Non-Linear Vibration
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Module - 5
Numerical Techniques
Lecture - 2

Welcome to today class of non-linear vibration. So, in this module we are discussing about the non-linear techniques used for studying this non-linear vibration systems. So, last class we have studied how to find the roots of the characteristic equation and also how to use this Runge-Kutta method to find the solution of a differential equation.

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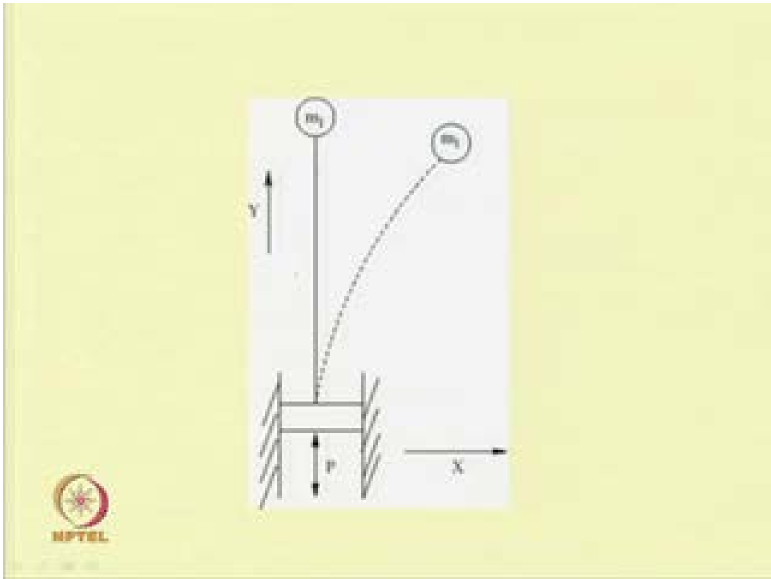

So will continue in that example, let us take this example, last class we have studied. So in this example this is a base excited beam. So where this base is excited by a force P and It is moving from its non trivial state. So this is the non trivial state the displacement can be written as w.

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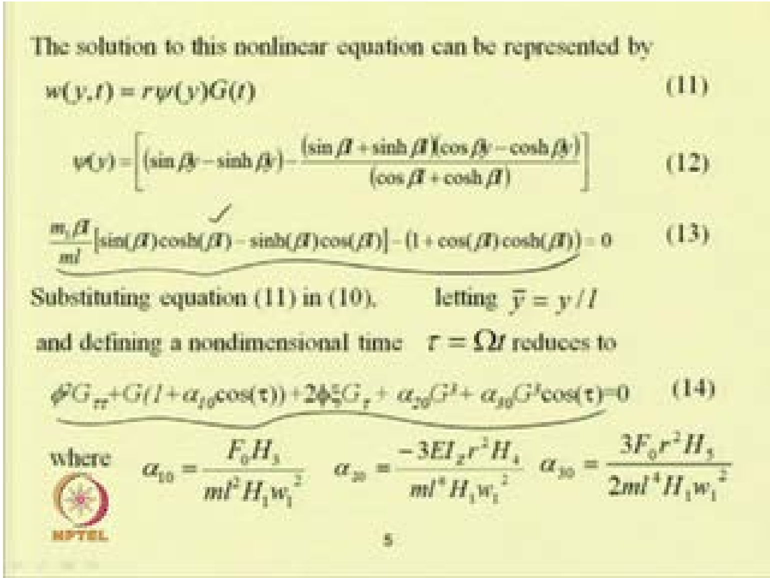

And we have we know that the equation motion can be written in its temporal form by this equation, and our objective was to find the roots of this characteristic equation. So this is the characteristic equation. So in this characteristic equation our objective is to find beta l. So last class we have studied different methods to find this beta l. So these methods are method of bisection.

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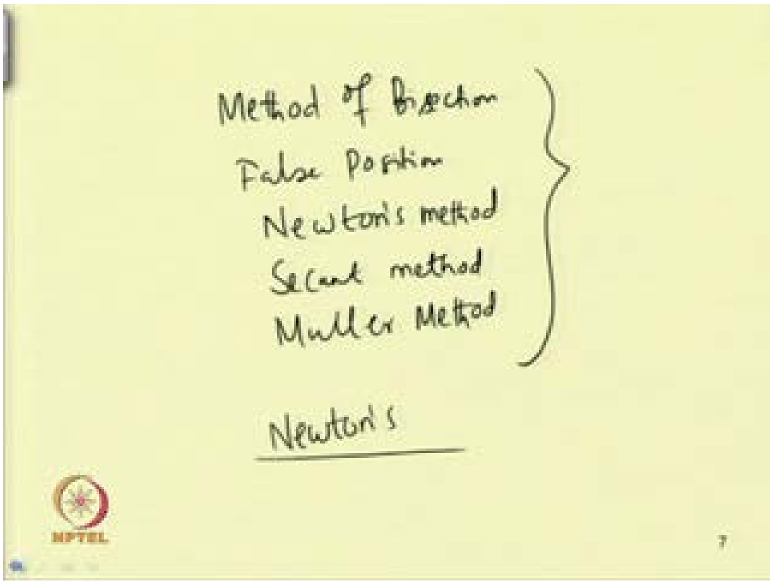

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So we have studied the method of bisection and method of false position and also we studied this Newton's method, then we studied the Secant method and also we studied this Muller method, for finding the roots of the characteristic equation. So when we have a single root to find so we can use these methods and also when we have multiple roots we may modify this Newton's method. So for multiple roots one can modify this Newton's method to find the roots of the algebraic or transcendal equation. So after the finding the roots so one can find the shape functions of the system.

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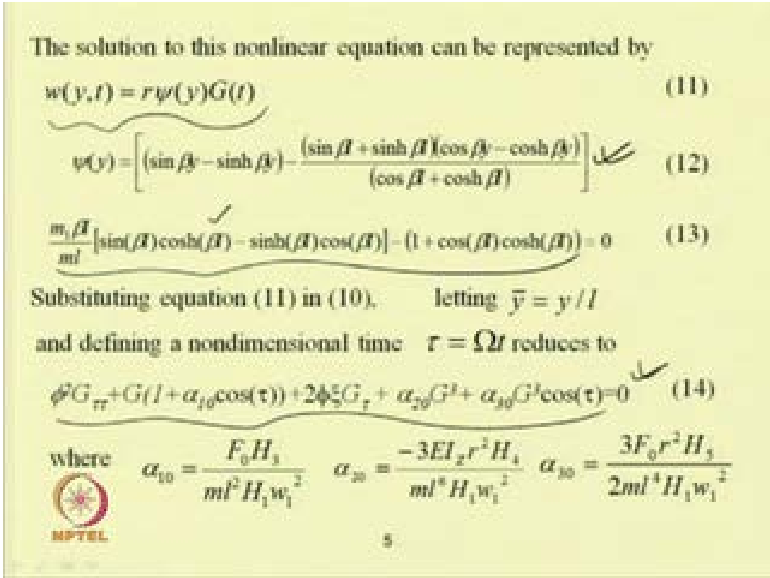

So using this beta value from this characteristic equation one can find the shape functions of the system and then using the shape function in the approximate function, where r is the scaling factor, psi y is the shape function and G t is the time modulation, then one can apply this Galerkin method to reduce this spatio temporal equation to its temporal form. So this is the temporal equation motion for a continuous system but, in case of discrete system or multi long parameter systems already we get or we can get the equation in this form. So in case of continuous systems we have to reduce this system equation to that of its temporal form, which is similar to the equations we are using for a discrete system. So now our objective is to find these coefficients.

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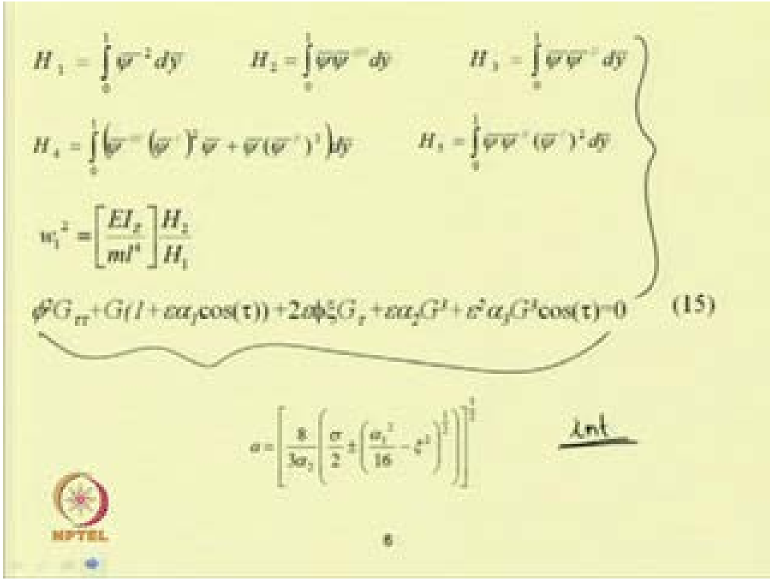

So one can find these coefficients by using these integrals and already we know there are several numerical techniques available to find this integral of the system. So one can use this Gauss quadrature method, Simpson rule, Trapezoidal rule and similar methods to find the integration of these functions in Matlab one can simply use this function int to integrate and find the coefficient of the equation. So after finding the coefficient of the equation the next step is to find the solution of the system, already we have studied the solution of the systems can be obtained by using this Perturbation method or by directly solving this equation. Now, we are studying the numerical method to solve this equation to find the response. So last class we have studied about this Runge-Kutta method and today class let us see what are the other methods available to study these type of ordinary differential equations.

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So we will we can see that there are Runge-Kutta method already we have studied. So one can take this difference method, finite difference method. So in finite difference method or one can take this forward, backward or central difference method then one can use this Houbolt method, Wilson method or Newmark method. So these three methods we will study now and we will see how these methods can be used for find the response of the system.

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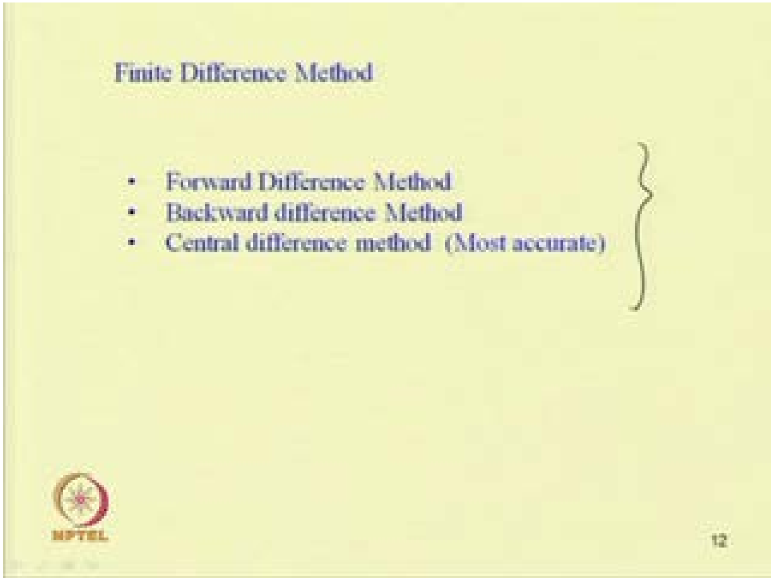

So in case of the finite difference method already we know there are 3 difference type of methods, one is the forward difference method or backward difference method and third one is the central difference method.

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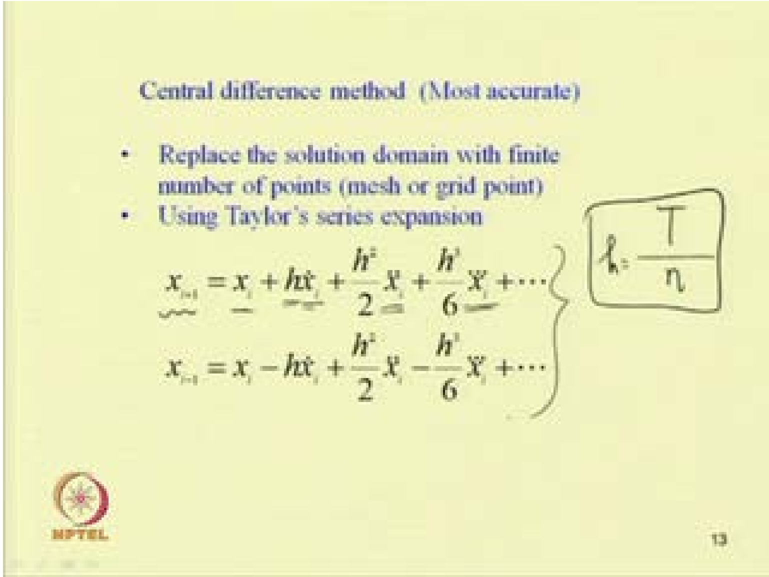

The central difference method is most accurate and most commonly used method. So in case of the central difference method so, we can replace the solution domain with finite number of points or mesh or grid points and using Taylor's series expansion we can write this x i plus 1 that is the solution in i plus oneth iteration. So if you are starting at ith iteration then this i plus oneth iteration can be written in terms of i'th iteration by using this formula. So x i plus 1 will be equal to x i plus h h is the increment, so h is the time so we can divide the time interval into several increments and it is so let we divide the whole time domain into n discrete points, then h will be equal to so if we are taking the time to be t. So h will be equal to and we have n number of points, then this h will be equal to T by n. So here x i plus 1 will be equal to x i plus h x i dot. So this is the velocity term in ith iteration, then h square by 2 x i double dot, this is the acceleration term plus h cube by 6 x i triple dot and the higher order terms.

