Non-Linear Vibration
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Module - 6
Applications
Lecture - 9
Parametrically Excited System
Elastic and Magneto-Elastic Beam Subjected to Periodic Base Excitation

So, welcome to today class of non linear vibration. So, today class we are going to study more examples on parametrically excited system. So, we will see the example of elastic and magneto elastic beams subjected to periodic base excitation. So, here we will study, how this periodic elastic and magneto elastic beam subjected to periodic excitation, behave as a parametrically excited system and how the response of the system changes with system parameters. Last class, we have studied about this sandwich beams. So, in which, we have applied axial load and we have studied the parametric instability regions of those systems.

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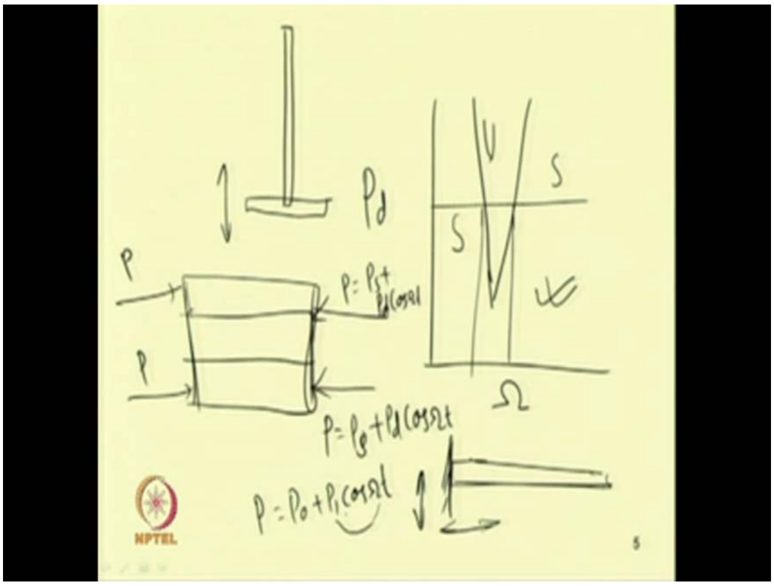

So, in case of parametric instability region or the region inside the… So, we have obtained the parametric instability region. So, in which we have plotted this P d versus omega. So, we have taken this symmetric and unsymmetric, unsymmetric sandwich beams, these are the, this is a sandwich beam so, subjected to periodic axial load. So, here we have applied the load to be in this form that is P equal to P 0 or P s plus P d cos omega t. Similarly, this is P equal to P s plus P d cos omega t so, here also we have applied P. So, when we have applied this force P s plus P d cos omega t. So, we have seen for certain values of this P d and omega for example, for this value of P d, the system become unstable. At a frequency, this correspond to this again, the system becomes stable at a frequency greater than this. So, this region is stable; this region is stable and inside this curve it is unstable. So, these are the parametric instability regions, we have studied in case of sandwich beams, in the last class. So, today class we are going to take some more examples in which, we will take this elastic beam.

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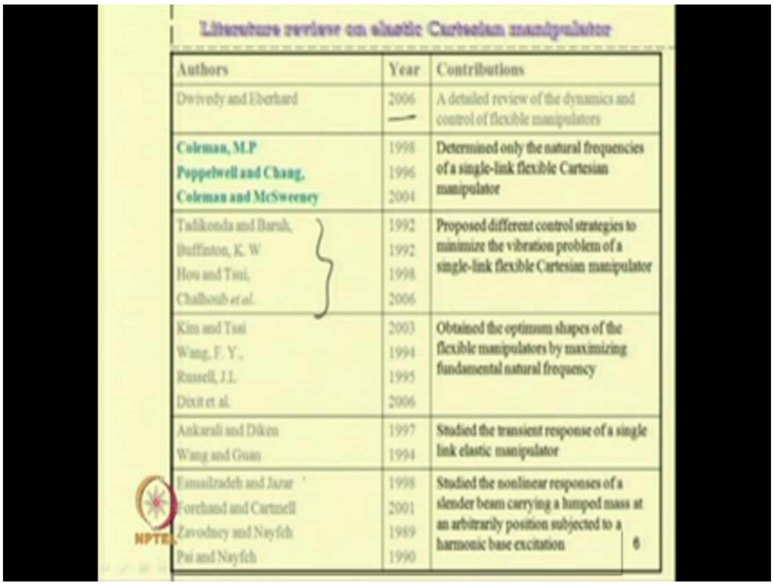

So, there are many works available on this elastic beams, which are or which can be used as Cartesian manipulator basic excited. So, one can take this basic excited beam. So, one can see this literature review on a detailed review on dynamics and control of Flexible manipulator by the Dwivedy and Eberhard in 2006. This is published in the general of mechanism and machine theory similarly, there are many other works related to cantilever beam, related to beams where it is subjected to base excitation. So, some works by Colemn M P then Poppelwell and Chang, Colemen Mcsweency and so they have determined for example, Colmen M P determined only the natural frequency of a single link Flexible Cartesian manipulator. Similarly, one can find this papers by Tadikoanda and Baruah Buffinton Hou Chalhoub.

So, where one can see this proposed different control strategy to minimize the vibration problem of a single link Flexible manipulator. Similarly, many other literatures are given here. So, where this base excited cantilever beams have been taken so, in this base excited cantilever beam. So, for certain value of this frequency and amplitude, the systems will vibrate in a. So, the system vibration can be, so for example, by taking a system. So, if one take a base excited cantilever beam. So, either this way base may may be excited or one can take a system in which it will excited in a vertical plane or either it can excite in a vertical plane or it may move in this horizontal plan.

So, in all these cases for the periodic force P equal to P 0 plus P 1 cos omega t. So, for certain value of this P 1 and omega. So, no longer, it will be in the trivial state, it will vibrate in a transverse. So, for those cases, so one has to find the parametric instability region and also one has to study, what will happen to the system character.

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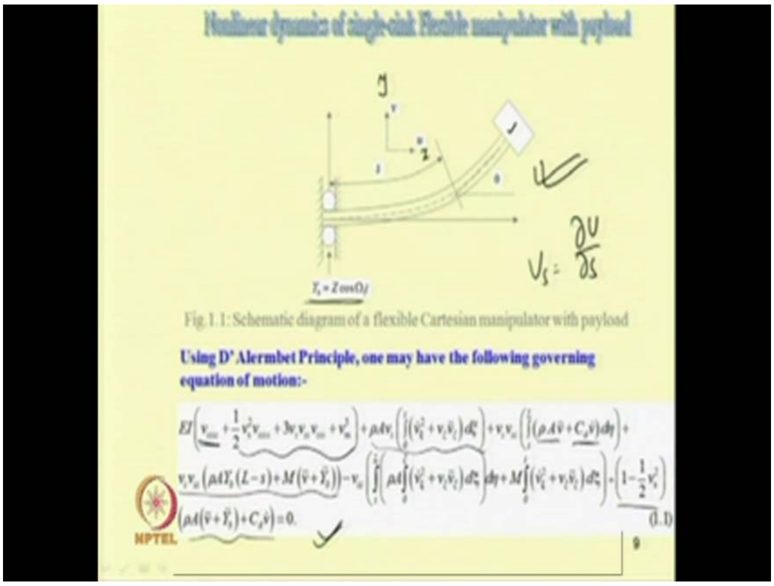

So, Let us see some of the example. So, here let us first take a cantilever beam. So, this is a cantilever beam with a tip mass and it is subjected this base excitation y b equal to z cos omega 1 t. So, here taking a section at a distance s. So, one can write the equation of motion. So, the equation of motion can be derived using this D’alembert principle or detailed derivation, one can see the work by Baruah and Dwivedy. So, which, I will show in the last page of this presentation. So, in the reference section, I will show details about this publications. So, this, this system, this base excited system, the equation motion can be written. So, by this equation that is e i del fourth b by del s fourth plus half v s square into v i v plus 3 v s into v s s into v s s s plus v s s q, where this v s is nothing, but this is the del v by del s, where v is the transverse deflection of the beam.

So, the transverse deflection of the beam, if it is v then v s is del v by del s and v s s equal to del square v by del s square. So, if one take this as a simple Euler Bernoulli beam then the equation will contain only the first term, that is e i del fourth e by del s fourth, but due to taking moderately large displacement. So, one can write the equation by adding these terms. So, this equation can be easily derived by using this D'alembert principle by considering different forces acting on the system and taking this moderately large deformation of the system. So, so this term will come from this stiffness of the system, then one can write this row a b s into integral 0 to s v zeta dot square plus v zeta into v zeta double dot into v zeta plus v s into v s s integral s to l rho a v double dot plus c d. So, this is damping due to damping and this part row a v double dot, this is due to inertia.

So, so this is the inertia force, this is the damping force similarly, these are the non-linear forces acting on the system and this rho a y b l minus s. So, this represent, so into v double dot. So, this represent the base excitation. So, y b double dot is the acceleration of the mass acceleration of the system. So, y b is the displacement of the base and as it is moving in this guide. So, one can have that inertia force similarly, these are the other terms present in the system. So, this terms are non-linear term. So, this Spatio temporal one can derive this Spatio temporal equation by writing different forces for example, 3 different forces, one can write. So, one is due to inertia force in x direction or direction u, it is shown here. So, direction x and 1 is in y direction.

So, let the displacement along x direction is u displacement along y direction is v then. So, one can write the inertia force in x direction inertia force in y direction in this damping forces and also the inertia force, due to the steep mass and taking a section at a distance s one can find the bending movement and differentiating that term, one can get differentiating that term twice. So, one can get this equation. So, after getting this equation, so this it may be noted that this equation is in the form spatio temporal equation; that means, it is in both space and time because v is a function of space and time.

