Non-Linear Vibration
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Module - 1
Introduction
Lecture - 2
Phenomena associated with non-linear systems

So, in today's class we are going to study about different phenomena associated with the nonlinear systems.

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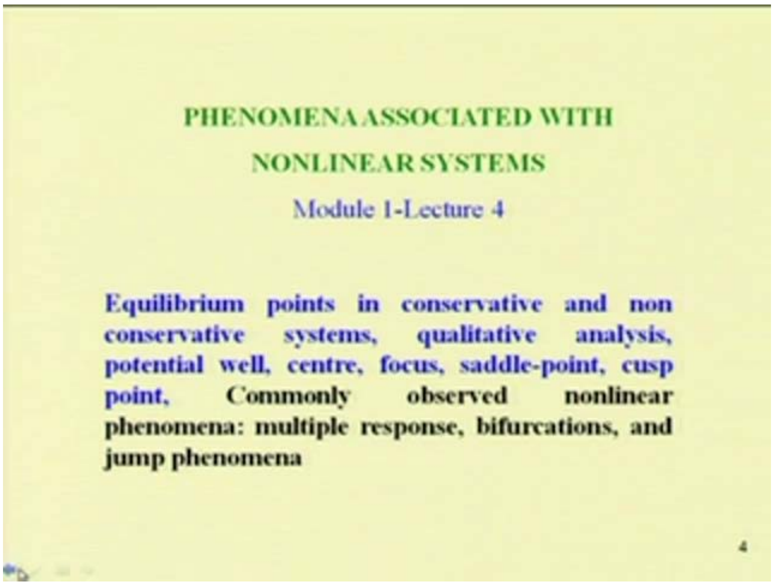

So, here we are going to tell about this equilibrium points in case of conservative and non conservative systems, also some qualitative analysis, what I told in the last class, I will elaborate on that; will develop the potential well. And discuss about the center focus saddle point, cups point and commonly observed non-linear phenomena such as, multiple resonance conditions, bifurcations and jump phenomena and other associated phenomena in non-linear systems.

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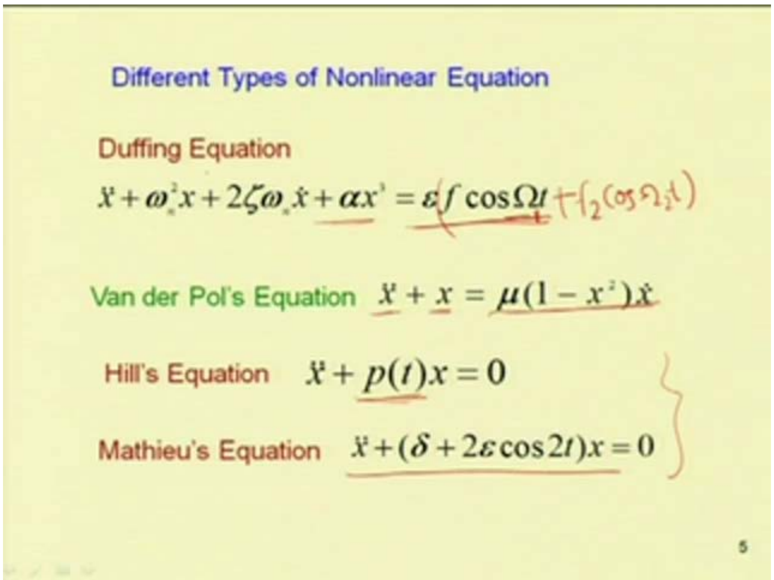

So, last class we have seen four different types of equations: one is Duffing equation, which can be commonly derived from a spring mass damper system with by taking a spring mass damper system with non-linear damp non-linear spring.

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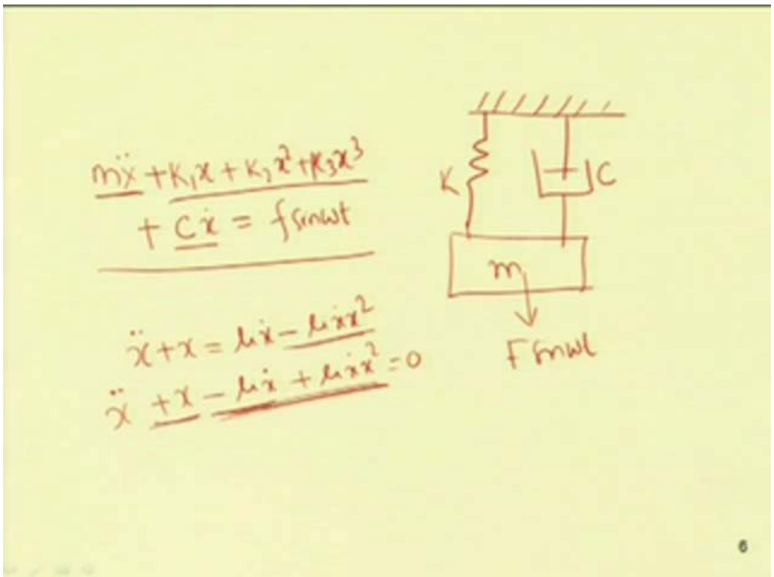

So, let us take the system. So, this is a spring mass damper system, this is mass m, spring constant k, this is c. So, if the spring constant, if the spring is not linear spring or it is a non-linear spring then, the equation motion for this system can be written in this form.

So, for the mass we can have the inertia force that is m x double dot plus, for the spring so, we can have a linear part and a non-linear part.

Let us assume we have a cubic non-linearity and a quadratic non-linearity. So, in that case I can write this is equal to k 1 x plus k 2 x square plus k 3 plus k 3 x cube then, plus the c x dot plus or this will be equal to the external force that is F sin omega t. So, in this m x double dot that is the inertia force and these are the 3 terms are the spring force and this is the damping force which is equal to the external force. So, this equation is of Duffing type. So, I can write this equation in this form x double dot plus omega n square x plus 2 zeta omega n x dot plus alpha x cube, I can add some other non-linear term like beta x square will be equal to f epsilon f cos omega t.

So, last class also, we have studied about the scaling parameter and scaling parameter to order the non-linear equation. So, in this case the forcing can be of this form so, this is the inertia force that is x double dot and then this is the spring force and we have a non-linear damping term here. Also, we have studied about this van der pol equation so, this equation can be written x double dot plus. So, in this case the equation can be written x double dot plus x equal to so, mu x dot minus, this is mu x dot minus mu x dot into x square mu x dot into x square. So, we have a non-linear damping term here. So, I can take this term to left hand side and I can write this x double dot plus x minus mu x dot plus mu x dot x square equal to 0.

So, we know, we have a positive stiffness and we have the damping is negative but, the resultant force from this minus mu x dot and plus mu x x mu x dot x square, if this is positive then, the system will be stable otherwise, the system will be unstable. So, depending on the relative magnitude of which term is more we can have a stable or unstable response of the system.

So, when this part is this non-linear part is more so, this x that; that means, the amplitude is more then, we can have a negative term here and we will have a positive damping, overall positive damping in the system also. But when this is small this amplitude is small then, the damping term will be mu x dot and will have a negative damping.

So, due to the presence of negative damping the system will vibrate with high amplitude or the system will be unstable and when this term is small that is the amplitude is small so, we have a positive damping and the system response will be stable or system will come to an equilibrium position. So, in this case for steady state we can have the steady state response. So, steady state response for steady state response I can take this x dot and x double dot equal to 0. So, the steady state response will be x equal to 0.

So, we can have a limit cycle in your x equal to 0 in this case because when we are increasing the damping, the system will be stable and for less when you are increasing the amplitude, the system is stable and when the amplitude is less the system is unstable. So, there exists a response that is the limit cycle for which the or there exist a limit cycle which exhibited by this type of system.

Then, we have studied about this Hill's equation and Mathieu equation or commonly this equation is known as Mathieu hill equation. So, in case of a hill equation, these term p t is coefficient of x that is the response of the system as this used as a parameter of the response of the system that is why this is known as a parametrically excited system. And incase p t equal to delta plus 2 epsilon cos 2 t that is we are taking a periodic function with 2 t and this delta is a small parameter then this equation is reduced to that of a Mathieu equation. So, both these equations are for parametrically excited system. So, in case of duffing equation by taking this force equal to 0.

So, we can have this duffing equation for a free vibrated vibrating system and when you are considering the force this is the equation for a forced vibrating system. So, in this case instead of taking a single force one can take a number of forces also. So, in that case the system will be subjected to multi frequency excitation. So, in this case we have a single excitation omega, but, one can add some other terms like, f 2 cos this is omega 1 t. So, one can put this omega 2 t.

