Non-Linear Vibration
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Module - 6
Applications
Lecture - 6
Nonlinear Forced-Vibration of Single and Multi-Degree-of-Freedom System

Welcome to today class of non-linear vibration. So we are discussing about non-linear force vibration of single and multi-degree of freedom systems. So already we discussed about the free vibration of single and multi-degree of freedom unlike in case of linear system, so in case of non-linear system as super position rules as are not applicable, so here different resonance condition has to be studied and one has to study this multi degree of freedom system from by studying this single degree of freedom system or by using the analysis what we have studied in case of single degree of freedom system same principle can be used for studying the multi degree of freedom system. So in the previous class, we have studied about the Duffing oscillator or (( )) force vibration of Duffing equation with cubic nonlinearity. So, today class will study the Duffing equation with both cubic and quadratic nonlinearity, considering both cubic and quadratic nonlinearity will study the response of the system.

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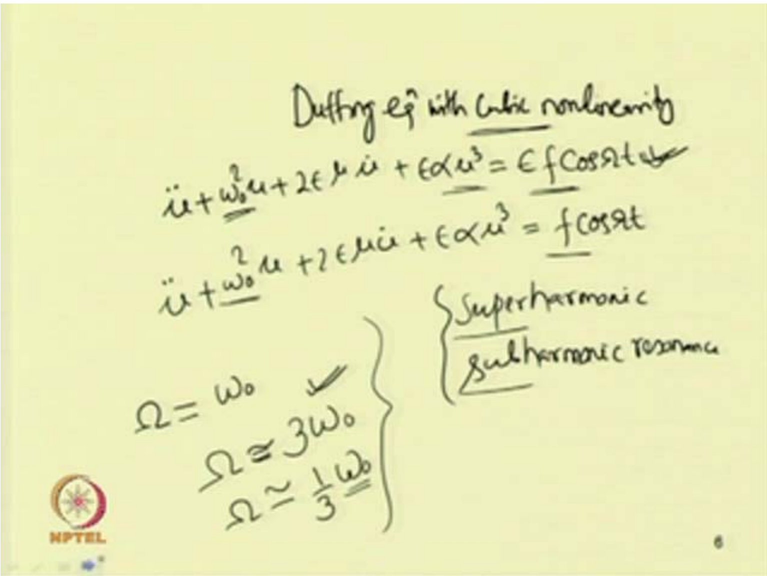

So first we briefly review about the buffing equation with cubic nonlinearity, Duffing equation with cubic nonlinearity, so here we have taken so the equation is given in this form. That is u double dot plus omega 0 square u plus 2 epsilon mu u dot plus epsilon alpha u cube. So this is for free vibration equal to 0 and in the force vibration case we have considered 2 cases. In the first case we have taken this forcing equal to epsilon f cos omega t and in the second case we have written the equation motion in this way u double dot plus omega 0 square u plus 2 epsilon mu u dot plus epsilon alpha u cube equal to f cos omega t. So in the first case as the forcing term is 1 order is less that of the linear term, that is the omega square, so we have studied only the principle parametric resonance in this case. But, in the second case as this forcing term f cos omega t is of the same order as that of the omega 0 square u, that is the linear term, so in this case last class we have seen we have 2 more different type of resonance conditions.

So one condition is the super harmonic condition super harmonic and second case we have studied is the sub harmonic resonance condition sub harmonic resonance condition, so in case of cubic nonlinearity, already we have studied about the principle, we have studied about the primary resonance conditions. So in case of primary resonance condition omega that is the external frequency nearly equal to this natural frequency or the linear frequency of the system and in case of the sub harmonic we have so when this so in case of sub harmonic and super harmonic, we have seen depending on the nonlinearity that means in this case we have cubic nonlinearity. So we have the resonance condition like this so omega 0 or we can write we can write external frequency equal to in1 case this is equal to 3 times omega 0 on the other or nearly equal to 3 times omega 0 and in the second case it is equal to one third omega0. So in this case as the natural frequency is less than one third of the frequency, so this is the sub harmonic resonance case and in this case this natural frequency is nearly the 3 times the external frequency. So this is the super harmonic resonance case so we have studied the resonance condition.

In case of the cubic nonlinearity for this resonance condition, also in addition to the system having single frequency one may multiple numbers of frequencies. So in case of the system having multiple frequency that means multiple forcing parameter or let us take 2 forcing parameter in that case the first equation can be written u double dot plus omega 0 square u plus 2 epsilon u mu u dot plus epsilon alpha u cube equal to epsilon f 1 cos omega 1 t plus epsilon f 2 cos omega 2 t. So in this case, so one may in addition to this principle and super harmonic and sub harmonic resonance condition we can get a number of other resonance conditions also.

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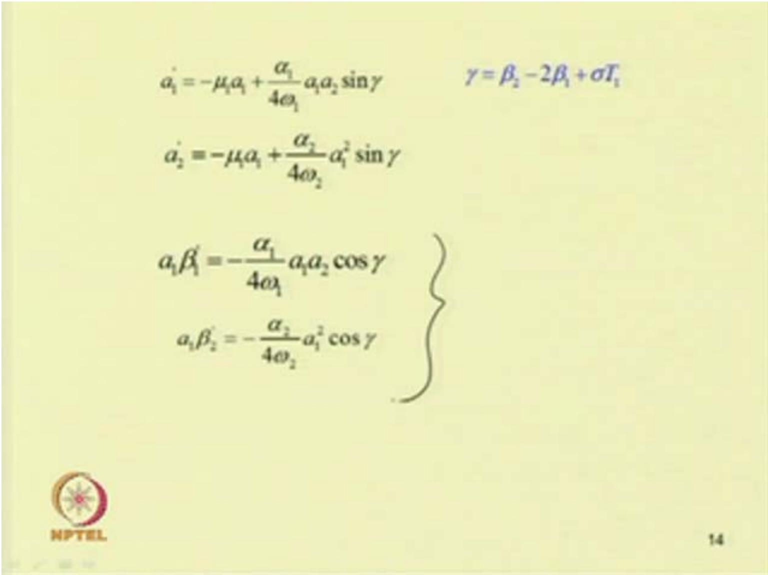

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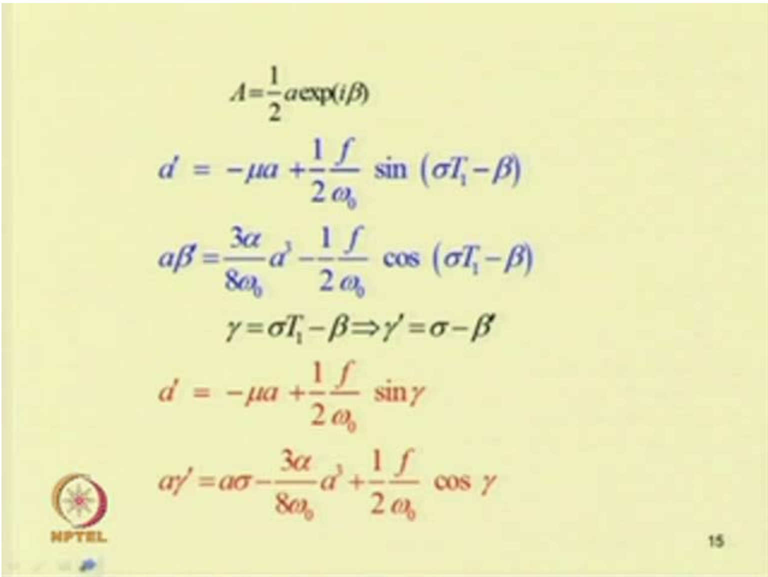

So we have studied those resonance conditions in the last class and recall part we have studied, so one may refer the equations what we have obtained. So in case of principle parametric resonances we obtained these are the reduced equation and from this reduced equation we got the frequency response equation.

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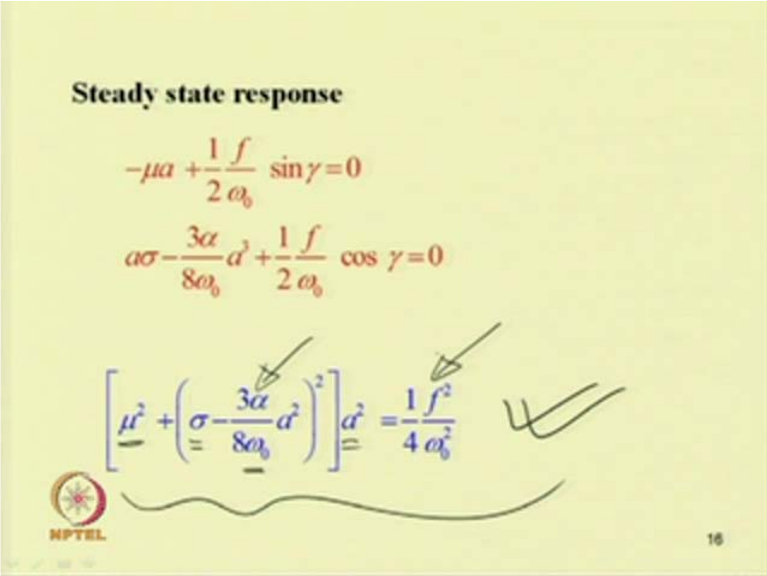

So this is the frequency response equation for the steady state condition, so in this case we obtain an equation in which is quadratic in terms of sigma and sixth order in terms of a. So easily one can solve this equation quadratic equation to get the frequency amplitude relation. So in case of the frequency amplitude relation and also one can have the one can have the response that is u in this form that is u equal to a cos omega t minus gamma, where a is the amplitude.

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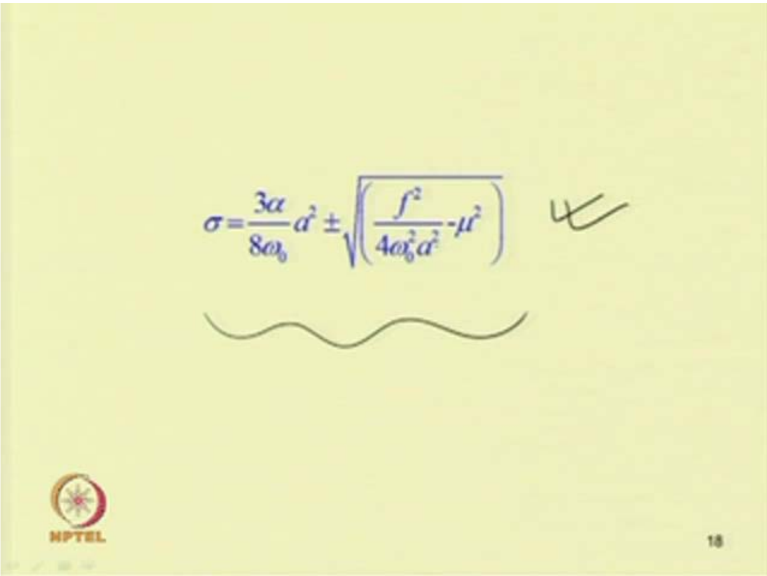

So this amplitude one can obtain from this by using this expression. So this expression mu is the dumping parameter, sigma is the parameter, alpha is the coefficient of the cubic nonlinearity, omega is the natural frequency or frequency of the linear part of the system, a is the amplitude of oscillation f is the external amplitude of the external forcing

