Non-Linear Vibration

Prof. S.K. Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 6
Applications
Lecture - 5
Nonlinear Forced-Vibration of Single and Multi Degree-of-Freedom System

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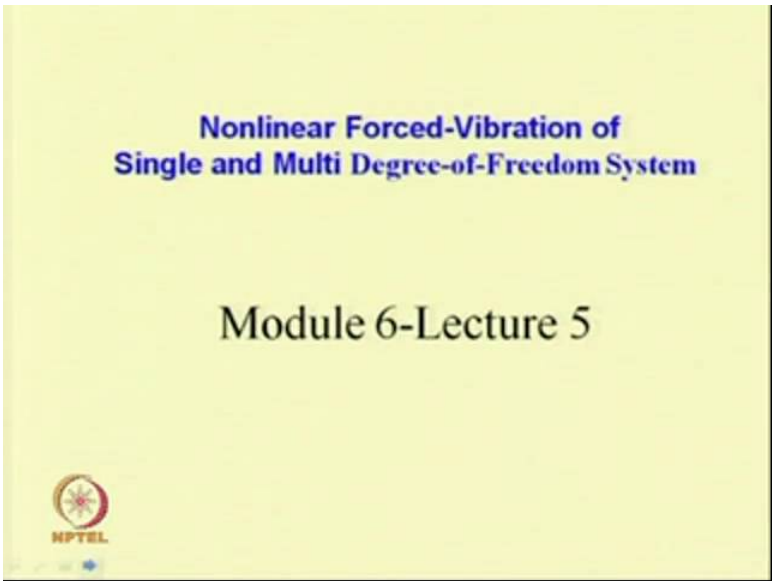

So, welcome to today class of non-linear vibration. Today class we will discuss about the non-linear force vibration of single and multi degree of freedom systems. Already we have or are we are disusing the single degree of freedom systems, so where we have seen how one can apply a weak forcing or strong forcing in this system, and we have studied up to the supper harmonic resonances condition in case of hard excitation. So, today class we will make revision of this single degree of freedom system, and also briefly we will introduce about this multi degree of freedom system. And next class will study in detail about the multi degree of freedom system force vibration.

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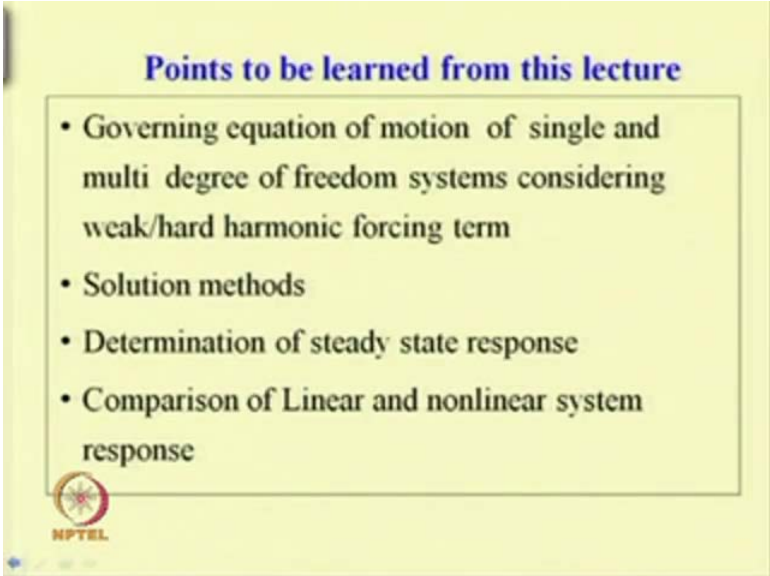

So, as we know that so in this class will see how the governing equation motion of single and multi degree of freedom system considering weak and hard forcing term has been derived. Then solution method determination of steady state response comparison of liner and non-linear system response.

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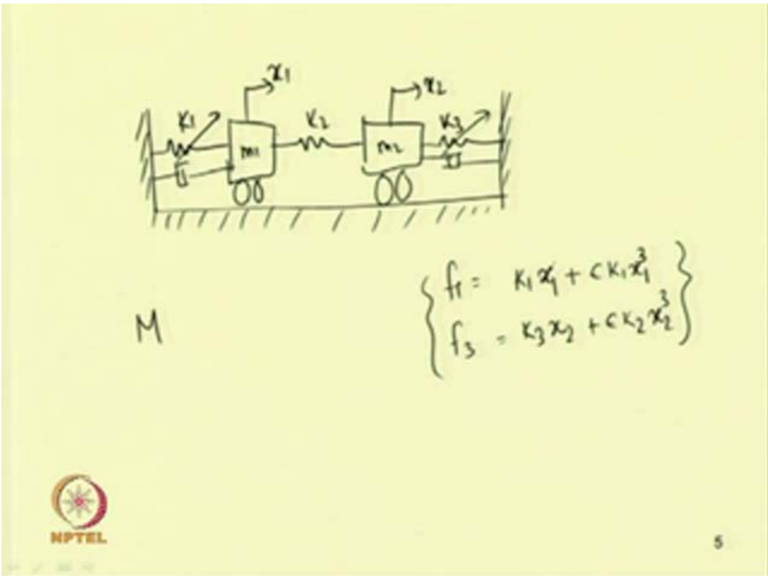

So, we know that let us start from a multi degree of freedom system and will see how our this single degree of freedom system analysis can be reduced to that of a multi, or how the knowledge of the single degree of freedom systems can be extended to that of a multi degree of freedom system. So, for example, let us take this three spring mass system. So, this is a mass and this is the second mass and we have springs and dampers attached to the systems. So, or in so let us let us put a system like this simple system.

So, in this case this is k 1 this is m 1 this is k 2 m 2 and k 3. So, if we make the spring k 1 or this first spring non-linear and the third spring non-linear. So, we can write the equation motion. So, the equation of motion for this case can be written in this form that is a mass matrix mass matrix. So, if we are writing this is x 1 and this is x 2 then for this for this spring it will be the spring force will be we can write this spring force equal to k 1 into let me put in this way. So, k 1 into or k 1 x k 1 spring force f 1 equal to k 1 x plus as it is we are assuming to be non-linear, then I can put this is equal to epsilon k 1 let it is cubic non-linear. So, it will be epsilon k 1 x cube.

Similarly, this for the third spring we can write this spring force equal to k 3, k 3 x 2 plus. So, this is k 1 x 1 and this is x 1 because this spring first spring will be extended by an amount the x 1. So, k 3 x 2 plus let me put it epsilon k 2 x 2 cube. So, this way we can introduce the non-linear, nonlinearity in the system. So, this first mass or if you write the free body diagram of the first mass.

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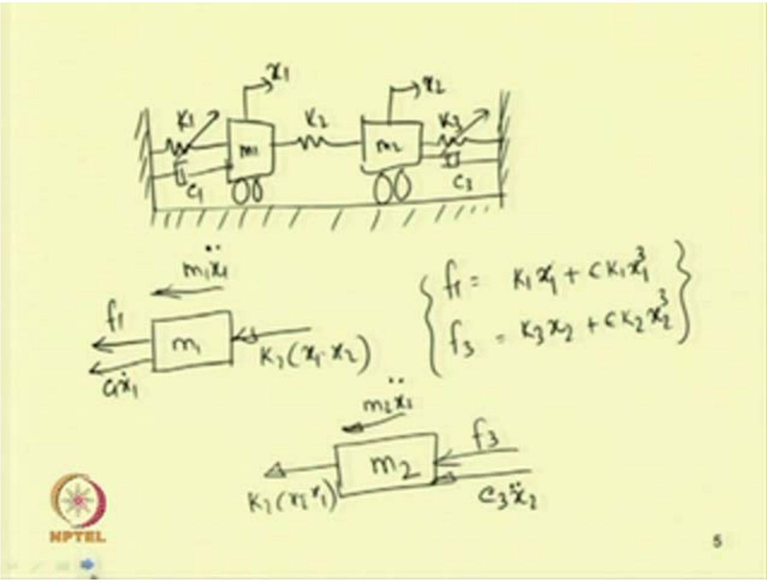

That is m 1 this case can be written in this form. So, this is subjected to a displacement x 1 towards right. So, it will be subjected to a inertia force of m 1 x 1 double dot towards left. Similarly, this spring force f 1 already we have spring force f 1 which is non-linear and this damping force also. So, damping force which will be. So, if this is c 1. So, this will be equal to c 1 x 1 dot and to as this spring k 2. So, let us assume a linear spring. So, assuming a liner spring. So, this will have a relative motion of x 1 minus x 2. So, this x 1 minus x 2 force will be equal to in this direction x 1 minus x 2.

Similarly, we can write the equation motion of the second mass. So, this is mass m 2. So, due this inertia force as it moving towards right, the inertia force will act towards left that is m 2 x 2 double dot and then the other forces will be equal to. So, this is f 3 that is the non-linear spring force and we can have this. So, let me write this is c 3 then it will be c 3 x 2 double dot. Similarly, this force will be equal to, this force I can write. So, this will be equal to k 2 into x 2 minus x 1. So, k 2 into x 1 minus x 2 force will act towards right and we can write this force towards left like this. So, m k 2 into x 2 minus x 1. So, combining this. So, we can write or we can write this equation motion in this form.

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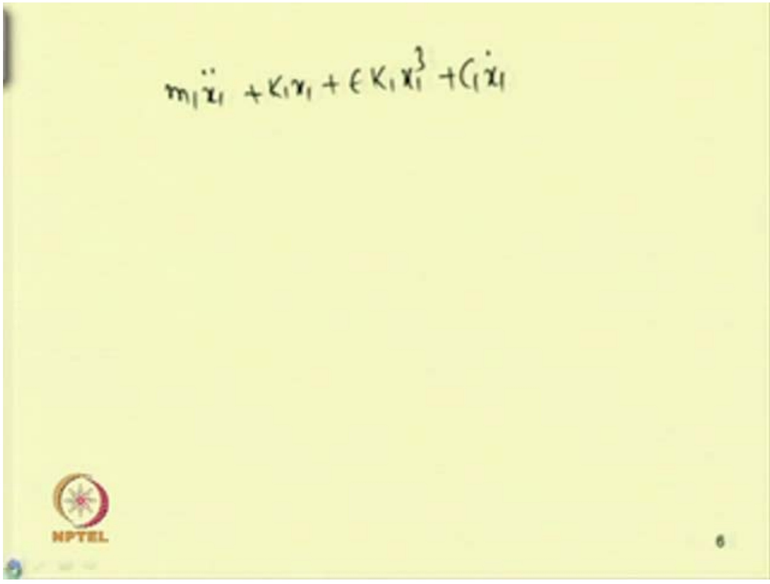

That means, m 1 x 1 double dot m 1 x 1 double dot plus f 1. So, for f 1 I can write this equation in this form that is k 1 x 1 plus epsilon k 1 x 1 cube. So, this is f 1 then plus c 1 x 1 dot c 1 x 1 dot. So, if I am applying a force.

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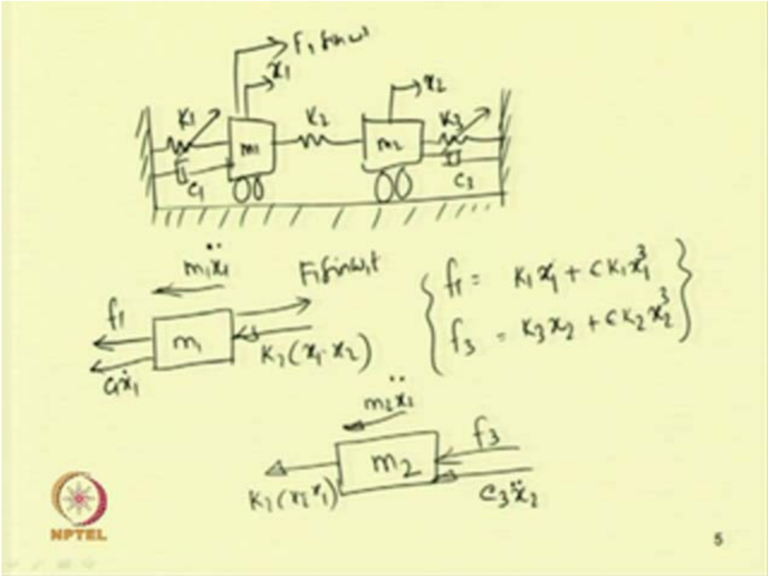

Let me apply a force capital F 1 F 1 sin omega t. So, that force, this will be equal to F 1 sin omega 1 t. So, I can write this equation.

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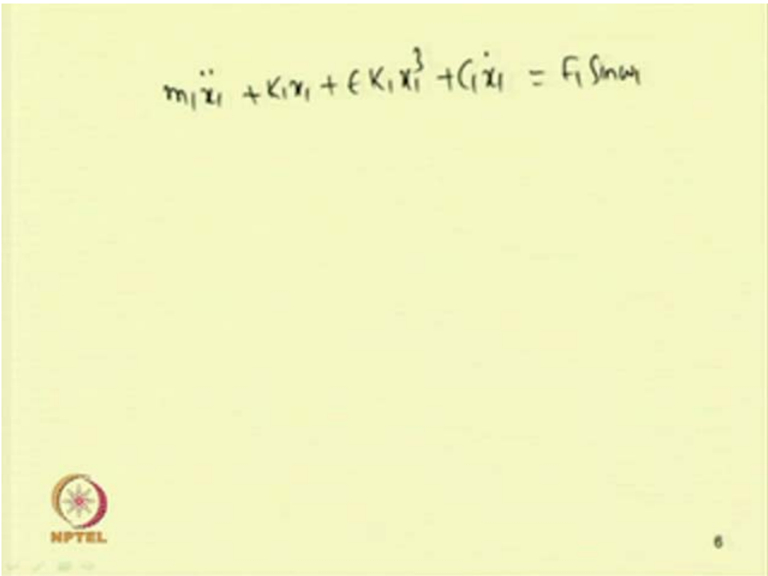

Equal to F 1 sin omega 1 t, so this way 1 can write the first equation and the second equation can be written from this Which is m 2 x 2 double dot plus k 2 x 2.