So one can take up to the second order term or one can keep up to the third order terms to define this or to find this x i plus 1 similarly, x i minus 1 can be found from this equation x i minus h x i dot plus h square by 2 x i double dot minus h cube by 6 x i triple dot. So by knowing the initial conditions and using these expressions one can use the central difference method to find the solution of the system. So one can iterate these equations still one obtain a converge solution.

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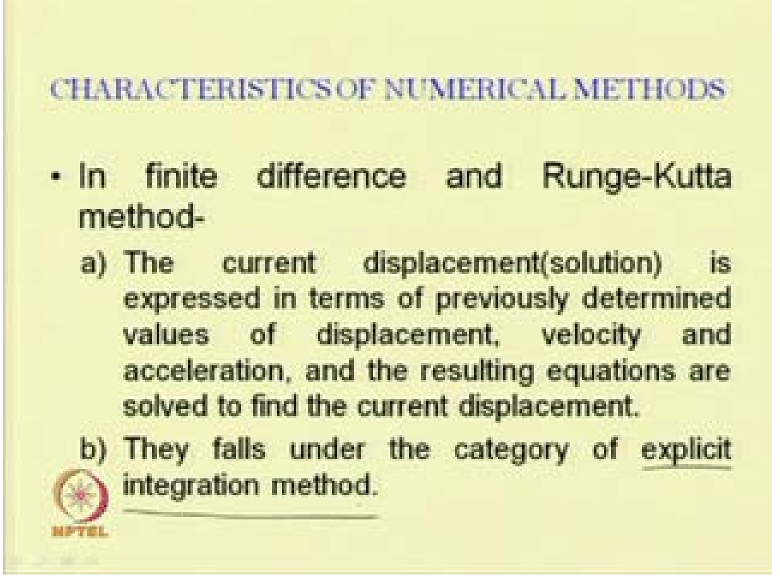

So in finite difference and Runge-Kutta method, the current displacement solution is expressed in terms of the previously determined values of displacement, velocity and acceleration, and the resulting equations are solved to find the current displacement. They fall under the category of explicit integration methods.

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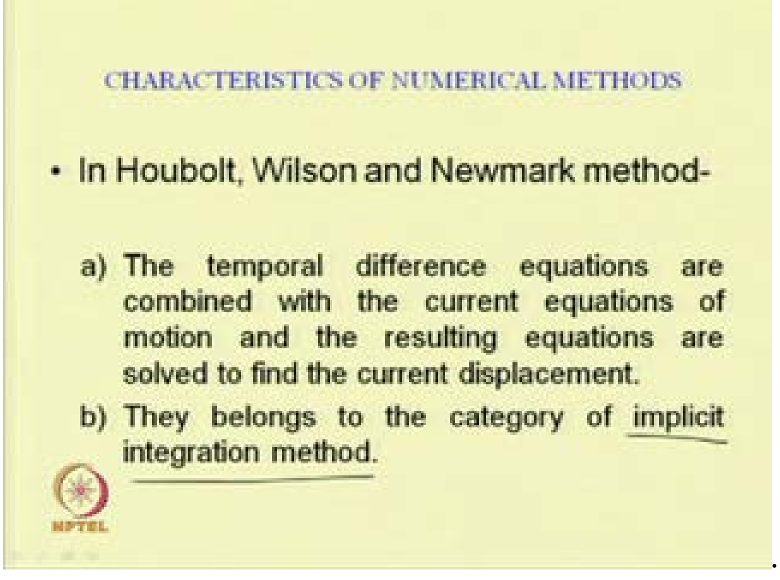

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So we will now see this Houbolt, Wilson and Newmark method. So in this method the temporal difference equations are combined with the current equation of motion and the resulting equations are solved to find the current displacement, they belong to the category of implicit integration method.

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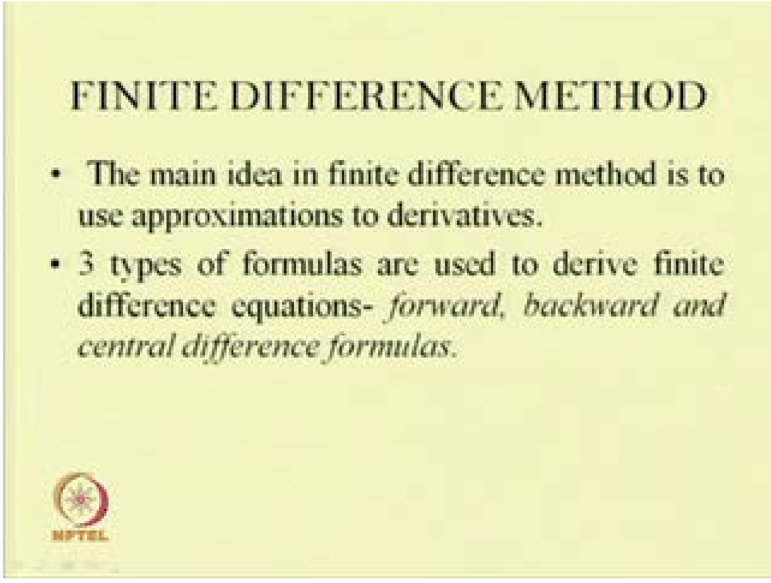

So in case of finite difference method, the main idea of finite difference method is to use approximate approximation to the derivatives. So already we know so we have this forward, backward and central difference method. So last class we have studied about this Runge-Kutta method.

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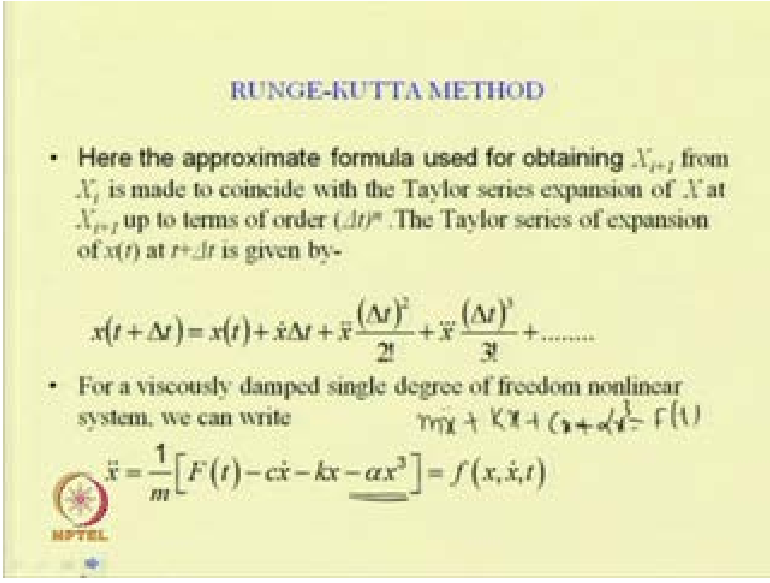

So in case of Runge-Kutta method, so this nest iteration so, let us take this example. So for example, let us take the viscously damped single degree of freedom equation. So we have taken this non-linear equation here. So this x double dot expression can be written our original equation is in this form m x double dot plus k x plus c x dot plus alpha x cube equal to F t. So now this acceleration term x double dot can be written in this form so, 1 by m F t minus c x dot minus k x minus alpha x cube.

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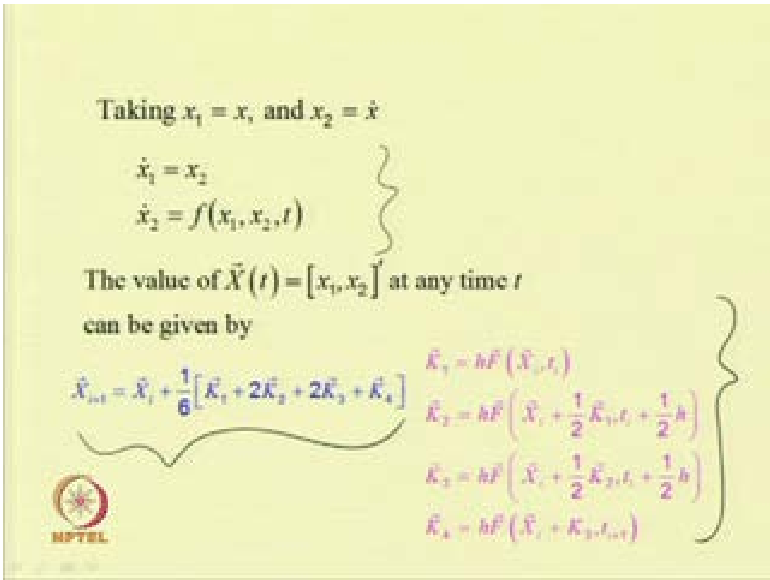

So from this we can write or we can find this x double dot and we can write a set of first order equation. So these are the set of first order equation. Now using the set of first order equation so, we can use this formula. So where we required this four constants that is k 1, k 2, k 3 and k 4. So where, k 1 expression for k 1, k 2, k 3 and k 4 are given here to find this x i plus 1. So here we have to take one initial condition. So initial condition in terms of the velocity, initial velocity and displacement by taking this initial velocity and displacement of the system so, we can use this formula to find the response of the system.

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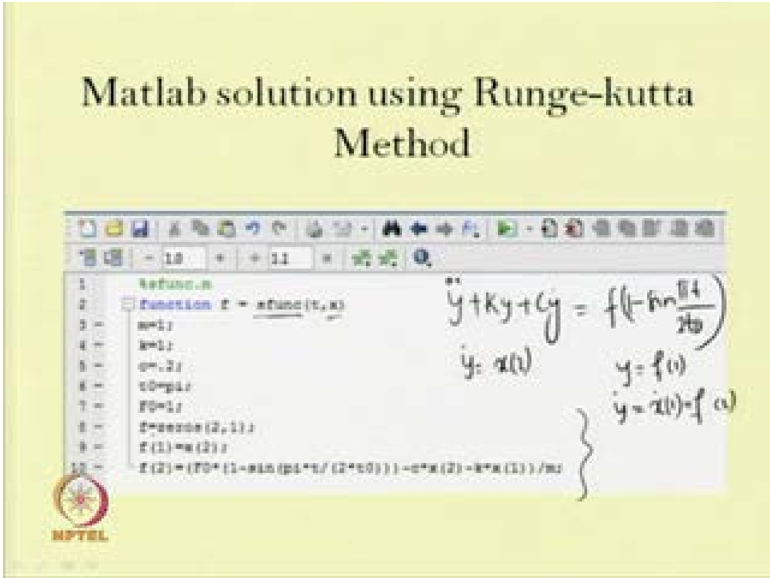

So for example, so this is a Mat lab code written to find the solution of a single degree of freedom systems without this non-linear term. So this is the function file. So in this function file one has to write this function. So function f equal to so this is the file name one can put s func. So here the input is t and x. So x will give the initial input. So let us take m equal to 1, k equal to 1, c equal to 0.2, t 0 equal to pi and f 0 equal to 1. So here the equation so, the first that is so we have taken this x dot equal to x 2 and or let us have a equation in this form that is y double dot plus k y plus c y dot equal to f. So here it is taken f into let us take the function f into 1 minus sin pi t by 2 t zero here taken pi by 2 t zero is omega. So it is taken omega 1 minus sin omega t in this form. So first one can write this y dot equal to x y y dot equal to x 2 and or we have taken this y equal to x 1. So y dot equal to y dot equal to x 1 dot equal to our x 2 or so x 2 or here it is written in terms of f so here it is written y dot y equal to f 1. So this is f 1 and this term is f 2. So then f 2, so this way one can reduce this equation to a set of first order equation. So after reducing to a set of first order equation so one can find the solution.