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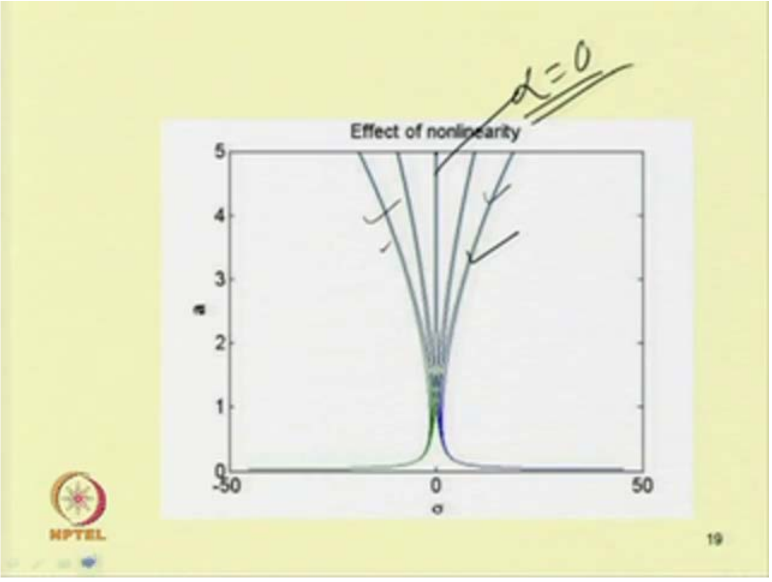

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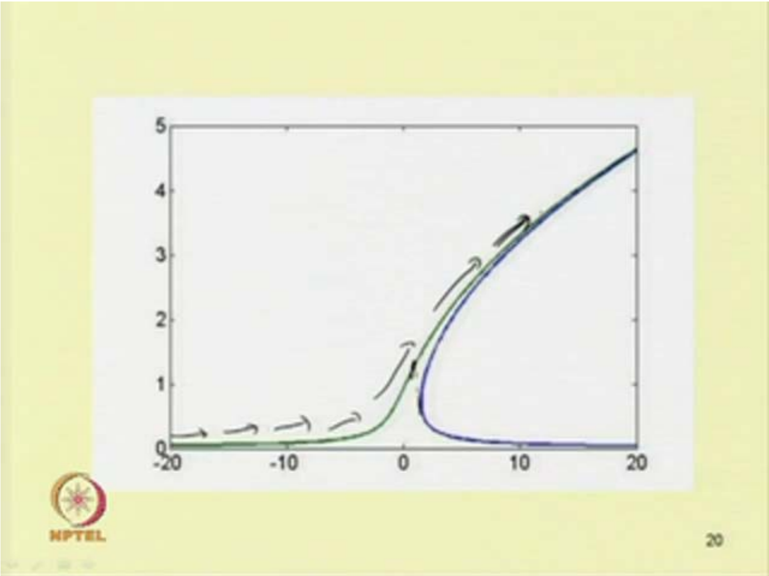

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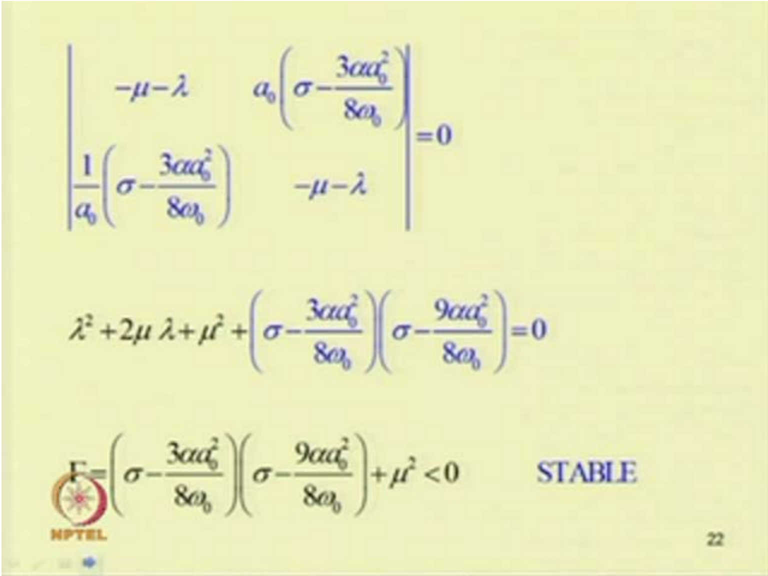

So using this thing, so we obtain the response equations by solving that equation we can write this sigma in terms of a and using this equation one can study the response of the system. So the purpose of recalling this equation is that while will solve for the system both with cubic and quadratic nonlinearity, we will get a similar equation in solution also will give us or give rise to this type of a response, where the response may be of sustaining type or depending on the value of alpha. So if alpha is positive one can get this saddening type of response saddening type of response and if alpha is negative one get this obtaining type of response.

So this corresponds to alpha equal to 0 so where the response is linear type. So in this we already discussed about the jump of and jump jump down phenomena here while reducing the frequency the system may so one may observe the jump of phenomena or while increasing in the frequency the system will flow this path and from the trivial response one will get the non-trivial response of the system. So already we have discussed about the stability analysis, so one can form this stability analysis by pottering the reduced equation.

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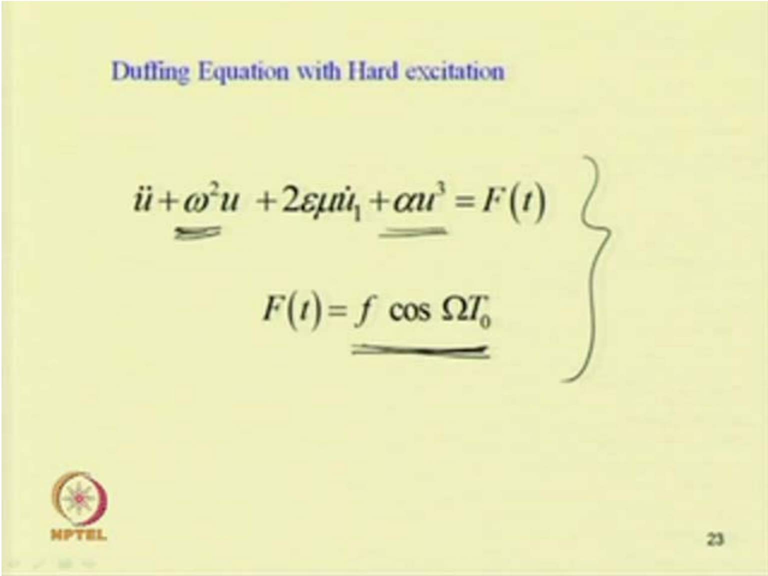

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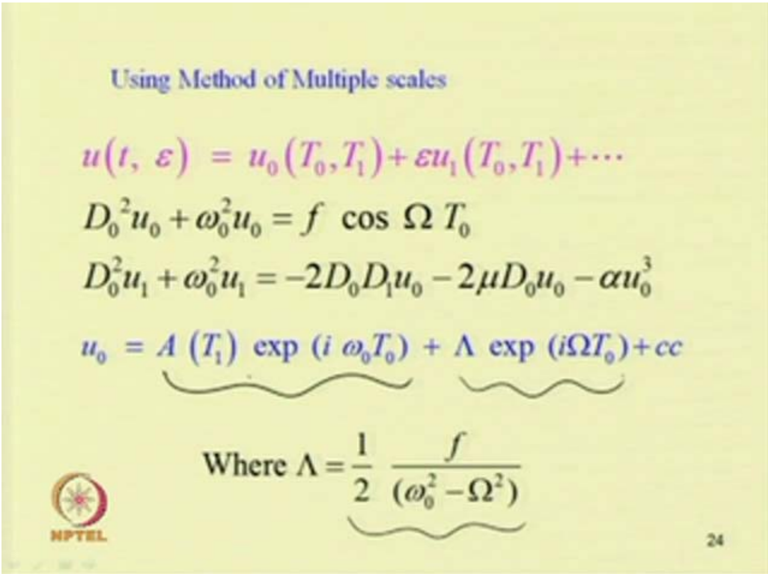

And finding the eigenvalues of the matrix, so if the eigenvalues so the real part of the eigenvalue is negative the system is stable otherwise the system is unstable. So in case of Duffing equation with quadratic Duffing equations, this quadratic linearity and when we have taken the hard excitation that means this coefficient the forcing term and this linear part of the or coefficient of the linear part there of the same order. So if there of the same order then when we use the method of multiple scale, so this term will appear in the order of epsilon and so we have the particular solution in addition to the in addition to the free vibration term or in addition to the complementary part of the response. So u 0 this contain the complementary part this, plus this part is the complementary part so this is the complementary part.

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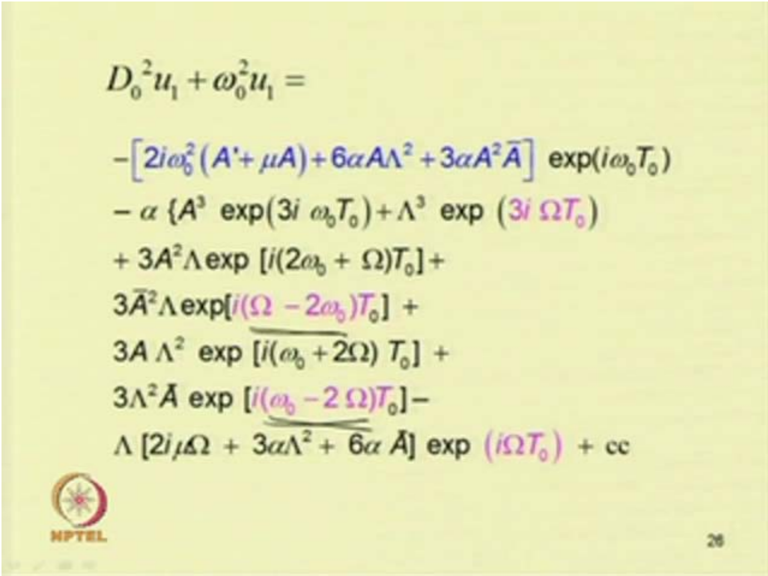

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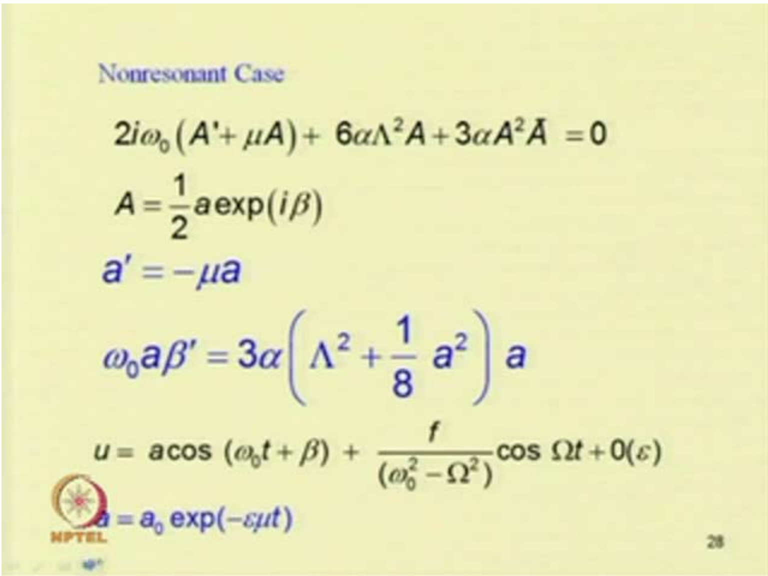