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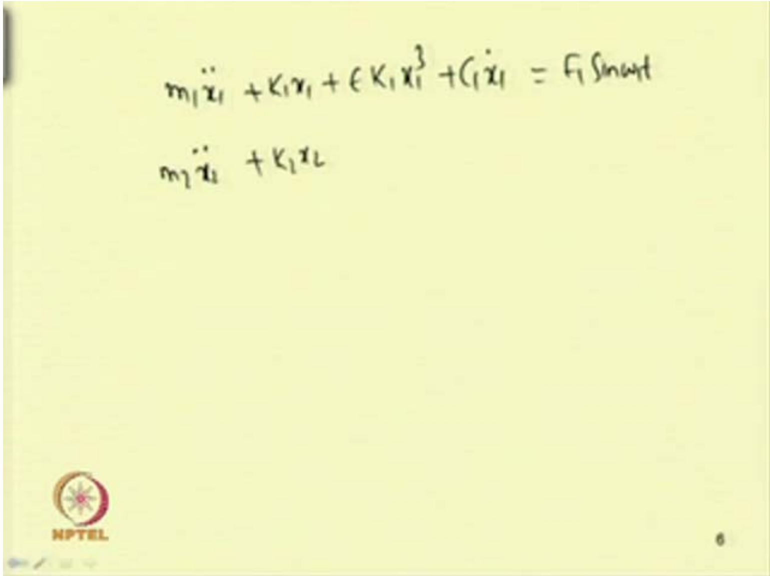

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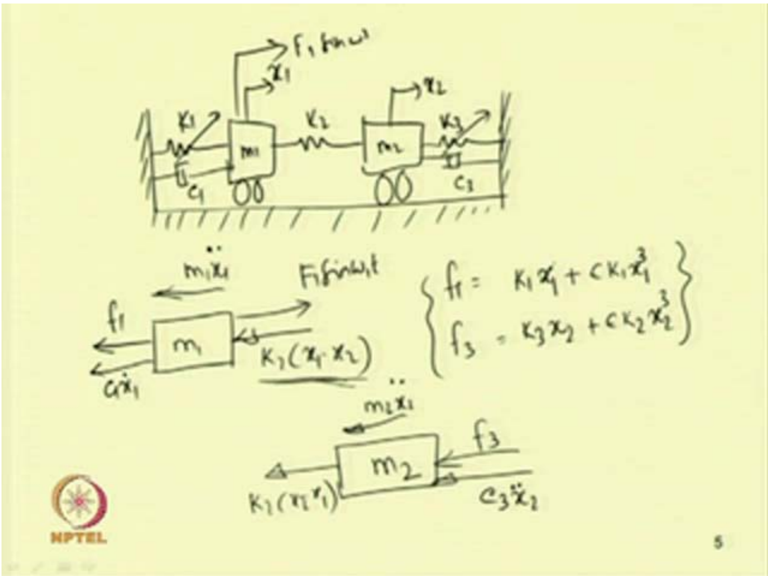

So, one can write this force. So, in the first equation I have to add this force k 2 into x 1 minus x 2.

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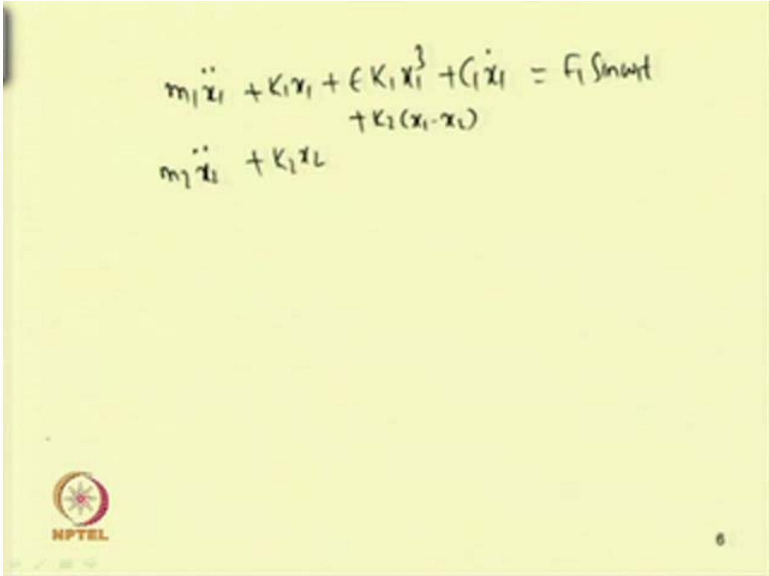

So, it is not been added. So, plus k 2 into x 1 minus x 2. So, this will be equal to F 1 sin omega t. Similarly, for the second equation m 2 x 2 double dot plus k 2.

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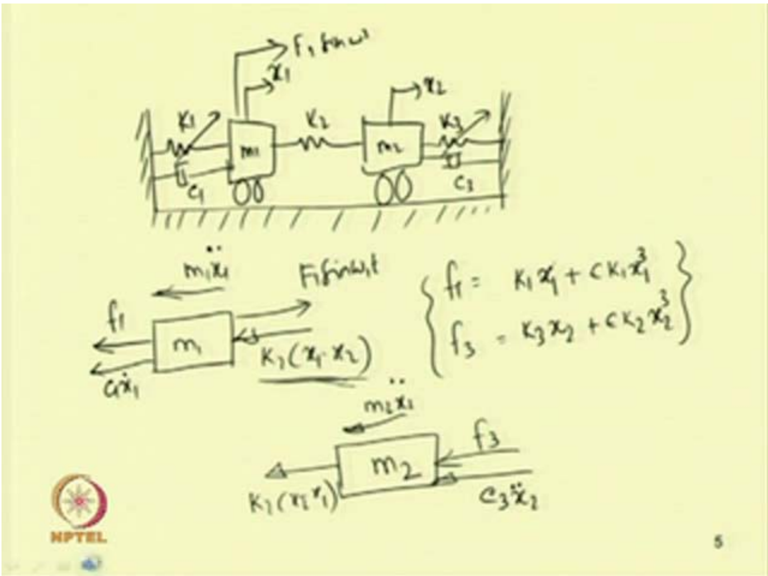

k 2 x 2 minus x 1 plus f 3 f 3 is k 3 x 2 Plus k 3 x 2 plus k 3 x 2 plus epsilon k 2 plus epsilon k 2 x 2 square x x 2 cube.

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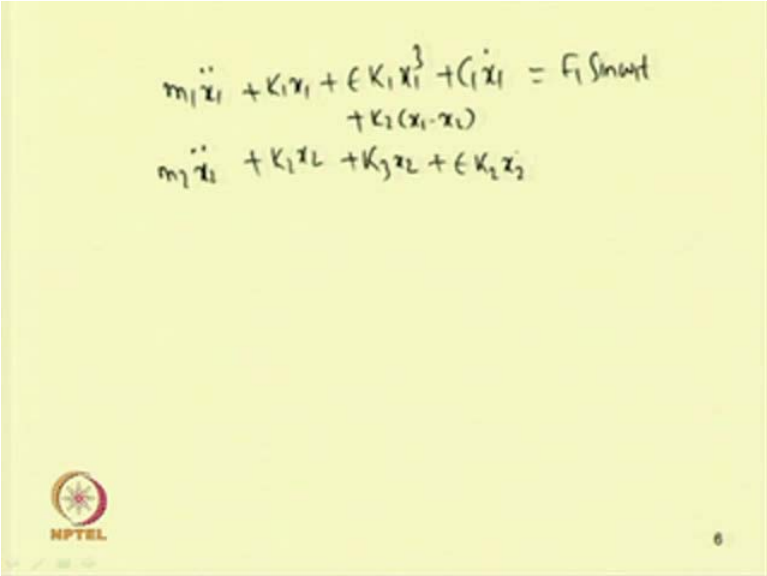

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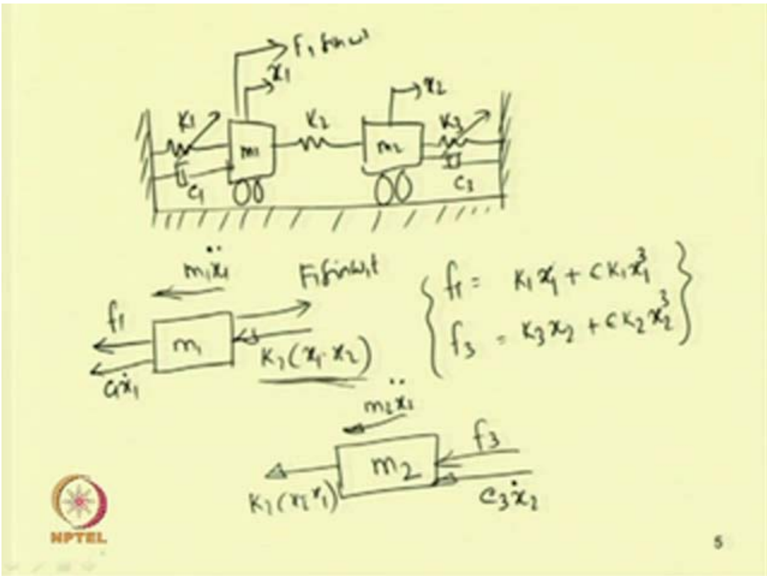

Let us see. So, what are the other forces acting. So, damping force plus c 3. So, damping force is x 2 dot not x 2 double dot c 3 x 2 dot.

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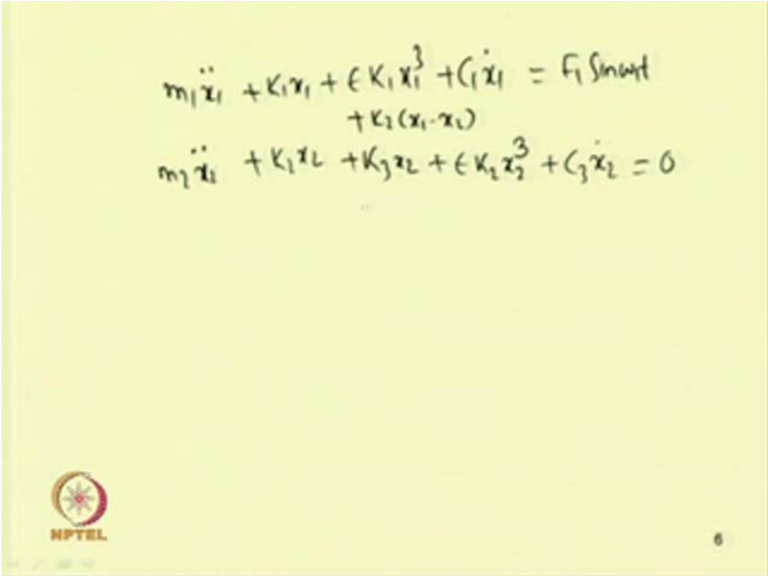

So, plus c 3 x 2 dot. So, this will be equal to 0. So, let us see what we have written.

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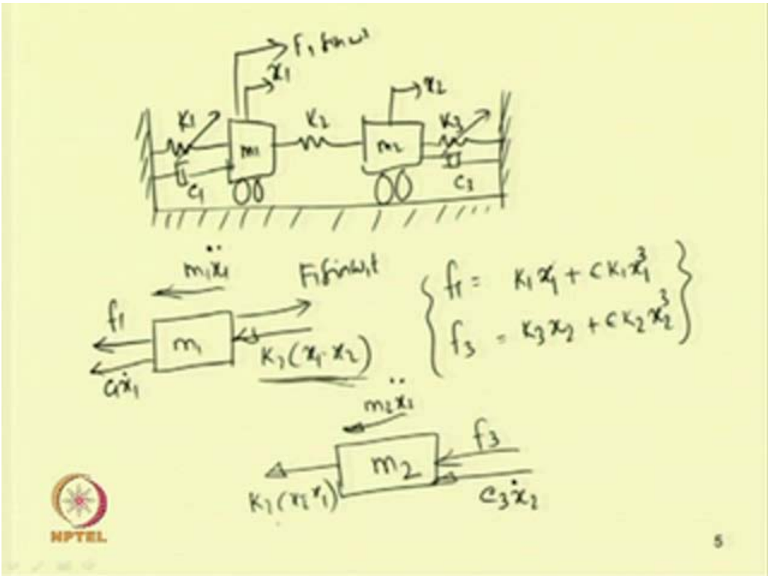

So, m 2 x 2 double dot. So, this is the force acting this way plus k 2 into x 2 minus x 1 k 2 into.

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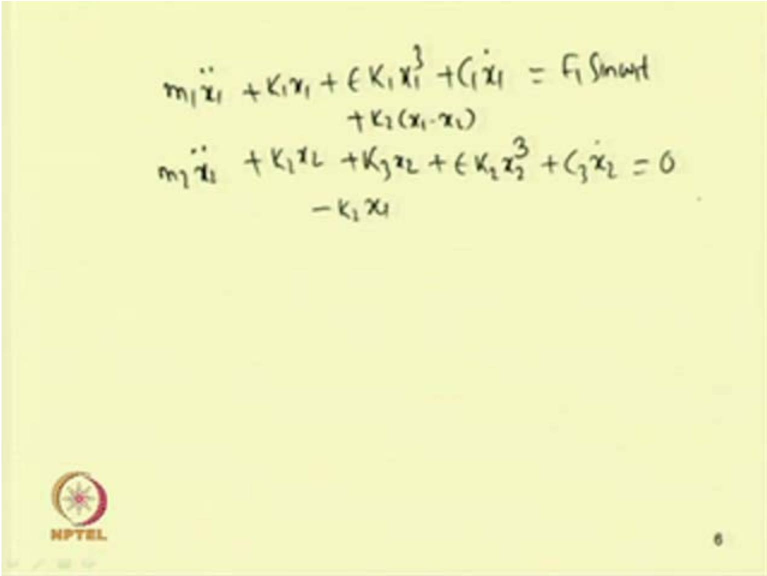

Minus k 2 x 1 is there, so minus k 2 x 1. k 2 into x 2 minus x 1. So, minus k 2 minus x 1. So, this way we have written. So, this is the equation motion. So, this equation motion can be written in matrix form also.

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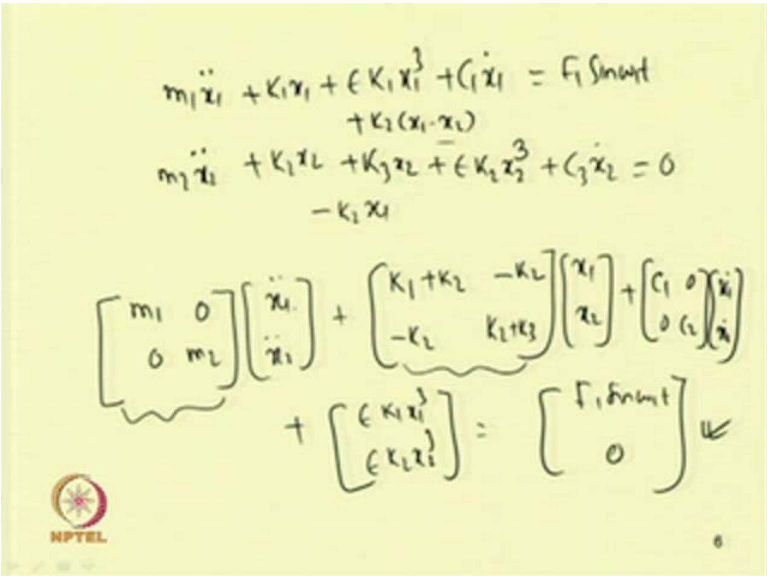

So, in matrix form this will equal to m 1 0 0 m 2. So, this becomes x 1 double dot x 2 double dot plus let us write the linear part in this way. So, this will be k 1 k 1 x 1. So, this is plus k 2. So, k 1 plus k 2 x 1 minus k 2 minus k 1 minus k 2 into x 2. So, minus k 2 , so then. So, this is the only linear term that is x 1 x 2 and for the second equation it can be written minus k 2 and this becomes k 2 x 2 plus let us see k 3 x 2.

So, this will be equal to k 2 plus k 3. So, plus the damping term can also written. So, damping term this is c 1 0 0 c 2. So, this x 1 dot x 2 dot. So, plus then the non-linear terms let us write. So, the non-linear terms in the first equation becomes epsilon k 1 x 1 cube and in this second equation. So, in the second equation this equal epsilon k 2 x 2 cube. So, this will be equal to F 1 sin omega 1 t and 0. So, this is the equation motion of a non-linear two degree of freedom system, and this equation and in this equation the analysis can be made similar to that of the single degree of freedom system also. So, in case of single degree of freedom system. So, we have studied about the weak forcing and also about the strong forcing.

