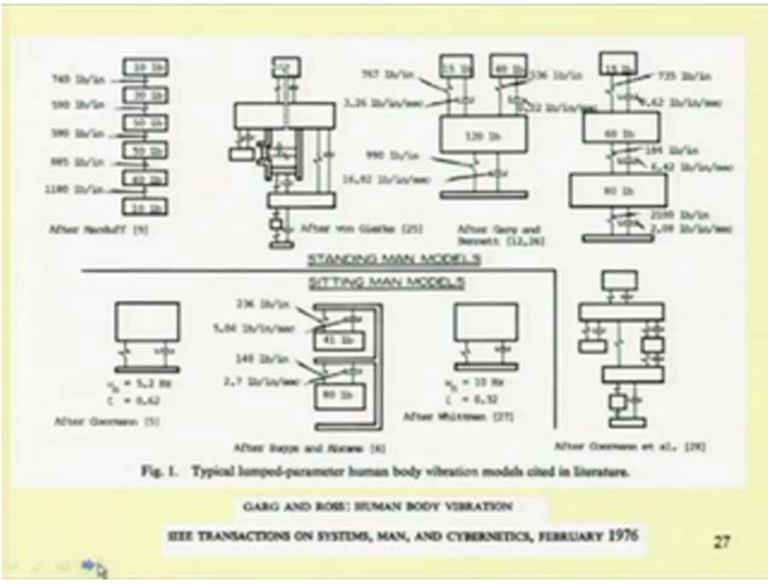

So, let us see. So, this is the different methods for converting a physical human body to different dynamic systems. So, mathematical modeling can be obtained by initially using the simplified models of a human body. So, one can attach different or one can write the whole human body as different spring and mass system. So, these are different spring and mass system from the leg position to the head position of the body or one can develop another type of model where, this portion can be represented as the spinal cord of the system then, these are different spring and mass damper system.

So, this is the head starting from the leg and head you can attach different spring and mass and you can convert the human body to a different model, different dynamic model. And this dynamic model can be simplified by putting this simplified model. Also one can take, one can combine this to and you can write a single mass and the spring damper and this part is here and one can have this 3 mass and 3 spring system. So, this is in standing position and in sitting position also one can develop different models. So, these are 4 different models developed in case of the sitting position and these are the 4 different models developed in case of the standing position and this work has been taken from this Garg and Ross human body vibration, Iterably transaction on the system man and cybernetics 1976.

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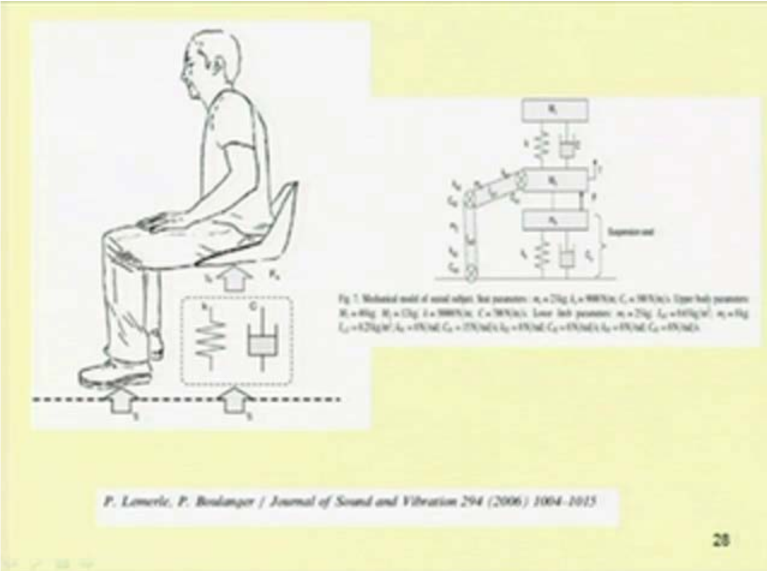

So, this is in the sitting position. So, the person is sitting on a chair so, if one can model this thing one can model the whole system by this spring and damper system. So, the leg part is modeled as true body to rigid body with the joints here. So, by taking all the spring and damper, spring mass damper one can develop a lumped parameter model for this system. Similarly, one can develop different type of lumped parameter model for different other different types of system.

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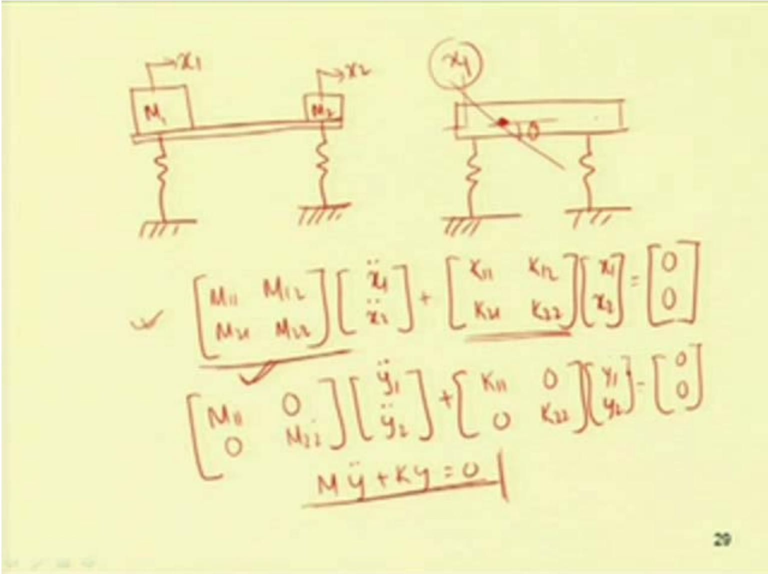

So, for example, in case of a leg machine, the leg machine can be modeled as a. So, this side one can model this as a spring and mass damper model spring and mass model. So, this is the mass of the head stuck m 1 and this is the mass of the tail stuck side. So, this is tail stuck side and this is head stuck side and this is the lathe bed. So, we are considering the mass to or the bed to be 0 or one can model this as a mass a centrically placed mass also. So, the mass can be put at this position.

So, in all these cases one can have different coordinate systems. So, in this coordinate system one can put different physical coordinate system with respect to a particular coordinate frame and develop the equation motion. So, depending on the coordinate system used one will obtain a coupled equation or uncoupled equation of motion.