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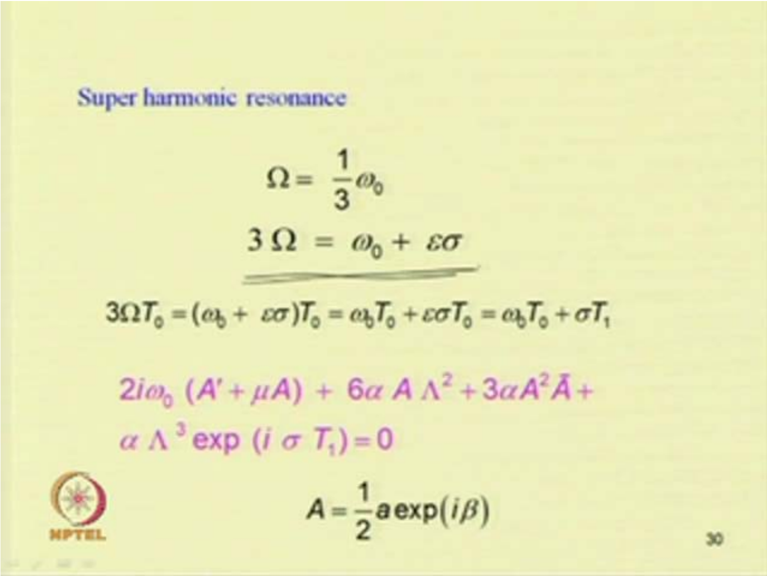

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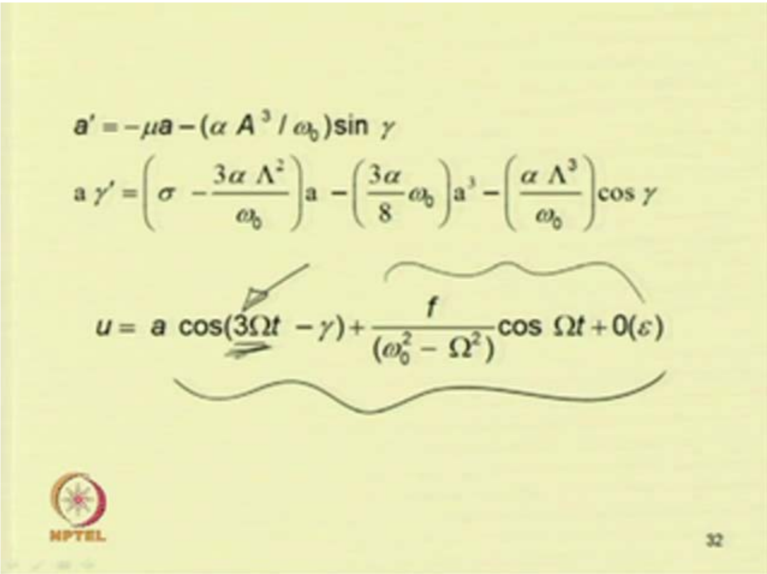

This, due to this when will put this homogeneous or right hand side equal to 0 so will get this solution and we are considering the forcing term. We have this part of the solution now substituting this equation in the equation, so one can see that two different types of resonance condition will be obtained depending on the, so this will lead to one type of resonance condition and this will lead to other type of resonance condition. So we have two types of resonance conditions, so we have studied the super harmonic. So in super harmonic case this 3 omega equal to omega 0 plus epsilon sigma and in this case we have obtained the solution of the system which can be given by this so free vibration part that is cos 3 omega t minus gamma, so that the response the free vibration will have a periodic solution or periodic response whose frequency will be 3 times that of the external frequency of the system and this part is the force vibration and this is part due to forced vibration of the system.

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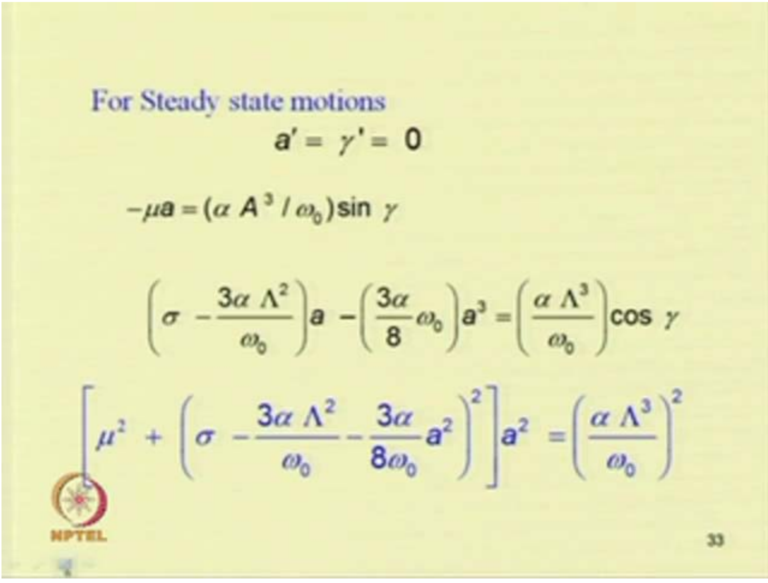

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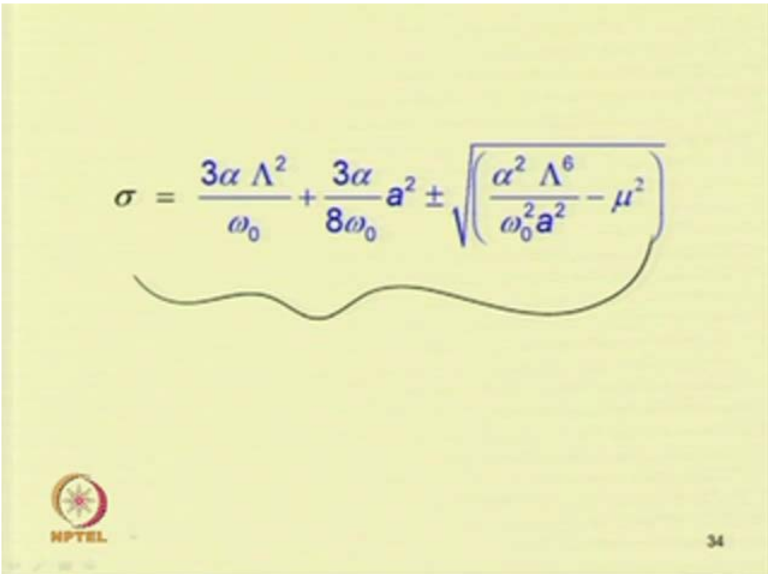

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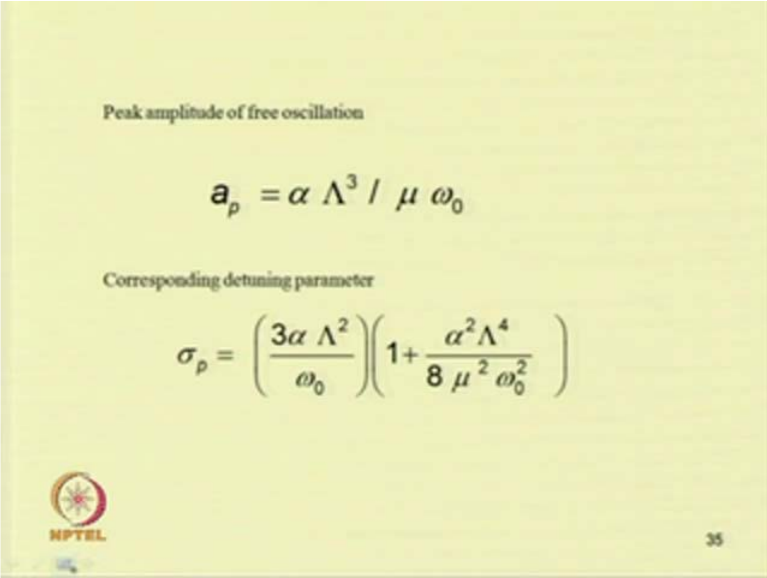

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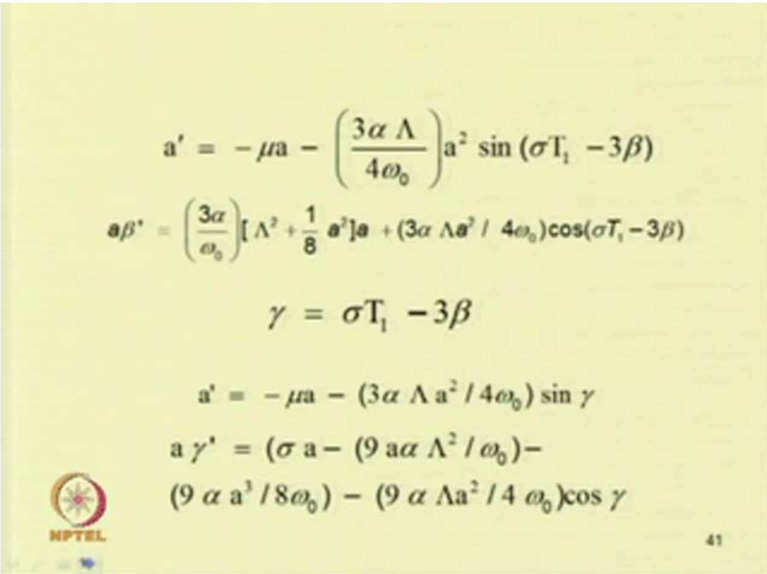

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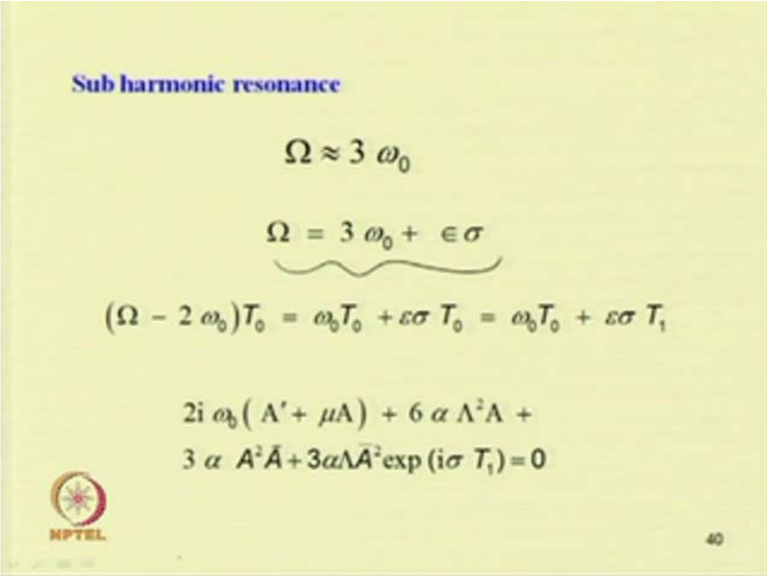

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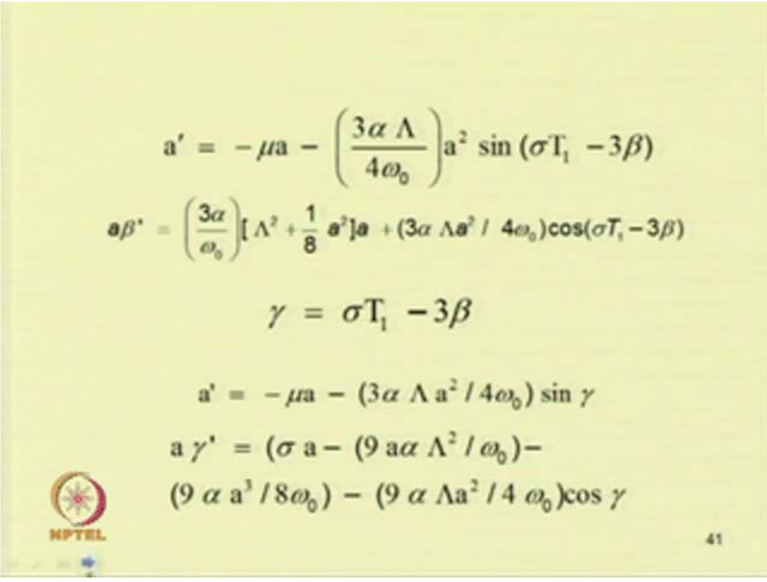