So, if now we can see that this equation is a coupled equation here though the mass matrix is uncoupled, mass matrix is uncoupled the stiffness matrix is coupled. So, here the stiffness matrix is coupled because in the first equation itself we have the term with x 2. The first equation, the first equation we have the term with x 2 so that is why we have couple stiffness matrix and one though the coupling is not mass matrix. So, we have coupling in the stiffness matrix. We can write this equation in a uncoupled form or the linear part can be written at least the linear part can be written in an uncoupled form by using this model analysis method; that means, we can find the dynamic matrix.

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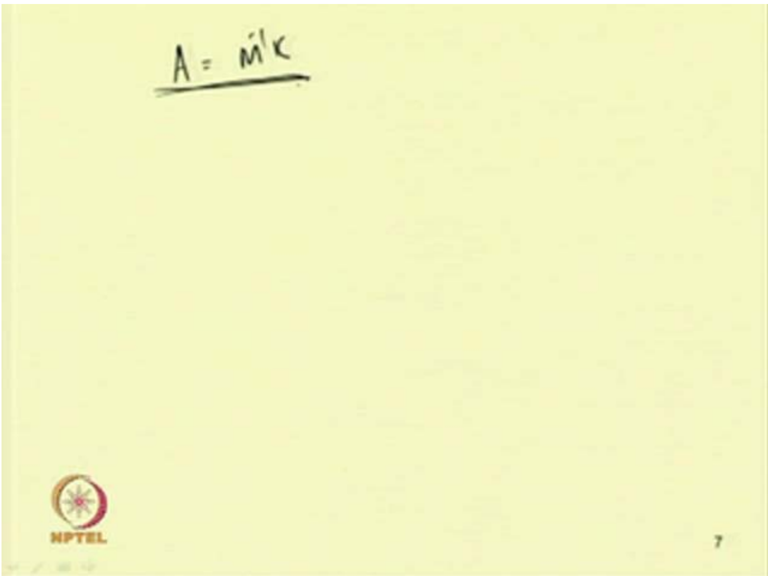

A let us find dynamic matrix A equal to M inverse k. So, if you write A equal to M inverse k and find the Eigen value of this. So, if we find the Eigen value of this and Eigen vector of this then we can write the model matrix.

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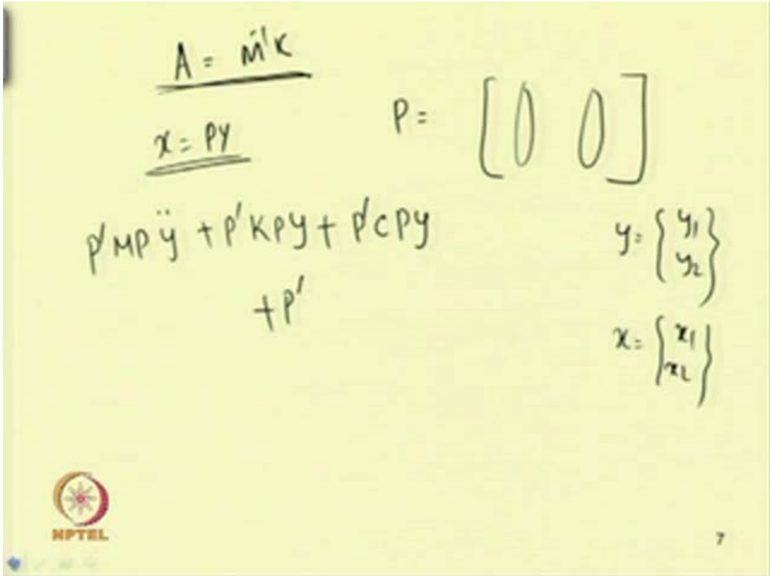

So, we can write the model matrix, in this case corresponding to these two. So, this is a two degree of freedom system. So, will have two Eigen values and corresponding to this two Eigen value will have two Eigen vectors. So, using this two Eigen vectors. So, we can uncoupled this equation by writing by using this thing P transpose M p p. So, by taking let us take this x equal to P Y.

So, if you take x equal to P Y then we can have this P transpose M p y double dot plus P transpose K p y. So, here y equal to y 1 and y 2 like x equal to x 1 and x 2. So, y equal to. So, this x equal to x 1 and x 2 similarly y equal to y 1 and y 2. So, then plus we can have this P transpose c. So, P transpose c P Y plus P transpose. So, we can write this non-linear part also by substituting this thing. So, now, by substituting this.

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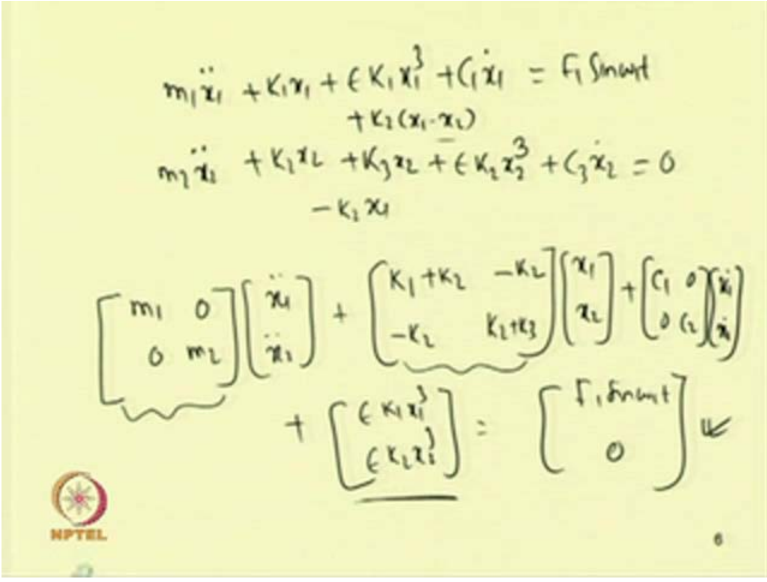

So, we can write this non-linear part epsilon k 1 x 1 cube and epsilon k 2 x 2 cube. So, p we can write the non-linear part also by multiplying this.

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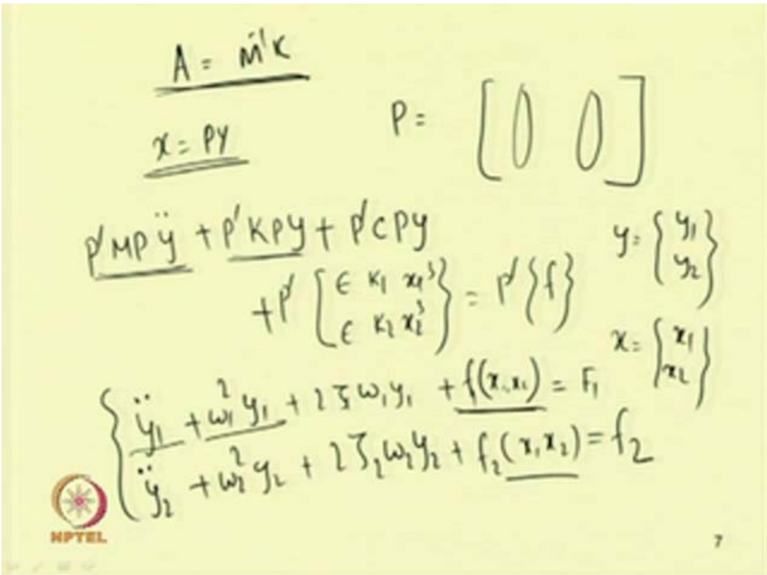

P transpose into p 1. So, that is equal to epsilon k 1 epsilon k 1 epsilon k 1. So, we can substitute that thing. So, by substituting we can write or we have two substitute this part epsilon k 1 x 1 cube and epsilon k 2 x 2 cube. So, x 2 cube. So, we have to substitute the corresponding value of x 1 and x 2 here similarly, we can have to write this p p 1 transpose. So, you have to multiply this P transpose here. So, p 1 transpose, this is the forcing function.

So, this way we can uncouple the mass matrix and stiffness matrix and we can have a equation uncoupled equation at least in mass and stiffness and this equation can be written in this form that is y 1 double dot plus omega 1 square omega 1 square y 1 plus 2 zeta omega 1 y 1 plus. So, will have a forcing term. So, this forcing term may be may contain this x 1 x 2 and derivate of that. So, this will be equal to a forcing term F 1 similarly will have second equation y 2 double dot plus omega 2 square y 2 plus 2 zeta zeta 1 zeta 2 omega 2 y 2 plus this is f 1 this is f 2. So, this is a function of x 1 x 2.

So, these are the non-linear terms in the system and this will be equal to F 2. So, we will have a set of two equations, now where this first two terms that is coefficient of the coefficient of y double dot and coefficient of y 1 are uncoupled. So, here we can now we can use the method, what we have studied for the single degree of freedom system and we can solve this two non-linear equations. So, we can solve this two non-linear equation simultaneously by applying the different methods, what we have studied.

So, tomorrow class will apply the method of a harmonic balance to the system and today class will just now revise how we have use, this method of multiple scale to a single degree of freedom system and found the response of the system. So, up to super harmonic resonance conditions we have studied last class, and today will revise about or will study how we can apply this sub harmonic resonance also in case of combination or if we have more than one forcing, or multi frequency excitation terms will be there then how we have to find the response by using method of multiple scale. So, here . So, now, these equations are reduce to that of a single degree of freedom system and now we can apply the method of multiple scale to study this thing. So, in our previous class we have we have taken.

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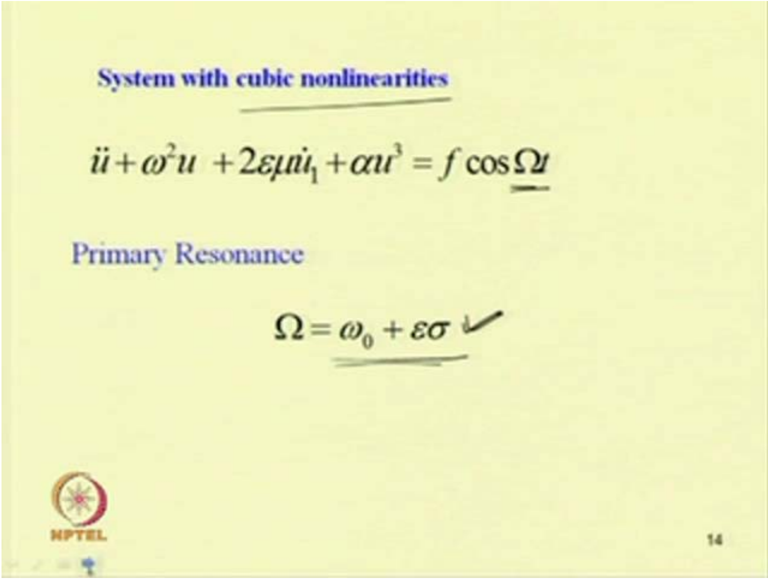

So, if the system is of weak nonlinearity, taking only a cubic non-linear term. So, we can have this primary resonance primary resonance means this omega external forcing when it is equal to near the natural frequency of the system, then we can add a detuning parameter to the system and by substituting.

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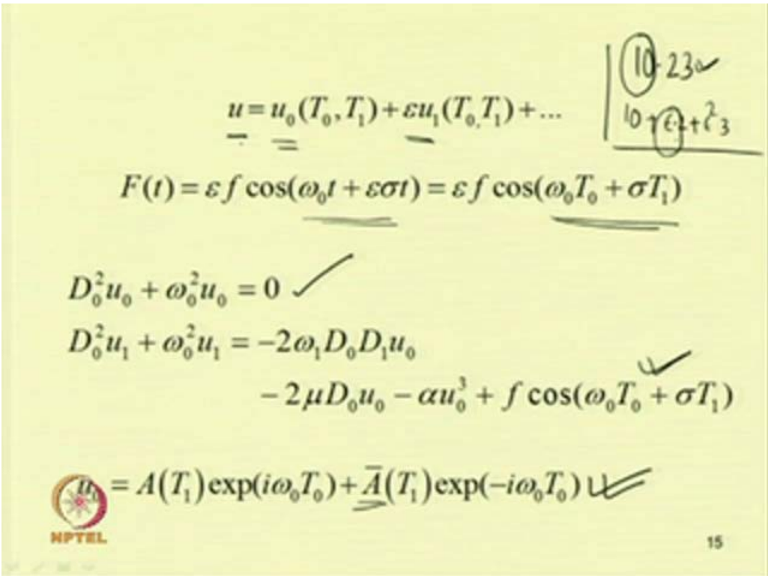

So, if our equation is reduce to this form then we can substituting this u equal to u 0 plus epsilon u 1, so will get this forcing term. So, this forcing term by using the detuning parameter we can write in this way and then by using method of multiple scale and separating the order of epsilon terms. So, we can have this D 0 square u 0 plus omega 0 square u 0 equal to 0 then the solution of this term equal is this. So, where we can write u 0 equal A T 1 e to the power i omega 0 t 1 T 0 plus A bar t 1 e to the power minus i omega 0 T 0.

So, here so A bar is the complex conjugate of A. So, by substituting this equation in the second equation where we have this term with u 1; that means, we have written this u that is the displacement of the system in by using two terms one is u 0 and another one is epsilon u 1. So, this is similar to let us take a number let us a take a number 10.23. So, here so we can write this number in this form that is the solution can be obtain plus epsilon into 2 plus epsilon square into 3.

So, if you are taking this epsilon equal to point 1. So, then this becomes 10.2 the solution will be 10.2 and if you are taking. So, you are as we are taking epsilon equal 0.1. So, epsilon square equal to 0.01 and multiplying this thing this become 0.03. So, the solution will becomes 10.2 plus 0.03. So, 10.23 if we are not taking this epsilon term this; that means, epsilon u 1 plus epsilon square u 2 then we may get a solution equal to 10, but if we want to get the precise solution then we have to take the higher order terms.

So, when will take the higher order terms we get this addition term that is epsilon that is 0.2, 0.2 will correspond to u 1 similarly point three will correspond to u 2. So, this way we can take different addition additional term in u to get the precise solution of the system. So, we can write in here we have written this u equal to u 0 plus epsilon u 1 and here we have assumed this u is a function of T 0 and T 1. Where T 0 equal to the time t and T 1 is equal to epsilon T 0 or epsilon t as T 0 equal to T 1. So, T 1 as A epsilon as T 0 equal to the time. So, T 1 becomes epsilon t. So, now, substituting this second equation this equation in this equation, so which is written in terms of u 1.