So, for example, in this case one can have two different type of coordinate system, one can use this physical coordinate system or one can use generalized coordinate system. So, as this is a 2 degree of freedom system one can have the displacement x 1 here, displacement x 2 here or one can represent this thing by displacement of this side and the rotation of this side also. So, one can represent this equation in terms of x 1 and theta. So, either one represents by only a translational displacement at this two ends or by using this 1 translational and 1 rotation. Also these points can be changed and one can get different type of equation. So, depending upon the chosen coordinate system one can obtain a coupled or uncoupled equation motion.

So, this equation motion can be or a generalized equation motion for a multi degree of freedom system can be written in this form. So, let for 2 degree of freedom system, it will be M 1 1, M 1 2, M 2 1, M 2 2 into x 1 double dot x 2 double dot plus k 1 1, k 1 2, k 2 1, k 2 2, x 1, x 2. So, in case of a free vibration this will be equal to 0 0. So, in this case, if the half diagonal terms are present, then the system is coupled. So, this is dynamically coupled if the mass matrix is coupled then, the system is known to be dynamically coupled and if the stiffness matrix is coupled that is the half diagonal terms are present then, the system is known as statically coupled.

So, if it is uncoupled then, we can get the advantage of this like this. So, let the half diagonal terms are 0, in that case I can write this system equation in this form. So, it can be written in this form y 1 double dot y 2 double dot plus k 1 1 0 0 k 2 2 this is y 1 y 1 and y 2 so, this will be equal to 0 0. So, for the coordinate y 1, y 2 the equation is uncoupled. So, in this case the equation can be written as M 1 1 y 1 double dot plus k 1 1 y 1 equal to 0 and the second equation will be M 2 2 y 2 double dot plus k 2 2 y 2 equal to 0.

So, it can be reduced to, it can be reduced to 2 first order, it can be reduced to 2 single degree of freedom equations. In this case the equations are coupled but, in this case the equations are uncoupled, as it is uncoupled one can solve this equation as m y double dot plus k y equal to 0 that is, that of the single degree of freedom system. So, one can get the solution easily by decoupling these equations. So, there are several methods to decouple the equation. So, one can use this model analysis method to decouple this equation.

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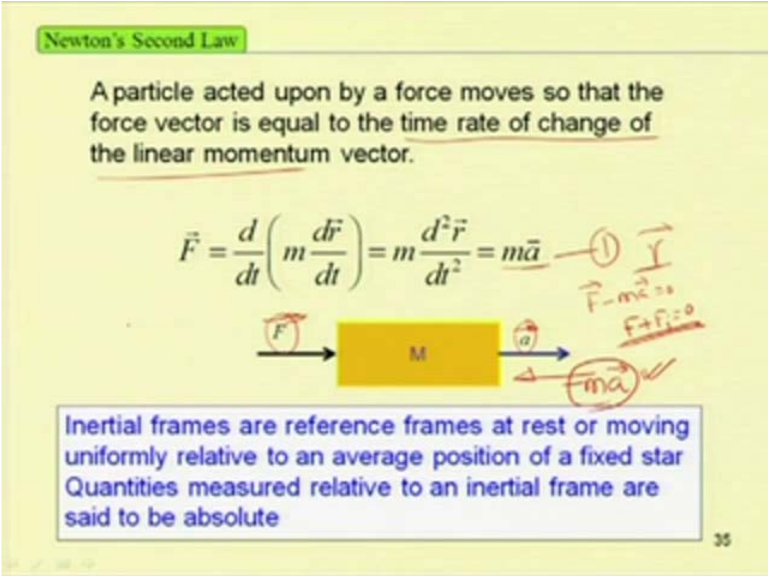

So, in today's class we are going to study about the Newton second law and d'Alembert principle and we will apply this thing to different systems. So, according to Newton second law a particle acted upon by a force move so that the force vector is equal to the time rate of change of linear moment of vector. So, let this mass m is acted upon by an external force a and an acceleration of a is produced in the system. So, in this case according to this second law, F will be equal to time rate. So, this vector is equal to the time rate of change of linear momentum. So, linear momentum equal to m. So, if displaced by a vector r with the position vector r represent the displacement of this body of mass m then, the change in momentum so, this is equal to m d r by d t.

So, one can write this equation as F equal to m a where, 'a' is the acceleration, F is the external force. So, external force equal to mass into acceleration this is second law Newton's second law. But if in a moving system, let the system is moving with an acceleration a, if a force of minus m a is applied to the system then, the body can be in equilibrium. So, F minus m a will be equal to 0. So, from this equation one can write F minus m a equal to 0. That means, summation of the external force summation of the external force and the inertia force so, summation of external force F plus this inertia force equal to 0. So, this is d'Alembert principle. So, one can obtain the d'Alemberts principle from this Newton's second law by adding this inertia force to the moving system.

So, in this way one can convert a dynamic system to one static system by adding the by adding the inertia force. So, the inertia force is a force which acts in a direction opposite to that of the acceleration and by adding the external force with the inertia force we can convert this dynamic system to static system. So, the disadvantage of the system is that the parameters we are using that is this force and this acceleration or all vector terms. So, it depends on both magnitude and direction.

So, depending on the direction if there is some ambiguity while calculating or while analyzing the system in the consideration of the direction then, the equation obtained equation motion will be erroneous. So, it is advisable to use scalar terms to develop this equation motion. So, by using, but using the scalar term that is for example, the potential energy and kinetic energy terms to write equation motion one can use this virtual work principle. But virtual work principle is applicable only for the static system. So, by applying this d'Alembert principle as we are converting this dynamic system to an equivalent static system now, one can apply this virtual work principle to this equivalent static system.

So, by using this d'Alembert principle so, after converting this dynamic system to a static system now, we are in a position to apply this virtual work principle. So, that is generalized d'Alembert principle which will study in detail. So, while applying this Newton's second law, one should know about the inertial frame. So, this inertial frame are the reference frame at rest or moving uniformly relative to an average position of a fix star quantities measured relative to this inertial frame are set to be absolute.

