**Non-Linear Vibration**
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**Module - 6**
**Applications**
**Lecture - 2**
**Nonlinear Vibration of Single Degree of Freedom System with Damping**

Welcome to today class of non-linear vibration. So today class, we are going to study about non-linear vibration of single degree of freedom system with damping. So last class, we have discussed about the single degree of freedom system without considering damping, and we have solved different equations, such as this duffing equation with quadratic and cubic damping. And we have found the response or we have compared the response in that case with that obtained from the linear system.

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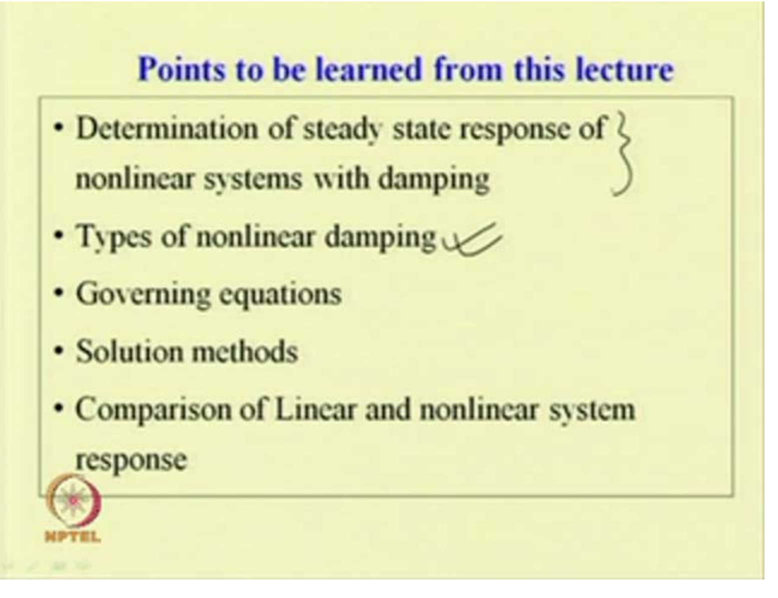


Today class also, we are going to study about this determination of steady state response of non-linear systems with damping. So, what are the different types of damping, governing equations also, then solution methods and comparison of linear and non-linear system response. So first, we will know about what are the different types of non-linear damping present in the system or in a single degree of freedom system how you can consider this non-linear damping, then how these governing equations can be written and then the solution methods.

So from the solution method will finally, know how we will determine the steady state response, their stability, already we have discussed about the stability and response of the non-linear systems and using those general principles; so we can discuss about the response obtained in case of this system with damping. Then we can compare these equation linear and non-linear systems with damping and we will take few examples to study the steady state response of the system.

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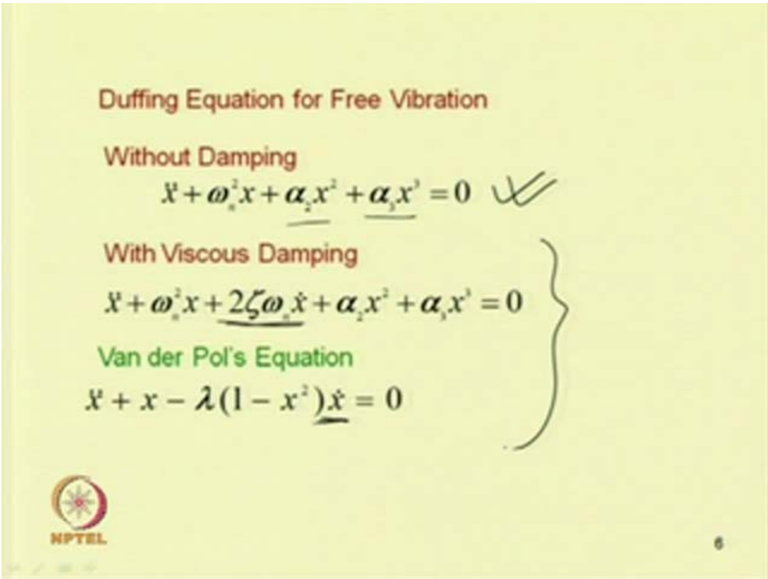


So for example, let us take the doffing, the duffing equation, so without damping, we can write this equation in this form, that is x double dot plus omega n square x plus alpha 2 x square plus alpha 3 x cube if you are taking quadratic and cubic damping. Then if you add a viscous damping term to this, so this term is the viscous damping term added to this duffing equation. And in case of Van der Pol equation also, this is the term with damping. So instead of taking these equations we can still take few simpler cases where we can have this non-linear damping.

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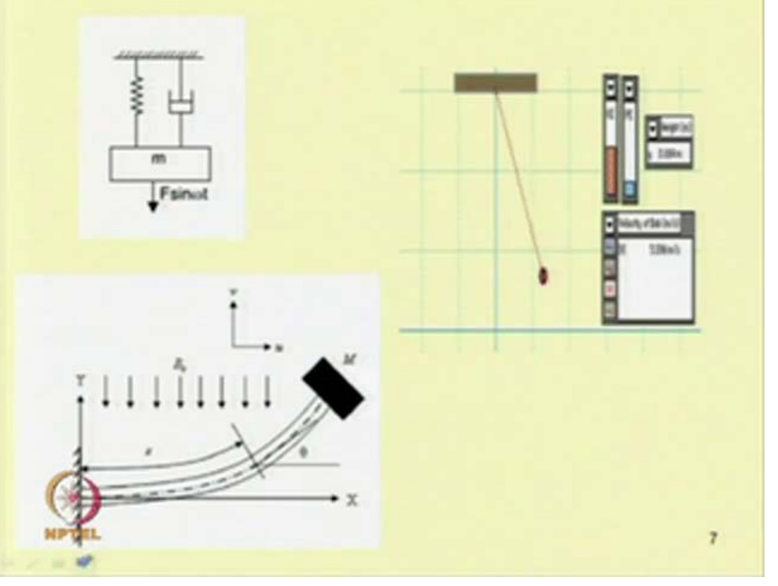


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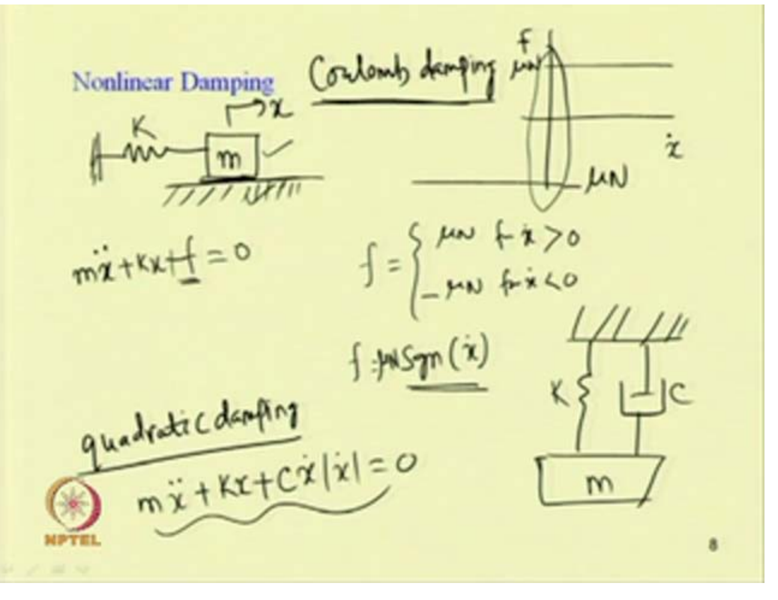


For example let us take this coulomb damping, so this spring mass damper systems or let us take a spring mass damper system with coulomb damping so we have a this is a spring, this is the mass and so this mass is constrained to move e or this is moving on this surface, so due to this dry fiction, so let k is the spring stiffness and here we have the we have the surface and m is the mass of the system. Then as there is dry fiction between this mass and the support position so there will be coulomb friction between these two. So this coulomb friction if one plot this force versus Dexter so if one plot force so this is the coulomb force versus this velocity x dot. So where x the displacement of the mass from the equilibrium position, so this will be equal to plus mu n and this is equal to minus mu n minus mu n. So this is equal to mu n so where mu is the coefficient of friction between these two surface and n is the normal force. So this normal force in this case will be equal to mg, m is the mass of the system and g is the acceleration due to gravity and for minus that means for minus x dot. So when the velocity is negative then this becomes minus mu n and when the velocity is positive then this force becomes mu n. So one can note that, so when this change the sign that is the velocity change the sign so this force change from minus mu n to plus mu n.

So, there this jump between this minus mu n to plus mu n and hence the system is no longer linear, so it is a non-linear damping case. So this equation can be simply written as m x double dot plus k x, so then so we can add this damping force so or we can write this damping force plus. So when it is moving towards right the force when x dot is towards right, the damping force will act towards left or you can write this force equal to plus f. So this is the damping force, so this will be equal to 0; so this f we can write equal to, that means this damping force coulomb damping, you can write equal to plus mu n or x dot greater than 0 and equal to minus mu n for x dot less than 0 or one can use this or one can write this is equal to f equal to signum Sgn x dot, so signum x dot multiplied by mu n, mu n into signum x dot signum x dot equal to plus one for x dot greater than 0 and equal to minus one for x dot less than 0. So one can write this coulomb friction coulomb damping or coulomb friction, so in this way also we can have other different type of forcing; other different type damping.

So let us take a quadratic damping, so we can have a spring mass system with this quadratic damping. Quadratic damping means so the damping term will be damping force will be quadratic so let us take the spring mass damper system, so this is the and here we have a non-linear damping term so this is bas m and this is this damper, so this is spring and if this damper is non-linear then we can write this quadratic damping in this form or this equation can be written m x double dot plus k x plus c. We can write this is equal to c x dot into mod x dot, so this will be equal to 0. So this is this x dot into mod x dot equal to the or is the quadratic damping present in the system.