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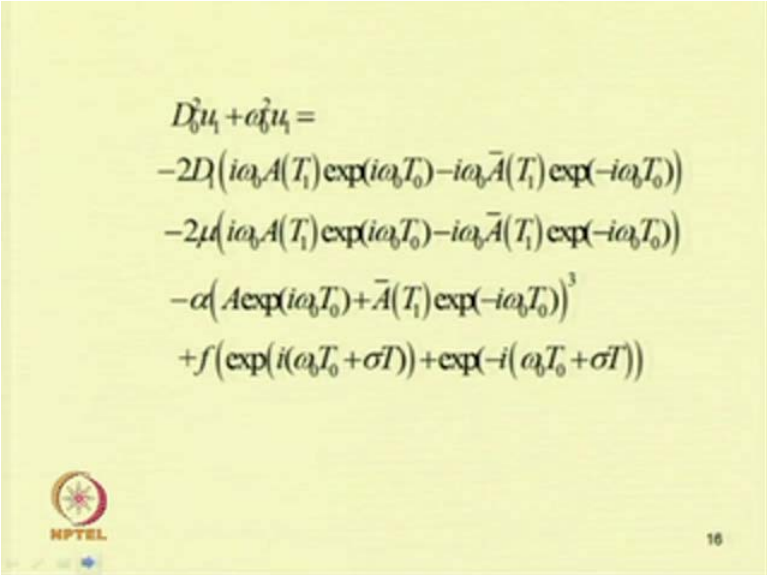

So, we can have some terms which leads to infinite resonance condition or infinite response of the system or unbounded response of the system. So, this thing happen if a term. So, from the left side we can see that the it contains D 0 square u 1 plus omega 0 square u 1. So, any term or any term in the right hand side containing e to the power i omega 0 t, T 0. So, will. So, will be a secular term or will be two infinite resonance or infinite response, but in actual case of the system is bounded. So, we have two eliminate these terms which are known as the secular term. So, here. So, one can see that this is secular term. So, we can write this equation using this complex conjugate also.

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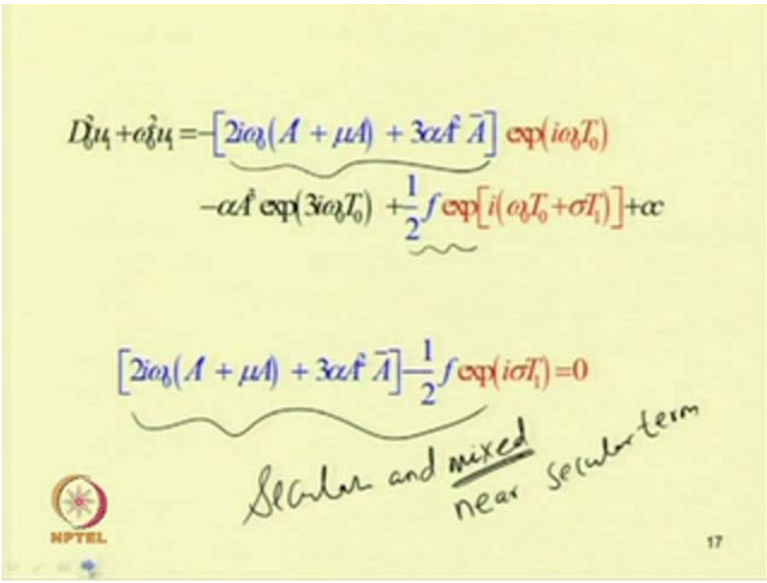

So, writing this equation in terms of complex conjugate. So, we can one can write in this way. So, one can see that the terms this has to be eliminated as this is coefficient of e to the power i omega 0 T 0 which will lead to unbounded solution similarly, here similarly this term. So, for small value of sigma. So, this term is also a secular term or this term will be unbounded. So, one can take this term and this term. So, to get the total secular and mixed secular term and eliminate this secular, secular and mixed or mixed or sometimes it can be near secular. So, one can eliminate this secular or mixed secular term to get the response of the system. So, now, eliminating this one can obtain.

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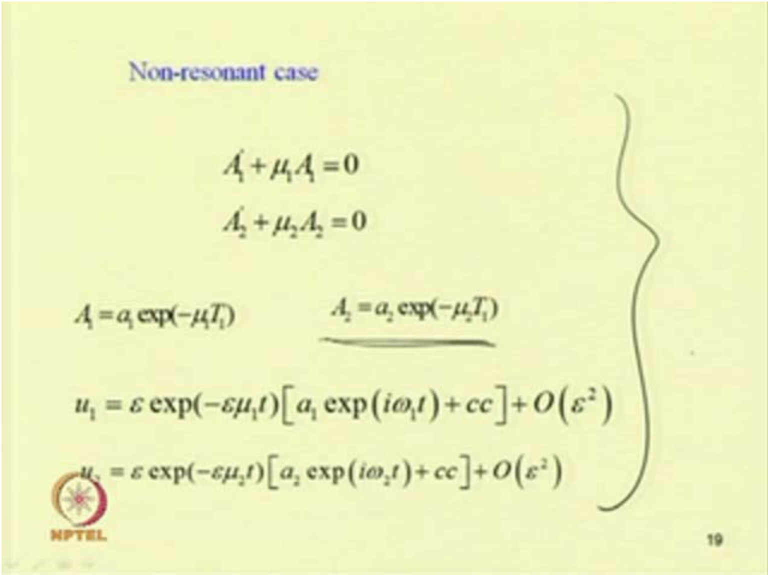

So, one can have two things one can get the resonant condition or one can study the non resonant condition. So, in case of non resonant condition all as already we have disused due to the presences of damping. So, it will lead to the it will decay and one can get the steady sate solution in this form. So, it will decay. So, A 2 equal to a 2 e to the power minus mu 2 to T 1. So, it will exponentially decay to the study state solution or trivial response.

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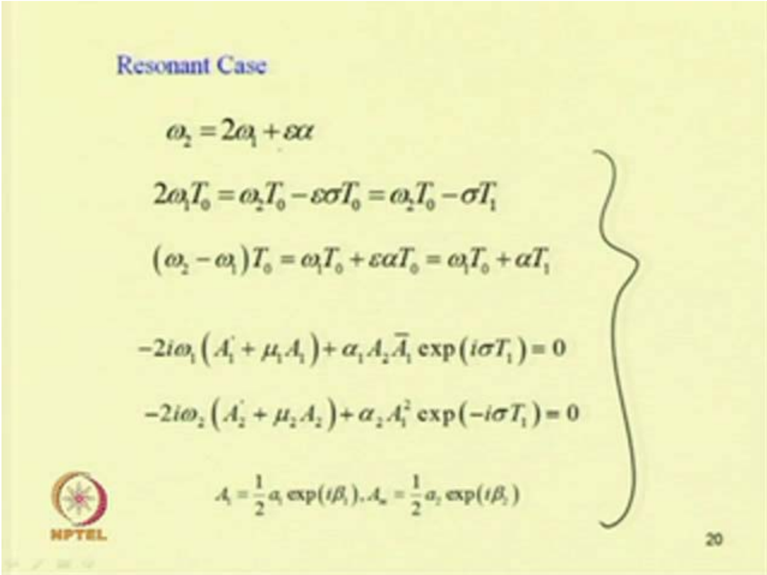

And in case of non resonance condition, we can take. So, if you are considering this internal resonance type we can take omega 2 equal 2 omega 1. So, depending on the resonance condition internal resonances condition if someone take then he has to take in this form.

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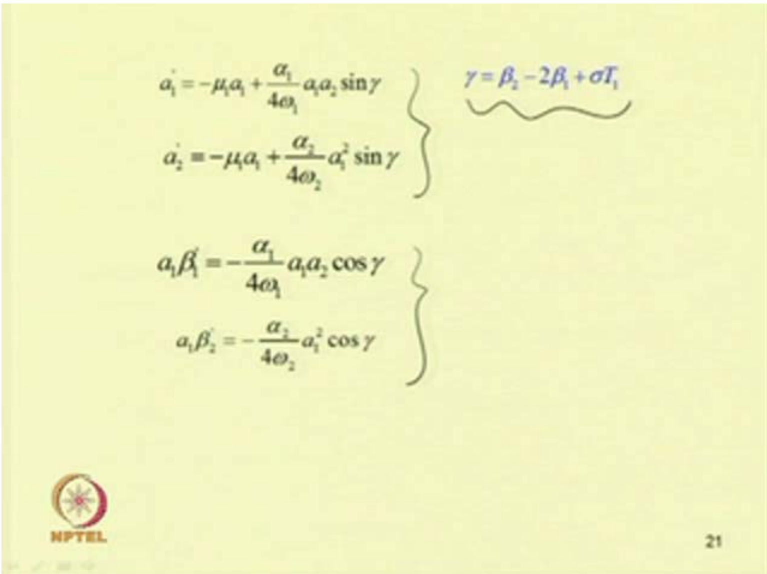

And already we have discussed this thing and it will reduce to a set of reduced equation from which we can get. So, these are the reduced equation and to write in autonomous form, we can use this gamma.

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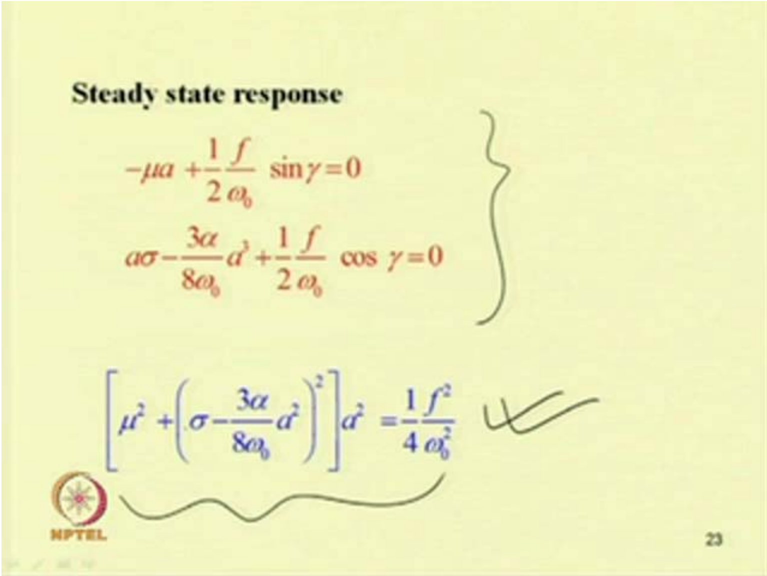

So, it will reduced a set of equation. So, for steady state it reduce to this equation. So, one can get a close form solution this. So, for the non resonant condition. So, one will get a steady state solution. So, this equation one can see this is sixth order in a or quadratic in sigma, sigma is the detuning parameter a is the response of the system. So, as it is quadratic in terms of sigma. So, one can find the response of the system.

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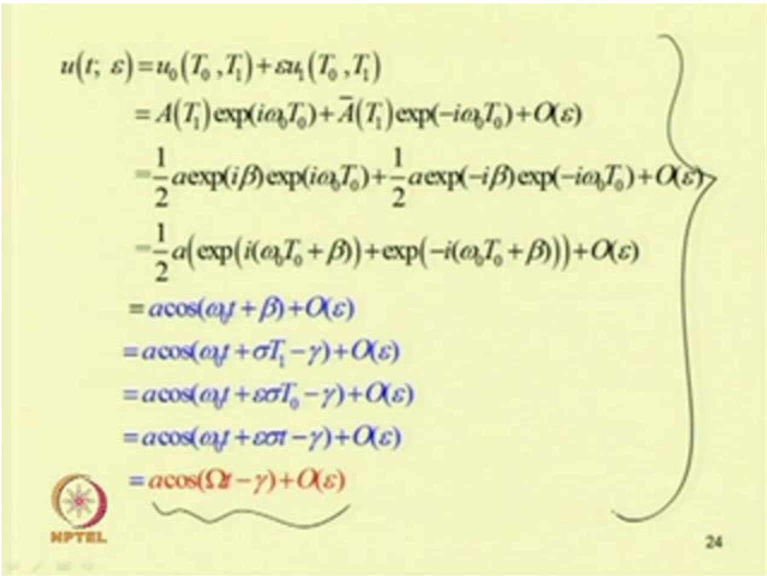

By using the quadratic equation and the solution of the overall solution of the system can be written in this form. So, u equal a cos omega t minus gamma; that means, the system will oscillate with a frequency of external excitation in case of the in case of the non resonant condition or. So, this is the thing what we have discussed for a system if it is weakly non-linear.

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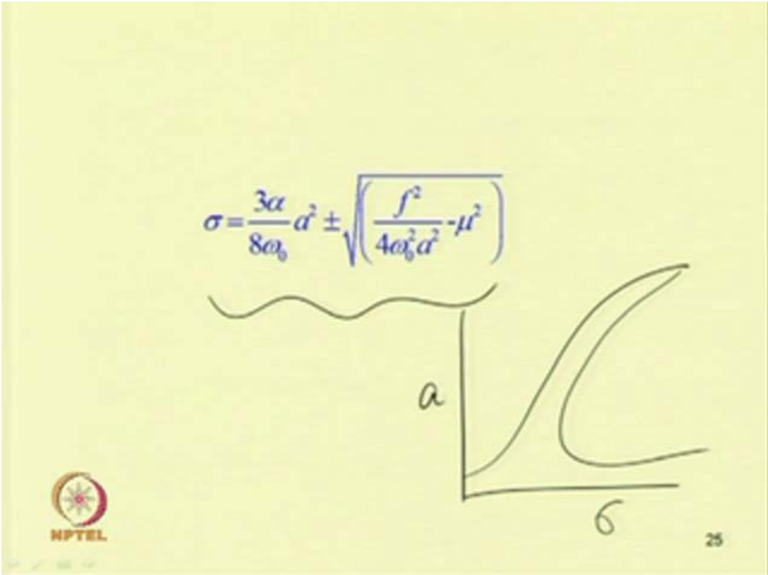

And this is the relation between the detuning parameter and response amplitude. So, by plotting the response amplitude verse detuning parameter, detuning parameter sigma. So, one can get. So, one can get the response like this. So, where it can observe the jump up or jump down phenomena and also one can study the stability of the system by finding the Eigen value of the equation these two reduced equation.

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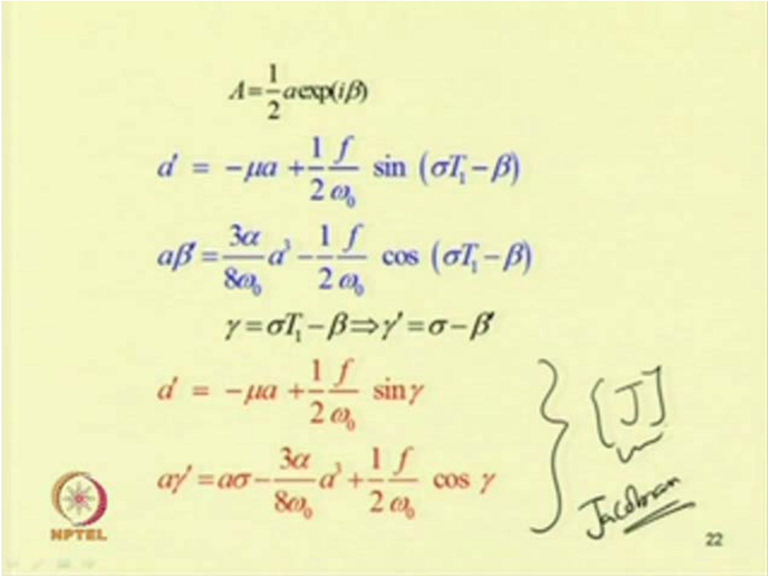

So, by finding the Eigen value of this two reduced equation. So, by finding the Jacobian matrix and finding the Eigen value of the Jacobian matrix. So, one can find or one can study the stability of the system.