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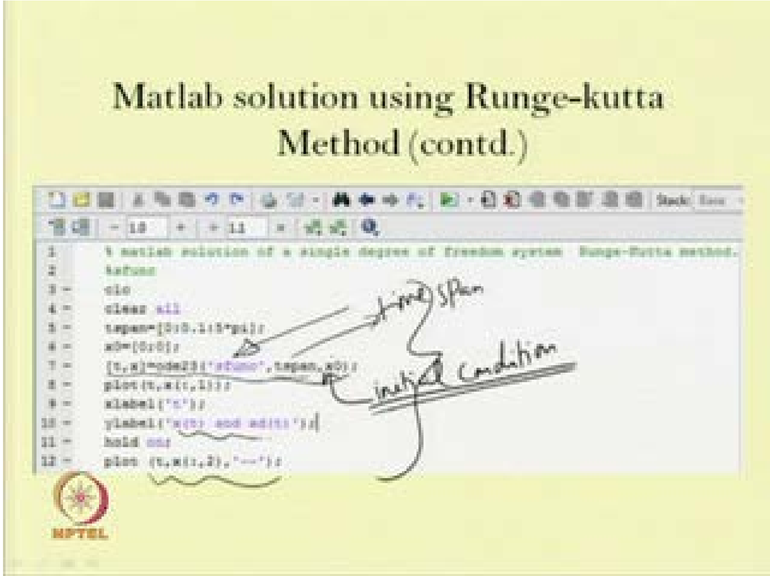

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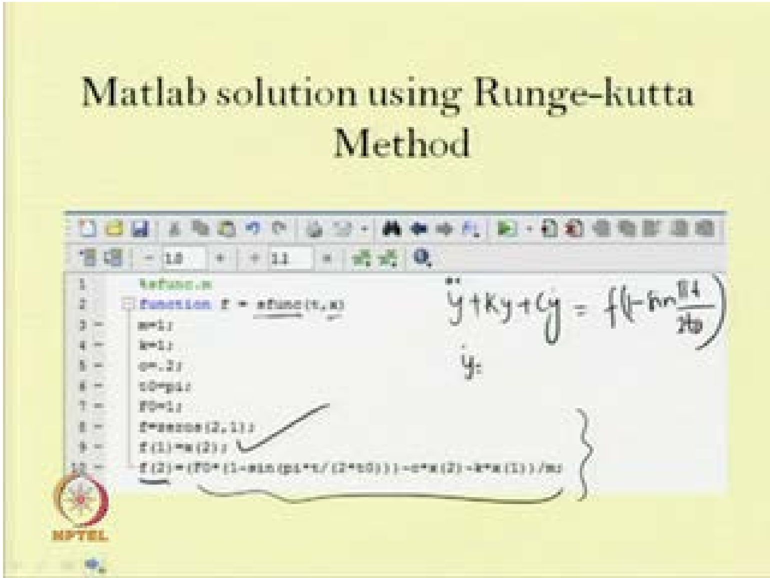

So this is the main file. So in this main file so this is the function file in which we have written, is the first function and this represent this y double dot this is y, y dot. So y dot equal to x 2 and y double dot equal to this function. So after getting that functional value so one can use this syntax that is, t x equal to so here ode23 is used one can use ode45 also.

So ode23, so this is the function file, so function file and this is the t span. So span is so one can take the time, so here the time is taken from 0 to 5 pi and this x 0 is the initial condition. So this is the time span. So one can take this time span. So this is the initial condition, so initial condition will give in terms of displacement and velocity, then one can obtain this x and t or t and x and one can plot this expression. So here one can plot the time response or one can plot the phase portrait so to plot the time response.

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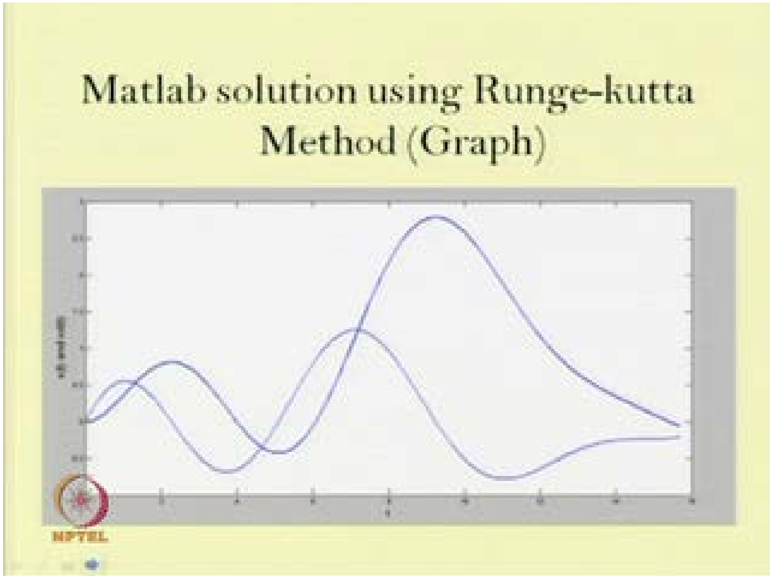

So this is the time response and this one is the phase portrait. So this curve shows for two different values of this equation, so it has been or two.

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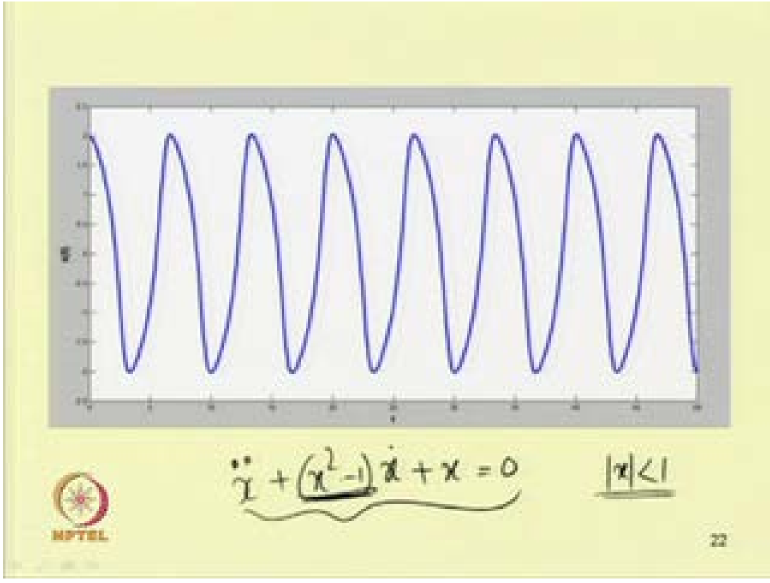

So this is x t and x dot t has been plotted here similarly, so this is plotted for the so if one take the Vander poll equation. So this time response is for the Vander poll equation were the equation is written in this form. So the Vander poll equation is written in this form that is, x double dot plus x square minus 1 into x dot plus x equal to 0. So one can observe that this term which is coefficient of x dot that is dumping similar to dumping. So this will be negative if this.

Mod x dot is less than 1 and it is positive if mod x dot is greater than 1. So when it is negative, so the system will have a tendency to become unstable or the response will grow and when it is positive, the system will have a tendency to the response to decay that is why one can have the swelling and swelling in case of or increase in response in case when it is negative, this term whole term is negative and it will at that time it will expand.

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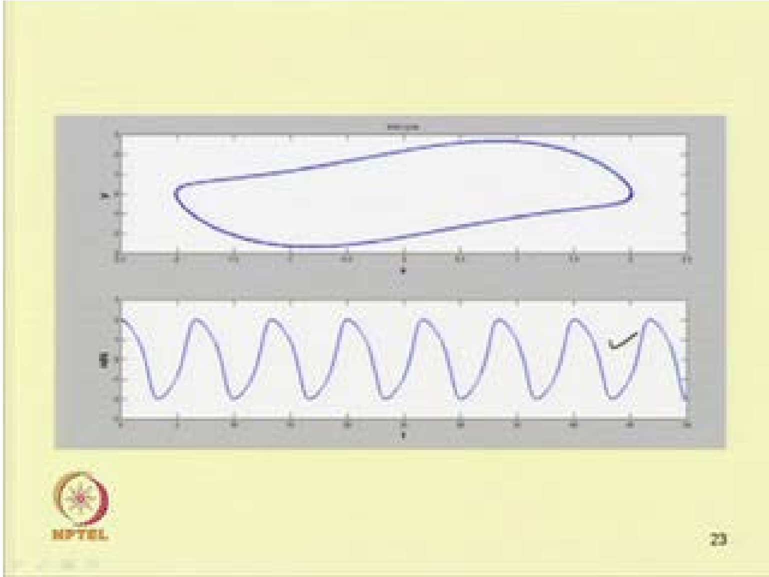

So one has this expansion and contraction of this curve. So one can observe this thing from this phase portrait. So the phase portrait y versus x is plotted here. So this is the time response for this curve so, by using this Runge-Kutta method. So one can find this response in this way.

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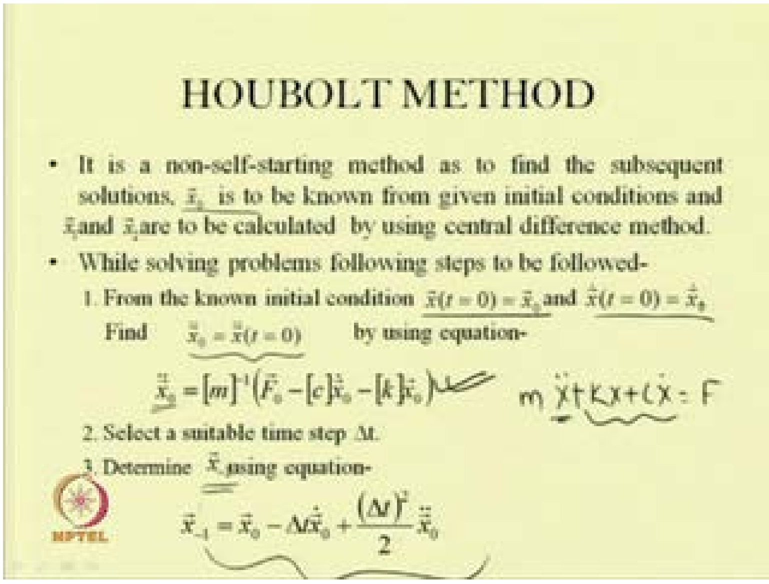

So let us see the other method that is the Houbolt method. So in this method it is a non-self starting method as to find the subsequent solution, x 0. So x 0 to be the known solution, so let us take this initial condition x 0 and x 1. So we can find one can find this expression for this acceleration at t 0. So from a known value of displacement and velocity so, we can first find this acceleration so, by using so let us take for a multi degree of freedom system, the equation can be written in this form m x double dot plus k x plus c x dot equal to F.

So in this case this x 0 double dot at time t equal to 0 can be obtained from this equation by substituting t equal to 0 from the known initial condition of x 0 and x 0 dot. So one by taking these two terms to right hand side and putting this m inverse, so one can so here we can put this small m as it is written here. So by putting this so, one can get the initial condition and one can obtain this x minus 1 from this equation. So by using this equation one can get this x minus 1 which we have already seen from the central difference method. So x i minus by using this expression for x i minus 1 as you are taking i equal to 0 so, one can obtain this x minus 1 from this equation.

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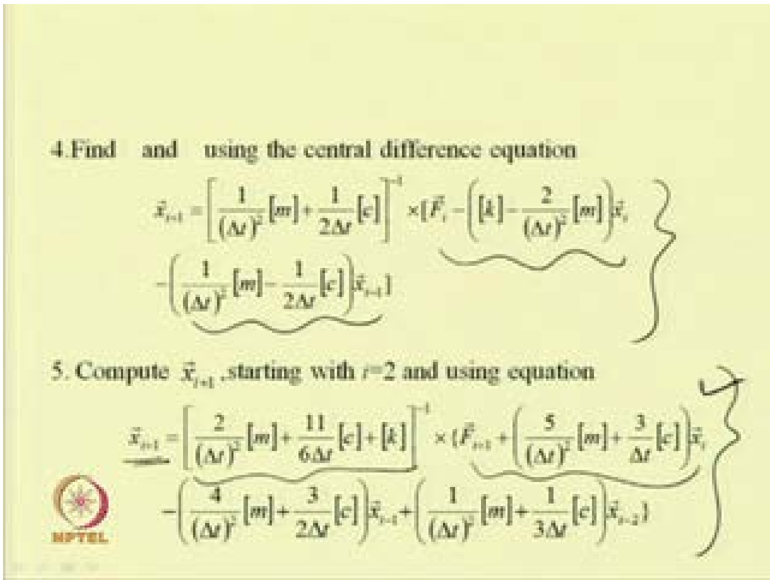

So by taking this these two so, now one can compute this x i plus 1 by using this expression. So x i plus 1 will be equal to 1 by delta t square m plus 1 by 2 delta t c minus. So this is the inverse into so, using this expression for this and this. So from a by taking a known value of delta t so, one and taking this known displacement and velocity so, by using this expression so one can find the x i plus 1. So one can compute this x i plus 1 starting from i equal to 2 so if one start from i equal to 2, then one can use this expression. So this will be equal to x i plus 1 will be equal to 2 by delta t square into m plus 11 by 6 delta t c plus k into k to the power minus this minus 1 whole to the power minus 1, then into f i plus 1 so one has to compute this i plus 1. So what is the force vector at i plus 1 so into f i plus 1 plus 5 by delta t square into m plus 3 by delta t c into x i. So from known value of x i and system parameter.

