Non-Linear Vibration
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Module - 6
Applications
Lecture - 1
Single Degree of Freedom Nonlinear Systems
With Cubic and Quadratic Nonlinearities

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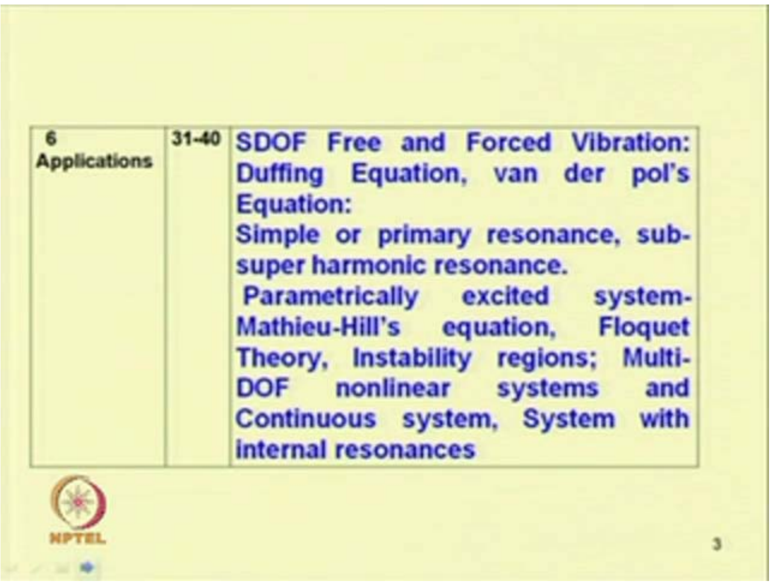

Welcome to today class of non-linear vibration. So, already we have completed 5 modules in this non-linear vibration. And in this module we are going to study about the applications of non-linear phenomena what we have studied till now. So, in the first module we have known about the introduction of the non-linear systems, in the second module we have studied about or how to find the governing differential equation of motion of the non-linear systems, in the third module we have studied about different approximation methods to find the solution of the governing equation of motion and in fourth we have studied about the stability analysis of the obtain response, in the fifth module we have studied about the numerical techniques used in this non-linear phenomena or non-linear vibration systems and in the last module we are going to study about the applications of this non-linear studies. So, we will apply the methods what we have studied still now to single degree of freedom system with free and force vibrations.

So, there we will discuss about the Duffing equation Van der pol equations also, in case of the forced vibration will discuss about the simple primary resonance simple resonance or primary resonance then, super harmonic and sub harmonic resonance. Then, we will study about the parametrically excited systems in which we will see about this Mathieu hills equations also, already we have discussed about the Floquet theory in stability analysis, we will see in case of the parametrically excited system how the solutions obtained by this Floquet theory.

Then, we will discuss about this instability regions and then, we will go for this multi degree of freedom non-linear systems also, in this examples we will take the continuous systems with and without internal resonance cases also, we will study. So, in this applications so, we will start with the single degree of freedom free vibrations or will study about the conservative systems first so, then next class we will study about the non-conservative systems so, single degree of freedom system conservative and non-conservative systems in the first two class. That means in the first class, today's class we will study about the single degree freedom non-linear systems with quadratic and cubic nonlinearities.

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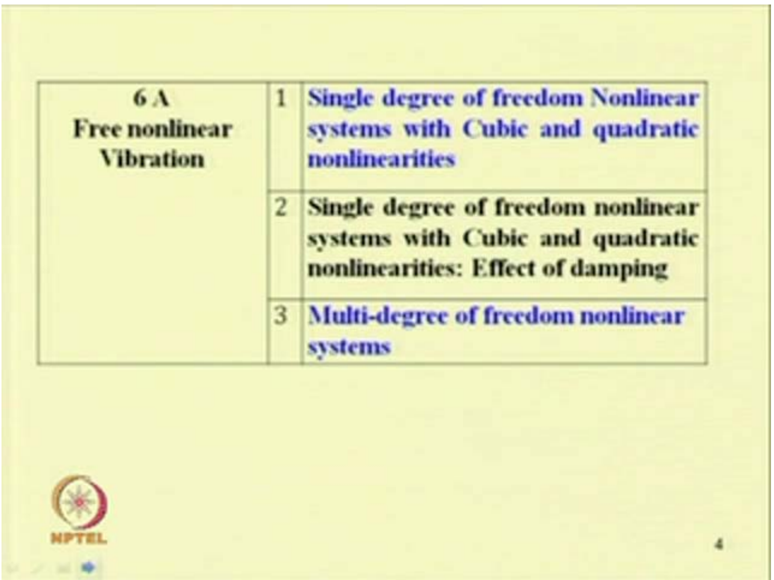

Particularly, will study about the conservative systems by using this Duffing equation; then, we will study next class about the single degree freedom, non-linear systems with cubic and quadratic nonlinearities so, here we will study about the effect of dumping that is will go for this non conservative systems then, in this free vibration we will study about the multi degree of freedom system. So, thus then after studying this free vibration we will study about the forced vibration of the system and then parametrically excited vibration of the systems.

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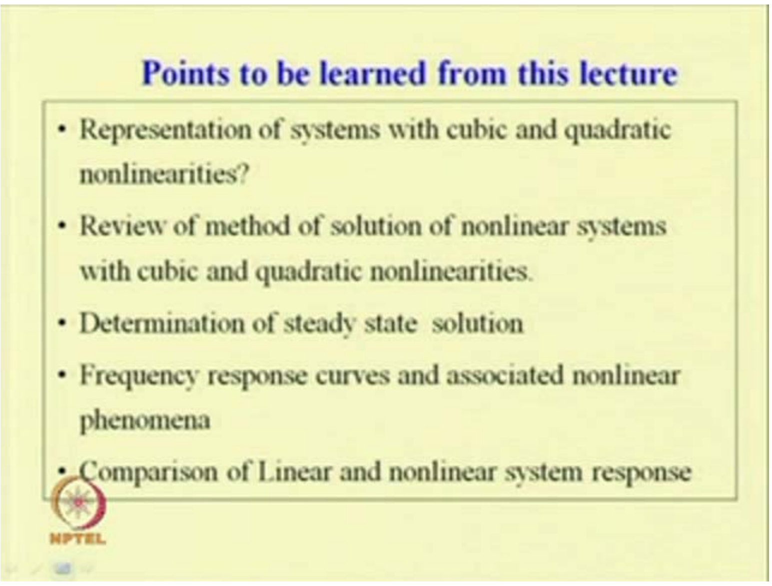

So, today class we will know about representation of the systems with cubic and quadratic nonlinearities so, some physical systems we will study where we will see about the cubic and quadratic nonlinearities then, review of the methods of solution of non-linear systems with cubic and quadratic nonlinearities.

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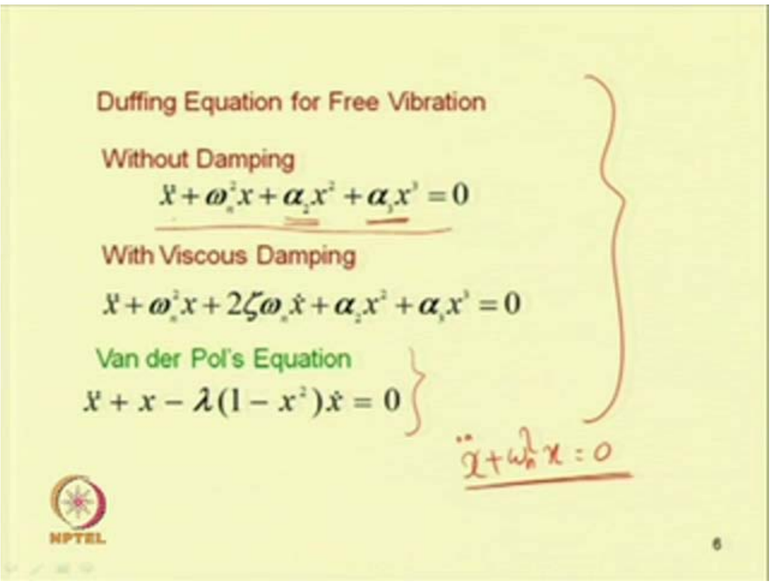

Then, we will discuss about the determination of the steady state response steady state solution then, from this equation so, we will see how this frequency response curve or the how the frequency and amplitude are related in this non-linear system then finally, will compare the linear and non-linear system response. So, in case of this free vibration so, will going to study or will study about this Duffing equation for free vibration so, without dumping the equation can be written as x double dot plus omega n square x plus alpha 2 x square plus alpha 3 x cube equal to 0. So, this is quadratic nonlinearity and these are the cubic nonlinearity and in case of the systems without nonlinearity it can be reduced to x double dot plus omega n square x equal to 0. So, in case of viscous dumping so, we can write this Duffing equation in this from that is x double dot plus omega n square x plus 2 zeta omega n x dot plus alpha 2 x square plus alpha 3 x cube equal to 0.