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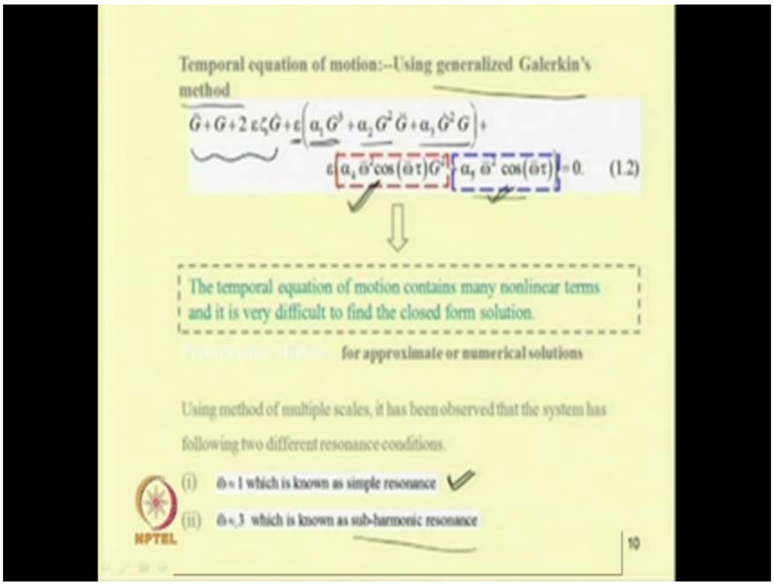

Now, by applying this generalized Galerkin method by taking the mode shape of a cantilever beam with steep mass. So, one can find the equation of motion or temporal equation of motion of the system. So, here this G is the time modulation taken. So, this equation reduce to this form that is G double dot plus G plus 2 epsilon zeta G dot plus epsilon. So, where epsilon is a bookkeeping parameter, which is very very less than 1 so, plus epsilon into alpha 1 G cube. So, this is the geometric non-linear term and these 2 are the inertia non-linear term. So, alpha 2 G square G double dot plus alpha 3 G dot square G plus. So, here it may be noted that. So, we have 2 more terms, which are, which represent the non-linear parametric term.

So, here the coefficient of this G square is alpha 4 omega square cos omega t, which is a time varying term. So, due the presence of this term, the system become a parametrically excited system and also due to the presence of this term that is alpha phi omega square cos omega t. So, the system is a combination of both force and parametrically excited system. So, the system, what we have considered now is reduced to that of a forced and parametrically excited system. So, this term, the last term with alpha phi. So, is due the forcing function, this is force this will represent the force vibration of the system and due to this it will behave as a parametrically excited system. So, this equation contain, so up to this these are the liner term then, we have this cubic non-linear terms along with a non-linear term in of quadratic type with parametric Excitation and we have a forced vibration term. So, as this temporal equation of motion contain many non-linear terms, it is difficult to find the close solution.

So, one can go for a numerical solution. So, here method of multiple scales has been used to find the solution of the system. So, while applying the method of multiple scale with different time scales. So, one can see that the system will have different resonance conditions. So, one resonance condition will be omega bar nearly equal to 1 and the other one is omega bar nearly, equal to 3. So, when omega bar is nearly equal to 1. So, this is known as simple renounce and omega bar equal to 3. So, this is the sub harmonic resonance. So, one can observe these 2 renounce in this case. So, in case of simple resonance so, one can get a set of reduced equation so, these are the set of reduced equation in a dot and a gamma dot. So, so in this case, one can observe that the system has only nontrivial response. So, the trivial state is not present in this case similarly, in case of sub harmonic renounce condition. So, when that means the omega bar equal to 3. So, omega bar equal to omega by that is the external frequency of excitation by the mode frequency.

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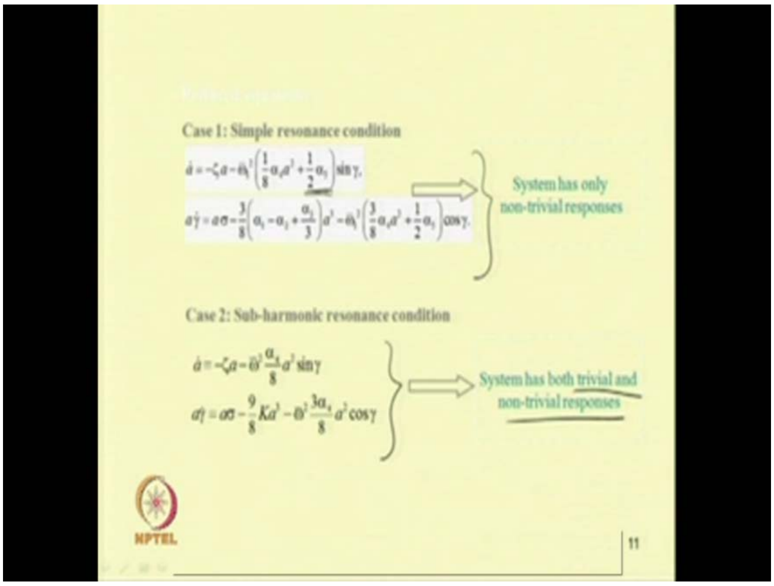

So, in that case, in that case, one will get this; one will get this reduced equation. Now, in this reduced equation by substituting this a dot and gamma dot equal to 0 for the steady state. So, we can have, so one can take this a common from this equation and also one can take a also common from this equation. So, one can see that a equal to 0 is also a solution of the system. So, a equal to 0 represent the trivial state of the state, but a not equal to 0, that response is the nontrivial response of the system. So, in case of sub harmonic resonance so, the system has both trivial and nontrivial response, but in case of simple resonance condition, the system has only nontrivial response as a equal to 0 is not a solution of the system. So, by substituting a dot equal to 0 and gamma dot equal to 0 so, one can see that a equal to 0 is not a solution because, one can see this zeta a minus this omega 1 square. So, this term this last term does not contain any a term. So, due to that the solution will not contain any trivial state.

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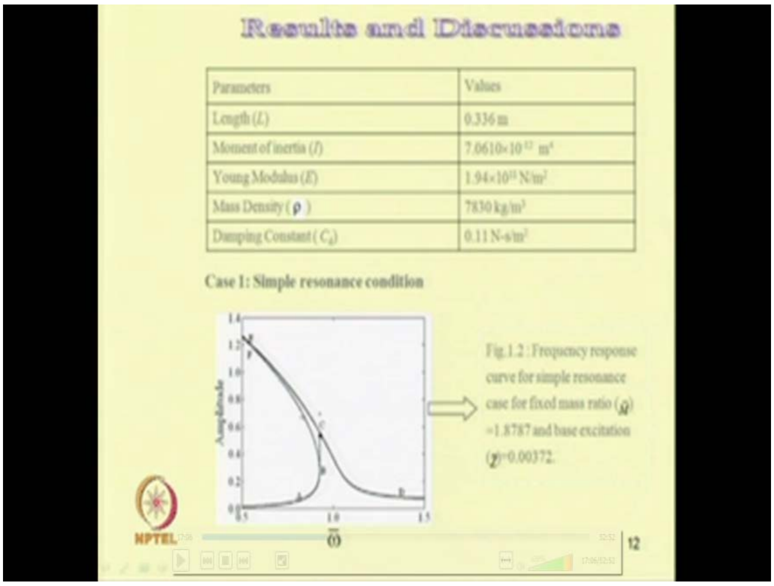

So, in both these cases so, one can let us take some numerical examples and see how the system will behave. So, here the length of the system has been taken 0.336 movement of inertia 7.0610 into 10 to the power minus 12 m's modules 1.94 into 10 to the power 11 Newton per meter square mass density rho equal to 7830 KG per meter cube and damping constant. So, 0.11 Newton per second, Newton second for meter square. So, in this case, so for simple resonance case it can be seen that the system has only nontrivial response.

So, up to this the system has a stable branch and then the system has a unstable branch, unstable is shown by this dotted line and then we have a stable branch. So, that means, when we increase the frequency sweeping of the frequency. So, up to b as the system is stable. So, it will follow this path that is a b and at b with further increase in this omega bar, what the system will subjected to a jump up phenomena and then one can get the response of the system stable response of the system, but while sweeping down. So, one can follow this path that is d c e and it may jump down some e as, as this unstable and stable branch was there or it may go on increasing. So, with higher value of damping so, this response, so instead of ending at this point so, it may end at some other point before e. So, then one, one may observe this jump down phenomena also, while sweeping off the frequency, one can have a jump up phenomena and while sweeping down. So, one may have a jump down phenomena. So, this correspond to different forcing frequency.

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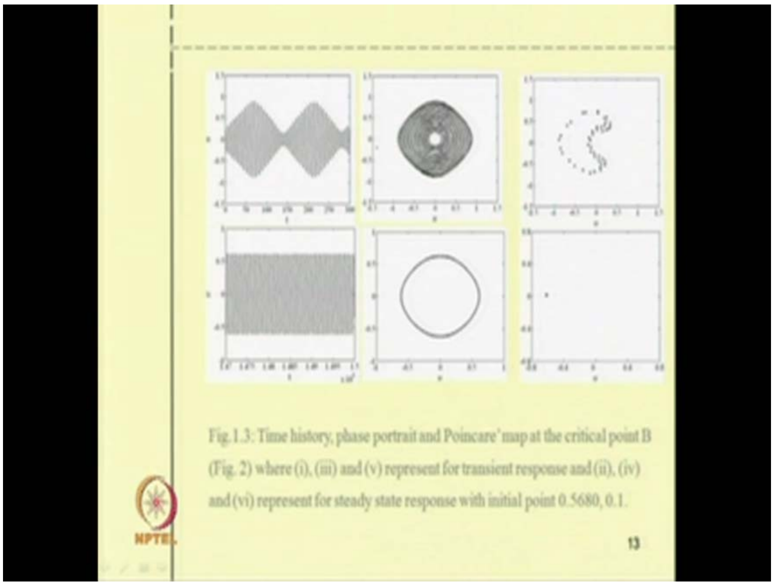

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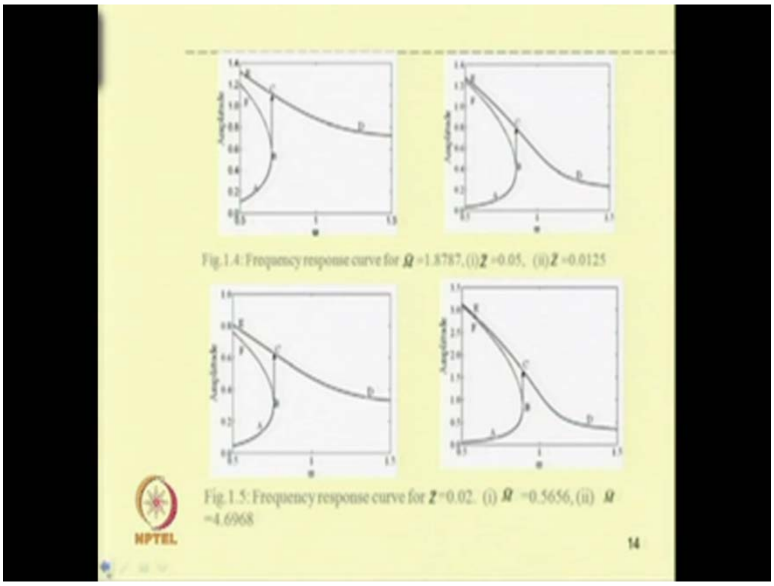