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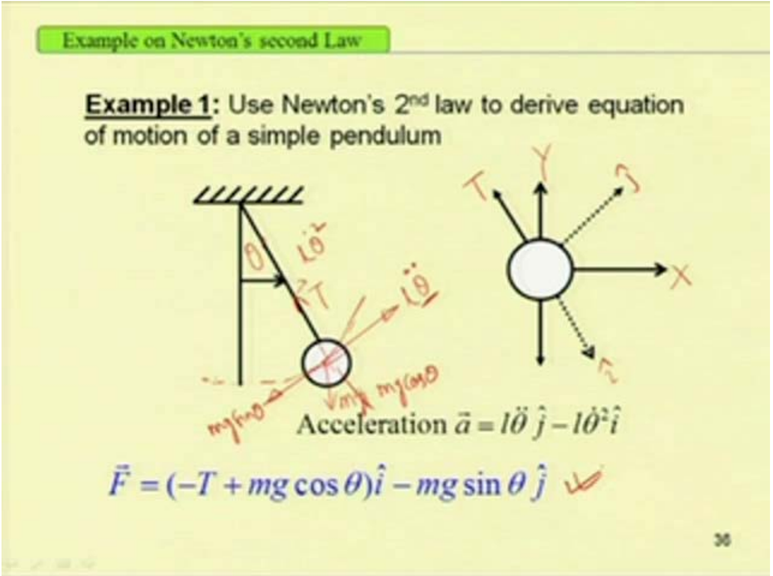

So, let us take or let us use this Newton's second law to derive the equation motion of this simple pendulum. So, in case of a simple pendulum this is bob with a massless rod. So, let me write this is my coordinate frame x y I can use a coordinate frame x y or I can use another coordinate frame also. So, I can take a unit vector along this direction that is and along this direction.

So, I can take. So, this is the let me take this is unit vector along this and perpendicular to this is the j and this represent the tension in the string. While moving, this is the three body diagram of this is 3 body diagram, this box in which the physical coordinate system can be shown either in terms of x y or using another coordinate system that is, in terms of unit vector i and j. So, the total force acting on the system can be written. So, this is the force tension force t and another force will act on the system.

So, this is mg and at the action of these two forces the system will be in equilibrium if we add the inertia force to the system. So, the acceleration at these points is of two types. So, if it is rotating by an amount theta it has a tangential acceleration, if it is rotating in this direction, you will have the tangential acceleration on this, that is equal to L theta dot square. And another term also we can have this term equal to L theta double dot and another term which is towards the center of rotation that is equal to L theta dot square. So, for a moving or rotating, this rotating buff so, we have 2. So, for this rotating buff we have 2 terms in acceleration: one is the tangential term that is, L theta double dot L theta double dot that is L alpha theta double dot is the angular acceleration and theta dot is the angular velocity.

So, we have the centripetal acceleration that is, L theta dot square or this is the acceleration towards the center that the centripetal acceleration that is L theta dot square and another term that is the tangential term this is l theta double dot. So, this is subjected to a force T that is tension and mass M and this mg force. Mass into acceleration due to gravity mg force can be divided into 2 parts so, 1 is tangential to the path and other one act’s along this direction.

So, this is equal to mg cos theta and this term equal to mg sin theta. So, as T will be equal to mg cos theta then, the resulting force acting on the system equal to mg sin theta and the inertia force also acting on the system. So, total force acting on the system external force on the system equal to. So, as I am taking a coordinate system i in this direction, it will be minus T and mg cos theta minus T plus mg cos theta i minus mg sin theta j. So, this direction as this direction is taken positive j. So, this is negative j so, this is minus m g sin theta j.

So, this equation becomes minus T plus mg cos theta i minus mg sin theta j. So, this is the external force acting on the system. So, according to Newton’s second law this external force equal to mass into acceleration and already we have written this acceleration in terms of i j like this. So, a will be equal to as we have taken this as the positive i it will be minus l theta dot square i and this is positive direction of j so, this becomes L theta double dot j. So, L is the length of the simple pendulum. So, now, by substituting this F equal to m a we can write the equation like this.

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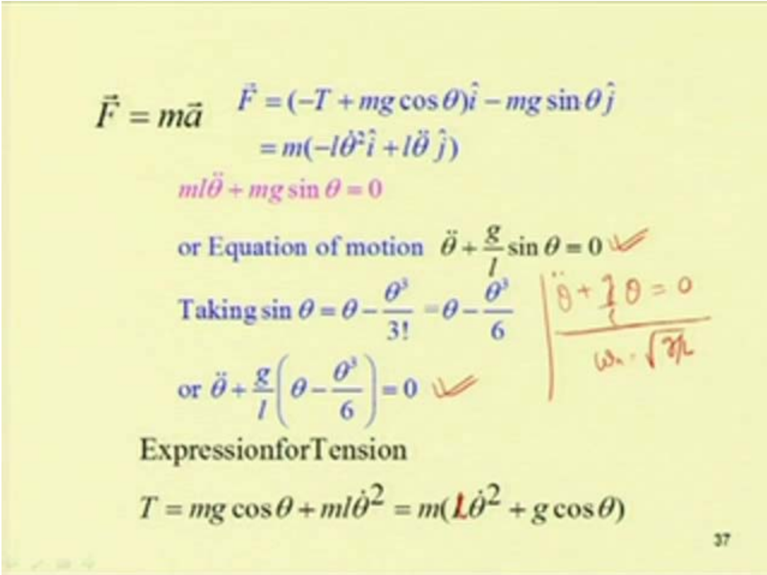

So, F equal to m a, F equal to minus T plus m g cos theta i minus m g sin theta j so, this will be equal to m into minus l theta dot square i plus l theta double dot j. So, by equating the ith component and jth component. So, I can write the equation motion of the system it will be m l theta double dot plus mg sin theta equal to 0 or by dividing this by m l this equation reduces to theta double dot plus g by l sin theta equal to 0. So, when theta is very small one can write this equation in this form theta double dot plus g by l theta equal to 0.

So, this will lead to simple harmonic motion where, omega n equal to root over g by l. But if theta is not small one can expand this theta in this form theta minus theta cube by factorial 3 or sin theta equal to theta minus theta cube by 6. So, this resulting equation becomes theta double dot plus g by l theta minus theta cube by 6 equal to 0. So, one can write the expression for tension in this form also so, T equal to mg cos theta plus m l theta dot square. Or one can write so, this is equal to m l theta dot square plus g cos theta by taking m common.