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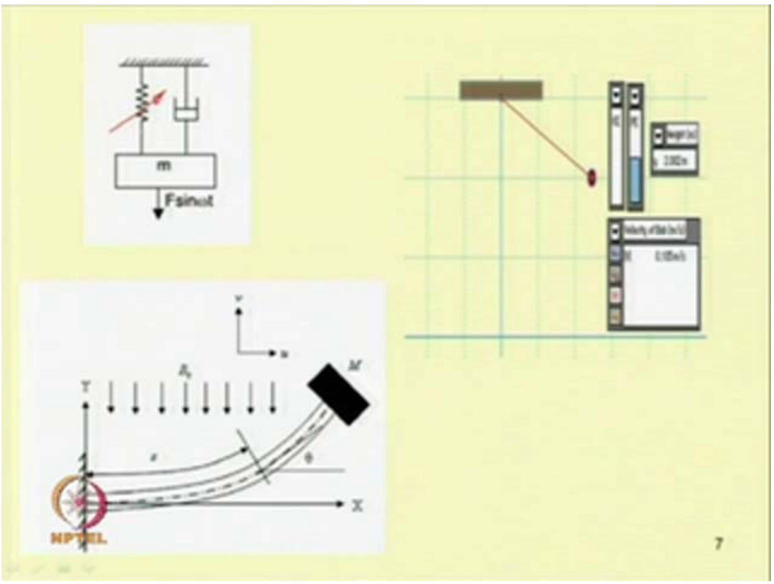

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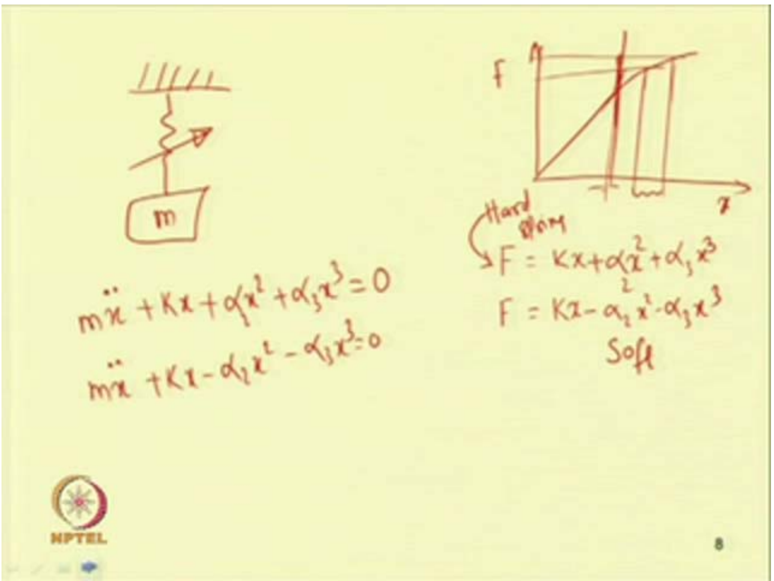

Also, if we will study the Van der pol equation this can be written in this form that is x double dot plus x minus lambda into 1 minus x square into x dot equal to 0. So, in these two classes we are going to study this conservative and non-conservative system. So, today class we will mostly discuss about the systems with quadratic and cubic nonlinearity that is of this type that is x double dot plus omega n square x plus alpha 2 x square plus alpha 3 x cube equal to 0. So, first let us see what are the systems which will give rise to these type of equations so, let us first consider this spring mass damper systems. So, in this spring mass damper systems considering the systems without dumping so, one can write the equation of motion of the systems, let us take the systems without damping so, that is spring and mass and let us take the spring to be non-linear. So, generally in case of a linear spring so, if we plot this force versus x the system will be for linear system this is the curve and for non-linear curve, non-linear systems either it will be soft or hard so, in case of soft spring so, in case of soft spring, there will be more elongation with lays force. When the force is applied when small force is applied so, we can have large deformation in case of soft spring and in case of hard spring so, with small with large force we will have small deflection.

So, with so, let us take two points on this so, this is one point and this is another point and let us draw the so, in this case the so, for the small deflection so, this is the small deflection so, for the small deflection so, we have to apply a large force and in this case with the same forcing so, let us take this force so, this the large deflection. So, this is x that is deflection so, the deflection is large and so, for same amount of force the deflection is large here so, this is soft spring and in this case the deflection is small with the same amount of force so, this is a hard spring. So, one can have a soft spring or hard spring so, in this cases. So, the equation no longer be linear or one cannot write this equation F equal to k x. So, in this case either we have to add so, for the hard spring let us add this quadratic and cubic so, alpha x square or you can write alpha 2 x square plus alpha 3 x cube.

Similarly, for the soft spring or this is for hard spring and in case of soft spring we can write this equation k x minus alpha 2 x square minus alpha 3 x cube. So, this is for soft spring and this equation is for hard spring so, either by taking a soft spring or hard spring. So, the equation of motion for the system can be written in this form that is m x double dot plus k x this is the linear part plus alpha 2 x square plus alpha 3 x cubes. So, as no external force is acting on the system so, this will be equal to 0. Similarly, for this soft spring we can write m x double dot plus k x minus alpha 2 x square minus alpha 3 x cube equal to 0. So, instead of taking a spring so, if you take a simple pendulum case also, in simple pendulum case also we can write the equation of motion.

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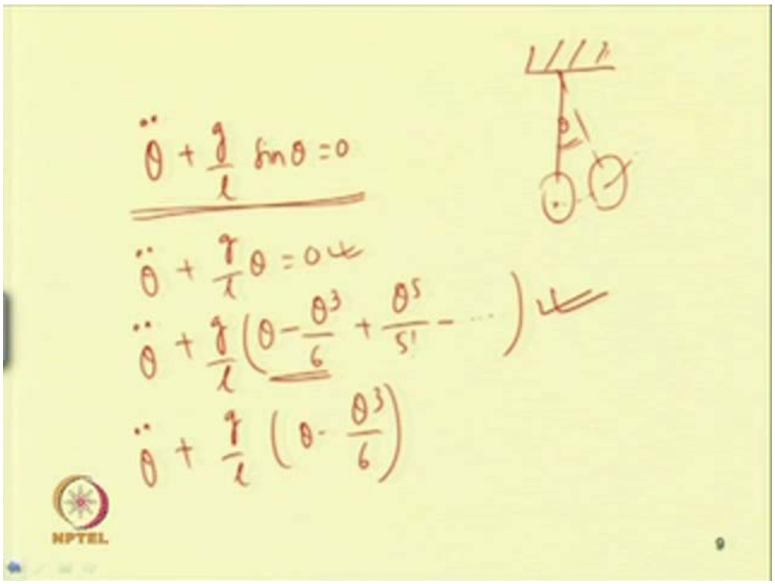

So, in case of the simple pendulum already we have derived the equation motion and this equation motion can be written as theta. So, the generalized coordinate we can take with theta and already we have derived this equation so, this is theta double dot plus g by l sin theta equal to 0. That means if we shift this mass to a position here and leave it then it will oscillate about this position about the mean position and if one write the equation motion then the equation motion is this theta double dot plus g by l sin theta equal to 0 so, where l is the length of the pendulum and theta is this oscillation. So, for small value of theta so, this equation reduce to that of a linear system that is, theta double dot plus g by l theta equal to 0 but, if theta is not small so, this equation can be written in this form that is theta double dot plus g by l so, this sin theta we can write theta minus theta cube by factorial 6 plus theta 5 by factorial 5 so, in this way we can write so, this will be equal to factorial 5 and one write this equation. So, one can write the high order terms also.

So, by taking only two terms then, we have a system with cubic non-linearity and if you take up to three terms then this will be a systems with quintic non-linearity also. This system is a system with cubic and quintic non-linearity and if we limit the systems up to the second term then, this becomes theta double dot plus g by l theta minus theta cube by 6. So, these two are the conservative systems as we are not considering damping in the systems.

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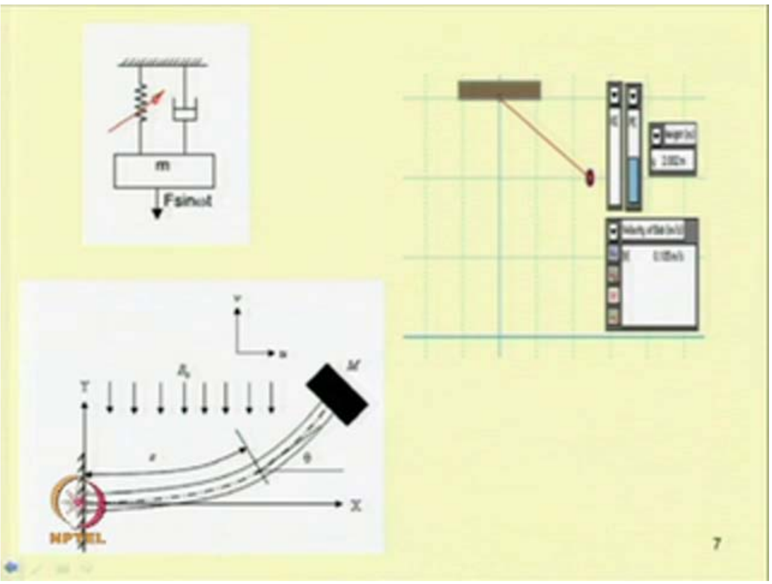

So, in this conservative system one can study the solution of these cases either by doing qualitative analysis or by doing this quantitative analysis. So, in case of qualitative analysis so, one can find the potential well of the systems and from the potential function one can find the type of response or one can plot the phase portrait and one can find what will be the response. Similarly, by taking a continuous system we can reduce to reduce that system to that of single degree of freedom systems by doing single mode approximation and in that case also, we can have conservative systems where, the system response or system equation can be written in the form of quadratic or cubic nonlinearity. So, in case of the pendulums one can observe that so, let us start from equilibrium position so, at this equilibrium position all the energy are kinetic energy. So, let us take to let take the mass up to this and then when it is released so, when it is released the potential energy goes on decreasing and the kinetic energy goes on increasing so, kinetic energy become maximum here and it reduces and kinetic energy goes on reducing so, the velocity becomes 0 here. So, one can see this velocity and so, velocity and this at a different position. So, when we consider the system to be non-linear so, let us see how the system will behave. So, in case of the linear systems already we know the system behaviour will be like this.