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$$u = a\cos\left[.333(\Omega t - \gamma)\right] + K\left(\omega_0^2 - \Omega^2\right)^{-1}\cos\Omega t + 0(\varepsilon)$$

$$-\mu a = (3\alpha\Lambda a^2 / 4\omega_0)\sin\gamma$$

$$(\sigma a - (9a\alpha\Lambda^2/\omega_0) - (9\alpha a^3/8\omega_0) = (9\alpha\Lambda a^2/4\omega_0)\cos\gamma$$

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Eliminating γ

$$[9\mu^2 + ((\sigma - (9\alpha\Lambda^2/\omega_0) - (9\alpha a^2/8\omega_0))^2]a^2 = (81\alpha^2\Lambda^2 a^4/16\omega_0^2)a^4$$

$$a^2 = 0,$$

or

$$9\mu^2 + ((\sigma - (9\alpha\Lambda^2/\omega_0) - (9\alpha a^2/8\omega_0))^2 = 81\alpha^2\Lambda^2 a^2/16\omega_0^2$$

$$a^2 = p \pm \sqrt{(p^2 - q)}$$

$$p = (8\omega_0\sigma/9\alpha) - 6\Lambda^2$$

$$q = 64\omega_0^2/81\alpha^2[9\mu^2 + ((\sigma - (9\alpha\Lambda^2/\omega_0)^2]$$

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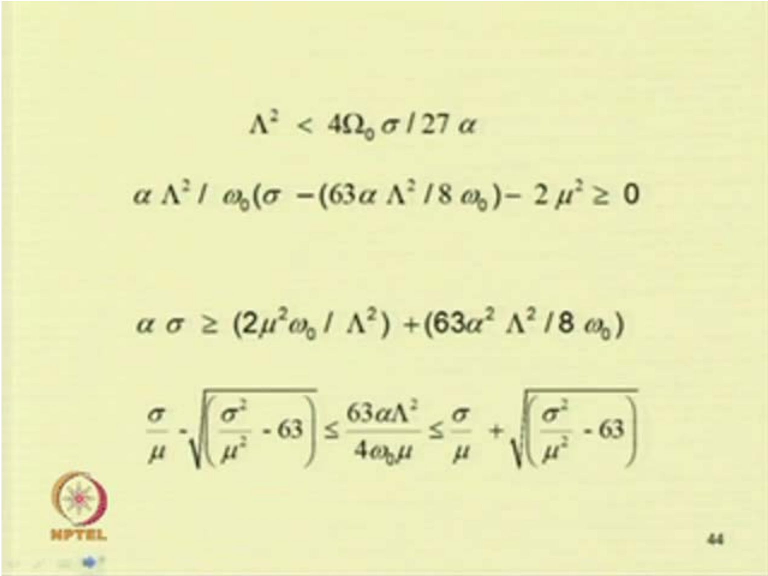

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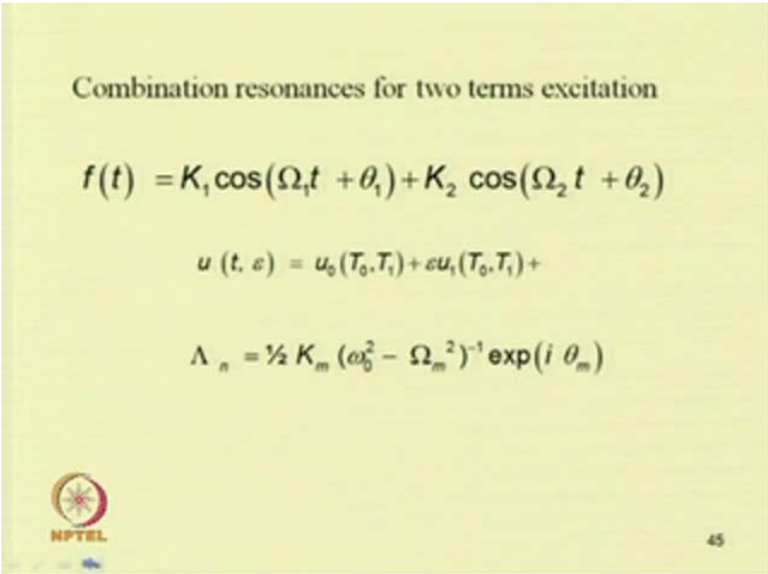

Similarly, one can study the stability and this expression. This is the expression for frequency response curve, so using this equation one can plot the frequency response curve of the system. So using this equation one can plot the frequency response of the system. Similarly, in case of sub harmonic resonance condition where one can take this omega 3 equal to 3 omega 0 plus epsilon sigma and obtain the reduced equation and the frequency equation and this case also the free vibration part contain a frequency which is one third of the natural (( )), one third of the external frequency of the system.

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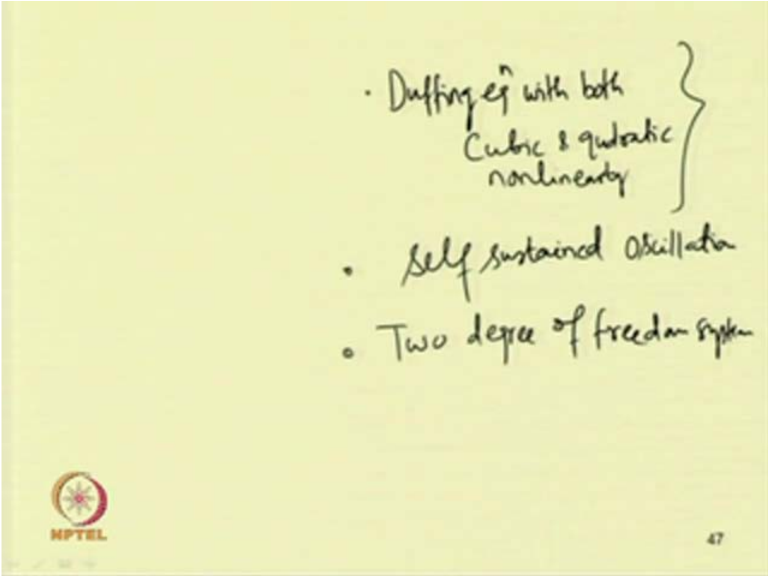

So let us now consider the case when we have or will consider the case when will consider both the cubic and quadratic nonlinearity. So today class particularly will study about 3 different cases. In one case we are doing to study with Duffing equation with both, so Duffing equation with both cubic and quadratic nonlinearity quadratic nonlinearity. So here we will discuss about the primary resonance condition and it will be left as an exercise problem to study the but sub harmonic and super harmonic condition. Second we will take another system that is self-sustained oscillation.

So here will take Van der Pol type of equation so self-sustained oscillation, so will discuss about the self-sustained oscillation of a single degree of freedom system. Third we will consider a 2 degree of freedom system with forced vibration.

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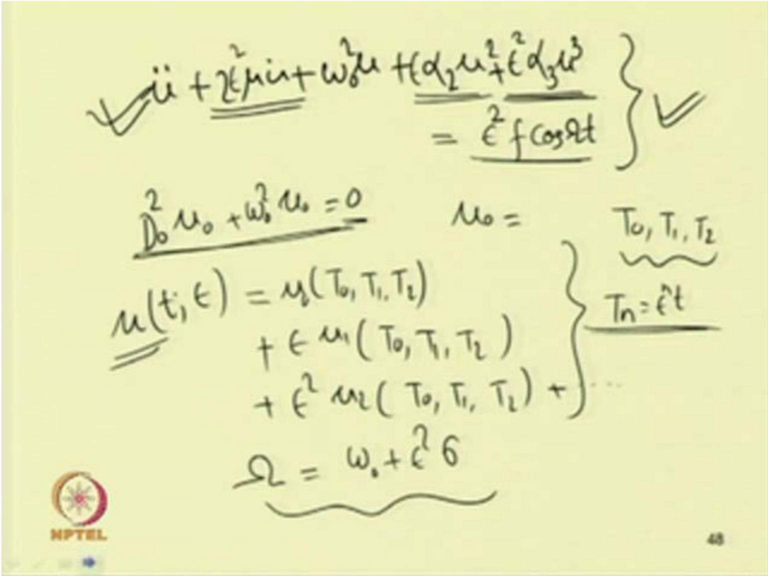

In all these three cases studied the force vibration response, so in the first type Duffing equation considering both cubic and quadratic nonlinearity the equation can be retained in this form, that is u double dot plus 2 mu u dot plus omega 0 square u plus. Let me take this is quadratic non-linear alpha 2 u square plus alpha 3 u cube, so let us take the forcing term, so the forcing term can be written in this form that is; so let us take the forcing term we can take of the different order, so let us take it is for the order of epsilon square f cos omega t. So let me take this dumping of the order of, dumping you can take of the order of epsilon square and this quadratic nonlinearity let us take it of the order of epsilon and this cubic non-linearity let us take of the order of epsilon square. So we can order this thing, so of the order of epsilon square.

So, one may actually note that depending on the problem; one can order the coefficient of different non-linear parts. So in this case we are considering the system where the dumping is of the order of epsilon square, then we are taken the forcing term is also of the order epsilon square quadratic nonlinearity of the order epsilon and cubic nonlinearity of the order of epsilon square, the purpose of making these ordering is that when we apply the method of multiple scales. So in the first order that is in the order of epsilon to the power 0, so we can have a equation which or so in that case will take a solution or will get equation D0 square u 0 plus omega 0 square u 0 equal to 0. So in this form, so right hand side will get a term 0 will be it will lead to the free vibration response or the solution will be simple or the solution u 0, you can write as we did in the previous case. It will be a u 0 equal to a cos omega 0 t or we can write using this exponential form that is a e to the power i omega 0 t plus a bar e to the power minus i omega 0 t. But if are making this forcing term as the same order as that of the linear term, so in that case in the other hand side will have this equal to f cos omega t.

In that case in addition to this free vibration part or in addition to the complimentary part one can add this particular integral. So depending on actual nonlinearity of the system, one can write this equation by using different order of different ordering and in that case one can get a set of a wide range of equations and solving those equations one get the actual response of the system.

So in this particular case we are considering a simple condition in which we have taken this dumping of the order of epsilon square, where dumping and forcing and this cubic non-linear term of the order of epsilon square and this quadratic term order of epsilon. So in that case we can the solution u t epsilon, so here we note that this epsilon is the parameter, so u t epsilon by using different time scale that T 0, T 1, let us take up to T 2 by taking this 3 times scales where this T n is nothing but epsilon n t.

So where the epsilon is the book keeping parameter and t is the time. So you can write this u t epsilon equal to u T 0T 1T 2 higher order terms also one can add plus epsilon u 1, this is a function of 0 T 1T 2plus epsilon square u 2T 0T 1T 2, so one can add the high order terms also, so by taking this u equal to in this form, so by as we considering the primary resonance condition, so you can take the external forcing equal to omega 0 plus epsilon square sigma where sigma i the t tuning parameter.