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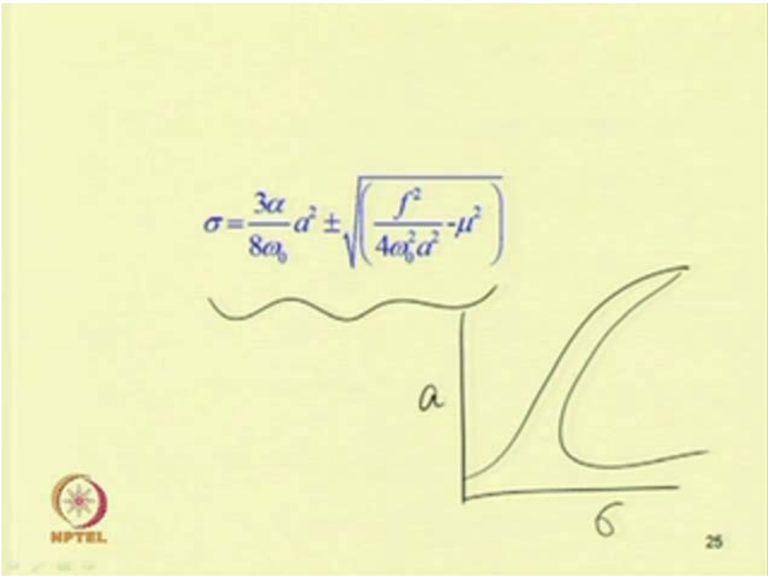

The response steady state response will be stable if the real part of the Eigen value becomes negative and it is unstable if the real part of the Eigen value is positive. So, one can plot.

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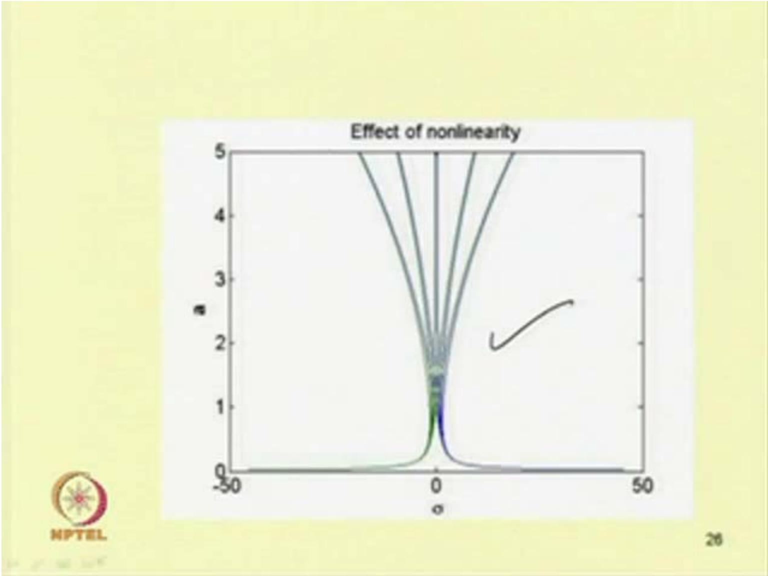

So, one can study the effect of nonlinearity.

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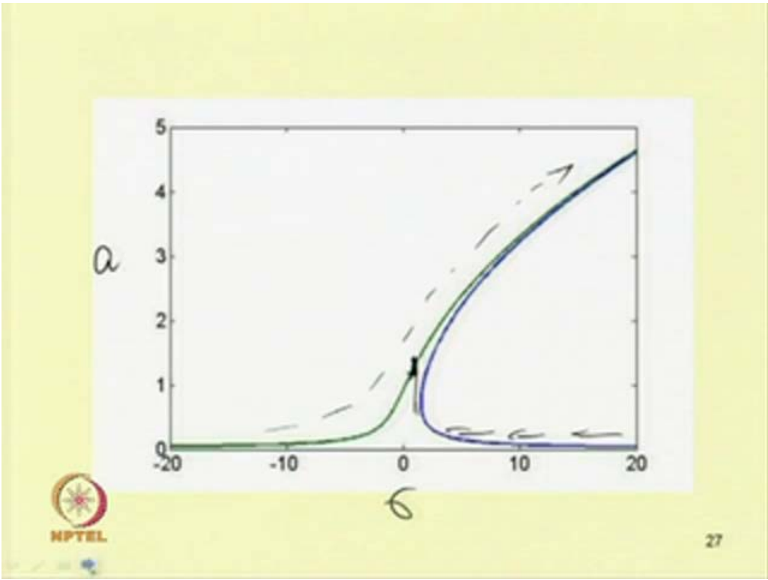

So, here by sweeping of so, one can. So, if it is super critical. So, one can follow this curve. So, it will go up, but while sweeping down the frequency. So, this is a versus sigma. So, by sweeping down the frequency one can observe. So, as here this branch is unstable. So, by sweeping down. So, one has as jump up phenomena here. So, it will the response will jump up and it will follow this curve again. So, one can have this jump up and jump up jump down phenomena.

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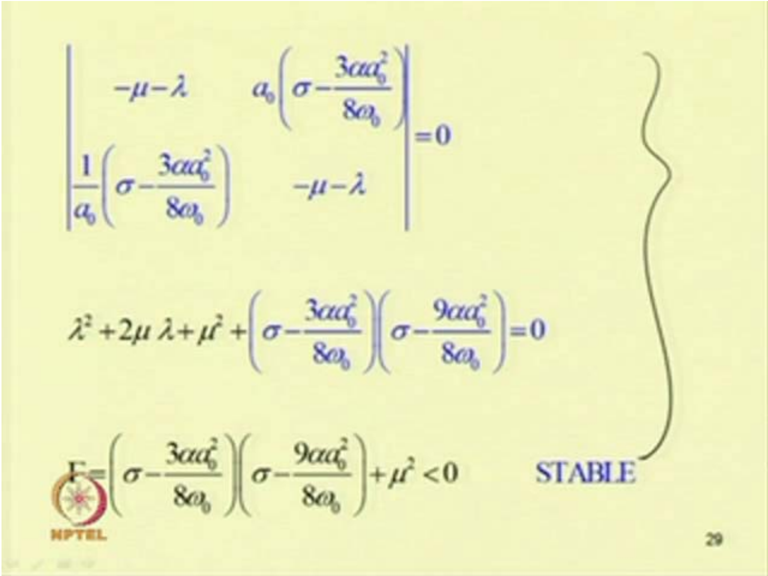

Already we have discussed about the stability of the system. So, in this way in case weak non-linear system one can study this system.

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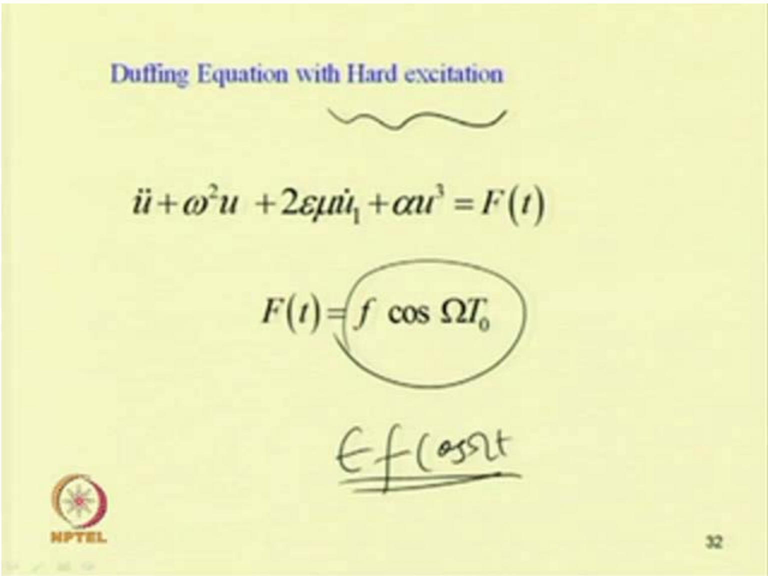

So, when the equation is hard excitation; that means, when we cannot write the forcing term as epsilon f cos omega t in the equation motion. So, here we are not using this epsilon. So, previous case we have written it is equal to epsilon f cos omega t, but in this as this terms no longer small.

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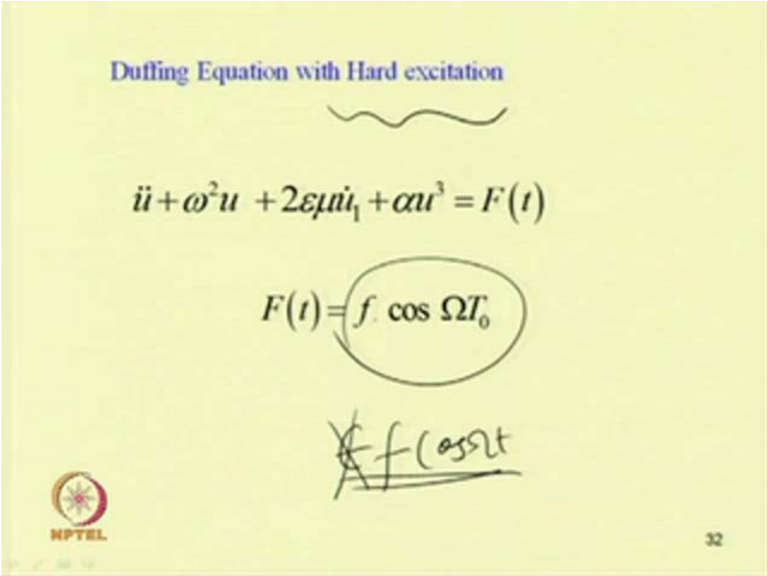

So, we cannot use this word epsilon and. So, this equation can be written f cos omega T 0.

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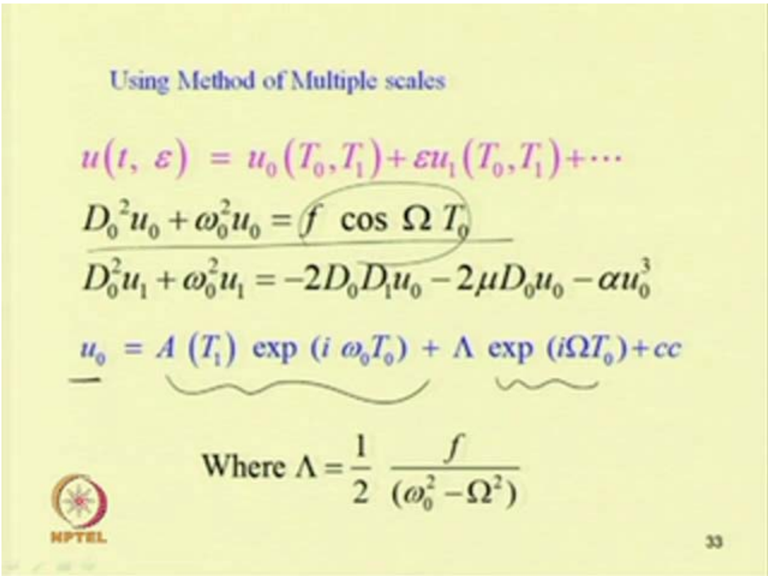

So, by proceeding in the similar way so, using method of multiple scale. So, we can see that the solution of this equation will contain two part previously as this right hand side was 0. So, we have only the complimentary part, but now in addition to this complimentary part we have to add the particular integral the system, particular integral of the system. So, the complimentary part becomes A T 1 e to the power i omega T 0 plus its complex conjugate that is A bar e to power minus i omega 0 T 0.

And plus the particular integral part of this will be equal to half f by omega 0 square minus omega square into e to the power i omega T 0 plus its complex conjugate. So, this way one can write. So, this is the additional term in equation u 0 when we are considering the excitation to be hard. So, in case of hard excitation we have seen the response u 0 can be written in this form, now by writing this equation in this form and substituting this is in the second equation.

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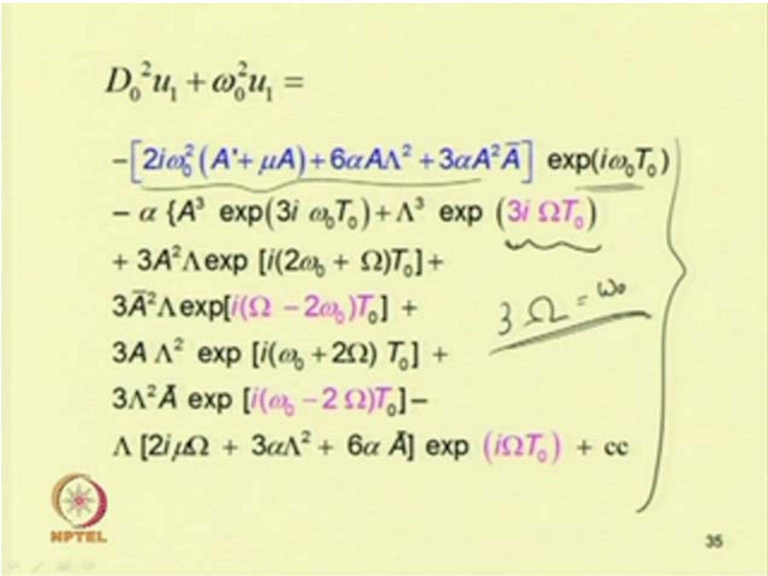

And we can see there several resonance conditions available in the system or several terms will lead to unbounded solution. So, one has to study. So, what are the condition for which the system response becomes unbounded. So, here what we have seen. So, this is one term the first term is the secular term because it is coefficient of e to the power i omega 0 T 0 and the second term in the second term we have the condition; that means, when this capital omega. So, this is equal to or this 3 omega nearly equal to omega 0 So, will have resonance condition. So, we can we have seen the different type of resonance condition.

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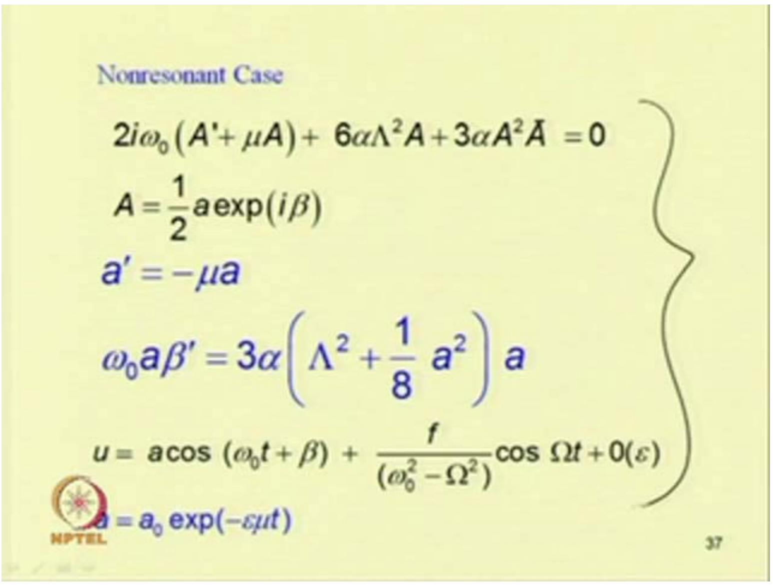

So, first one can study the non resonant case in non resonant case similar to previous.