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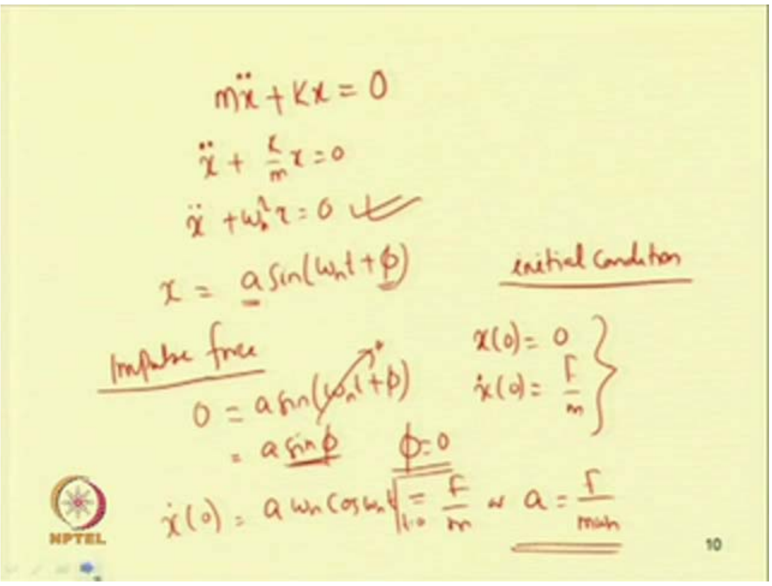

So, for example, in case of the linear spring mass system so, when you are writing the system equation m x double dot plus k x equal to 0 the or the equation can be written in this form x double dot plus k by m x equal to 0 or x double dot plus omega n square x equal to 0. So, already we know the solution of this linear systems can be written in this form x equal to a sin omega n t plus phi.

So, where this a and phi can be obtained from the initial condition. So, for different initial condition we can obtain the response of the system. For example, so, if you take at so, let us apply impulse force to the system so, if we apply impulse force and leave it. That means so, if we apply impulse force and leave the system then the resulting vibration will be a free vibration with this initial displacement equal to 0 that means x 0 equal to 0 and x dot 0 will be equal to the applied force by m so, this is for the initial. So, called initial condition we can write this way and substituting this initial condition in this case as x equal a sin omega n t plus phi so, at it at t equal to 0 so, x 0 becomes 0 so, 0 equal to a sin omega n t plus phi t equal to 0 plus t equal to 0 this term become 0 so, this become a sin phi as a cannot be 0 so, if a equal to 0 then the response is prevail solution or the system is not vibrating.

So, for that case this in phi should be equal to 0 as sin phi equal to 0 so, you can substitute this phi equal to 0 so, taking phi equal to 0. So, now by substituting this differentiating this things so, you can write x dot so, x dot 0 equal to so, this becomes a omega n cos omega n t so, a omega n cos omega n t now this will equal to F by m so, we can write at t equal to 0 so, t equal to 0 so, this cos omega n t equal to 1 so, a omega n equal to F by n or a will be equal to F by m omega n. So, as a equal F by omega n so, the solution in this becomes a x equal to…

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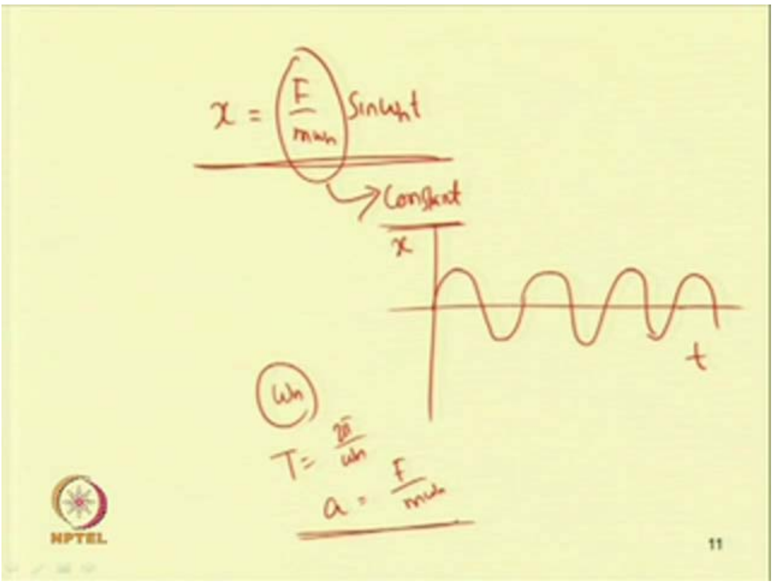

So, we have this solution x equal to F by m omega n F by m omega n into sin omega t sin omega n t. So, the solution in case of the linear system when we are considering the systems to be linear so, this is the solution. So, here the amplitude of the response amplitude of the response that is F by m omega n is constant so, this is constant. But, as we see in this lecture so, in case of the non-linear systems this amplitude of the response is not constant so, it will vary with the frequency of the system. So, in this the response resulting solution is F by m omega n so, that is constant factor. So, the system will vibrate with constant amplitude. So, the system response we so, this is t and this is x so, omega so, here frequency equal to omega n. So, the time period t equal to 2 phi by omega n and the amplitude the amplitude of the response equal to F by m omega n.

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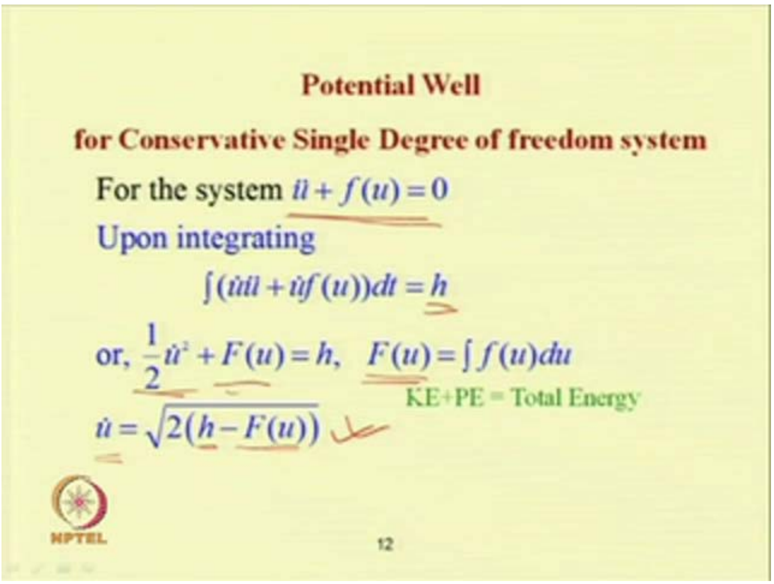

So, in case of the linear systems we have this thing. So, but, in case of the let us see in case of the non-linear systems other system behaviour will be there. So, we can do the qualitative analysis or quantitative analysis so, recall the quantitative analysis. So, in case of qualitative analysis if the equation is retained in this form that is u double dot plus f u equal to 0 then, by integrating this thing then we can write u dot u double dot plus u dot f u d t equal to so, integrations this become constant. So, from this one can write this becomes half u dot square plus F u equal to h so, half u dot square is nothing but, the kinetic energy and F u is the potential energy so, h will be the total energy. That means this is the conservation of the energy of the system so, in this conservative systems the kinetic energy plus potential energy equal to total energy.

So, from this total energy so, we can find this potential energy equal to so, potential energy will be equal to which is equal to integration F u d u and from this we can write this u dot equal to root over 2 into h minus F u. That means, for a particular energy total energy if you know the potential energy of the systems then, we can find the velocity of the system. So, by using this qualitative analysis we can find the velocity of the system for a given potential of the system.

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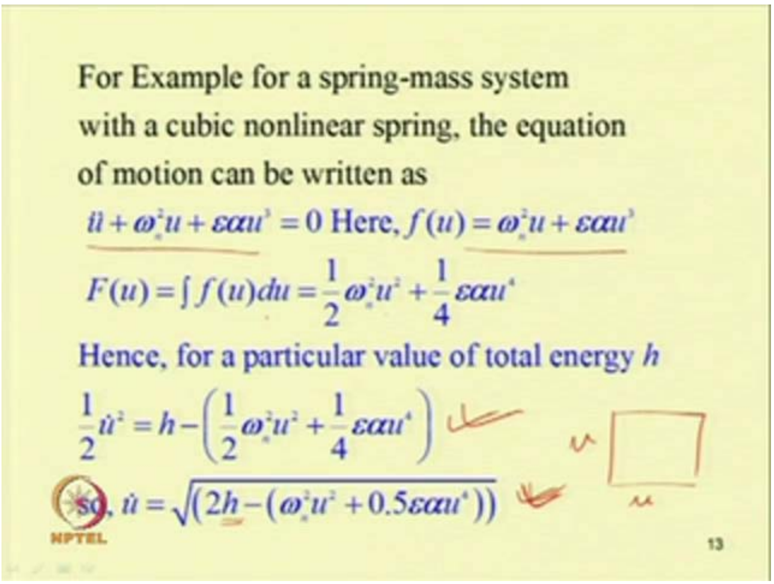

So, for example, in this spring mass systems with cubic non-linear spring so, the equation can be written in this form that is u double dot plus omega n square u plus alpha epsilon alpha u cube equal to 0 so, here F u equal to f u equal to omega n square u plus alpha epsilon alpha u cube so, this capital F u becomes f u d u integration f u d u so, this becomes half omega n square u square, integrating this u this becomes half omega n square u square and this u cube so, this becomes u to the power 4 by 4 so, 1 by 4 epsilon alpha u to the power 4. So, from this so, equating this 2 half u dot square so, we can have this half u dot square equal to h minus this term so, u dot can be obtained so, u dot equal to root over 2 x minus omega n square u square minus 0.5 phi epsilon alpha u to the power 4.

So, for a given potential we can find for a given potential, we can find the relation between u and u dot that is displacement and velocity of the systems. So, we know the plot between u and u dot that is displacement and velocity is the phase portrait of the systems.