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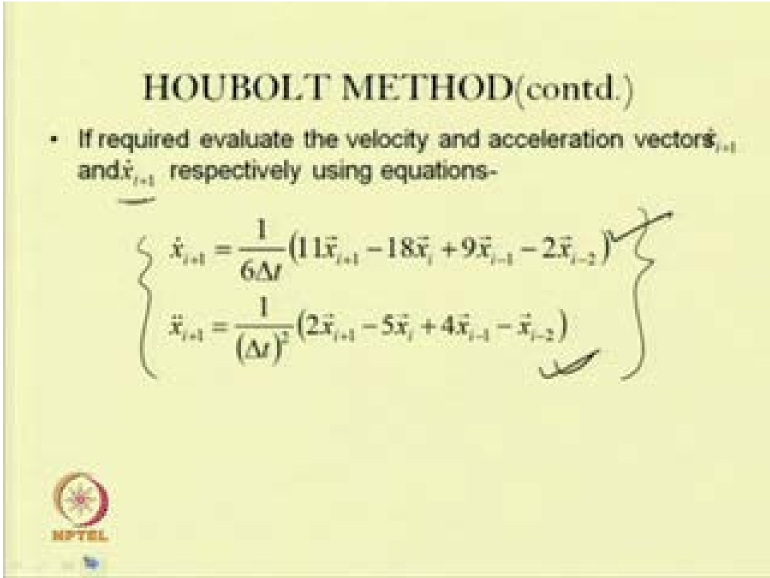

By using this expression so, one can find compute this x i plus 1. So in this way one can obtain this expression for displacement at x i plus 1. So here if required evaluate the velocity and acceleration vector at t i plus 1 and x i plus so, this is the velocity, this is the, so one can use these expressions to find the velocity at i plus 1 and i acceleration at i plus 1. So after finding this displacement at i plus 1, one can find the velocity using this expression and the acceleration using this expression so, one can obtain this expression by numerically differentiating this term x i plus 1..

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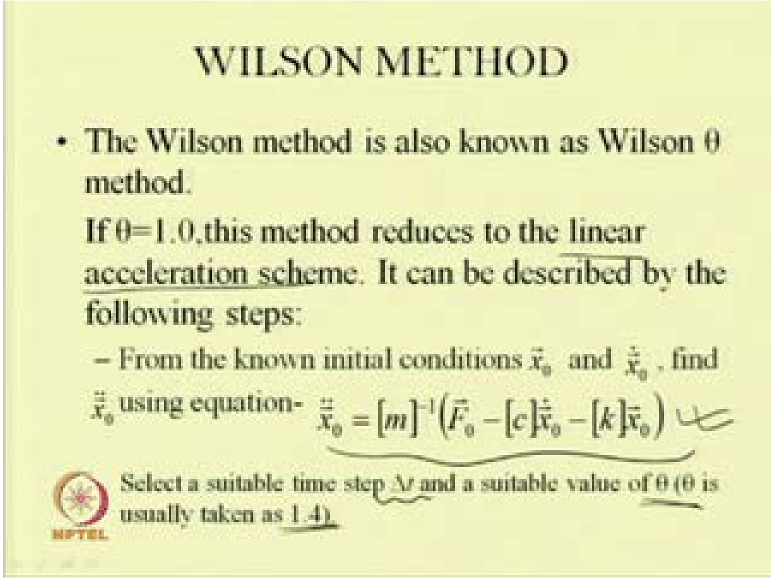

So let us see the another method that is Wilson method. So in case of Wilson method so, in previous method we have taken one initial condition. So here in Wilson method we are assuming a linear acceleration scheme. So we are assuming that the acceleration to be linear between two different points. So in this case so, as we are assuming the acceleration to be linear within a within two points. So we can use this equation directly that is the equation of the acceleration. So acceleration equation is x 0 double dot equal to m inverse F 0 minus c into x dot minus k minus x 0. So for a multi degree of freedom systems we can write this, so we have to select a time step delta t and suitable value of theta generally, this value of theta is taken to be 1.4.

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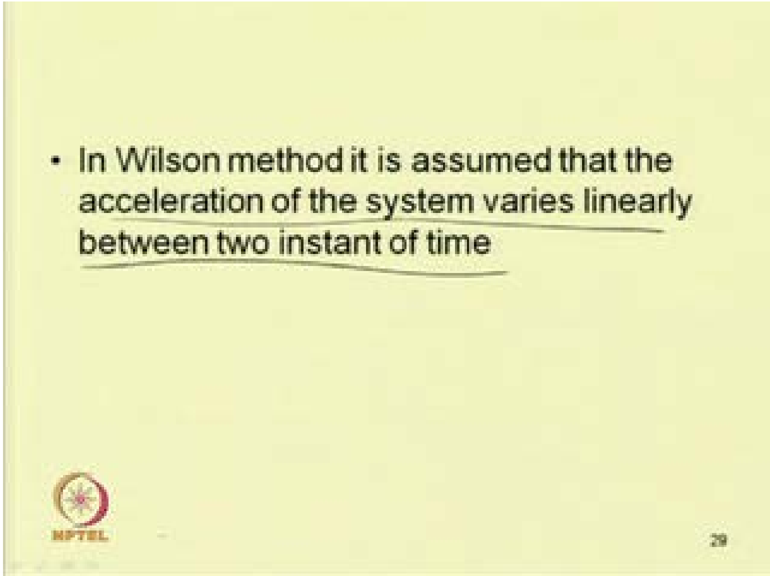

And we can here we are assuming linear acceleration, that is the acceleration of the system varies linearly between two instant of time.

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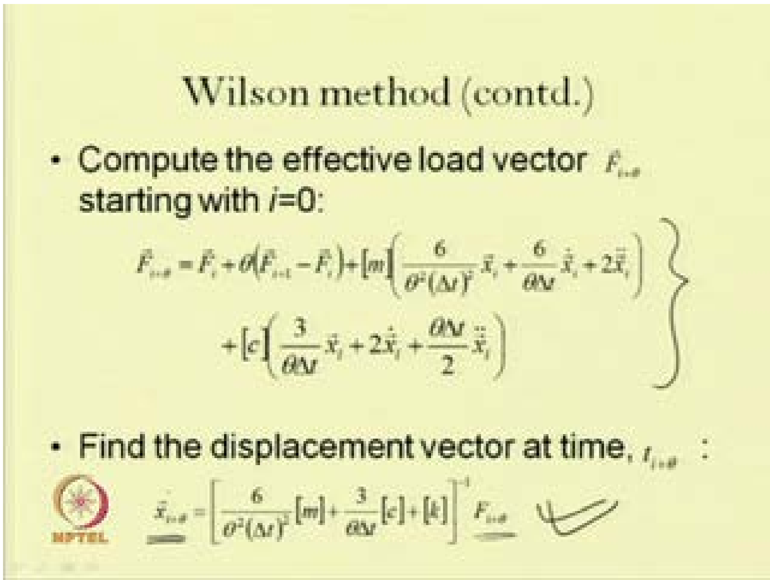

And we are finding this thing. So F i plus theta can be obtained using this expression, then as we are assuming acceleration to be linear between two points, then we can find this x i plus theta instead of finding x i plus 1 in as in case of this Runge-Kutta method here we are finding after theta. So x i plus theta can be obtained using this expression and where x i plus theta equal to one can obtain this 6 by theta square delta t square into m plus 3 by theta delta t c plus k to the power minus 1 into f i plus theta. So, one can obtain this value of this next iteration after theta interval.

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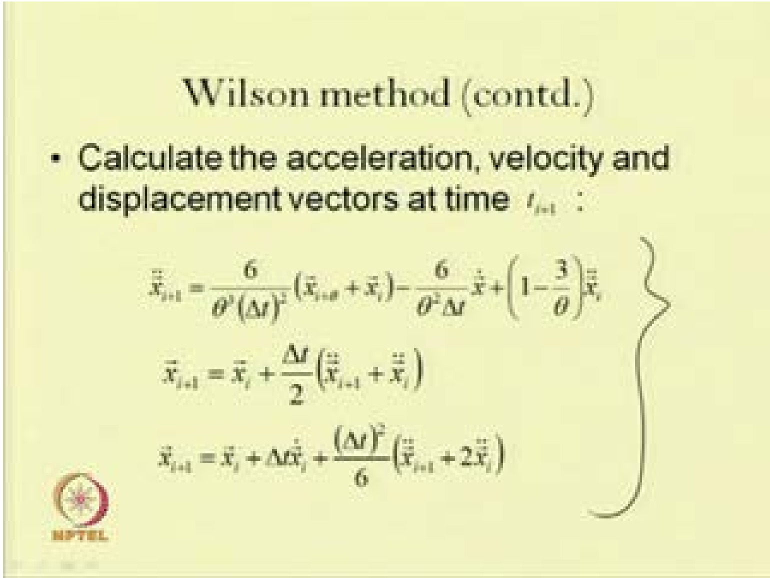

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So x i plus theta so, after getting so, this is the expression for x i plus theta which can be obtained from these expressions.

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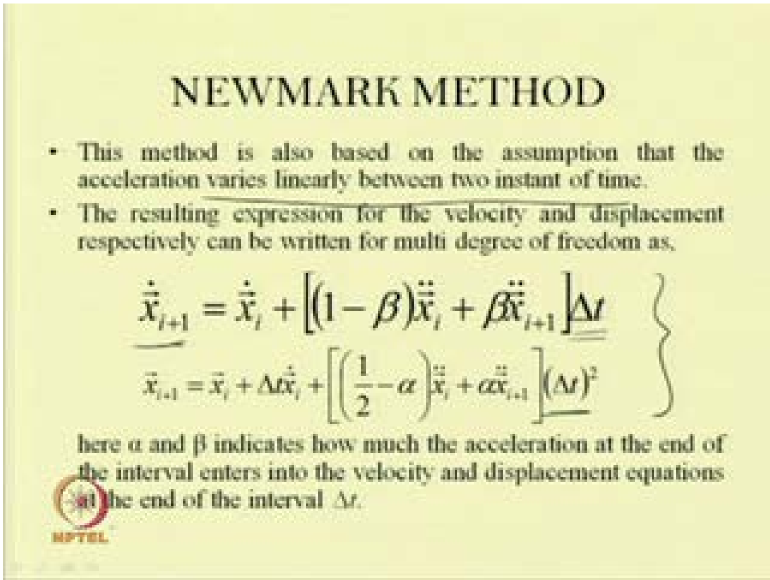

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Similarly, in case of the Newmark method. So in case of Newmark method also we are assuming the acceleration to vary linearly between two instant of time, the resulting expression for velocities and displacement respectively can be written for a multi degree of freedom system using these expressions. So here we are assuming the acceleration to vary linearly between two instant of time. So here x i plus 1 can be written using this expression, this is the velocity. So x i plus 1 dot equal to x i dot plus 1 minus beta into x i double dot plus beta into x i plus 1 double dot into delta t.

And x i plus 1 can be written as x i plus delta t x i dot plus half minus alpha into x i double dot plus alpha into x i double dot delta t square. So using this expression here alpha and beta indicate how much the acceleration at the end of the interval enters into the velocity and displacement equation, at the end of interval delta t. So by using these expression.

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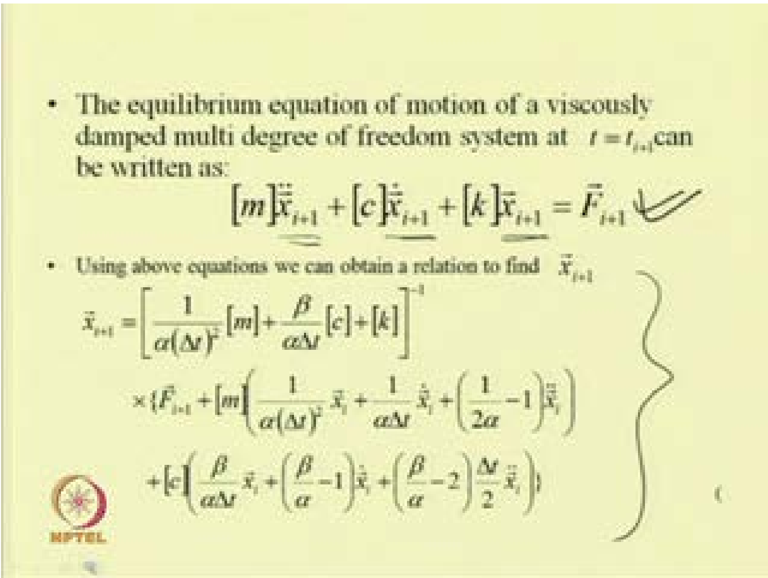

So we can find the response of the system for example, in this case. So the equilibrium equation of motion or the let us take this equation. So in this case m x double dot this x i plus 1. So this is x velocity at i plus 1, this is the displacement at x i plus 1. So if x i plus 1 is the equilibrium position so, it will satisfy this equation. So one can obtain this x i plus 1 from this expression using this expression so, x i plus 1 can be written by 1 by alpha delta t square into m plus beta alpha delta t into c plus k to the power minus 1 into F i plus 1 plus m into 1 by alpha delta t square x i plus 1 by alpha delta t x i dot plus 1 by 2 alpha minus 1 x i double dot plus c into beta by alpha delta t x i plus beta by alpha minus 1 x i dot plus beta by alpha minus 2 into delta t by 2 into x i double dot. So in this way one can use this different types of methods for example, one can use this Runge-Kutta method Houbolt method Wilson method and this Newmark method to find the solution of this ordinary differential equation.