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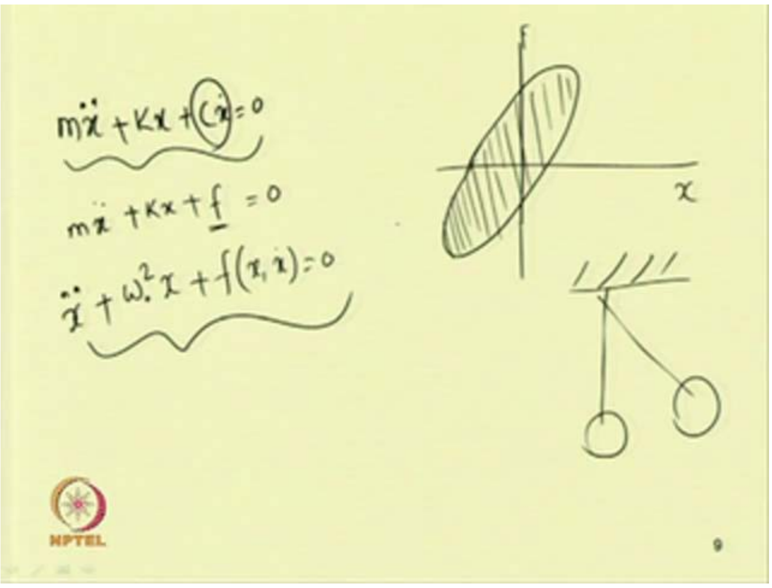


Similarly we can have other different type of damping such as hysteresis damping, so in case of hysteresis damping if you plot the force versus dis displacement curve you can have a hysteretic loops, so we are hysteretic loop, so f versus x. If one plot the force versus the displacement; so this is the hysteretic loop and this loop represent the energy dissipation due to damping. So this damping energy or this damping force, so this damping force is no longer linear and one can have a non-linear damping viscous. So this hysteretic damping may be into this material, different type of material produces this hysteretic type of damping and due to the presence of or in the many structure due to the joints, so one can have the structural or hysteretic damping also. Hysteretic damping, material damping and structural damping can be combined to have non-linear dampings in the system.

So now, we know we can write different type of damping present in the system so they may be linear or non-linear. So for example, in case of the linear damping, we have a simple viscous damping; so that is linear and we have written the equation motion in this form, m x double dot plus k x plus c x dot equal to 0 so this is for the linear damping. So in case of non-linear damping, so instead of this linear force c x dot; so we can have some non-linear force m x double dot plus k x plus f. So this force will be non-linear or we can write in this form also x double dot plus this k by m will be equal to omega n square so or omega 0 square we can write x double plus omega 0 square x plus f, so this f is a function of x x dot you can write, so this way also we can write general equation of a system and as this damping reduces the energy of the system.

Or if there is loss of energy in the system due to this damping, so these systems are non-conservative systems; unlike in the case we have studied before where we have only simple spring or we have taken this system of the pendulum. So in this pendulum without considering the damping effect; so we have seen that the system reduce to that of a so or the system equation is that of a conservative system, but in this case when we are considering damping in the system no longer the system becomes conservative and this becomes a non-conservative system. So in this non conservative system similar to the conservative systems, one can study or we can find the solution of the system qualitatively and quantitatively. Qualitatively or one can use this numerical methods to solve these equations and for qualitative study so one can proceed in the similar way as discussed in case of the conservative system and one can find the response.

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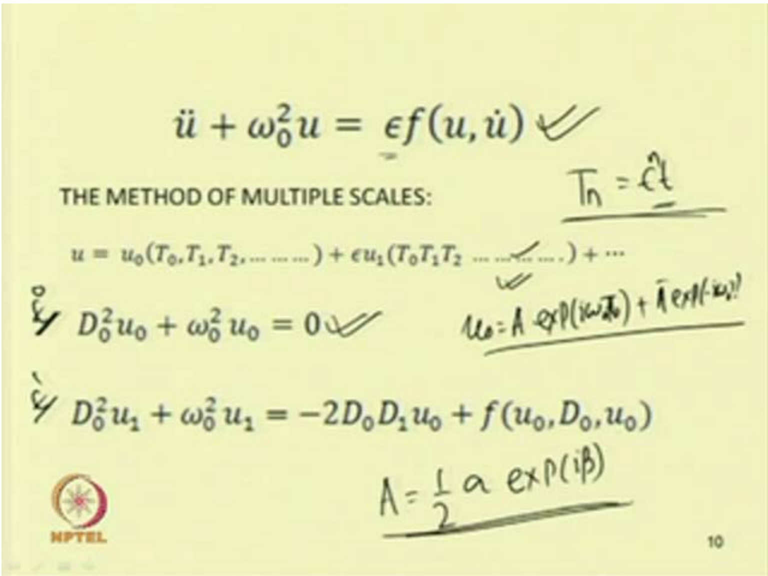


And in case of qualitative study, quantitative study. One can make one can make approximate solutions to find the response of a system. For example, let us take this equation u double dot plus omega 0 square u equal to epsilon f u u dot, here we have assume this forcing so which is a function of this damping is very small that is why one has used this epsilon or the book keeping parameter. So this book keeping parameter epsilon is always less than very very less than one. So writing this equation in this form one can use different types of solutions for example, one can use this method of multiple skills, method of harmonic balance, method of averaging lindstedt poincare technique. And many other different approximate methods what we have studied in module 2 to find the response of the system. For example, in this case by using this method of multiple skills, where you can take different time scales such as this t 0 t 1 t 2 so where t n equal to epsilon to the power n t, t is the time, epsilon is the book keeping parameter so the nth time scale t n can be written as epsilon n t and so that this t 0 equal to time and t1 equal to epsilon t and one can write this u that is the displacement using these two parameters, or one can take more number of parameters also depending on the accuracy of the solution required. So by taking this u equal to u 0 which is a function of t 0, t 1, t 2,plus epsilon 1 which is also a function of this t 0, t 1, t 2. So and substituting this equation in the original equation, that is u double dot plus omega 0 square u equal to epsilon f u u dot, so one can and separating the order of epsilon so this is for the order of epsilon to the power 0 and this is order of epsilon to the power 1 so the equation reduces to this form that is d 0 square u 0 equal to omega 0 square u 0 equal to 0. Similarly d 0 square u 1 plus omega 0 square u 1 equal to minus 2 d 0 d 1 u 0 plus f u 0 d 0 u 0.

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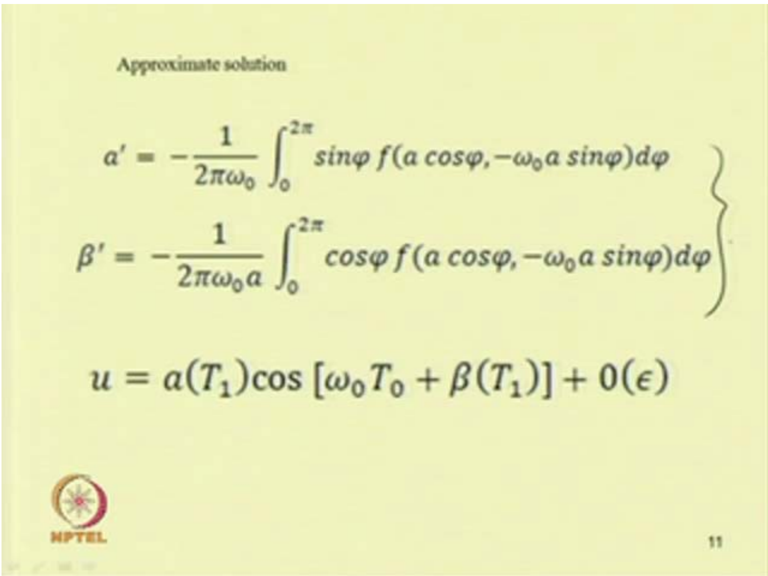

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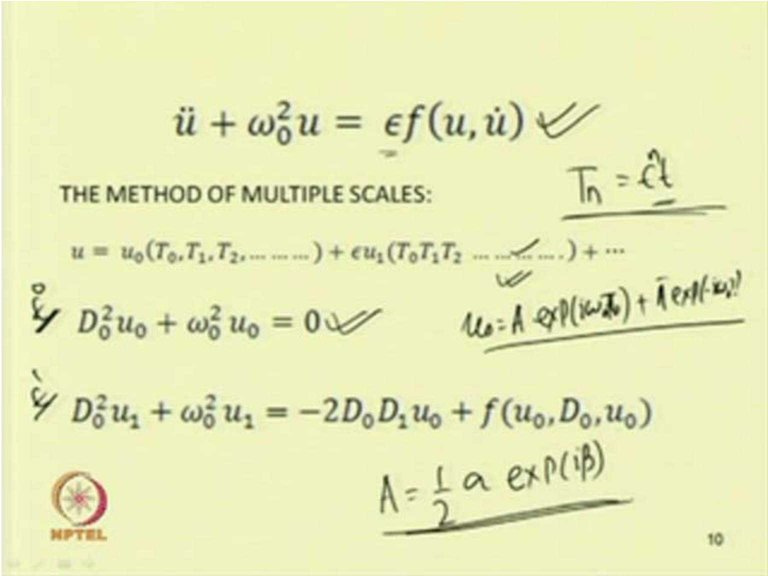


So now the solution of this equation can be written in the form of a e to the power i omega 0 t plus a bar e to the power i omega 0 t and substituting that equation in this equation and separating the terms separating the so by substituting the solution in this equation and killing the secular terms one can get a set of reduced equation. So those reduced equation can be written in this form, so these are the reduced equation so the solution of u 0 is written in this form, u 0 equal to a e to the power i omega 0 t i omega 0 to 0 plus a bar e to the power minus i omega 0 t 0, so by substituting this equation in the second equation and separating or killing the secular terms one can obtain a set of equation, and in that equation substitute a equal to half a e to the power i beta and separating the real and imaginary parts.