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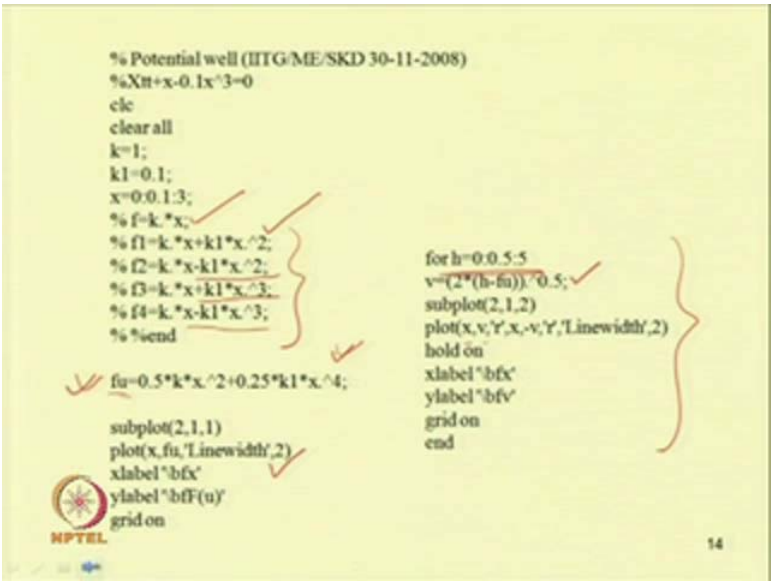

So, with time how evolves can be obtained or can be analyzed from this equation qualitatively. So, first one has to plot this potential function of the system so, from the potential function taking a potential. For a given potential one can find the response so, this is a code written to find the potential well of the different systems. So, one can take different values of this f that is small f u one can take this small f u equal to k x or k u and one can take this equal to so, this is for the linear systems then, for non-linear systems with quadratic damping one can take f 1 equal to k x plus k 1 x square and then, for f 2 can be taken another function can be taken with so, this is with hard spring with quadratic nonlinearity one can take this soft spring with quadratic nonlinearity minus k 1 x square or one can take with cubic nonlinearity only and with hard spring and this is cubic nonlinearity with soft spring.

So, taking different type of function so, one can write for example, so, here this function is written for this F u equal to 0.5 so, this is capital F u capital that is the potential so, half k x square plus 1 by 4 k 1 x to the power 4 .So, if one plot this x versus f u so, that is the potential function so, one can find the potential function of the system so, then from the potential function so, for a given potential one can find so, for different value of x so, this is potted for different value of h. So, different value of h so, one can find this v so, v is or the u u dot so, u dot is the velocity so, one can plot the displacement versus velocity to find the phase portrait.

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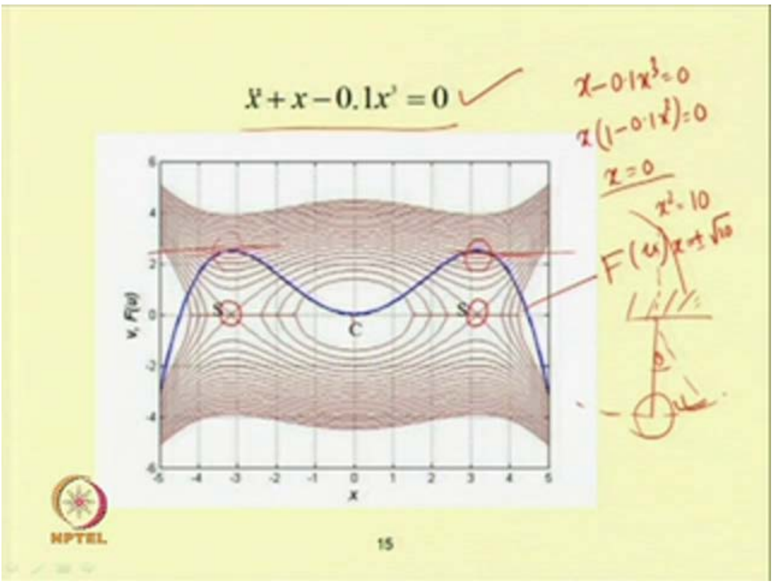

So, in this way by writing a simple code one can find the potential well and the corresponding phase portrait of the systems. So, for example, in this case with cubic nonlinearity x double dot plus x minus 0.1 x cube equal to 0 so, one can have this, this is the potential function capital F u. So, by taking different value of F u for example, one can take this to be so, this is maximum so, it has a maxima here and it has another maxima here and it has this function has a minima here. So, in these conservative systems corresponding to this maximum potential energy corresponding to the maximum potential energy one can obtain a point which is known as the saddle point. So, this is the saddle point that means these points are practically not achievable that means the systems will be unstable. So, in this case we have three equilibrium points so, to find the equilibrium points so, we can put this x double dot equal to 0 or our equation becomes x minus so, x minus 0.1 x cube x minus 0.1 x cube equal to 0 or i can write x if i will take common 1 minus 0.1 x square equal to 0.

So, from this either x equal to 0 or we can have this x square equal to 1 by 0.1 that is 10 or x equal to. So, we can have x equal to plus minus root 10 so, we have x equal to plus minus root 10. So, we have three equilibrium position so, 1 is x equal to 0 that means in case of the spring mass systems we have three equilibrium position or in case of the simple pendulum also, so, in case of the simple pendulums also, we have two so, in this case we can find that we have to equilibrium position this is one equilibrium position and another equilibrium position will be in the top position.

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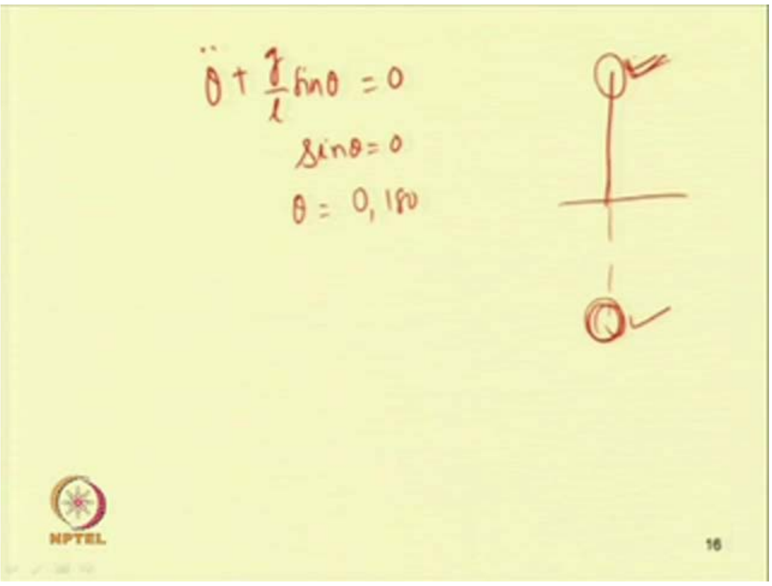

So, in vertically down ward or vertically top ward up ward so, or this is pendulum and we can have the inverted pendulum. So, in case of the inverted pendulum position that is if we will take this angle equal to theta so, when theta equal to 0 so, that is one equilibrium position and when theta equal to so, you can find that thing so, theta double dot plus g by l sin theta equal to 0. So, we can have two equilibrium positions by putting this theta double dot equal to 0. So, we have sin theta equal to 0 or theta equal to when sin theta equal to 0, theta equal to 0 or 180 degree

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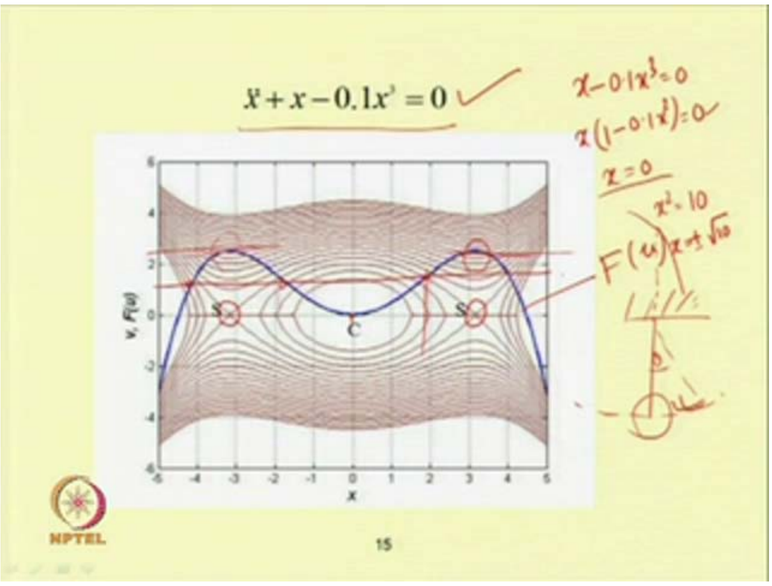

So, corresponding to theta equal to 0 so, this is the position and corresponding to 180 degree so, the position is vertically upward; the system is vertically the pendulum is vertically upward. So, in this position already we have seen or one can show that this position will be a unstable position and this position is a stable position. So, theta corresponding to corresponding to 0 is a stable position so, this will give the minimum potential energy and in other case the potential energy will be maximum. So, similarly, in this case we have three equilibrium position 1 x equal to 0 so, x equal to 0 is the minima so, corresponding to minima we have a centre. So, we have periodic response near to that point.

So, if you take if one take this F u let us take F u equal to this line so, corresponding to this value of F u so, we have three positions this 1 two and this. So, near the so, this two so, in between this point and this point that means near the minimum point so, the response will be periodic so, we will have periodic response and corresponding to these two, corresponding to this maximum point we have saddle points so, in between these two saddle points the response of the systems is periodic and outside the saddle point so, we have two solutions. So, in this case near the centre we have periodic response of the system. So, in this way by doing this qualitative analysis we can study the response of the system. So, corresponding to a particular point so, we can have the velocity and displacement.