So, this way one can find the governing equation of the system by applying Newton’s second law. So, while applying Newton’s second law one can write the equation motion in this better form first one can write the acceleration and then one can write the force term. And by equating the force equal to mass into acceleration one can find the equation motion. So, this is this simple example illustrate how one can use this Newton second law to derive the equation motion. So, one can have, in this case we have taken or we have derived this equation motion by considering force equal to mass into acceleration but, if you can consider this as a rotating system.

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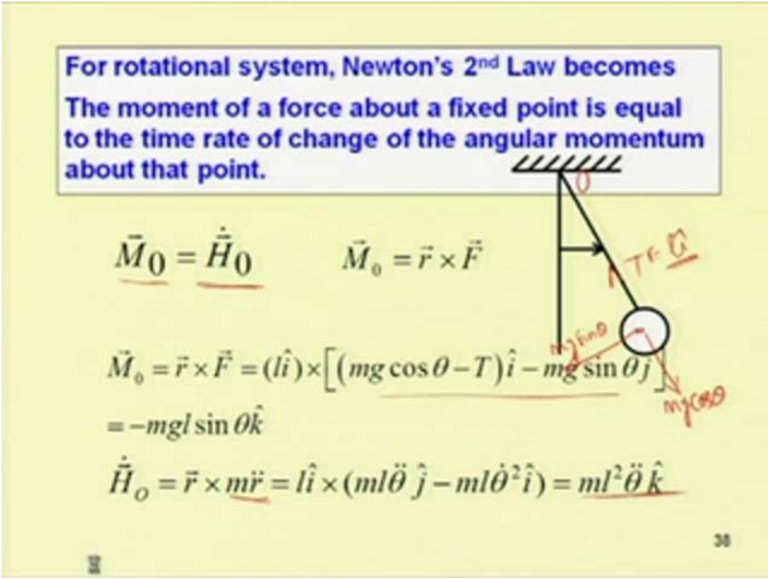

So, in that case also we can write the Newton's second law or Newton's second law can be written in this form. The moment of a force about a fix point is equal to the time rate of change of the angular momentum about that point. So, after finding the external force so, this external force acting on this, this is T and this is mg sin theta and this term equal to mg cos theta. So, we can write this M 0. So, this equation can be written M 0 that is moment about this point due to this external force will be equal to rate of change of angular momentum.

So, this is rate of change of angular momentum. So, we can find. So, this M 0 equal to r cross F. So, already we know F and r equal to in vector form if you write, r will be equal to l i so, r equal to l i and we can write this M 0 equal to r plus F so, this becomes l i cross mg cos theta minus T i and then, minus m g sin theta j. So, as we are having this cross product then, i cross i this becomes 0. So, we will have i j that will be differentiated by k i cross j equal to k. So, this becomes minus m g l sin theta k. So, m 0 becomes m g l sin theta k. So, now we can H 0 so, H 0 dot will be equal to r cross acceleration term. So, acceleration equal to m r double dot. So, l I cross so, r equal to l i.

So, cross m l theta double dot j minus m l theta dot square i. So, this becomes m l square theta double dot k. Now, by equating these two terms one can find the equation motion to be this.

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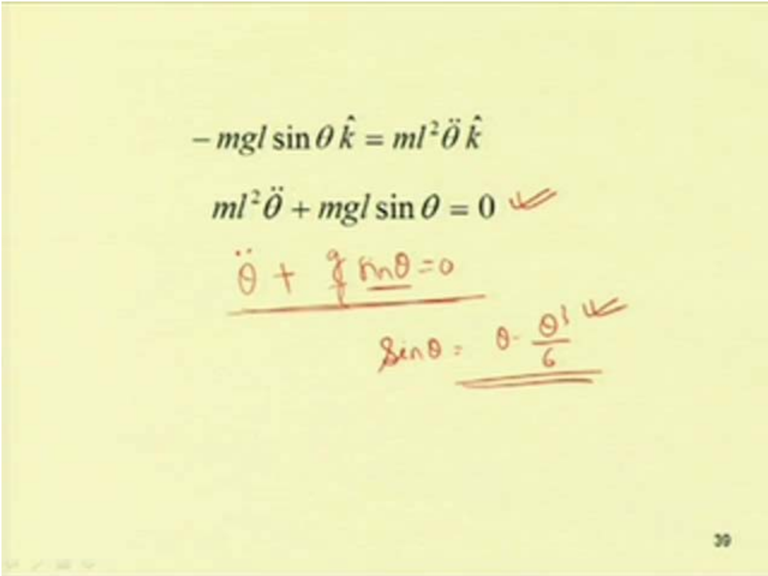

So, what we obtained before the equation motion equal to theta double dot g by l sin theta equal to 0. So, one can write the non-linear equation motion by expanding the term sin theta equal to so, sin theta equal to theta minus theta cube by 6. So, in this way one can write the equation motion. So, as we have discussed previously we can or one can have or one can obtain the potential well and one can study the response of this system qualitatively.

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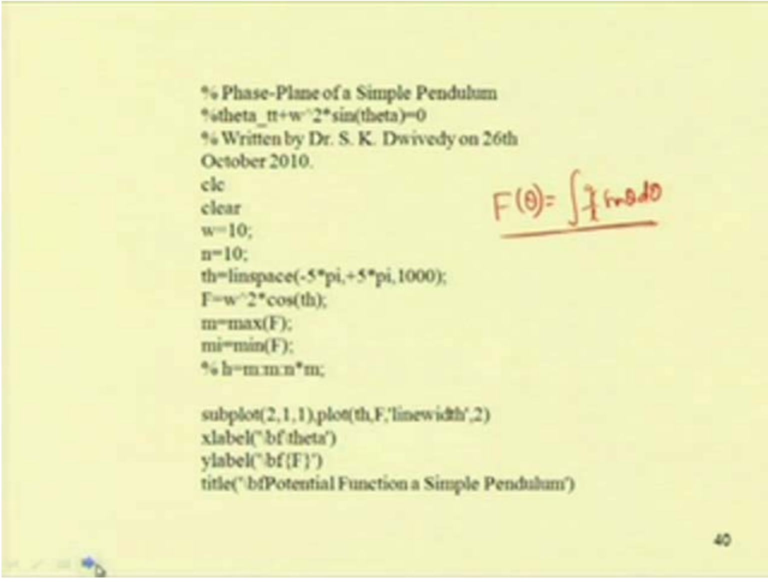

So, one can, one can write a program, simple matlab program to find the potential function. So, potential function F theta will be equal to. So, in this case F theta equal to g by l sin theta so, capital F theta will be. So, this will be equal to g by l sin theta b theta. So, this is 0 to l. So, one can find this F theta. So, this will be g by l minus g by l cos theta. So, in this case it is written so, F equal to omega square cos theta.