Particularly when we are using this finite different, finite element method. So we have a large number of equation motion and this Newmark beta method is more suitable for solving the response of the system in that case. So after obtaining the response of the system now, our objective is to analyze the response of the system. So how to analyze this response so, one can analyze the system response from the so after getting the time response.

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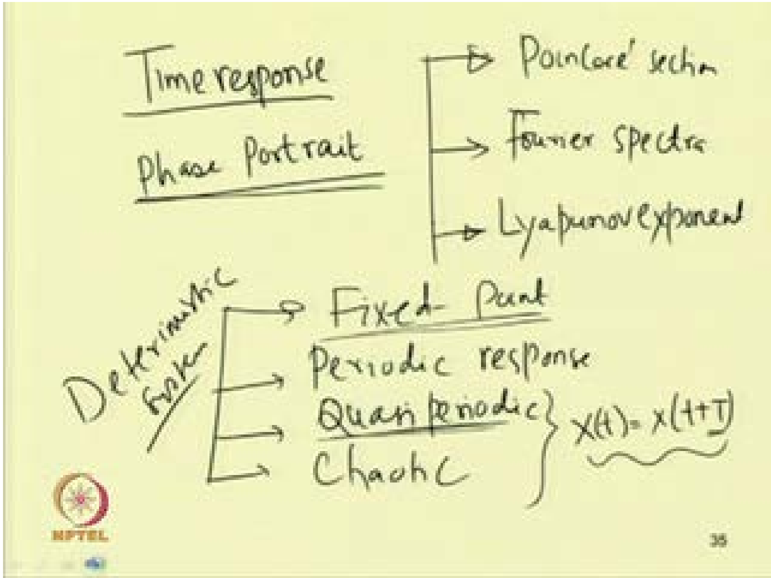

So after getting the time response of the system so, one can analyze this time response either by using this Poincare section, one can also use this Fourier spectra Fourier spectra, also one can use the Lyapunov exponent method to characterize the response of the system also, after getting the time response one can plot the phase portrait to observe the response type already we know that we have for deterministic system we have four different type of response. So these are fixed point response, also we have this periodic response, also we have quasi periodic response and we have this chaotic response.

So for deterministic system so for deterministic system we have this four different type of response of the system. So fixed point response in case of fixed point response the response becomes either a trivial fixed point or non-trivial fixed point, in case of periodic response the response x t equal to x t plus t. So we can find a minimum time period after which we can observe that the system response repeats, in case of quasi periodic response also we can find or one can observe this response of the system. So this but, one can note that this quasi periodic and chaotic response as they are not periodic. So it is difficult to distinguish these responses from the time response and the phase portrait.

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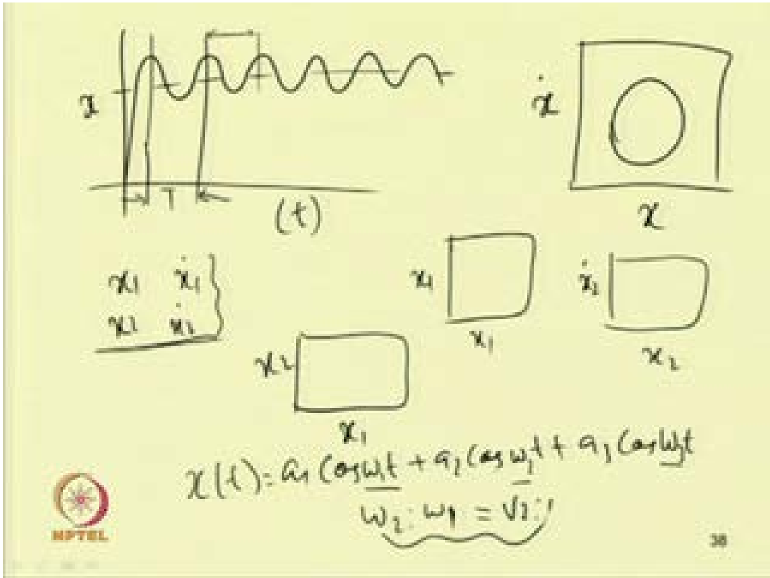

So if one plot the phase portrait of the system so, if one plot the phase portrait so let us plot the phase portrait of the system in case of a fixed point response, phase portrait for example, this is x versus x dot if one plot, so in case of the in case of a fixed point it will the response will either come. So if it is the system is stable so, it will come and this is the final fixed point response of the system but, if the system is unstable so, one can have so the resulting phase portrait will look like this. In case of time response so, in this case the time response will be so, starting from a point. So, this is the time response so this is x t one can plot this x t and x dot t.

So this is the time response of the system so initially, one will have or one can plot more times also and then finally, it will come to the steady state system. So this is particularly similar to the response of an under damped system and in this case the response of the system will grow and one can plot the response of the time response of the system like this.

So this is for the fixed point response but, in case of the periodic response so, in case of periodic response one can observe that the response will be periodic, the resulting response is periodic and if one plot the phase portrait. So this is x versus t similarly, one can plot this x dot versus t and one can obtain the phase portrait like this. So, in case of the periodic response so, one can obtain the phase portrait as a closed loop, so one can obtain a closed loop in this case.

So this is x, this is x dot versus x so one can obtain a closed loop in this case. So for n dimensional system so one can so, it is difficult to plot the phase portrait as will have n state vectors and in that case one can plot so for example, if we have four state vector let we have four state vector x 1, x 2, x 1 dot, x 2 dot. So in this case one can plot the phase portrait with x 1 dot x 1 and x 2 dot x 2 dot or one can plot also this x 1 and x 2.

So this way there will be different representations of the state space and by using different state space so, one can find or one can analyze the different type of response and it is very difficult. So for n dimensional system so, it is difficult to represent the phase portrait of the system. So in case of the quasi periodic response so, already we know in case of the periodic response so this is one period. So if one find so, this is one period and so after each period one can observe that this to this also one period. So one can observe that the system response repeats in this case.

So in this case one has distinct frequency. So one can write this frequency of the system or one can find the frequency of the system. So the system may be a single periodic or it may be k periodic, k periodic means it may have k number of frequencies. So in that case the response can be written x t equal to a 1 cos omega 1 t plus a 2 cos omega 2 t plus a 3 cos omega 3 t. So in this way one can have a number of frequencies also present in the system. So if these frequencies this omega 1, omega 2 and omega 3 be your integer relationship, then one can find this periodic response but, if these ratios are ratios are not rational number for example, this omega 2 by omega 1 so let it is root 2 by 1.

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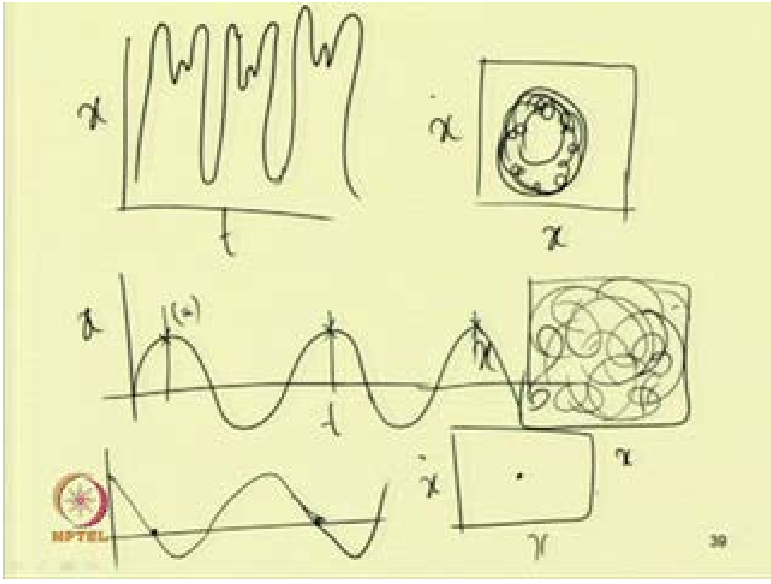

So this is not a rational number. So if they are irrational number then instead of getting periodic response one can obtain quasi periodic response and already we have seen. So in case of the quasi periodic response the phase portrait may look like this. So it may look like so it is known as the torus, so the phase portrait will look like this. So this is x dot versus x and the time response corresponding time response is a-periodic and one can observe the response may be like this and it will it may repeat.

So one can get a-periodic response and by plotting this a-periodic response one can tell whether it is, whether the response is periodic, quasi periodic or fixed point similarly, in case of chaotic response the response so this will be, so this phase portrait will so if one plot this phase portrait so, one can see that is can filled up this phase portrait and now to distinguish this from the time response so, one can either plot this phase portrait or one can go for this Poincare section. So in case of Poincare section so in case of Poincare section we have to sample the time response at regular interval of time. So in case of the periodic response as we know the period of the system so one we can sample the time response with minimum period. So for example, in this case let us take a sine curve.

So it is, so this is x versus t. So the velocity will be a cosine curve. So let us take so if we are taking this point as the starting point, then after time t. So this is the same point one can observe and similarly, after one more time. So this is the time. So if one plot this Poincare section of this so one has to sample this. So this is x versus x dot similarly, in our x dot curve, so x dot curve we can have a cosine curve. So in this case we can we can observe that so, as we are taking this point here so this is the point at 90 degrees so this is the point similarly, we can have after one period this is the point. So we have this let this value is a so in Poincare section will have a point which will have a value a and 0.

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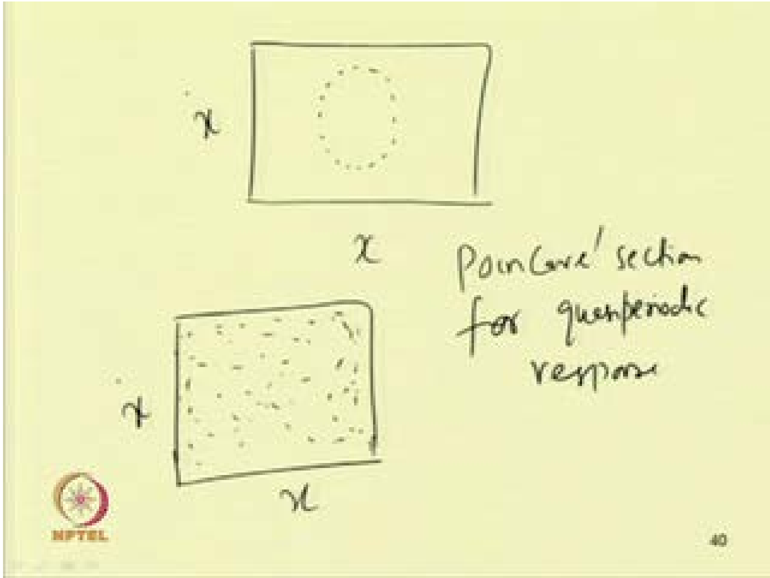

So this is the point so; that means, in case of the periodic response the Poincare section is a point similarly, in case of quasi periodic section, quasi periodic response one can find the Poincare section and this Poincare section will be in the form of a closed loop. So one can observe a close loop in case of the so, this is Poincare section for so Poincare section for quasi periodic response similarly, one can plot the Poincare section for a chaotic system. So in case of the chaotic system so, one can observe that this Poincare section will fill up this whole space so, one can get a series of points which filled this state space in Poincare section.

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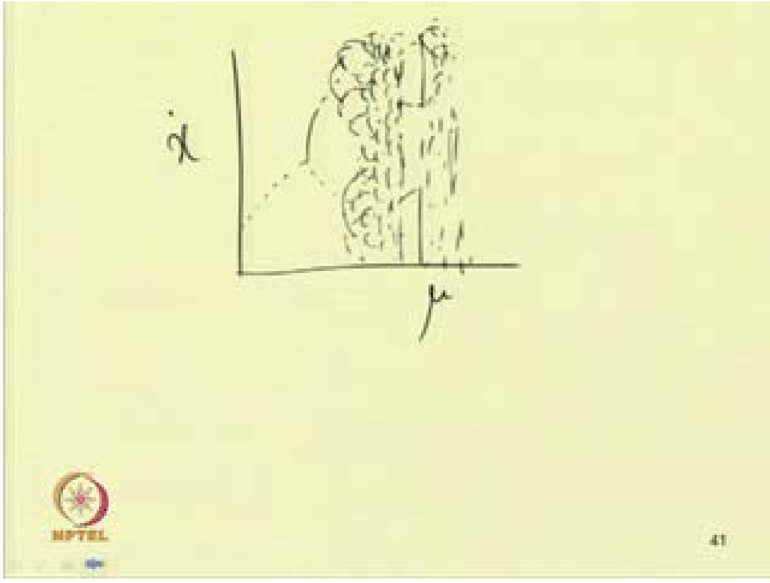

So if one plot let us take one example, that is period doubling route to chaos. So in that case if one plot the Poincare section so as initially, the response. So this is x dot versus x initially, the response is single periodic so, one can get these points. So by varying this control parameter so one can plot this curve. So this is the control parameter mu and so by varying this control parameter. So one can find the points so, then two period. Now this two period becomes four period so, one can get four period and then this four period again they will becomes eight period and one can obtain a series of points.