So, in this case the system is subjected to a 2 frequency excitation. Similarly, one can go on adding the number of forces to have multi frequency excitation. So, in case of parametrically excited systems also one can add the number of forces to reduce this equation to Mathieu Duffing type or Duffing Mathieu equation. Similarly, in case of van der pol equation also one can add this forcing term to have this forcing term and the non-linear cubic non-linear term or non-linear terms to have a Duffing van der pol equation. So, one can have a combination of equations taking this duffing equation, Van Der Pol equation, Hill's equation and Mathieu equations. So, let us take one example how we can order a non-linear system.

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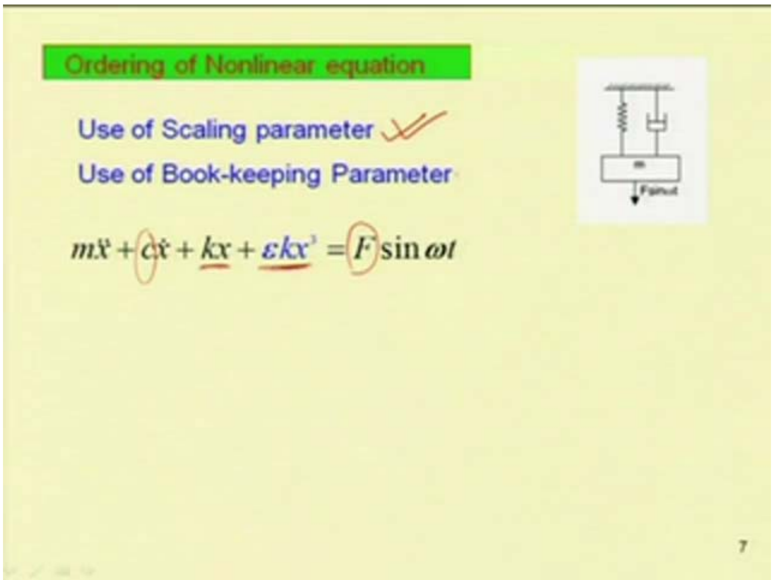

So, given a non-linear differential equation we can use the scaling parameter and book keeping parameter to order these systems. So, to make the system of the same order generally, this non-linear term already we have seen so, this is the non-linear, this part is the non-linear term associated with this duffing equation. So, generally this part is a small addition to the force, the spring force. So, if it is a small addition of the spring force, then by taking a linear the solution of a linear system we can find the equation or the solution for this non-linear system.

But if this term is large, we cannot take this linear system or we cannot use the linear part of the solution to find the resulting solution of the system. So, for that reason we showed first know how to order these terms. So, to make this term and or to make the non-linear term of the same order or the forcing term of the same order or the damping term of the same order so, we can use the scaling parameter and sometimes we have to use this book keeping parameter also to order the equation. So, let us take one example.

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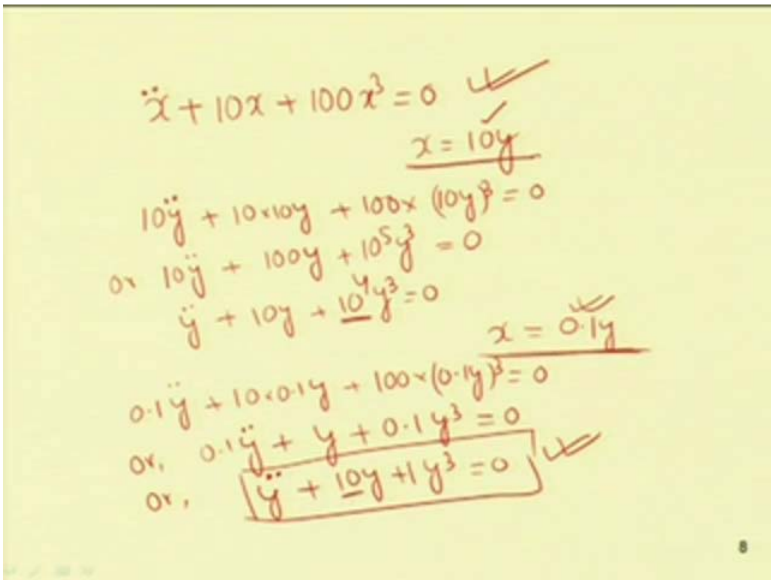

So, in this case let x double dot let us take a single x double dot, let this is 10 x plus 100 x cube equal to 0. In this case, if I will take x equal to let me take x equal to 10 y so this equation will be reduced to 10 y double dot plus 10 into 10 y plus 100 into so, this is 10 y cube, this is equal to 0 or this equation will becomes 10 y double dot plus so, this becomes 100 y plus so, this is 10 to the power 3 and 2 so, 10 to the power 5 y. So this is equal to 0 or y double dot plus 10 y plus 10 to the power 4 y cube so, this is equal to 0.
So, by taking x equal to 10 y using the scaling factor of 10 then, in this case you have observed that this coefficient has increased. But if I will take a term let I am taking a term this is equal to 0.1 y x equal to 0.1 y. So, in this case my equation will reduce to so, this is 0.1 y double dot, this is 10 into 0.1 y plus 100 into 0.1 y cube this is equal to 0 or this is 0.1 y double dot plus. So, this becomes y and this term becomes.

So, this is 1001 by 1000 this is 100 so, this becomes 0.1 y cube so, this is equal to 0 or I can write this equation in this form so, if I will divide this 0.1 everywhere, then, this equation reduces to y double dot plus 10 y plus y cube equal to 0. So, if you compare this equation with the original equation you can see the coefficient of this non-linear term is a small. So, coefficient of non-linear term is 1, while the coefficient of the linear part that is the spring stiffness that is equal to 10. So, 1 is very-very, 1 is 10 time less than this 10. So, in this way by using proper scaling parameter here we have used a scaling parameter of 0.1. So, by using a proper scaling parameter we can scale down or we can rewrite the equation of motion in a different form. Now, to make this term and this term of the same order, I can use one book keeping parameter.

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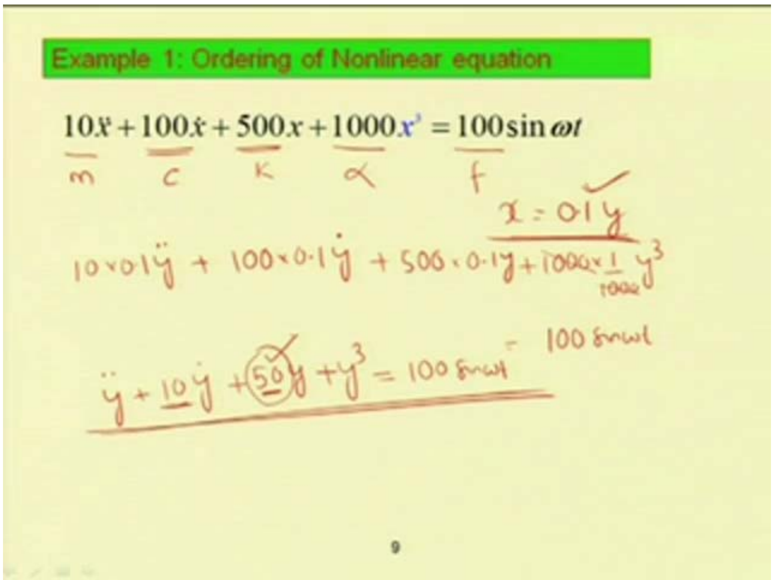

So, I can use some book keeping parameter. So, in this case let us take one more example, this is 10 x dot plus 100 x plus let us take 5, so, in this case 500 is k so, this is c and this is our m and this is alpha and this is the forcing term. So, if I will take let me take x equal to 0.1 y. So, if I will take x equal to 0.1 y, so this equation will reduce to, it will be 10 into 0.1 y double dot plus 100 into 0.1 y dot plus 500 into 0.1 y plus 1000 into 1 by 1000 so point cube will be equal to 1 by 1000 y cube. So this will be equal to 1000 sin omega t let us take so, omega t. So now, this becomes y double dot plus. So, this reduces the damping term so, 10 y dot plus here the stiffness term is also reduced from 500 it becomes 50 y plus this you can check that this non-linear term has reduced a large amount. So, this becomes y cube only. So, the coefficient is 1 now and this becomes equal to 100 sin omega t.

So, we have reduced the coefficient of the non-linear term by using the scaling parameter x equal to point y. Similarly, by taking suitable value instead of taking 0.11 can take a different value so that these coefficients can be comparable. So, generally the damping term is very-very small. So, we should require the coefficient of this damping term should be smaller than the coefficient of this stiffness term similarly; the coefficient of the non-linear stiffness term should be very-very less than that of the stiffness term.

