Non-Linear Vibration

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Module - 6
Applications
Lecture - 7
Nonlinear Forced-Vibration of Multi-Degree-of-Freedom System

Welcome to today class on Non-linear vibration. So, in the last few classes we are discussing about the application of the non-linear vibration systems, what we have studied in our earlier classes. So we are discussing about the response of the non-linear systems and their stability, so initially we have studied about the free vibration response of non-linear systems, now we are continuing with force vibration of non-linear systems in which we have discussed about the single degree of freedom systems and multi degree of freedom systems also.

So, in multi degree of freedom system, so we have just discussed about a few degrees of freedom system, but now a days many systems which can be obtained by analysing the finite element methods will contain a large degrees of freedom system, and the matrix size or the equation motion will be n-dimensional or it will contain large number of equation motions. So in that equation motion if non-linear terms are present along with the boundary conditions, and also the loading vectors, then analysis of those systems cannot be, cannot discuss the analysis of those systems by the systems what we have studied before. So, in such cases, so we to use some model reduction method or model reduction method and we have to discuss some other techniques to solve those systems.

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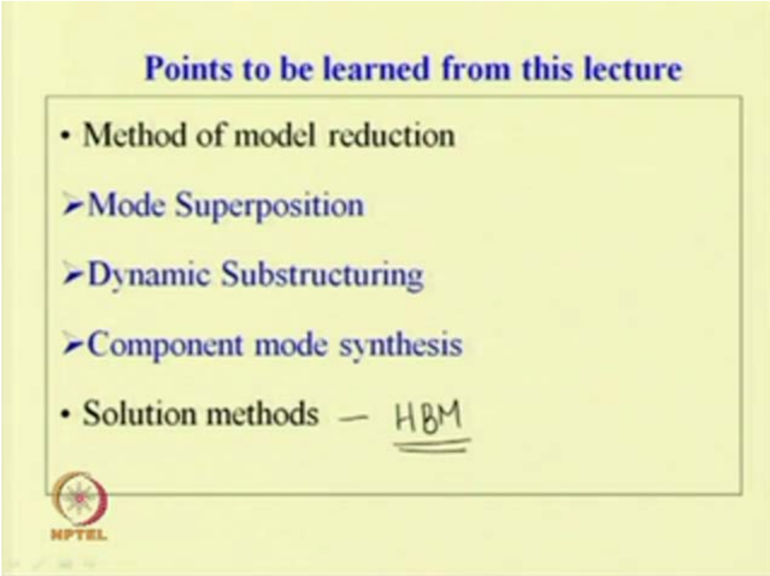

So, today class we are mainly focusing on the study of the multi degree of freedom systems where will discuss about the method of model reduction, then so in which will discuss about this modes super position method, dynamics of structuring method and component modes synthesis and the solution will see by using this harmonic balance method. So will use this harmonic balance method, what we have discussed before to find this solution of the system. So this harmonic balance method can be used along with these modes per position method, also it can be used with this dynamics of structuring and component modes synthesis method.

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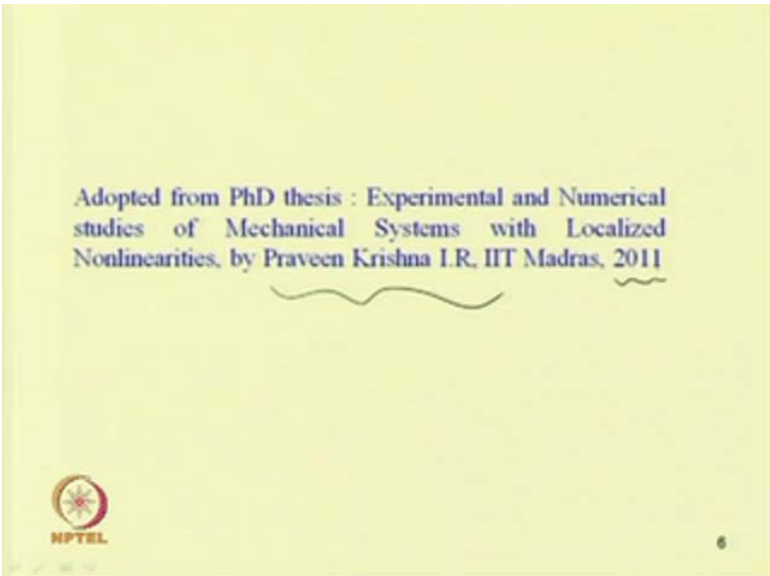

So the analysis what we are going to study today, so it is adopted from the PhD thesis of that is “Experimental and Numerical studies of Mechanical Systems with Localized Nonlinearity” if which is carried out by Doctor Praveen Krishna I.R of IIT Madras which is submitted in 2011. So if we have a system multi degree of freedom system with several nonlinearities and also boundary conditions and external forcing conditions then in that case we can use this finite element method to write the equation motion.

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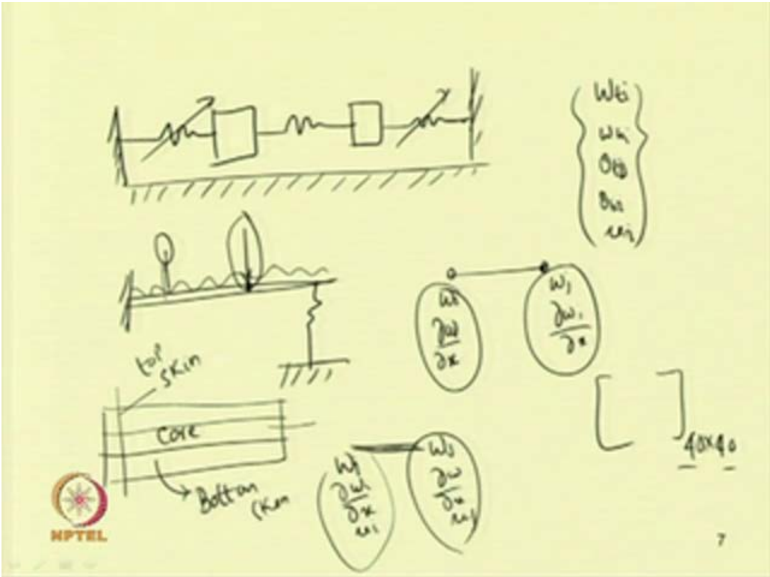

For example let us take, we can have many different type of structures, also this systems can be used for solving the system with localized nonlinearities. So we can let us take a simple structure what we have seen in the last class also, where we have taken a simple spring mass system, so this simple spring mass system. So here we can make this spring non-linear and so this spring can be made non-linear this spring can made or all the springs can made non-linear and in that case and this nonlinearity may be of quadratic or cubic type. So in that case we can write this equation with 2 degrees of freedom system only. But, if we have a system; let us take a continuer beam, so in this continuer beam, so let it supported by some spring.

So in this case or so it is supported by some spring and applied by some load so in this case this continuer beam as it is a continues system, so this can be modelled as infinite degrees of freedom system. Also one can take infinite formulation, so for transfers duplication of the system one can so for each node if take if one taken beam element then for each node one can take this displacement and slop slope as degrees of freedom. So for the i th node so one can write this the displacement, displacement i and slope that is del w by del x so at i similarly, so it will be at j node w j and del w j by del x, so displacement and slope one can take as the degrees of freedom at this node.