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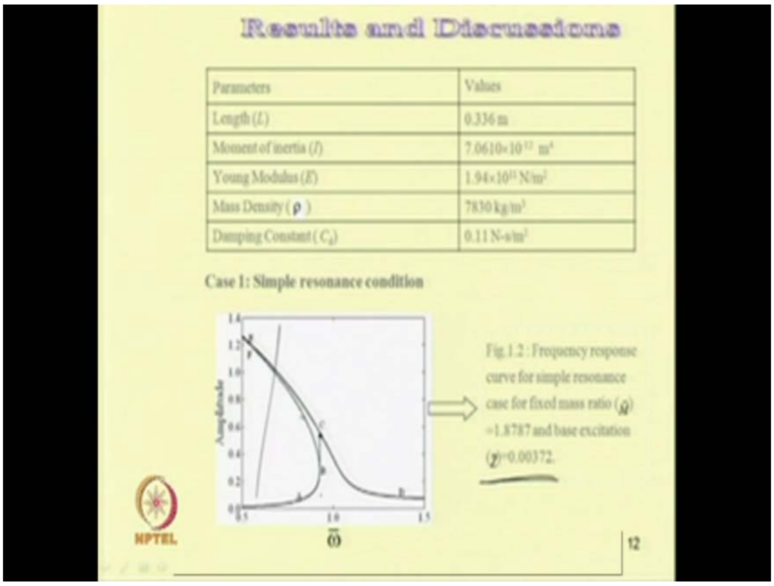

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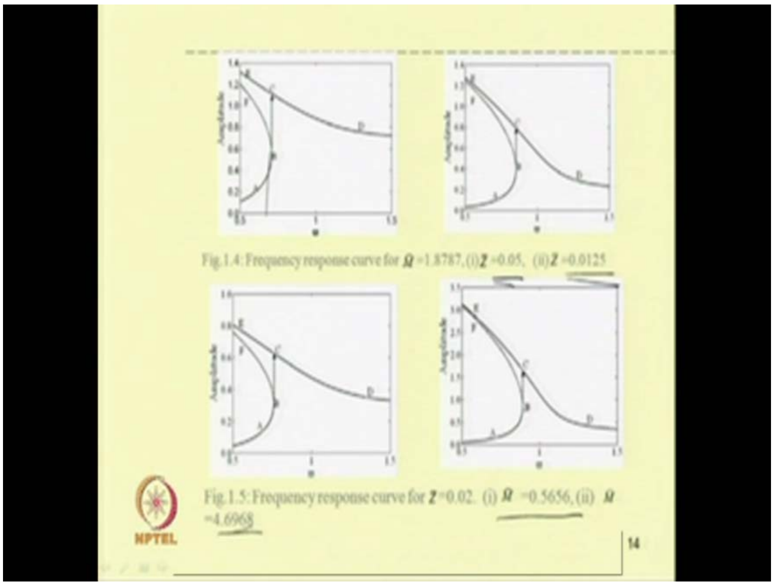

So, one can find so, for these different positions. So, the time response has also been plotted and it can, one can see that the system behave in quasi periodic response as the Poincare section, which shows a closed loop, the system behave as a quasi periodic response. So, for some value, so the system response is periodic also. So, system has a periodic response and also quasi periodic response because, in this case, one can have a bistable region, also one can have a bistable region here in this range up to this b. So, one can have a bistable region and after b. So, the system has a single table region similarly, this frequency response has been plotted for different value of mass ratio. So, one can see the system response for mass ratio equal to so, for the first one.

So, this is this curve is for different mass ratio and the upper curve is for different base excitation so, by decreasing the base excitation. So, here the base excitation is 0.05 and this is 0.01 to 5. So, one can see the position of this. So, one can see this position, this is d. So, this position, so with higher value of excitation, this position left to left. So, at a lower value of omega, the system will have a jump up phenomenon, but for a system with less value of this z that is the amplitude of base excitation. So, one can have this jump up phenomena or the system will become stable up to a higher value of omega similarly, by changing the mass ratio, one can observe that for lower value of mass ratio. So, for lower value of mass ratio, the b point the jump of phenomena takes place at a lower value than by taking a mass ratio higher than higher mass ratio. So, here m bar equal to 4.69.

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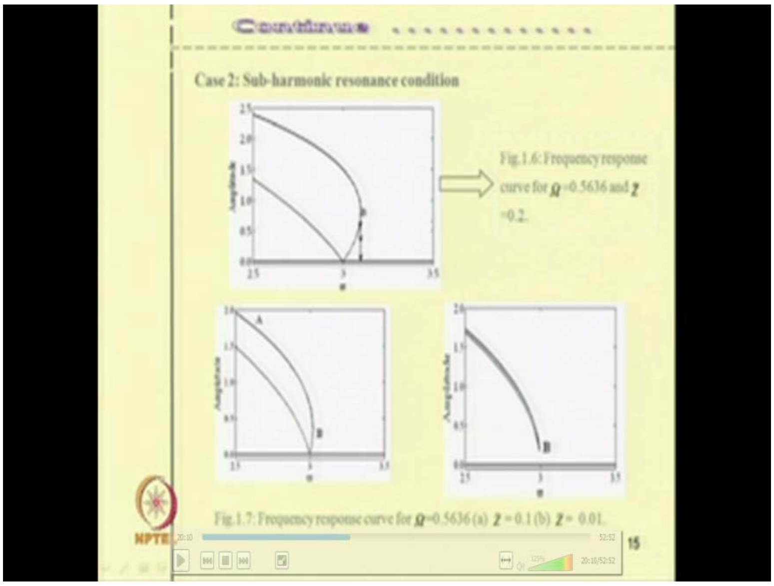

So, for sub harmonic resonance case so, in sub harmonic resonance case it been observe that the system has both stable and unstable region. So, as the system has a wide range of stable region wide range of trivial stable region. So, the system, so while sweeping the system for a wide range of frequency, the system become stable, but depending on the initial condition up to this point v as the system as bistable region. So, it will go to this nontrivial state depending on the initial condition.

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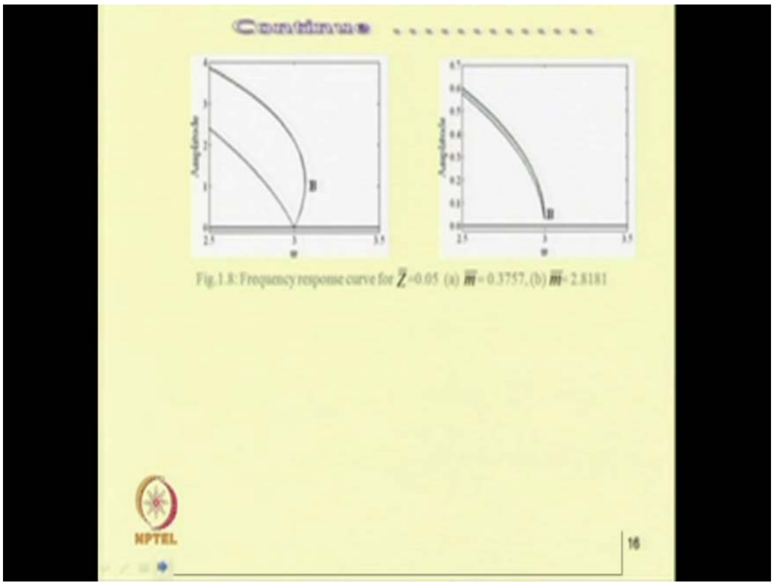

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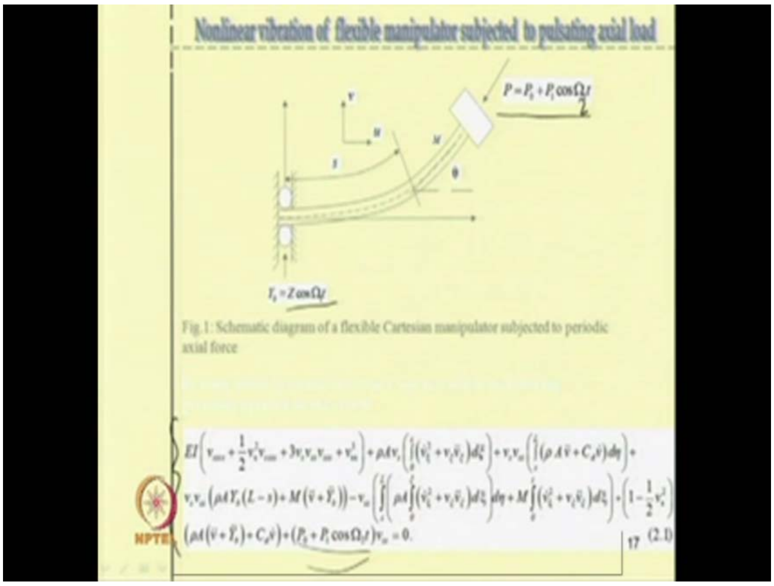

So, for a different value of mass ratio and base excitation, these curves have been plotted. So, let us take another example. So, in this case, we have taken a base excited cantilever beam, which can be molded as a Cartesian manipulator with n mass subjected to a Periodic load. So, periodic load, so in this case the system is subjected to 2 frequency excitation, 1 due to this base excitation, that is omega 1 and the second frequency is due to the second frequency is due to this axial loading. So, due to axial loading, we have this omega 2 and due this base excitation, we have the frequency omega 1. So, due to this now, we have some additional term. So, this is the additional term present in the system that is p 0 plus p 1 cos omega 2 t into v s s following the similar procedure as before using this D’alembert principle or by using this Hamilton principle, one can get a set one can get this equation. So it may be noted that in these cases, we are considering only single mode discritization in the Galerkin method.