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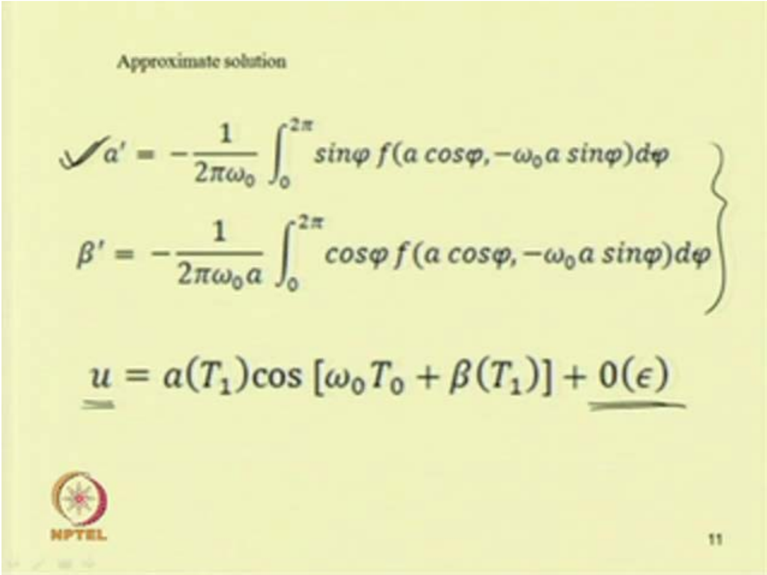


So one can obtain the set of reduced equation, so following similar principle what we have discussed in module 2, so we can find the solution to be in this form that is a dash equal to minus half 1 by 2 pi omega 0 integral 0 to two pi sin pi f a cos pi minus omega 0 a sin pi t pi and beta dash equal to minus 1by 2 pi omega 0 a 0 2 2 pi cos pi f a cos pi minus omega 0 a sin pi t pi, so one can find this amplitude, so for steady state solution one can substitute a dash equal to 0 and beta dash equal to 0 as they are not function of time. So this will reduce to, so one can obtain the response of the system. And the response of the system in terms of u can be written as a t 1 a is a function of t 1 cos omega 0 t 0 plus beta t 1 so which is of the order of epsilon plus order of epsilon. So in this way by using method of multiple scales one can find the response of the system.

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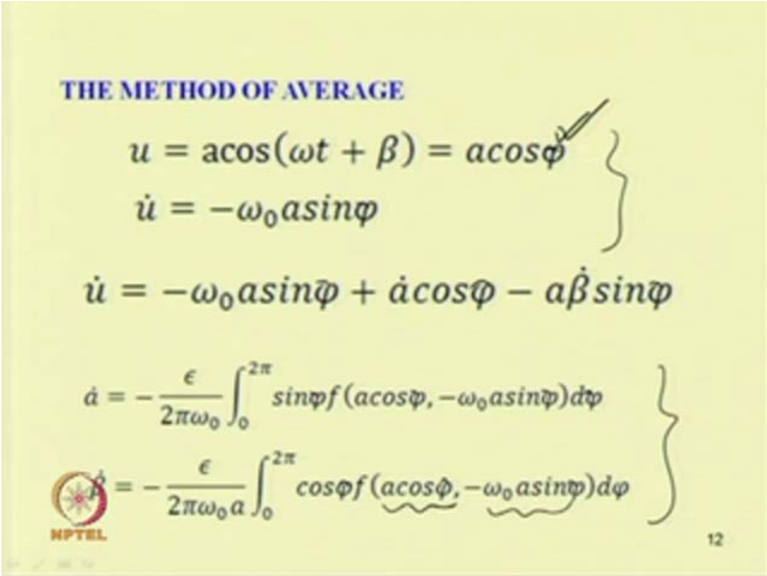


Similarly, by using method of averaging also one can find, so this is a just a revision of the method of averaging where we can use a solution u equal to a cos omega t plus beta and this omega t plus beta can be written in terms pi or this is equal to a cos pi. So one can write the solution u equal to a cos pi and this velocity in a simpler form by writing this equal to minus omega 0 a sin pi, but here unlike in case of the linear systems where this a and beta are concerned in this case in this case of non-linear system no longer a and beta will be constant. So a and beta should be function of time, so a and beta are function of time.

Now differentiating this, one can obtain this u dot equal to minus omega 0 a sin pi plus a dot a cos a dot cos pi minus a beta dot sin pi and substituting this equation and following similar procedure what we have discussed in module 2, so we can obtain this reduced equation in this form that is a dot equal to minus epsilon by 2 pi omega 0 0 to 2 pi sin pi f a cos pi and minus omega 0 a sin pi d pi and beta dot equal to minus epsilon 2 pi omega 0 a 0 to 2 pi cos pi plus cos pi into f a cos pi and minus omega 0 a sin pi here this a cos pi is u and minus omega 0 a sin pi is u dot, so substituting u equal to a cos pi and u dot equal to u dot equal to a sin pi so we obtain this equation. Let us take few examples, so taking few examples we can find. So let us take the linear damping.

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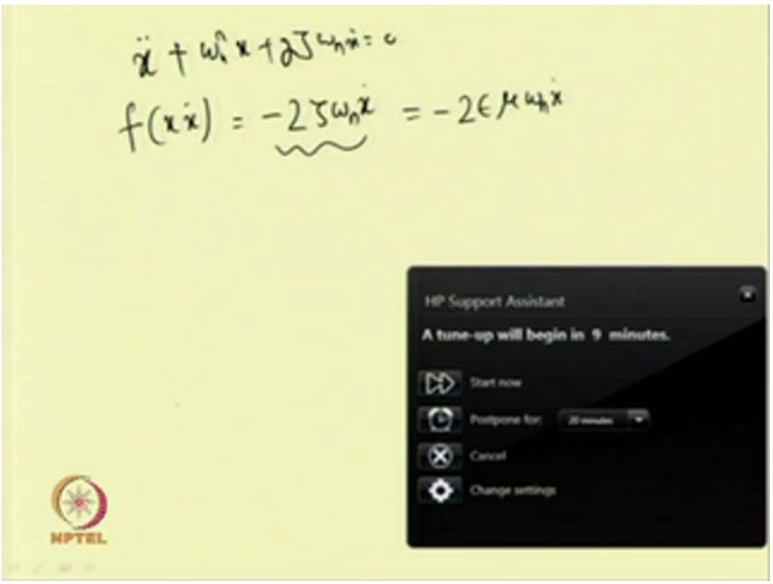


For example taking the case of linear damping, so we can write the equation in this form; equation in case of linear damping will be x double dot plus omega n square x, so in this case this plus we can write this thing equal to 2 zeta omega n so this is equal to 2 zeta omega n x dot equal to 0 so in this case this f function x x dot is nothing but, minus 2 zeta omega n x dot, so this thing can be written using this so this thing can be written using, using epsilon so you can write this equal to minus 2 epsilon mu omega n x dot.

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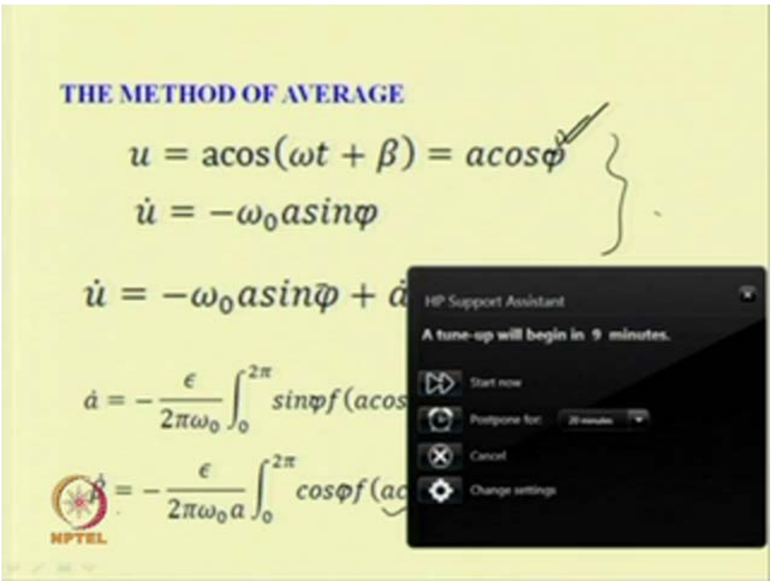

And by substituting this equation in this a dot and beta dot so we can find the solution of the system. So this is minus 2 mu omega n x dot, so in this case so let us so by substituting this f.