So for each element it will have 4 degrees of freedom, so taking several elements, so one can let, if one take 10 elements then the assembled matrix will contain assembled mass matrix stiffness matrix will contain 40 is to 40 or the size will be 40 is to 40. Similarly, if one can take a sandwich beam, so let us take a sandwich beam, so in which will have so this is the skin layer and so this is the skin layer; if we are considering only 3 layer this is the core and this is the. So, this is the top skin core and this is the bottom skin so in this case one can take the classical theory.

So if one can take the classical theory then one can write the displacement of top bottom and core to be same in there transfer direction at a particular section. So one can write or if one takes the high order theory where taking a flexible core, if the core is flexible then in that case the displacement of the top and bottom layer transform displacement of top and bottom layer will be different. So in those cases, so when one is taking classical theory so one can take for 1 element, so one can take this W as the deflection W at the W i is the defluxion then del w by del w i by del x at the slope then one can take this u i as the u i is the actual displacements.

Similarly, for the j th node one can take w j del w by del x or theta j w j theta j and u j, so in this case one can have one can for element one can have 6 degrees of freedom and taking number of elements so it will multiplied by this number and one can get a matrix or elemental matrix or or the assembled matrix of flower size. Similarly, in case of the flexible course sandwich beam so at the top and bottom skinned defluxion will be different, so in that case at the i th node one can have W t, W t then so one can write this W t W b then theta so 1 theta t theta b and then u is so these are at the i th node. So in case of I, so it will reduced to so at the i th node one will have this 4, 1, 2, 3, 4 and 5 degrees of freedom. Similarly, at j th one can have this 5 again 5 so 10 degrees of freedom system or element and then in that way taking several elements.

So one can find the number of equations motion a large number of equation motion, so when one has a large number of equation motion so in such cases; so one has to use these model deduction technique. So in this case in case of the continuer beam so if one use this Euler-Bernoulli beam theory only when write. So the equation is liner but taking a large deflection one can have non-linear nonlinearity representing the system. Also when taking a force, so this force may be periodic or a-periodic so if one take this force at this node so this force may be concentrated or this force also may be, so this force may be uniform distributed or the force may be so one can have a moving load type of force. So this force may be that of the moving load so in all such cases one has to write the equation motion with this external force taking into account.

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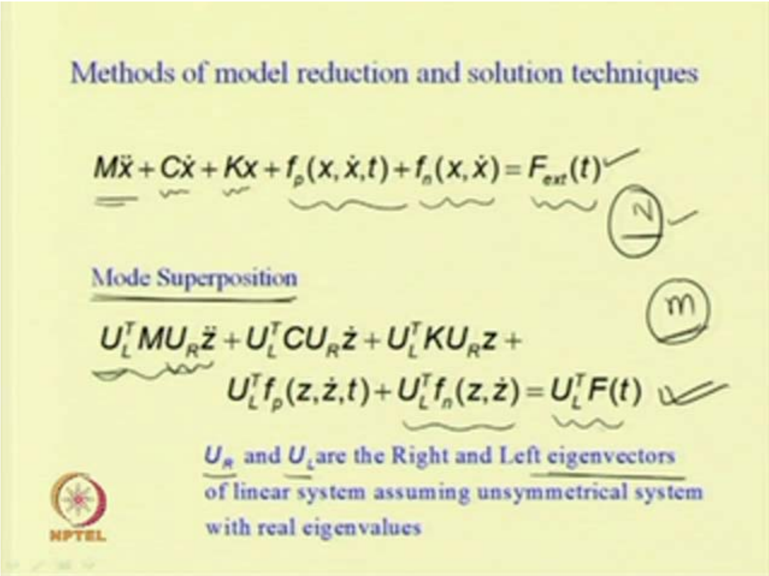

So let us have a generalized equation motion. So this generalized equation motion in such case can be written in this form m x double dot plus c x dot plus k x plus f p f p is the force containing these x x dot and time varying force. So this may be that of a parametric excited system also, when it it is functions of time and x then this the non-linear term. So one can write this non-linear term also, with this, so this the external force. So this is the external force acting on the system, this is the non-linear force we are taking and this is the localized force localized parametric excitation force, which is defined in terms of displacement velocity and time and then this is the stiffness. So this is the force due to stiffness and this is the dumping force and this is the inertial force of the system. So these are the forces acting one the system and one can write the equation motion by using any method including these finite element method, so this equation can be of n cross n where n can be, n can take any dimension.

Now this n dimension equation where this mass matrix stiffness matrix and dumping matrix are of n cross n size and these are vectors with n cross 1, so one can write now using these modes super position method which is this simplest one reduce the size of this equation. So this equation can be reduced to that of order m where m is very very less then n by using these right and left eigenvector of the linear system taking the liner system m x double dot plus k x equal to 0 and a equal to m inverse k so one can find the eigenvalue, if the matrix are unsymmetrical one can find these left and right, right and left eigen values and eigenvectors one can find these left and right eigenvectors of the system corresponding to the eigen values. So after finding these left and right eigenvectors, one can use those things in this equation so taking this x equal to U R Z taking x equal to U R Z and substituting in this equation. So one can write M U R Z double dot plus C U R Z dot plus K U R Z plus then other terms equal to so this f t. Now pre multiplying that thing by U L transpose, so we have this equation so by pre multiplying this equation so we have the equation U L transpose M U R Z double dot plus U L transpose C U R Z dot plus U L transpose K U R Z plus U L transpose f p z z dot t plus U L transpose f n z z dot equal to U L transpose f t, so here one can note that when the matrix is symmetric, so U R will be equal to U L.

And if it is asymmetric, so will have this left and right are, left and right eigenvectors. So one so from the properties of this eigenvectors, so one can know that these will reduce to so by taking let by taking let us take m mod, so if you are taking m mod then it will reduce to size m cross m. So by taking only first few modes let us take only 2 modes then this equation will reduce to that of 2 is to 2. But, the disadvantage of this thing is that so by calculating while this matrix is left side it is reduces to so these 3 reduced to 2 is to 2, this forcing vector, these non-linear terms and the forcing vector will so during simulation so one has to find these modes and it will it will rotate between or it will rotate between the physical coordinate and the model coordinates. So for these non-linear terms this forcing terms so when we are multiplying this thing so this transformation, so these transformations now will contain the model vectors, so this the disadvantages of this method and in but in this way we can reduced, we can reduce the n dimensional equation to m dimensional equation. So the non-linear function forms a few physical coordinates to all non-linear coordinates. So it will move between this model coordinate and bet between the physical coordinate and the model coordinates.