So here so one can see from this period doubling route to chaos so, one can have many different points in this Poincare section. So by changing this control parameter initially, one has a single period. So it is represented by a single point, then it becomes two period, then this two period becomes four period and eight period, 16, 32 and in that way so one can obtain so one can find a window. So in this window one may have this three windows or five window and after that again one can have this chaotic response.

So this way one can or by using Poincare section one can distinguish this periodic, quasi periodic and chaotic response. So while taking this Poincare section it is important to know how to sample this time response. So to sample this time response for example, in case of periodic response, one should know the minimum time period of the system so, one can find the minimum time period of the system by studying the resulting time response of the system similarly, in case of quasi periodic. So in case of torus, one will have two frequencies. So these two frequencies will be commensurate so, by taking these frequencies the minimum frequency one can sample the space and or sample the time data and then plot the Poincare section of the system.

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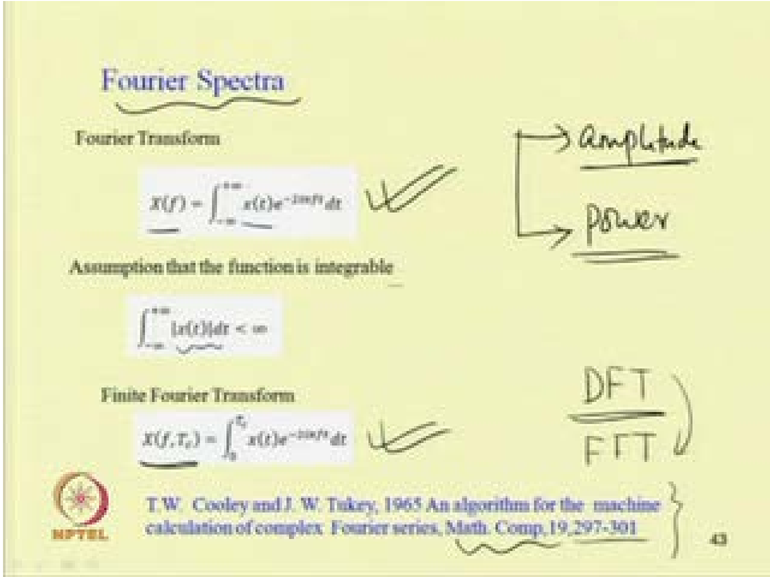

So this way one can characterize the system by using time response phase portrait and Poincare section. So let us see how what are the other way one can represent, also one can use this Fourier spectra to represent the response of the system. So in case of the Fourier spectra one can use this Fourier transform. So this equation can be used to find the transform of the time response, let x t be response of the system. So the Fourier transform can be represented by using this equation. So X f equal to minus infinity. So this is the two sided Fourier transform. So that is X f equal to minus infinity to plus infinity x t into e to the power minus 2i pi f t d t. So as the negative frequency has no meaning so, one can take this thing.

Only by taking the positive also positive side of this thing and here we are assuming that this function, that is the function should be integrable. So if it is not integrable then one cannot use this Fourier transform to find the spectra of the system. So in case of the Fourier spectra either one can use this amplitude spectra either one can use this amplitude spectra or one can use this power spectra, either amplitude spectra or power spectra to represent the motion of the system in frequency domain.

So here the as it is difficult to obtain this function to be integrable in the whole range of minus infinity to plus infinity so, one can take this finite Fourier transform in between which the system is integrable so, one can write this finite Fourier transform using this expression that is, X f t c. So let it is integrable up to time t c, then this expression can be used. So X f t c, f is the frequency, t c is the time up to which the function is integrable by using this expression integral 0 to t c x t into e to the power minus 2 i pi f t d t.

So in this way one can find this amplitude spectra of the system. So this using this one can so for discrete points one can use this DFT discrete Fourier transform or a powerful method is FFT also fast Fourier transform which is a special form of this DFT because in case of the experiment and in case of also this numerical.

Response of the system so, one can get this response at discrete times. So instead of getting the response in a continuous form we can get the response in discrete time. So we can use this DFT or FFT fast Fourier transform. So this method was proposed by Cooley and Tukey in 1965, in this paper and algorithm for the machine calculation of complex Fourier series, published in mathematical notation in volume 19, 270 page number, 297 to 301 and so this way one can obtain the powers amplitude spectra of the system similarly, one can obtain the power spectra. So to find the power spectra so which are otherwise also known as auto spectrum or Auto spectral density function or power spectral density function.

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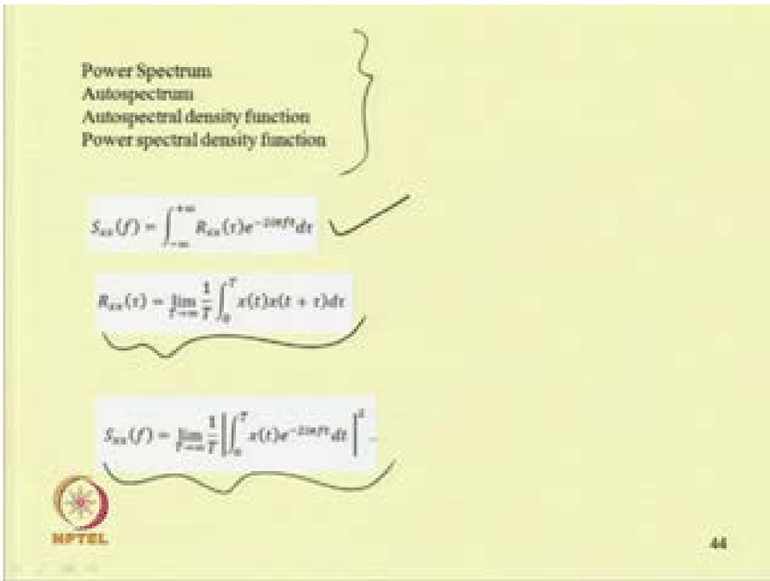

So, one can use this equation that is S x x f equal to minus infinity to plus infinity, R x x tau e to the power minus 2 i f t tau d tau, where this R x x is known as the auto correlation function, which can be given by this expression. So R x x tau equal to limit t tends to infinity 1 by t. So t is the time up to which we are finding the spectra. So limit t tends to infinity 1 by t, 0 to t x t x t plus tau d tau, where tau is the delay of the system and one can find also or one can write the power spectrum also by using this expression that is, x x f equal to limit t tends to infinity 1 by t integration 0 to t x t e to the power minus 2 i by f t d t whole square.

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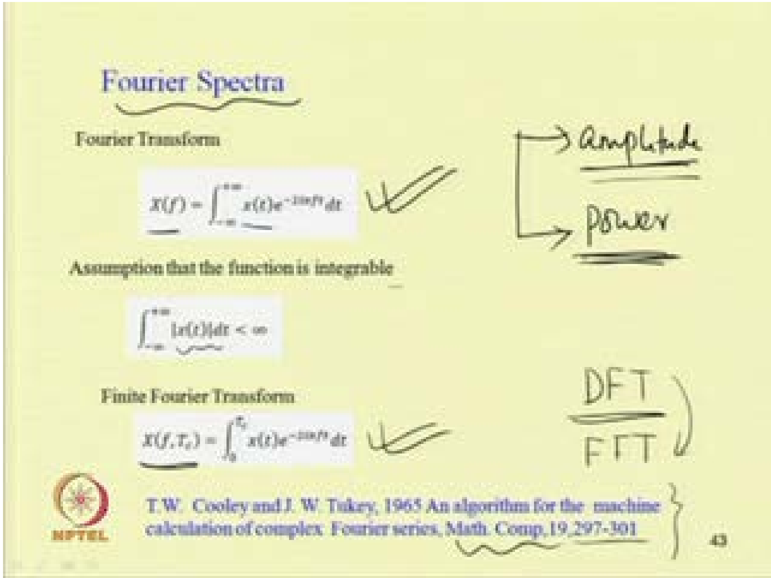

So, one can note that the power the square of the amplitude and so one can either obtain this Fourier spectra in terms of amplitude or power, we represent the motion of the system.

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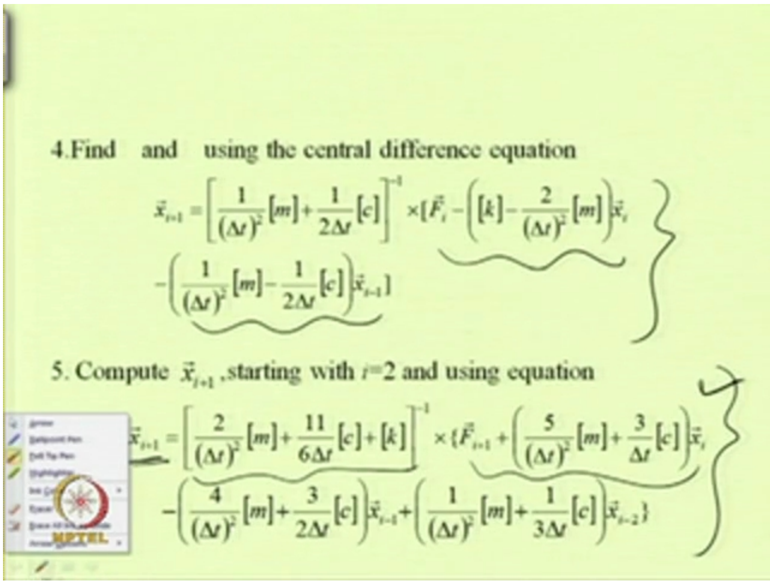

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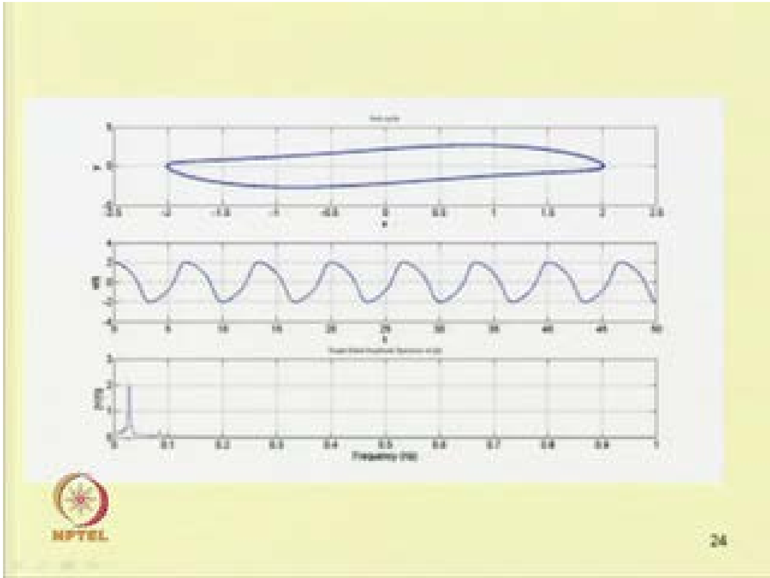

So here in case of the so in case of the.. in this case we have already shown, so this is the power spectra shown for this system. So in this case, in case of the Vander poll equation this shows the power spectra of the system, in case of the limit cycle as we have a limit cycle one can observe that we have a distinct peak here. So this peak represent the frequency of this limit cycle similarly, in case of the quasi periodic response one can plot this spectra.

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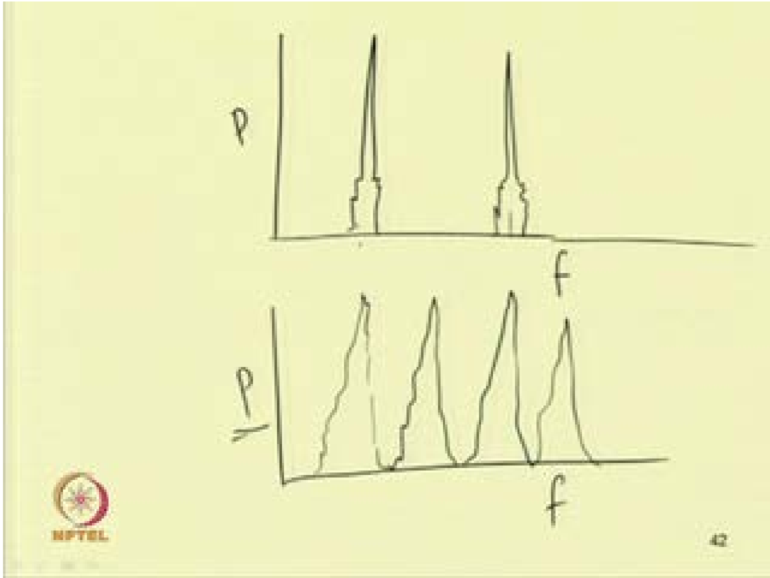

So in case of the quasi periodic response, the spectra will look like this. So we will have a close, so we can have two frequencies but, these frequencies one can find these frequencies and one can find from this frequency that they are not commensurable, one can find different frequencies this is your p versus f, this is the frequency in case of chaotic response. So this is for the quasi periodic response. In quasi period response one can find the frequency and find whether they are commensurable or not and but, in case of this chaotic response.