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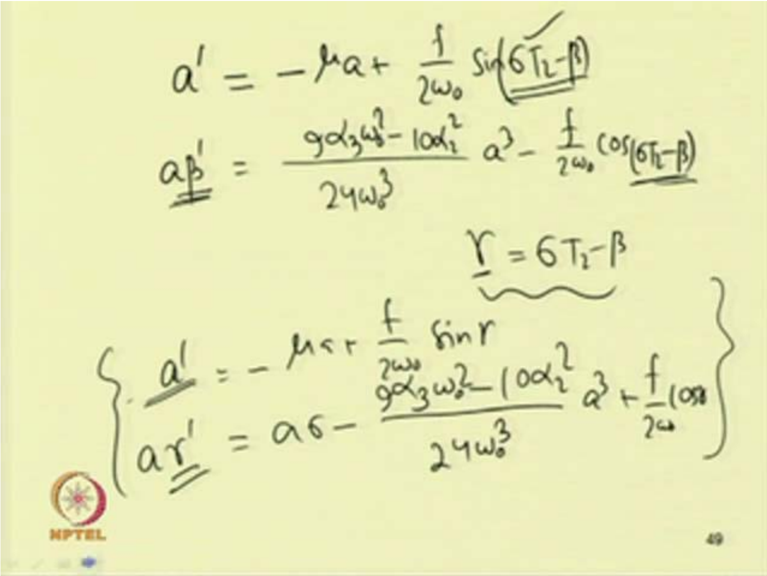

And proceeding in the previous case, as we have analysis in the previous case by substituting initially in this equation originally equation and separation in order of the epsilon different order of epsilon; we can get different equation. One such equation is written here so and then proceeding further, so we can get the reduced equation. So here the reduced equation one can get in this form, that is a dash will be equal to, so this may be this is D a by D t 2, so D equal to minus mu a plus f by 2 omega 0 sin gamma and beta dash will have equal to 9 alpha 3 omega 0 square minus 10 alpha 2 square by 24 omega 0 cube a cube minus f by 2 omega 0 cos gamma. As we did in the previous case here to, so we can we can instead of writing this beta dash we can write in terms of the gammas, so we use this gamma equal to sigma T 2 minus beta.

And this second equation we can write in this form that is a gamma dash. So here it will be actually a dash will be equal to sin beta beta and we can write in terms of gamma. So a gamma dash we can write in first equation a dash equation will so gamma dash will be equal to, so we can write, so this is actually this equation will be sigma beta minus beta. So it will be sin sigma T 2 minus beta, this is also cos sigma T 2minus beta. Soto eliminate this time term or to write in its autonomous form, so this written in non-auto autonomous form. That is terms of T, so to eliminate or to remove this T you can use this equation in their autonomous form by introducing this gamma. So gamma equal to sigma T2 minus theta, so if you introduce this thing, so this equation first equation will become a dash equal to minus mu a plus f by 2 omega 0 sin gamma and the second equation. So you can differentiate this thing with respect to T 2, so this T gamma by T will be equal to sigma minus beat dash. And substituting this equation we can write this a gamma dash will be equal to a sigma minus 9 alpha 3 omega 0 square minus 10 alpha 2 square by 24 omega 0 cube a cube plus f by 2 omega 0 cos gamma. So this 2 reduced equation for steady state condition one can write this a dash times a dash equal to 0 and this gamma dash equal to 0 so by putting a dash equal to 0 and gamma dash equal to 0 for steady state condition.

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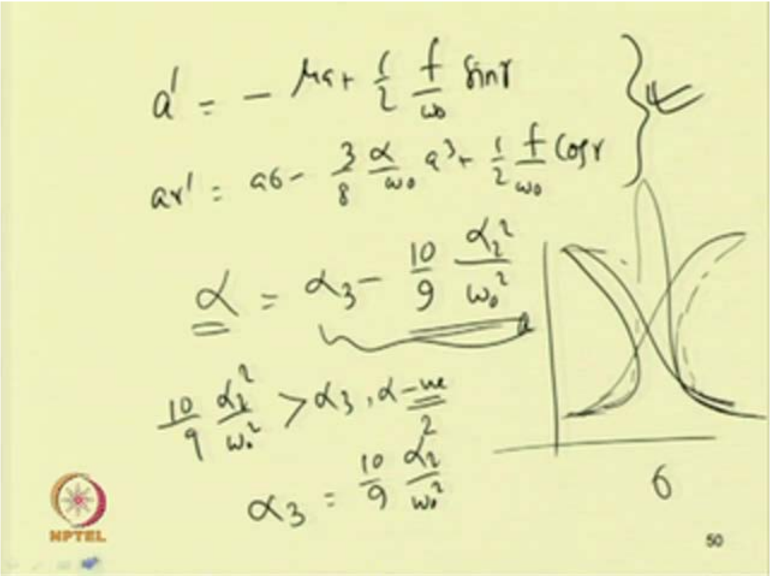

So one can get the frequency response equation, so one may note that if one can compare this equation with that of the cubic nonlinearity so one can see that this equation is similar to that of cubic nonlinearity in which we have this a dash equal to so a dash equation was minus mu a plus half f by omega 0 sin gamma and this a gamma dash equal to a sigma minus 3 by 8 alpha by omega 0 a cube plus half by omega 0 cos gamma. So by compare, so this this equation is for the system Duffing equation with cubic nonlinearity or a similar type of oscillation.

So comparing this equation with previous equation, one can note that this alpha can be substituted by alpha 3 minus 10 by 9 alpha 2 square by omega 0 square. So what we have studied before in case of this system with cubic nonlinearity if this alpha become negative, so we can have softening effect and if alpha is positive then we have this hardening type of effect. So from this equation we can see if the right hand part this part that 10 by 9, 10 by 9 alpha 2 square by omega 0 square greater than alpha 3, then this part also become negative and in that case we can have softening type of response. So in case of softening type of response, if one plot a verses sigma, so that response curse will like this. So in this case we have softening type that means it tends towards or tilts towards the lower frequency range.

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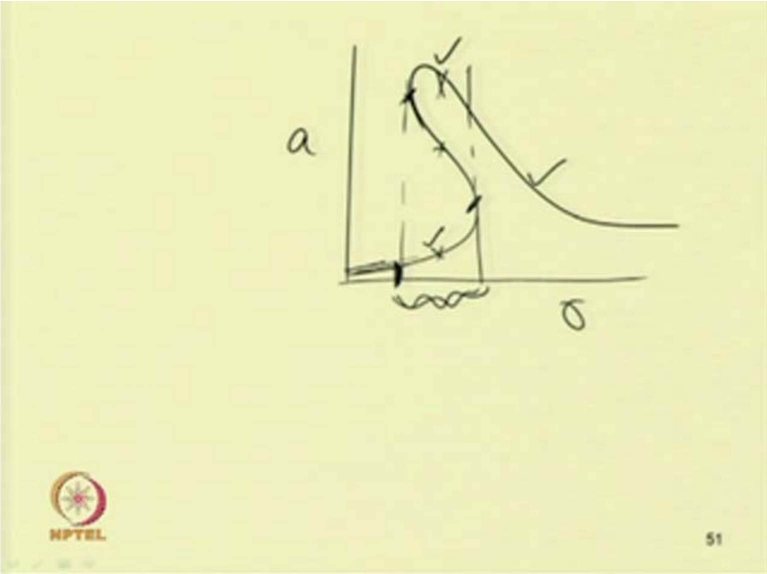

And in case of hardening type of response the tilting will be towards the higher frequency range. So in this case what we have seen if this 10 by 9 alpha 2 square by omega 0 square greater than alpha 3, that means this part becomes negative alpha becomes negative so we have softening type of response. So if this is equal to 0 so alpha 3 equal 10 by 9 alpha 2 square by omega 0 square. That means if one take the non-linear term in such way that this alpha 3 equal to 10 by 9 alpha 2 square by omega 0 square, so in that case this alpha becomes 0, so alpha equal to 0 means, so we have a linear response of the system. So the system behavior will be that of a linear system that means irrespective the value of the quadratic and cubic nonlinearity, so the system will behave as a linear system. So if alpha 3 equal to 10 by 9 alpha 2 square by omega 2 square and if alpha 3 greater than this 10 by 9 alpha 2 square by omega 2 square. So in that case this alpha will become positive and we can have a hardening type of response.

So the response will be that of a hardening type of resonance. So in this case, so you have seen that in this case we have 3 or let us take a particular case and so we have 3 joules. Already this part has been discussed, so up to this value this is a verses sigma, so up to this value so if have a single response and in this range we gave in12 and 3 response out of which one can observe this part this to this part is unstable, so we have a this part is stable and this branch is stable. So we can have or we have two stable branches and after this we have a single stable branch. So between this ranges we have a bi-stable region, so in this by stable region depending on the initial condition one can have the one can have the lower branch of the response or the upper branch of the response. So one can or one can plot the basin of attraction to know so what will the system basements.

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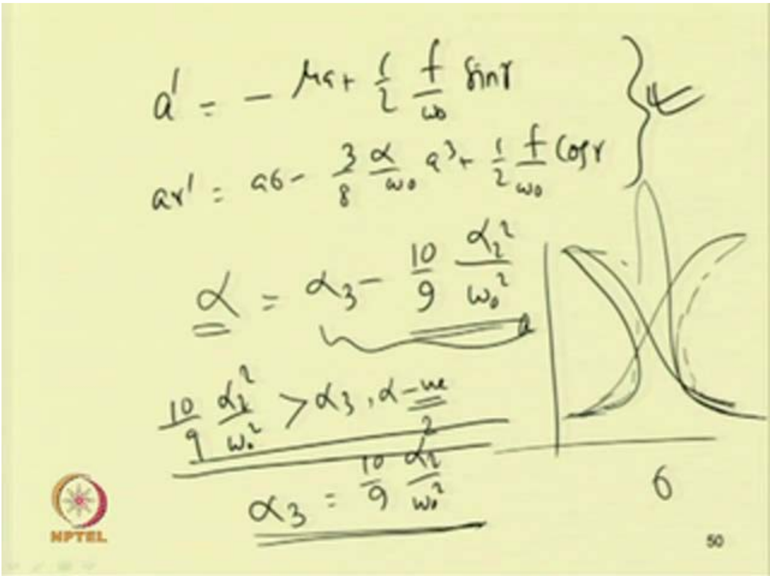

So in this case now you have studied the effect of cubic and quadratic nonlinearity that is if the condition is like this alpha 3 is greater than, if alpha 3 is greater than 10 by 9 alpha 2 square omega0 square. The system will show hardening type of effect, so if it is if this condition holds good then the system behave so a soft spring type of thing then the system response will behave or so this softening effect and if this is equal to 0 that is alpha 3 equal to 10 by 9 alpha 2 square by omega 0 square, in that case we have a linear type of response or we have linear response of a systems.

