Non-Linear Vibration
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Module - 6
Application
Lecture - 8
Nonlinear Vibration of Parametrically Excited System

So, welcome to today class of non-linear vibration. Today class we are going to study few examples related to parametrically exited systems. So, previous classes we have studied about the free and force vibration of non-linear systems; and today class we are going to study about the parametrically excited system, which differs from the force vibration of a system, in mainly two different ways.

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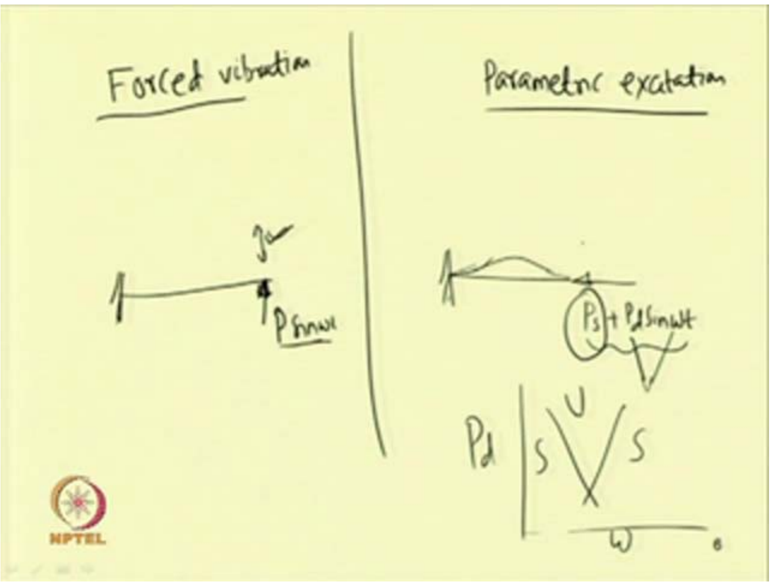

So, we can have a force vibration system and parametrically excited system; let us see, what is the difference between these two? Force vibration, so parametric excitation is also a kind of force vibration, and in this forced excitation and parametric excitation. So, let us take a cantilever beam and so if we are applying a force, let us apply a force in this transverse direction that is P sin omega t, and the vibration will takes place in this direction. So, the direction of application of force and the excitation or the response takes place in same direction in case of the forced vibration of the system.

And also in this case we have seen that when the frequency equal to the natural frequency of the system then the system have a resonance condition, so system attains a resonance condition. But in case of parametrically excited system; so let us this same cantilever beam and let us apply one axial load. So, in this axial load let me apply this is equal to P s plus P d sin omega t. So, in this case, as we are applying a axial load, as we are applying a axial load to the system, so so if it exceeds the Euler buckling load, then it will start buckling. But when a dynamic force is acted to this system, then for some value of this dynamically dynamic amplitude of the load and the frequency omega, so it will start buckling before its attain the critical euler buckling load. So, if we are applying only the static load to the system then when it is equal to this euler buckling load it will buckle, but when a dynamic load is added to the system it will start buckling or it will start vibrating in a non-trivial state when this P d and omega that is the amplitude and the frequency of the excitation term. So, depending on some value of this amplitude and frequency of this excitation term. So, we have to find for what value of this P d and omega the system start vibrating in a non trivial state or the system becomes.

Unstable in the trivial sense. So, in this systems. So, it is very very important find the parametric instability region that is if you plot a curve between this P d and omega. So, we can find a region for which the systems will become unstable. So, this is this this is the unstable state. So, this is the stable state and this is the stable state. So, in this study mainly our focus will be to find the parametric instability region of the system also one more difference between this force and this parametric excited vibration is that in first case I I told that is a direction of the forcing a response takes place in the same direction, but in this case in this case though the force is applied in a axial direction the response will takes place or it will vibrate in the transverse direction. So, it will vibrate in the transverse direction. So, when. So, the applied forcing and the response are orthogonal to each other and also it will takes place at a frequency.

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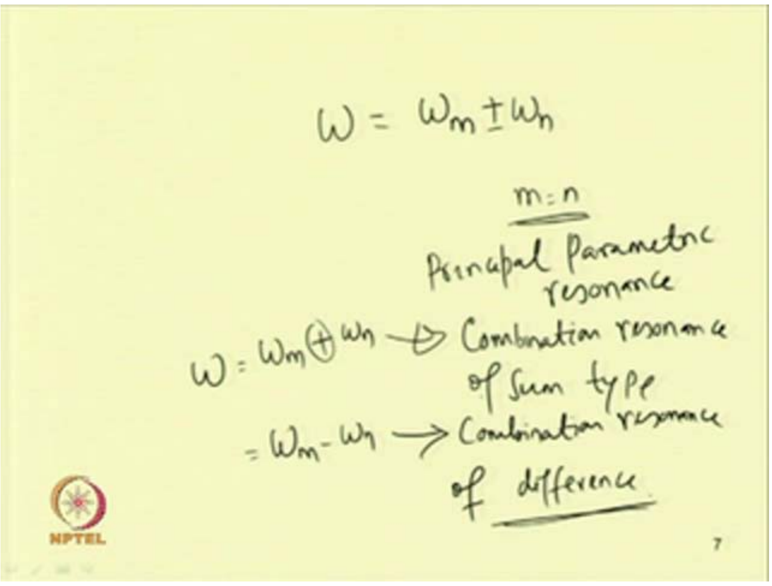

So, this parametric excitation will takes place at a frequency omega. So, so this will be equal to omega m plus minus omega n of the system. So, when m equal to n. So, this is known as principle parametric resonance principal parametric resonance condition and when m not equal to n then will have two conditions. So, one will be omega m plus omega n. So, where m.

And n at the m th and n th mode natural frequency. So, in this case when you are taking the sum type. So, this will be combination resonance of sum type. So, this called combination resonance of sum type and another one when omega equal to omega m minus omega n. So, this is combination resonance of difference type . So, already we have studied the equation governing this type of system. So, that is mainly known as Mathieu hill type of equation.

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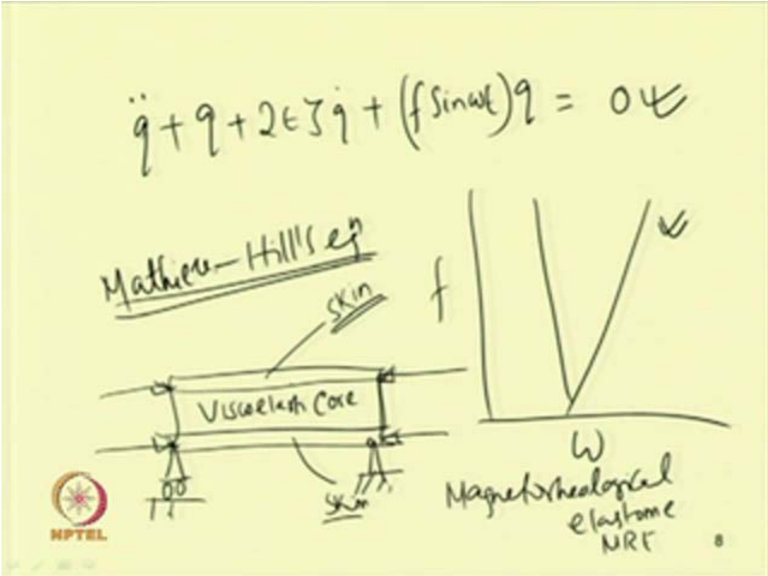

So, in case of the Mathieu hill type of equation the equation can be written in this form that is q double dot plus q plus if we are adding damping to the system it will 2 epsilon zeta q dot plus. So, we have a term this is q sin omega. So, will have a term. So, this is f sin omega t. So, the this will be the coefficient of q and this will be equal to 0. So, if you are not considering the non-linear terms then the most linear equation will be in this form and it is known as the Mathieu hill equation and already we have studied this equation or we have we know how.

To solve this equation by applying different methods such as method of multiple scales or method of harmonic balance method of averaging. So, we can use different methods to find the solution of this or we can find the transition curve which separate this stable and an unstable region for. So, for this for some value of. So, for some value of f and omega. So, this is the instability region. So, we can plot. So, this equation is in the form of Mathieu hill equation Mathieu hill equation. So, so today class we are will take a multi degree of freedom system for example, will take a sandwich beam and will apply. So, will take sandwich beam with a different boundary condition. So, this the sandwich beam. So, in this sandwich beam it may have different boundary conditions for example, let us take a boundary condition like this and will apply this period axial load.

So, the periodic axial load will apply and will find the resulting response of the system. So, particularly this sandwich beam consists of this skin layer and. So, these two are skin and will have a viscoelastic core. So, so it will contain skin to skin. So, this this skin may be of metallic type this skin may be of composite material or the skin may be of functionally greater material and this viscoelastic core may be may contain this rubber material or it or recently people are using this magnetorheological elastomer magnetorheological elastomer which is in short known as MRE magnetorheological elastomer. So, one may use this magnetorheological elastomer patch also in this viscoelastic core and by applying this magnetic field and by applying magnetic field one can increase the stiffness of this magnetorheological elastomer and increase this stiffness of overall structure. So, will today class will study about or will develop the equation motion of a parametrically excited sandwich beam with soft core the core may be soft or it may be stiff. So, when it is stiff then the displacement of top and bottom layer will be same and one can use this anti plane concept or classical sandwich beam theory and if the core is considered to be soft then one can take higher order theory to derive the equation of motion. So, let us first derive the equation motion of a sandwich beam.

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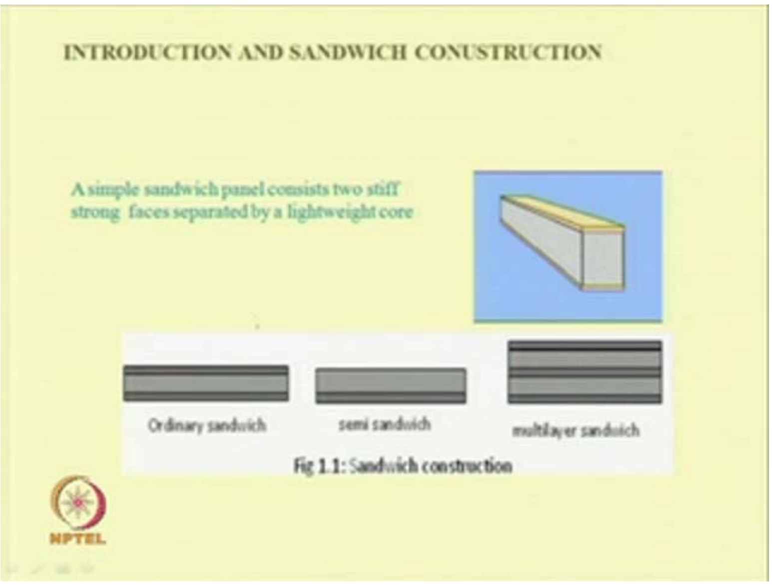

So, a sandwich beam. So, it can be of a ordinary sandwich beam or it is a semi sandwich where will have one singles skin and this is the viscoelastic layer. So, one can have multi layer sandwich beam also the sandwich beam may contain foam type of.

So, it may contain this a viscoelastic rubber type of material or sub foam type of material. So, this core may be of different constructions also this may be honeycomb type also one may use this honeycomb core and any other different types of core.