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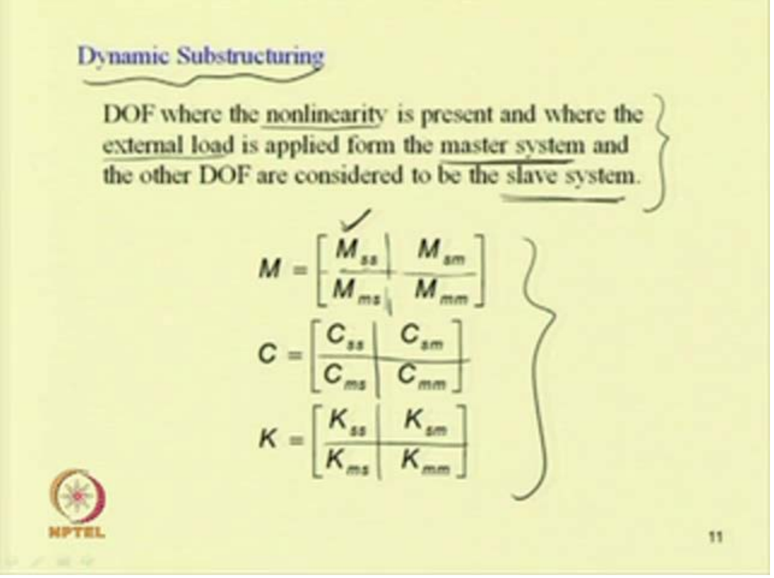

Now we can use another method which is known as dynamics sub structuring, so in which we can divide this mass matrix stiffness matrix and dumping matrix by using the slave and master degrees of freedom. So here we can use the degrees of freedom where the nonlinearity is present and where the external node is applied we can take those nodes or we can take those degrees of freedom as the master degrees of freedom and all other degrees of freedom as the slave degrees of freedom. So we can we can take the degrees of freedom where the non-linear terms are present, where the nonlinearities is present and where external nodes are applied as the master degrees of freedom and all other degrees of freedom as slave degrees of freedom by taking that way. So we can write the mass matrix, dumping matrix and stiffness matrix these ways we can partition this mass matrix like this.

So we can partition the mass matrix as M s s that is which slaves then M s m and M a m s and M and m m. So this contain all the master coordinate or master degrees of freedom this contain all slave degrees of freedom and this a coupled between this master and slave degrees of freedom. Similarly, the dumping matrix can also be partitioned by this slave and master and stiffness matrix also can be partitioned by slave and master and while partitioning these thing we so take node of the thing that this nonlinearity and external nodes should be in master degrees of freedom and all other degrees of freedoms will be in this slave degrees of freedom. Now taking this or dividing this mass stiffness and dumping in this master and slave form so we can write the original equation.

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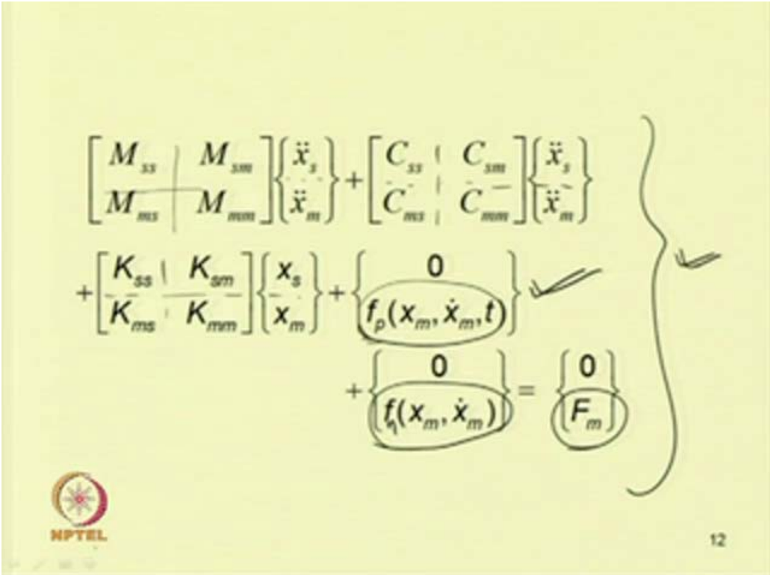

So our original equation this M x double dot plus C x dot plus K x plus f p plus f n equal to e f external in this form. So by substituting this thing, we can write our equation in this form, that is so it is partitioned now and so where this is X s also this is partitioned X s and X m. So here it may be noted that the parametric term, so the terms associated with this X m X m dot and t can be written so in this form as we are putting all this things in master degrees of freedoms only the lower part will contain this thing. So the upper part where slave things there, so we are putting that thing equal to 0. Similarly, for this non-linear term so this is in the master degrees of freedom and slave degrees of freedom we are putting equal to 0, and this is the external nodding, so this is the external nodding of the system. So we can write the original equation in this way by taking the master degrees of freedom and slave degrees of freedom and we can partition and we can write this equation in this form. Now by writing in this from so we can solve this equation by using different method.

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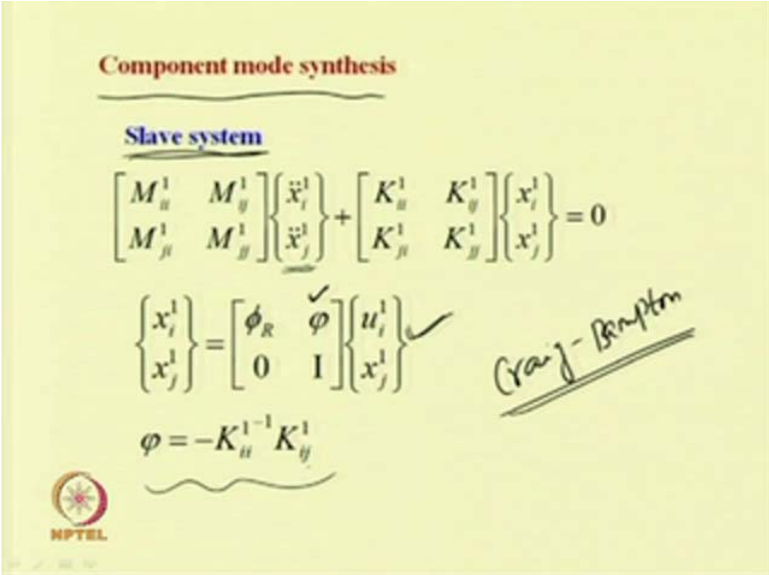

Another method what we can use, so in this in the previous two equations, so the non-linear terms again we are using this model reduction method, the non-linear terms forcing vector rotates between the physical parameter or physical coordinates and the model coordinates. To avoid that thing one can use this component modes synthesis method. So in component mode synthesis method a Craig, a Craig bump ton type of bump ton types of structure with more number of degrees of freedom represent using generalized coordinates while the other sub structure with nonlinearity is model with physical coordinate. So we can use a sub structure technique.

So so will use two different types of i sub structure in which we can use this generalized coordinate and physical coordinate. That means in this type of things, so we can use both generalized coordinates and physical coordinate. So will keep further non-linear trans that degrees of freedom associated with non-linear terms will keep them in the physical coordinate and the other terms will keep or write them using this generalized coordinates. So, initial physical coordinate can be transformed to the generalized coordinate; by using different techniques or by using some methods.

So that thing will discuss here, so here will have a slave system. So as we have discussed before will have a slave and master degree of freedom, so in case of this slave degrees of freedom so here initially we have divided our structure into 2 parts, one with generalized coordinates another with physical coordinates so initially we have this all with generalized coordinate or physical coordinate x and we can de replace them by the generalized coordinate u in this way.

So our let us take only the un-damped and un-damped liner systems, so in case of the un-damp linear systems. We can write the equation in this form M x double dot plus K x equal to 0, so here we have taken only this one that is the slave part. So in this case this so i i represent the interior coordinate and j represent the junction degrees of freedom, so we can partition this stiffness and mass matrix of structure into interior and junction degrees of freedom. So these are interior degrees of freedom and j represents junction degrees of freedom, so the normal mode of the above system found using eigenvalue analysis the physical coordinate of the slave system are transformed into the set of normal coordinates u 1 and the degrees of freedom x j using the transformation, so x j is the. So this the junction coordinate and u is the so we have taken this u is the normal coordinates.