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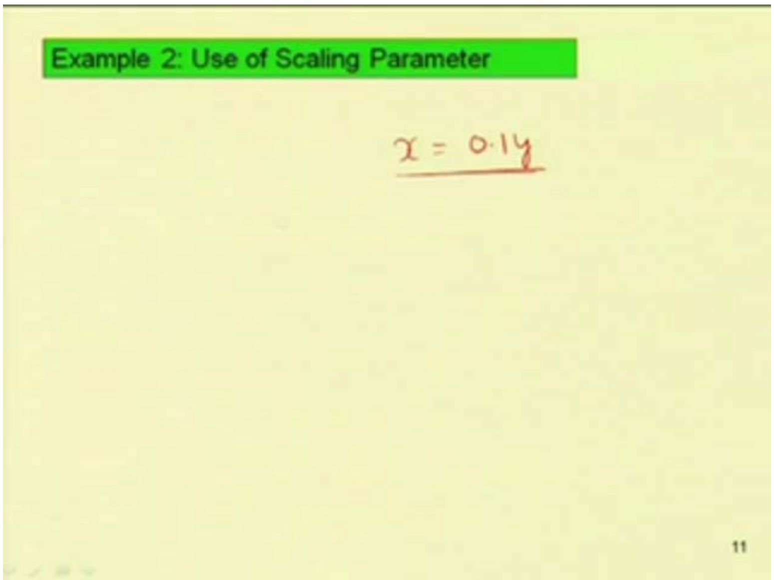

So, let us see how to use. So now, you have seen the use of scaling parameter. So, we have taken in both the cases we have taken x equal to we have taken a term x equal to 0.1 y and we have found the equation. But in this case, we have seen that the forcing term is unchanged by using that expression.

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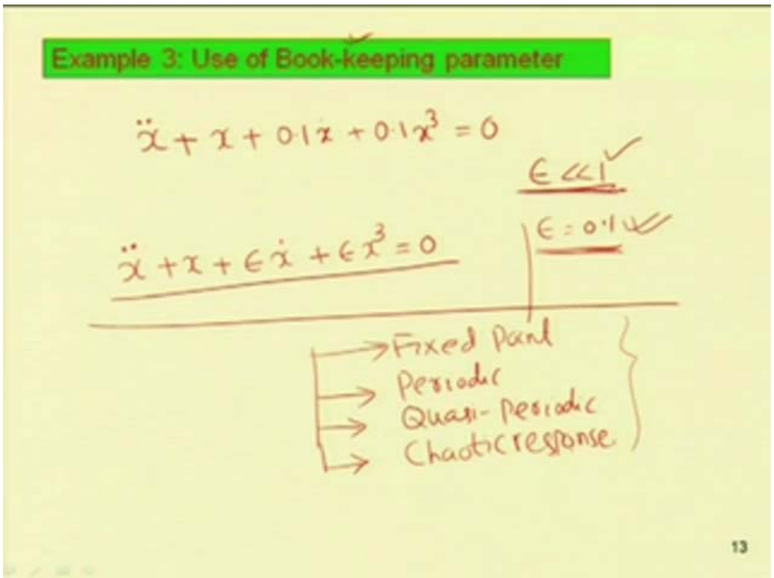

So, let us take the equation what we have obtained or let us take another equation. So, this is x double dot plus x plus let it is 0.1 x dot plus 0.1 x cube equal to 0. Now, by using a book keeping parameter epsilon, which is very-very less than 1, we can order this equation and make the coefficient of all the terms of the same order. So, if I will take a term let epsilon equal to 0.1 so, if epsilon is taken to be 0.1 in this equation. So, this equation will reduce to x double dot plus x or this term I can write this equal to epsilon x dot plus. So, for this also I can write this epsilon x cube equal to 0.

So, by using this book keeping parameter epsilon the coefficient of x cube is 1 now so, here also it is 1 which is equal to the coefficient of x. So, by using this book keeping parameter we can make the coefficient of the non-linear terms of the same order as that of the stiffness term.

So, this is required when we want to find the solution of the non-linear system by using that of the linear system. So, in this case one should note that this book keeping parameter epsilon is very-very less than 1. But while taking this booking keeping parameter one can make some conversion study that is by varying this epsilon and checking the response one can study for what value of epsilon there is no appreciable change in the response of the system. So, one should not take this parameters arbitrarily otherwise, the resulting response will be different than the required response. So, in all this type of non-linear systems we can obtain different type of response of the system. So, one type of response is the fixed point response, one is the fixed point response, the second one is the periodic response, third one is the quasi periodic response and the fourth one is the chaotic response. So, today we will use some programs to find these different types of response.

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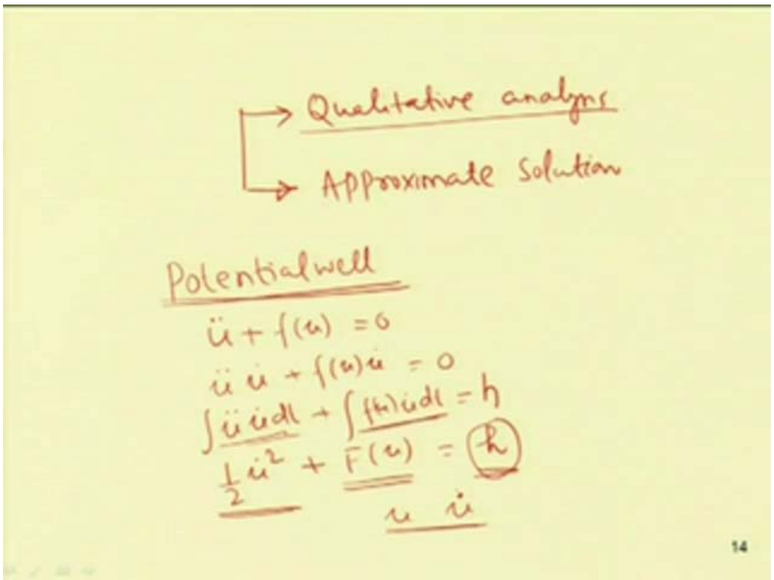

So, in case of all the systems, one can do a qualitative analysis qualitative analysis or one can go for the approximate solution to find the response of the system. So, last class briefly we have studied about this qualitative analysis by using the potential well method. So, we have determined the potential well so, we have determined the potential well or we have developed the potential function and from this potential function one can study the phase portrait or the flow of the system.

So, let us take the conservative system. So, in case of conservative system our equation in this form; u double dot plus f u equal to 0. So, in this case by multiplying u dot one can write this equation in this form u double dot into u double dot into u dot plus f u into u dot equal to 0.

And now by integrating, one can find u double dot u dot D t plus f u u dot d t so, this will be equal to a constant h and this is nothing but the kinetic energy that is equal to half u dot square so, plus this is f u. So, this is the potential function and this is the total energy of the system. So, this is kinetic energy of the system, this is potential energy of the system and this is total energy of the system. So, for a given total energy of the system one can find the relation between u and u dot. By plotting this u versus u dot, one can obtain the phase portrait of the system.

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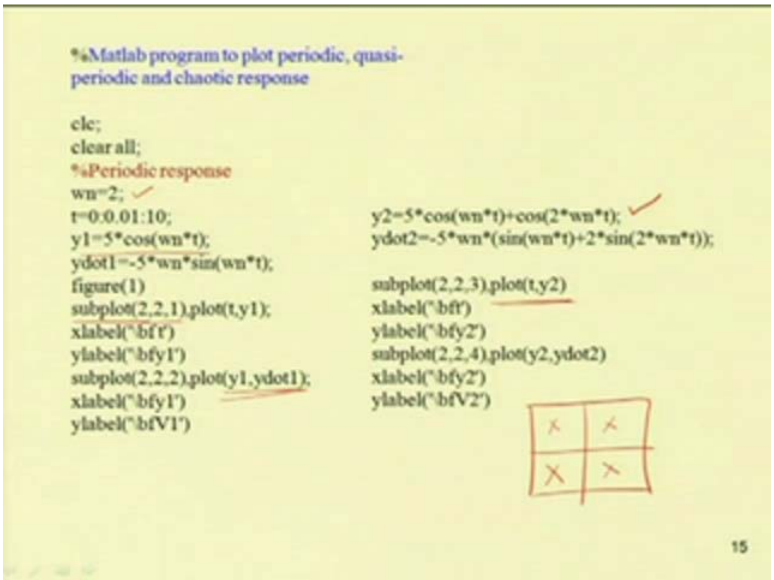

So, here a matlab program is given for finding the periodic, quasi periodic and chaotic response. So, for example, let us take y 1 equal to pi cos omega n t so, here omega n is taken to be 2 so, its velocity will be by differentiating that thing one can find the velocity that is equal to minus 5 omega n into sin omega n t.