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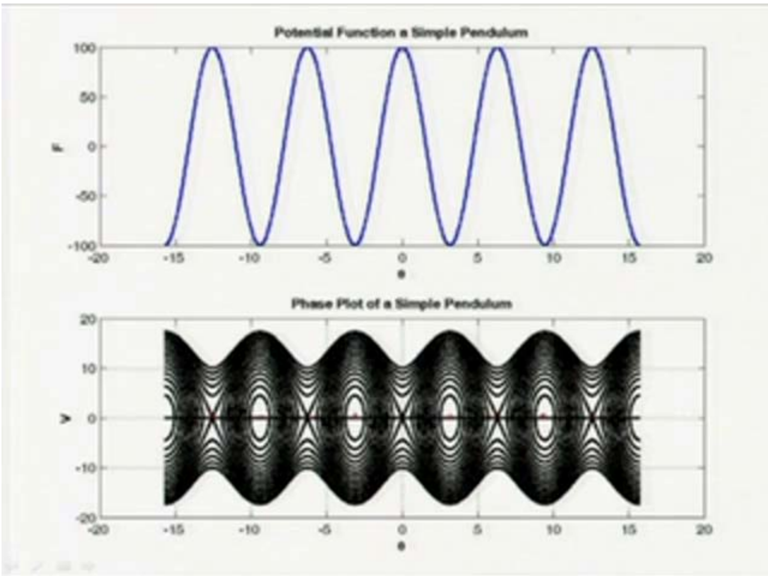

So, one can similarly find the theta dot term and if one plots so, this is the potential function and the resulting phase plot for the simple pendulum. So, these are the center point and these are the saddle node points. So, it has been plotted for different angular position of the pendulum. So, these are the center point and these are the saddle node point and these are the separatrices which separate and in between separatrices we have this Homoclinic and Hetroclinic orbits. So, these are homoclinic orbits and these are the hetroclinic orbits and one can study the flow around this saddle node and center points also.

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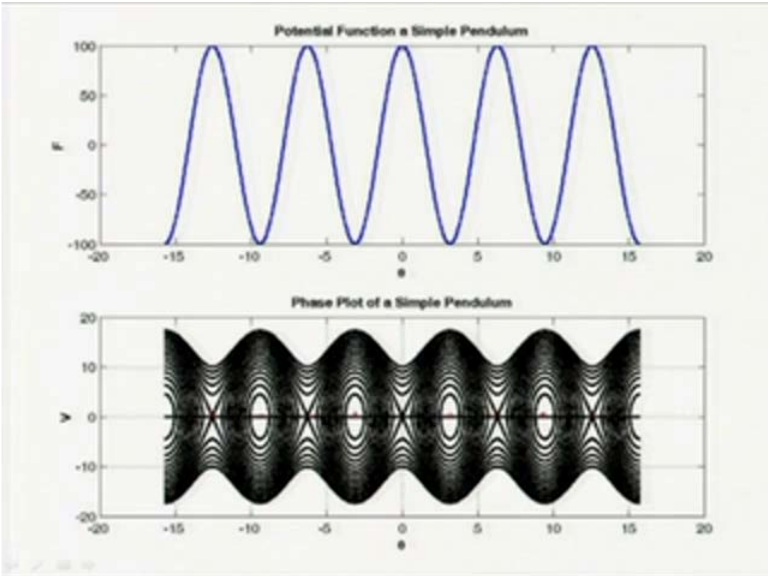

So, one can study also the work energy principle which is required to derive the generalized d’Alembert principle, which we required for finding or we required for converting this vector form of equation motion to a scalar form or to derive the equation motion using the scalar form like potential energy, kinetic energy. So we can convert or we can use this work energy principle. And according to work energy principle the work performed in moving a particle from position 1 to 2 is equal to the change in kinetic energy.

So, work done equal to F dot d r, from position r 1 to r 2 so, one can find this is potential work done. So, this work done equal to the change in kinetic energy. So, work done if one can write this F equal to m r double dot. So, one can write this thing equal to d of half m r dot r dot and this thing can be written in this form. So, this is equal to t 2 minus t 1. So, the work done in moving body from or particle from position 1 to 2 equal to change in kinetic energy of the system so, force for which the work performed in moving a particle over a closed path is 0.

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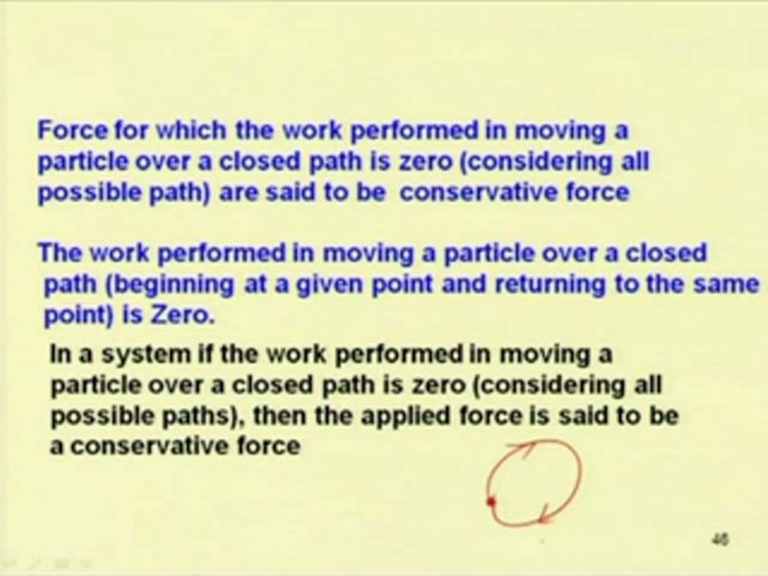

So, considering all possible paths are said to be conservative forces. The work performed in moving a particle over a closed path beginning at a given point and returning back to same point. So, if it has started from this and it has come back to this same point then, the work done is 0. Similarly, in a system if the work performed in moving a particle over a close path is 0 considering all possible paths then, the applied force is said to be a conservative force.