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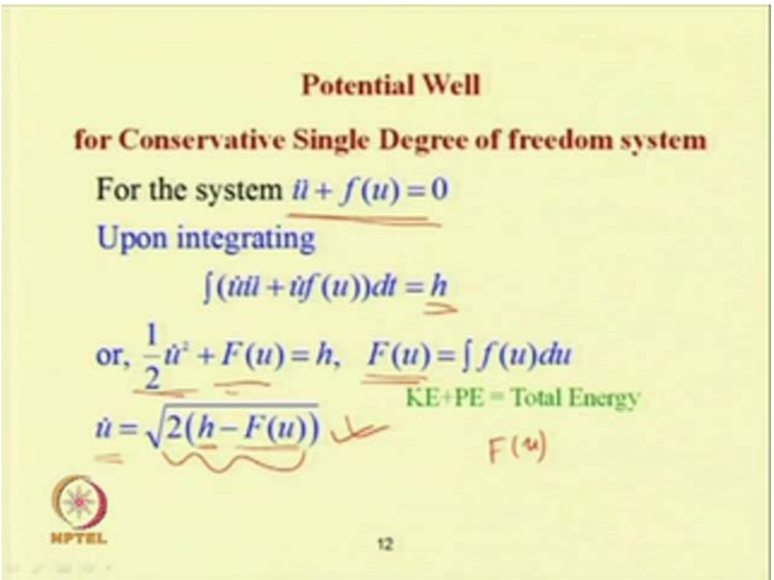

So, at this point so, for example, in this point so, will have so, for this value of this so, we have this and this. So, we have two solutions two and by taking this thing so, we have a simple periodic response. So, if we are taking let this is F u so, if this F u is less than so, if F u is less than total energy of the system then or from this equation we can see so, u dot equal to 2 into h minus F u so, if F u is less than h then, we will have a real number u dot so, if f u is less than h. So, for a particular value of h that means h is the total energy so, for the particular value of h if F u is less than h then, we will have two velocity terms and if h is less than F u then, this term becomes imaginary and the flow will not exist. So, that means we have to take F u less than already we know this total energy this h is the total energy so, F u should be less than this so, this potential function is greater than if we are taking a potential function which is greater than h then the system response will not be there.

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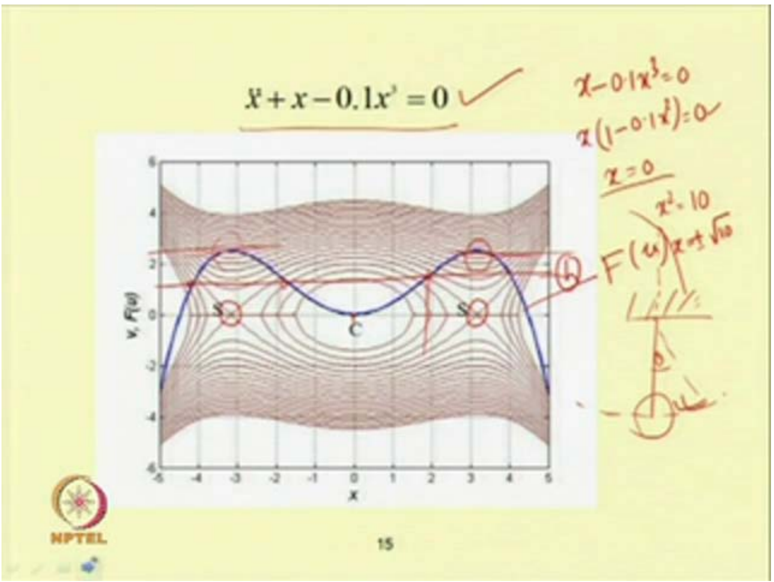

So, this is the line we have taken total h so, this is the h line so, this is the potential function. So, for this total energy h here so, we just see that these value up to this F u has a value less than h so, the motion will be so, we have corresponding to this value so, we have some motion and corresponding to this to this so, there will be no motion of the system and no so, as h is more than F u so, we have motion so, here F u is greater than h.

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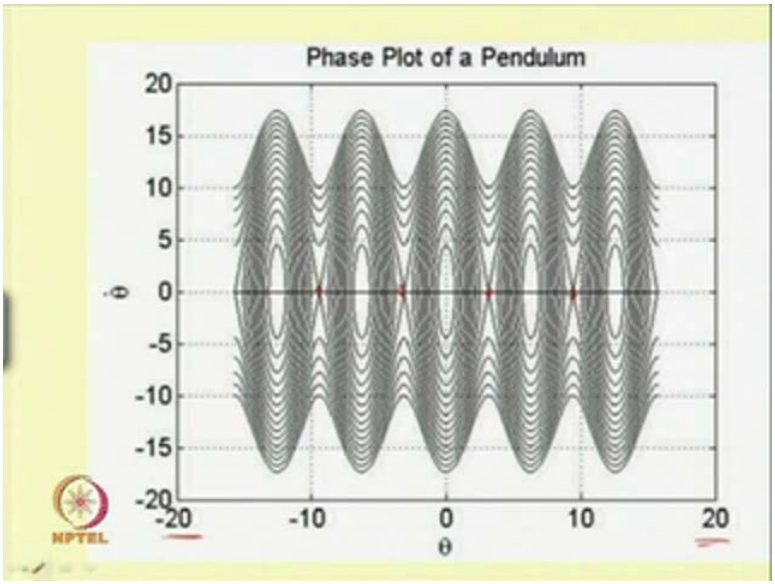

So, there will no motion and here f u is less than h so, we have periodic motion and similarly, here there we will no motion of the systems and then again here we have periodic motion of the system. So, will have alternate periodic solutions of the systems corresponding to these centre points. So, in this way one can do the qualitative analysis to study the response of the conservative systems also, we have one can do this quantitative analysis. So, in case of this pendulum, the phase portrait is plotted.

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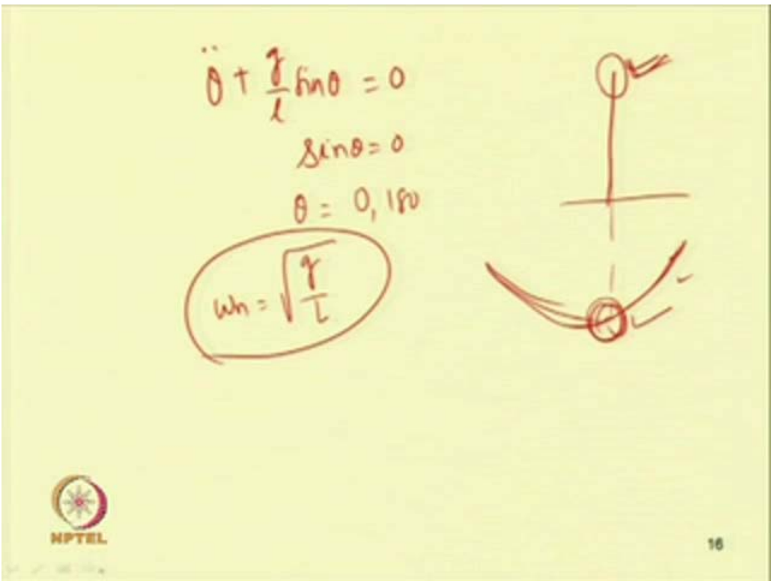

So, this periodic response so, these are the saddle point so, theta minus 20 radiant 2 plus 20 radiant it is potted so, this are the saddle point. So, one can have the homo clinic and hetero clinic orbits so, this is the homo clinic orbit then so, in between this orbit so, we have this periodic response, isolated periodic response. That means, if you leave the systems at some positions then it will try to oscillate with different amplitude depending on the initial condition. So, it depends on the high at which we are leaving this thing. So, let we leave it at this position so, it will come to this equilibrium position and then again it will oscillate.

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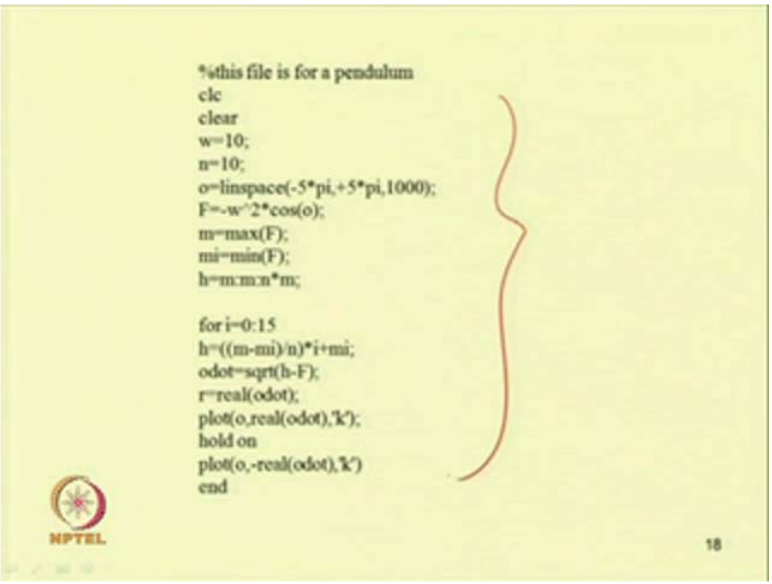

So, this amplitude depends so, amplitude depends on the initial disturbance. But, the frequency you can note the frequency does not depend on that so, the frequency is constant. But, in case of the non-linear systems we are going to see that the frequency depends on the or the amplitude depends on the frequency in case of the linear systems so, this omega n equal to root over g by l but, in case of the non-linear systems we will see that it will be no longer g by l but, a factor multiplied with this g by l. So, after doing this qualitative analysis so, this code is written for this pendulum to obtain the phase portrait and already we know.