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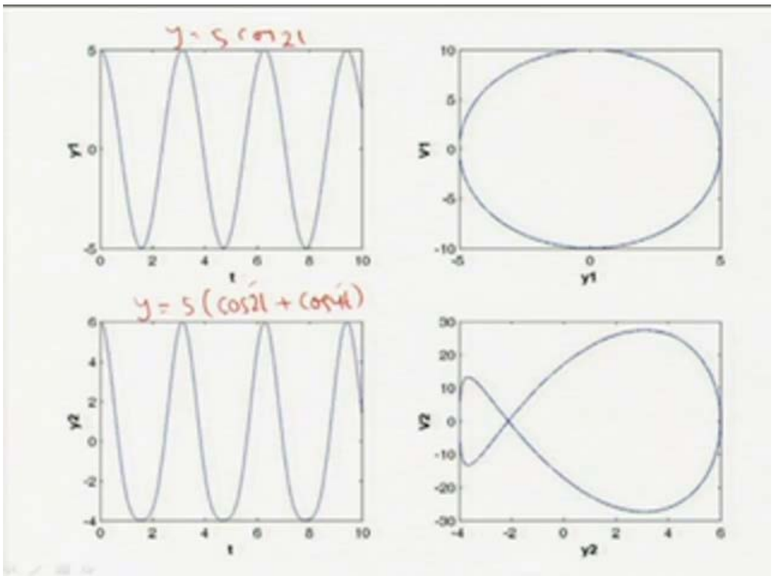

So, by plotting one can see the plot and later also we can show this plot by running this program. So, this is for y equal to pi cos 2 t. So, by taking y equal to pi cos 2 t one can find the response and the corresponding phase portrait one can obtain as a close curve.

So, now, by taking 2 periodic that is, if y equal to 5 cos 2 t plus cos 4 t so, if one take y equal to 5 plus 2 t and cos 4 t then, this y dot will be equal to 5 into minus this 2 sin 2 t and minus 4 sin 4 t. So, one can have a period or one can find the period of this time period of this and it shows the response to be periodic and incase of a phase portrait due to the presence of this 2 frequencies one can find 2 loops. So, one loop is corresponding to this frequency 2 and other loop is corresponding to the frequency 4. Now, by taking different amplitude so, in this case we have taken the amplitude to be same for both this term.

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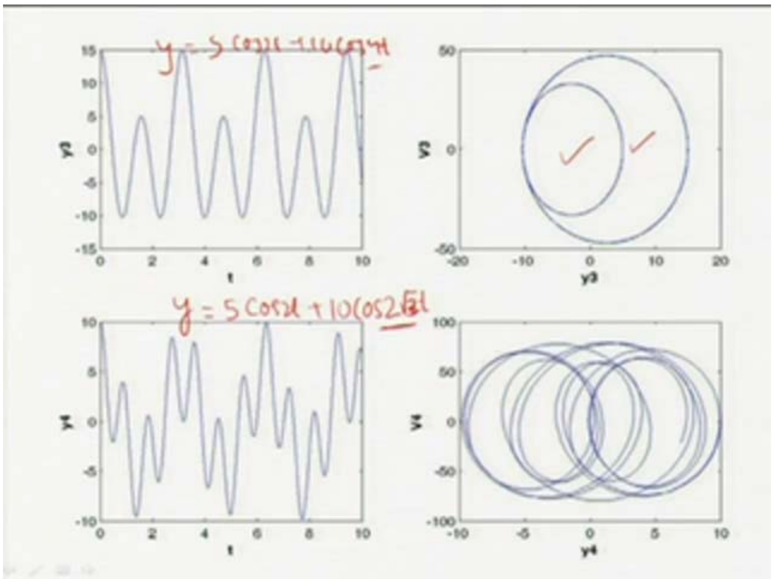

So, if one take different amplitude. So, let the rest. So, this is plotted for y equal to 5 cos 2 t plus 10 cos 4 t. So, if one plot this for 5 cos 2 t plus 10 cos 4 t so, in that case the amplitude is different and one can clearly observe the 2 frequency response at the system. So, in this case one can clearly observe also 2 loops in the phase portrait. So, in case of a periodic response one can observe that the ratio of this frequency that is this omega 2 by omega 1 this 4 by 2.

So, they are integer numbers. But in case of a quasi periodic response, the ratios between these frequencies are irrational numbers. So, one can take y equal to so, in this case y equal to 5 cos 2 t plus 10 cos 2 root 2 root 3. So, in this case it is taken to be root 3 t. So, this 2 root 3 by 2, this root 3 clearly root 3 is an irrational number. So, by using irrational number you can see the response is not periodic, it looks a periodic approximately periodic it looks approximately periodic but actually this is not periodic response.

So, if one plots the phase portrait, one can observe that phase portrait is not similar to that in case of a periodic response. So, in this case for 2 periodic responses you can have 2 loops; in case of a 4 periodic response one can have 4 loops. But in case of a quasi periodic response when the ratio are irrational number that is this is 2 root 3 t by 2 this is root 3 or if one take this is 4 root 3 by 2 then, it will be 2 root 3. So, if these ratios are irrational number then, the response will be a periodic or quasi periodic and the corresponding phase portrait will be a torus.

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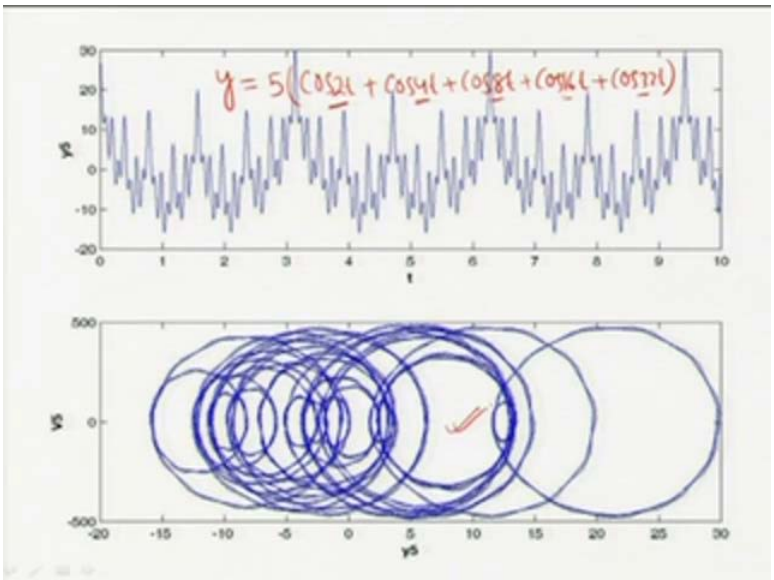

Now, one can consider a period doubling root to chaos or one can take the response y equal to let me take this way, 5 into cos 2 t plus cos 4 t plus cos 8 t plus cos 16 t and plus cos 32 t. So, you just see I have taken the period first period is so, this frequency is 2 then, it is 4 next it is 8 then 16 and then 32. I have doubled the period in this case. So, by doing or by taking a response in which I have taken the frequencies in this way then, the corresponding response you can see this is neither periodic nor quasi periodic nor it is fixed point response. The resulting response is chaotic response. So, the phase portrait for the corresponding chaotic response is shown here.

So, this is the phase portrait of the corresponding chaotic response. So, you can find the response by writing a small matlab program which is written here. So, you can use this subplot command to divide the screen into different region. So, for example, this subplot 2 2 1 means you have defined divided the window into 4 parts. So, this is number of rows, number columns so, you have divided into 4 parts so, 2 rows 2 rows and 2 columns and then you are plotting each plot at a particular position.

So, for example, you have taken this 5 cos omega n t. So, time response of this 5 cos n omega t at this position. So, this 2 2 1 then, you have plotted the second one at this position that is 2 2 2 so, this is the phase portrait. Similarly, you have plotted the time response for the 2 periodic response at 2 2 3 so, this is 2 2 3 position and finally, the phase portrait corresponding to this 2 periodic response are this. So, by using the subplot command you can plot the, plot the different subplots at different position in a single figure.

So, in case of figure one I have plotted force of figures which are shown here. So, these are the 4 subplots plotted here. Similarly, in this case you have 4 subplots, but, the third one I have taken only 2 plots so, there I have used this command. So, you can see this command for subplot here.

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So, in the third case that is in figure 3 for the chaotic response we can find, for the chaotic response let us see it is not there. So, for the chaotic response you can divide the subplot into 2 parts.