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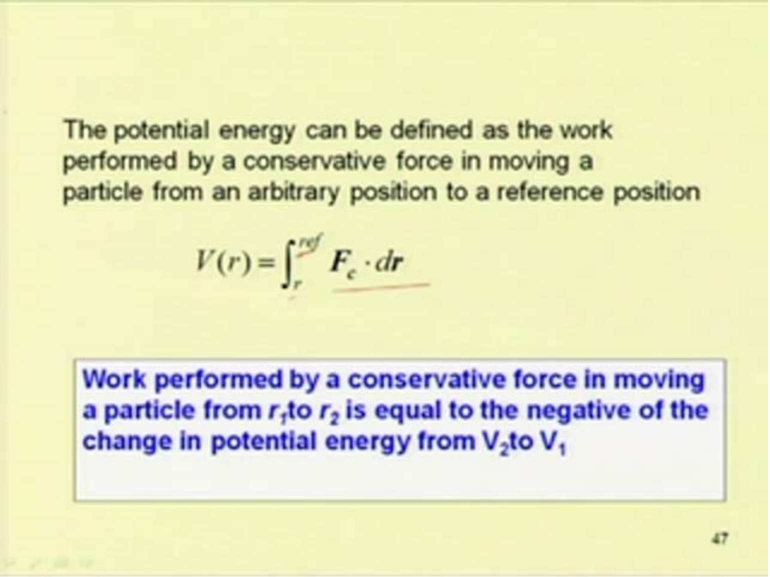

The potential energy can be defined as the work performed by a conservative force in moving a particle from an arbitrary position to a reference position. So, the potential energy is defined by the work done or work performed by a conservative force in moving a particle from a arbitrary position so, arbitrary position r to the reference position this. So, the work done one can find and that will be the potential energy of the system. So, work performed by conservative force in moving a particle from r 1 to r 2 is equal to the negative of the change in potential energy from V 2 to V 1.

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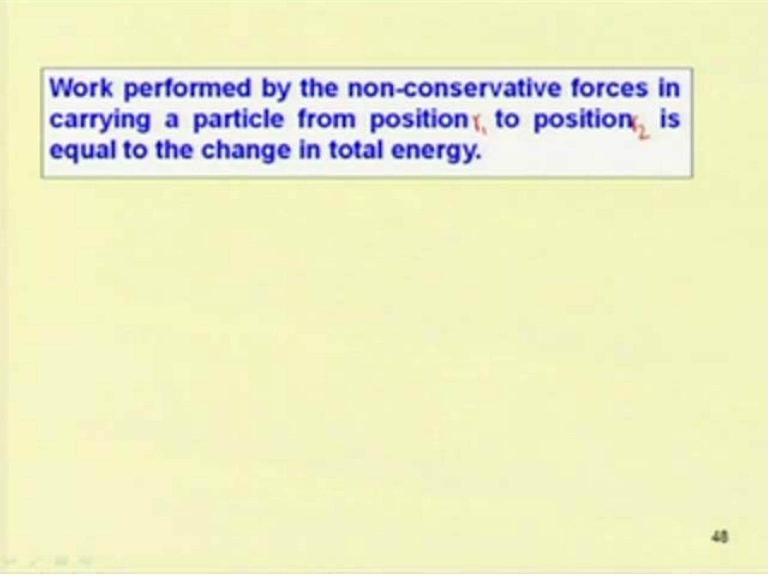

Work performed by non-conservative forces in carrying a particle from position r 1 to position r 2 is equal to the change in its total energy. Work performed by non conservative forces in carrying a particle from position r 1 to r 2 is equal to the change in total energy of the system.

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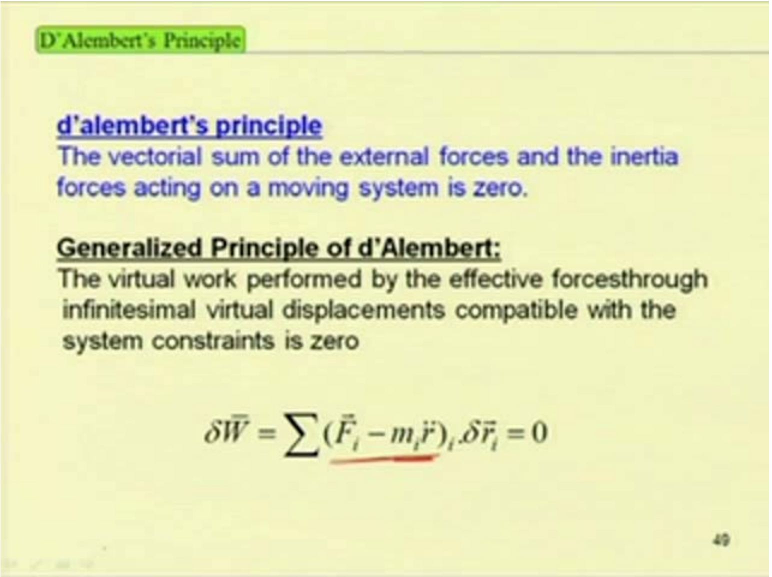

So, already we know about d'Alembert principle. So, it states the vectorial sum of the external forces and inertia forces acting on a moving system is 0. And now, one can use this to state the generalized principle of d'Alembert like the virtual work performed by effective forces through infinitesimal virtual displacement, compatible with the system constraints is 0. So, one can write the work done equal to. So, this is the inertia force, F i minus. So, this is this force this is the equivalent force of the system dot delta r i equal to 0. So, the virtual work equal to 0. So, this is d'Alembert principle.