So we can write this i so we are using this so this x i 1 and x j 1 i is interior degrees of freedom and j is the j is the junction degrees of freedom. So we are using this 1 for the slave part, so this will be equal to 5 are psi 0 i into u i 1 and x j 1 so here psi equal to minus K i i 1 inverse into K i j 1, so this 1 for the slave system. So in this way we can write this equation in this form.

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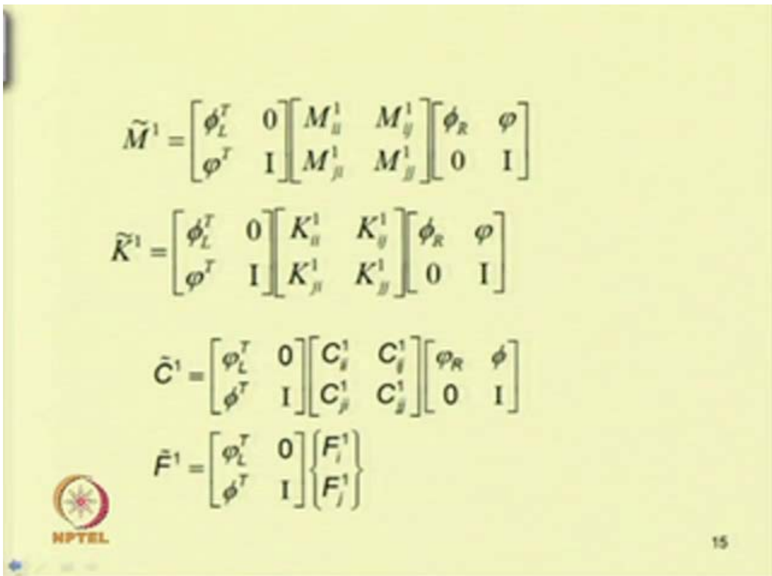

Then, we can write this M 1, so here so we have written in this equation this M 1 1 or M 1 can be written.

So the mass stiffness and dumping matrix of the slave system 1 transpose into new coordinate system. So we can transform into the new coordinate system u i u i 1 and x c 1 in this way so you can use this coordinate transformation. So these into, so this is the mass matrix into this. Similarly, for the stiffness matrix also, we are using so this is the left eigenvector and this is the right eigenvector, so using the left and right eigenvector so we can write this. And for the dumping also we can write and for the force also we can write this way.

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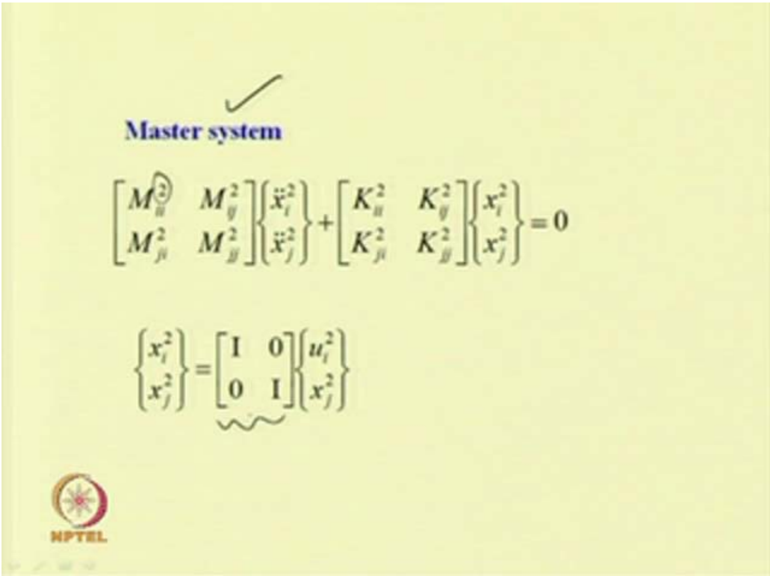

So now for the master system similarly, for the un-damped and unforced equation of the master system can be written using this mass matrix and stiffness matrix. So these 2 represent the master system and 1 was for the slave system. So in this master system we can write, so as we are not changing the coordinate so of the master system so they are transform to this new coordinate system by this i 1 so i 0 and 0 i. So i is the identity matrix so this identity matrix is a use for the transformation to keep the procedure of the component mod synthesis same as that for the slave system. So in this way so you can write or we can replace this x coordinate in terms of this u coordinate

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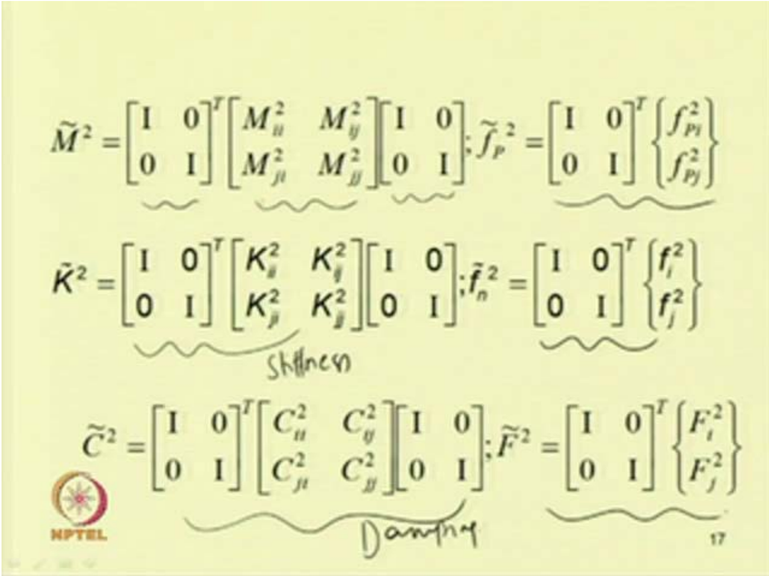

So here x j is the junction coordinate now for the similar to the slave for the master we can write this or we can transform this x coordinate to the u coordinate system or we can write this M 2 in this form so this is 0 i 0 0 1 transpose this is the mass matrix into i 0 0 i. Similarly, this is the forcing function can be written with by multiplying this transpose of i 0 0 i. Similarly, this is the K 2 and this is for the non-linear force and this is for the external force, and this is the dumping matrix. So this is the dumping matrix, so it is changed in this way dumping master matrix so this is the stiffness master matrix.