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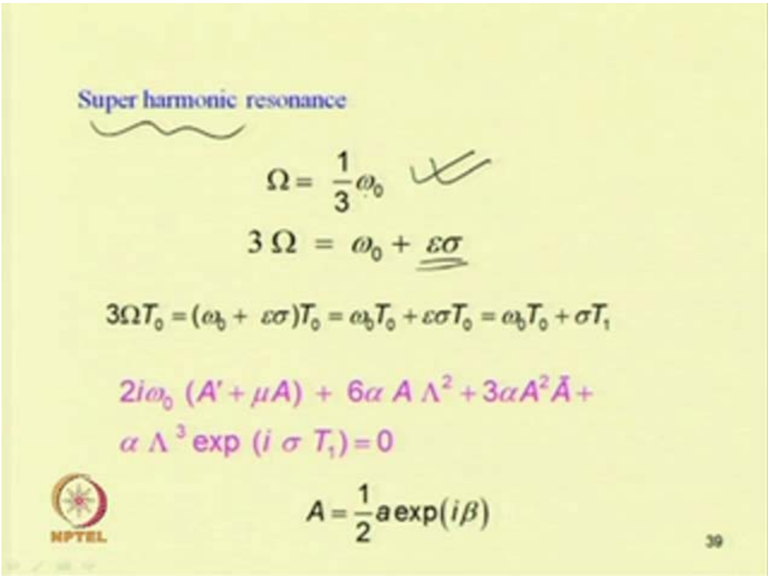

So, it will decay. So, we have the super harmonic resonance condition. So, we have seen the case 3 omega equal to omega 0. So, that is known as super harmonic resonance condition because in this case the system will oscillate at a frequency equal to omega which is equal to one third omega 0. So, this is super harmonic resonance condition. So, proceeding in the previous way, we can write this 3 omega equal to omega 0 plus epsilon sigma. So, sigma is the detuning parameter now eliminating the secular term for this resonance condition.

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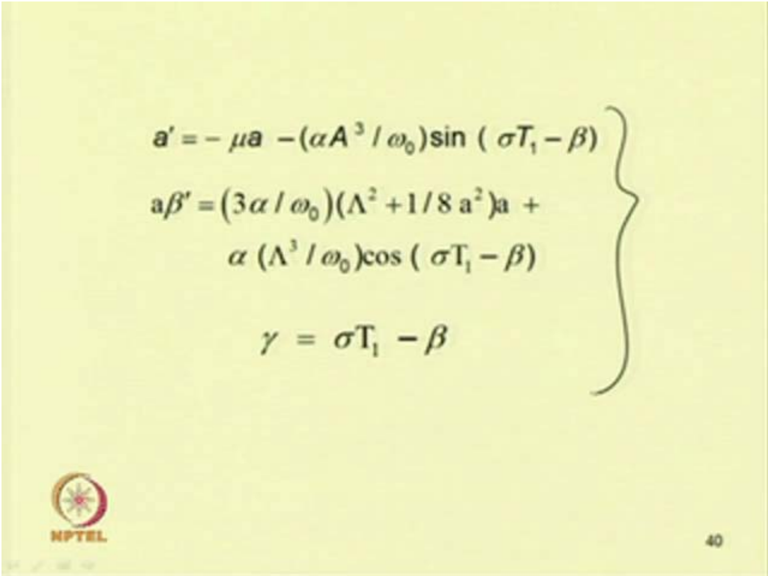

So, we can have a set of a reduced equation. So, these are the set of reduced equation. So, and by solving. So, for steady state solution steady state are the steady state is not the function of time. So, this time derivative term should not be present in this system. So, by putting this a dash equal to 0 and beta dash equal to 0. So, we can have a set of equation. So, will have a set of equation.

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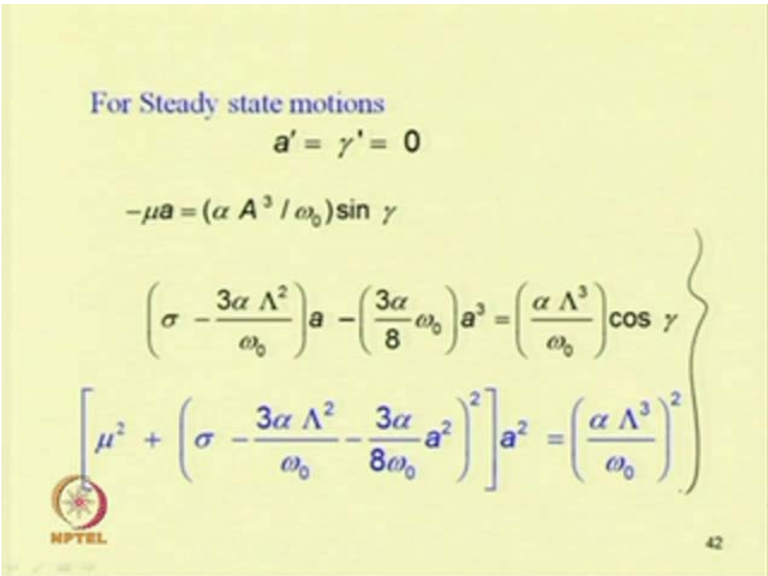

So, these are the equation. So, here to make the system autonomous. So, we have substituted this gamma equal to…

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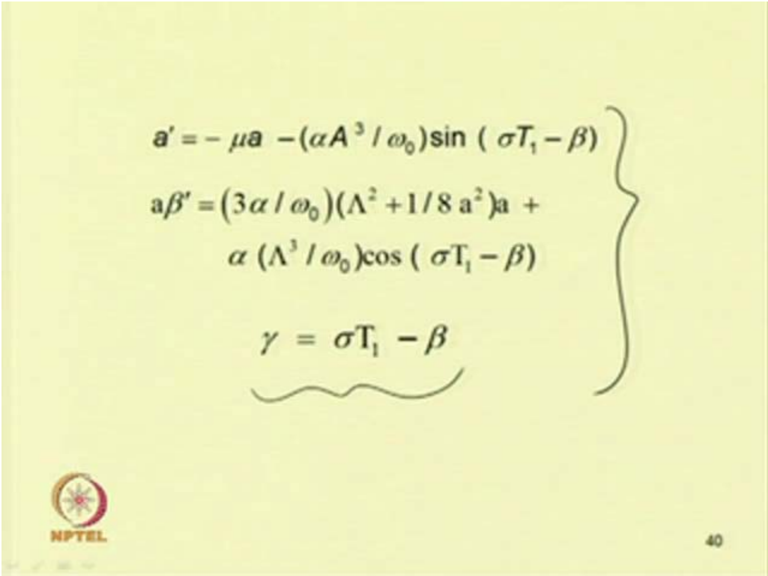

So, gamma equal to t gamma equal to sigma T 1 minus beta by substituting gamma equal to sigma T 1 minus beta.

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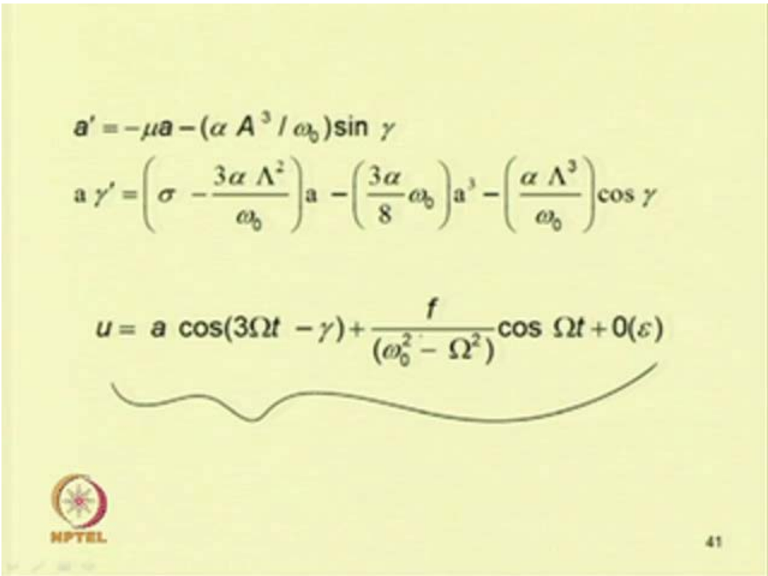

So, we have written this equation. So, this is the first order solution of the system. So, u equal to a cos 3 omega. So, this is a cos 3 omega t minus gamma plus f by omega 0 square minus omega square cos omega t plus order of epsilon.

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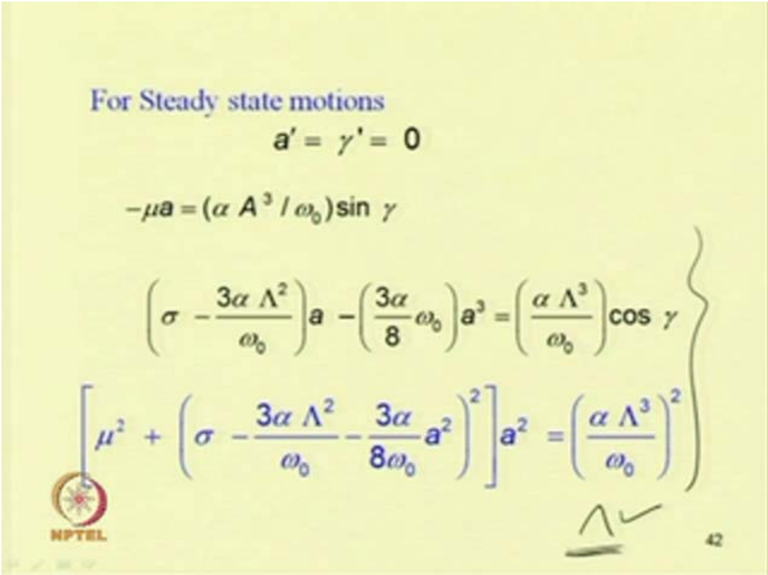

So, for steady state already we have seen. So, it will reduce to that of a close form solution. So, this is the close form solution written in terms of sigma and a sigma is the detuning parameter of the system and a is the amplitude here mu is the damping parameter of the system and this, this term represent the forcing parameter of the system. So, one can plot this equation for different amplitude of forcing different amplitude of mu that is damping and plot the frequency response of the system.

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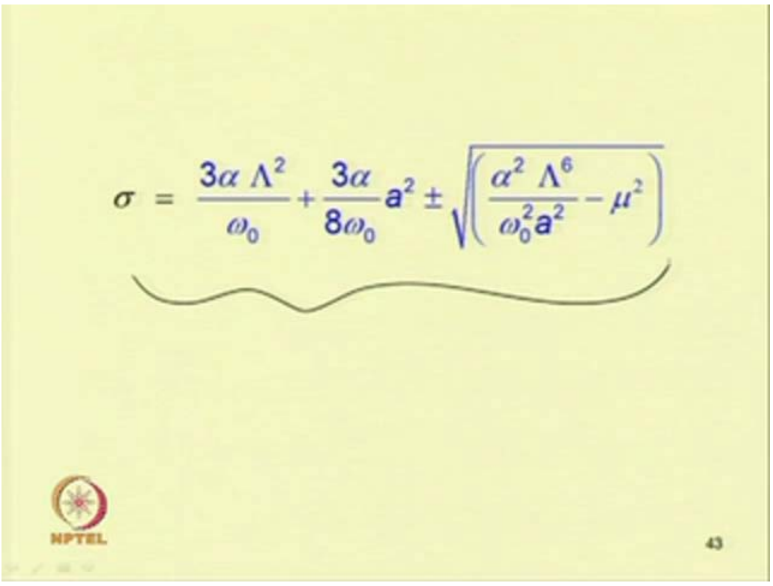

So, here sigma, this is the close form solution of sigma because in this equation we have written sigma as sigma is quadratic in this equation it appear as a quadratic form of sigma or sixth order form of a. So, one can either numerically solve this equation for a or write or solve this quadratic equation and solve to write the sigma equal to this by getting sigma in this form. So, one can find the response of the system.

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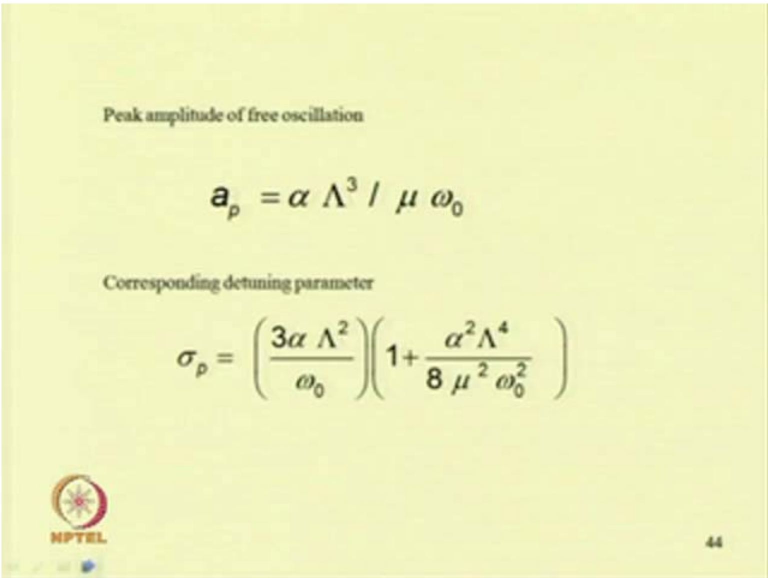

Last class we have seen the response of the system. So, here one can see or observe from this equation.

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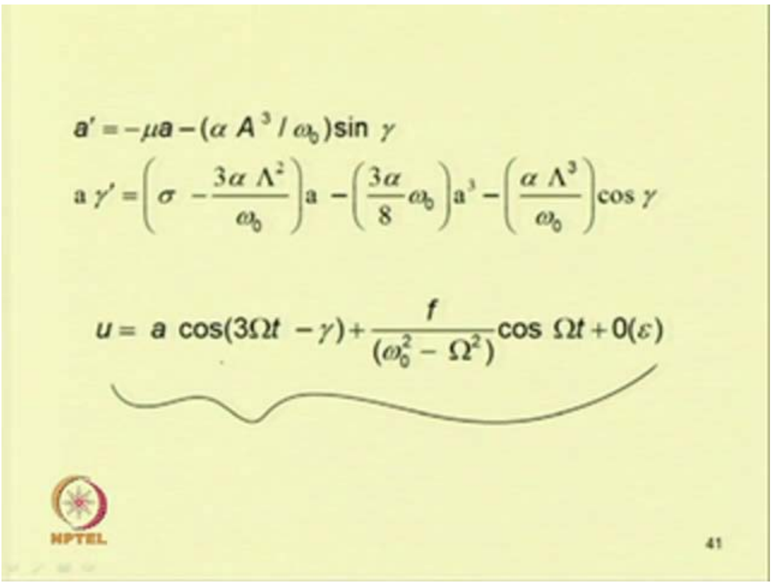

The free vibration term of the system and the force vibration term of the system and one can observe that of free vibration amplitude.