Now, by using this d'Alembert principle one can derive the Lagrange principle and the extended Hamilton principle of the system. So, in extended Hamilton principle or Lagrange principle the terms used are potential energy and kinetic energy. So, these are scalar term that does not depend on these two terms, does not depend on the direction of the system.

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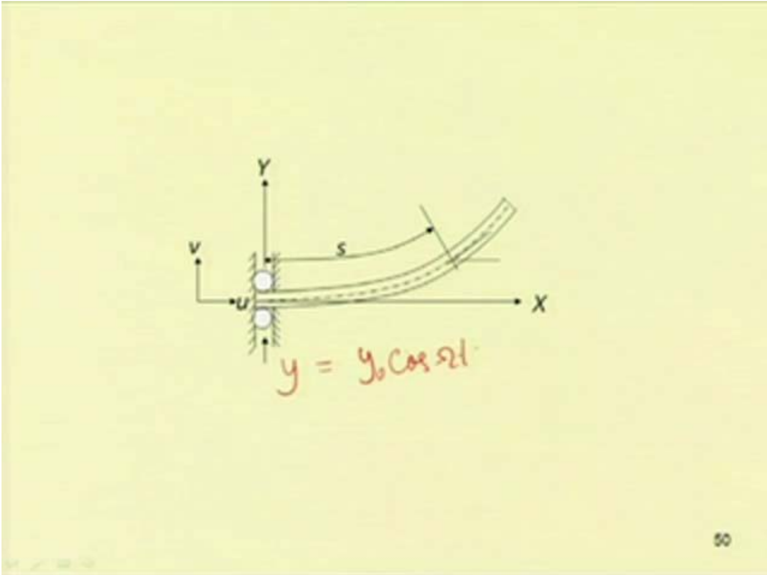

So, one can easily find the equation motion by using those scalar terms. Now, one more example we can see. So, in this case we have to derive the equation motion for cantilever beam, large cantilever beam which is subjected to a periodic force y equal to y 0 sin or cos omega t. So, let this y 0 cos omega t.

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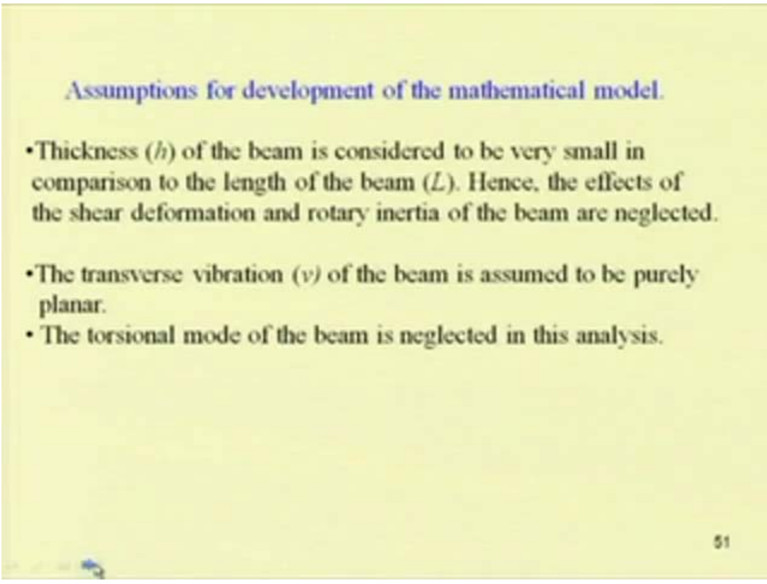

So, in this case one can take these assumptions, you can assume this as an Euler Bernoulli beam and derive the equation motion. So, by using Newton's second law one can write the bending movement equals to e i.

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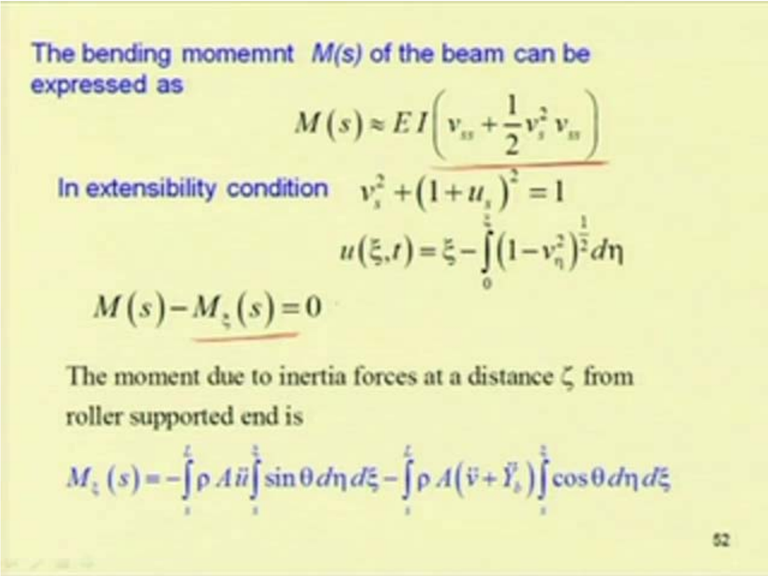

So, this is the non-linear. So, assuming large deflection or moderately deflection one can write the bending moment. And now by using this inertia term one can write this bending moment minus this inertia, bending moment due to this inertia equal to 0 and one can derive the resulting equation motion.

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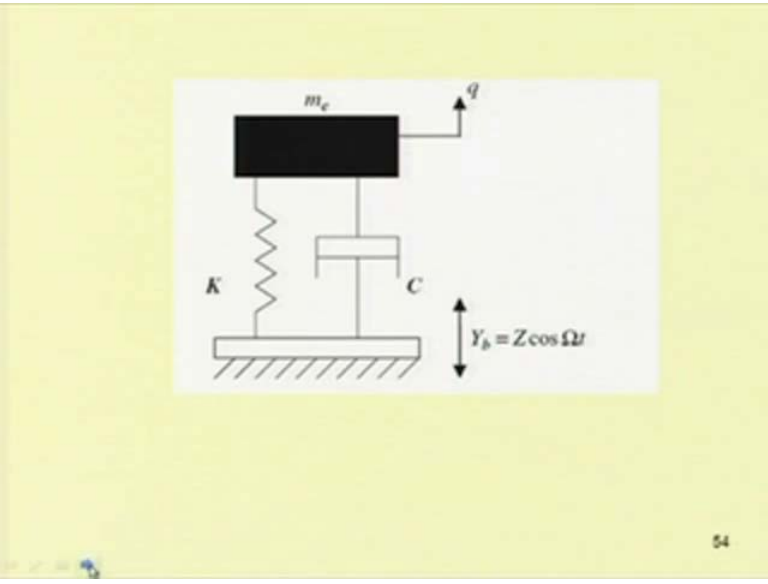

So, one can model the system like a spring mass damper system with moving base and find the equation motion also. So, let us see some quiz problem.