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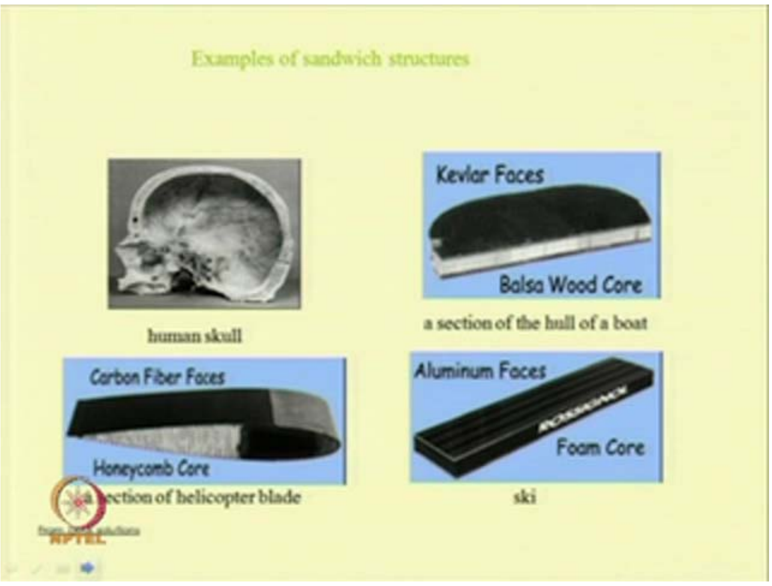

So, example of the some of the sandwich beams. So, human skull is the natural sandwich beam and. So, these are the carbon fiber faces one can use this honeycomb core. So, this is section of the helicopter blade where one can use this sandwich beam similarly this is a balsa wood core kevlar face also this is foam face with aluminum. So, foam core aluminum face and one can use this magnetorheological elastomer also.

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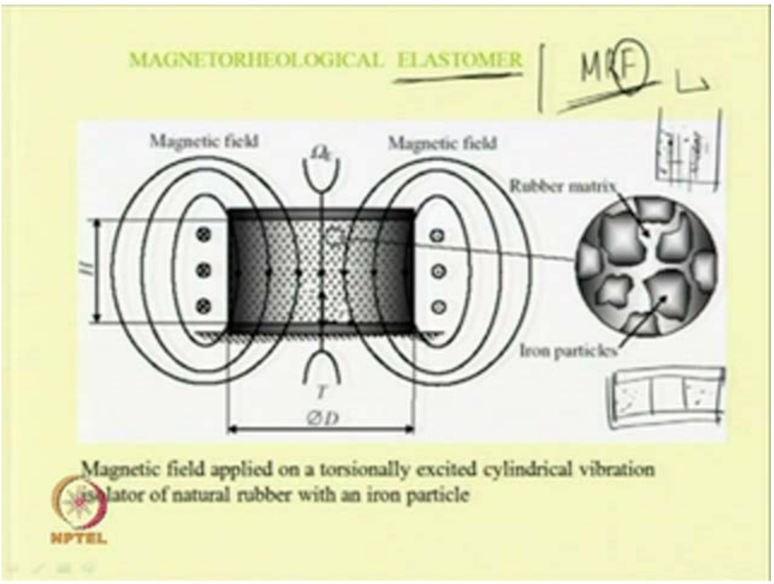

So, as the core material. So, in case of magnetorheological elastomer. So, so in this either in natural rubber or in the silicon rubber. So, one can add the Carbonyl iron particles and apply magnetic field during curing process. So, if one apply magnetic field then this iron particle will aligned during curing and they will subsequently increase the stiffness of the system. So, the elastomer one can note that this elastomer are solid substances unlike this magnetorheological fluid MRF. So, in case of magneto rheological fluid. So, which will act as a damper. So, this magnetorheological fluid will act as a damper. So, there in case of magnetorheological fluid. So, in this fluid this iron particles are embedded and then one applied magnetic field. So, then one apply magnetic field to align the iron particle the the iron particles will be aligned and in that case it will act as a damper also no a day’s some researchers are also working or using this magnetorheological fluid inside this sandwich structure. So, in the in between this viscoelastic layer one may use, one may use this magnetorheological fluid also.

So, in case of fluid, it will use as a damper by applying this magneto magnetic field, but in case of elastomer when this magnetic field is applied. So, it will increase the stiffness, but today class we will study only on the soft core magneto only soft core sandwich beam.

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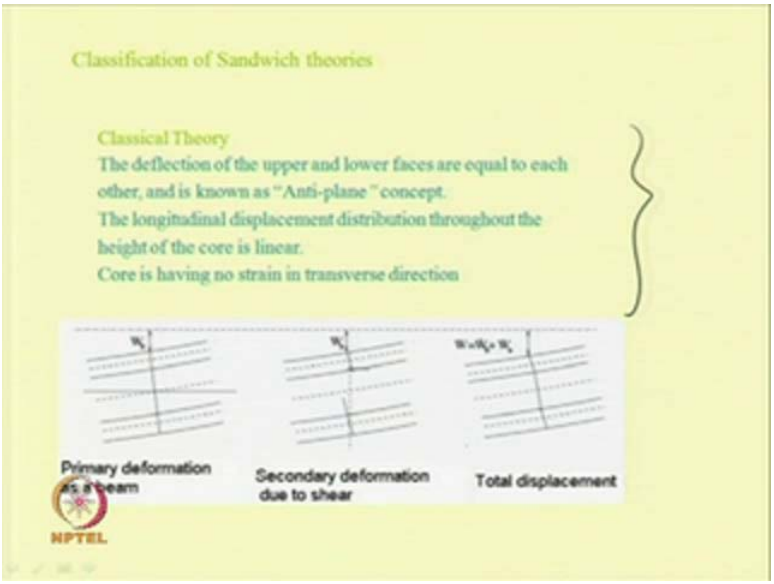

So, already I told you there are two mainly two different types of sandwich beam theories. So, one is the classical theory and second one is the higher order theory. So, in between may also use the superposition theory, but mainly this classical theory tells. So, this is the primary deformation and secondary deformation of a beam. So, this combination of deformation one can find this the total deformation one can find, in case of classical theory.

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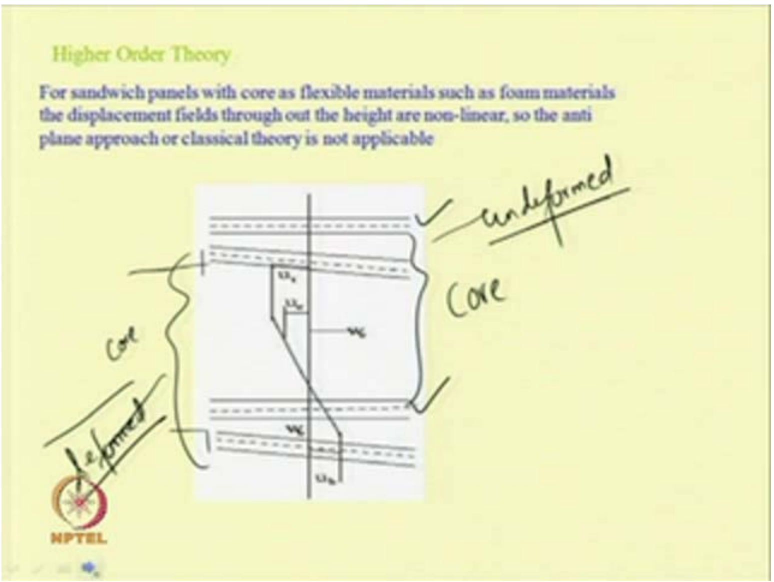

So, it is assumed that the top that the core is stiff and one can use the same transverse deflection for both top and bottom layer, but in case of. Higher order theory this is the this and this, this is this is the core part. So, this is the undeformed. So, this is the undeformed undeformed sandwich beam. So, now, let us take the deformed sandwich beam. So, this is this and this the deformed sandwich beam. So, here. So, this the core layer. So, this the core and this the deformed sandwich beam. So, in case of soft core. So, the top layer and bottom layer deflection will be considered to different. So, one can take this top layer deformation as w t and bottom layer deflection as w b and one can find the deflection of the core layer with respect to this top and bottom layer.

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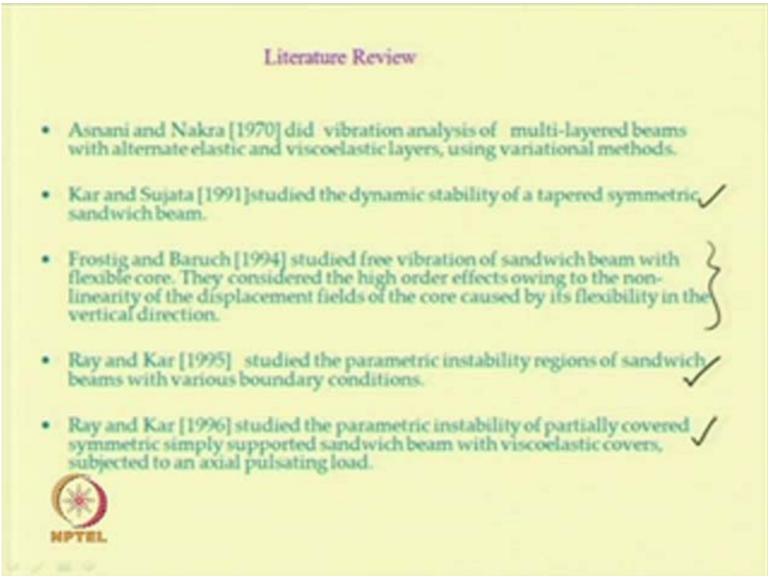

So, there are many literature available on this sandwich beam particularly one can see this paper by this Asnani Nakara or Sujatha Frostig Baruch. So, in 94 95... So, they have used this higher order theory incase of Kar and Sujatha. So, they have used core. Sujatha and Ray and Kar. So, they have used classical theory Frostig Baruach.

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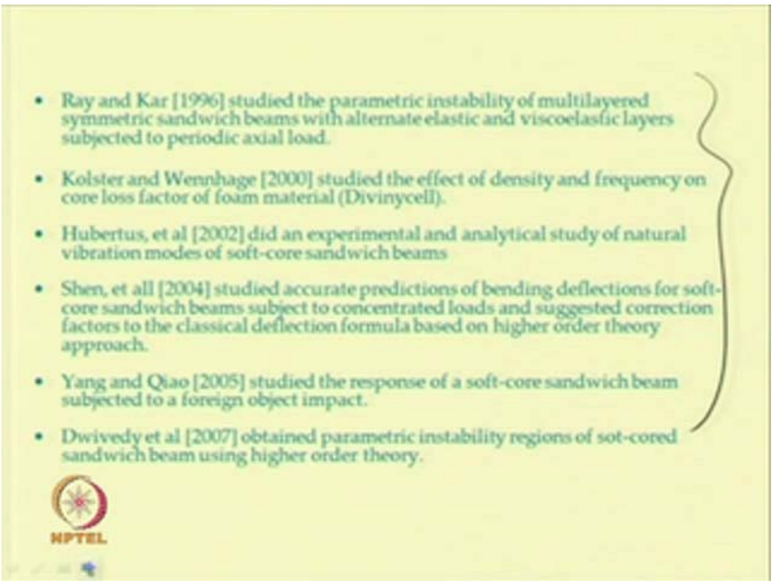

So, they have used this higher order theory also there are many other paper by other authors which considered mostly classical and high order theory for the sandwich beam.

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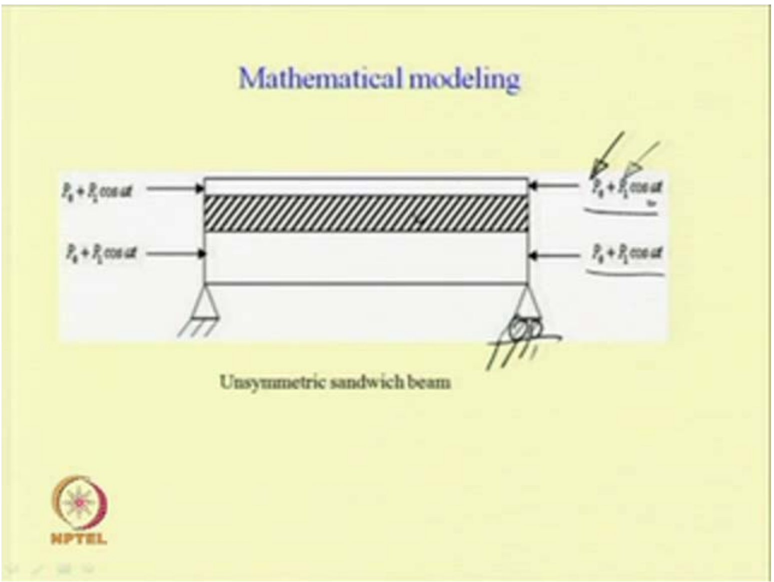

So, let us see the mathematical modeling of this sandwich beam. So, here a force P 0 cos omega t is applied. So, let us take unsymmetric sandwich beam with simply supported at the ends. So, it is subjected to axial force of P 0 plus P 1 cos omega t. So, P 0 is the static force and P 1 is the amplitude of the dynamic force with frequency omega. So, this is the core layer. So, we can consider both aluminum and steel for the skin and viscoelastic foam type core for this sandwich beam.