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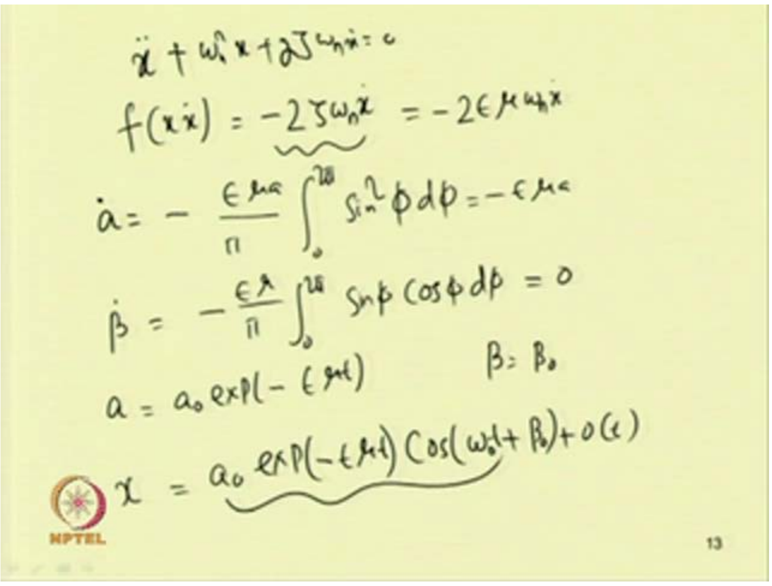

And so in this case so one can obtain this a equal to, so by substituting this minus 2 epsilon mu x dot in that equation, so in this equation so one can obtain this a equal to minus epsilon mu a by pi, integration 0 to 2 pi sin square pi into d pi, so this becomes minus epsilon mu a. Similarly one can write this beta dot equal to minus epsilon mu by pi integration 0 to 2 pi sin pi into cos pi d pi, so this is equal to 0. So the steady state solution as we got this beta dot equal to 0 and a dot equal to this is a dot a dot equal to minus epsilon mu a so a will be equal to so you can write this a equal to a 0 e to the power minus epsilon mu t and this beta equal to beta 0. Beta dot equal to 0 so beta becomes beta 0. So we can write this x, so we can write x equal to a 0, so we can write this x equal to a 0 e to the power minus epsilon mu t cos omega 0 t plus beta 0. So this is the order of epsilon. So this is for the linear systems already we know. So the solution in case of the linear systems becomes a 0 e to the power minus epsilon mu t or epsilon mu t into cos omega 0 t plus beta 0.

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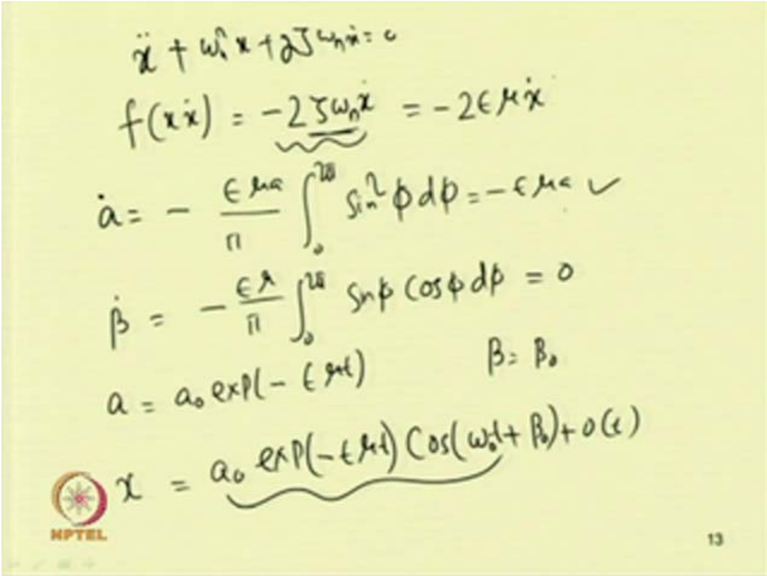

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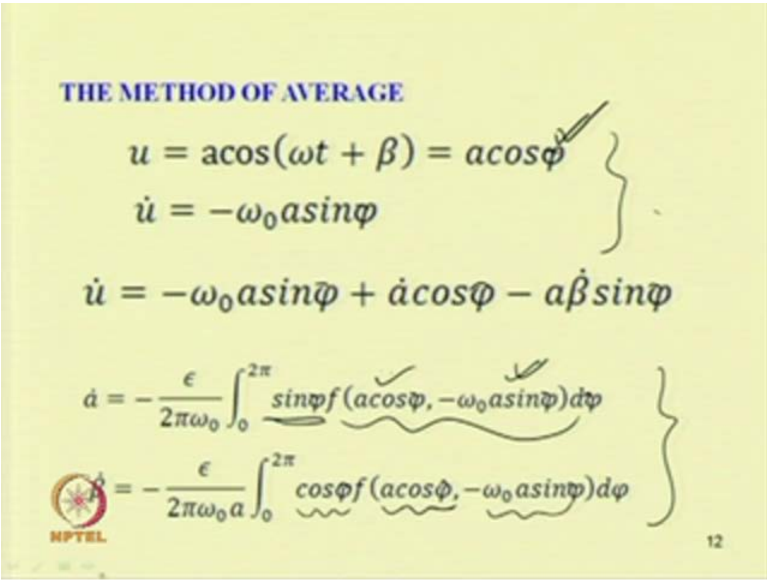


So, we have taken this here, we have taken this zeta omega n so let us take this zeta omega n equal to mu, so here zeta omega n is taken equal to mu not zeta so in this case 2 zeta omega n t so we have substituted this zeta omega n equal to mu. So this becomes minus two epsilon mu x dot, so the a dot equation we have written so from this so a dot so in a dot sin pi is multiplied with this function and in case of beta dot this cos pi is multiplied with this function. So, this is a cos pi and minus omega 0 so this is this is x and this is x dot, so for x dot one has to substitute this term and for x so if there is some x term then one has to substitute this.

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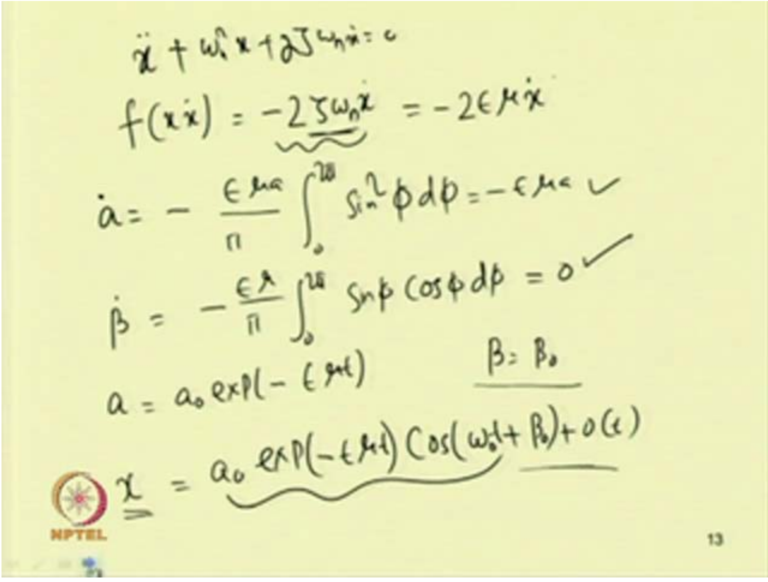


So in this case we have the x x dot term so one has to substitute this is minus omega 0 a sin pi, so by substituting that thing. So this equation reduces to a dot equal to minus epsilon mu a by pi 0 to 2 pi sin square pi d pi.

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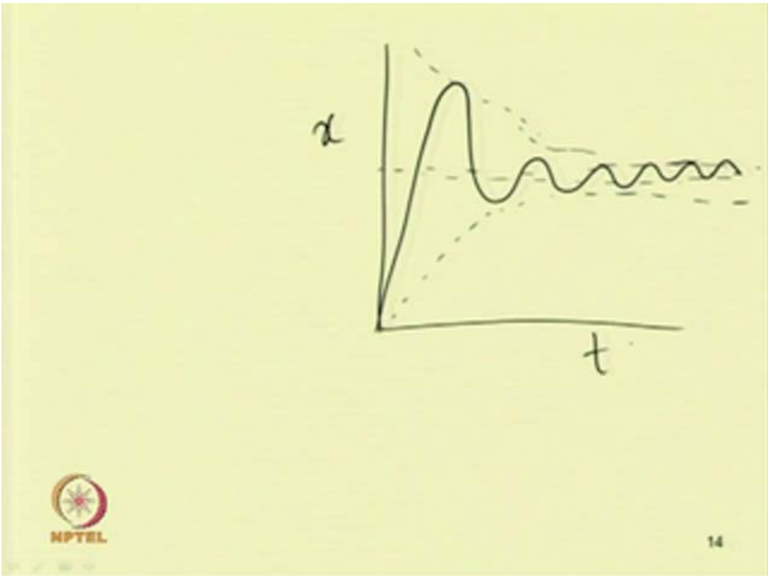


And this is equal to minus epsilon mu a and beta dot becomes 0, so beta integrating beta becomes beta 0 and integrating this one. So one obtain a equal to a 0 e to the power minus mu t. So the overall solution x becomes a 0 e to the power minus epsilon mu t cos omega t plus beta, so one can plot this equation to find the response of the system and already one familiar with the in case of the linear systems, so if one plots the response of the systems so it becomes; let us take this term so, it goes on reducing and so this reduction is exponentially so one so it reduces exponentially with time so this reduces exponentially with time, so this is the x versus time so if one got x versus time so this is for the under damped system so in case of the under damped systems.

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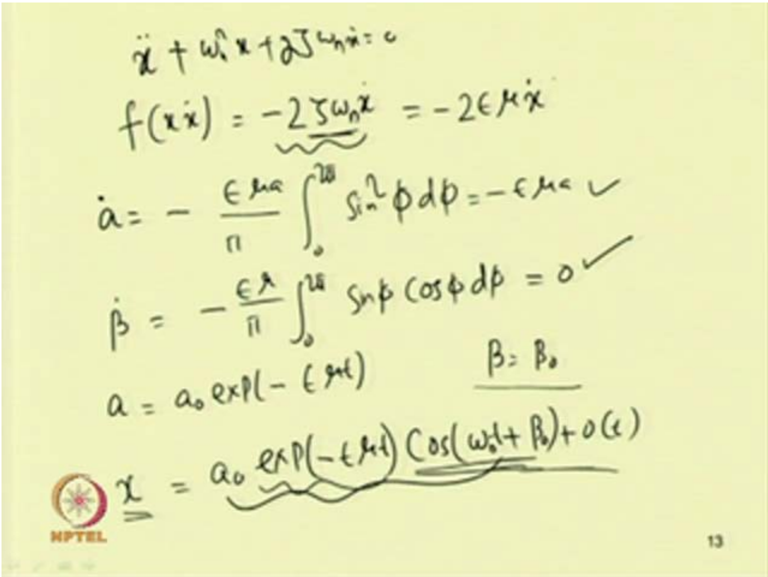


This reduces with time exponentially due to the presence of this term. So this is e to the power minus mu e to the power minus epsilon mu t so this reduces it is exponentially, and also there is variation in harmonic with a frequency omega 0. So here this 0 not a function of is not a function of this omega, so but if you take a non-linear systems.