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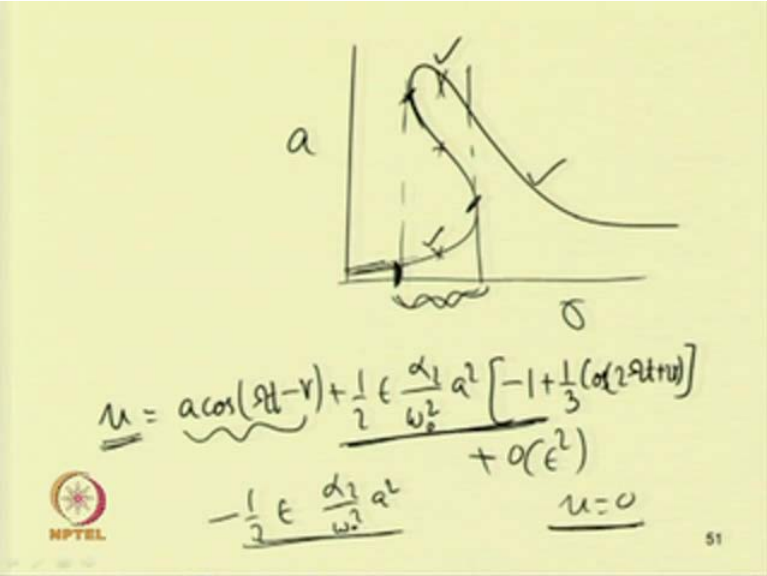

So in this case we can write this response of the system u equal to a cos omega t minus gamma plus half epsilon alpha 2 by omega 0 square a square minus 1 plus 2 gamma plus order of epsilon square.

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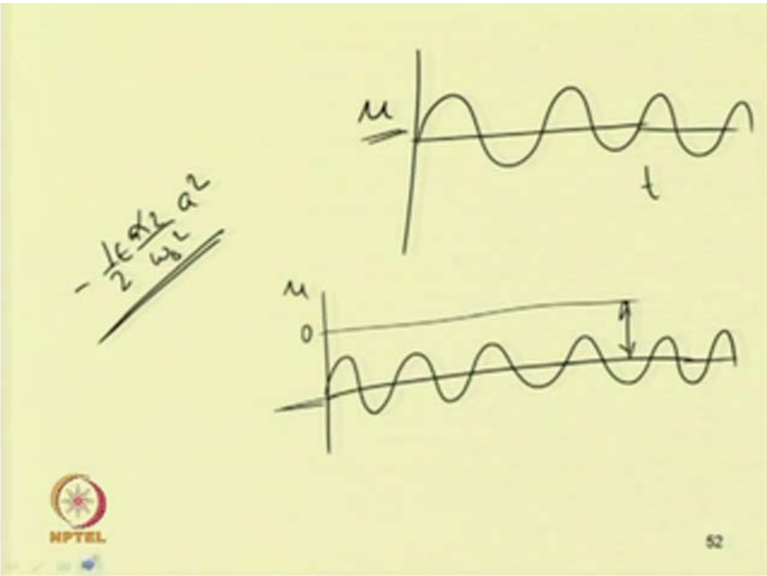

So one can note that in case of cubic nonlinearity, if have the same term but in this case in this case one may note that due the presence of this term, that is due the presence of minus epsilon half alpha 2 by omega 0 square a square, so this response will know response will not be 0 or it will not oscillate about the u equal to 0. There will be adrift from the u equal to 0line and that is due this presence of this quadratic nonlinearity. So due the presence of the quadratic nonlinearity, so before one can plot the response like this, now due to the presence of this quadratic nonlinearity, so this is when quadratic nonlinearity was not there; now due the presence of this nonlinearity there will a drift that is this u equal to 0 and the response will be, so one have the response of the system this way. So one can plot the response of the system which will be a combination of cos omega t and cos 2 omega t.

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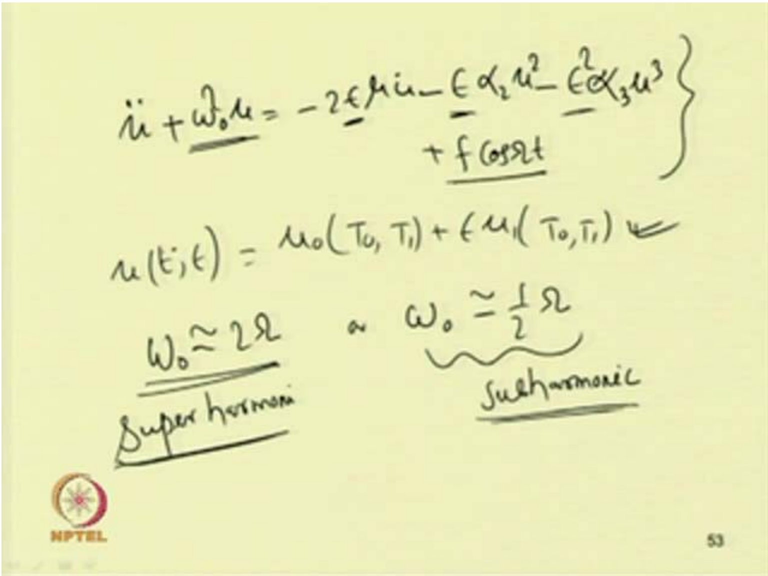

There is a drift here, so this amount or this drifting is nothing. But, so this magnitude equal to half so this is equal to half minus half alpha 2 by omega 0 square epsilon into a square. So the oscillatory motion no longer oscillate about the line mu equal to 0 but it will oscillate about the point that is minus half epsilon alpha 2 by omega 0square a square. So in this case also one can study the sub harmonic and super harmonic resonance condition. So instead of taking the forcing term equal to or of the order of epsilon square, so one can take of this order of this linear term. That is by writing the equation in this form u double dot plus omega 0 square u equal to minus 2 epsilon mu u dot minus epsilon alpha 2 u square minus epsilon square alpha 3 u cube plus f cos omega t. So here we have taken this forcing term of the same order as that of the linear part and this dumping of the order of the epsilon quadratic linearity of the order of epsilon and this cubic nonlinearity of the order of epsilon. So in this case we can by proceeding in the similar way one can see that by taking this u t epsilon equal to u 0 T0, T1plus epsilon mu one T0, T1, so one can consider so one can proceed and one can see that one can obtain the resonance condition when omega 0 nearly equal to 2 omega or omega 0 nearly equal to half omega. So in this case omega 0 nearly equal to 2 omega, so this is the super harmonic resonance condition.

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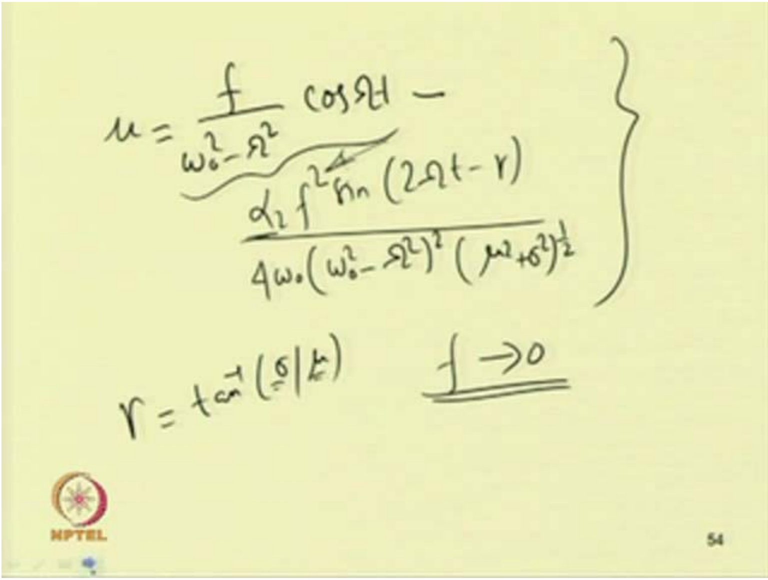

Super harmonic resonance condition and this case is the sub harmonic resonance condition. So similar to the case of cubic nonlinearity in this case of both cubic and quadratic nonlinearity also, one can study this effect of super harmonic and sub harmonic resonance condition is in this case. In case of the super harmonic conditions, if one proceeds one can see the response can be obtained in this way.

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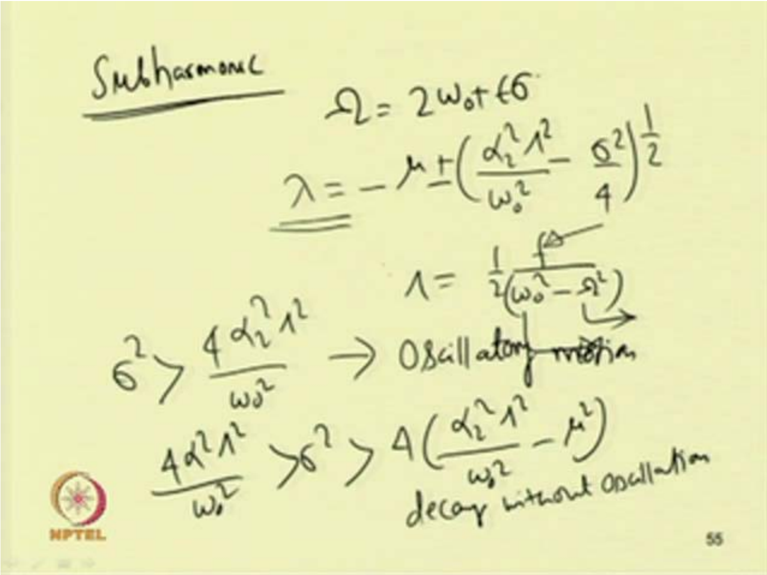

So u will be equal to f by omega 0 square minus omega square cos omega t minus alpha 2 f square sin 2 omega 2 omega t minus gamma 4 omega 0 into omega 0 square minus omega square whole square into mu square plus sigma square whole to the power half. So here one may note that the alpha 3 term is not present in this considering of the order of epsilon only, so here if one has a week forcing term then at, so let's take a condition when the forcing term tends to 0.

So, if the forcing term tends to 0, so this f square term this term this f square term tends to 0 in comparison to the first part. So one can obtain a response, so if tends to 0; so one can obtain a response similar to that of the linear system of the linear system. Similar to the of the linear system, so here one can write this gamma equal to, so here gamma equal to tan inverse sigma by mu, where sigma is d tuning parameters and mu is the mu is the dumping of the system. Similarly, in case of sub harmonic resonance, so in case of sub harmonic resonance one take this omega equal to 2 omega 0 plus epsilon sigma and one can see or the roots of the characteristic of equation or roots of the response can obtain in this way. So this will be minus mu plus minus alpha 2 square lambda square by omega 0 square minus sigma square by 4 to the power half. So here this lambda equal to half f by omega 0 square minus omega square.

So here this omega is the external frequency, this omega is the natural frequency of the system, f is the amplitude of the external excitation. So depending on this root lambda, so if this root is positive then if the real part this root is positive, then the system is unstable, if the real part of the root is negative then the system is stable and we can have the different conditions; that is f sigma square greater than 4, alpha 2 square lambda square by omega 0 square. Then one can get imaginary roots also, so due the presence of the imaginary root the motion will be oscillatory, so one can get oscillatory motion. Similarly, when this sigma square or this part let me write this way so if 4 alpha square lambda square by omega 0 square greater then sigma square greater than 4 into 4 into alpha 2 square lambda square by omega 0 square minus mu square.