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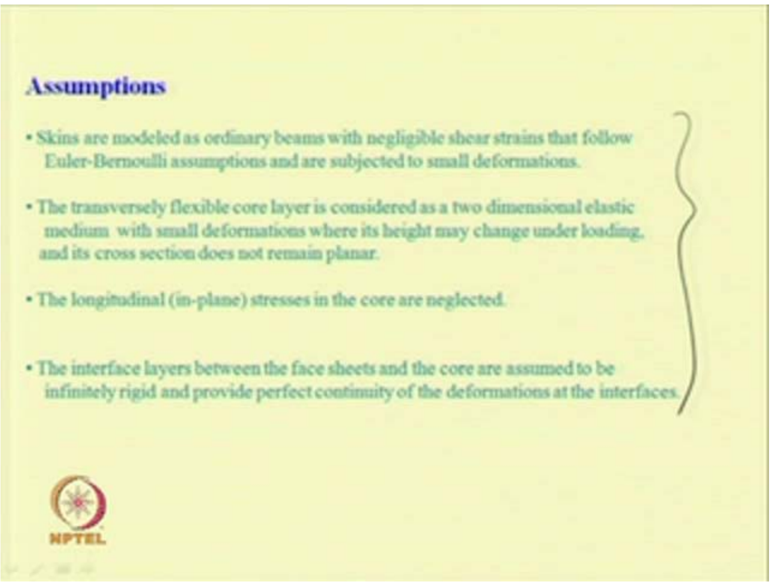

So, let us take this as the assumptions. So, skins are modeled as ordinary beam with negligible shear strains that follow Euler Bernoulli assumptions under subjected to small deformation the transversely flexible core layer is considered as a two dimensional elastic medium with small deformation where its height may change under loading and its cross section does not remain planner the longitudinal stresses in the core are neglected. So, the interface layer between the face sheet and the core are assumed to be infinitely rigid and provide perfect continuity of the deformation at the interface. So, taking this assumptions now we can derive the equation motion of the system by using extended hamilton principle and by writing the kinetic energy of the system and potential energy of the system.

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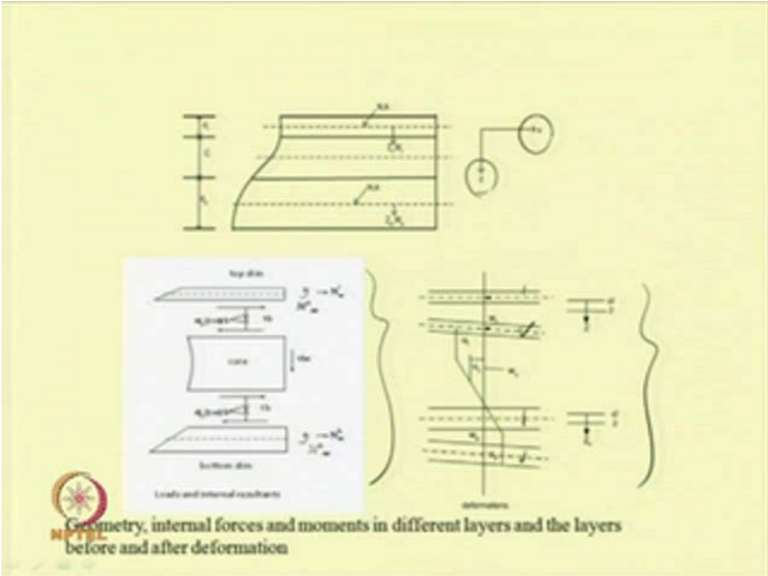

So, this is the sandwich beam considered sandwich beam. So, this is the neutral axis of the top layer and this is the neutral axis of the bottom layer. So, w t that is the deformation of the top layer is with respect to this neutral axis and w b also the transverse deflection of the bottom layer with respect to this neutral axis and one can write. So, this is the z direction and this is taking this as z direction and this as x direction. So, we have taken this d t as the thickness of the top layer d v as the thickness bottom layer let see if the thickness of the core. So, one can take this symmetric sandwich beam where the both top and bottom layer thickness will be same or on can take on one symmetric sandwich beam where the top bottom thickness will be different. So, now so, these are the. So, as we are considering only plain as we are considering Euler Bernoulli beam theory. So, it is subjected to pure bending. So, we are considering the case when the system is subjected to pure bending only. So, here the forces are shown forces and moments are shown in the top and bottom layer and. So, this is already it is shown the deformed and undeformed state. So, this is the deformed and undeformed and this is the deformed state. So, here u t and u b. So, at the axial displacement of the top and bottom layer. So, u c is the axial displacement of the core layer. So, one can write. So, this is the save before deformation and after deformation.

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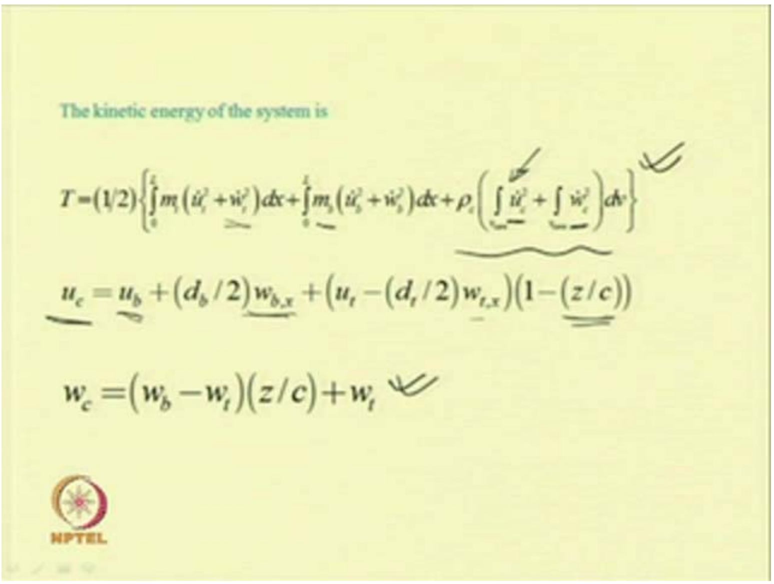

So, now, one can write kinetic energy of the system as the kinetic energy of the top layer plus the kinetic energy of the bottom layer plus the kinetic energy of the core. So, the kinetic energy of the top layer can be written as m t into u t dot square u t is axial displacement and w t is the transverse displacement similarly for the bottom layer can be written this way and for the core it can be written. So, rho c d b is the volume. So, that will give the mass and it will be the axial velocity. So, u t dot and this is the transverse. So, this is due to axial displacement and this is due to Transverse displacement taking the velocity term. So, one can write the kinetic energy of the system.

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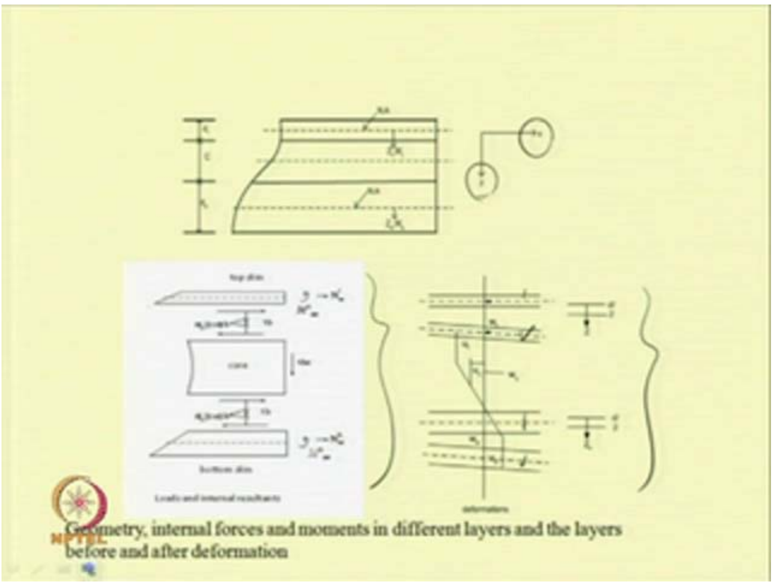

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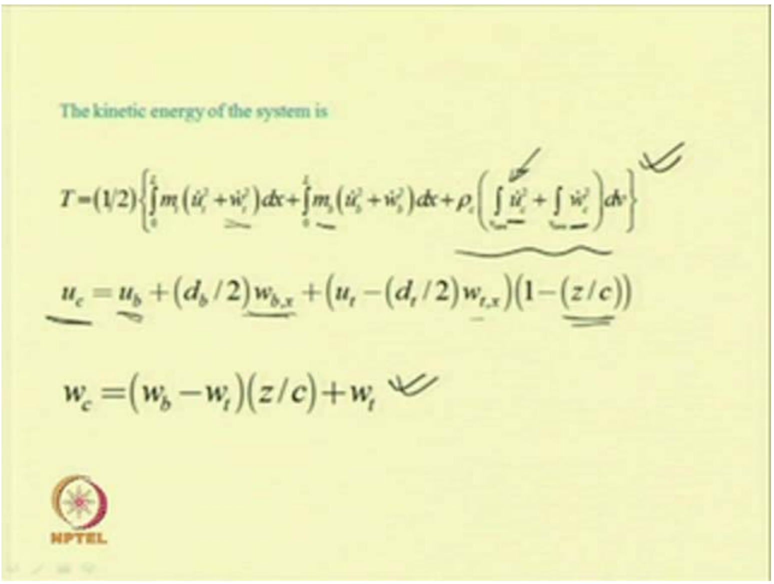

So, here from this figures one can write this u c that is the displacement of the. So, u c the displacement of the core layer as in terms of the bottom layer and top layer axial displacement. So, u c can be written as u b plus d b by 2 d b is the thickness of the bottom layer. So, d b by 2 into del w b by del x. So, this is del w b by del x plus u t minus d t by 2 del w t by del x in to 1 minus z c z by c. So, 1 minus z by c. So, z. So, at a distance z from the bottom layer. So, if we are finding what is u c that is the displacement. So, we can write the expression in this way similarly this w c w c that is the transverse deflection of the poor can be written in terms of the transverse displacement of the top and bottom skin. So, it can be written as w c equal to w beam minus w d into z by c plus w t. So, so it is w t plus the additional term this similarly the core layer axial displacement can be written as the displacement of the bottom layer plus this this is the additional term. So, writing this core axial Displacement and transverse displacement in terms of that of the top and bottom layer.

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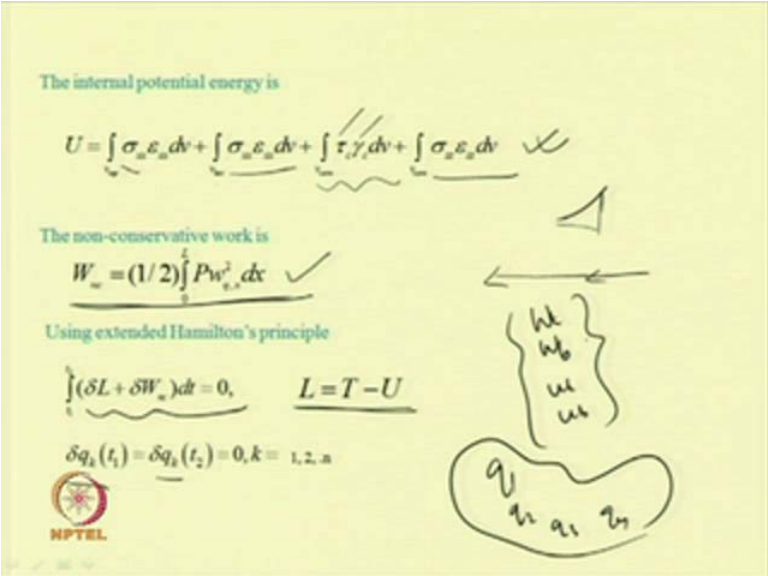

So, one can write or one can write this equation the for this kinetic energy of the system similarly the internal potential energy of the systems can be written using this stress and strain. So, for the top layer it is equal to sigma x x into epsilon x x into d v. So, this is strain energy into d v. So, that will give U the potential energy or internal energy for the top layer similarly for the bottom layer one can write and for the core as the as it is assume that the core is subjected to sear. So, as the core is subjected to sear. So, it can be written the strain energy due to sear can be written using this sear stress and sear strain and also due to this longitudinal stress and longitudinal strength.