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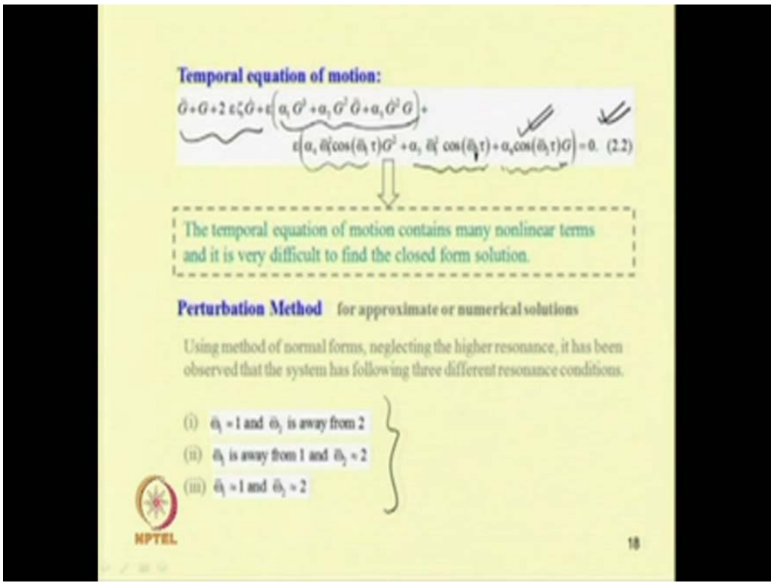

So, by taking single mode discretization in the Galerkin method so, our Spatio temporal equation reduced to this temporal equation. So, in this temporal equation so, in addition to, these linear term so, these are the cubic geometric and inertia non-linear term. So, we have several other terms. So, here this is the term associated with these 2 terms are associated with our terms. So, already we have these alpha 4 and alpha 5. So, these 2 terms, where before due to this base excitation and now we have this additional term, this is omega 1 bar. So, we have additional term alpha 6 cos omega 2 bar tau. So, this is due to this base excitation, this is due to the axial loading.

So, it may be noted that a part of the axial loading, that is the static force will be in will be the coefficient of the G term that is the stiffness term. So, the stiffness of the system and hence the natural frequency of the system will be effected by the static part of the axial loading and the due to the dynamic part of the axial loading. So, we can have the instability region. So, here it may be noted though due to this base excitation, we have a non-linear parametric term that is the coefficient of G square equal to alpha 4 omega 1 square cos omega 1 tau, but due to this axial loading, we have a term alpha 6 cos omega 2 bar tau into G. So, this is the coefficient of the time modulation this alpha 6 cos omega 2 tau is the coefficient of G.

So, this is a linear parametric equation, but this term is the non-linear parametric term. So, as so here also we can use this perturbation method, but in this case we can have this resonance condition. So, we can take this resonance condition omega 1 bar nearly equal to 1 and omega 2 bar is away from 2. Similarly, this omega 1 bar is away from 1 and omega 2 bar nearly equal to 2 and this omega bar equal to 1 and omega 2 bar equal to 2. So, in the first case, it will be simple resonance like previous 1 and the second case, when omega bar omega 2 bar is nearly equal to 2. So, we can have this principle parametric resonance and when this omega 1 nearly equal to 1 and omega 2 bar nearly equal to 2. So, we can have the simultaneous resonance condition. So, this 3 resonance conditions, we can have in the system and for these 3 case.

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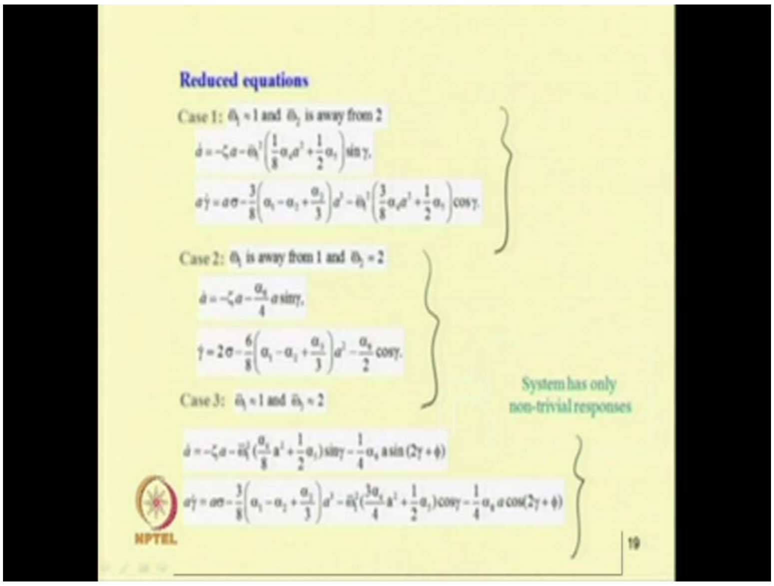

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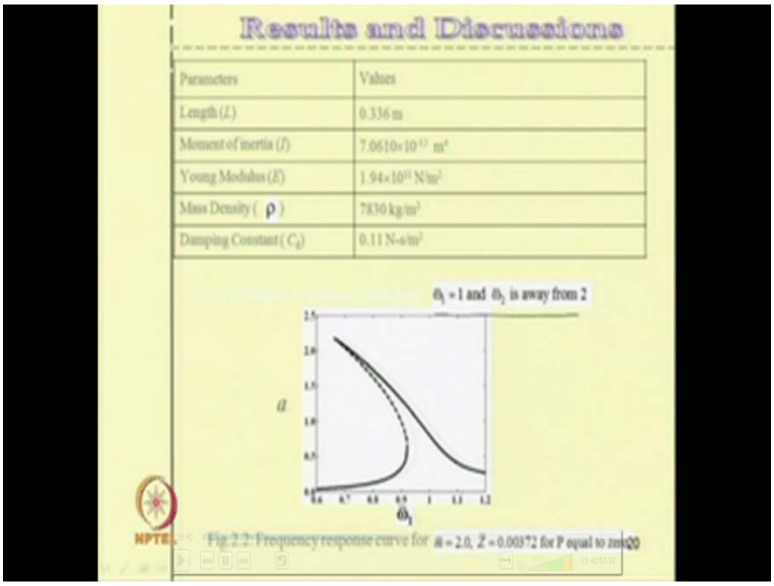

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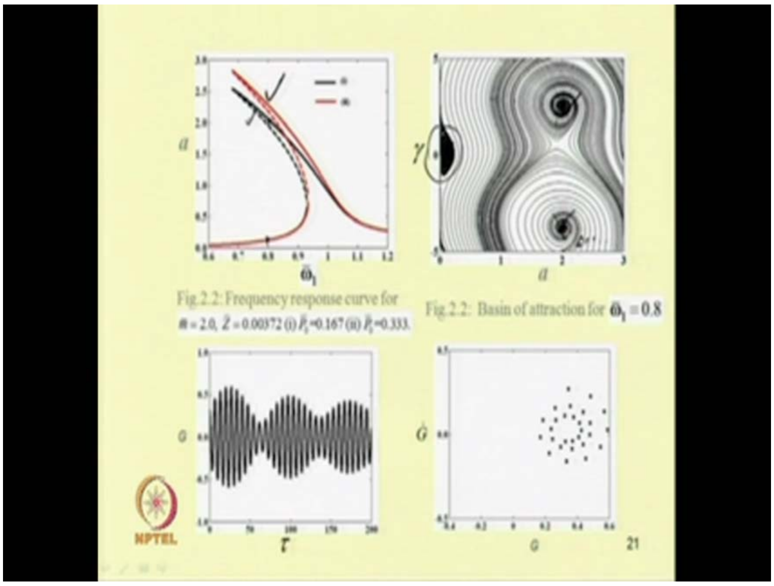

So, we can have this reduced equation. So, this is the reduced equation for the first case and this is the reduced equation for the second case and this is the reduced equation in the third case, the system has only nontrivial responses. So, similar to the previous case, here also we can plot the frequency response curve. So, in this case like the previous case, when in case of the simple resonance. So, we can have similar response. So, we can have a jump up phenomena while increasing this frequency here, we are considering this omega 2 away from 2. So, we are considering the case when omega 1 bar is nearly equal to 1.

So, in this case, we can see at for example, at omega 1 bar. So, this figure shows two cases. So, 1 case or taking this mass ratio equal to 2, this z bar that is the non dimension base excitation equal to 0.00372 and 2 cases of static loading that is 1 case is P 0 bar equal to. So, P 0 bar equal to 0.167 and P 0 bar equal to 0.333, the second case that is the rate case. So, here the response is plotted for a higher static load and this is the responses plotted for a lower static load. So, in this case, one can observe for this omega 1 bar equal to nearly equal to 0.8. So, when omega 1 bar is nearly equal to 0.8. So, we can have so, so we can have two solutions. So, we can have two solutions and those two solutions can be plotted. So, one can see, so for this case we can have a solution for example, so this is 1 stable state and this is the other stable state and this branch. So, we have a unstable state. So, we can have two stable solution and 1 unstable solution which is shown.

So, this near nearly about, for this case, it is nearly about 2. So, this, this is the nearly about 2. So, we can have 2 h. So, a is the amplitude and gamma is the phase. So, this is, this so the basin of attraction. So, for a different initial condition for example, so if you take this these this point as the initial condition, always it will go to this branch, but if somebody takes another initial condition one can see it will go to this branch, which is nearly equal to 0.0 point some value very small value. So, one can have this, so this correspond to, so this point correspond to 1 solution and this and this correspond to the trivial state solution, nontrivial sates solution. So, in the nontrivial state solution, one can have 2 different type of phase.

So, one can see or one can observe the basin of attraction or one can plot these basin of attraction to know for what initial condition, the system will be in which state. For example, so if one take an initial condition of this that is 2 point amplitude equal to 2 point something and a phase nearly equal to minus 5 minus 4 point something, then it will lead to this upper branch, nontrivial branch though both the branches are nontrivial. So, this branch is with higher amplitude and this branch is with lower amplitude. So, one can plot the time response also in those cases. So, the time response can be plotted by solving this either by solving this temporal equation or by solving this reduced equation. So, by solving the temporal equation, one can find the response and one can compare this response with other response obtained using this method of multiple scale and one can find that both yield the same result.