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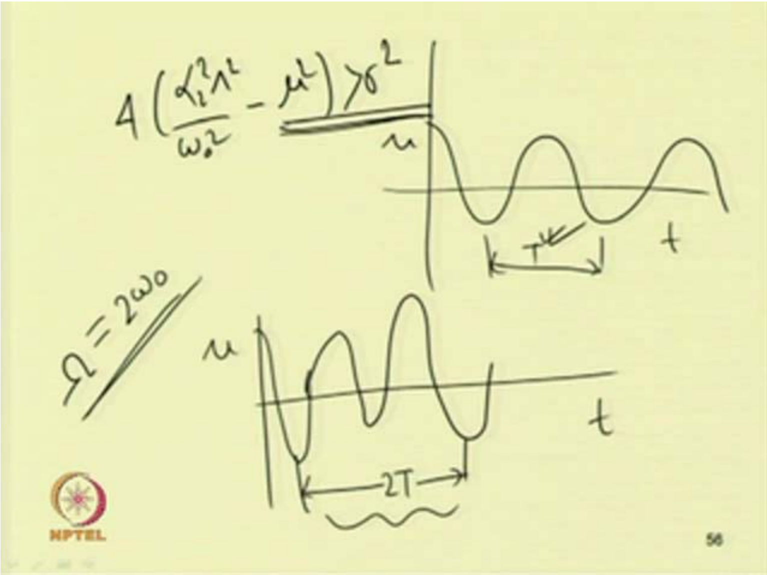

So in this case one can get positive response or one can one can observe that the system will decay. So it will be over dumped, so it will be the system will decay without oscillation. So the last one when will get the positive root that means if we consider the condition 4 alpha 2 square lambda square by omega 0 square minus mu square greater than sigma square, so one can observe that the system response will be unstable as the response will grow they would not sub bound.

But, in actual case the system cannot be are unbounded, so one has to perform the linear non-linear analysis to obtain or one can perform for that analysis one can take additional terms to study the system response and stability of the system. So in this case one can note that if one plot u verse t, so when this sub harmonic resonance condition is not considered the response will have a time period of t. So it will have a time period of t but when we are considering this sub harmonic resonance condition so here the response will be if one numerically integrate and plot it so one can get the response of the system this way . This to this now this is 1 period of the system, so this period is 2 t unlike here the period is t, so when one consider the sub sub harmonic condition, then one can see the time response. If one plot so this becomes 2 t so in this case one can see this external excitation or the pre oscillation adjust itself to that of the external excitation which is nearly equal to twice omega 0 of the system.

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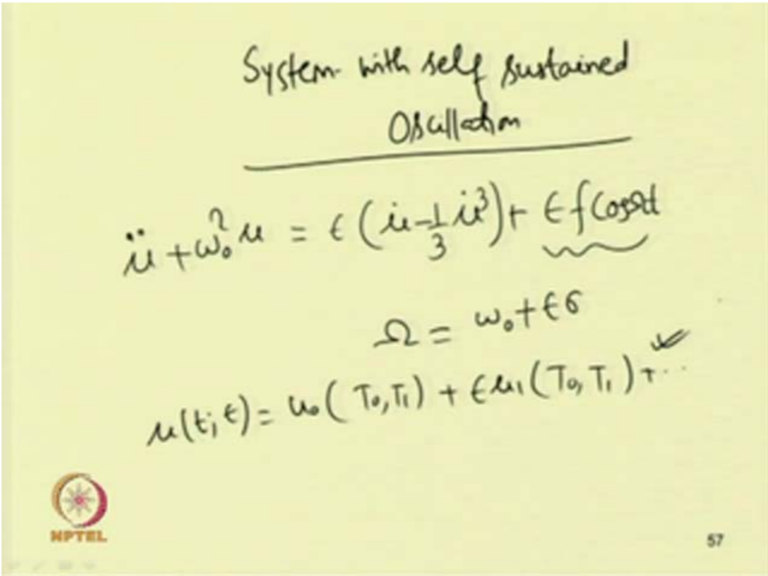

So one can either one can solve this equation using a thorough multiple scale or using other types of methods or one can solve this equation numerically to obtain theresponse of the system. So let us consider another case, so in which we have the system with self-sustained oscillation system.

With self-sustainedoscillation, so in this case; so the equation is similar to that of the Van der Pol equation. So let considerthe equation in this way in this form that is u double dot plus omega 0 square uthis equal to epsilon u dot minus one third u dot cube plus, let us take the forcing term of the order of epsilon, epsilon f cos omega t. During free vibration of the system with have studied the free vibrationresponse of the Van der Pol type of oscillator, so there we have seen the resulting solution is a limit cycle. So in this case we have considering a similar equation but we have taking the forcing into account. So considering the primary resonance condition, that is omega linearly equal to omega 0, so let me write using determine parameter also, omega equal to omega 0 plus epsilon sigma. So i am taking this u t epsilon equal to 0, so let me take this plus so epsilon u one T 0, T 1, and high order terms so and following the usual procedure of method of multiple scale so we can obtain the reduced equation.

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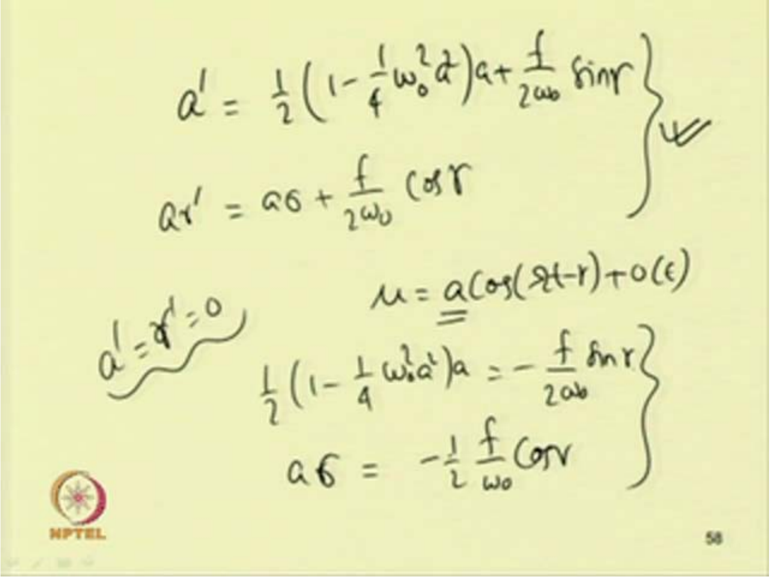

So in this case the reduced equationare of this form this a dash equal to half one minus 1 by 4 omega 0 square a square into a plus f by 2 omega 0 f by 2 omega 0 sin gamma and then a gamma dash equal to a sigma plus f by 2 omega 0 cos gamma. So this are the reduced equation one can obtain by using the method of multiple scale. So in this case one can eliminate the this gamma term, so by eliminating gamma term for steady state solution; first let us see for steady state solution this a dash equal to be 0 and gamma dash will be equal to 0. This u can be writtenin this form u equal to a cos omegat minus gamma of the order of epsilon, so by obtaining this a response one can plot the response of the system. Now one can see for steady state response this a dash equal to gamma dash equal to 0, so by putting a dash equal to gamma dash equal to 0 in this reduced equation on can write half.

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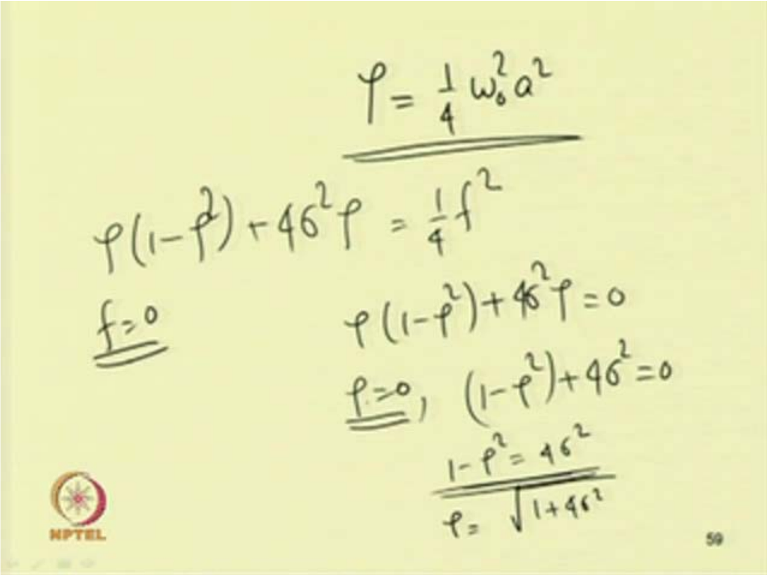

1 minus 1 by 4 omega 0 square a square into a will be equal to minus f by 2 omega 0 sin gamma, then a sigma equal to a sigma equal to minus half f buy omega 0 cos gamma. Now by squaring and adding, so we can get this equation. So let us take oneterm that is rho equal to 1 by 4 omega 0 square a square. By taking rho equal to 1 by 4 omega 0square a square and squaring and adding this previous 2 equation, so 2 equations, so we obtain this equation. So rho in to 1 minus rho square plus 4 sigma square rho equal to 1 by 4 f square so in this case one can plot the rho verses sigma to obtain the frequency response curve of this system. So forequal to 0for f equal to 0, so one can see this rho in to 1 minus rho square plus 4 sigmasquare rho equal to 0. So in this case either rho equal to 0 or by taking common rho either rho equal to 0 or one can have this 1 minus rho square plus 4 sigma square equal to 0 so in rho sigma so either rho will be equal to 0.

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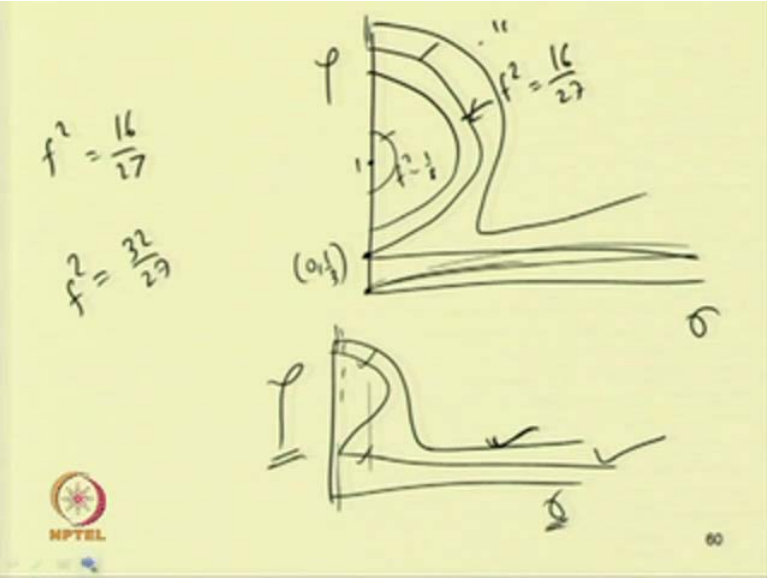

Or form this equation one can findwhen sigma equal to, so one can find 1 minus rho square minus equal to 4 sigma square andrho will be equal to1 plus 4 sigma square root over. So for sigma equal to 0, so we have the solution rho equal to0 and rho equal to 1 so rho equal to 0 and rho equal to 1. So if one plot thisrho verse sigma so for sigma equal to 1 already we have seen, so we have 2 solutions; so one solution equal to rho equal to 0 and other solution equal to rho equal to 1. So if we go on increasing this value of f, this forcing parameter so if you increase this value of forcing parameter

So one can see, so one can find the value 2 different value of rho for a particular value of sigma and one can plot this. So this is one value one can solution, so one can get3 solutions here, so in one branch you can obtain this thing and here also one can, so one branch will start near this rho equal to 1 and other branch will start from rho equal to 0. So it will continue, so one branch will be like this and other solution will be this so if for example, this is this value is for f square equal to 1 by 8. Similarly, for f square equal to 1 by 4 one can plot another curve so 2 branches will be there. So this will continue till, so this will continue we have thisf square equal to 16 by 27, so in that case when it is16 by 27 these two branches will merge and they will merge at this point that is 0 one third. So this is 0 one third, so will have double point. Here we have a double point, here at this point and with further increase of so this is f square equal to 16 by 27f square equal to 16 by 27. So by taking higher value of sigma, that is k square nearly equal to 32 by 27, f square equal to 32 by 27, so we can have branch like this.