So, the core is subject to both sear and longitudinal strain. So, one can write this total internal energy of the system by using the internal energy associated with the top layer bottom layer and due to the core now as the axial load is applied to the system. So, as we are applying a axial load. So, this is the axial load applied to the system. So, one can consider a small deformation of the system and taking this in extensibility condition. So, one can write the work done due to this load equal to half 0 to l integral 0 to l P into del w by del x whole square. So, in this way one can find the non-conservative work done and also the potential energy and the kinetic.

Energy and now by using this terms in the extended Hamilton principle. So, which is written as integration t 1 to t 2 delta L plus delta W n c d t equal to 0 where t L equal to T minus U. So, by taking this q k as the generalized coordinates which one is at t t 1 and t

2. So, one can find the equation motion. So, now, one can find a set of equation motion in terms of w t w b u t and u b. So, one can get. So, taking this q 1 q 2. So, will have 4 q q 1 q 2 q 3 and q 4. So, taking this four q generalized coordinates as w t w b u t and u b. So, one can find a set of equation governing equation motion. So, in terms of by taking the term which are which. So, when we are applying this Hamilton principle. So, will have this terms. So, integral t 1 to t 2 some term with delta w t d x d t.

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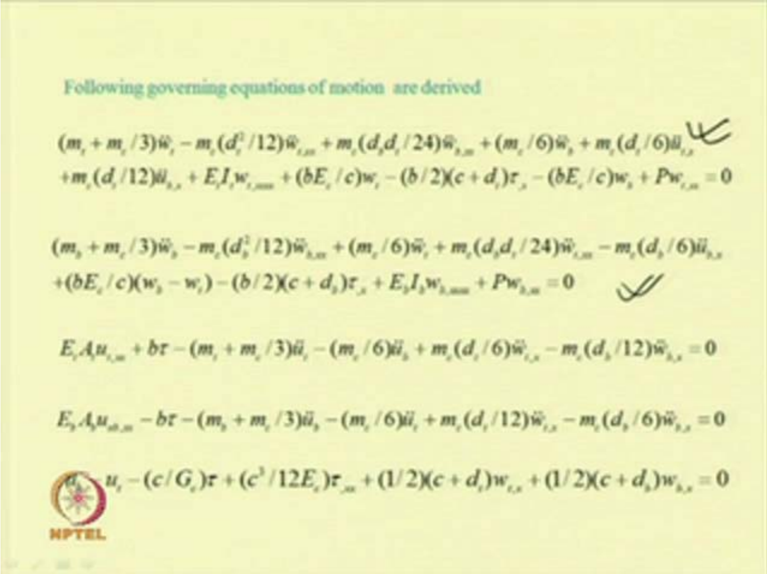

So, the coefficient of delta w t d x d t will be first equation similarly the coefficient of delta w b into d x d t will be the second equation similarly the term with u t and u b are u t double dot and u b double dot will be the other terms.

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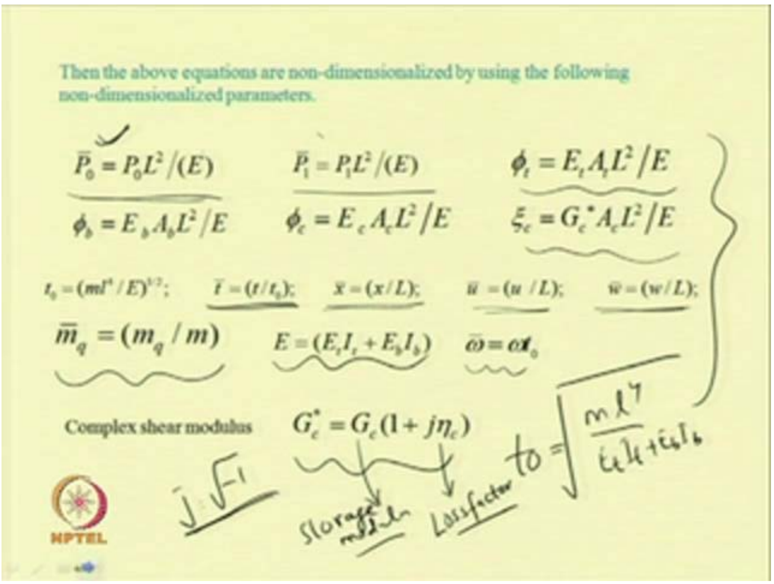

So, now, one can use these are the non dimensional parameter for example, one can use this non dimensional. So, this is static loading. So, which is equal to P 0 bar equal to P 0 L square by E where E equal to E t I t plus E b I b. So, E t is the young’s modulus of the top layer I t is the moment of inertia of the top layer E b is the young’s modulus of the bottom layer and I b is the I b is the moment of inertia of the bottom layer. So, so this will give the non dimensional static load factor. So, similarly P 1 bar will be P 1 L square by E where E is already it is described. So, this is the non dimensional dynamic load factor and one can non dimensionalize the time also. So, taking this time t 0. So, one can take this t 0 equal to. So, t 0 equal to m l forth by this E t I t plus E b I b root over. So, taking this non dimensional time. So, one can write this omega bar equal to omega t 0 and this mass ratio one can write m cube by m.

So, q. So, q can the top layer and bottom layer. So, m is the total mass of the system and one can take this I t equal to E t A t L square by E and zeta c equal to. So, zeta c equal to G c star A c L square by E. So, it may be noted that this for viscoelastic layer as the viscoelastic layer as the core is the viscoelastic layer. So, the sear modulus will contains the storage modulus and the loss modulus or one can write this using this complex number G c into 1 plus j into eta c where G c is the storage modulus and eta c is the loss factor. So, one can take this loss factor and this is the storage modulus storage sear modular. So, this is the storage sear modulus where j equal to root over root over minus 1 this is the these two represent the imaginary quantity. So, the coefficient of the sear modulus of the viscoelastic layer can be retained using this complex number G c into 1 plus i eta c and we are non dimensionalzing this x term that is the displacement x bar equal to x by L time non dimensional time t bar equal to t by t 0 u bar equal to u by L and w bar equal to w by L. So, using this non dimensional parameters.

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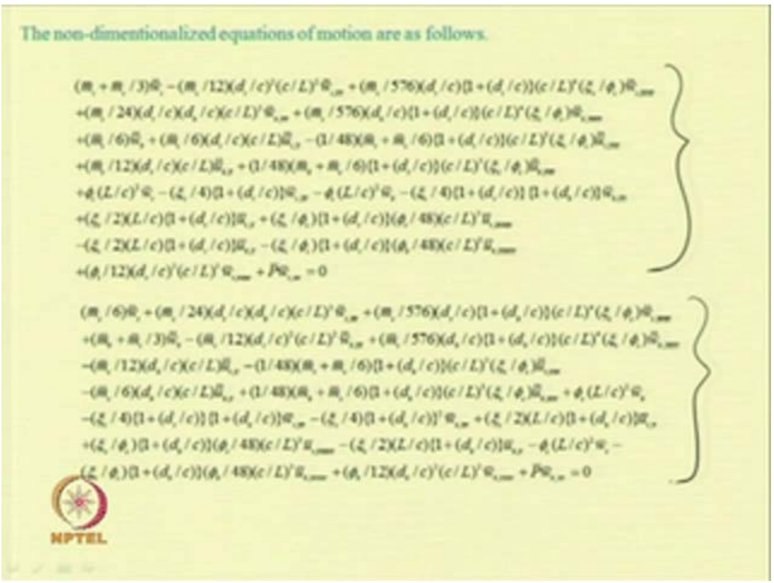

So, we can write the previous governing equations. So, in this forms. So, this is the equation in terms of w t. So, this equation contain start with the terms w b. So, w t w b and u t u b.

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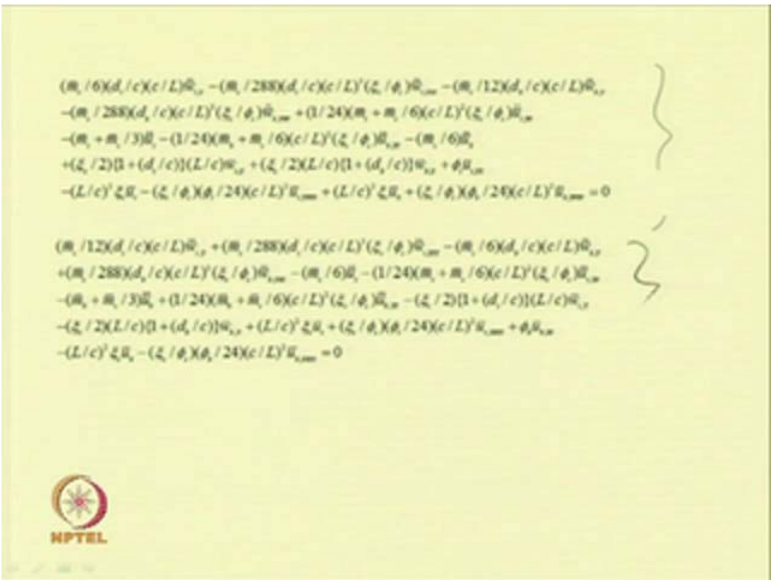

So, will have four equations. So, which are derived from this by using this Hamilton principle and using this non dimensional parameter.

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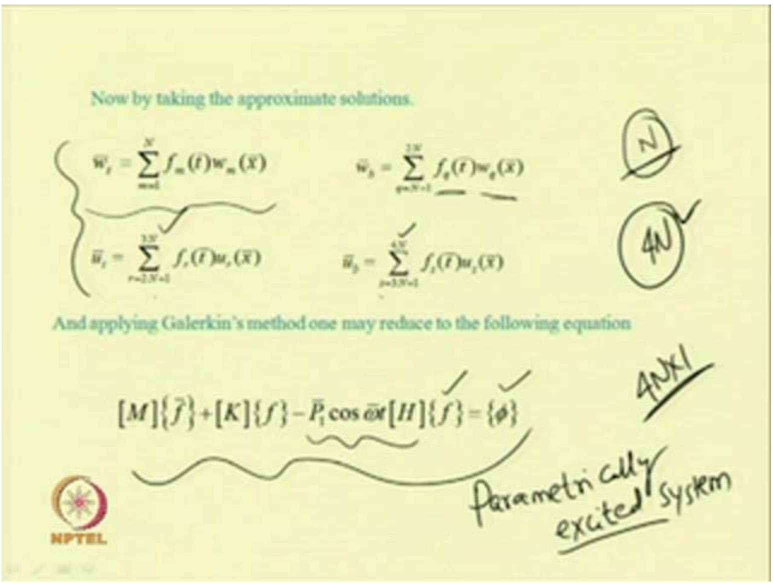

So, now, taking this w t in this form writing or taking few modes of the system. So, as this is a continuous system. So, one can have infinite degrees of freedom, but for analysis purpose we can truncate this system to a few finite degree of freedom. So, let us take first n degree of freedom of the system. So, by taking the first n mode of the system. So, we can write this w t bar equal to summation m equal to 1 to N f m t bar w m x bar. So, here f m is the time modulation and w m is the safe functions of the systems. So, the safe functions depends on the boundary conditions. So, we can see for or we can study the system for different boundary conditions similarly this w t w b bar can be written. So, w b bar can be written by using this f q and w q. So, one may note that here m is taken from 1 to N and next parameter we have taken q equal to N plus 1 to 2 N similarly for u t we have taken this parameter from 2 N plus 1 to 3 N and for u b we have taken this 3 N plus 1 to 4 N. So, will have 4 N state vectors present in this equation motion. So, we can have 4 N state vector. So, now, substituting this equation w t w t bar w b bar u t bar and u b bar. So, these equations are multi with multi modes. So, by substituting this equations in our previous equations.