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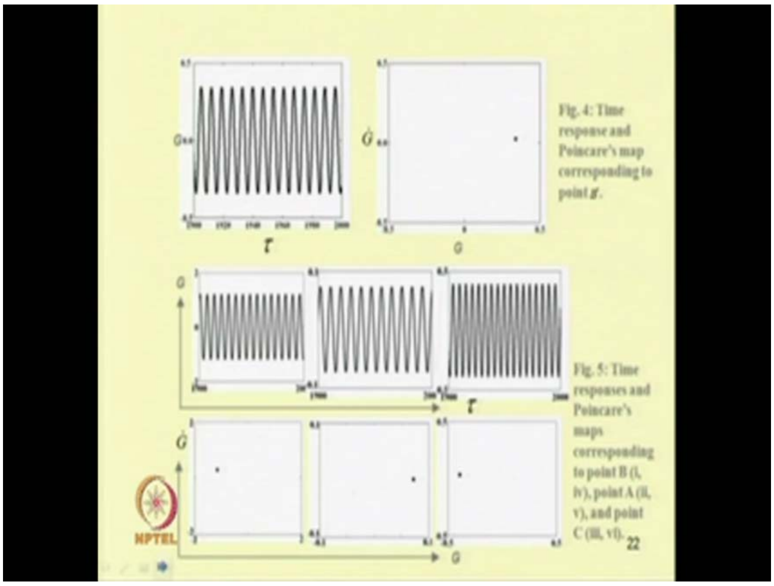

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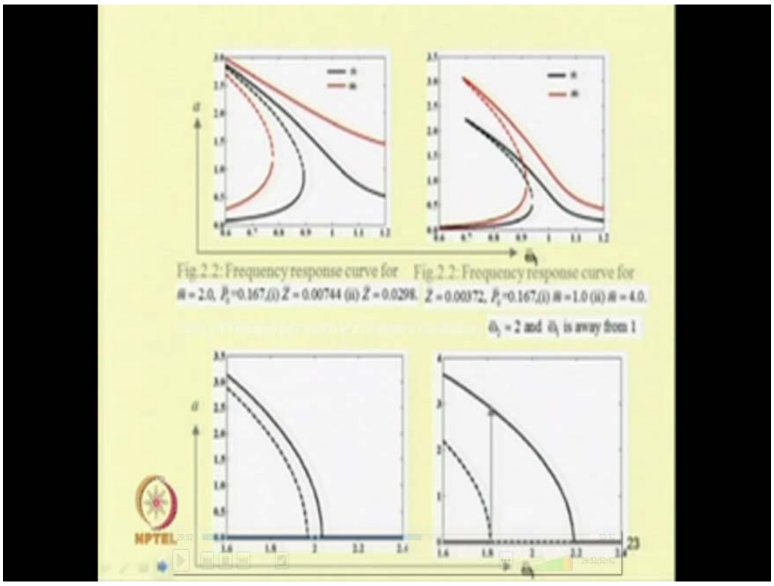

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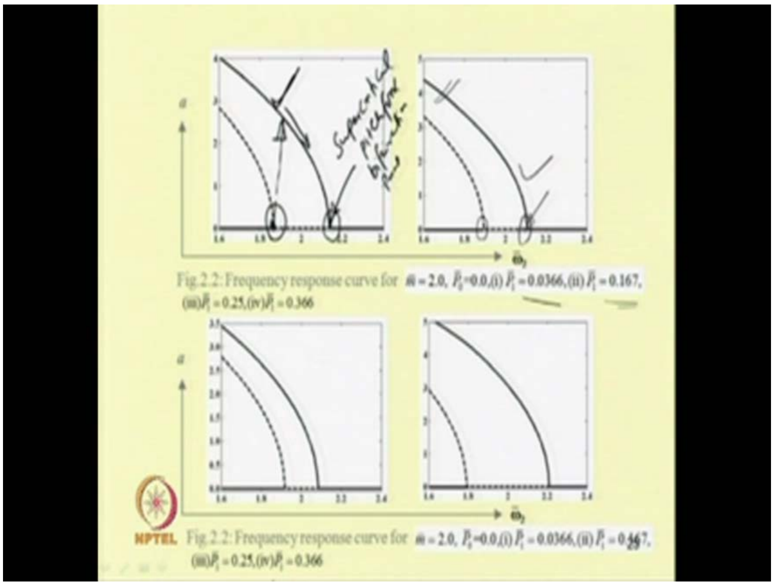

So, here some comparisons have been given and these are some more results obtained for these cases. Now, one can see the frequency response curve, when the system that is omega 2 is nearly equal to 2. So, when omega 2 is nearly equal to 2, the system will have principal parametric resonance. So, in case of this principle parametric resonance so, one can observe that, so here the trivial state become unstable due to this pitch fork bifurcation. So, while sweeping up, so one can observe this trivial state become, unstable so or the trivial state and the, so there is, so at this point; so at this point, these trivial stable trivial branch and unstable trivial an untrivial branch merge to form a; to from a unstable form an unstable trivial branch.

So, this is; so this is a sub critical pitch fork bifurcation point similarly, this point is a super critical. So, this is a super critical pitch fork bifurcation point. So, one can have, so while sweeping of the frequency at this point, that is at sub critical pitch fork bifurcation point. The system will be subjected to a jump of phenomena because, only the stable branch available is the nontrivial state and so the system then will follow this nontrivial branch, till it reach the trivial state and after that the system will have a single state solution. So, up to this point up to this sub critical pitch fork bifurcation point, the system will have bistable region and after the sub critical pitch fork bifurcation point, the system has nontrivial response up to the supper critical pitch fork bifurcation and after the supper critical pitch fork bifurcation the system has only a trivial state. So, depending on the initial condition; so depending on the frequency of excitation and amplitude of excitation the system response will be different.

So, one can while sweeping down the frequency similarly, one can follow, so up to the supper critical pitch fork bifurcation point the system will not; will not, will be in its state of trivial state. And after that the system will have a nontrivial state that means the system will vibrate at a frequency at an amplitude equivalent to the response amplitude of the non trivial sate similarly, by changing the system parameters. So, for example, in the first case by taking this non dimensional amplitude of dynamic loading equal to 0.0366 and the second case in this case it is taken to be 0.167 that means the amplitude is increased. So, one can see, so when the amplitude is increased the response amplitude of the nontrivial state also increases and also increases and one can study.

So, where the pitch fork bifurcation takes place and to design the system, one should note that either it has to operated at a frequency more than this omega 2 bar equal to the supercritical pitch fork bifurcation point or if it is required to be required to be operated at a frequency less than the sub critical pitch fork bifurcation point. So, one has to take note of the initial conditions and between the sub critical and supper critical pitch fork bifurcation point. The system will always vibrate at a nontrivial state. So, similarly, one can plot the response for different values of P 1 bar.

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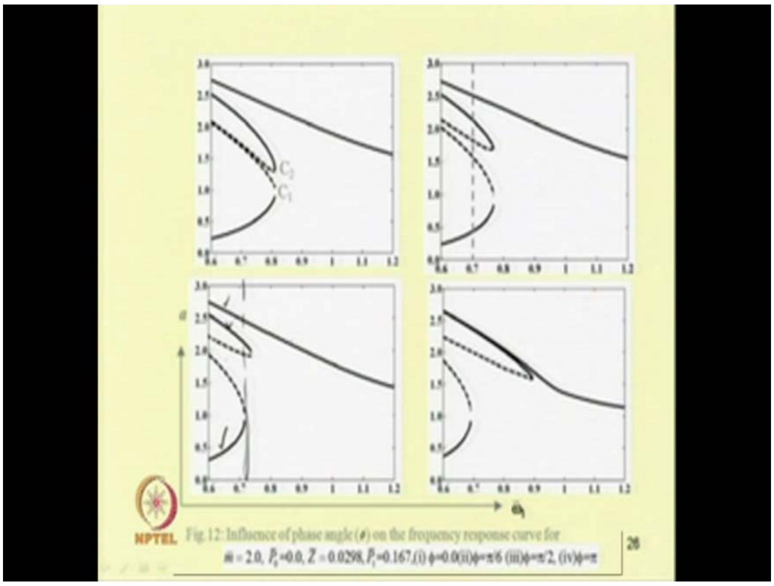

And so, next one can study the case when the system will excited at a frequency which contains which is near to both 1 and 2. So, if it is near to both 1 and 2. So, in that case that means when it is subjected to simultaneous principle parametric and simple resonance case. So, one can observe that the response of the system like this so, up to certain part so, up to this the system has a multi stable region. So, for example, this is a stable state is the second stable state, and this is the third stable state and after this point; after this point the system will have is only the; system will have only the nontrivial study nontrivial solution. So, one has to check for what initial condition, it will have less amplitude of excitation or amplitude of response because, the system will have 3 stable state. So, for a different system parameter, so one has to study, so for what parameter the response will be less.