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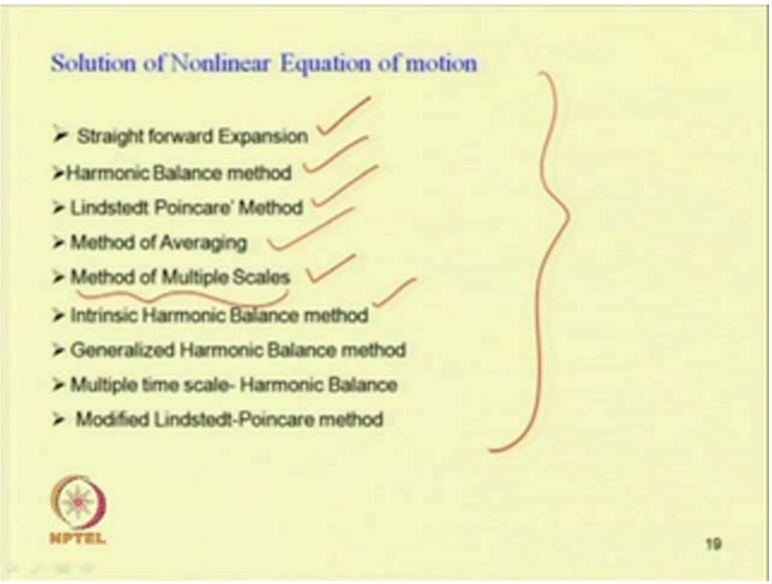

So, these are the different methods used for quantitative analysis so, in module two we made detailed discussions about how to find the solution of the non-linear equations by straight forward expansion method, harmonic balance method, Lindstedt poincare method method of averaging, method of multiple scales, intrinsic harmonic balance method generalized harmonic method, multiple time sale harmonic balance method and modified Lindstedt poincare methods.

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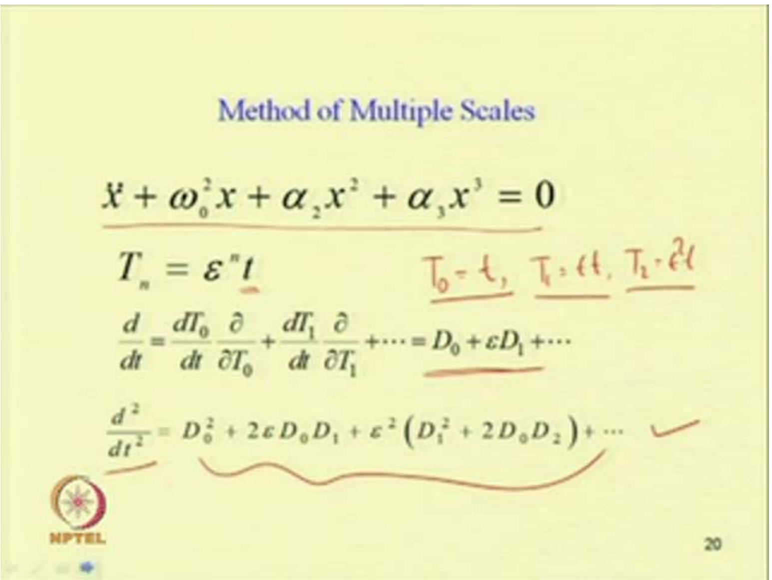

And to know or to review the systems with cubic nonlinearity let us take the method of multiple scales and in case of the method of multiple scale already we know. So, let this is the equation for the system with quadratic and cubic nonlinearities so, in case of method of multiple scale we should take T n equal to epsilon to the power n t so, where T is the time and t n we have n equal to 0 1 2 so, n equal to 0 so, we have t 0 so, t 0 equal to t so, and t 1 equal to epsilon t t 2 equal to epsilon square t so, t 1 and t 2 t 0 t 1 t 2 are different time scales corresponding to or similar to our second hand, minute hand or hour hand.

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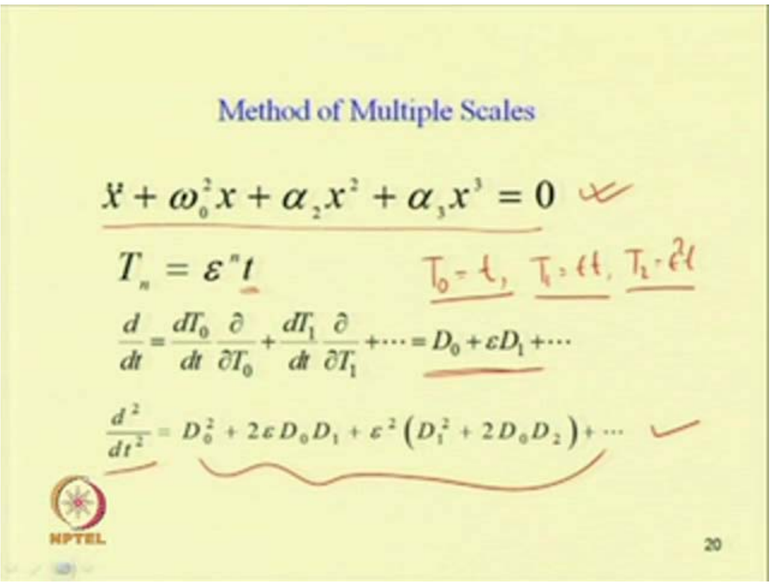

So, t 0 equal to t 1 equal to epsilon t and t 2 equal to epsilon square t. So, by taking different time scales so, already we know this d by d t can be written in this form that is d 0 plus epsilon d 1 and d square by d t square can be written in this form that is d 0 square plus 2 epsilon d 0 d 1 plus epsilon square d 1 square plus 2 d 0 d 2 and high order terms. So, by substituting these two equation also, we have to write this x so, writing x as a function of t and epsilon is the parameter so, can be written as epsilon x 1 plus epsilon square x 2 plus epsilon cube x 3 where, this x 1 x 2 x 3 are function of different time scales that is t 0 t 1 t 2. So, taking these three equation that x and d square by d t square by d t in this equation.

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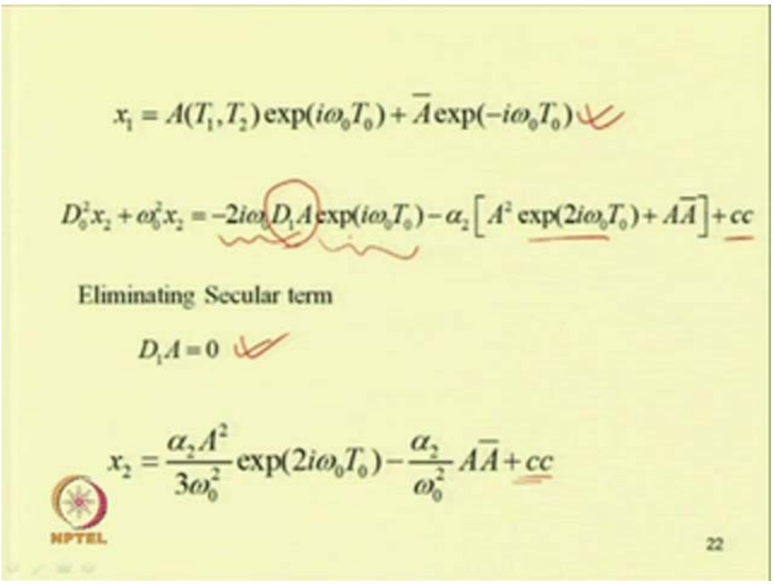

So, we can write or we can separate the terms with order of epsilon 0 so, this is order of epsilon 0 this is order of epsilon 1 and this equation is order of epsilon 2. Separating the equation with epsilon to the 0 so, one can write this equation that is d 0 square x 1 plus omega square x 1 solution of this equation is x 1 equal to one can write this x 1 equal to A e to the power i omega 0 t 0 plus a bar e to the power minus i omega 0 t 0. So, d 0 square x 1 plus omega 0 square x 1 equal to 0 so, the constant a so, should not be a function of t 0 so, this is a function of t 1 and t 2. So, a t 1 and t 2 e to the power i omega 0 t 0 plus a bar e to the power i minus omega 0 t 0 so, by substituting this x 1 equation in the second equation that is d 0 square x 2 plus omega f 0 square x 2 equal to minus 2 0 minus 2 d 0 d 1 x 1 minus alpha 2 x 1 square so, we can find this equation. But, in this equation one can note that this term this minus 2 i omega 0 d 1 a is coefficient of a to the power i omega 0 t 0 so, if this term is present in the solution in this equation.

Or this term will lead to a solution so, this term will lead to a solution which will be infinite as omega up to which is omega 0 or the solution becomes so, this is exponentially as we are taking this e to the power i omega t 0 so, this will be circular term or this will be non bound un bounded so, but, as the actual solution is bounded so, we have to eliminate this term so, this term is known as circular term. So, eliminating this circular term so, we can write this d 1 a equal to 0 so, putting d 1 a equal to 0 so, this equation reduces to d 0 square x 2 plus omega 0 square x 2 equal to minus alpha 2 a square e to the power 2 i omega 0 t 0 plus a a bar plus it is complex conjugate.

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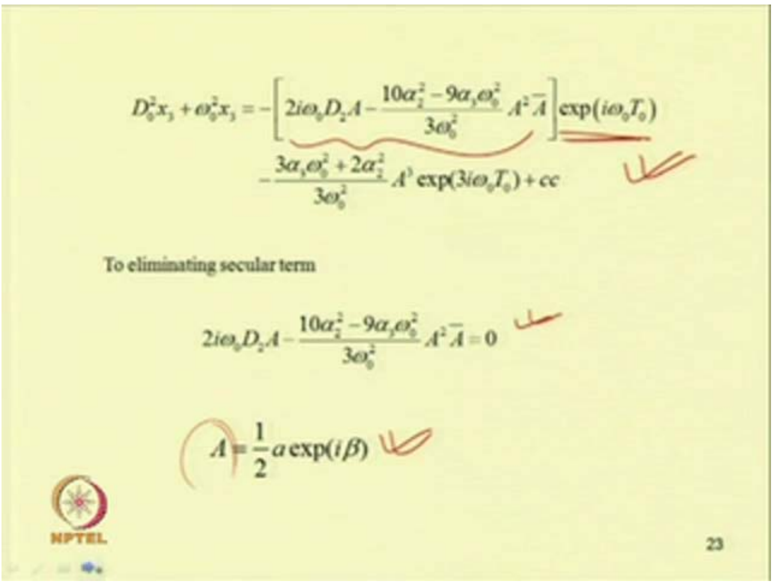

So, the solution will become so, from this we know so, the solution becomes alpha 2 a square by three omega 0 square e to the power 2 i omega 0 t 0 minus alpha 2 omega 0 square a a bar plus its complex conjugate. So, from this by or substituting this x 0 x 1 and x 2 in this epsilon square equation so, we can write the equation in this form. So, in this case also, as it contains a term e to the power 2 i omega 0 t 0 so, this is a secular term. So, one has to eliminate this term to limit or make the solution bounded so, to eliminate this terms so, we can substitute 2 i omega 0 d 2 a minus 10 alpha 2 square minus 9 alpha 3 omega 0 square by 3 omega 0 square a square a bar equal to 0.