So, this two and this two, this four equation, now, substituting this in this equation and applying Galerkin method. So, we can write this equation in this form. So, already we know. So, this equation is in the form of Mathieu hill equation, but here unlike in case of Mathieu hill equation which is written for the single degree of free system. So, this equation is written for a multi degree of freedom system. So, where this f contain. So, f is 4 N cross 1 vector. So, here we can have M as the mass matrix f as the time modulation K is the stiffness matrix P 1 cos omega t into f equal to pi. So, pi is the non vector P 1 bar already we know. So, this is the load parameter dynamic load parameter and omega bar is the external frequency acting on the system. So, here it may be noted that this term this P 1 cos omega t that this the time varying periodic term is a coefficient of this f f is the time modulation. So, which will be found which will be used to find the response of the system. So, as this term is a coefficient or parameter of this f term that is why this system is known as a parametrically exited system .

So, this is known as a parametrically exited system as, this is a parametrically exited system with. So, we have 4 N equations. So, this contain 4 N equations unlike the simple single degree of freedom system what we have studied earlier. So, in this case also we can non dimensionalize this mass and stiffness and this load vector and we can have this uncoupled equation which will be similar to that of a single degree of freedom systems then after non dimensionalizing or after uncoupling this equations when we get 4 N uncoupled equation then we can individually apply this methods what we have studied for finding the instability regions.

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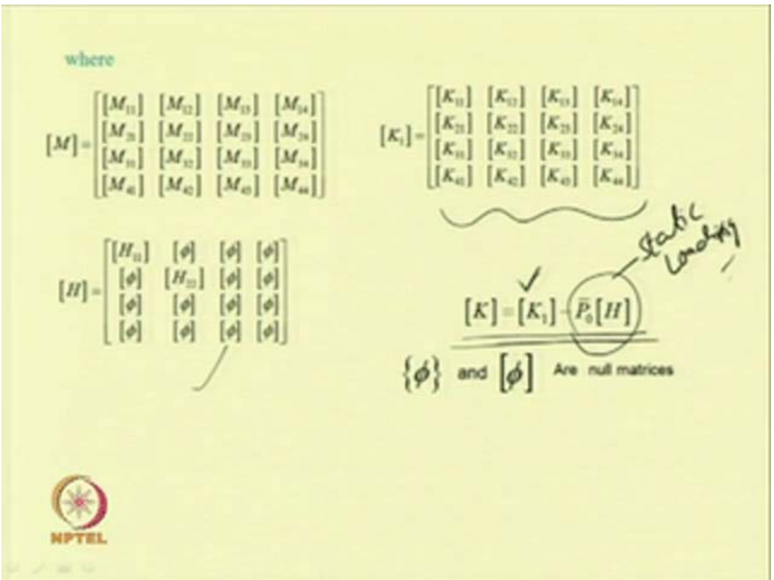

So, now, here this M can be written in this form. So, M equal to M. So, one can write this M 1 1, M 1 2, M 1 3, M 1 4, M 2 1, M 2 2, M 2 3, M 2 4 this correspond to this w t w b u t and u b so; that means, this will be w t double dot terms will be multiplied with this and w b double term. So, w u t double dot and u b double dot. So, in that ways, we have this each will be of 4 is to 4 and. So, we can each will be of. So, depending on the number of modes we have taken.

So, each sub matrix can be determined. So, similarly this stiffness can be written in this form K 1 1 K 1 2 K 1 3 K 1 4 up to K 4 4 mass matrix M 1 1 2 M 1 4 similarly this H contain this H 1 1 H 2 2 all other terms are pi; that means, null matrix and here it may be noted that this K that is the stiffness.

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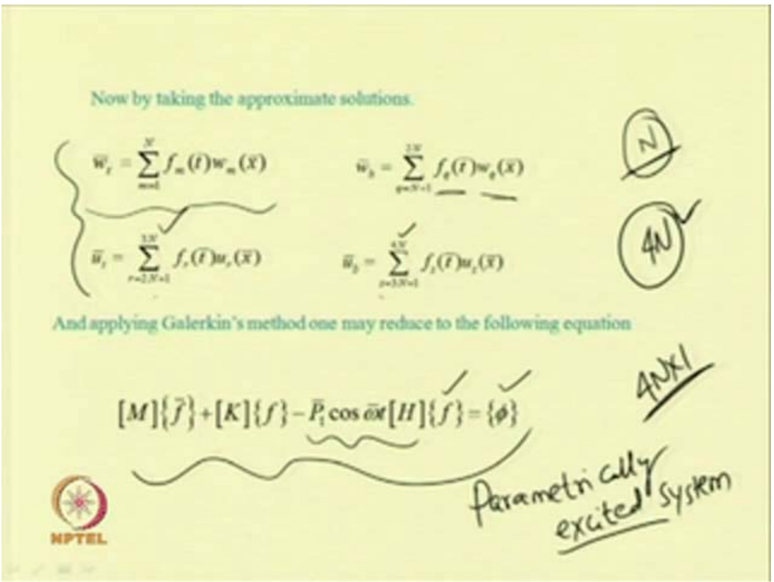

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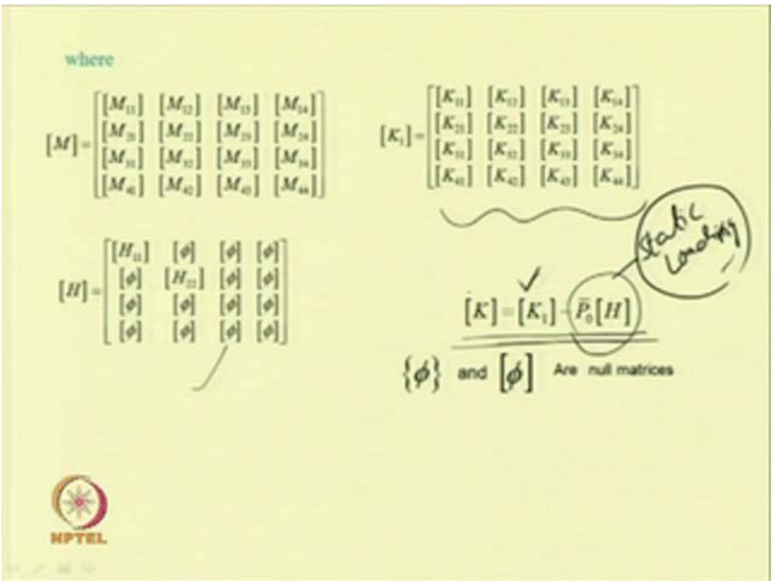

K which is coefficient of f is nothing, but it is equal to K 1 minus P 0 bar into H. So, it may be noted that this term. So, this term comes from the stiffness, but this term is coming from the static loading. So, static loading so; that means, the static loading influence. So, the static loading influence the stiffness of the equivalence stiffness of the system and one can find the equivalence stiffness of the system and one can find similarly one can find the model frequencies for this system. So, model frequency can be obtained by finding this this K inverse M.

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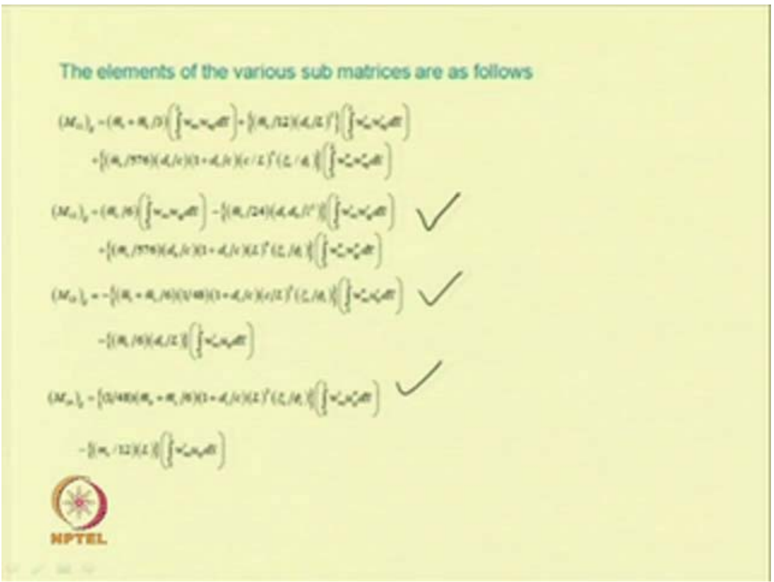

So, let us first see what are this terms what M 1 1 stands for. So, all these terms can be written for example, M 1 1 i j; that means, i equal to 1 j equal to 1 also one can take or this i and j vary from 1 to 4. So, this M 1 1 i j can be written as this is M t bar plus M c bar by 3 integration 0 to l w m t in to w m w m i into w m j into d x bar plus M c bar by 12 d t by l square integration 0 to l w dash m i into w dash m j d x. So, this dash is differentiation with respect to x similarly this is plus M c bar by 576 into d t by c into 1 plus d t by c into c by l to the power 4 into zeta c by 5 c into integration 0 t l w m i. So, this is double dash. So, single dash is differentiation single differentiation with respect to x and this double dash is double differentiation with respect to x similarly one can find M 1 2 using this expression M 1 3 i j can be found from this expression and M 1 4 i j can be found from this expression.

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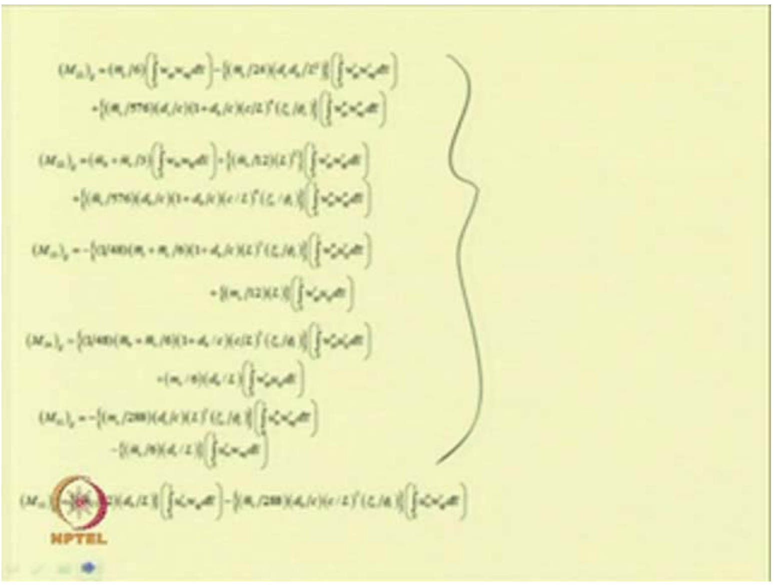

So, one can find this mass matrix using this expressions which are obtained by using method of multiple scale.

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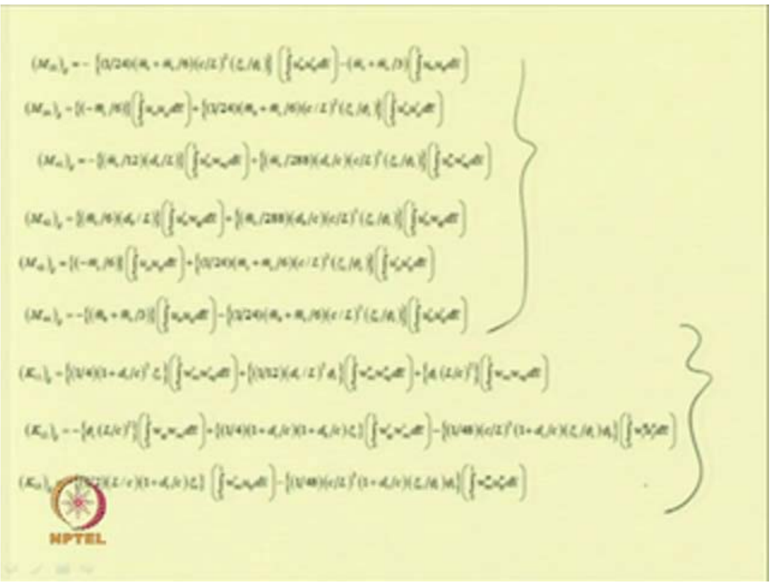

So, these are up to M 4 4 similarly one can find this K K 1 1 after finding K 1 1 K 1 2 K 2 1 K 2 2 then up to K 4 4.