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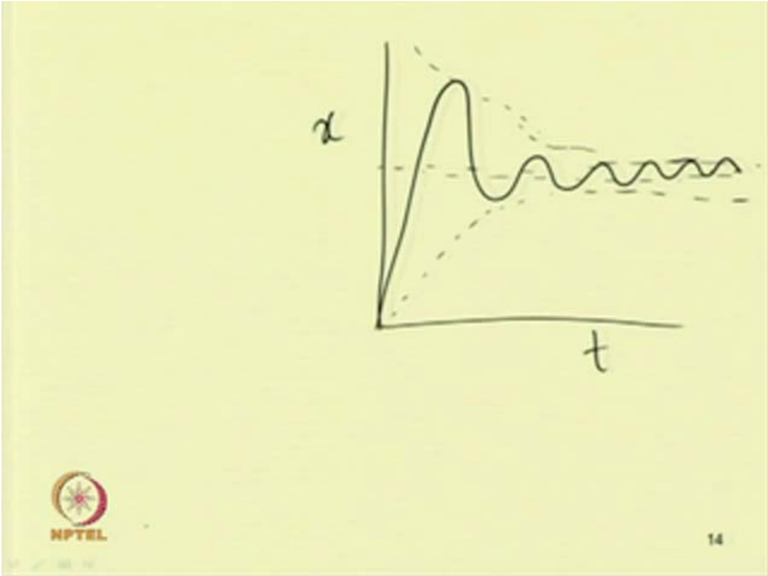


Let us take non-linear systems with damping, coulomb damping, so in case of coulomb damping we can write the equation in this form with coulomb damping.

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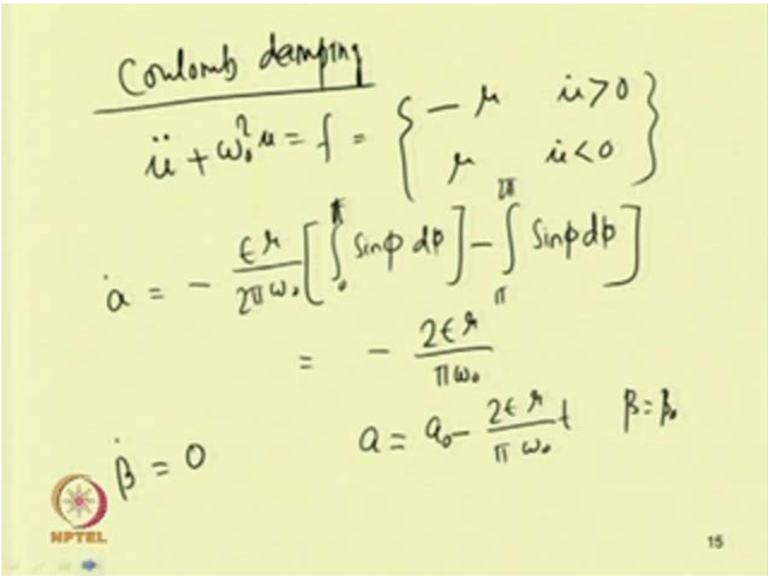


So taking coulomb damping we can write the equation in this form that is u double dot plus omega 0 square u equal to f, so this thing we can write equal to minus mu, so so when u dot greater than 0 so plus mu when u dot less than 0. So in this way we can write this function and now applying this method of averaging, so we can write this equation equal to a dot equal to minus epsilon mu by 2 pi omega 0. Integration 0 to 2 pi sin pi so sin pi into, so this term is constant so i as this is constant so you can write minus mu. So this is minus mu and then we have this is d pi, so this whole cycle 0 to 2 pi we can divide into 2 parts. One will be 0 to pi, so this is 0 to pi and then minus pi to 2 pi minus pi to 2 pi. We can write this is equal to sin pi d pi sin pi d pi so this integration is coming to be 2 epsilon mu by pi omega 0. Similarly, we can find this beta dot equation so beta dot equal to so beta equal to 0.

Like previous case beta dot here also equal to 0, so the solution becomes as a dot equal to this. So a become so integrating this a becomes a 0 minus two epsilon mu by pi omega 0 into t and beta equal to beta 0. So one can plot this function, so for coulomb damping, so you just one can observe the solution to be in this form.

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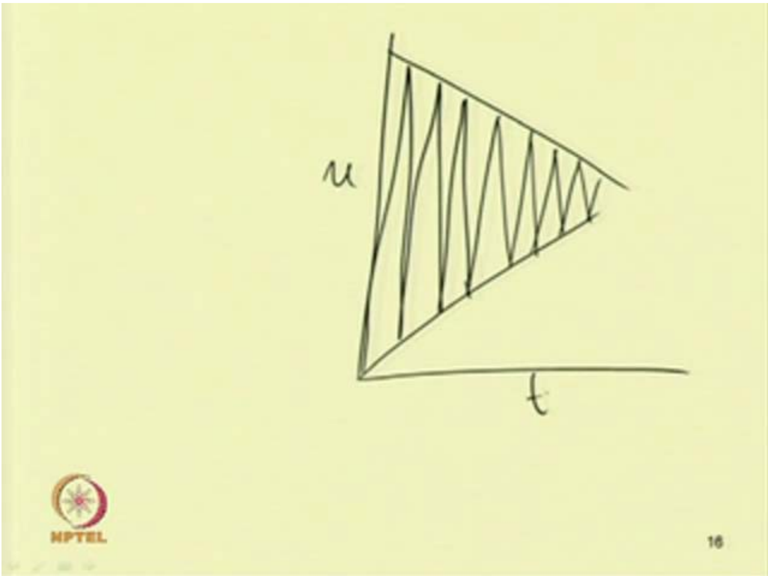


So no longer it is exponentially dickering decaying, so as its not exponentially decaying so one can plot this solution which will be in this form. So it reduces, so this goes on reducing in a linear way unlike in case of, unlike in case this is this is x u versus t. So unlike in case of linear system where it is exponentially decaying so in this case it reduces in a linear way.

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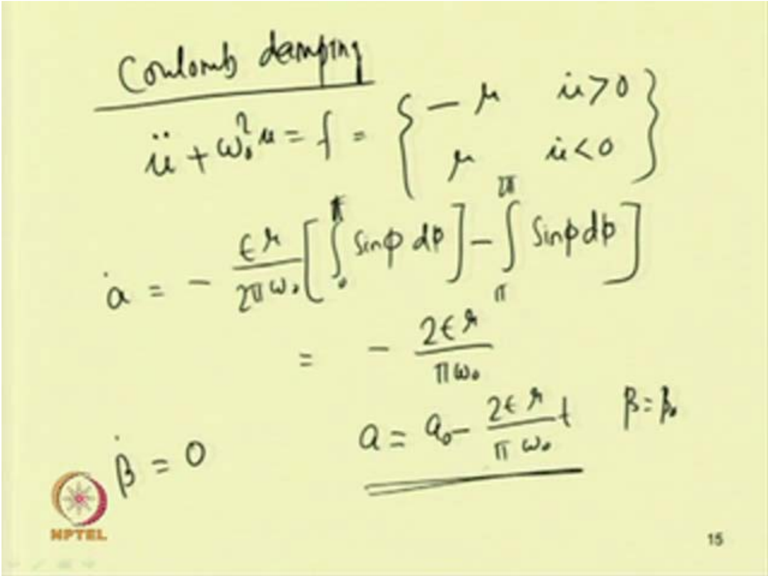


So this a equal to a 0 minus 2 pi mu by pi omega 0. So this is time so this constant, so it is in the form of m x plus c y equal to m x plus c that is that of a linear equation and 1. So that means it reduces linearly and one can obtain the response in this way. And in case of let us take the example of a quadratic damping.

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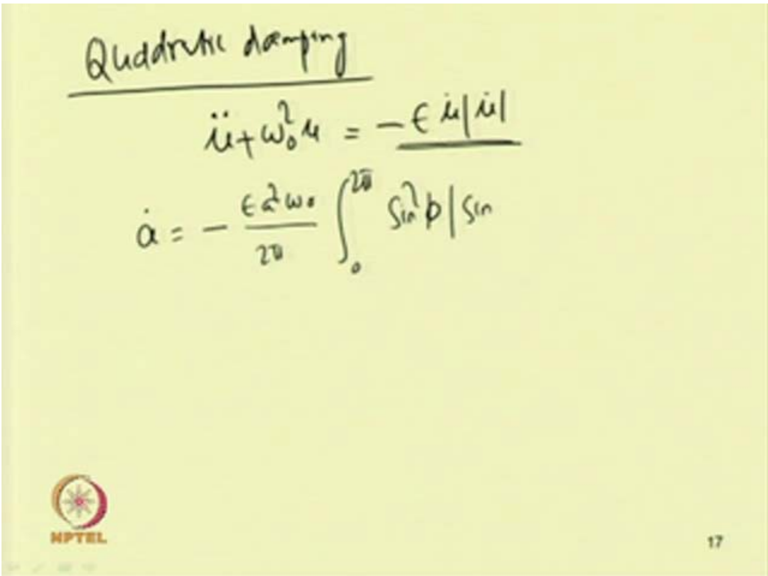


So in case of a quadratic damping, so in case of system with quadratic damping the equation motion can be written. So the equation motion can be written in this form that is u double dot plus omega 0 square u equal to minus epsilon u dot mod u dot. So in this case one can write this a dot equation in this form, a dot will be equal to so a dot equal to minus epsilon a square omega 0 by 2 pi integration 0 to 2 pi sin square pi into mod sin pi. So this is sin pi into u dot into u dot and for u dot where one can substitute this so for u dot one has to substitute minus omega 0 a sin pi, so by substituting that thing so one can obtain.