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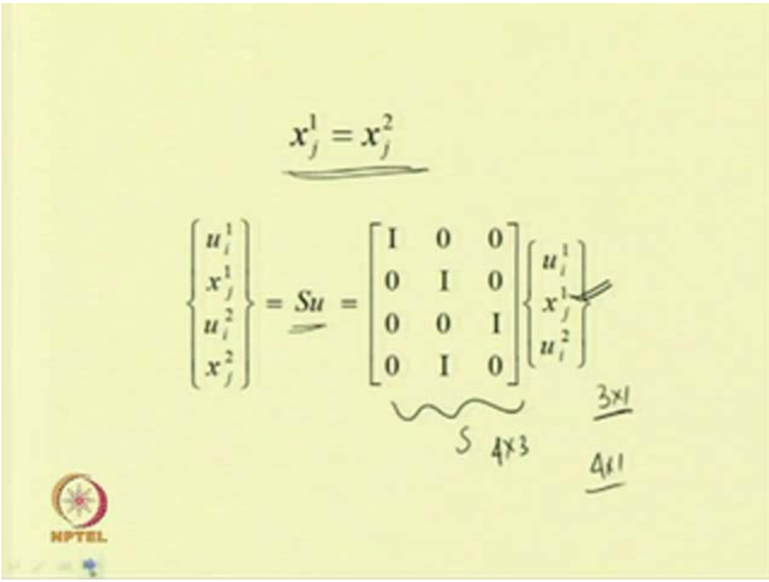

And this is the mass matrix of the master component, so here x j, so we are taking this x j 1 equal to x j 2. So by taking this x j 1 equal to x j 2, so we can replace this u i 1 x j 1 u j 2 and x j 2 to this form that means by writing this is equal to S u, where S can written this. So here u equal to u i 1 x j 1 as we are putting x j 1 equal x j 2, so we can write using this x j 1 then u i 2. So this a 4 is to, so this 4 is to 3 matrix and this is 3 is to 1. So finally, this becomes 4 is to 1, so this is u i 1 x j 1 u i 2 x j 2. So we can replace this vector by u i 1 x j 1 and u i 2.

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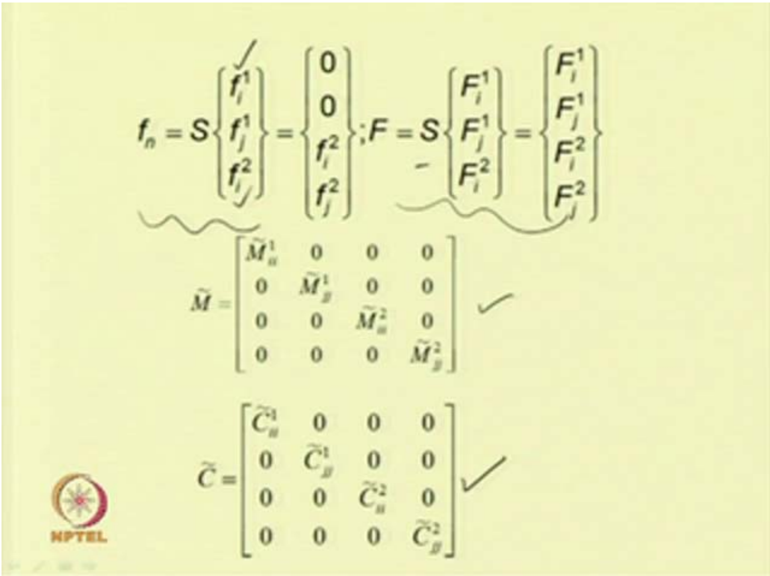

So writing this way, so we can now write this non-linear force in this for so f n equal to S into f 1 1 f j 1 and f i 2. So this is for the slave component this is for the master component and the middle 1 is for the junction force. So which is equal to 0 0 f 1 2 and f j 2. Similarly, we can write f in this form using this S matrix and the mass matrix and the transform mass matrix and dumping matrix.

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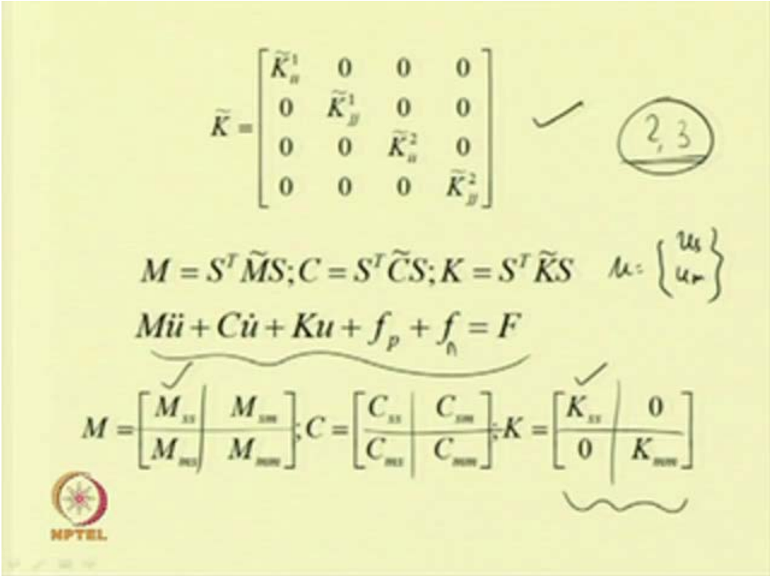

And the k matrix then also is written in this. So here u may note that this becomes diagonal matrix so here M equal to a S transpose M S, C equal to S trans, C equal to S transpose C S and K equal to S transpose K S. So our equation reduces to this for that is M u double dot plus C u dot plus K u plus f e plus f n equal to f, so it has reduced to this form and now after it is reduce to this form so one can write this where this M equal to M s s and m s m and this is equal to so this is similar to the dynamic sub structuring what we have studied here so one can reduce. But, in this case it may be noted that in this equation so the size of the matrix can be reduced very much.

So by using these model methods, so you can reduce the size of the matrix to the required level of the equation motion. So and here also it can be noted that by this method so by the stiffness matrix is uncoupled, both the mass and dumping matrix are coupled. So here the stiffness matrix is uncoupled K S S and K M M, but the other matrixes are coupled. So here we are using this u s and u m, so u s will be the slave displacement degrees of freedom and so this u will be equal to so will have this u s and u m.

So where u s is the slave displacement degrees of freedom and u m is the master displacement degrees of freedom. So slave degrees of freedom consist of only the included nodes of the sub structure 1 and master degrees of freedom consist of the boundary degrees of freedom and the physical degree of freedom of the sun structure 2 the mass matrix M s s and k s will be diagonal because they represented in the natural modes. So this mass matrix M s s and K s s, so they are diagonal so M s s and K s s are diagonal, so as they represent the natural modes.

So in this way first one can reduce the equation motion to a required degrees of freedom, so instead of use very higher degrees of freedom. So after reducing this equation to the required level, for example, if one reduce the equation to a 2 degrees of freedom or 3 degrees of freedom, then one can use the methods what we have studied or one can use the quart basin method what we have studied earlier to find the solution of the system and but for higher degrees of freedom when it is of higher degrees of freedom then it will be difficult to apply those perturbation methods but one can still use this perturbation method by using some symbolics up to here.

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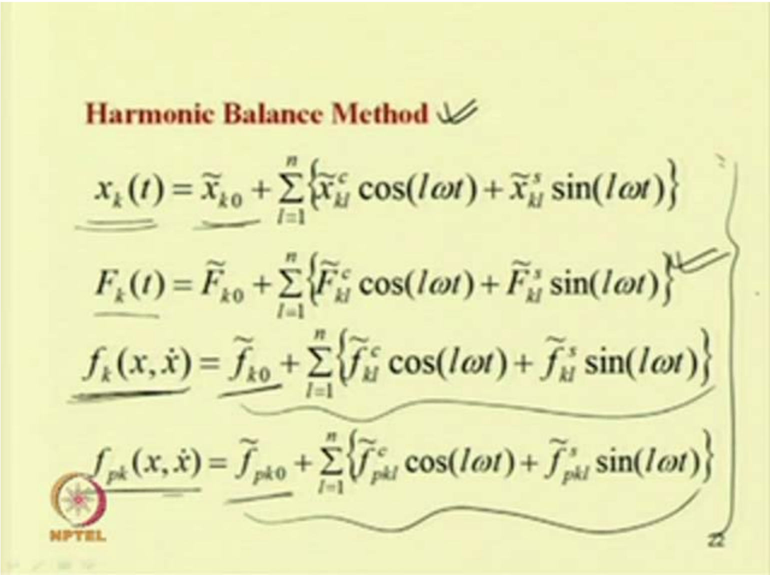

And now let us see or will use this harmonic balance method. Now we use this harmonic balance method or will describe this harmonic balance method what we have studied in our module 2 again to use this thing in this method to find the solution of the system. So here while using this harmonic balance method.