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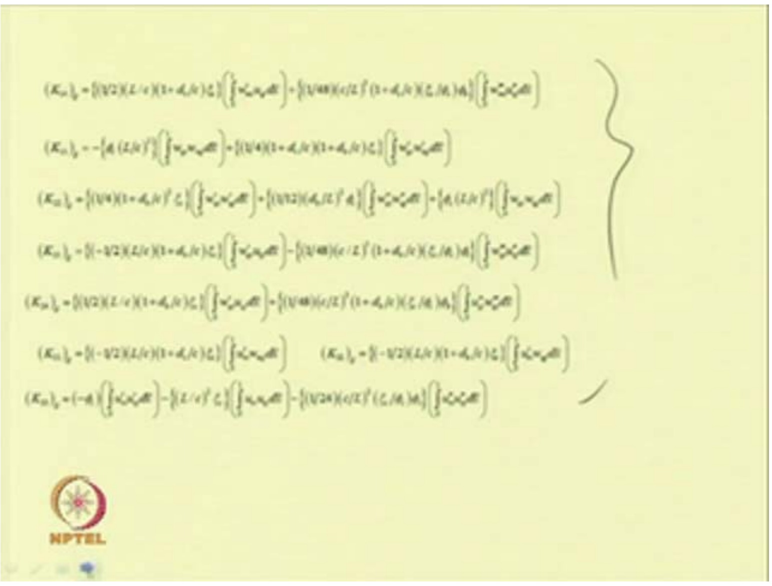

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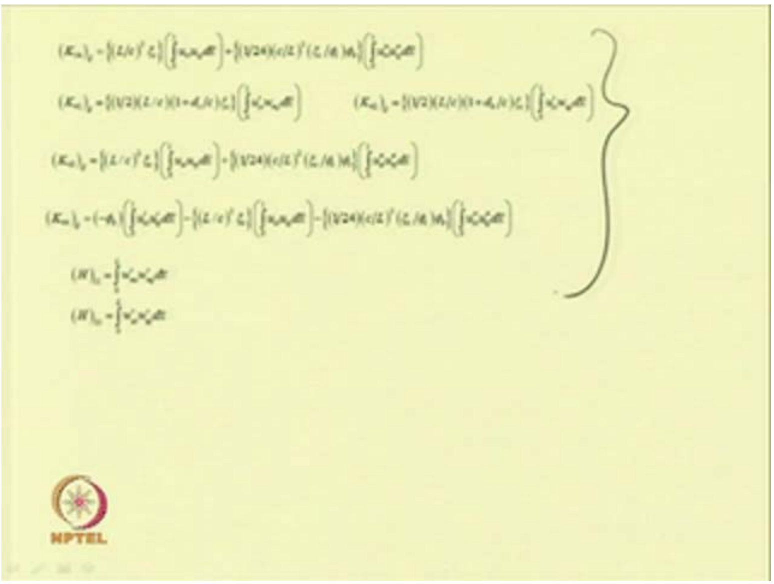

So, one can find this mass matrix and stiffness matrix by using. So, by using the load vector also. So, here H 1 1 can be written as w m i dash into w m z dash d x bar and this H 2 2 equal to integration 0 to l w q dash into w q j dash into d x bar here it may be noted that. So, we are using M.

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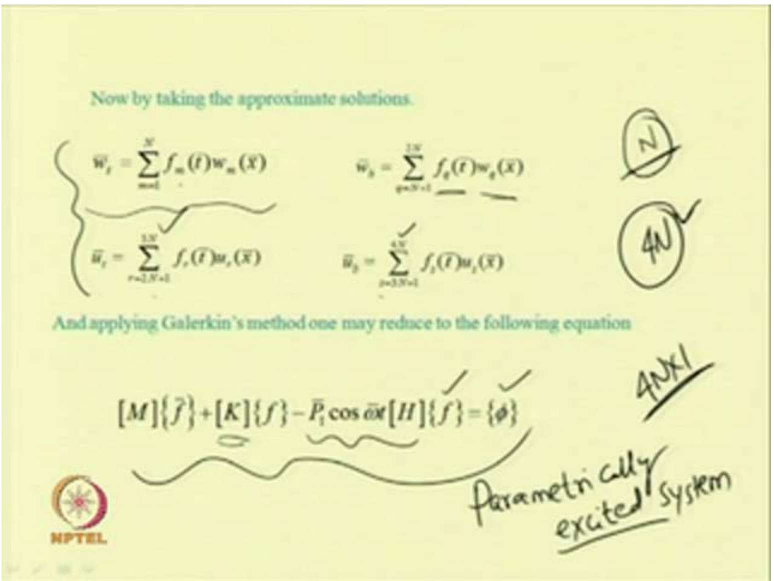

So, we are using from this one may note that we are using m for t that is is top layer q for bottom layer r for the. So, this m and q are for the transverse deflection and r and s for the axial deflection of the beam. So, in this way we can write the equation motion and find this coefficient by using this integrals.

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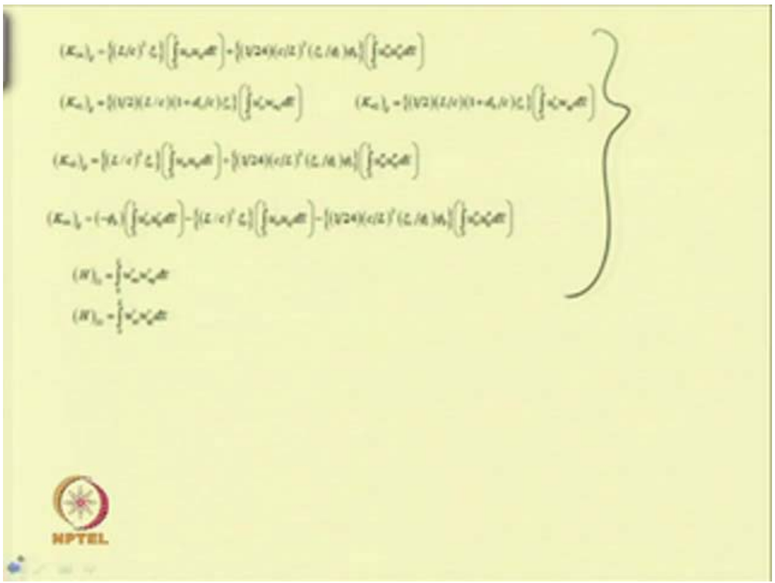

So, here one has to note. So, what is the expression for w t w b. So, the expression for this pi terms minus 2 c.

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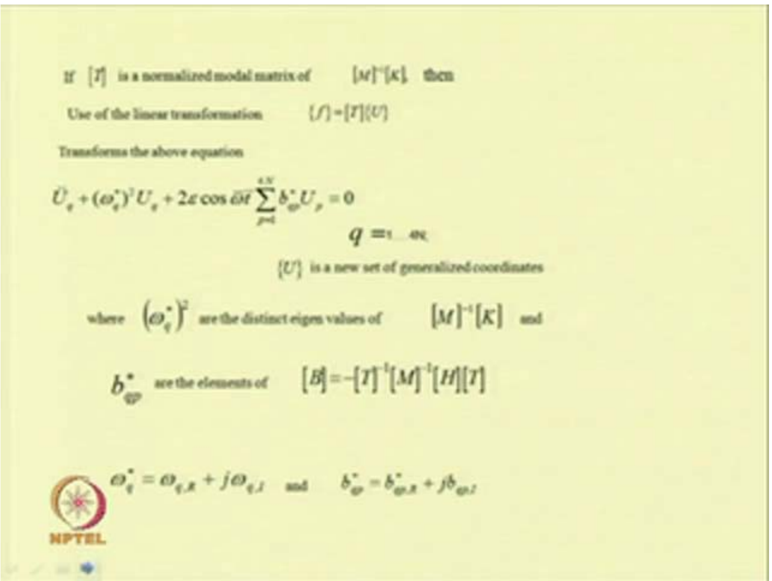

So, these expressions actually are obtained from the eigen values of the corresponding boundary conditions.

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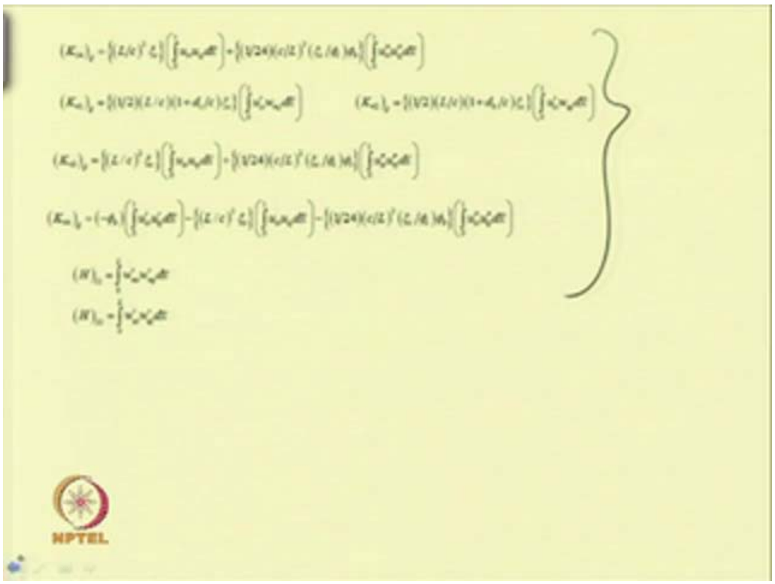

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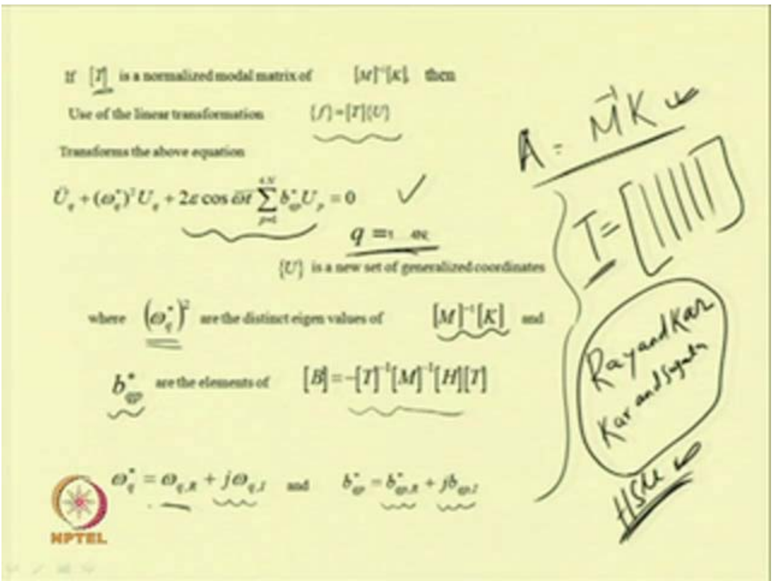

So, now one can after getting these equations or after getting this coefficient one can find this mass matrix and stiffness matrix by integrating this terms and after find this mass matrix and stiffness matrix one can find this T matrix that is known as the dynamic matrix. So, T is the dynamic matrix which can be written as M inverse K. So, this is the dynamic matrix now by finding the eigen value of this dynamic matrix that is M inverse K. So, one can get the eigen vectors and eigen values. So, for. So, taking this assuming these are distinct. So, one can use or one can transform the original equation to another form that this uncoupled form by taking this transformation. So, by taking f equal to T into U where T T is the normalized T is the A. So, let us A is M inverse K. So, we can find the eigen vector. So, T is the model matrix which contain the eigen vectors. So, these are the eigen vectors corresponding to different modes.