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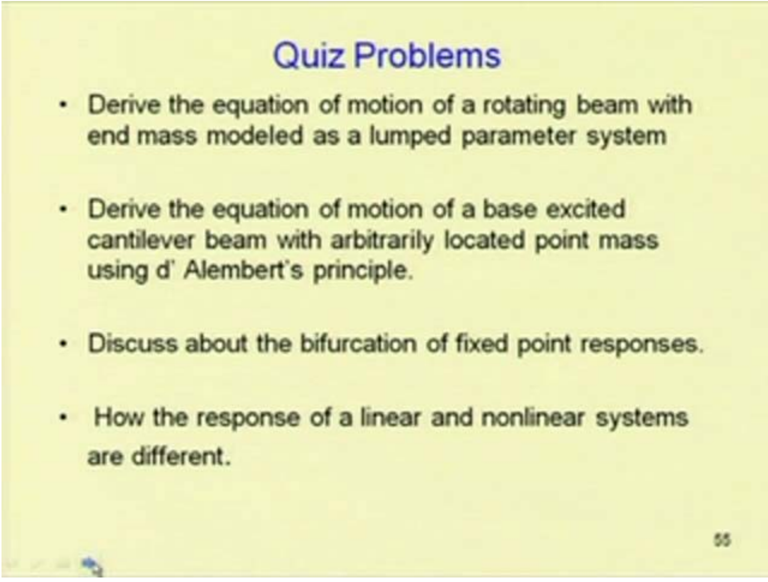

So, derive the equation motion of a rotating beam with end mass modeled as a lumped parameter system. Then, derive the equation motion of a base excited cantilever beam with arbitrarily located point mass using d’Alembet principle. Discuss about the bifurcation of fixed point responses, how the response of a linear and nonlinear system are different? So, for the first one you may refer. So, this is the system.

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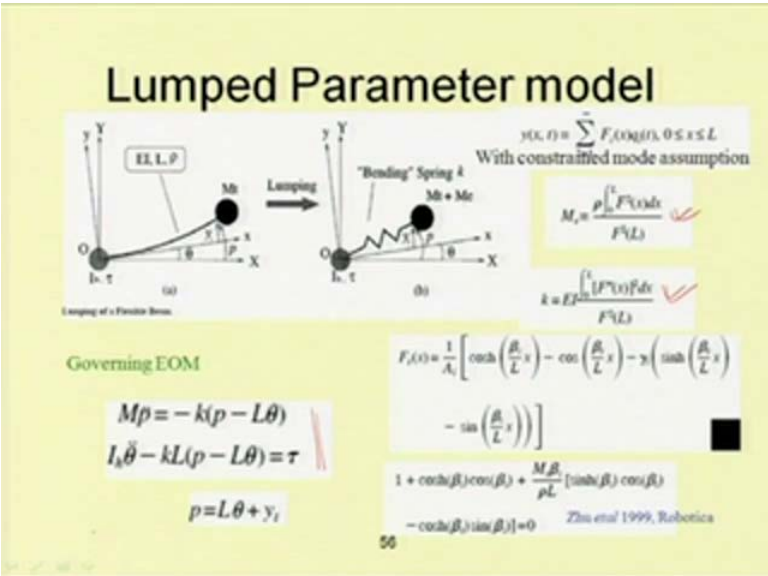

So, you can model the bending of the system like a bending spring and this mass you can take this mass of the mass of this part empty plus mass elastic mass. So, it can be taken as the elastic mass. The lumped mass is present here, that is M t plus M e and this is the bending spring. So, this M e can be obtained from this equation. Similarly, the k can be obtained the stiffness can be obtained from this equation and the governing equation can be written in this form. So, these are the hints given for deriving the equation. Similarly, for a base excited system.

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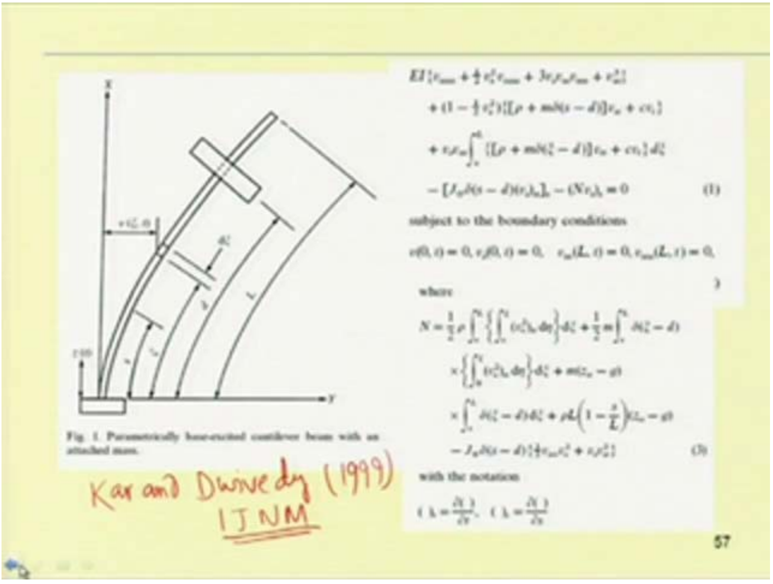

So, one you can refer the paper by Kar and Dwivedy 1999. So, this paper is from International Journal of Non-linear mechanics. So, the equation motion can be derived using this d’Alembert principle, by taking a small element, by writing the bending moment at this point one can derive the equation motion. So, in this way you can derive the equation motion. So, in the next class, we will use the Lagrange principle and extended Hamilton principle to derive the equation motion of different systems.

Thank you.