So we will discretize this or we will write this equation this displacement, vector x k in terms of different components different harmonics. Also we can write this forcing term this forcing term which is the function of time or the forcing term like the parametrically excited term. So this x x dot or this the terms with concentrated loading or called terms with loading vector or the terms with non-linear terms. So these terms with non-linear terms, so we can expand them in terms of different harmonic terms by using this harmonics. So for example, x k that is k th displacement we can write equal to x k plus x k l cos l omega t plus x k l sin l omega t where l varies from 1one to n. For example, n equal to 1, so it will be x k 1 cos omega t plus x k 1 sin omega t. Similarly, for higher modes it will be x k 2 cos 2 omega t plus x k 2 sin 2 omega t.

So in this way, so we can write the terms with difference of up to certain level of frequency. So for example, we can take only the 3 modes, so in that case l will be equal 3 so in addition to this constant term so we can have 3 more term 3 plus 3, 6 more terms in this expansion. And similarly, for the forcing term also we can write f a equal to f k 0 plus i equal to 1 2 n f k l cos l omega t plus f k l sin l omega t so this for the generalized force if we have a generalized force so then we can write that generalized periodic force in terms of this harmonic forces. Similarly, in case of these non-linear terms we can also write the non-linear terms using different harmonics.

Where k 0 is the constant part of the non-linear terms and this contains the harmonic, so if the non-linear terms are periodic; then we can write using these harmonic terms and so this represent the loading vector also. The the loading may be a static loading or it can be a loading with periodic force, so in that case of static loading we can use this f p k 0 and in case of periodic we can write using different harmonics. Now using different harmonics so one can use or one can use this mod synthesis method or dynamic sub structuring method or this component mod synthesis method to write the, write this equation motion or substitute this equations in those equations and one can.

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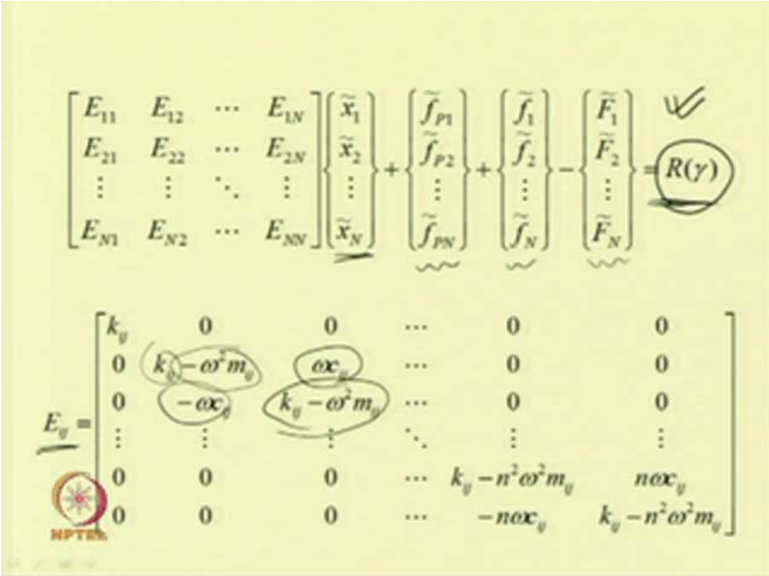

For example so in original equation by substituting these equations, so one can write or that equation will reduce to this form. So where this R gamma is a residue, so this equation is reduce to this form that is e i j so where E 1 1, E 1 2, E 1 n, so this at the x 1 x 2. So here we have this x. These x terms are there, so it will x 1 bar x 2 bar x n bar and these are the forcing term. So this is the parametric forcing this is the non-linear term associated with that and this external force. So we have taken all the force to the left hand side and substitute these equations in the original equation and as this are approximate solutions we have taken so the right hand side will never be equal to 0 but there will be some residue part to this. So this residue is written by writing this R gamma.

So this equation it reduced to this form where this e i j equal to so e i j equal to k i j 0 0, so this the this reduce to 8 diagonal form so k i j, k i j minus omega square m i j so the diagonal term one can see this is k i j this k i j minus omega square m i j so then here we have this minus omega into c i j this is from the dumping term and this is from this mass matrix and this the stiffness, so when we substituting these modes or when are substitute this harmonics; then from that harmonic by differentiating twice.

So we have this minus omega square term coming into there so here we have this dumping term then this is the k i j minus omega square m i j. So each so for example, E 1 1, so E 1 1 will be equal to k 1 1 and this will be k 1 1 minus omega square m 1 1 c 1 1 minus omega c 1 1 k 1 1 minus omega square m 1 1. So in that way so one can write this matrix. So e i j will contain or e i j terms will contain this way.

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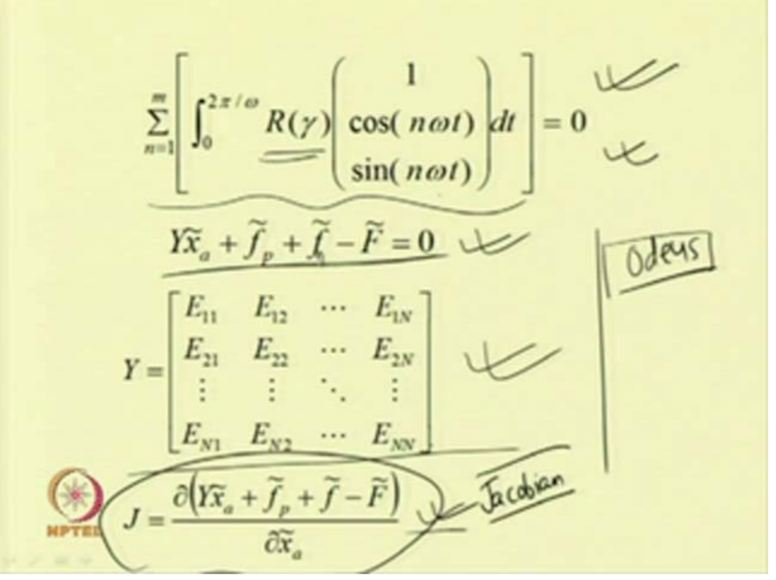

Similarly, these so one can minimize this residue so the residue one can minimize this residue by using this method or one can minimize this residue, so this residue; so integrating this residue 0 to 2 phi by omega so that is 1 cycle. So taking this n equal to 1 to m and multiplying this R gamma with 1 cos n omega t sin n omega t d t, so one can minimize this residue and minimizing this residue this equation will reduce to this form. So it will reduce to Y x a plus f p plus f n bar minus f bar equal to 0 so where y equal to this so y equal to E 1 1, E 1 2 , E 1 n, E 2 1, E 2 2, E 2 n and E n 1, E n n. So one can obtain this equation which is a set of algebraic equations and the set of algebraic equation can be, so the set of algebraic equation can be solved by using different methods. For example, one can use this Newton- Raphson procedure to solve this set of algebraic equation.