So one can observe a continuous band in power spectra, one can observe a continuous band. So a continuous band represent a chaotic response also one can find similar type of response in case of case of noisy response. So to distinguish whether the response to be chaotic or noisy one can use different other tools.

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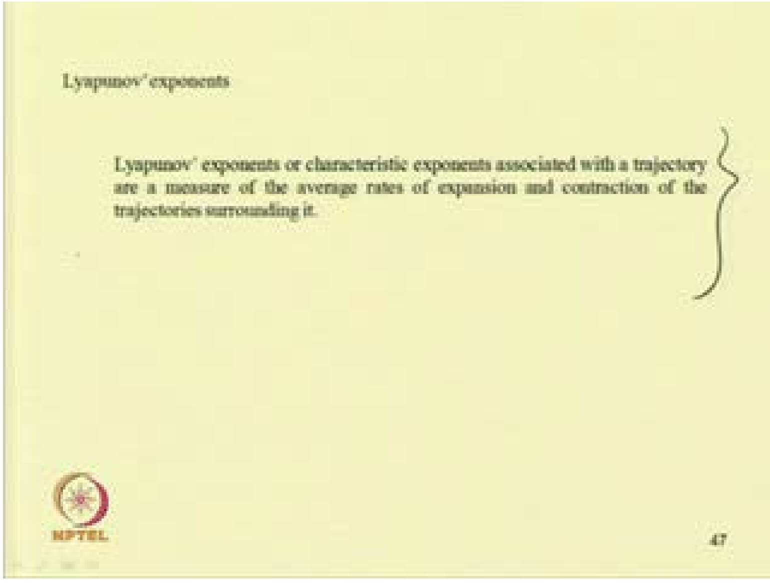

For example, one can go for this Lyapunov exponent method. So by using Lyapunov exponent method one can find whether the response is chaotic, periodic or fixed point. So the Lyapunov exponent or characteristic exponent associated with a trajectory or a measure of average rate of expansion and contraction of the trajectories surrounding this.

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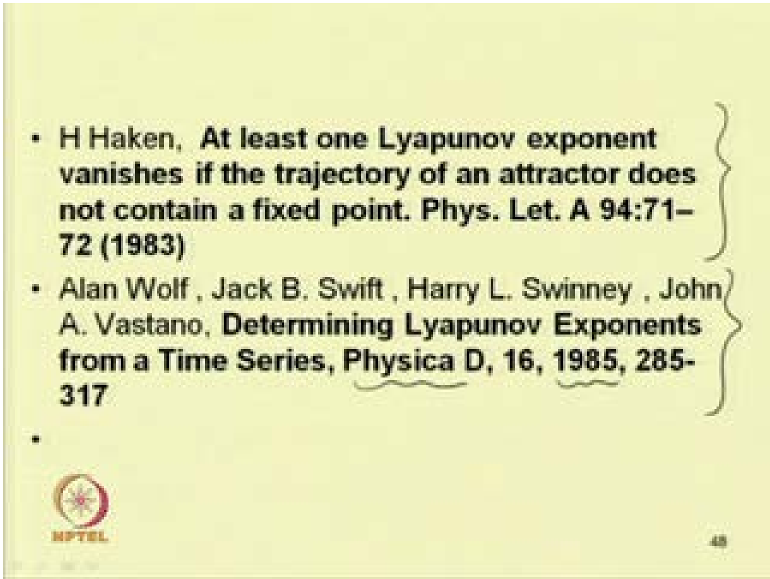

So one can use this method by Wolf, Swift and Swinney and Vastano determinate determining Lyapunov exponent from a time series. So this is published in Physica D in 1985 to obtain the Lyapunov exponent for the systems with experimentally obtained response also one can study this paper. So here it is observed by Haken that at least one Lyapunov exponent vanishes if the trajectory of an attractor does not contain a fixed point. So if the attractor does not contain a fixed point so, at least one of the Lyapunov exponent vanishes.

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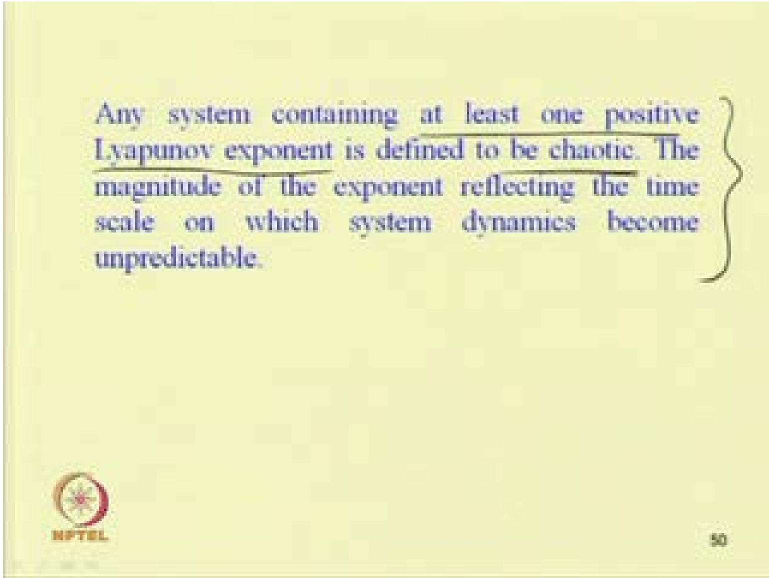

So one can observe also that any system containing at least one positively Lyapunov exponent is defined to be chaotic, the magnitude of the exponent reflecting the time scale on which the system dynamics becomes unpredictable.

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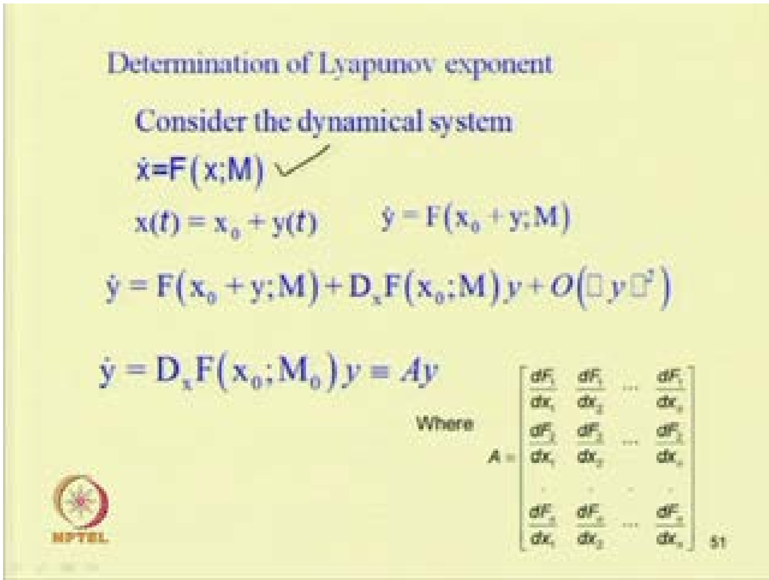

So one may note that so, any systems containing at least one positive Lyapunov exponent is defined to be chaotic. So in case of chaotic response at least one of the Lyapunov exponent will be positive. So let us now know how to determine this Lyapunov exponent. So the Lyapunov exponent let us have a dynamical system which is defined by this equation x dot equal to F x M. So for an autonomous systems it can be defined in this way for an on non autonomous system.

So one can write the equation also in by using this autonomous by adding one more term, one more variable or one more equation. So a nth order non-autonomous systems can be written by M plus 1 autonomous systems that thing we know before. So let us have this equation x dot equal to F x M. So in this case now the flow so, let the flow x t, so let we perturb this flow by using this y t.

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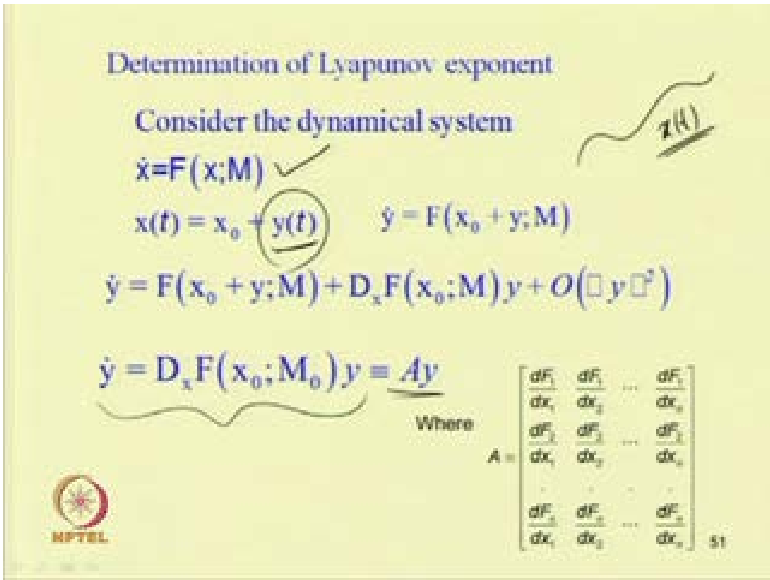

So we are perturbing this. So we should know how this perturbation will expand or contract and depending on that we can find the Lyapunov exponent. So we can write this by perturbing this equation. So we can write this y dot equal to Ay. So this is similar to the stability analysis what we did in case of the fixed point response and in case of the periodic response.

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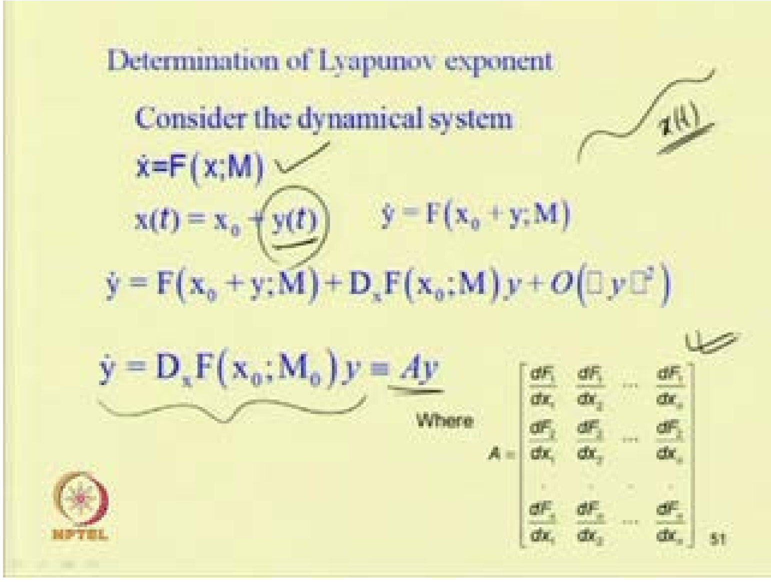

So after finding this equation y dot equal to Ay, where a is the Jacobean matrix.

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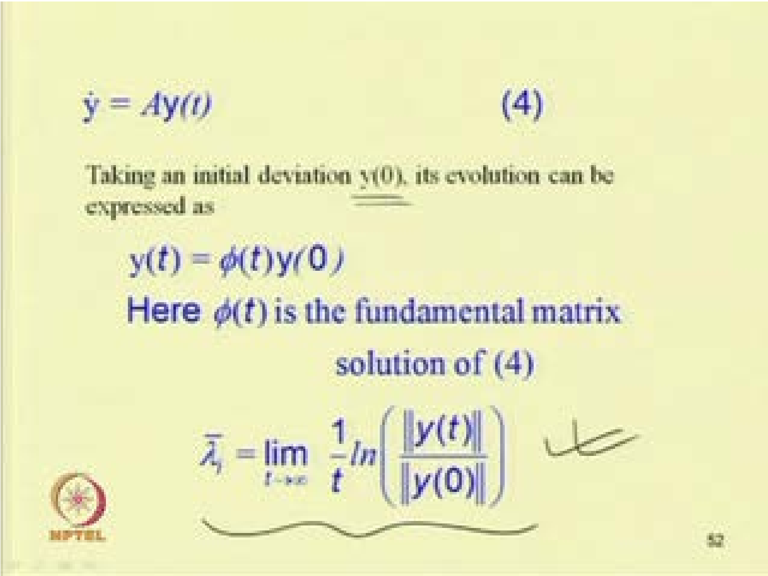

So we can write this y dot equal to Ay t and so taking some initial condition y 0 its evolution can be expressed in this form y t equal to phi t into y 0, here phi t is known as the fundamental matrix of solution and in case of periodic system we know this phi t can be or we can find this monodromy matrix using this phi t and here the Lyapunov exponent is defined after getting this y, we can define this Lyapunov exponent by using this expression that is, lambda i equal to limit t tends to infinity 1 by t ln y t by y 0. So one can define this Lyapunov exponent by using this y t and y 0.