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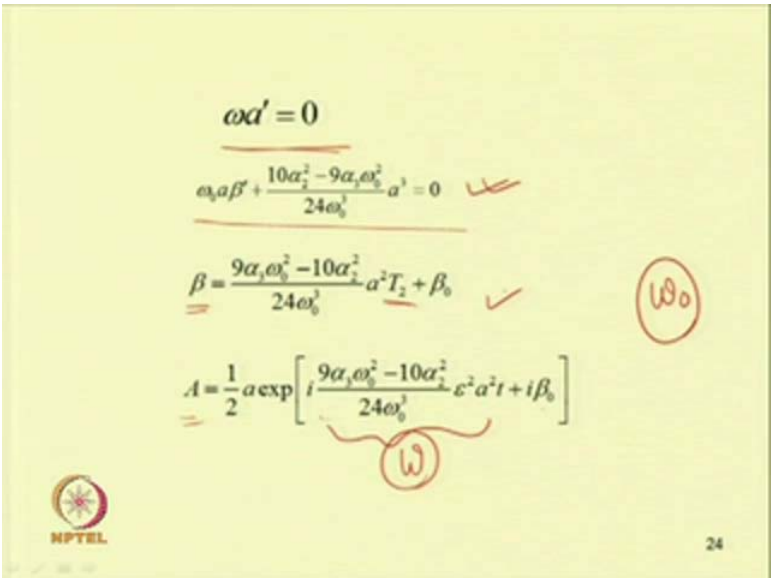

Now, substituting this as we have taken this A, A can be written in polar form like a equal to half A e to the power i beta. So, by taking this thing and separating the real and imaginary parts so, we can write this omega a dash equal to 0 so, this is one equation and second equation becomes omega 0 a beta dash plus 10 alpha 2 square minus 9 alpha 3 omega 0 square by 20 for omega 0 cube a cube equal to 0. So, we can have this beta equal to so, from the second equation we can write this beta equal to 9 alpha 3 omega 0 square minus 10 alpha 2 square by 24 omega 0 cube a square t 2. So, one a has gone so, because a is multiplied with this so, plus beta 0 and as omega a dash equal to 0 so, we can write this a dash equal to 0 or a is constant.

So, you can write A this capital A equal to half a e to the power i this term i 9 alpha 3 omega 0 square minus 10 alpha 2 square by 24 omega 0 cube s epsilon square so, t 2 equal to replacing t 2 by epsilon square t so, we can write this epsilon square a square t plus i i beta 0. So, one can note that this term is the frequency resulting frequency of the system that means, in case of the non-linear vibration so, with this frequency the systems will vibrate. So, the frequency is no long equal to this omega 0 unlike in case of the linear system in case of the linear system so, the response frequency was omega 0 but, in this case the response frequency is this omega which is no longer omega 0 but, equal to 9 alpha 3 omega 0 square minus 10 alpha 2 square by 24 omega 0 cube epsilon square a square.

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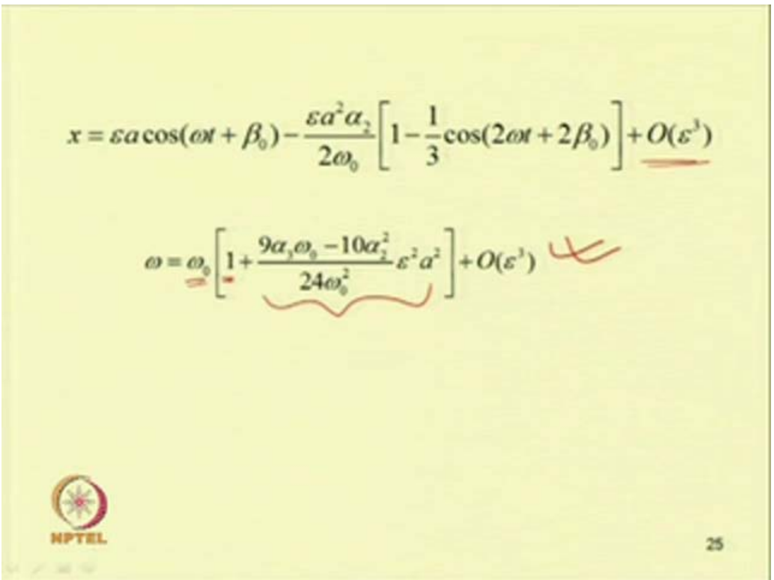

So, one can note that the resulting solution becomes so, as x equal to epsilon x 1 plus epsilon square x 2 taking only the first two terms so, we can write x equal to epsilon a cos omega d plus beta 0 minus epsilon a square alpha 2 by 2 omega 0 into 1 minus 1 by 3 cos 2 omega t plus 2 beta 0 plus order of epsilon cube.

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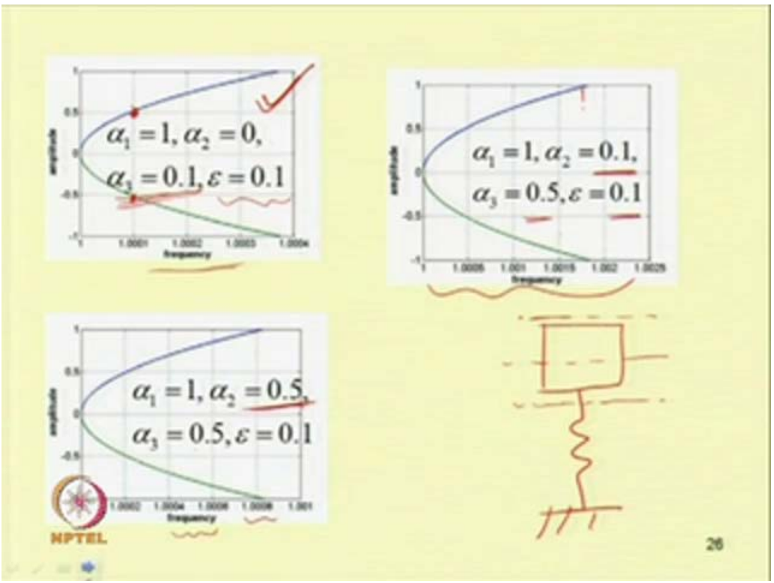

Or we can write this omega equal to omega 0 into 1 plus 9 alpha 3 omega 0 minus 10 alpha 2 square epsilon square a square by 24 omega 0 square. So, we have taken this omega common omega 0 common so, for linear systems only this one is there and for non-linear systems so, this is the part added to the frequency of the systems. So, in our module 2 so, we have used different methods to find this equation of the system so, in case of the system with quadratic and cubic nonlinearity so, let us take some examples and see how this omega is varying with omega so, with amplitude. So, if one plot or if one take alpha 1 equal to 1 alpha 2 equal to 0 alpha 3 equal to 0 that means or alpha 1 equal to 1 alpha 2 equal to 0 alpha 3 if take 0 so, then it will lead to a linear systems and we know that the amplitude is constant and that will be equal to F by m omega n if applied an impulse force. But, if we take let us take only the cubic nonlinearity that is alpha 3 so, if you alpha 3 equal to point 1 so, let us take this epsilon equal to point 1 also. So, if you take thing so, one can find so, with frequency with increase in frequency the amplitude increases.

So, for a particular frequency for example, for this frequency 1.0001 so, either the amplitude will be this or this that means in case of this in case of a spring with cubic non-linear system. So, this is a spring with cubic nonlinearity so, this is the equilibrium position static equilibrium position so, it means either it will be the position will be at this frequency the amplitude will be this or the amplitude will be this. So, it will have 2 value of the amplitude. So, with increase in so, with increase in frequency no longer the response or with increase in amplitude no longer the frequency is constant but, the frequency is a function of the amplitude. Second let us take so, in this case we have seen this and if you take this alpha 1 equal to 1 alpha 2 equal to point 1 and then alpha 3 equal to 0.5 so, we have taken alpha 3 so, this is third case we are taking and epsilon equal to 0.1.

So, in this case also, one can see the frequency so, here corresponding to you just see so, here corresponding to amplitude 1 so, 1 has frequency nearly equal to 1.004 but, in this case this frequency decreases. So, in comparison to this case one can have a higher amplitude here but, with higher amplitude so, this is the frequency so, in this case the frequency so, the maximum so, for a amplitude same amplitude 1 same amplitude 1 the frequency is decreased for the same amplitude so, by taking this alpha 2 equal to point 1 so, it decreased. Now, let us take this alpha 2 equal to 0.5 so, by taking alpha 2 equal to 0.5 now, increasing this alpha 2 one can see this frequency again increases. So, here comparing this and this so, here alpha 3 equal to 0.5 here also, alpha 3 equal to 0.5 but, here alpha 2 equal to 0.1 here alpha 2 is taken to be 0.5 so, here alpha 2 is increased. So, by increasing this alpha 2 that is the quadratic non-linear terms so, one can see the frequency increases.