So instead of solving this n dimension equation using this using this Runge Kutta method or using this Runge Kutta method, so one can use this Newton-Raphson method because this Newton’s Raphson procedure require the calculation of this Jacobian and which will converge very fast. So instead of using this odeys so if one use this mat lab then one can solve this differential equation by using odeys 4 5 which is fourth and fifth order Runge

Kutta method to solve those equation, but solving these large degrees of freedom system; so it will require huge memory and space and computational time also.

So one can instead of doing that thing, so one can solve this equation. So, this reduced equation, which are set of algebraic equations and one can get the solution very fast. So while using this Newton-Raphson method one required this Jacobian matrix, so the Jacobian matrix this is the Jacobian matrix which is required or; which is to be evaluated is in each iterations. So this is the so this is the Jacobian matrix which is required to be solved in each iteration and this is equal to j equal to del y x a plus f p plus f n bar minus f bar by del x a.

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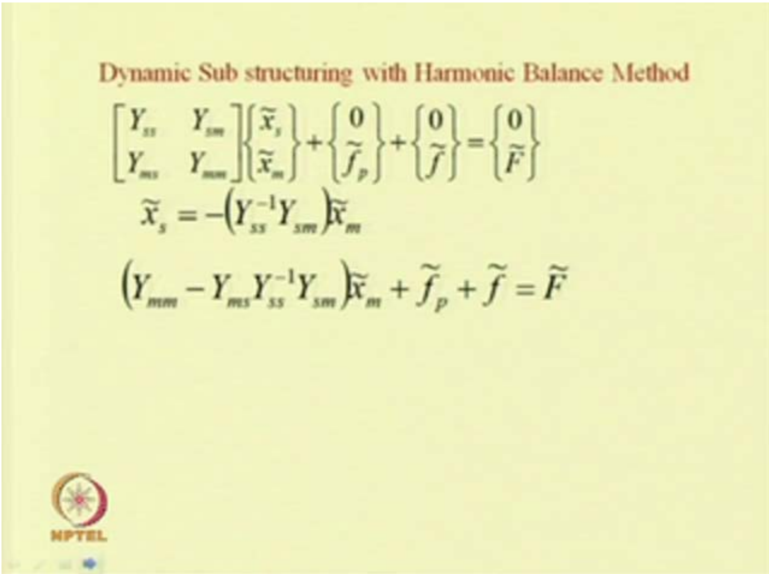

Solving this thing so one can find the equation or one can find the response that this response amplitude x a of the system for different system parameters. Similarly one can use this dynamics sub structuring with this harmonic balance method also, in dynamic sub structuring we previously we have seen that we can divide this structure in to slave and master. Where this slave coordinates can be written using this generalized coordinates and the master coordinates or the master degrees of freedom can be kept using this physical coordinates. So we have a set of generalized coordinates and physical coordinates, so by using this dynamics of structuring so we initially we obtain this equation and after getting this equation.so what we can do.

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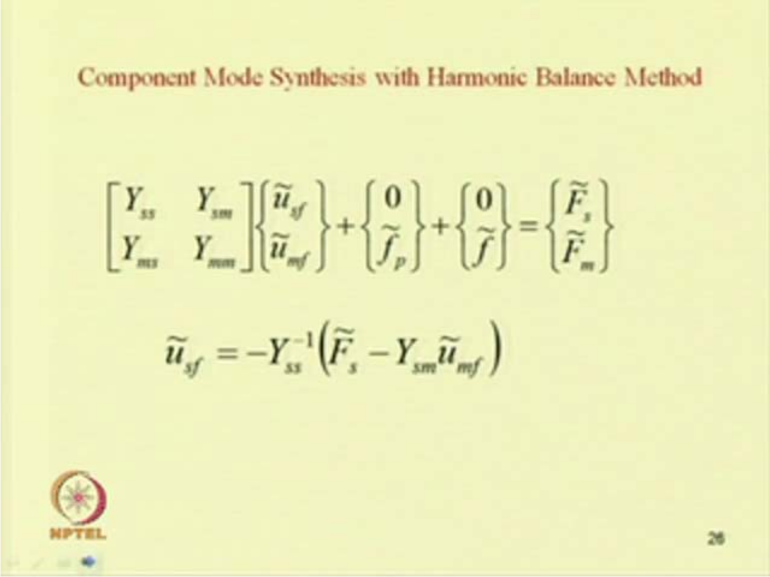

So we can apply this method of harmonic balance to reduce this equation to solve. So in this case, so in dynamic sub structuring, so initially in dynamic sub structuring we have divided the system into this slave and master components. And here also by using this harmonic balance method it can be reduced to this form, so which can be solved using this Newtons-Raphson method to find the solution.

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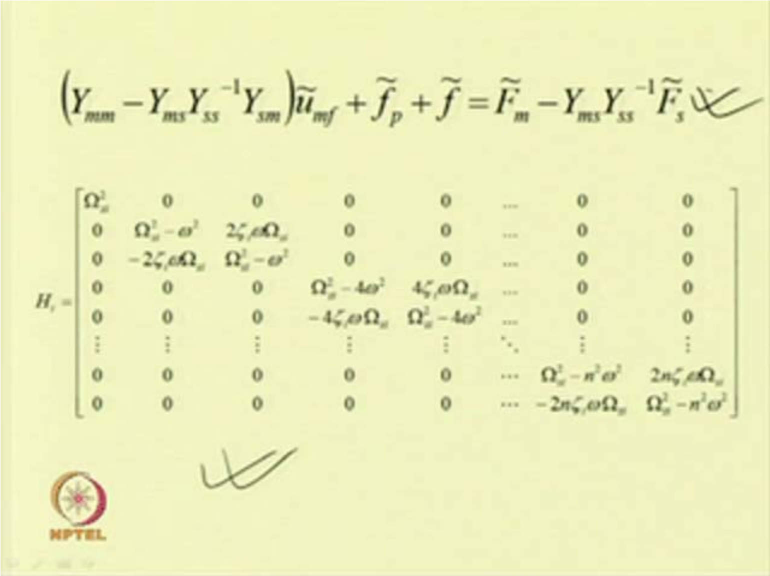

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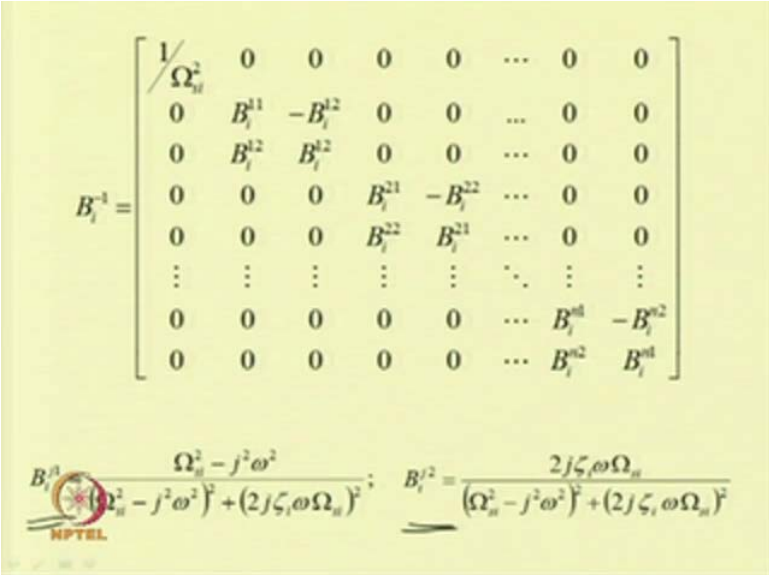

Similarly in component mode synthesis, so in compound mod synthesis we are diving the system into slave and master; where the slave systems are retained in terms of the generalized coordinates and masters are retained using these physical coordinates. So after getting these equations now we can find or we can use this harmonic balance method and by using this harmonic balance method so we can find the equation. So this is the equation we can obtain where H i can be written in this form.