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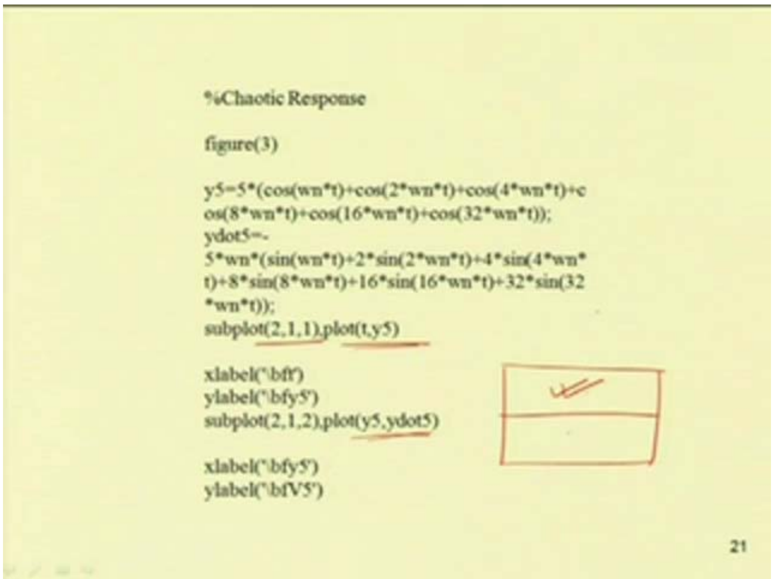

So, one will be in the subplot you can write it 2 2 1 so, the whole screen is divided into 2 rows and 1 column. So, in the first row that is 2 2 1, a chaotic response is plotted here and the time response is plotted here and the phase portrait is plotted here. So, phase portrait can be plotted by using the displacements and the velocity and the time response is displacement versus the time. So, in this way by using different response you can plot periodic, 2 periodic, 4 periodic, 3 periodic or a number of periodic response then, quasi periodic response and chaotic response.

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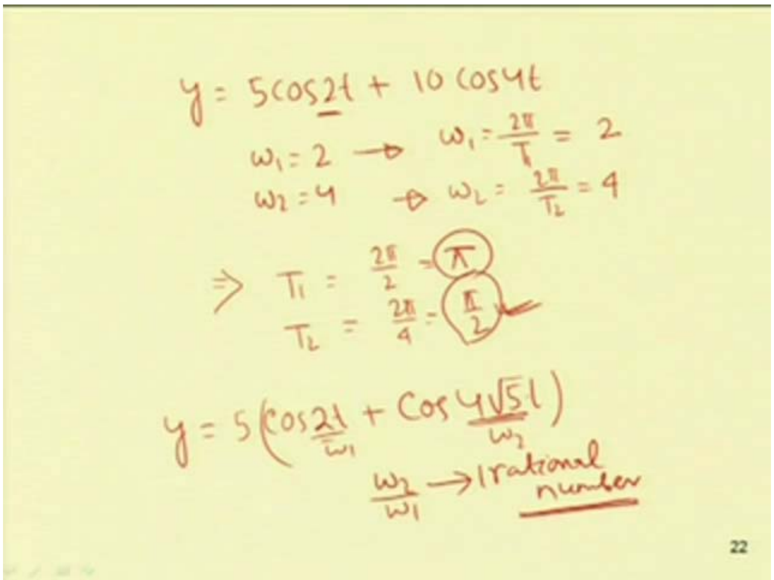

So, in case of periodic you may have, let this equation is y equal to 5 cos 2 t plus 10 cos 4 t. So, in this case you have 1 period. So, you know, your omega 1 equal to 2 and omega 2 equal to 4 so, omega 1 equal to omega 1 equal to 2 pi by T 1 so, this becomes 2 and omega 2 that is equal to 4 so, omega 2 equal to 2 pi by T 2 so, this is equal to 4. So, this implies this T 1 equal to 2 pi by 2. So, this is equal to pi and T 2 equal to 2 pi by 4 so, this is equal to pi by 2. So, you can observe in case of a 2 periodic response. So, the period is pi by 2 here and this is pi. So, the lowest period is pi by 2 as I have taken the amplitude to be 5 so, in this case you can observe just see, this is pi by 2 so, you can have the period the repetition at pi by 2 intervals.

But, in this case we can see the response to be 2 t so, the time period equal to 2 pi by 2 that is pi and incase of a quasi periodic response it is written in this for 5 cos let it is 2 t plus cos. I can take a irrational number root 2 root 5 root 3 where, the different where, the division between this and this that is omega 2 this is omega 1 so, omega 2 by omega 1 should be irrational number. So, if it is irrational number then, will get quasi periodic or a periodic response of the system.

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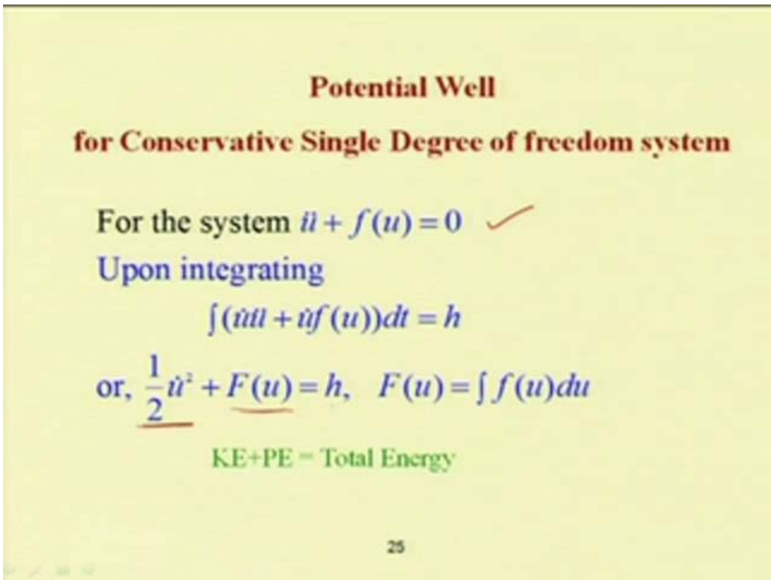

So, last class we have seen about this potential when well for the conservative system so, in this case, the equation is written in this form. And we have seen that this kinetic energy plus potential energy plus this or will be equal to the total energy of the system. So, let us take the potential well for the spring mass system we have, we have plotted this thing. So, let us take, so here more different examples have been shown.

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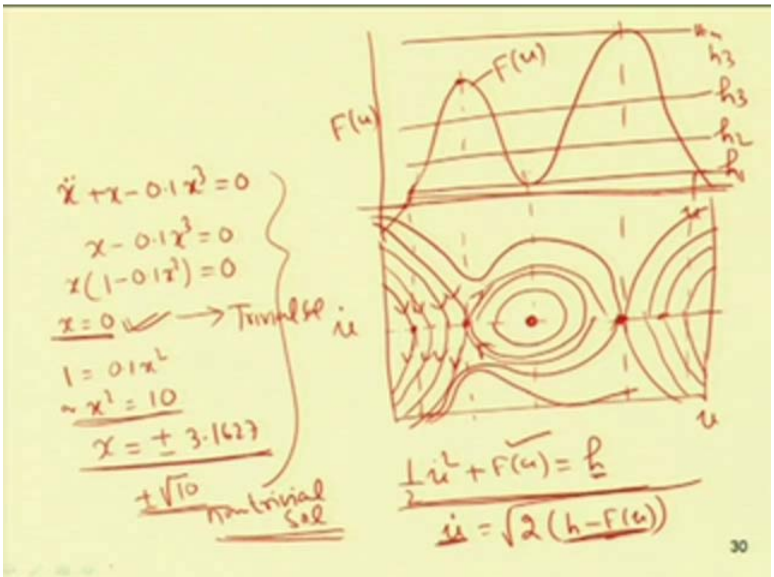

So, let us draw one more potential well and see how it will behave in this h portrait. So, this is f u and this is u so, let us extend this thing and plot the corresponding phase portrait. For example, in this case, in this case x double dot plus x minus 0.1 x cube so, we have the equilibrium position so, x double dot plus x minus 0.1 x cube equal to 0. So, the equilibrium position will be found by putting this x double dot equal to 0. So, x minus 0.1 x cube equal to 0 or I can take this x common. So, this is 1 minus 0.1 x square, this is equal to 0 or x equal to 0 this is one solution and the other solutions are 1 equal to 0.1 x square or x square equal to 10.

So, this gives x equal to plus minus 3.1627 or x equal to root plus minus or this is plus minus root 10. So, for x equal to plus minus root 10, we can have the equilibrium position. So, these are the 3 equilibrium position we can obtain from this equation. So, x equal to 0 is known as the trivial solution and x equal to plus minus this so, these are the non trivial solution of the system. So, in the last class we have seen corresponding to these positions we can have different types of equilibrium position, equilibrium solution. So, for example, let us take the response or let us take the total potential a total energy let us take the total energy of the system. So, this is h 1 and this is h 2 then, we can take h 3 this is h 4 and you can take higher values also.

So, in this case I can divide this zone. So, when this kinetic energy that is half u dot square plus this f u so, our equation is this. So, this is the potential function, this is the kinetic energy and this is the total energy. So, the system will or I can write this u dot equal to 2 into h minus f u root over. So, in this case we will have a positive solution or this u dot will be feasible if this h is greater than f u. That is the total energy of the system, if it is greater than the potential function, then only the system, we have a oscillation; otherwise, the system will not oscillate or system will not vibrate or the velocity will be 0, the system will have a velocity 0 or these term will be imaginary 2 into root over so, if this part is negative then, the square root of this will be imaginary quantity so, the velocity will be an imaginary number. So, we can have the solution or u dot only when this term is positive.