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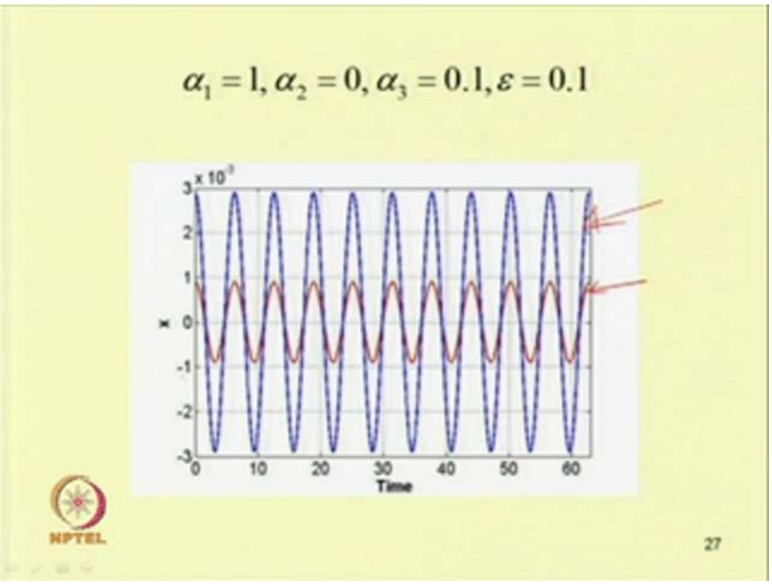

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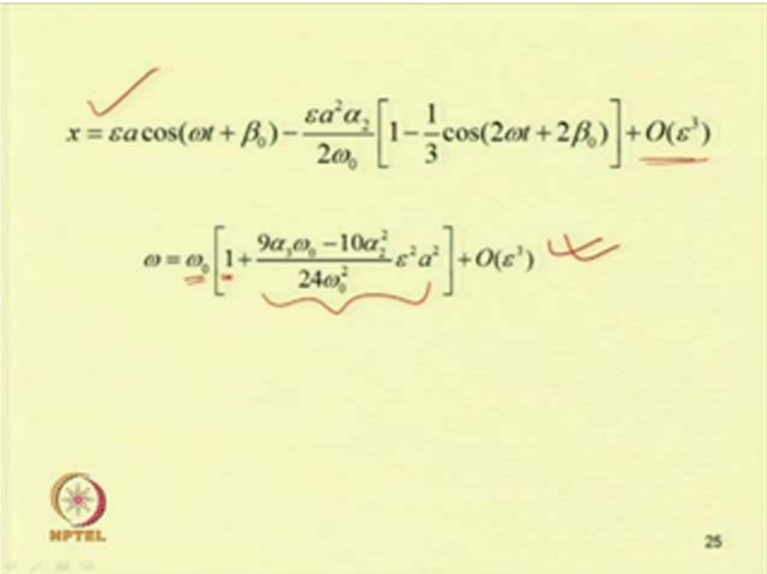

So, let us take so, for 2 so, the time response is plotted for 2 value of frequency so, this corresponds to this correspond to 1 frequency and this corresponds to another frequency. So, corresponding to that frequency we obtain the amplitude and then the response is plotted the response is plotted from this equation x equal to epsilon a cos omega t plus beta 0 minus epsilon a square alpha 2 2 omega 0 into 1 minus 1 by 3 cos to omega t plus 2 beta 0.

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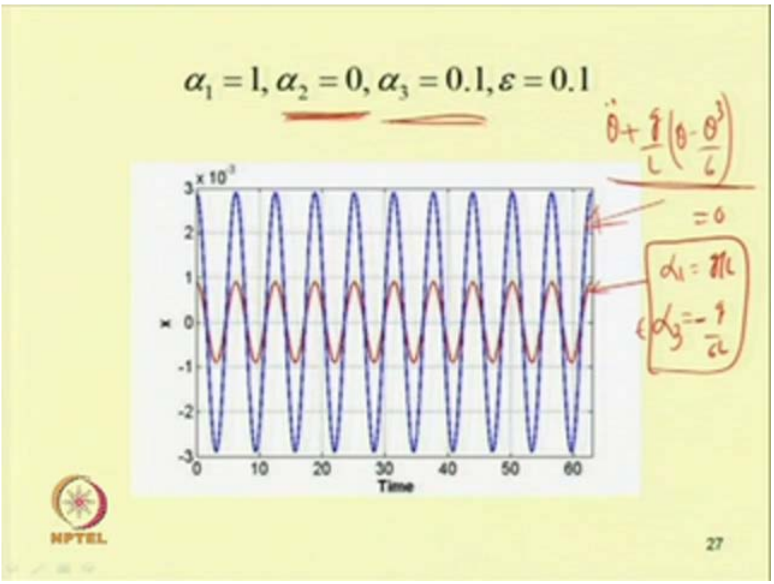

So, from this equation this curve has been plotted. So, in this case so, this equation by taking this alpha 2 equal to 0 so, this can be reduce to that of a simple pendulum equation that is, theta double dot plus g by L so, plus g by L theta minus theta cube by 6 equal to 0. So, this is a system with cubic nonlinearity. So, here if we take so, this equation is alpha 1 so, here alpha 1 that is omega square equal to g by L so, this g by L if 1 take equal to 1 then the corresponding thing if 1 so, g by so, the coefficient will be so, this in this case alpha 3 alpha 3 so, here alpha 1 equal to g by L and alpha 3 equal to alpha 3 equal to minus g by 6 L. So, by taking and epsilon term so, this is epsilon alpha 3 so, epsilon alpha 3 equal to minus g by 6 L so, by substituting this equation in this frequency equation one can find this frequency and also one can find this x. So, in this case one curve corresponds to one frequency and the second curve corresponds to another frequency. So, in this case we have taken alpha 3 equal to 0.1.

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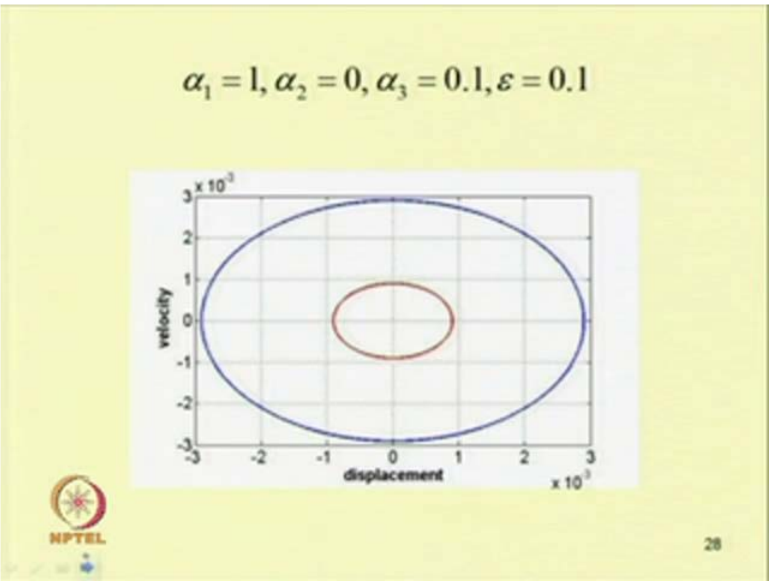

Similarly, let us take another case so, in this case the phase portrait so, one can plot the phase portrait also, phase portrait one can see the phase portrait is periodic so, which we have obtained in case of the already we have seen this phase portrait in the qualitative analysis.

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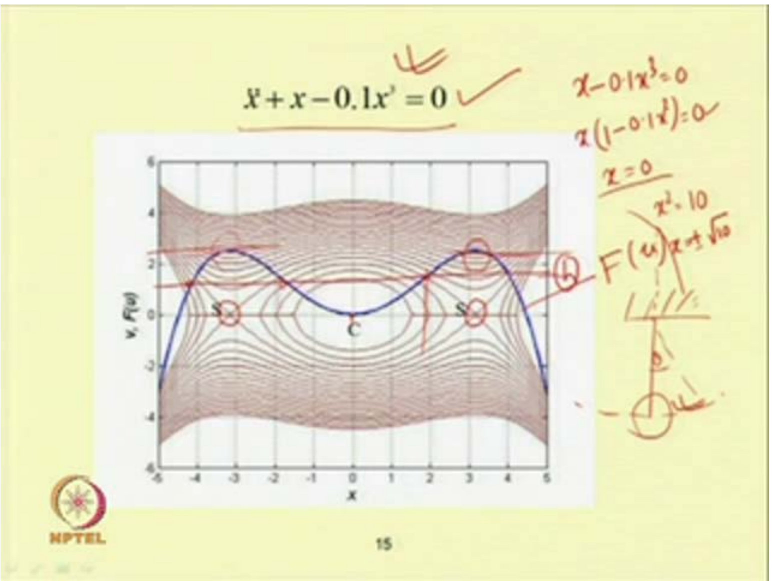

So, in qualitative analysis also, we got the same thing so, this is the example same example we have taken. So, here we have a periodic response here so, near this equilibrium position that is 0 0 so, we have this periodic response. So, in this way for conservative systems one can apply either the qualitative analysis or the quantitative analysis by using different perturbation methods to find the response of the systems. So, we have seen the response of the linear system while the in linear systems the amplitude of the response does not depend on the frequency. In case of non-linear systems the frequency and amplitude are related so, with increase in frequency or with increase in amplitude the frequency changes so, depending on the nonlinearity the frequency will either increase or decrease. So, in this case in case of non-linear conservative systems the frequency is a function of amplitude in case of non-linear system and in case of linear system it is constant.

So, next class will see or will discuss about the non-conservative systems where, we take different type of damping. For example, will take the viscous damping, coulomb damping and hysteresis damping and will plot their response curve and phase portrait and will discuss about the response of the system so, will take both Duffing equation and the van der pol equation to study the non-conservative system.

Thank you.