T is a normalized model matrix. So, if are using T as the normalize model matrix and then by taking this f equal T into U; that means, we are transforming this coordinate from f to U. So, then we can write this equation in this uncoupled form. So, we can write the original equation in this uncoupled form like this that is U q double dot plus omega square omega q star square q plus 2 epsilon cos omega bar t into summation p equal to 1 to 4 N into b q p star into U p equal to 0. So, where q equal to 1 to 4 N are the 4 N number of generalized parameters which depends on the modes of the system. So, U is a new set up generalized coordinates we are using here. So, in this way we are transformed this coupled equation for the sandwich beam to a set of uncoupled equation and in this uncoupled equation one can note that this term. So, this term the coefficient of U p is a periodic term that is why this equation is a parametrically excited system, but here we have a number of force forcing term associated with this term depending on the number of modes we are taking in this solution. So, now, to find the instability region. So, here we can follow the method developed by.

So, for Mathieu hill equation with complex coefficient and we have this case as also been used extensively by Ray and Kar and Ray and Kar. So, in their work they have used extensively this method also the work by Kar and Sujatha. So, this method developed by Sito and Otomi and. So, this method is developed by Hsu, but used for the parametrically excited system by Sito Otomi for viscoelastic or equation with complex coefficient and. So, in this case this w q star square which is the distinct eigen value which of M inverse K and b q star. So, this is a complex number. So, which is coefficient U p. So, here b q star are the elements of this B matrix.

So, square B matrix is nothing, but minus T transpose T inverse into M inverse into H into T. So, here we can write this w q star as w q real part plus w q imaginary part similarly b q star we can write as b q real part b q p real part plus b q p imaginary part. So, writing this eigen values and this b terms in their real and imaginary parts.

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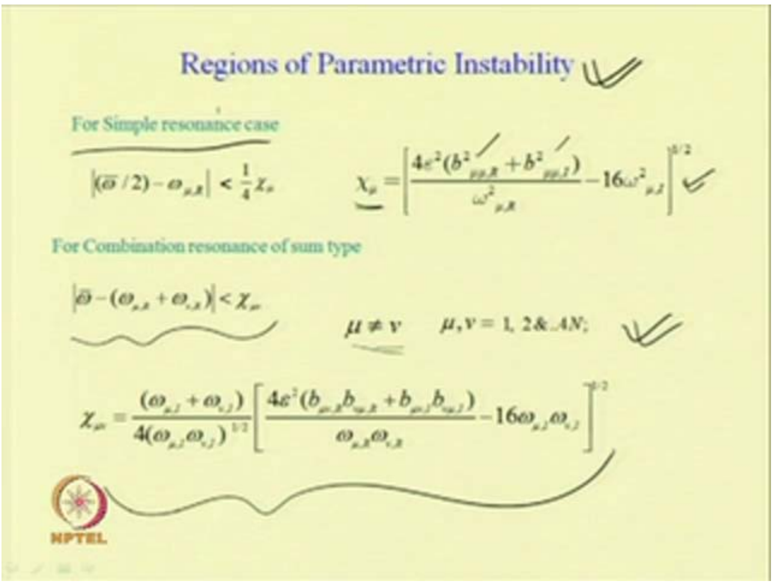

So, we can find the region of instability. So, for simple resonance case or simple resonance case or are the principle parametric resonance case. So, we can write this expression in this way; that means, that transition curve can be obtained from this equation that is w bar by 2 mod of w bar by 2 minus omega mu R less than equal to 1 by 4 x mu where x mu can be written in this way. So, this is equal to 4 epsilon square b square mu mu R plus b square mu mu I. So, this is the real part this is the imaginary part by omega square mu R minus 16 omega square mu I.

So, root over. So, this is for the principle parametric resonance case. So, here omega bar by 2. So, in this case it may be noted that the resonance will occur at a frequency twice that of the natural frequency that is why. So, one can take this omega bar by 2 minus omega mu mu R. So, this should be equal to for transition curve it will be equal to 1 by 4 x mu. So, taking or finding this mu value. So, one can plot the instability region similarly for combination resonances of some type. So, the equation is in this form that is omega bar minus. So, this is sum type omega mu R plus omega mu R mod of these should be less than x mu v. So, when mu not equal to v. So, this will be equal to. So, we can have this when mu equal to v we have the simple resonance case when this is not equal to this then only you will have this. Combination resonances of sum type.

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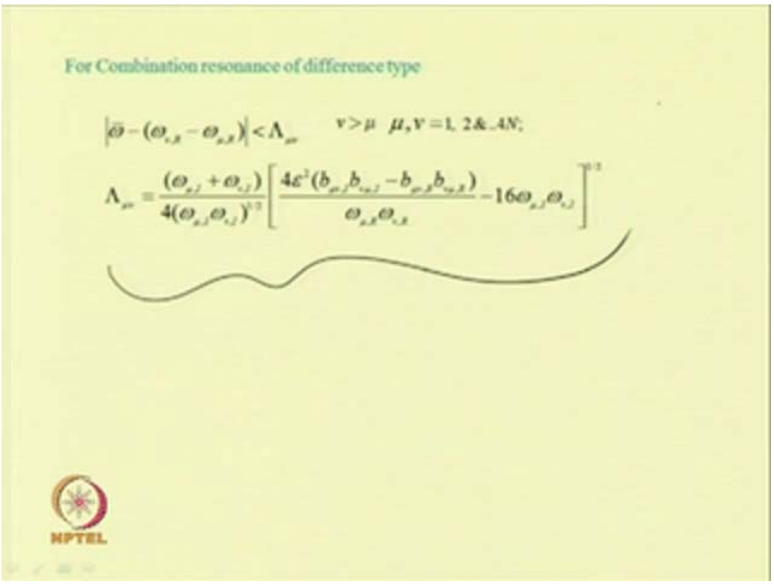

So, in this case this x mu v can be written in this form now for the difference type also we can have this expression. So, using this expressions now for. So, let us take a physical example and will see how will get the instability region.

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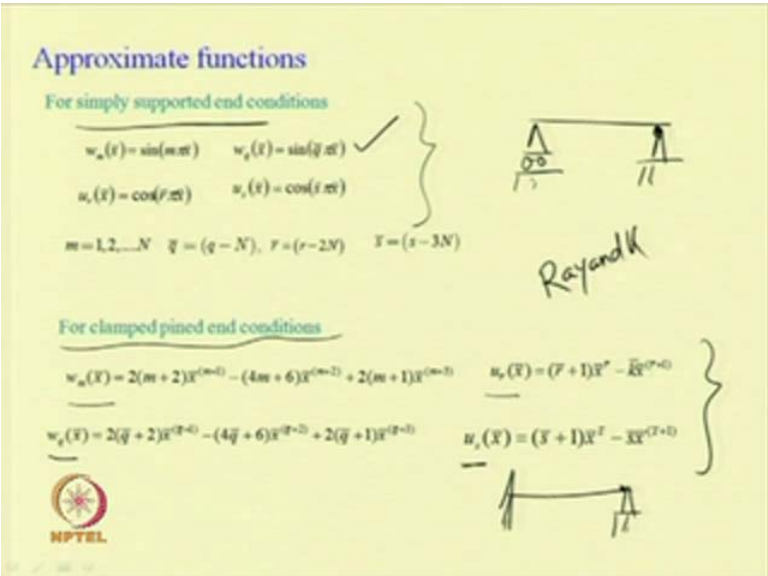

So, here we can take this w m w q u r and u s as the safe functions of the corresponding Euler Bernoulli beam with boundary condition. So, for example, in case of simply supported end condition. So, we can take this w m x bar equal sin m pi x bar similarly w q k bar these are the transverse direction thing. So, w q x bar equal to sin q bar pi x bar similarly u r x bar equal to cos r bar pi x bar and u s x bar equal to cos s bar m pi x bar. So, this equations will be used in those integrations to find this mass matrix and stiffness matrix of the system similarly one can use this equation for the clamped pined clamed pinned end condition. So, this is for simply supported simply supported means this end condition will be like this. So, this is simply supported and in case of clamped pinned. So, one side is clamped and other side is pinned. So, this for clamped pinned end condition. So, one can use this w m in this.

So, by using this polynomial one can use w m u r w q and u s. So, this equations have been followed from the work of Ray and Kar Ray and Kar paper. So, it has been followed from this paper on the sandwich beam vibration. So, they have used this classical theory to find the response of those the parametric instability region.

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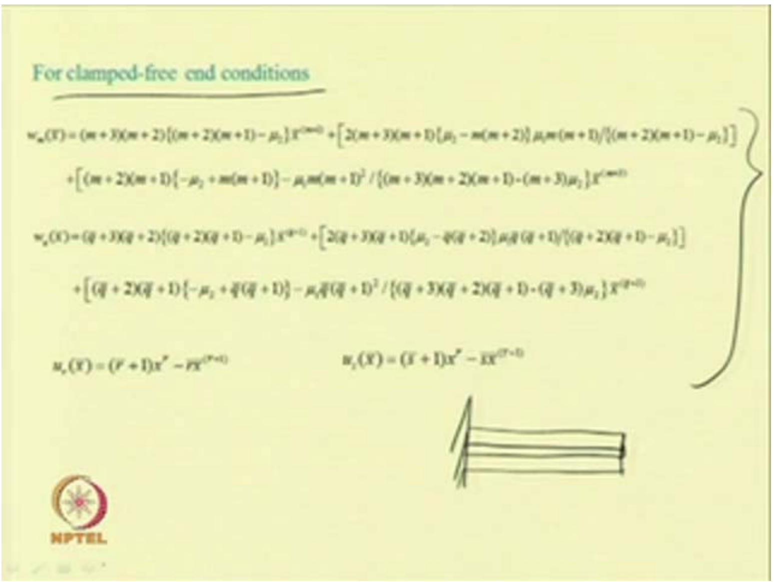

So, this is for clamped free end condition. So, in case of clamped free end condition. So, this is clamped and this is free or this is a cantilever type. So, in this cantilever sandwich beam this is a cantilever sandwich beam.

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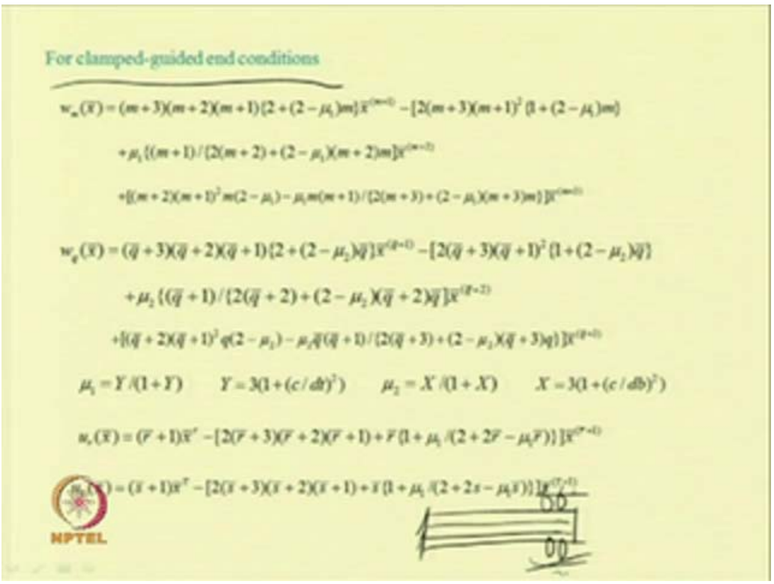

So, in this case one can use this equations and in case of clamped guided. So, one end can be guided also. So, in this case one end is clamped and other end. So, other end. So, we have a sandwich beam and other end is guided; that means, we have a guide this constrain to move. So, it will constrain to move in this guide.