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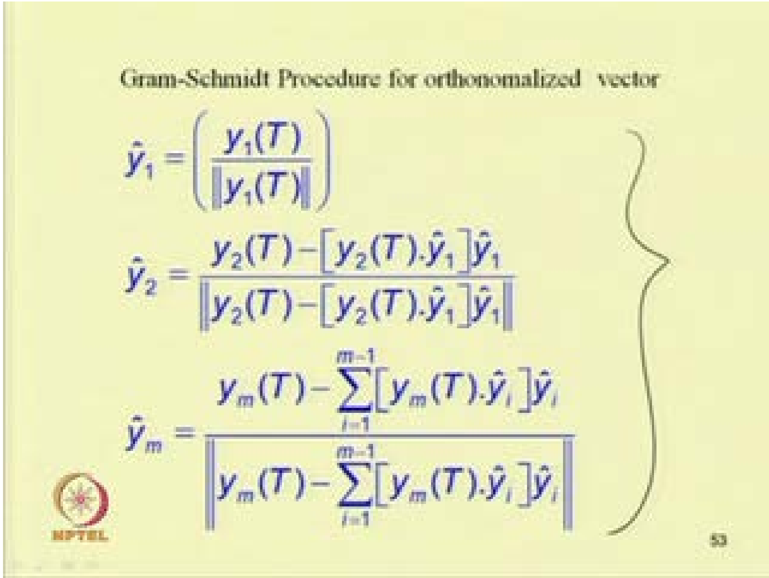

But, finding this y t by integrating this is sometimes difficult so, for that purpose one can use this Gram-Schmidt procedure for orthogonalization vector. So one can use this orthonomalization vector that is, y 1, y 1 is normalized to be 1, then y 2 can be written by using this expression. So where this dot is the dot product of so by using this expression.

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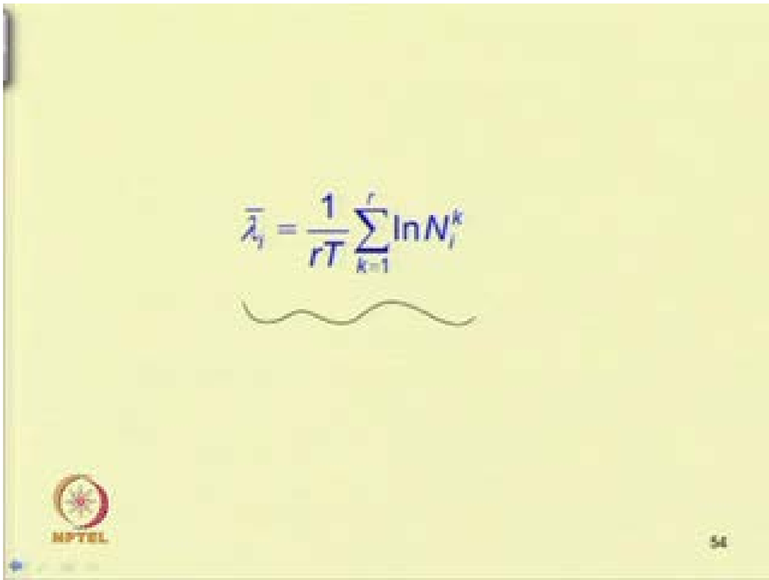

So one can find this for the rth iteration so, one can find this lambda i equal to 1 by r T k equal to 1 to r ln N i k. So where the expression for N i k one can find from this.

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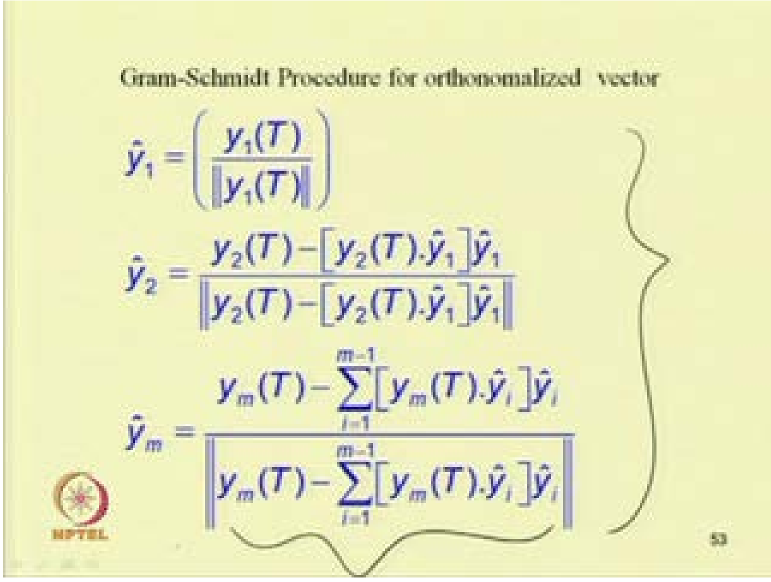

So this is Ni k, this denominator part represent this Ni k. So for the kth iteration this is for the mth iteration it is written. So for kth iteration one can find this part and one can obtain this Lyapunov exponent by using this formula.

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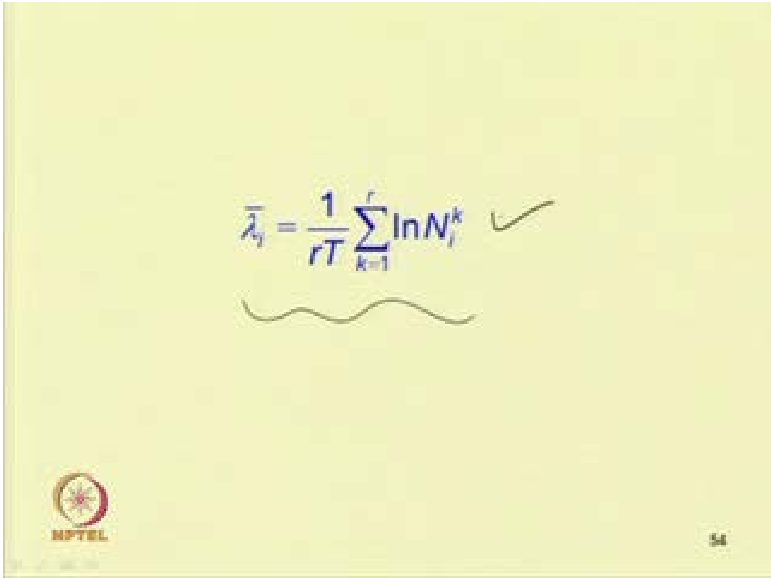

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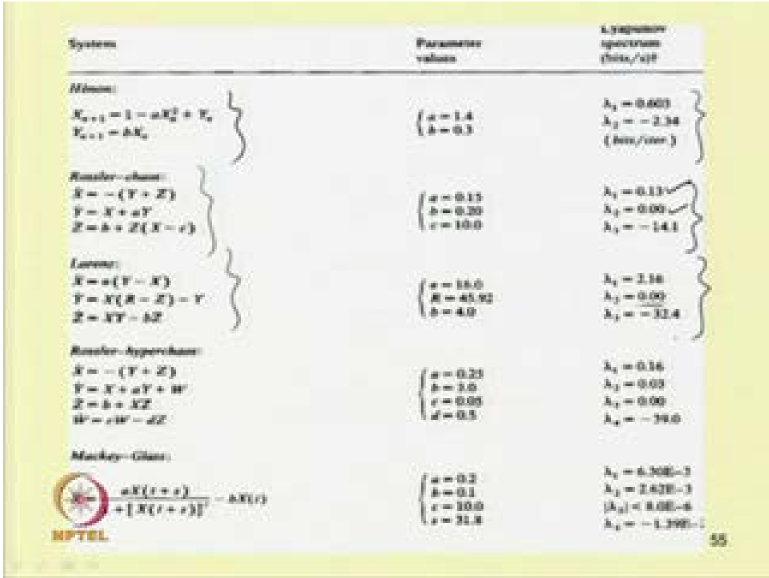

So this following this wolf paper one can find for the for the Henan equation. So one can find this for this parameter value a b d’s one can find this is two equations. So one can get two Lyapunov exponent. So one can see that one of the Lyapunov exponent is positive that is 0.6 that is why the system is chaotic similarly, for the Roseler chaos. So this is the equation one can find this 3 Lyapunov exponent and here one can see that one of the Lyapunov exponent is 0 and one is positive. So as one positive Lyapunov exponent is present. So the system is chaotic similarly, for Lorentz attractor one can see one of the one of the Lyapunov exponent is 0 other is positive and third one is negative due to the presence of this positive Lyapunov exponent. So the system is chaotic.

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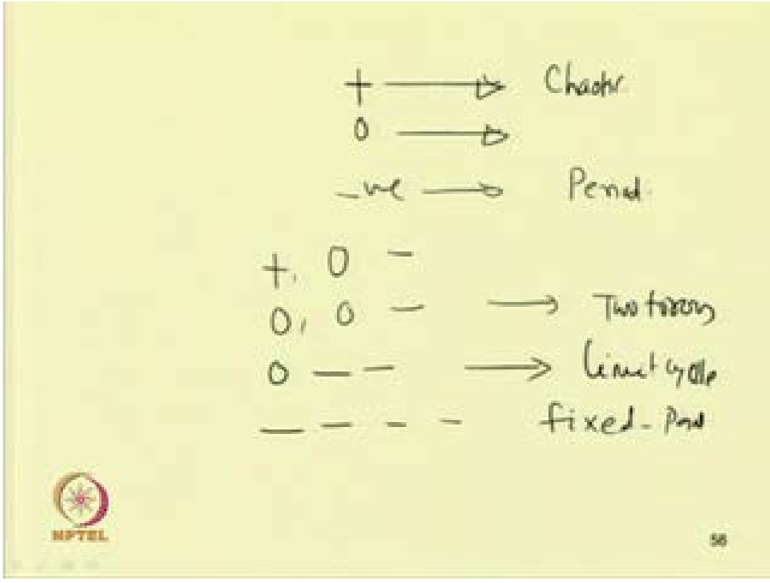

So if one has one dimensional map so, one can have one positive Lyapunov exponent and one 0 Lyapunov exponent. So in case of positive Lyapunov exponent so one can tell the response to be chaotic and if the Lyapunov exponent is 0 so one can tell that it is marginally stable and if it is negative, if the Lyapunov exponent is negative then one can get a periodic response. So and in case of three dimensional so, in case of three dimensional continuous systems so, one can have three Lyapunov exponent.

So either one will be positive so, these are the options positive, zero and negative. So in this case one have a strange attractor. So if it is zero, zero and negative. So one can have two torus and if it is zero, negative, negative so, one has a limit cycle and if it is negative all negative then one can have fixed point similarly, in case of four dimensional so one can tell the system to be chaotic.

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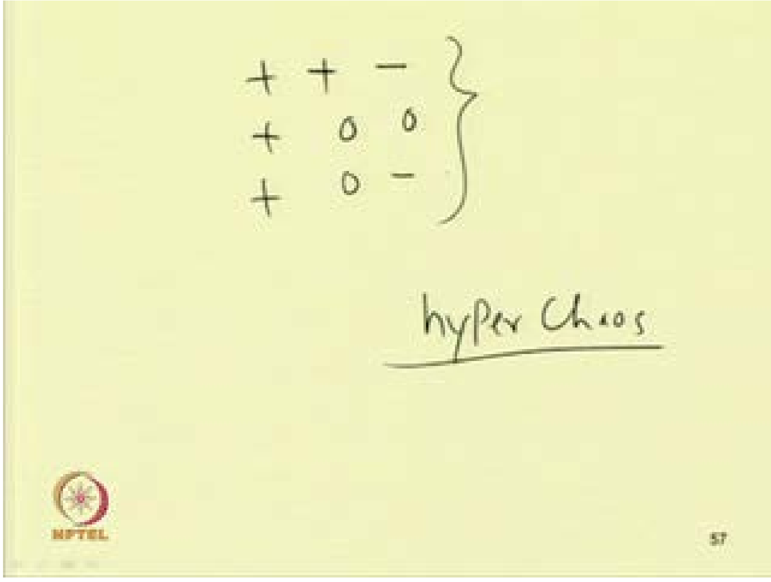

If so this is plus plus Lyapunov exponents are positive, positive, negative or positive, zero, zero or it may be zero, positive, zero, negative. So these are the three options for getting chaotic response. So in this so, if more than one Lyapunov exponents are positive, then it is known as hyper chaos. So, one can have this hyper chaos in case of more than one Lyapunov exponents are positive.

So today class we have studied different methods for finding the solution of ordinary differential equations also we know how to characterize these different these responses by using Poincare section, power spectra and also by using Lyapunov exponent. So using these methods one can characterize the system response. Next class we will study about the frequency response of the system and the basin of attraction of the system.

Thank you.