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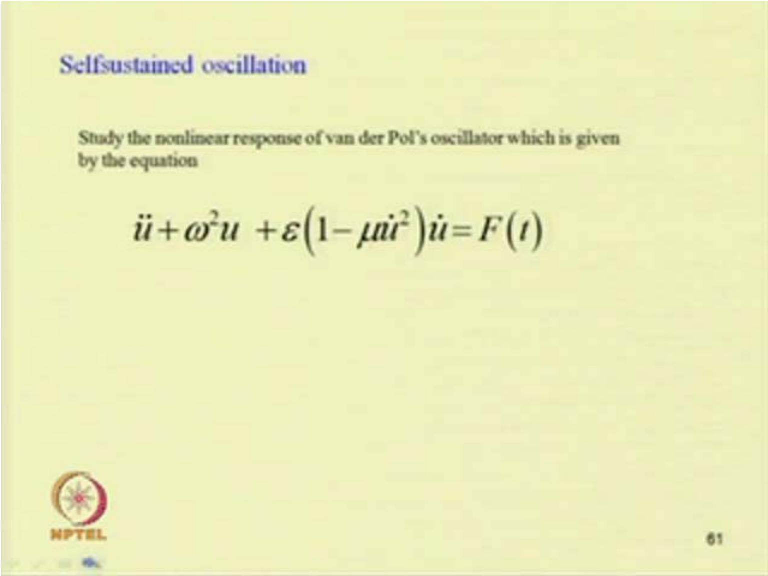

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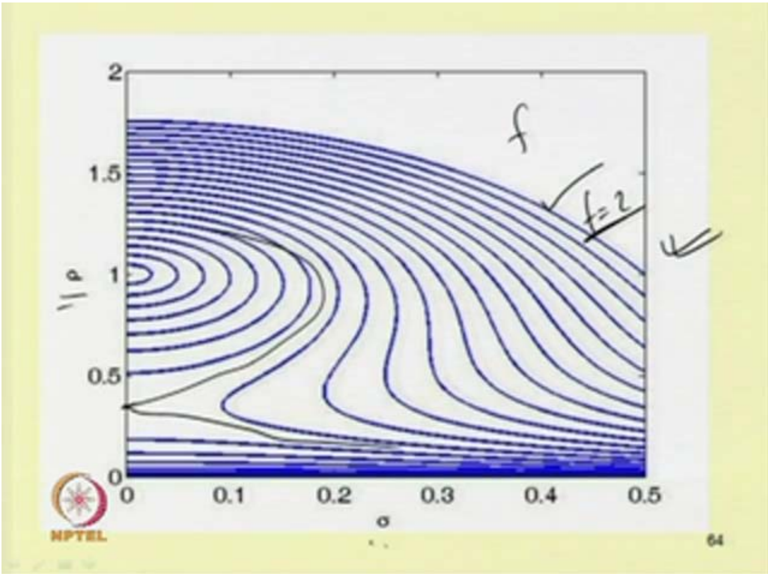

So here as we have a branch like this if one plot rho verses sigma, so one can distinguish between different zones. So one can observe three different zones, so up to some value of sigma one can have a single branch, then after that one can have three branches then after which one can have a single branch. So when we have single branch that is stable, when we have this 2 for 3 branches this upper and lower stable and the middle one is unstable. Similarly, the right hand sidebranch also one can see this is unstable. So actual stability of this curve can be studied by perturbing this equation and one can find for that this system is stable or unstable by plottingthe response of the system.

So here for the different value of rho and sigma, for different value of f this curve have been plotted. So one can observe that one has a one single response when f is, so for example, this is f nearly equal to 2, so this is for f equal to 2, so this value is f equal to 1 by 4, 1 by 8.

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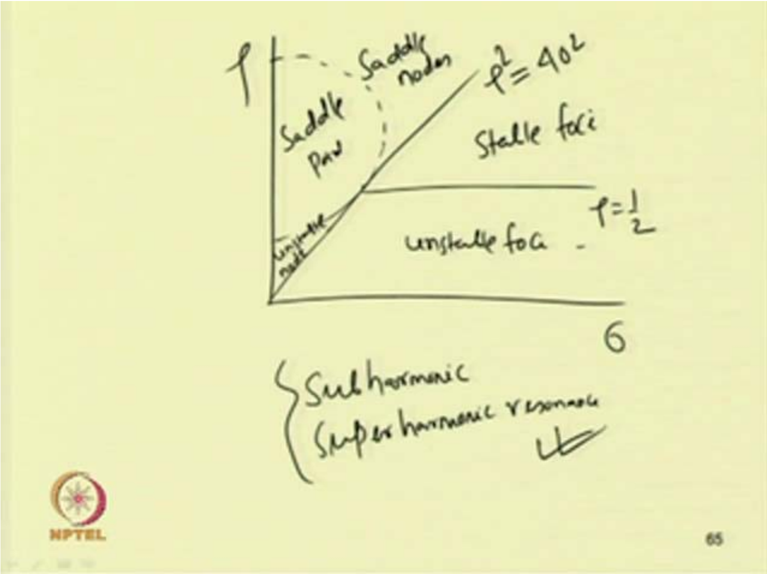

And in between one can have a branch where it will be, one will have adouble point. Soin this way one can study the response of the system for self-sustained oscillation and study the stability of the system. So to study the stability the system one can actually plot the response and one can see that in this zone, so this is rho verse sigma. So in this zone one can have t saddle point saddle point, so here you have, one will have this saddle nodes. So here one will have stable foci, so this is the line of rho square equal to 4 sigma square. So here unstable node, so one will have unstable node, so one will have stable node here unstable focirho equal to half, so this is this line.

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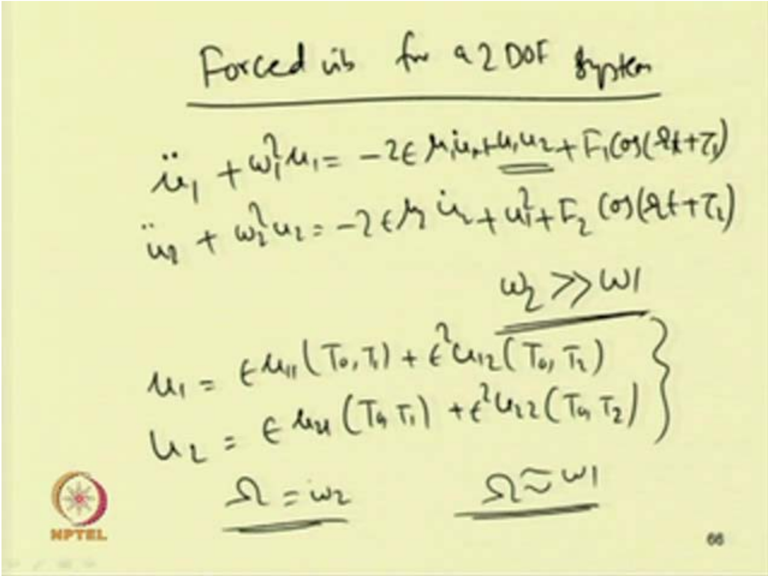

So this line is for rho equal to half, so similar to the case what we have studied for cubic nonlinearity and the both cubic and quadratic nonlinearity here one so one can study the sub harmonic and super harmonic resonance condition. So this is left as in exercise problem sub harmonic and super harmonic resonance condition. So this is left as an exercise problem to study the sub harmonic and super harmonic resonance condition in case ofthis self-sustained oscillation. Also one can take one can write the force oscillationforce vibration for a 2 degree of freedom system, so let us take this equation and it is left as an exercise problem also, u 1 double dotfor a 2 degree of freedom system. The equation can be written in this form u 1 double dot plus omega 1 square u 1 equal to minus 2 epsilon mu 1 u 1 plus u 1.

So taking quadratic nonlinearity one can write this is equal to u 1 into u 2 plus f 1 cos omega 1 t plus tau 1similarly, u 2 double dot plus omega 2 square u 2 equal to minus 2 epsilon mu 2 u 2 dot. So this u 1 dot, so this is u 2 dot plus u 1 square plus f 2 cos omega t plus tau 2 so where this tau 1, tau 2 are the phase difference, one can consider this omega 2 greater than omega 1 and study the response of the system. So similar to in case of single degree of freedom system, so here one canconsider this u 1 equal to epsilon u 11T 0 T 1 plus epsilon square u 1 2 T 0 T 2 and u 2 equal to epsilon u 2 1 T 0T 1 plus epsilon square u 22 T 0 T 1 and proceed for different resonance condition.

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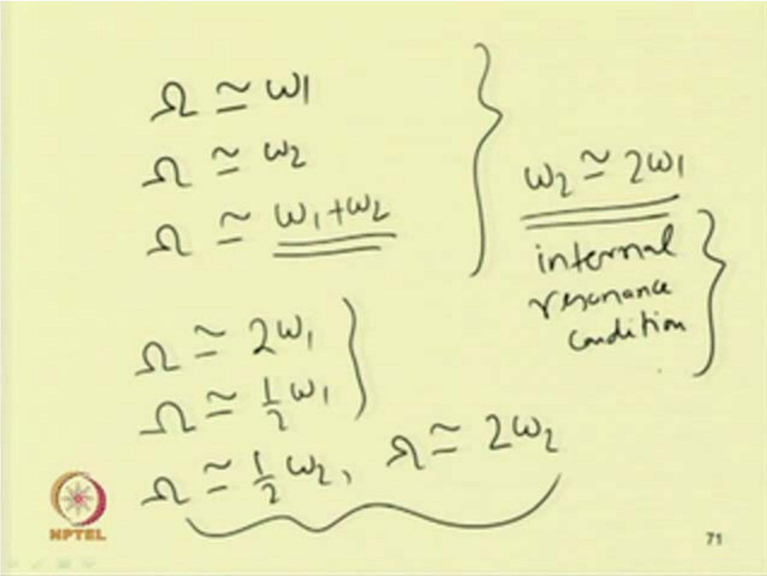

For example, if one take the resonance condition of omega nearly equal to omega 2 or 1 can the resonance condition of omega nearly equal to omega 1 also one may consider the resonance condition of combination of this omega 1 plus omega 2 combination of omega 1 plus omega 2. So different resonance condition one can study some of the resonance condition are written here, so omega 1 nearly equal to omega 2 then omega may be combination of omega 1 plus omega 2.

So this is the combination type of resonance so one can consider the case when this omega may be nearly equal to twice omega one so in that case one we observe the internal resonance conditioninternal resonance condition, so one can study several different types of resonance condition. So here also one can study this sun harmonic super harmonic resonance condition 2omega1, 1 by 2 omega 1.Similarly, omega nearly equal to 1 by 2, omega 2 omeganearly equal to twice omega 2. So all this resonance condition one can study for this 2 degree of freedom system. So we have observed like in case of the linear system, where we have only 2frequency in case of this 2 degree of freedom system, so here or 2 resonance conditions. So here in case of the non-linear system we can have several resonance conditions.

So, one of them can be primary others secondary, in secondary resonance condition one can have the sub harmonic super harmonic in addition. To that we can have this combination type of resonance condition, so and considering the internal resonance conditions also one can get other different type of resonance condition in the system, so one can study all those resonance condition to find the response of the system, so next class we will study the model reduction method to study the non-linear vibration of multi degree of freedom system.

Thank you.