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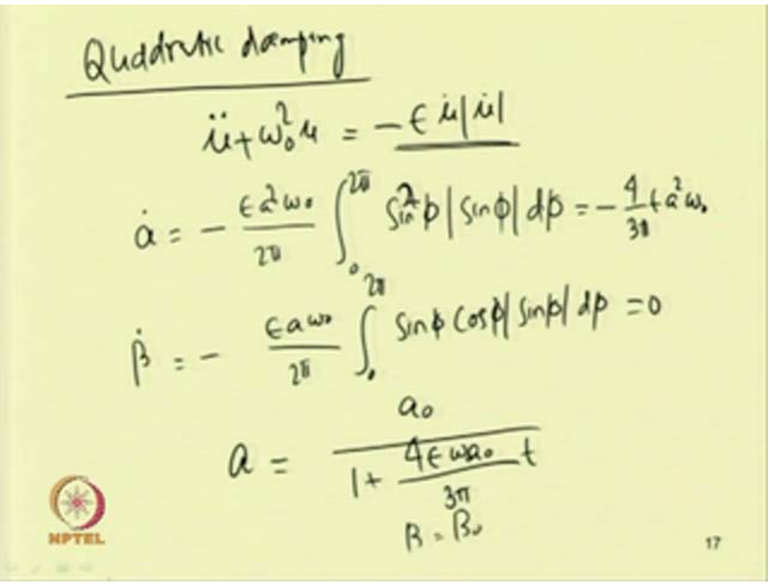


So sin pi and sin pi into sin pi so this becomes sin square pi and for this u dot so one have this mod u dot so this into d pi. Similarly, beta dot will be equal to minus epsilon a omega 0 by 2 pi integration 0 to 2 pi sin pi into sin so it is sin pi into cos pi into mod sin pi into d pi. So integrating these two, so one can write this beta dot equal to 0 and this a dot equal to minus 4 by 3 pi epsilon a square omega 0 so a dot becomes minus 4 by 3 pi epsilon a square omega 0. So one can write so by integrating this thing, so one can write this taking this initial a equal to a 0; so the solution a one can write 1 by a equal to or one can it so one can write this a equal to solution a equal to a 0 by 1 plus 4 epsilon omega 0 a 0 by 3 pi into t.

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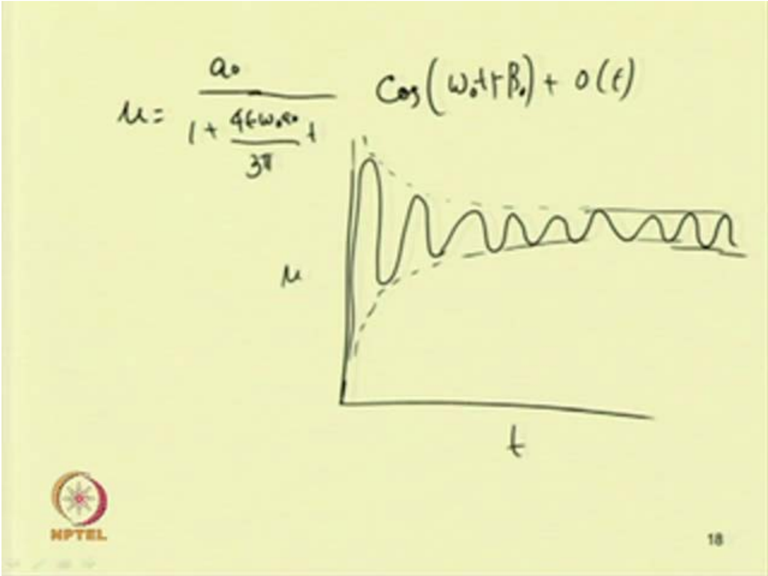


So in this case a equal to a 0 by one plus 4epsilon omega 0 a 0 by 3 pi t and this beta equal to, so beta equal to beta 0. So as beta equal to beta 0, so the solution total solution of the systems u can be written as a 0 by 1 plus 4 epsilon omega 0 a 0 by 3 pi into t into cos omega 0 t plus beta 0 so this is order of epsilon. So one can plot this thing also, unlike in case of the coulomb damping or in case of this linear case so here no longer it is exponentially decreasing, but this is one can plot this to find the response so u so equal to u versus t, so the solution will be, will vary between this.

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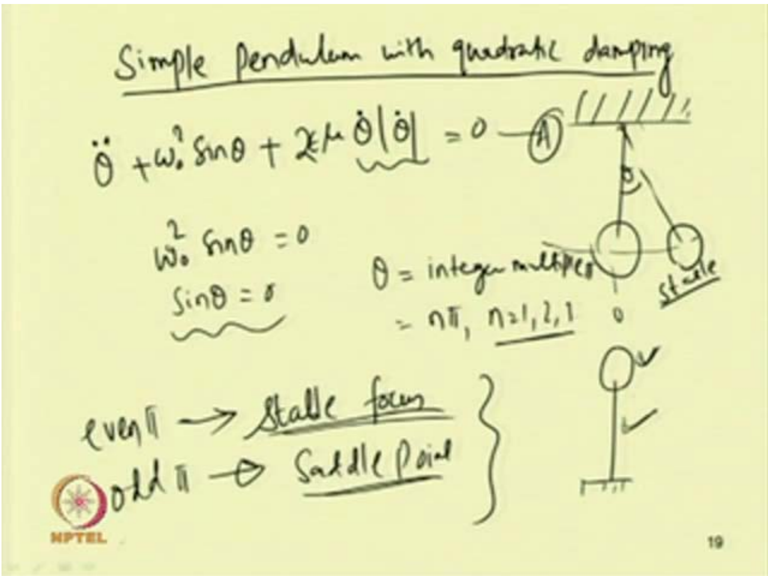

So similarly one can find the response for other different type of systems with different damping. So let us take one more example that is the simple pendulum, so in this case in case of the simple pendulum, let us take simple pendulum with quadratic damping. So in case of simple pendulum with quadratic damping, let us take the case of simple pendulum with quadratic damping. So previous case we have taken example where we have taken the restoring force to be linear, but in case of the simple pendulum already we know that the restoring force is not linear and we can write the equation motion in this form. That is theta double dot plus omega 0 square sin theta plus, so if you are considering this quadratic damping then we can write this equation in this form two mu theta dot mod theta dot or we can write this is equal to this mu dot.

So instead of writing mu cap we can write this epsilon mu theta dot into theta dot. So this is the quadratic damping term we have taken, so if the damping is considered to be quadratic in case of the simple pendulum motion; let us see what will be the response. So this is the equation, equation of motion in this case, so theta is the motion of the pendulum. So when we are considering this theta to be small then the sin theta becomes theta and the equation reduces to theta double dot plus omega 0 square theta plus. So if you are considering this damping then this damping terms will be there, so if damping is not considered then that that equation is theta double dot plus omega square theta equal to 0 and already we know the solution of the system.

But if you are considering this theta plot to be small, then we can go for this qualitative analysis and also approximate solution by using different methods. So if one performs this qualitative analysis then taking this, so (( )) one can obtain the singular point in this case by putting this time derivative terms equal to 0. So by putting this time derivative terms equal to 0, that means theta double dot equal to 0 and this damping term equal to 0. So one obtain this equation omega 0 square sin theta equal to 0 or one can have the sin theta equal to 0, so the singular points will be obtained corresponding to sin theta equal to 0 or theta integer multiple of theta is integer multiple of pi. So this is integer multiple of pi or this is equal to n pi; so now by taking even multiple, so one can take this even multiple of pi or odd multiple of pi.

So by taking even multiple of pi, so even multiple, so this is theta correspond to 0, so this is theta correspond to pi that is 180 degree. Then theta equal to pi correspond to the pendulum vertical pendulum and corresponding to theta equal to 2 pi or 0, so this is the equipped damp position. So we have this sin theta equal to 0 or theta equal to integer multiple of pi that is n pi n equal to n equal to one 1 2 3 4. So this corresponds to different equilibrium positions. So it has two set of equilibrium position, so one in case of it is vertically downward and the second case when it is vertically upward. So the vertically upward position, so which is, so the vertically upward position this is the vertically upward position. So in this case the system, it can be shown that the system is unstable and when it is in this downward position, the system stable; so it is in stable condition, when it is in downward position that is corresponding to even multiple of pi and for all multiple of pi so this becomes unstable and one during of stability analysis we have studied, so if one perform the stability analysis one can see in this case for even multiple of pi, so it reduces to stable focus so stable focus.