So, for example, for h 1, when you are taking h 1 term, let me extend it this part. So, in this case is h 1 is greater than this, this is this function, this is the potential function. So, h 1 is greater than h 1 is greater than this f u. So, for this region you can have 2 values of for this region you can have two values of u dot and for this position corresponding to this position when h 1 equal to f u this term becomes 0. So, corresponding to this, this term becomes 0. So, u dot will be 0. So, at this position u dot equal to 0 but, before that one can have 2 values of u dot one can have two values of u dot. So, in this way one can plot this. Similarly, here for this part also this it is greater than this; h 1 is greater than this f u so, one can have a response. Similarly, by taking this h 2 up to this so, the system will have a response like this here also up to this point.

So, at this position the velocity is 0 u dot equal to 0 but, any other position corresponding to any other point let this point, this point or this point as f u is less than h h 2 so, it will have 2 value of u dot. So, we are plotting u dot versus u here. And now for h h 3 similarly, for h 3 so, you just see up to this one can plot this. Corresponding to this point so, for h 1 corresponding to this point, this is 0. So, u dot equal to 0, but, for h 2, when you are taking this line h 2, this curve is within this h 2 is greater than this f u. So, as h 2 is greater than f u, one can have a periodic response here. But, at this point, this point or this point, one can seek the response to be or one can observe a separate Riggs in all these cases.

Corresponding to this point the flows gets separated here so, one can observe the flow gets separated. So, you can have a pair of response so 2 coming out and 2 going inside this thing. And similarly, one can have a separatrices corresponding to this position also. So, the corresponding response has been plotted. So, in this way you can observer that there are 2 separatrices corresponding to the maximum potential energy and one has a center corresponding to the equilibrium position that is the minimum potential energy. So, this potential wells have been plotted here.

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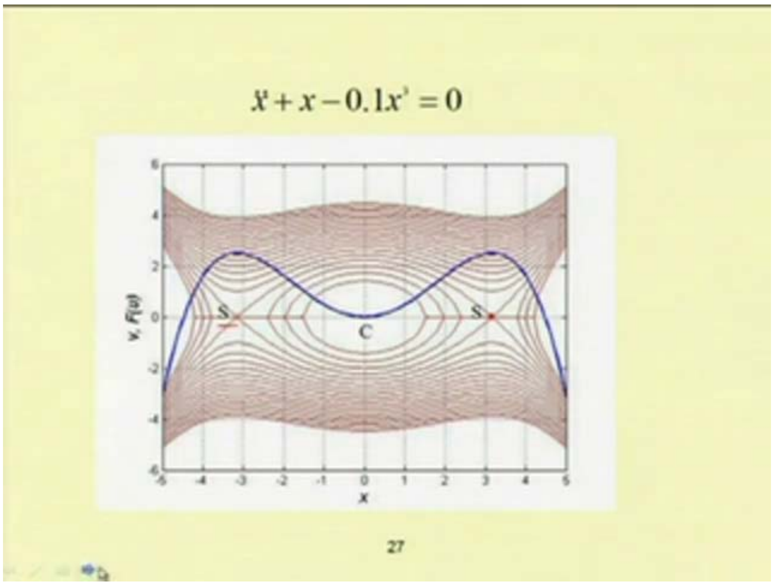

So, we have, these are the potential well corresponding to different values of h. So, this x refers to this saddle point or this separatrices. So, these are the four separatrices and x is the saddle point. So, near the saddle point the response is unstable and near the center, this point the response of the system is stable. So, the system has a stable response near this point but, the system has an unstable response near the saddle points. One can plot so here one can observe that this is plotted for x minus 0.1 x cube equal to 0. So, if one takes this negative stiffness.

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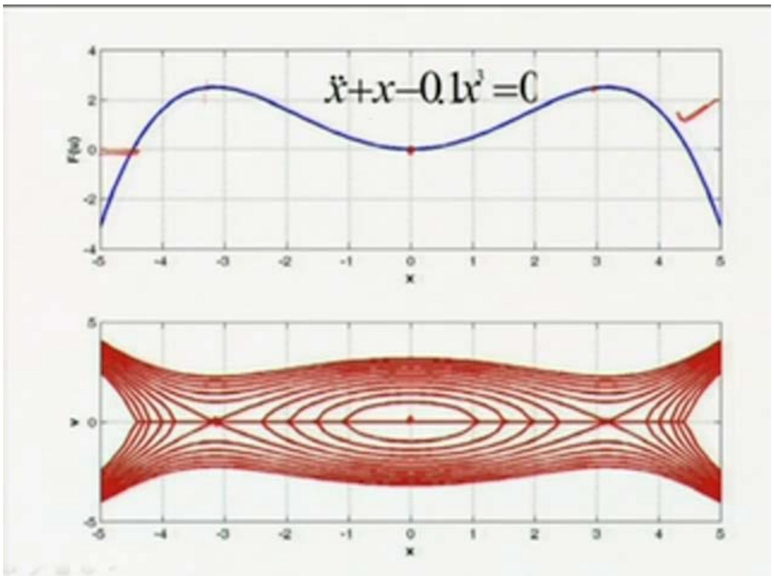

So, in case one takes. So, the same thing is plotted here in a. So, this is the potential function and the corresponding phase portrait is shown here. Corresponding to different values of x this curves have been plotted. So, for example, in this case, this to this point h is greater than this f so, it will have 2 values of 2 values of solution, but corresponding to this point that is the peak point it will have a separatrices. Similarly, corresponding to this point, this is a center and corresponding to the maximum value here, you can have a clear separatrices here.

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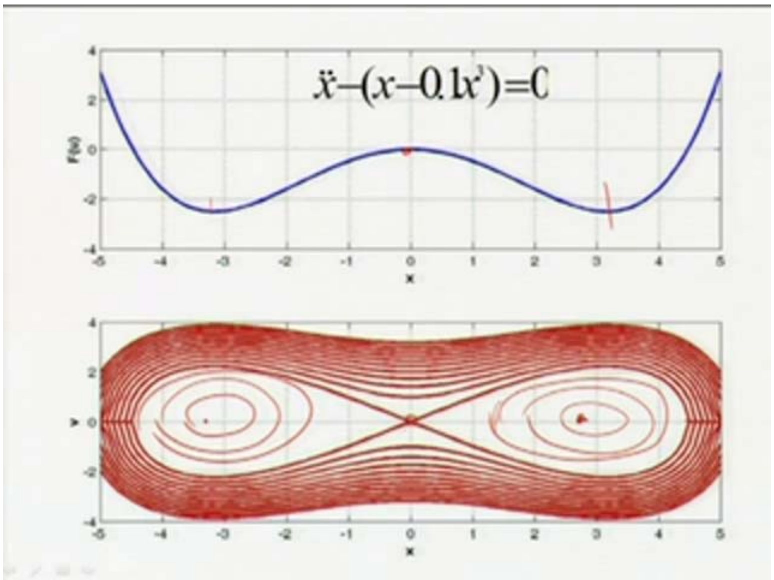

If one can take their negative stiffness so, in that case one can observer that the previous center point becomes unstable that becomes a separatrices and these two points one should have the center. So, one will have center in between here also corresponding to this one will have periodic responses in between this separatrices, between this point and this point. So, you have periodic response corresponding to the center and here you will have an unstable or a response which will grow with time.

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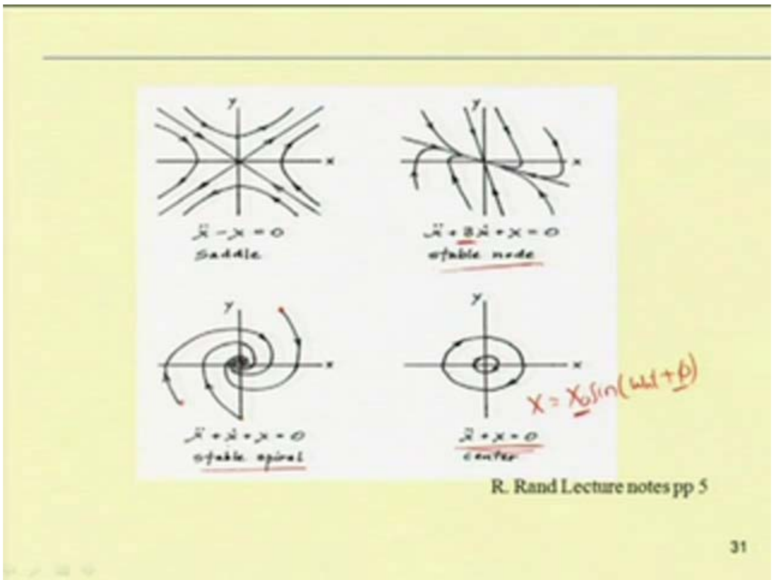

So, let us see some other examples also. So, in this case we have taken x double dot minus x equal to 0. So, already I told the response to be so in this case, the response will exponentially grow and this corresponds to a saddle point. And in this case, x double dot plus x dot plus x equal to 0 so, in this case the response will converts to this point so, whether you are starting from this initial point or this initial point or this initial point so, it will spiral to come to this equilibrium position.