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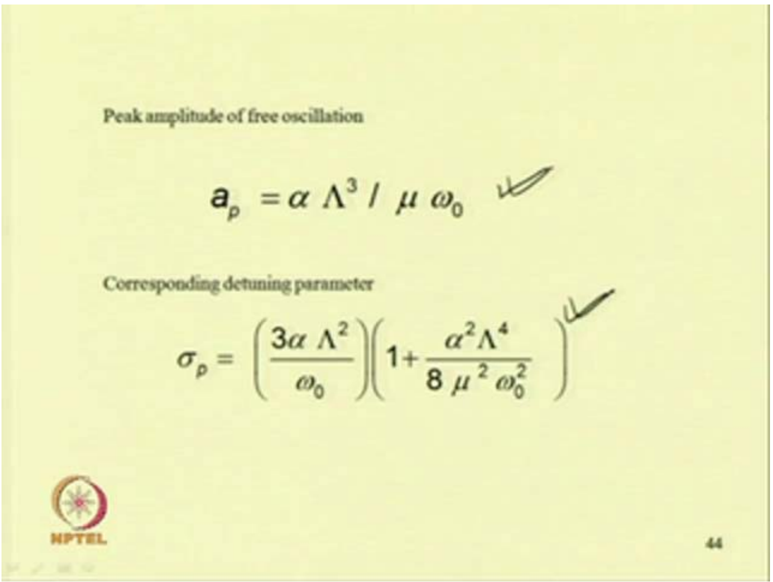

The free vibration amplitude of the system equal to alpha lambda mu cube by omega mu omega 0 and the corresponding detuning parameter is this. So, this is the super harmonic resonance condition and one can also study this sub harmonic resonance condition in. So, from this equation already we have seen the condition 3 omega equal to, So, 3 omega equal to by substituting 3 omega equal to omega 0.

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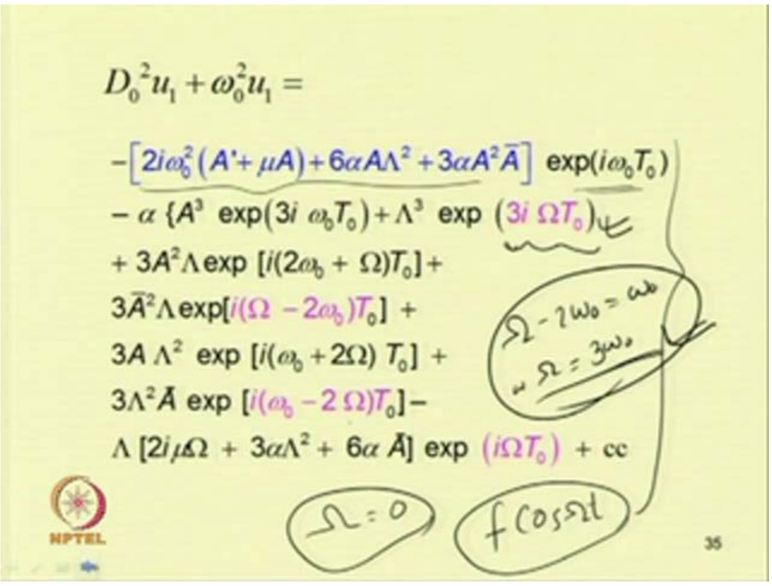

So, we have seen one resonance condition and the next resonance condition one can observe. So, when omega minus 2 omega 0 equal to omega 0 omega minus 2 omega 0 equal to omega 0 or omega equal to 3 omega 0. So, this also leads to a resonance condition, in this case. So, another condition is omega minus 2 omega equal to omega 0 or this omega equal to 0.

So, omega is 0 omega equal to 0 means we have a fixed amplitude. So, our forcing function as our forcing function is omega cos omega t, putting this omega equal to 0. So, will have a fixed amplitude forcing. So, we are not considering that term. So, now let us see this case when this external excitation frequency omega is near to 3 times omega 0 of the system.

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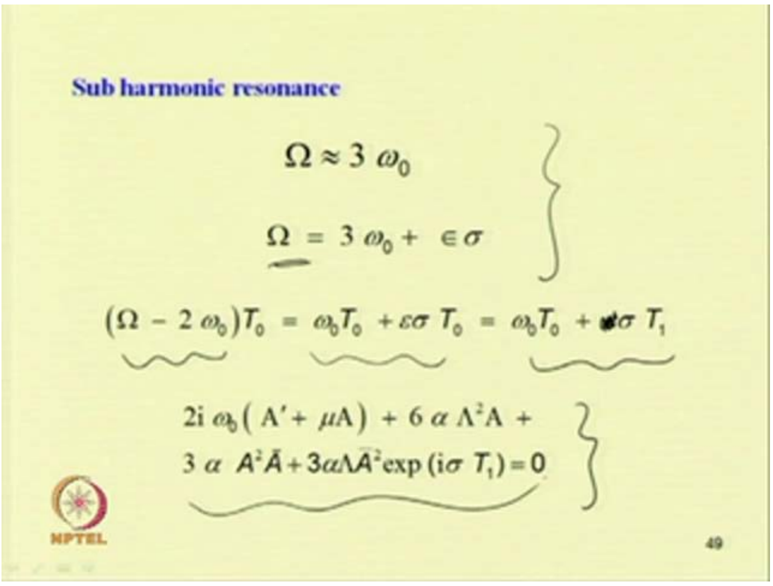

So, if it is three times then it is known as sub harmonic resonance condition. So, in case of sub harmonic resonance condition. So, we have taken omega nearly equal to 3 time omega 0 or this external frequency omega equal to 3 omega 0 plus epsilon sigma. So, here this forcing term. So, in the forcing term we have this omega minus 2 omega 0 T 0. So, that thing can be written as. So, 3 omega 0 into T 0 minus 2 omega 0 T 0. So, this becomes a omega 0 T 0 plus epsilon sigma T 0. As epsilon T 0 equal to sigma epsilon T 0 equal to T 1, we can write this equal to omega 0 T 0 plus. So, you can write this equal to omega 0 T 0 plus sigma T 1. So, this term can be written in this way omega 0 T 0 plus sigma T 1 and substituting that equation. So, this leads to the secular term. So, this term has to be eliminated from that equation.

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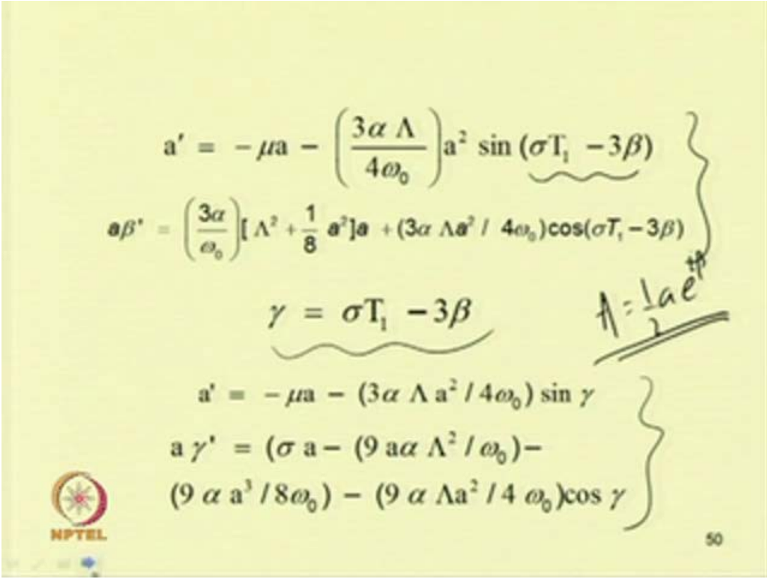

So, now, proceeding in the similar way by substituting A equal to half a e to the power i beta half a e to the power i beta and separating the real and imaginary part. So, we obtained this two equations that is a dash equal minus mu a minus 3 alpha lambda by 4 omega 0 a square sin sigma T 1 minus 3 beta and a b dash equal to 3 alpha by omega 0 into lambda square plus 1 by 8 a square into a plus 3 alpha lambda a square by 4 omega 0 cos sigma T 1 minus 3 beta.

So, two make it non autonomous. So, in this equation this time appear explicitly in this equation. So, to make the system non autonomous we can substitute the sigma T 1 minus 3 beta equal gamma and write this equation in this form. So, that is a dash where a dash is differentiation of a with respect to T 1. So, this writing this a dash and gamma dash equation you can write.

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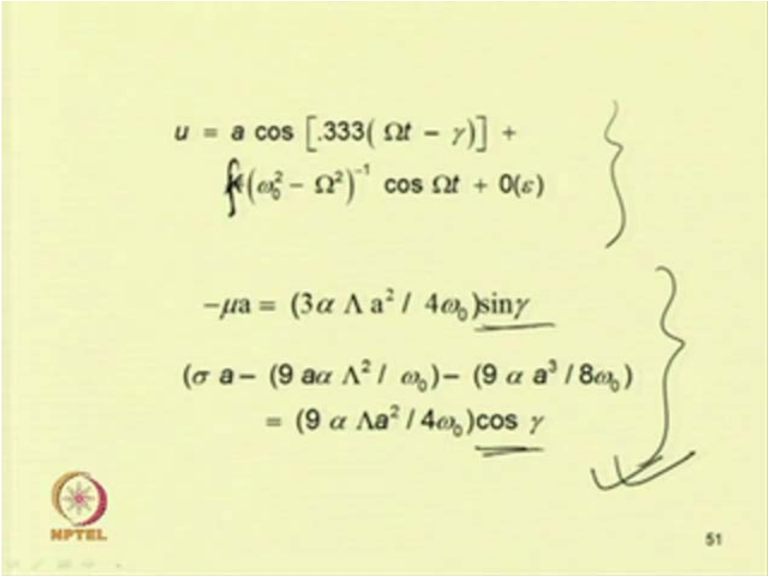

So, now, we can write this solution of the system in this form. So, solution of the system u equal to a cos one-third omega t minus gamma plus. So, plus we have this forcing term f f omega 0 square minus omega square to the power minus 1 cos omega t plus this. So, this way we can write the solution of the system where for steady state eliminating this a dash gamma dash or substituting a dash and gamma dash equal to 0. So, we have this two equation. So, in this equation eliminating this cos gamma and sin gamma.

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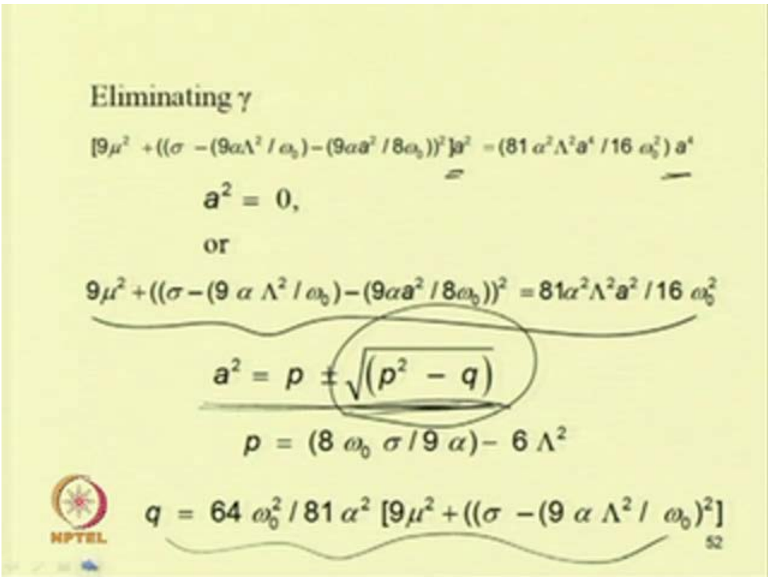

So, we can have this close form solution, in this close form solution one can see. So, this is a square this right side it is a fourth. So, one can take this a square term common so; that means, some term into a square equal to 0. So, either a square equal to 0 or this term equal to 0. So, for steady state eliminating this gamma either we can see this a square equal to 0 or this equal to 0. So, a square equal to 0 correspond to trivial state of the system, either. So, in this case; that means, a trivial state exist in the system. So, either the trivial state will be stable or unstable.

So, one can study by perturbing this reduced equation and studying the Jacobian matrix Eigen value of the Jacobian matrix and for the case of non trivial solution. So, in this case one can find this a square equal to. So, from this as it is in quadratic firm. So, one can this a square equal p plus minus root over p square minus q and where p equal 8 omega 0 sigma by 9 alpha minus 6 lambda square and q equal to 64 omega 0 square by 81 alpha square into 9 mu square plus sigma minus 9 alpha lambda square by omega 0 square.

So, it can be noted that. So, from this equation the solution will exist the solution will exist if this part is or we have a real solution if this part is positive. So, the solution will exist if this p square greater than q or if p square is not greater than q then we cannot have a real solution. So, to have a real solution. So, p square should be greater than q.

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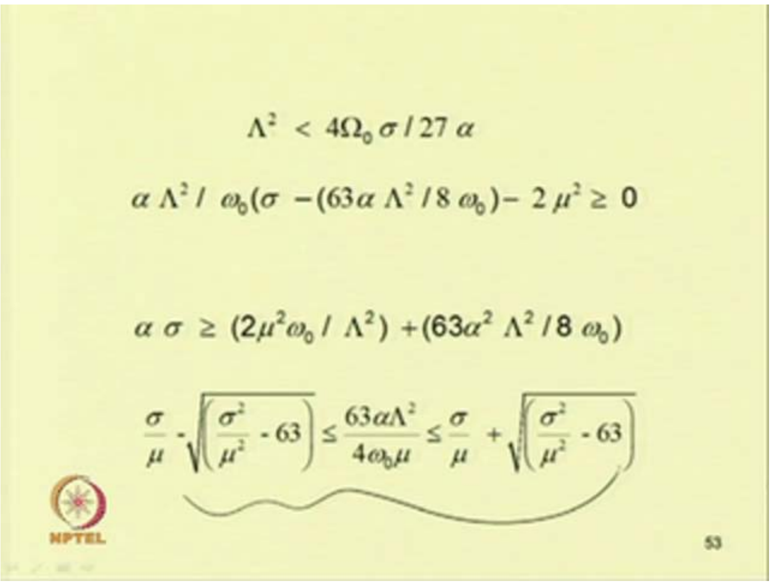

So, in this condition, one can obtain a condition from this. So, which will lead or which will give rise to the non trivial state of the system. So, one can find the trivial state. So, in this case both trivial and non trivial state of the system exist. So, one can plot the response of the system from this using this detuning parameter and one can study the stability of the system also, in this way. So, we can have the sub harmonic or super harmonic resonance condition along with the primary resonance condition when we have either a. So, when we have soft excitation or hard excitation of the single degree of freedom system and already we have seen.

So, we can reduce a multi degree of freedom system to that of a single degree of freedom system, and we can make the make similar analysis to obtain the response of the system also a continuous system can be reduced to that of a single degree of freedom system by using the by using the generalized Galerkin method, where we can take a single mode approximation. So, by taking a single mode approximation. So, we can reduce the continuous system to that of a single degree of freedom system or by taking multi mode approximation.

So, taking a multi mode approximation we can reduce the equation to that of a multi degree of freedom system. So, the single degree that. So, the single degree of freedom system analysis is very much important before studying the multi degree and continuous system. As one can use this single degree of freedom system analysis for finding the response of a system with multi degree of freedom let see. So, if a multi frequency excitation acts on the system what will happen to the system. So, previous where we have taken only a single term single forcing term. So, we can take instead of taking a single forcing term we can take multiple forcing term.