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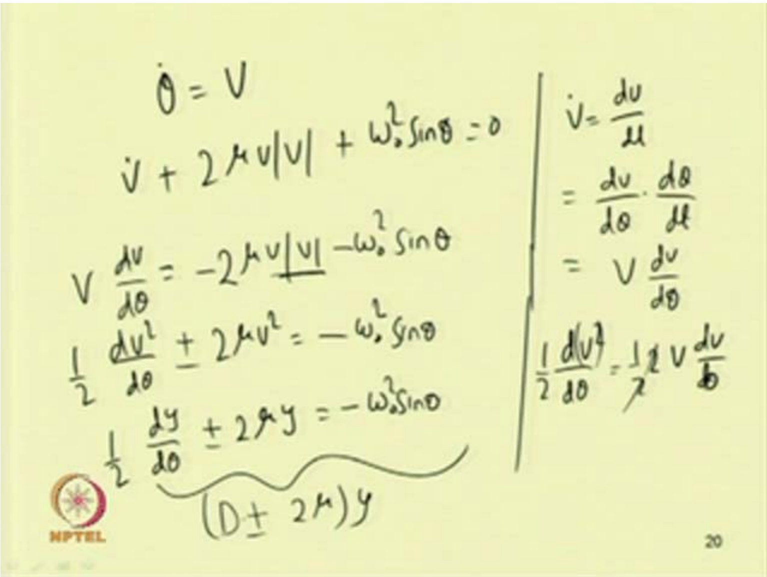

And in case of this odd multiple of pi, so this is for even even pi so this reduce to stable focus and for odd multiple of pi, so this reduce to unstable or the saddle point, so one can have a saddle point corresponding to this odd multiple of pi and for even multiple pi so this becomes stable focus. So one can perform this qualitative analysis now to find the response of the system, so for example, so by taking this theta let let us take theta dot equal to v, so if one take theta dot equal to v then this equation so this equation a reduces to so one it reduces to v dot plus 2 mu v v plus omega 0 square sin theta equal to 0. So one can write this v dot equal to dv by dt, so which can be written as dv by d theta into d theta by dt. So d theta by dt is nothing but, so we have taken already theta dot equal to v so this v dot can be written as v into d b v by d theta.

So this v dot can be written as v d v by d theta. So this equation then we can write or we can write this v dv by d theta equal to taking these things to right end side so it can be written equal to minus to mu v into mod v minus omega 0 square sin theta. So v dv by d theta equal to, so one can write v dv by d theta equal to minus 2 mu e multiplied by v minus 0 square sin theta. Now one can write this equal to the this v d v by d theta can be written in this form also, half dv square by d theta so one can write equal to half into 2 into v dv by d theta. So half d by d theta v square equal to half into 2v into dv by d theta so this this cancel so this v dv by d theta can be written equal to half d v square by d theta d v square by d theta so plus minus 2so if will take this thing to left hand side so this becomes by removing this mod.

We can write this v mod v equal to v if v greater than 0, and it is equal to minus v if v is less than 0. So that way you can write this equal to plus minus 2 mu v square equal to minus omega 0 square sin theta. So taking this v square equal to y so we can write this equation half dy by d theta so half dy by d theta plus minus 2 mu y equal to minus omega 0 square sin theta. So this equation, so you can write the solution of this equation, so taking, so it will have a homogenous part and a particular integral. So in case of the homogenous part you can write putting it equal to 0 and in case of homogenous part we can find the solution so here the auxiliary equation one can write the auxiliary equation.

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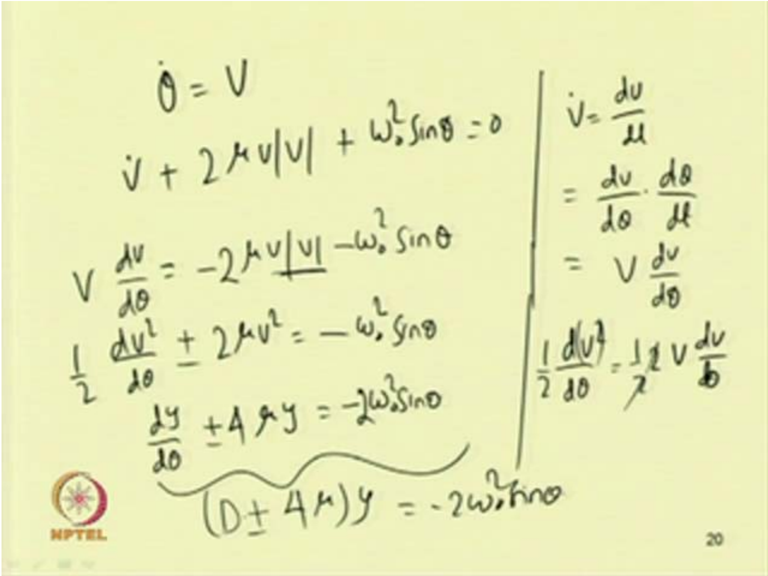

So writing the auxiliary equation in this form that is your d0, so this is d plus minus two mu into y or we can write this equation by removing this 2 so this will be written as 4 and this becomes 2 so d y by d theta plus minus 4 mu y equal to minus 2 omega 0 square sin theta. Now it can be written so d plus minus 4 mu y equal to minus 2 omega 0 square sin theta. Now finding the complimentary part and particular solution of this equation.

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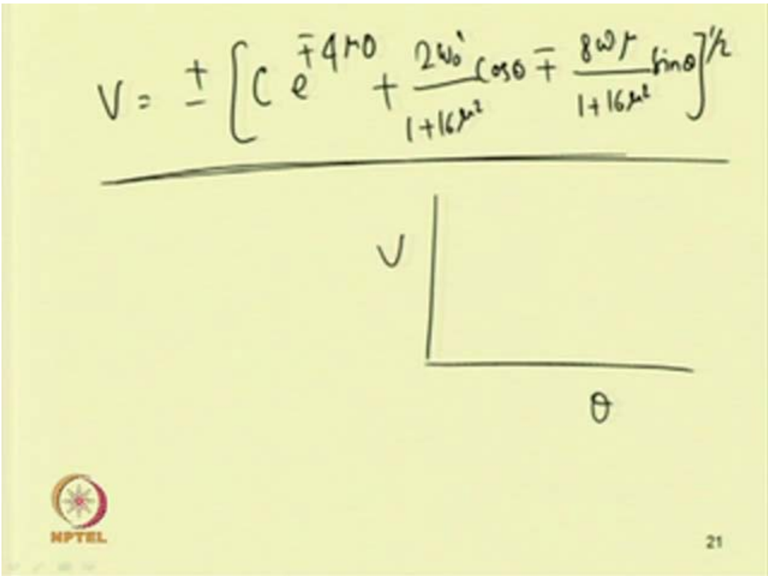

So one can write the solution of the equation, so the solution of the equation in terms of v can be written as... So v will be equal to so plus minus c e to the power minus plus 4 mu theta. So this come from the complimentary part and for particular integral one can obtain this is equal to 2 omega 0 square by 2 plus 16 mu square cos theta plus minus 8 omega 0 square mu by 8 omega square mu by one plus sixteen mu square sin theta to the power half so one can plot this equation for v and theta so one can plot this v and theta where v equal to the d theta by d t and one can find the response of the systems so in case of if one recalls so in case of the linear sys in case of the system conservative systems without considering damping.

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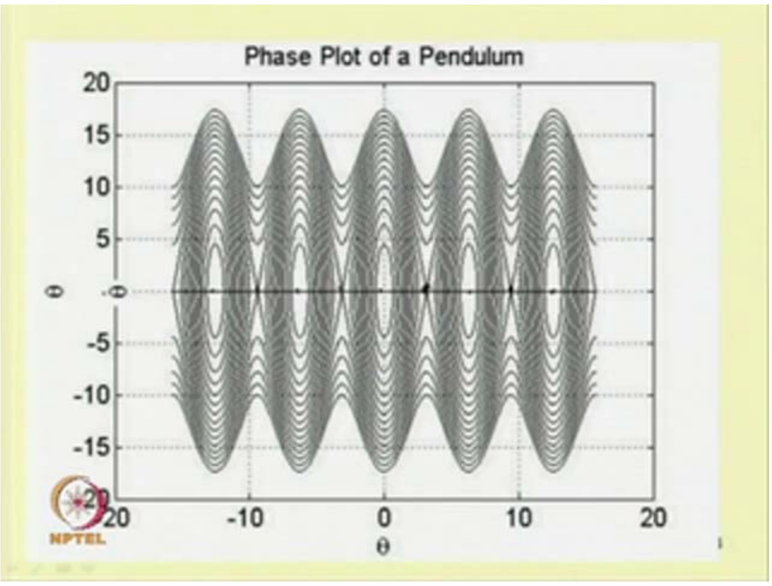


So the potential well has been plotted and one can obtain this stable node at these points. So these are the stable node where periodic responses has been obtained and one has saddle points also in between, so these are the saddle nodes corresponding to the unstable points so in this case also, in this case also one can obtain this 2 type, but the saddle node one can obtain the saddle points but, these saddle nodes are reduces to saddle focus so in between the saddle nodes, so in between the saddle points so one has a set of so if one plot this equation.

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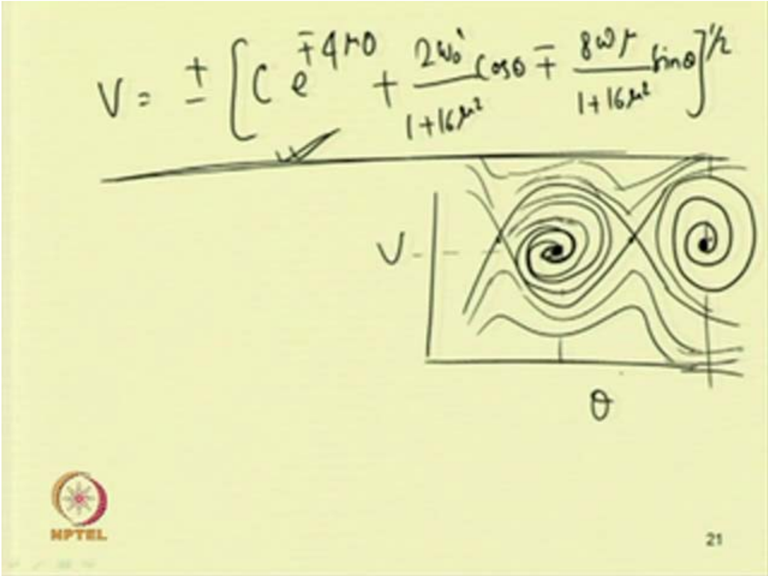


So, if it is plotted so one can see so in between the one can have the saddle, so these are if these are saddle points so corresponding to this so one has a set of saddle focus. So the system has come to rest at these points. Similarly, here one can have saddle, so for different initial conditions so it will come to the equilibrium position. This equilibrium or it will come to equilibrium position or it becomes 00, so this will be 00 corresponding to this so it becomes a stable focus.