So, here the equilibrium position is x equal to 0. So, by substituting x dot equal to 0 and x double dot equal to 0 the response is x equal to 0 so, this is a stable spiral in this case. So, this is similar to that of an un-damped, single degree of freedom system. So, solution equal to x capital X equal to x 0 sin omega n t plus pi. So, taking this initial condition one can find x 0 1 pi and this shows the center and one can take or a damped system so, here the coefficient is 1 and here the coefficient is increased. So, one has the damping is more if, the damping is more so, in that case, this become a stable node. So, this is stable spiral, the coefficient you just note the coefficient here and this is a stable node.

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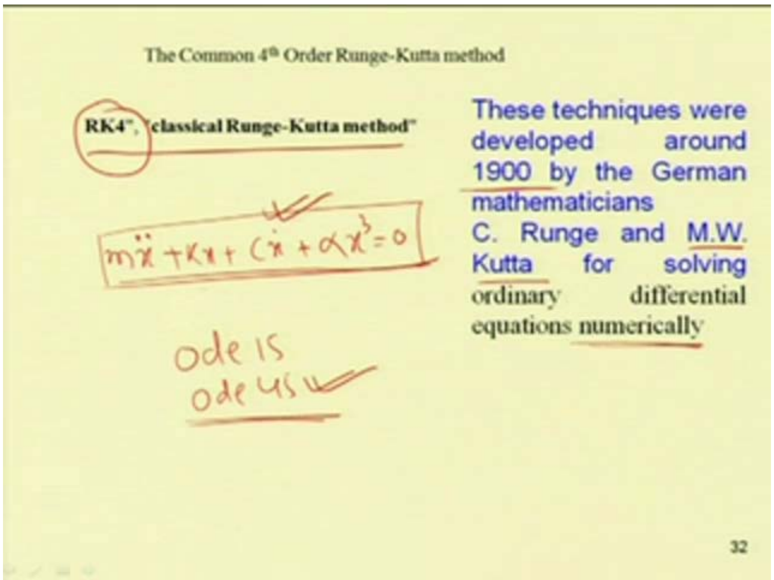

So, one can use to find the response of the system for this case. Let you have this equation m x double dot plus k x plus c x dot plus alpha x cube equal to 0. So, to find the response of the system differential equation one may use this classical Runge Kutta method or RK 4 method, commonly known as RK 4 method. So, this method is developed around 1900 by German mathematician C Runge and M W Kutta for solving the differential equation numerically. So, one can use this RK 4 method. In matlab so, you have the sub routine ode 15, ode 45. So, one can commonly use this ode 45 to solve the equation of this form. So, by using this RK 4 method one can find the response of the system when it is given in this temporal form. But in that case one will get the response correspond to corresponding to a particular initial condition.

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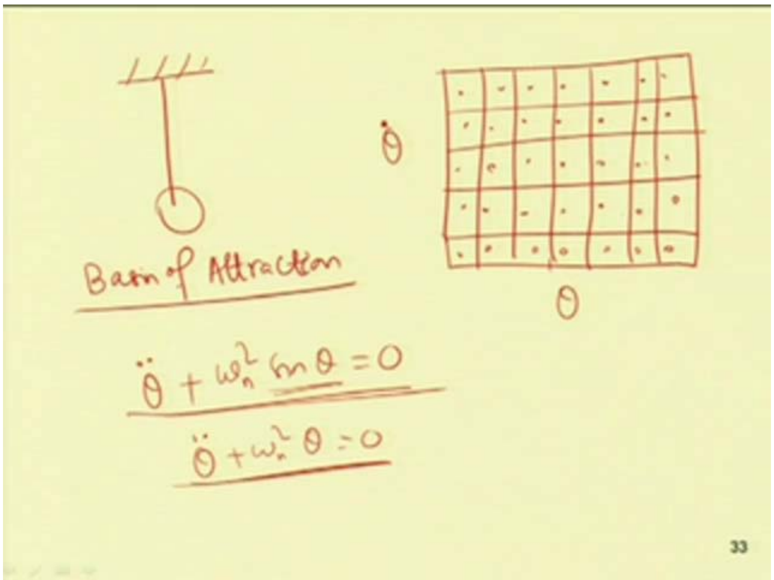

So, by changing the initial condition one may have different response of the system. For example, in the case what we have studied just now. So, one have three equilibrium solutions. So, to study different equilibrium solution, one can plot the basin of attraction. So, one can plot the basin of attraction. So, this basin of attraction can be plotted by different method. So, here a simpler method is depicted so, one can take is theta versus theta dot or for a simple pendulum. So, for a simple pendulum if you want to, let for a simple pendulum you want to plot the phase portrait for this simple pendulum so, it is known that the equation will be in this form. So, theta double dot plus omega n square sin theta, this is equal to 0 or small value of theta. So, it will be theta double dot plus omega n square theta equal to 0, where omega n square equal to root omega n square equal to g by l. So, for small value of theta this becomes sin theta, becomes theta and equation reduces to this form theta double dot plus omega n square theta equal to 0.

But for large value of theta one can expand the sin theta term to have a non-linear equation. So, in that case to plot the phase portrait, you can use this Runge Kutta method and in that case one requires different initial conditions. So, to find the initial condition, one can make a grid like this and take the initial points initial condition take different initial conditions and taking these initial conditions one can plot the response. So, as you have seen there are three equilibrium positions from these initial conditions so, the trajectory one can find the trajectory which is which is reading to these 3 different equilibrium position and can plot the basin of attraction.

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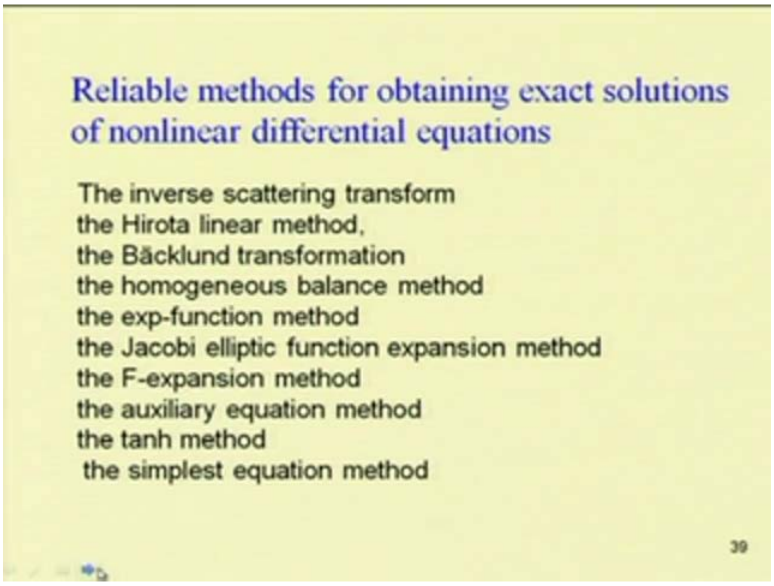

So, there are different reliable methods for obtaining the exact solution of non-linear differential equation. So, some of these methods are the inverse scattering transform the Hirota linear method, the Backlund transformation, the homogenous balance method, the exponential function method, Jacobi elliptic function expansion method, the f type F expansion method, the auxiliary equation method, the tan h method and the simplest equation method. So, their one can use this type of or these reliable methods to solve or find the exact solution of the differential equation. But in this course, we are going to study about the approximate methods and some numerical methods to find the solution of the differential equation non-linear differential equation.

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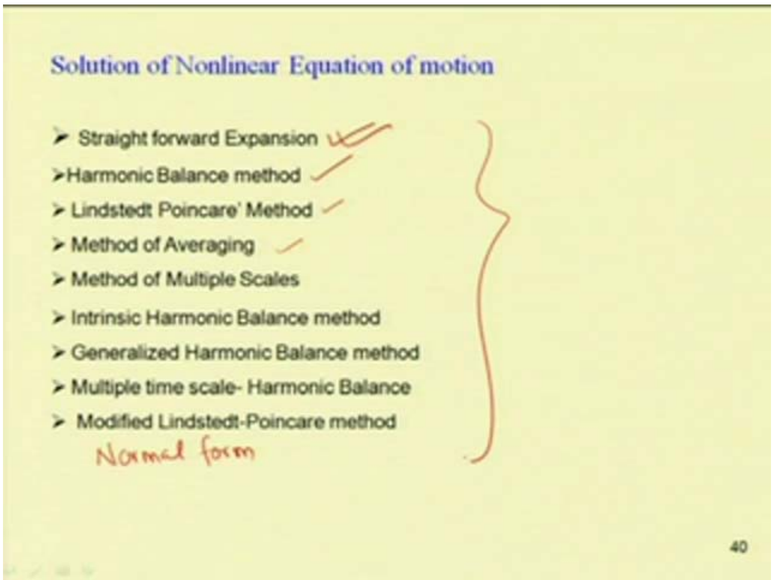

So, to find the approximate solution one can first use the straight forward expansion method. So, in straight forward expansion by taking some approximate function one can find the solution of the system. Later, we can see that this straight forward expansion give rise to Eronious result. So, one can use the other types of method. So, one can go for this harmonic balance method Lindstedt-Poincare method, modified averaging method of multiple skills, intrinsic harmonic balance method, generalized harmonic balance method, multiple time scale harmonic balance method, modified Lindstedt Poincare method. Also one can use this normal form method also normal form method or center manifold method can be used to find the solution of the non-linear equation of a particular system.