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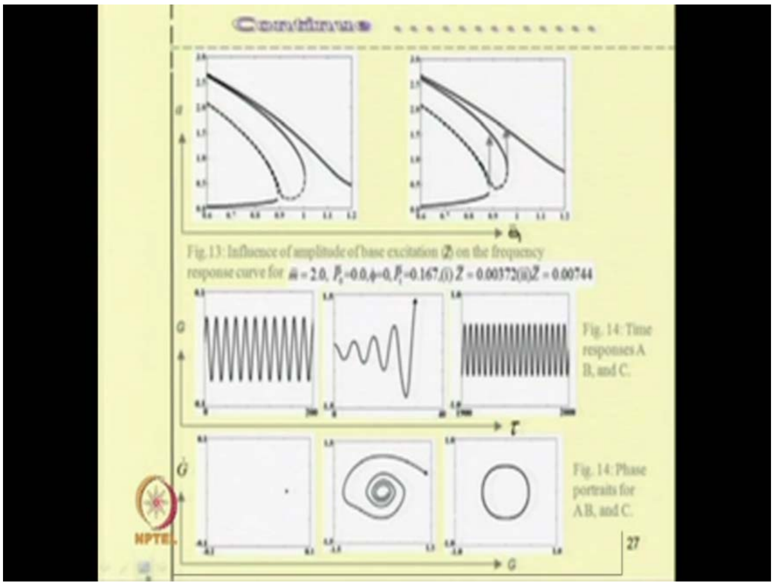

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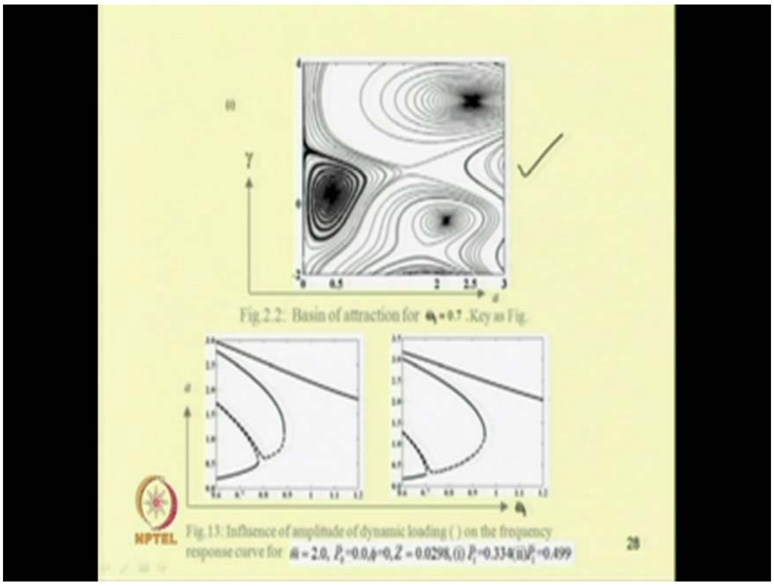

So, one can plot this things. So, here the basin of attraction has been plotted to show the 3 state so, the 3, 3 stable state of the system. So, this is one stable state, this is the second stable state; this is the third stable state of the system and depending on the initial condition. So, one can find to which stable state the system response will go.

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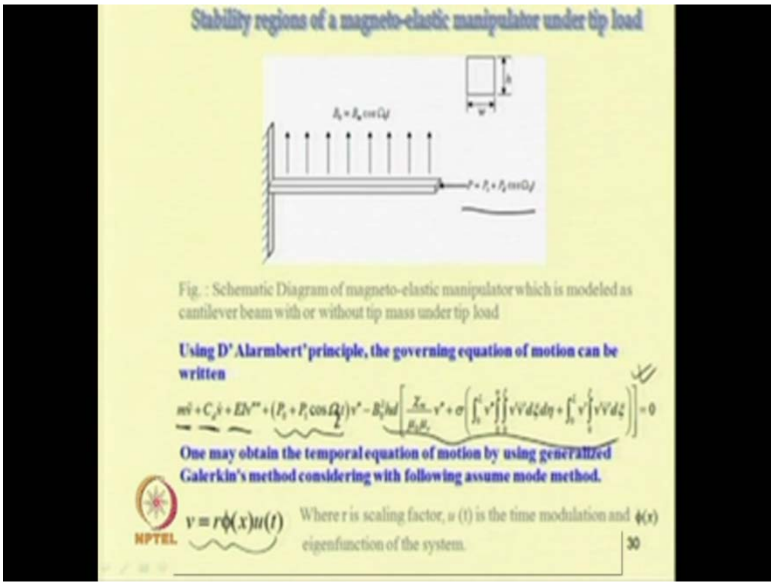

So, in this way, one can study different elastic system base excited elastic system. So, previous example, we have taken a metallic beam. So, we have taken some beam elastic beam and which is base excited at the same time it is subjected to an axial periodic load. So, let us take some example, where we will consider the magneto elastic beam. So, there are some hero magnetic material or some material, when it is subjected to this magnetic felid. So, there will be magneto elastic load on the beam itself. So, due to this magneto elastic load, the there will vibration of the system or if the system is vibrating so, by applying this magneto elastic load. So, we can control the vibration of the system.

So, let us take a simple example when we have a cantilever beam, which is subjected to periodic axial load P equal to P s plus P d cos omega 2 t also it is subjected to a magnetic felid, that is equal to time varying magnetic field so, b 0 equal to b m cos omega 1 t. So, here we have taken the cross section to be rectangular and so due this magneto elastic load and this axial load. So, we can derive this equation of motion of the system. So, the equation motion of the systems can be written in this form that is m v double dot, where v is the vibration in the transverse direction. So, the equation of motion can be written this form that is m v double dot plus by taking this damping c d v dot then e i del fourth e by del s 4, then we can have this loading term.

So, due to axial loading, we can have this term that is P 0 plus p 1 cos omega 1 t into cos P 1 cos omega 2 t into del square v by del s square, then due to this magneto elastic load.

So, we can have these terms. So, in this magneto elastic load, the term will be minus P 0 square h into d is the area. So, j m by mu 0 into mu r into del square v by del s square plus sigma into 0 2 l mu double dash integration 0 to eta 0 to zeta v dash v dot dash d zeta into d eta plus integration 0 to l v dash, integration 0 to zeta v dash v dot dash v zeta, where this eta and zeta are dummy variable, dummy integration variable and this mu 0 and mu r, mu r is the relative permittivity, permittivity and mu 0 is the permittivity of the free space. So, sigma is the conductivity of the material. So, the spatio temporal equation can be reduced to its temporal form by using this equation. So, let us use this equation v equal to r phi x into u t, where r is the scaling factor phi x is the. So, phi x is the safe functions; safe functions; safe function and u t is the time modulation. So, by using this safe function of that of a cantilever beam.

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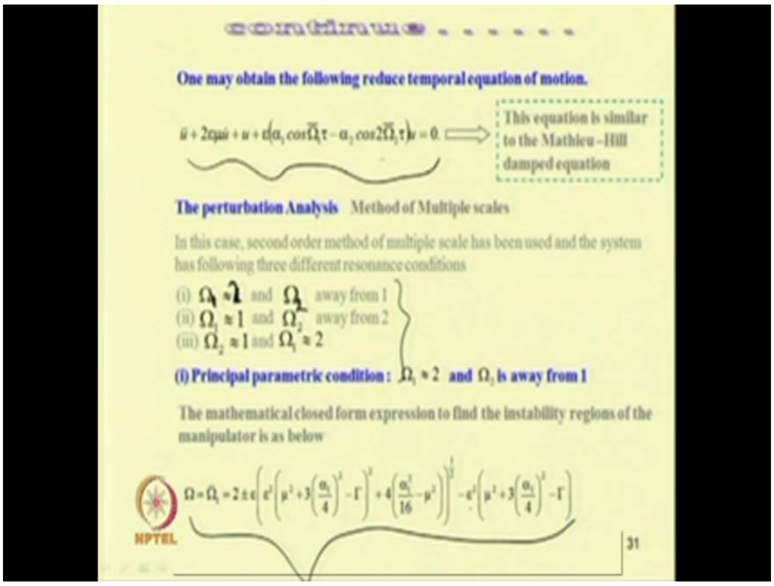

So, we can reduce this equation to its temporal form and one can get this equation in this form. So, in this case, one can note that equation is u double dot plus 2 epsilon mu u dot plus u plus epsilon into alpha 1 cos omega 1 bar tau in to u so, into u minus epsilon alpha tow cos 2 omega 2 bar tau into u. So, we can have 2 parametrically excited term. So, the fast one is omega 1 with frequency omega 1 and second 1 is with frequency 2 omega 2. So, this equation, so is similar that of Mathieu Hill Equation and one can solve this equation by using different methods and here the method of multiple scale has been used and, in this case one can note in ordinary Mathieu Hill Equation, one will have the single term that is epsilon alpha 1 cos omega 1 t into u, but as in non-linear systems.

The super position rule will not holds good. So, we cannot apply the supper position rule in this system by taking this forcing function or this equation equivalent to u double dot plus 2 epsilon mu u dot plus u plus epsilon alpha 1 cos omega 1 tau into u 1 system and the second system as u double dot plus to epsilon mu u dot plus u plus epsilon or minus epsilon alpha 2 cos 2 omega 2 bar tau into u. So, as super position rule is not applicable so, we cannot take this equation as 2 Mathieu Equation and solve this thing. So, but in this case, we can have 3 resonance conditions so, you can study those resonance condition so, in one case this omega 2 nearly equal to 1 and omega 1 away from 1 second case is omega 1 nearly equal to 1 and omega 2 is away from 2 and third case, we can take the simultaneous 1.

So, where we will consider this omega 2 nearly, equal to 1 and omega 1 nearly equal to 2. So, this is the simultaneous case and so we can have this when omega 1 is nearly equal to 2. So, when omega 1 is nearly equal to 2 and omega 2 is away from 1. So, we can have this principle parametric resonance. So, omega 1 nearly, so let us take the first 1 omega 1 nearly, equal to 1 and omega 2 omega. So, this is omega 2 away from 1 and omega 1 nearly equal to. So, this is the first case omega 1 nearly equal to 2. So, if omega 1 is nearly equal to 2. So, we will have this principle parametric resonance and in this case. So, we can applying this method of multiple scale. So, we can obtain the equation form the transaction curve. So, this is, so here we have applied the method of multiple scale up to second order.

So, by applying method of multiple scale up to second order, the equation can be written as this omega equal to 2 plus epsilon plus minus epsilon. So, we can have 2 branch 1 plus for this plus and another for this minus. So, we can have this omega or omega 1 bar nearly equal to 2 plus minus epsilon into epsilon square into mu square plus 3 into alpha 1 by 4 whole square minus gamma whole square plus 4 into alpha 1 square minus 16 minus mu square to the power half minus epsilon square mu square plus 3 alpha 1 by 4 square minus gamma. So, it can be noted, that if one take, so this is, one can see this is of the order of cubic of the epsilon. So, this is epsilon cube and this term is only epsilon and so this term contains only epsilon and this contain epsilon cube.