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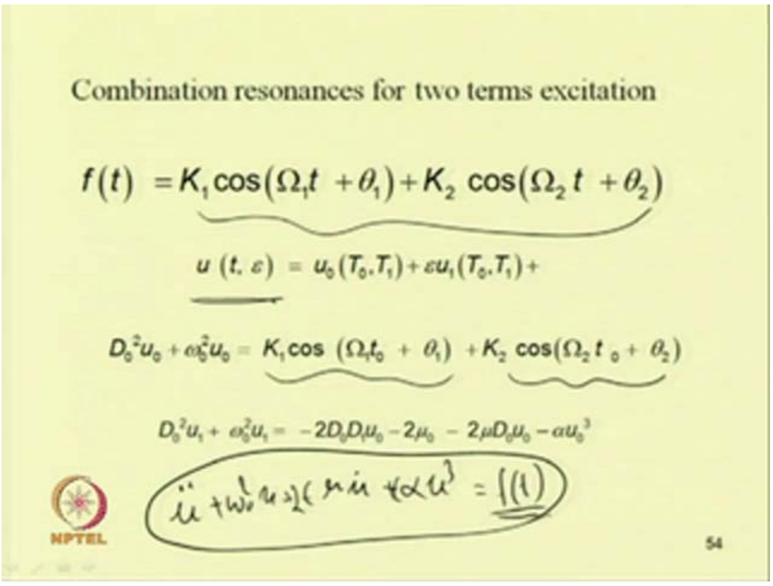

So, let us take the case when this forcing term equal to K 1 cos omega 1 t plus theta 1 plus K 2 cos omega 2 t plus theta 2. So, one can reduce this two forcing term to a single forcing term, either one can reduce this to a single forcing term or one can use this two terms and solve the response of a system to find the solve, the equation of the system to find the response of the system. So, like previous case here also we can take u equal to u 0 plus epsilon u 1 then our equation that is the equation, what you have used before that is u double dot plus omega 0 square u plus epsilon mu to epsilon mu u dot plus alpha epsilon alpha u cube equal to. So, here we are taking this equal to f 1 f t. So, where f t equal to combination of two harmonics or combination of two frequency.

So, this is two frequency excitation similarly one can take the system with multi frequency excitation. So, in this case either one can study the system by taking this forcing as weak forcing or one can study the system by taking this as hard excitation. So, in case of weak excitation. So, or in case of hard excitation one can write the system equation in this form that is D 0 square u 0 plus omega 0 square u 0 will be equal to K 1 cos omega 1 T 0 plus theta 1 plus K 2 cos omega 2 T 0 plus theta 2 . So, in this case in contrast to the single degree of freedom system one can observe that one can get a number of resonance conditions. So, here by taking u 0 equal to, similar to the previous case. So, here the u 0 will contain a compliment and the particular solution due to this one and plus due to this term.

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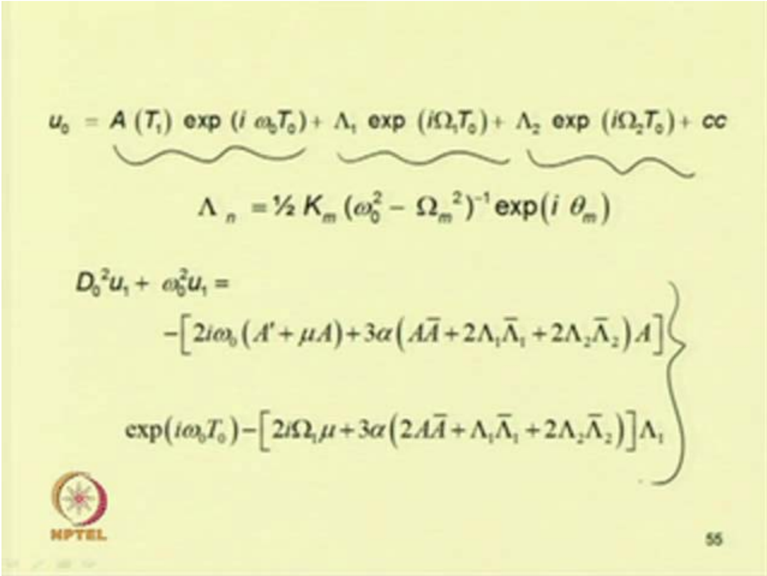

So, one can write the particular solution with omega 1 this and for this is for forcing K 1 and this is for forcing K 2 in addition to the complimentary part solution that is A T 1 e to the power i omega 0 T 0. So, by writing this u 0 or substituting this u 0 in this equation of u 1 and eliminating the secular terms one can get different resonance conditions. So, while eliminating the secular condition one should pay attention to the terms which lead to the secular term or mixed secular term or near secular term for example, we have studied.

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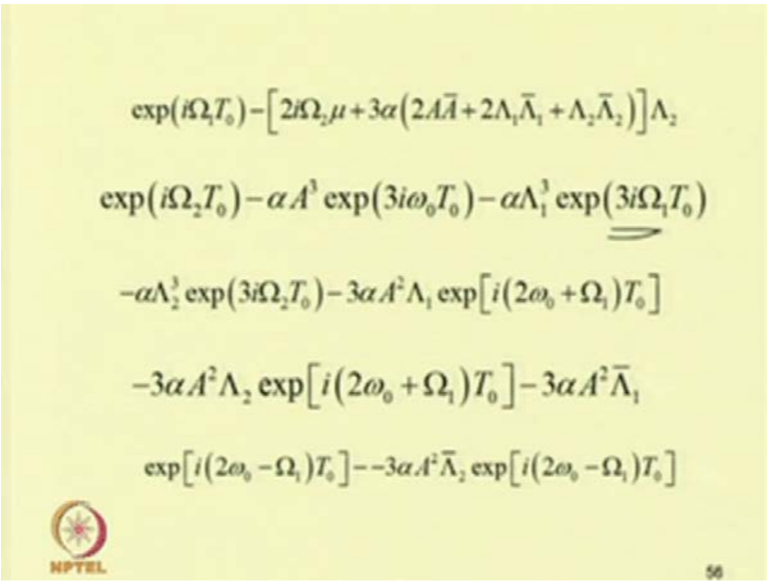

This is the super harmonic resonance; that means, when this 3 omega 1 nearly equal to omega 1 similarly we can have the terms. So, we can have one can see that we can have.

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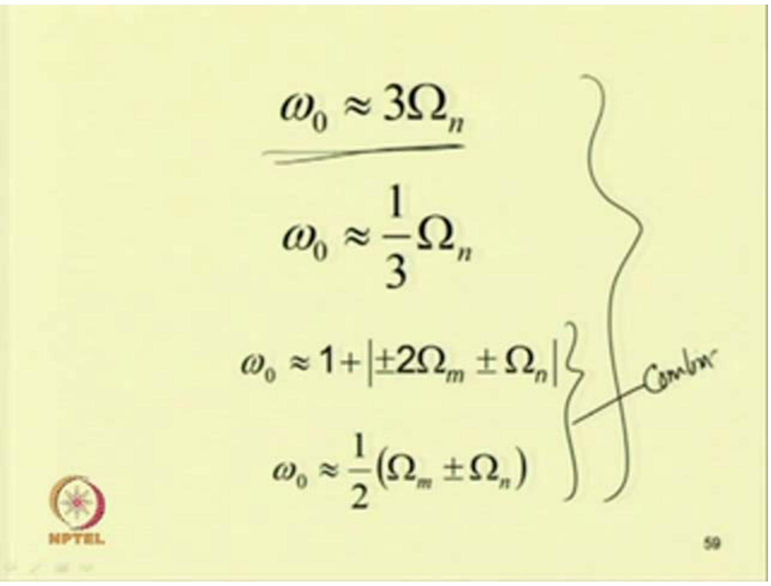

A number of resonance condition, these are the number of resonance condition one can obtain. So, these are super harmonic or sub harmonic and in addition to that one can have this secondary primary resonance condition, secondary resonance conditions. So, in case of primary resonance condition. So, the resonance will be near to the natural frequency of the system when it not near to the natural frequency of the system. So, we can get this secondary resonance condition. So, from the previous equation one can see the resonance will happen.

So, when this omega 0 nearly equal to 3 omega n similarly this omega 0 equal to one third of omega n similarly one can see this condition that is omega 0 equal to 1 plus plus minus 2 omega m plus minus omega n and omega 0 nearly equal to half omega m plus minus omega n. So, in this case. So, this is known as. So, these are primary resonance. And these are combination resonance. So, as we are taking combination of two frequency here. So, this is combination resonance. So, combination resonance condition. So, one can have primary resonance and combination resonance in this case.

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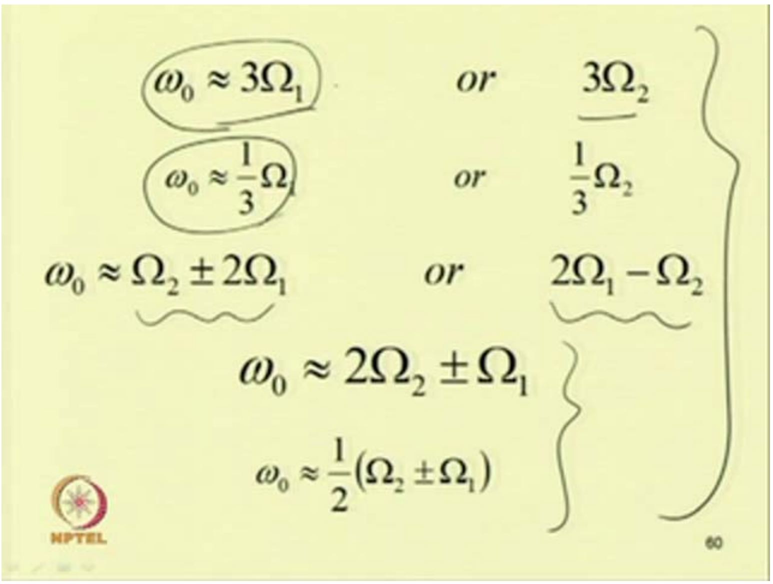

So, taking this resonance condition or taking these two condition. So, one see in this two frequency excitation one can have these different types of resonance condition; that means, omega 0 nearly equal to 3 omega 1 or 3 omega 2 we can get this response similarly omega 0 equal to one third omega 1. So, one case is the super harmonic resonance other case is the sub harmonic resonance condition and this leads to the combination resonance of the system. So, we can have this primary resonance or combination resonance of the system, in this way.

So, for each resonance condition as we have studied in the last or we have discussed in last section, here also for each resonance condition. So, we have to study the system separately and we have to find the response of the system. So, by taking different resonance condition first we have to write the reduced equation then in this reduced equation we have to substitute the time varying term equal to 0 to get this steady state solution and from the steady state solution we have to find the, we have to find the equation or we have to find the frequency amplitude relation equation and from that we can obtain a either a close form solution or in case of multi degree of freedom system we can get a set of equations for example, in case of two degree of freedom system as that of two equation. So, it can expand this u 1 and or x 1 and x two in terms of. So, in our case the example what we have taken in the in this case; that means, we have x 1.

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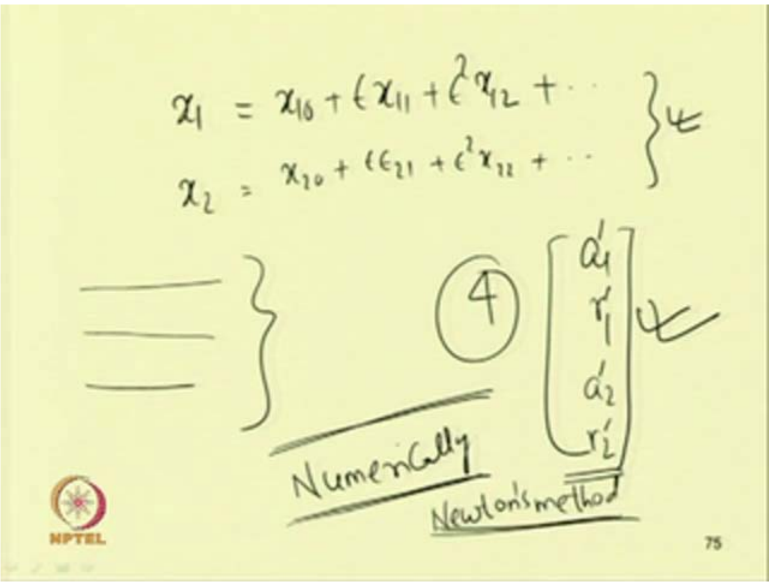

So, this x 1 we can expand it in this form x 1 can be written this x 1 0 plus epsilon x 1 1 plus x epsilon square x 1 2 plus higher order terms similarly one can write this x 2 equal to x 2 0 plus epsilon x 2 1 plus epsilon square x 2 2 and higher order terms. So, now, substituting this equation in the original equation and separating the terms of the order of epsilon and epsilon to the power epsilon to the power 0 epsilon to the power 1 and 2. So, we can get a set of equations. So, from this equations we can get. So, we can get the reduced equation. So, in this case will have 4 reduced equation.

So, one will be in terms of a 1 other will be in term of gamma 1 second one will be a 2 and gamma 2. So, will have a equation a 1 dash gamma 1 dash a 2 dash and gamma 2 dash. So, will have 4 will have 4 equation unlike in case of single degree of freedom system where we have two equations in this case will have four equations. So, getting a close form solution in this case will be difficult. So, in this case one has to solve this equation or by substituting this a 1 dash a gamma 1 dash a 2 dash gamma 2 dash equal to 0.

So, one can get the equation for the steady state, but this four equation this four non-linear algebraic and transcendal equations can be solved numerically. So, one has to solve numerically this equations. So, one has to solve numerically these equations to obtain the response of the system. So, to solve this equation numerically one may use the Newton's method. So, one may use the Newton's method to find the solution of the system. So, unlike in case single degree of freedom system where we are getting a maximum three branches. So, in this case we may get multiple solutions or multiple branch solutions.

So, one has to apply this Newton's method properly to find this solution of the system also one may note that in this Newton method. So, it depends on the initial condition. So, for different initial condition as different initial condition will leads to different equilibrium position. So, one should be very careful in applying this one Newton's method to obtain this multi branch solution of the system. So, next class will see some examples and also how to use this harmonic balance method for finding the solution of a multi degree of freedom forced response of a system.

Thank you.

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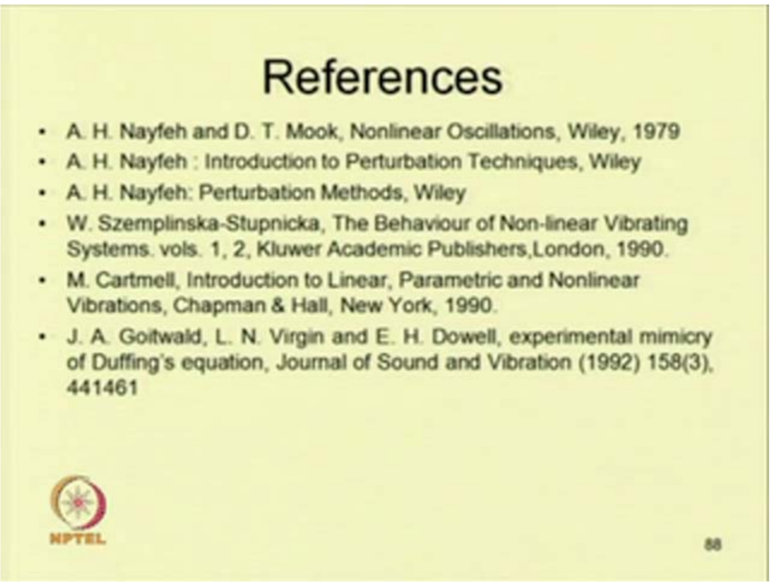

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