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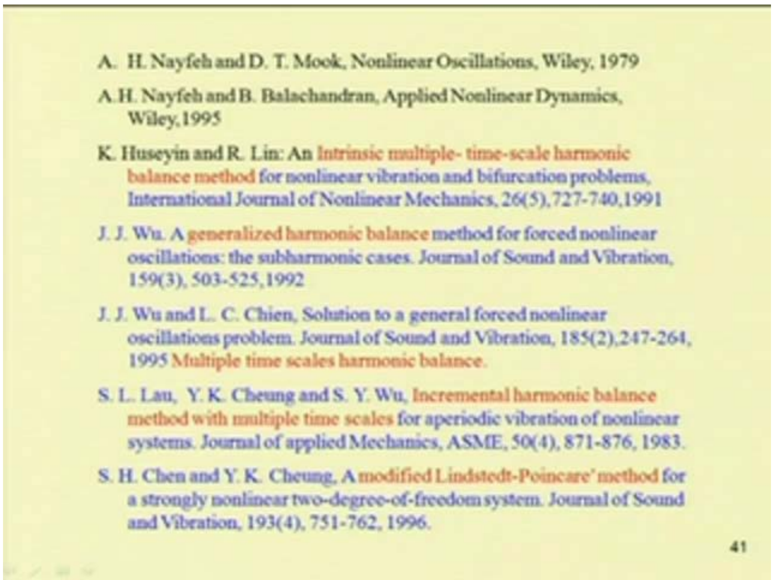

So, these are the books one can refer. So, Nayfeh and Mook non-linear oscillation, Nayfeh and Balachandran applied non-linear dynamics then, Huseyin and Lin they have developed this intrinsic multiple time scale harmonic balance method. Then Wu developed this generalized harmonic balance method, Wu and Chien developed multiple time scale harmonic balance, then Lau Cheung and Wu intrinsic harmonic balance method with multiple time scale and Chein and Chung modified Lindstedt Poincare method. Also, one can find a book by Nayfeh and this non-linear normal form method.

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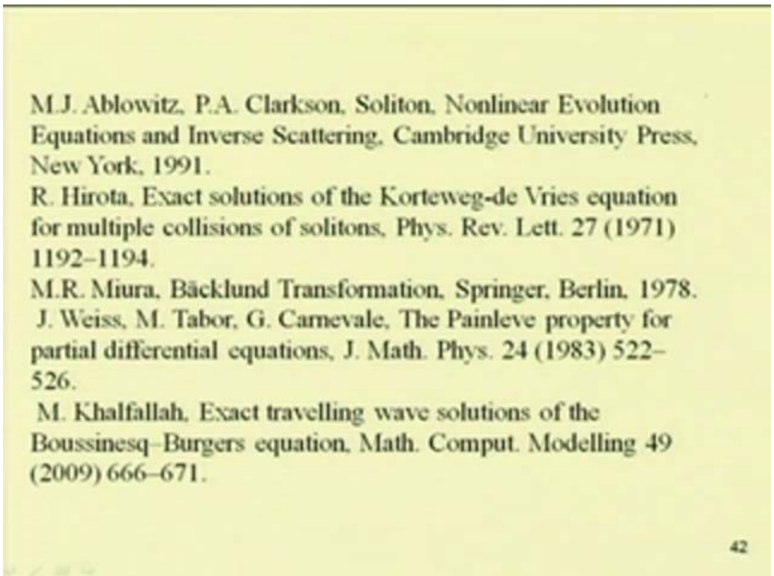

So, these are some of the reference for different type of exact solution method that is evolution method, non-linear evolution equation and inverse scattering then, this Hirota method so, what we have seen and this Backlund transformation method and other types of exact solution methods. So, these are some other references one can study to find the exact solution of the non-linear equation.

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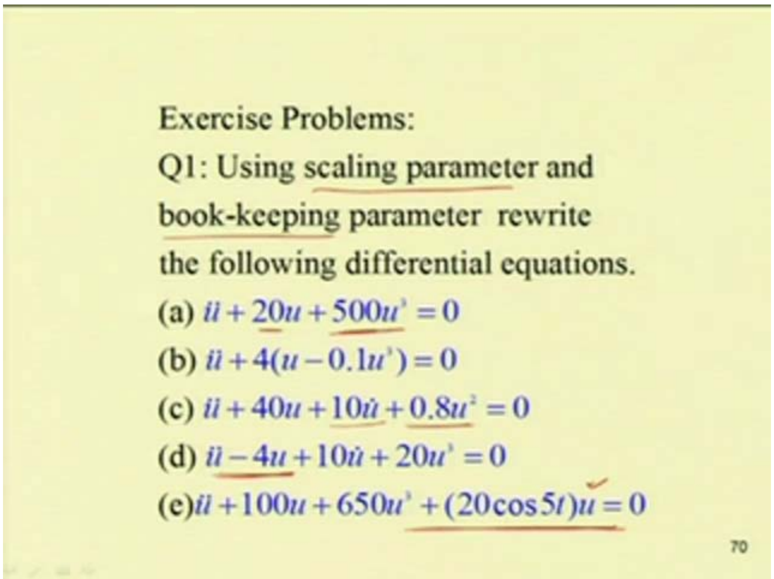

Here some exercise problems are given using the scaling parameter and book keeping parameter you have to rewrite these equations. So, that it can appear as a so that the coefficient of the non-linear term will be a small term that should be comparable with the coefficient of u term.

So, write the scale use the scaling parameter and book keeping parameter to rewrite these equations. So, here five equations have been given. So, this is that of a free vibration duffing type. Similarly, this is also a free vibration of duffing type. So, here you have a hard spring, here you have soft spring. So, in this case damping is added to the system and you have a quadratic non-linear term. And in this case, the stiffness parameter is negative and this case is for a parametrically excited system as this periodic term is function of u. So, in all these cases using scaling parameter and book keeping parameter, rewrite these equations.

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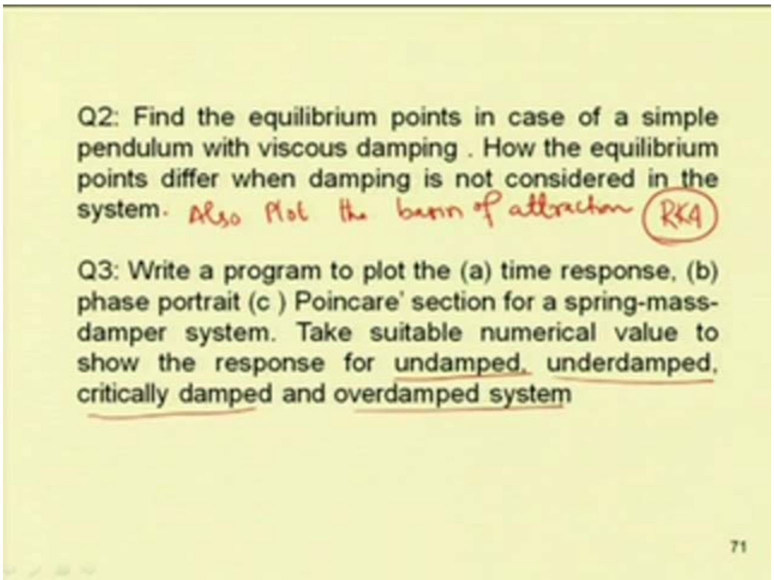

And also you can take these two different problems. So, find the equilibrium problems in case of a simple pendulum with viscous damping. So, how the equilibrium points differ, when damping is not considered in the system? Also you may plot the basin of attraction, basin of attraction by taking different initial point using, so you may use RK 4 method to find the solution. Similarly, you write a program to plot the time response, phase portrait, Poincare section for a spring mass damper system. Take suitable numerical value to show the response for un-damped, then under-damped critically damped and over damped system. So, initially you plot for a linear system, then take soft spring and find the response and then take a hard spring and find the response of the system. So, next class, we are going to study or we want to develop the differential equation for different type of structural biological or other type of non-linear systems.

Thank you.