And solving this where H i written in this form which contain this term B i and which can be obtained from this equation so where these are the terms. So by solving this algebraic equation so one can find the solution of the system. So in this way so one can use this harmonic balance method in these 3 cases that is mode synthesis or model reduction method, dynamics sub structuring method or component mode synthesis method to reduce this equation to a set of algebraic equation and those equations has to be solved to obtain the response of the system.

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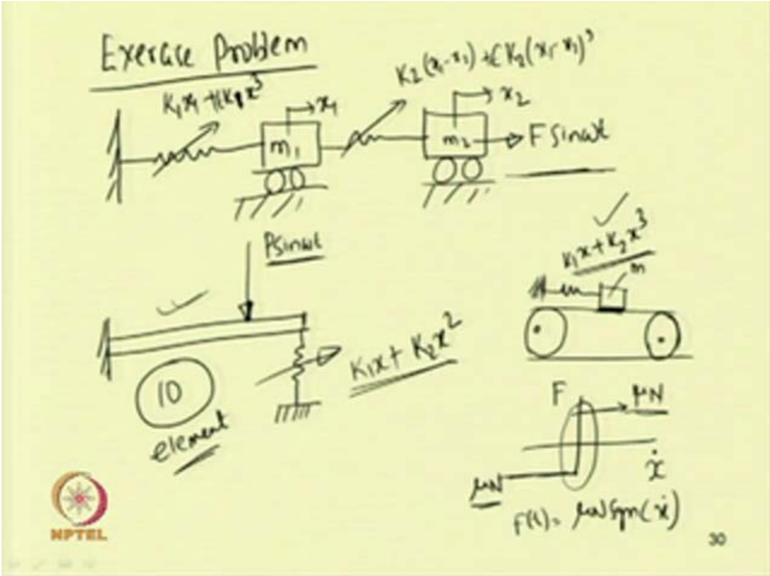

So after getting this response of the systems one can study the response of the system for different system parameters and one can study the effect of the nonlinearity or effect of forcing dumping nonlinearities system and for one can take this excise problems to exercise problems which have been so for example, take this system, so one can take this system with.

So we have 2 degrees of freedom system and here we have this, these are the non-linear stiffness. We are given so this is m 1 this is m 2, so this k 1 can be taken as, so this is k 1, so this is k 1 x 1. So this is x 2 so here one can take k 1 x 1 plus k 2 x cube cubic nonlinearity here also one can take this cubic nonlinearity and one can write this equation using this k 2 so the displacement will be x 1 minus x 2 plus.

So let us write this is equal to f epsilon k 1 f epsilon k 1 x 1 cube. So this is k 2 plus f epsilon k 2, so this term can also be written as f epsilon k 2 x 1 minus x 2 cube. So taking this nonlinearity one can so let force is applied here so this force equal f sin omega t so one can write the equation motion so this is very simple equation so one can write the equation motion and use the method and as the force is applied to the second mass one can reduce this thing to that of a single degree of freedom system by taking only the model matrix of the second mass or and one can write a single equation to find the equation motion. Next one can take a physical or a continuer beam with springs attached here, so let the spring is non-linear spring so here it is equal to k 1 x plus k 1 x plus k 2 x square as one take multiple degrees of freedom here so one can use a finite element method to solve this problems. So by taking 10 elements by taking 10 elements one can write the equation motion add this non-linear spring to the system also add a external force that is equal to P sin omega t, P sin omega t to solve this system. Also one can take a system belts system, so this is belts in which a mass is placed on this and so in this case, so this is a non-linear spring k 1 x plus k 2 x cube and here the dumping.

So here the dumping term one can take as that of pull on dumping, so in case of pull on dumping so it will be equal to a depends on velocity. So if one plot this x dot so this is equal to f so this will be f mu n and this minus mu n, so where n so let this mass is m. So one can find the so this will be equal to mu n and minus mu n and at x dot equal to 0 or when x dot change its sign so this change from minus mu into plus mu n. So the system will have a so the system we have a cubic nonlinearity in spring and also it will have a nonlinearity here due to this dumping, so this dumping force f t can written as mu n sigma x dot.

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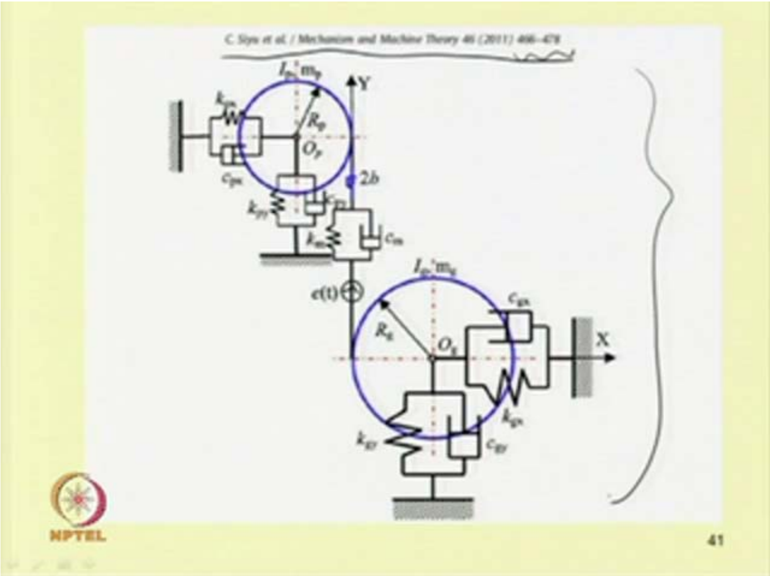

Similarly, we can have some Rotar system with are gear system with back class. So let us see this system, so this is a gear system with back class one can see the detail of the system from this paper by published in mechanism and machine theory in 2011 volume 46 and page number 466 to 478. So, here the back class and clearance is taken into account by taking the black class and clearance in a gear system. One can write the equation motion, so one can write this spring or take this spring as non-linear spring also, by write so one can write the equation motion of the system and one can solve these equations using the methods studied today.

So next class we study, so this finishes the force vibration in non-linear systems. So next class will study the systems parametrically parameter subjected to parametric excitation. So in that case we will take few examples, so mainly will take example of a sandwich beam subjected to periodical actual node. Then will take one electromagnetic system where the system is subjected to actual periodic role in addition to that magnetic felid and third will take another system with basic excited continuer beam with arbitral placed mass which is subjected to or which where will consider the internal resonance also in the system. So, will study these 3 systems in case of the parametrically excited system so next 3 classes will devote to parametrically exited system.

Thank you.