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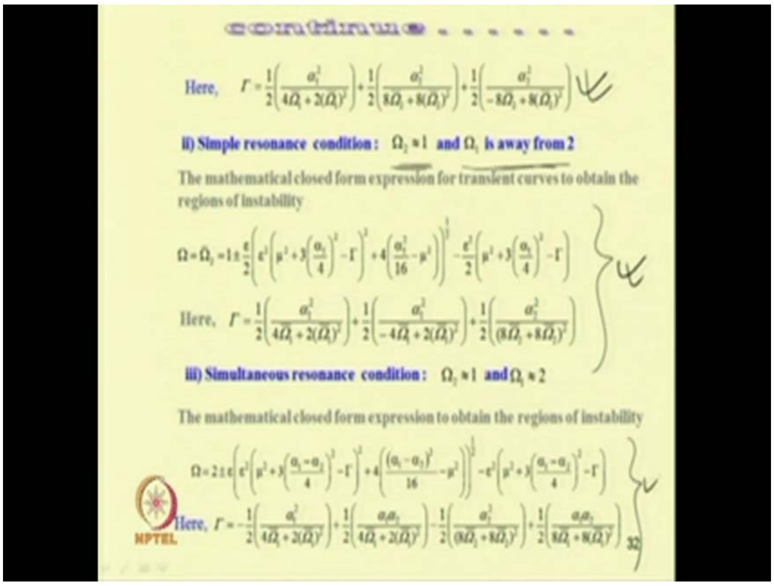

So, in this term this gamma, so this gamma is a term, which contain this alpha 1 alpha 2 alpha, then omega 1 omega 2, all these terms are present in this case. So, in case of simple resonance condition again, one can obtain this equation, where in case of simple resonance condition omega 2 nearly equal to 1 and omega 1 is away from 2. So, one can obtain this equation from the transaction curve similarly, for simultaneous resonance condition. So, one can obtain this equation. So, these equations are obtained using higher order method of multiple scales and so when 2. So, here when the system is subjected to 2 frequency excitation so, we will have these equations or these terms.

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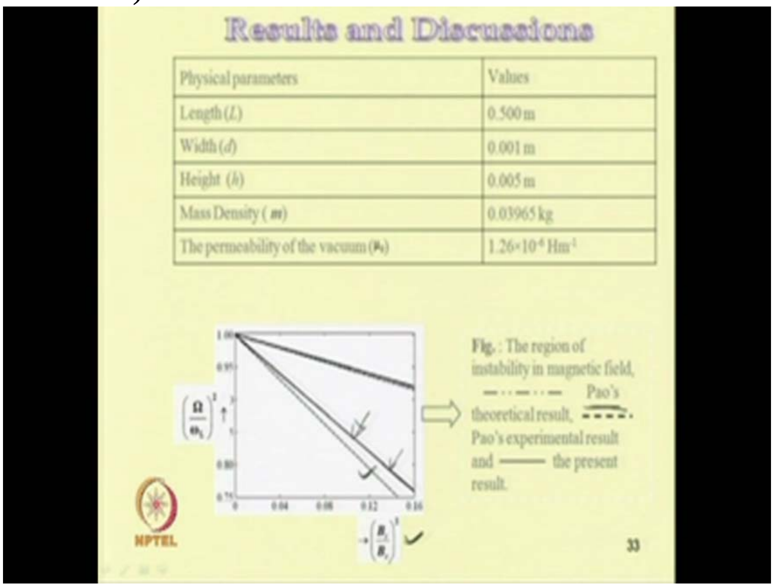

So this system has initial been discussed or studied by Pao and moon in tau and moon they have studied experimentally and theoretically, the system, a system subjected to a magnetic field and they have plotted this instability region. So, here so for comparison propose, this theoretical results. So, this shows the Pao's theoretical result and the present result. So, this is the Pao's theoretical result, this result show the Pao's theoretical result and inside this shows the Pao's experiential result and this result is the present result for the present analysis. So by using second order method of multiple scale. So, one can observe that by using the second order method of multiple scale, one can get a get an expression which is very closer to that of the experiential result. So, by taking higher, higher order terms, one can get very instability region, very closer to this experiential value. So, this way, one can plot the instability region and, in this instability region it is plotted, this taking this magnetic felid into account. So, this is the amplitude of the magnetic field, this is omega by omega l term.

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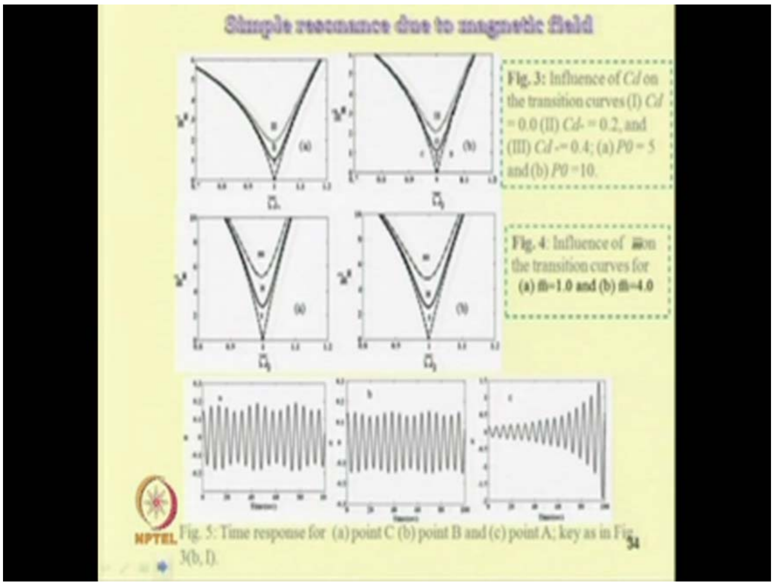

So, similarly, one can plot or one can study the influence of different system parameter due this magnetic felid due different value of static loading P 0, due to different values of mass ratio also, one can compare this results by plotting the time response of the system.

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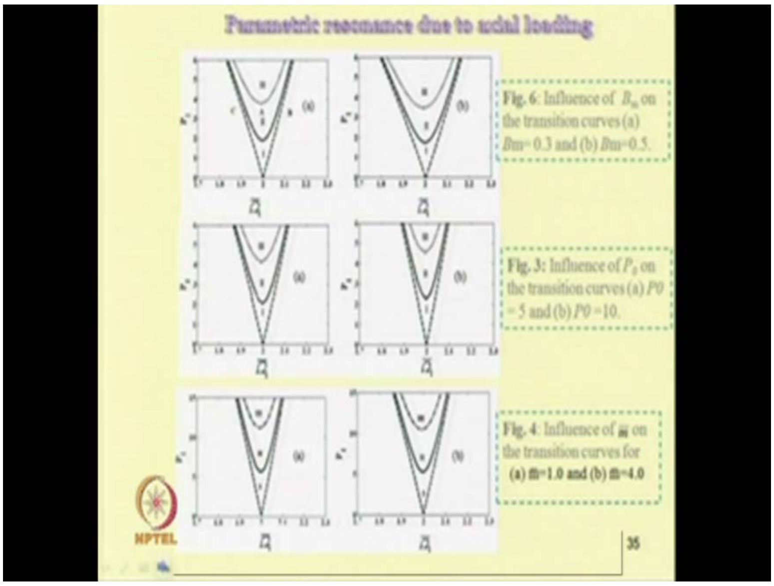

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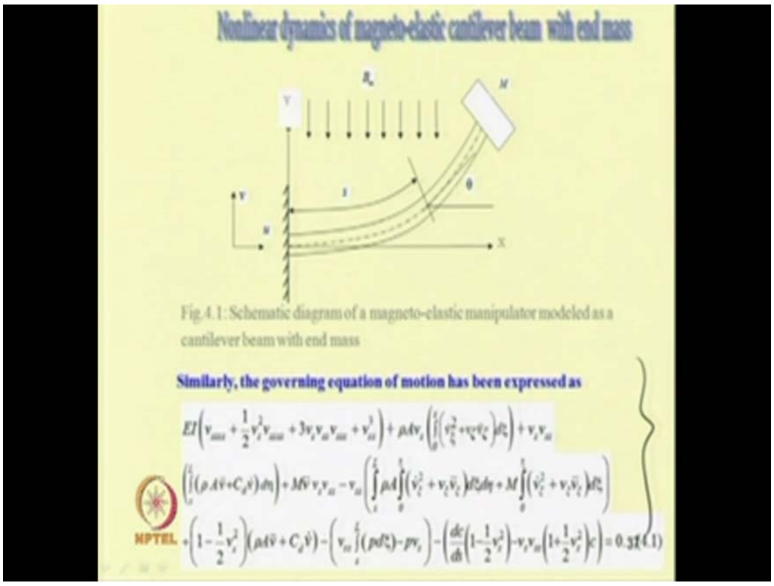

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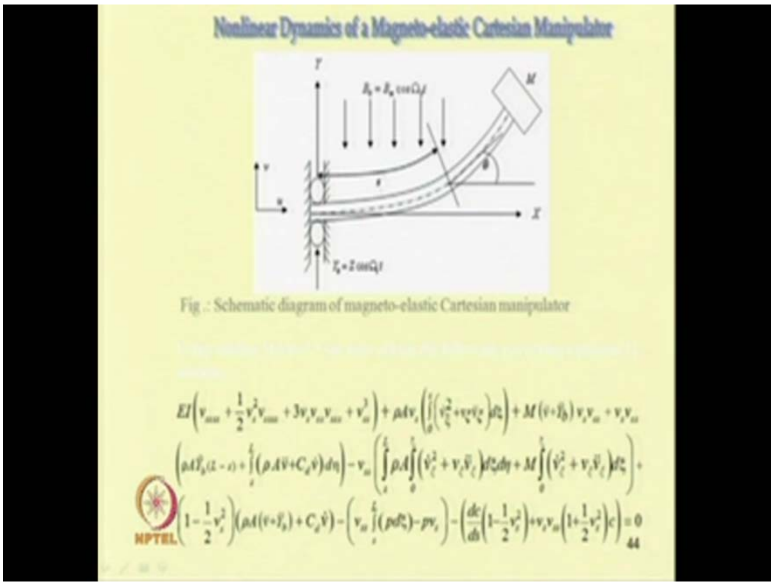

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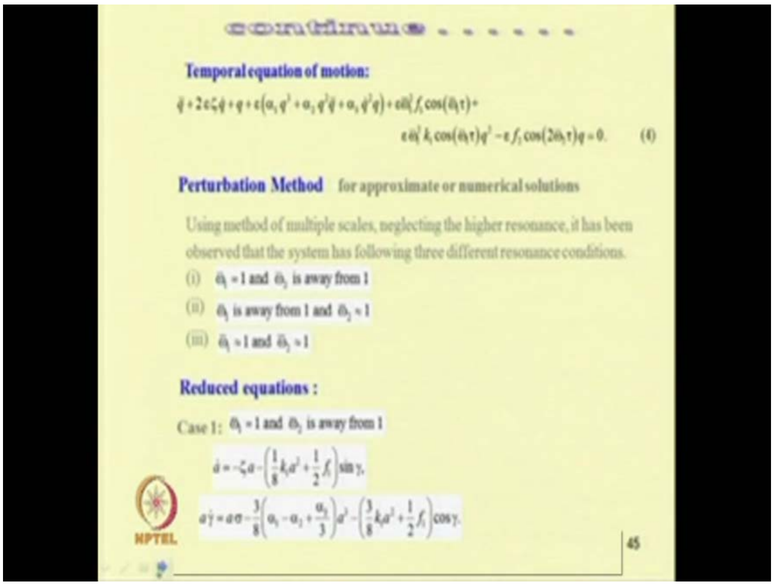

So, this way, one can study the influence of different system parameters on this instability region, when it is subjected to this both magnetic felid and axial load, one can study similarly, different other different systems, where a tip mass is considered also. So, in this case the response, one can obtain the response of the system also, one can study the system with base excitation and applying magnetic field. So, in this case, the system will be a two frequency excitation term. So, in this case, one can obtain different resonance conditions.