So instead of a stable node as in case of the system conservative system, that is without considering damping so in this case one has a stable focus. So if one expand this thing so also one can have a set of, so corresponding to this also similar to similar to in case of the conservative system so here also one can have the flow one can plot the flow diagram and similarly corresponding to this point, so we have a stable focus. In case of stable focus the response becomes so it comes to the trivial state, so that means due to damping the response come to the trivial state. In case of non con conservative system when there was no damping in the system the pendulum oscillates with frequency omega 0, but in this case it come back to the equilibrium position with the solution this. So one can use this method of multiple skill to to find the response of the system one can also use this approximate method so for example, by taking the approximate method.

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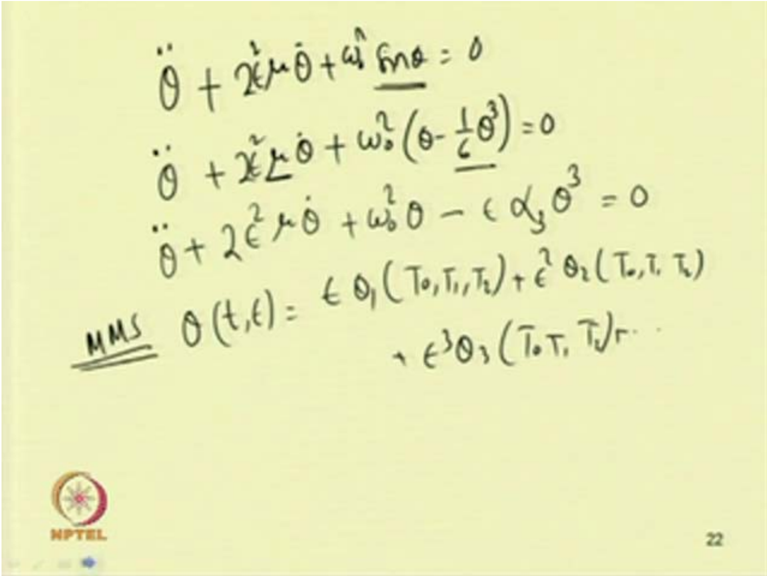


Let us write this equation. Equation in this form so if one use this original equation is theta double dot plus 2 epsilon mu, so one can write the equation and one can find the solution of this theta double dot plus 2 mu 2 epsilon mu theta dot plus omega 0 square sin theta equal to 0. So in this case one can by expanding the sin theta up to cubic term, so this can be written theta double dot plus 2 mu epsilon mu theta dot plus omega 0 square into theta minus 1 by 6 theta cube equal to 0. So now by substituting this mu epsilon mu or by substituting this mu equal to, ok so let us take this further small so we can write this things equal to theta double dot plus 2 epsilon square to take care of both the non-linearity. So here we have a non-linear term here and another non-linear terms also we can, this damping so we can write this equation 2 epsilon square mu so let us take this is order of epsilon square. So mu theta dot plus omega 0 square theta minus epsilon omega 0 square, so we can use this epsilon square theta 2.

So we can write this equation by using this epsilon term theta cube equal to 0. So we can substitute so this term epsilon alpha 3 we can write this thing as epsilon alpha 3 and we can write this equation or simply we can write this equation also by putting 1 epsilon. Now by using method of multiple scale using method of multiple scale, so we can write this theta t epsilon in this form epsilon theta 1 t 0 t 1 t 2 plus epsilon square theta 2 similarly, t 0 t 1,t 2 and epsilon q plus epsilon q theta 3 t 0 t 1 t 2.

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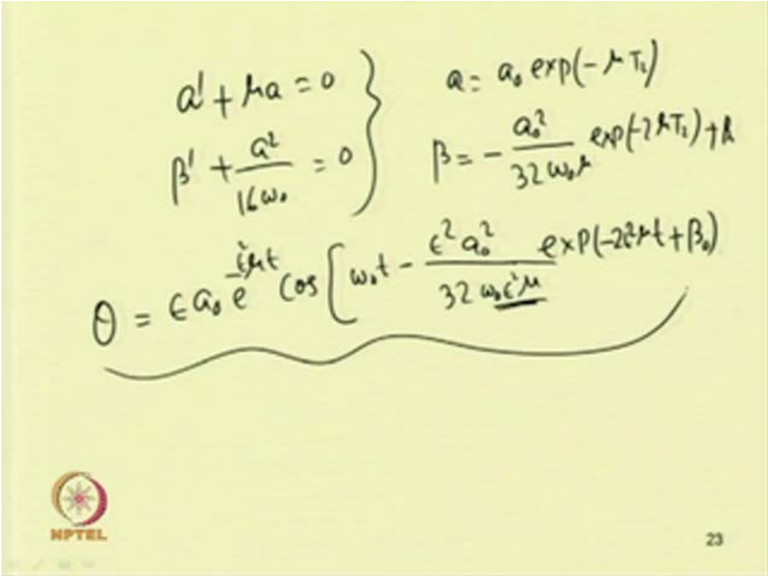


So this way we can write so by writing this so and following the standard procedure of method of multiple skills so we can obtain the reduced equation in this form that is a dash plus mu a equal to 0. So this is the first equation one can obtain and the second equation beta dash plus a square by 16 omega 0 equal to 0. So from this one can write this a equal to a 0 e to the power minus mu t 2 and this beta equal to minus a 0 square by 32 omega 0 mu e to the power minus 2 mu t 2 plus beta 0. So the theta term can be written as epsilon a 0 e to the power minus mu t cos omega 0 t minus epsilon square a 0 square by 32 omega 0 mu e to the power minus 2 mu so we can write the original mu.

So minus 2, so minus 2 mu, so this this case minus 2 mu t plus beta 0 or this is equal to minus 2 epsilon square mu t beta 0. So theta equals to epsilon a 0 e to the power minus epsilon square mu t cos omega 0 t minus epsilon square a square by 32 by 32 epsilon 0. So this is epsilon square, epsilon square mu, so this is original damping. Epsilon square mu e to the power minus 2 epsilon square mu to plus beta 0. So in this way one can write or one can find the solution of the equation by using by using this approximate solution one can use different type of approximate solution to find the response of the non-conservative systems.

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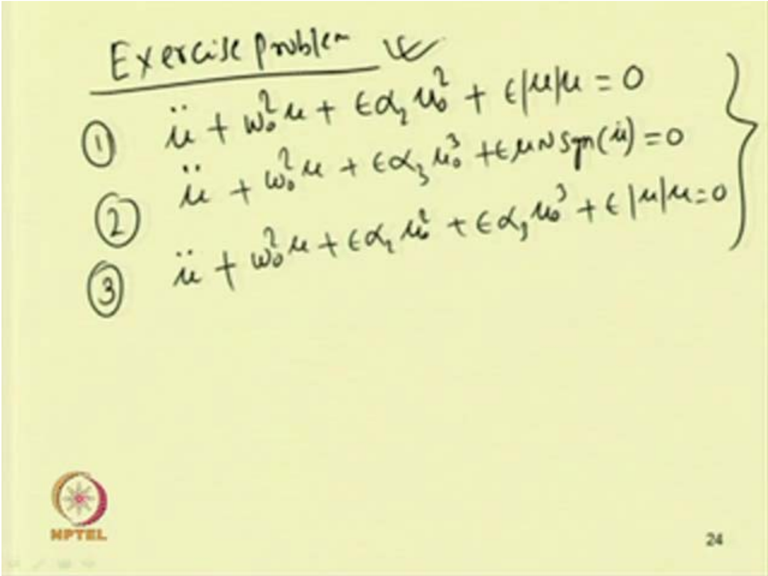


So one can take several examples to find or one can take this exercise problem to find the response of the system. So by taking different type of damping, so let us take this equation duffing equation u double dot plus omega 0 square u plus epsilon alpha 2 u 0 square. So let us take along with this this damping epsilon into u into u so this is equal to 0, this is 1. So one can take this relaxation equation, so one can take other type of equations also to find the solution, other equation let use another equation with cubic damping u double dot plus omega 0 square u plus epsilon alpha 3 u 0 q. So with coulomb damping also one can write; so in case of coulomb damping, one can write this is equal to epsilon mu n signum u dot so this is equal to 0, one can take one more example also and omega 0 square u, taking both epsilon alpha 2 u 0 square plus epsilon alpha 3 u 0 cube plus epsilon u u equal to 0.

So one can take these are the exercise problem to solve this non conservative systems. So we have seen in case of the linear, the difference between the linear and non-linear systems. So in case of linear system the amplitude no longer is a function of omega, but or the frequency does not depends on the amplitude. But in case of the non-linear system the frequency depends on amplitude. Also in case of this non-linear conservative - non conservative system, so in this non conservative system so instead of getting a stable node one obtain a stable focus and the response die down die down with time. So in case of linear systems one get a periodic solution but, in this case as a stable focus is found so it reduces to the trivial state. So next class we will study about this multi degree of freedom system and we will find the non-linear response of the multi degree of freedom system.

Thank you.