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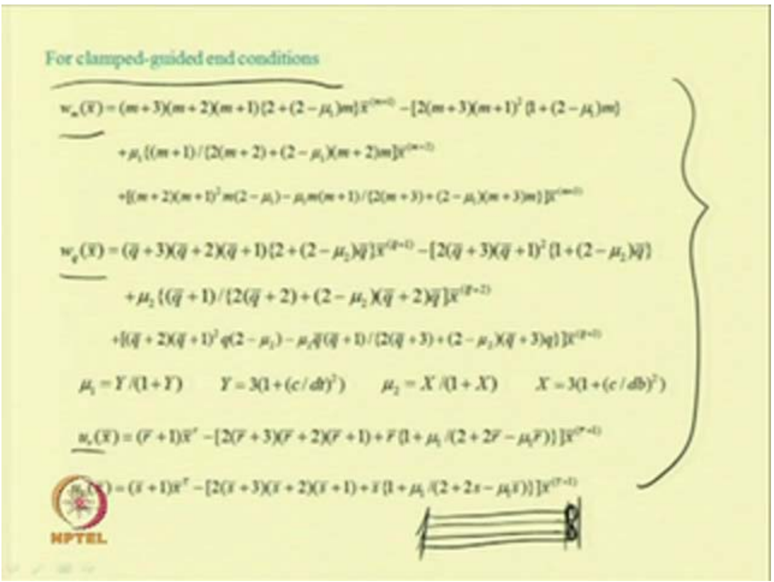

So, in that case we can have either we can have a guided this way or we can or we may. So, the guided condition we can have. So, it can be. So, it is constrain to move in this guide as we are considering the transverse vibration. So, it is constrain. So, this sandwich beam is constraint to move in this direction. So, we have roller support. So, this is for the clamped guided end condition. So, for the clamped guided end conditions you can use this w m w q and u s u r u s.

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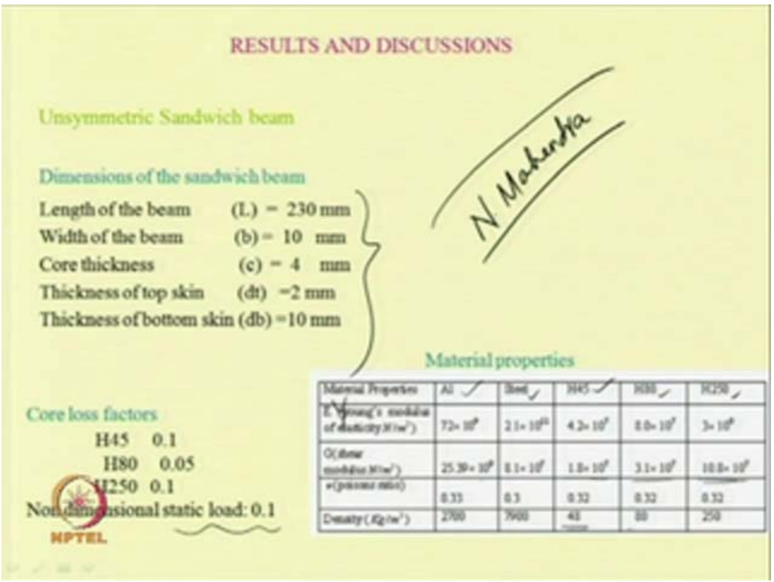

So, in this way and, let us say see some results. So, in this case this work is taken from the work by N Mahindra a MTech student of mechanical engineering department IIT Guwahati. So, this numerical results and discussion have been taken from his m tech thesis and this work also has been published in some conference and journals. So, this is for a unsymmetric sandwich beam. So, the length let us take this length width core and thickness in this way and here three different types of core has been taken. So, one is H 45 from core H 80 and H 250.

So, whose loss factors are taken .1 .05 and .1. So, the non dimensional static load factor is taken as 1. So, these are martial properties for aluminum steel and H 45 H 80 and H 25. So, young's modulus. So, young's modulus and then sear modulus sear this is the storage modulus has been taken. So, it may noted that is the storage modulus or this martial are taken very less. So, coil for storage modulus for aluminum is of the order of 10 to the power 9. So, here it is of the order 10 to then power 7. So, the density one can see the density of this H 45 is near to 45 k g per meter cube similarly H 80 it is equal to 80 k g per meter cube and for 250 it is equal to 250 k g per meter cube.

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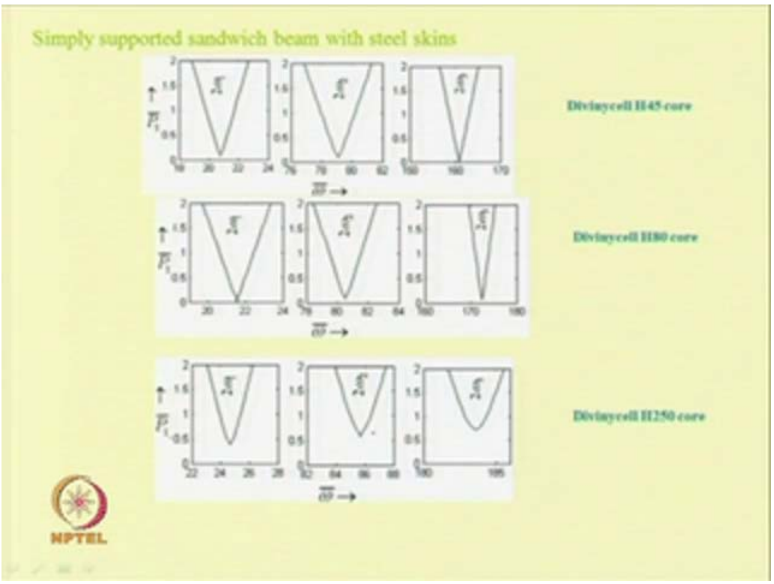

So, taking these values, one can plot this instability region. So, this are for this simply supported case. So, for different case and for different core and martial one can find for different end conditions one can find these instability region.

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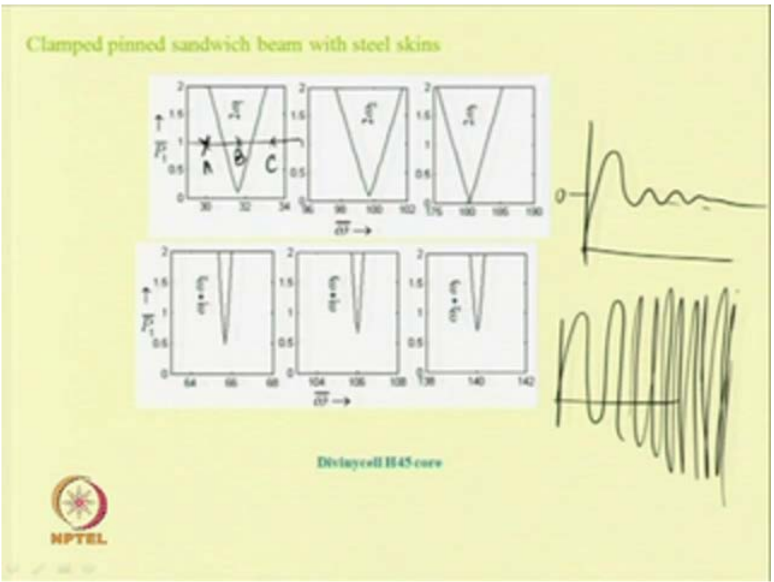

For example, let us take this instability region here. So, here if one take a point here here and here. So, this point corresponding to A. So, if one plot the time response. So, it will lead to a stable condition. So, it will finally, lead to stable condition which is 0 trivial state similarly for this C also it will lead to trivial state and for one can take B then the response actually will grow. So, the response will grow and finally, the system will be unstable. So, this way one can find the instability region for different types of core and for different end conditions.

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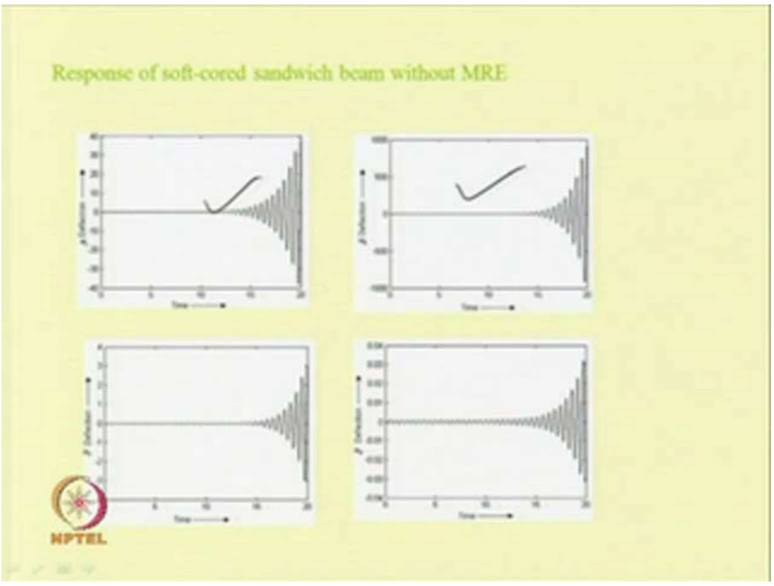

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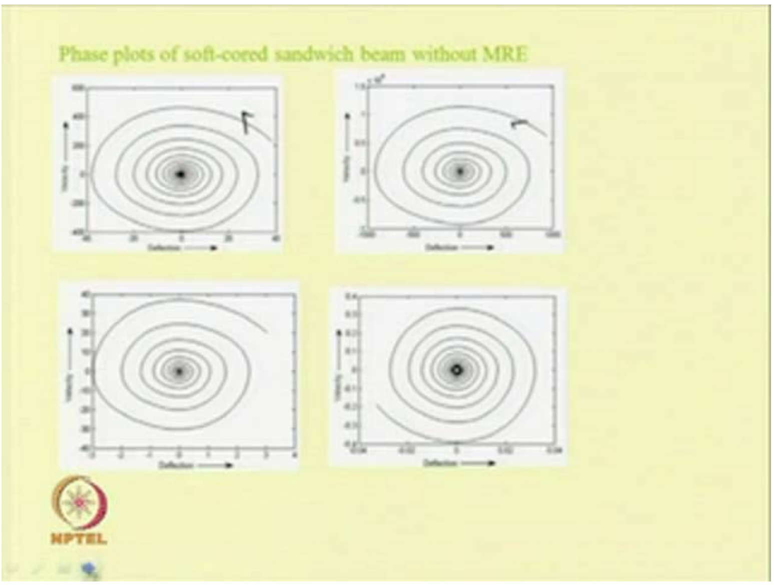

So, this has been shown in the, in case of unstable region. So, in case of stable region, it will be it will come to this 0 position in case of stable and in case of unstable it will grow and finally, one can have unstable region.

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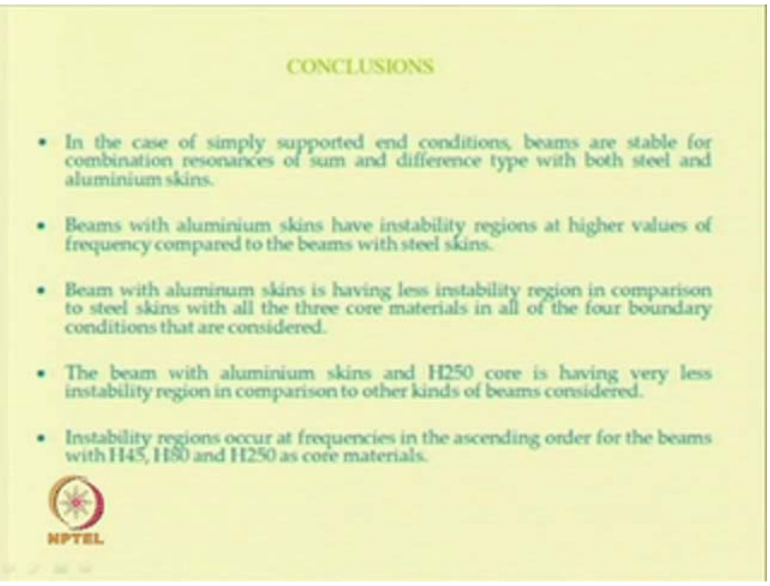

So, in this way one can study the parametric instability region and one can obtain the response of the system by using the equations developed here in this or the equation used in this work also one can take the different type of core material for example, one can take this magnetorheological elastomer core for finding the instability region also the skin martial can may be changed instead of taking metallic skin one may take this composite skin or functionally graded skin to find the parametric instability region of the sandwich structure. So, this is one of the application of the parametrically excited system we have see today. So, next two class also we study two more applications. So, in tomorrow class will study about the parametric instability region of an axial loaded elastic beam and elasto magnetic beam and in the last class or in the third class in the series will study the parametric instability region with internal resonance of the system.

Thank you.