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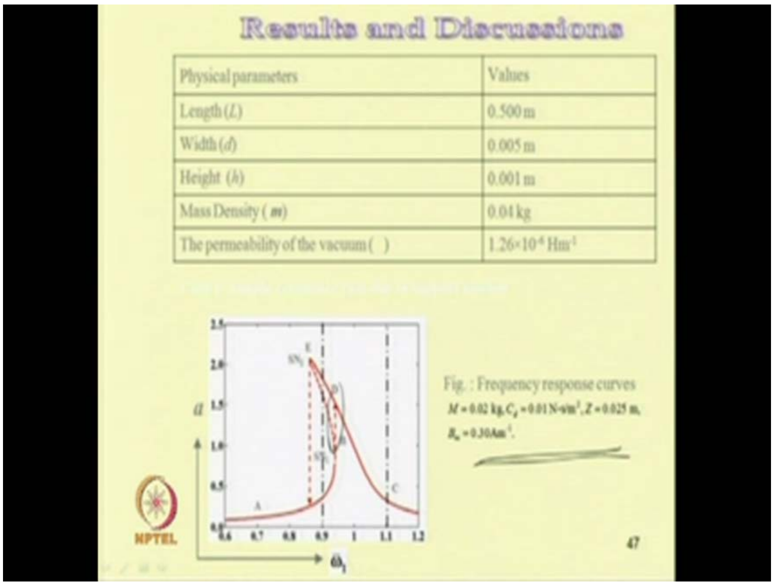

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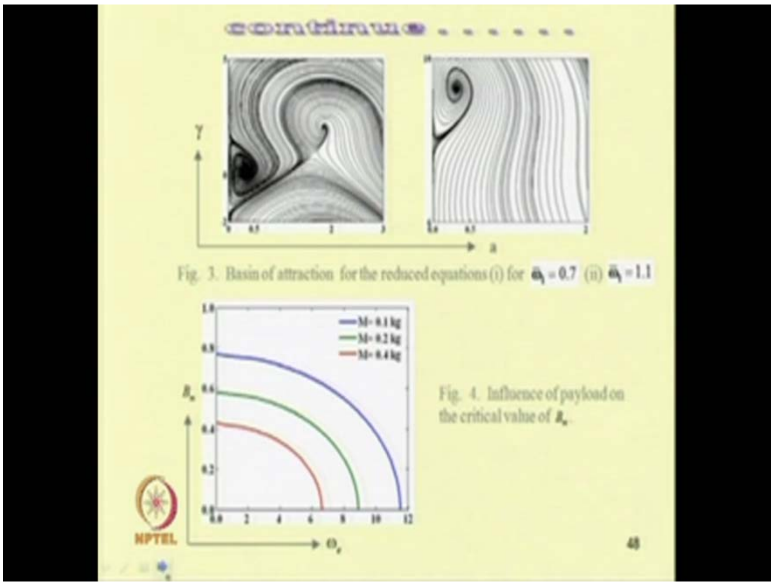

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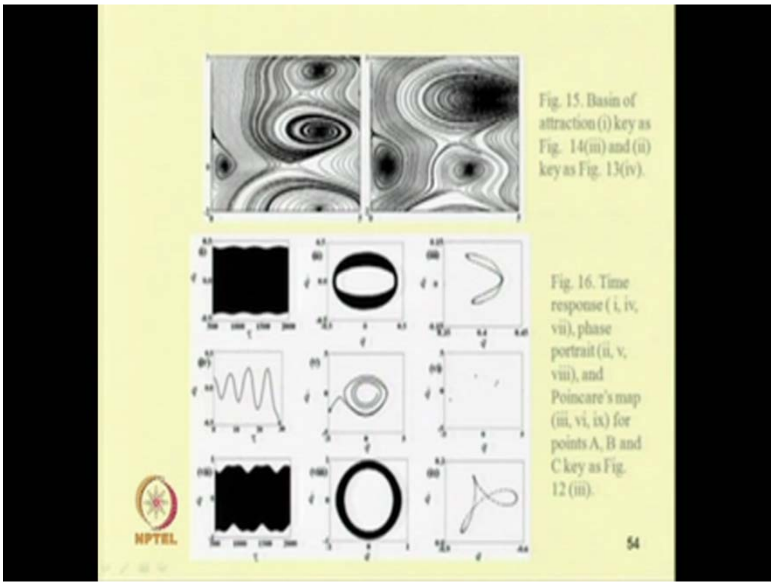

And getting this reduced equation one can plot the response of the system. So in this also, one can study, the response of the system, what we have observed in the beginning. So, one can have this from the response curve, one can have this jump up, jump down phenomena. So, these points are saddle node bifurcation points. So, for different system parameters, one can observe different phenomena in the system. So, in this way, one can study the parametrically excited system and for further study can obtain the, or draw the basin of attraction for when this multi stable regions are present and obtain the response curve for different cases. So, these are some of the basin of attraction obtained while applying or when multi stable regions are present in the system.

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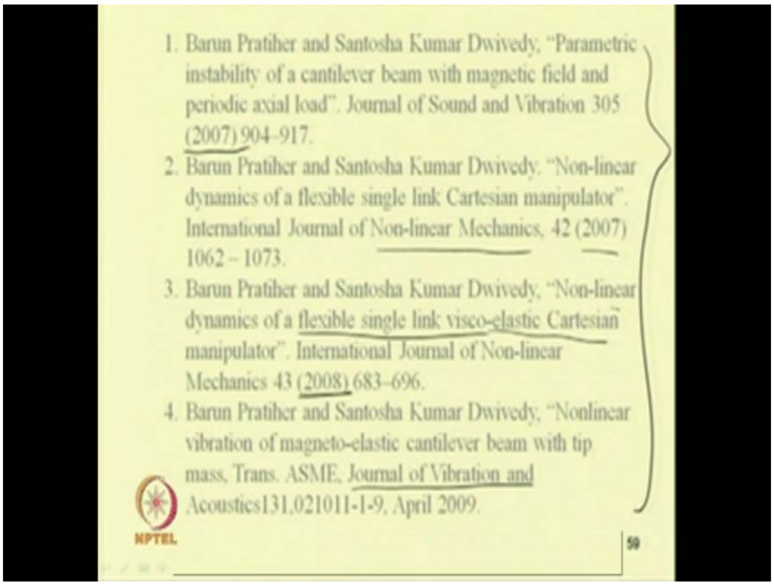

So, one can study this system. So, here some of the references are given. So, for further study, so these the today class the presentation was taken or adopted from the PHD work of my student Barun Pratiher. And these are some of the publications, which are already published in international journal for example, this is published in journal of sound and vibration parametric instability of a cantilever beam with magnetic felid and periodic axial load in 2007. Similarly, another paper non-linear dynamics of flexible single link Cartesian manipulator so, this is published in international journal of non-linear mechanics in 2007. So, another work this non-linear dynamics of flexible single link Visco elastic. So, if one take a Visco Elastic Cartesian manipulator also, this work has been. So, this is a parametrically excited system. So, one can study some other works related to this magnetic felid and elastic beam in this journal of Vibration and Acoustics which is published in 2009.

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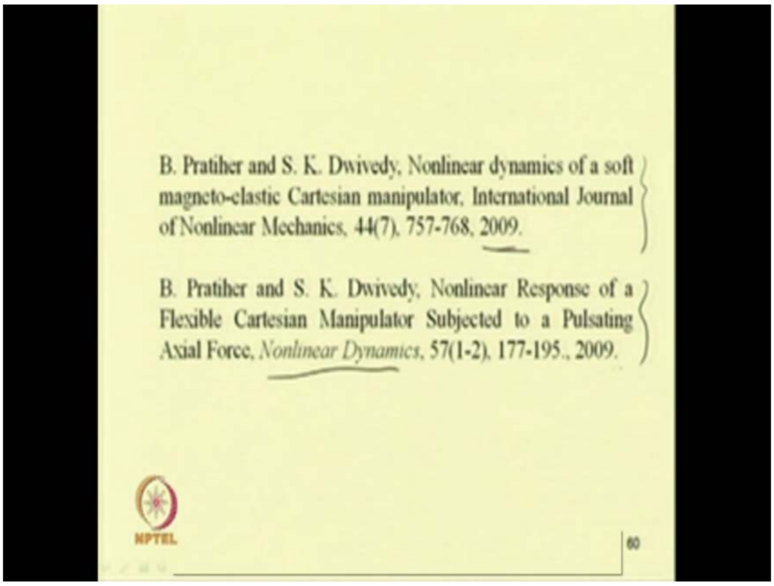

And some these are some other work on only elastic beam. So, non-linear dynamics of, so these are on magnetic. So, this work is on non-linear dynamics of a soft magneto elastic Cartesian manipulator. So, this is published in international journal of non-linear mechanics in 2009. So, another work also non-linear response of a flexible Cartesian manipulator subjected to pulsating axial load, which is published in non-linear dynamics. So, in 2009, so this is on the, this is only on elastic systems and other works are on magneto elastic system and also Visco Elastic System, when it is subjected to axial periodic loading and the system is modeled as a parametrically excited system. So, in this way, one can study the non-linear vibration of parametrically excited system with different resonance conditions. And next class, we will study about the parametrically excited system with internal resonance.

Thank you.